High CEO Compensation: Incentives for CEO or Managers?*

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Abstract

To analyze how two incentive schemes, promotion and pay for performance, interact to determine CEO compensation and within-firm wage inequality, I embed a pay for performance framework into a tournament structure. The model shows that when CEO and managers contribute to a firm's output independently, it is optimal for the firm to provide the CEO a compensation far beyond her reservation value if there are many managers who compete for the CEO position. In particular, promotion incentives for managers generate the high CEO pay and the wage gap between CEO and managers. However, I find that the promotion incentive motive can disappear if there is interdependency between the CEO's and managers' outputs. In this case, the main purpose of a high CEO compensation is to induce the CEO to exert effort, not to provide promotion incentives for managers. The tension between incentives for CEO and managers makes it difficult to interpret the meaning of within-firm wage gap. As a possible solution, this paper suggests the use of CEO's base salary to identify which incentive factor is driving the pay gap.

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1 Introduction

The provision of incentives has been considered as an important factor that contributed the rise in executive compensation since 1970s. In particular, researchers have focused on two incentive schemes: promotion and pay for performance.[3] However, each incentive scheme has a drawback in explaining the trend in executive compensation. First, pay for performance schemes have been criticized because theoretical analysis predicts a reduction in base salary with the rise in incentive pay, which contradicts the empirical evidence. On the other hand, promotion incentives do not capture the weak correlation between executive compensation and firm size. This paper studies whether a hybrid incentive scheme that includes both pay for performance and promotion can account for these empirical findings.

This paper contributes to the theoretical literature on executive compensation based on a principal-agent framework. The moral hazard literature attributes high CEO compensation to the need to provide incentives. There are two incentive schemes relevant to this problem. The first incentive scheme is pay for performance as studied by Grossman and Hart (1983) and Mirrlees (1999). These works suggest that firms need to provide high compensation to a CEO in order to make him/her exert more effort. The second incentive scheme is internal promotion associated with a rank-order tournament model proposed by Lazear and Rosen (1981). They argue that firms set up internal labor markets and promote workers who do well. In this setting, high wages at the top of the firms can make lower level employees work harder to increase their promotion probability. Following the analysis of Lazear and Rosen (1981), the CEO may well have a wage that is much higher than his or her contribution to the firm because it acts as the prize for lower level managers. The main difference between these two incentive schemes is that the former is based on absolute performance while the latter depends on relative performance. In this paper, I analyze how the two incentive schemes interact within a firm to understand when a firm might want to combine the two in a hybrid incentive scheme: a contract based on both absolute and relative performance.

[3]For example, Bognanno (2001) shows that CEO compensation increases with the number of competitors for the position. Also, Frydman and Jenter (2010) illustrate that the increase in incentive pay such as option compensation significantly contribute to the rise in CEO compensation.

To study how the hybrid incentive scheme works, it is important to understand which incentive scheme is relevant to each agent in a firm. In terms of incentives, the pay for performance scheme works for both the CEO and other top executives (from now on, I call these executives "managers") in the same way. However, the promotion based incentive scheme only matters for managers who have a possibility of being promoted. On the other hand, with respect to compensation level, promotion incentives affect two types of agents, CEO and managers, in opposite ways. When other conditions are fixed, raising promotion incentives increases CEO compensation, but decreases managers' compensation. The pay for performance scheme yields a higher compensation to both agents if a firm wants them to exert a higher effort. Given that raising promotion incentives impacts managers and CEOs in different directions, it is natural to ask how the two types of incentive payments constitute the executives' total compensation.

In order to answer this question, I consider a model with a hierarchical structure in which an infinitely lived risk-neutral firm hires N + 1 agents (one CEO and N managers) in each period. For tractability, I assume that the firm offers a contract based on agents' positions. That is, there are only two types of contracts which do not depend on time: one for CEO and one for managers.[4] Each agent conducts her/his own tasks and produces an outcome that depends on her/his effort level.[5] In the benchmark model, I assume that each task affects the firm's profit independently. That is, there is no complementarity between agents. All outcomes are observable and contractible. This structure closely follows Grossman and Hart (1983). In addition, I introduce a tournament structure to add promotion based incentives. If there are qualified managers for the CEO position according to a promotion rule, the firm fills the position by internal promotion when the previous CEO leaves.

The most important contribution of this paper is that I analyze how the two incentive schemes interact in a dynamic environment similar to Rogerson (1985*a*). In particular, the reward for promotion is not an immediate monetary remuneration, but a position that can provide high compensation in the future. Under this distinctive structure, the analysis of the model shows

[4]This is a stronger assumption than the commitment assumption in Lazear and Rosen (1981) However, this makes the firm's problem simple when I consider a complex situation in Section 7

[5]Throughout the rest of this paper, I use she as a personal pronoun for a CEO and he for a manager.

that firms can rationally employ the two incentive schemes simultaneously. Therefore, it is important to understand the interaction between the two incentive tools within a firm as well as the effect of each scheme.

An extensive literature investigates the two incentive schemes separately.[6] Since the seminal work of Lazear and Rosen (1981), promotion incentives based on the tournament theory have been extensively studied in various settings.[7] Although the tournament theory has broad applications[8], unlike athletic tournaments, it requires caution to apply the theory to a firm's internal organization problem. The distinction of the firm's internal labor market is that the winner of the tournament stays in the firm as a worker to whom the firm needs to provide incentives. Ke, Li and Powell (2014) and Goel and Thakor (2008) reflect this idea in their papers. However, Ke, Li and Powell (2014) only consider a risk-neutral agent and one of two incentive schemes, while Goel and Thakor (2008) do not explicitly analyze managers' compensation. Different from these works, I investigate the role of agents' risk aversion and the relationship between managers' and CEO's compensations in a hierarchical structure.

On the other hand, the pay for performance literature investigates how the optimal level of incentives changes according to numerous factors affecting the contract.[9] For example, Baker and Hall (2004) examine how the measure of CEO incentives could change according to the effect of a CEO on the firm's value, and Prendergast (2002) studies the effect of uncertainty on incentives and shows that a positive relationship between uncertainty and incentives can arise. In this paper, I examine the relationship between CEO incentives and the internal labor market.

A number of papers compare the two incentive schemes from a theoretical perspective. However, most papers do not consider a firm that uses both schemes together though most of firms implement an incentive system based on both schemes in practice. For example, Nalebuff and Stiglitz (1983) and Mookherjee (1984) investigate conditions under which one of the incentive schemes is optimal. That is, these papers consider the two incentive schemes as substitutes.

[6]See Prendergast (1999).

[7]See Lazear and Oyer (2012).

[8]Konrad (2009) and Dechenaux, Kovenock and Sheremeta (2015) review a broad literature on contests.

[9]For a comprehensive review, see Bolton and Dewatripont (2005).

A notable exception is Ekinci and Waldman (2015), where they combine pay for performance and promotion incentives. Different from my paper, Ekinci and Waldman (2015) use a market-based tournament theory, where the reward of promotion is determined by expected wage offers of competing firms.[10] Therefore, their paper focuses on how the market wage is determined, while I illustrate that a CEO can earn more than her market wage because of the internal labor market.

The formal analysis yields the following results. First, the firm provides compensation beyond the CEO's reservation value when there are enough candidates for the promotion and if agents are risk-averse. That is, the CEO's participation constraint can optimally be slack to maximize the firm's profit. I interpret the gap between the CEO's expected utility under the optimal contract and the CEO's reservation value as the promotion incentive because this utility gap, rather than the monetary gap, is the reward from the agent's point of view.[11] The non-binding participation constraint implies that a large wage gap between a CEO and managers could be optimal for the firm in order to incentivize managers below the CEO even when the firm employs an absolute performance based incentive scheme. Intuitively, when there is a small number of managers, the profit generated by the CEO accounts for a substantial part of the firm's profit. Thus, the firm does not want to raise promotion incentives beyond the CEO's reservation value since this makes it more costly for the firm to provide incentives to her (particularly when CEO is risk-averse). However, when the firm is large, the CEO's contribution is marginal compared to the profit created by managers. Thus, the firm uses its CEO position as the prize to the winner of internal competition rather than an output producer, which leads to the rise in CEO compensation and larger wage gap.

Second, for a fixed number of managers, analyzing the comparative statics gives interesting results for a CEO's utility and managers' wages. I show that there is a negative relationship between the promotion incentive and the CEO's effort level. In other words, the optimal contract provides a higher expected utility to the CEO when the firm requires less effort from her.

[10]See Waldman (2013) for a survey of the literature on the two tournament theories.

[11]Note that the monetary gap can widen if the firm requires a higher effort from the CEO since agents are risk-averse.

The contract could even offer her a higher expected compensation. This result captures how promotion incentives interact with CEO incentives. I will discuss this issue in more detail later. The other important comparative static is the relationship between promotion incentives and managers' reservation value. The positive correlation between them illustrates that the two extreme allocations between the two incentive schemes for managers can happen. That is, the firm can only adopt the promotion incentive without pay for performance scheme if managers' reservation value is high enough. The opposite situation (no promotion incentives) can occur when the reservation value is very low. In particular, I expect that the former case can provide an explanation of why performance-tied compensation is not prevalent in the workplace as illustrated in Lemieux, MacLeod and Parent (2009).[12] Firms may optimally mute the absolute performance channel even though they consider this incentive scheme. Namely, introducing relative performance-based incentives can make managers' compensation less dependent (even independent) on their performance.

As an extension of the benchmark model, I analyze a situation where there is complementarity between the CEO's task and managers' outputs. Specifically, I consider the CEO's task with a multiplication effect on the sum of other managers' outputs. This specification is closely related to Baker and Hall (2004), where the marginal product of the CEO's marginal effort is increasing in the size of the firm.[13] This extension indicates that the level of promotion incentive depends on the role of a CEO. Under the multiplication specification, the promotion incentive can disappear. That is, if the firm hires enough managers, the slackness of the CEO's individual rationality constraint can alter to the binding constraint. Because the CEO's marginal effort is much more valuable for the firm as the number of managers increases, the firm prefers to focus on the CEO's absolute performance based incentive rather than the managers' relative performance based incentive.

The extension provides a unique implication regarding CEO compensation. Although the benchmark model and the extension can produce a positive link between compensation and firm size, they make a different prediction about the optimal compensation structure. That

[12] Macera (2016) provides a behavioral explanation based on loss aversion on this problem.

[13] They use the market value of the company or firms' sales as the measure of the size of the firm.

is, the two specifications link the rise in CEO compensation to different channels. When the CEO's marginal productivity is independent of firm size, compensation grows with the size of the firm because of the raised promotion incentive. Under the alternative specification, however, the higher compensation stems from enhanced pay for performance incentives for the CEO because her demanded effort grows with the size. I connect this implication with a measure of promotion incentives in section 10.2.

In addition, this paper contributes to the literature on the trend in executive compensation. The hybrid incentive scheme model predicts that CEO compensation remains stable and the wage gap between lower-level executives and CEO does not expand until the size of the firm reaches the point that it begins to raise its CEO compensation to induce managers' effort. The main conditions for the result are executives' risk aversion and the independence of agents' outputs. Therefore, if these conditions are satisfied, CEO compensation has a non-monotonic relationship with firm size measured by the number of candidates for promotion. [14] In other words, the compensation remains at the same level although the number of competitors increases since CEO's participation constraint binds. However, beyond a certain point, the firm wants to raise its CEO compensation beyond her reservation value in order to provide incentives to its managers. Also, the wage gap widens when the firm starts providing compensation to the CEO beyond her reservation value but not before that point. These results suggest a possible explanation for two empirical facts shown in Frydman and Jenter (2010). First, they find a non-monotonic increase of executive pay: the rapid growth in executive pay only started in the mid-1970s. Also, their results show that the compensation gap between CEO and other top executives rapidly grown during the past 30 years but not before 1980. I discuss this issue in more detail in section 10.1.

The rest of the paper is organized as follows. Section 2 presents the basic model and the promotion rule. Before I introduce the formal model, I present a preview of the tension between promotion incentives and pay for performance in Section 3. In Section 4, I simplify the firm's problem and provide the primitive trade-off of the problem. In Section 5, I consider the problem without any agency problem to illustrate the effect of information asymmetry. In Section 6, I

[14] For instance, Acs and Audretsch (1987) use the number of employees as a measure of firm size.

discuss the main properties of the firm's problem demonstrating the effect of promotion on CEO's compensation. In Section 7, I introduce several dynamics into the basic model and analyze the effect of them on CEO compensation. Section 8 examines how the role of a CEO impact on promotion incentives. I compare two promotion rules in Section 9 to demonstrate the benefit of external CEO recruitment. I discuss the important implications of this paper in Section 10. Section 11 contains concluding remarks.

2 The Benchmark Model

2.1 Firm structure and executives

I start with a simple discrete time model, where an infinitely lived risk-neutral firm with a discount factor $\delta \in (0,1)$ maintains its employment structure. In particular, the profit maximizing firm employs one CEO and N managers over time. The firm hires the managers from a labor market every period, but it can promote one of the previous managers to be the next CEO according to a promotion rule. If there is no manager satisfying the promotion criteria, the firm hires a CEO from an external labor market. Throughout most of the paper, I consider the case where the CEO leaves the firm or retires after one period, and a manager remains in the firm if he is promoted to CEO. Otherwise, managers leave the firm after one period. I consider a moral hazard situation where agents' efforts (e) are not observable to the firm and incur a cost to agents, $g(\cdot)$. Agents are risk-neutral or risk-averse with an additively separable (von Neumann-Morgenstern) utility function $U(C, e)$, where C is consumption[15], in each period satisfying the following assumption:

Assumption 1. *Agents' utility function is of the form*

$$U(C, e) = u(C) - g(e),$$

where $u'(\cdot) > 0$, $u''(\cdot) \leq 0$, $u(\cdot)$ *is defined over the real interval* $(\underline{C}, \infty)$, *and* $g(0) = 0$, $g'(\cdot) \geq 0$, $g''(\cdot) > 0$ *over* $e \in [0,1]$. *Also, there exists* $\widehat{C} \in (\underline{C}, \infty)$ *such that* $u(\widehat{C}) > 0$ *and* $\lim_{C \downarrow \underline{C}} u(C) = -\infty$.

[15] I do not consider the possibility of private saving in this paper.

This assumption comes from Grossman and Hart (1983) and Rogerson (1985*b*), which guarantees the existence of solution and validates the first-order approach. Additionally, I assume that agents do not discount the future, and a CEO and managers have an outside option $\underline{U}_C \geq 0$, and $\underline{U}_M$, respectively, if they do not accept the offer from the firm. Also, in their second period, managers are assumed to obtain a reservation utility, normalized to zero, if they leave the firm. Hence, the positivity assumption on $\underline{U}_C$ assures that promotion is beneficial for managers. Agent i does an independent task X_i, $i = 0, \ldots, N$, which can end in a good or bad outcome.[16] The probability of good outcome depends on the agents' choice of effort level e_i by the function $s(e_i)$ in the following way:

$$X_i(e) = \begin{cases} \mathcal{G}_i & \text{with probability } s(e) \\ \mathcal{B}_i & \text{with probability } 1 - s(e) \end{cases}.$$

I assume that $s(e)$ is a linear function of the effort level e.[17] That is, $s(e)$ satisfies

$$s(e) = \alpha + \beta e,$$

where $e \in [0, 1]$ is the agent's effort level. For two parameters α and β, I assume that $\alpha \geq 0,\ \beta > 0$, and $\alpha + \beta \leq 1$, which guarantees that $s(e) \in [0, 1]$.

I assume that each manager's task has an identical effect on the firm's output. That is, $\mathcal{G}_i = \mathcal{G}_j$ and $\mathcal{B}_i = \mathcal{B}_j$ for $i, j \in \{1, 2, \ldots, N\}$. After observing managers' outputs, the firm makes the promotion decision according to the following rule:

(**Promotion Rule 1**) *Among managers whose outcome is good ($\mathcal{G}$), the firm chooses one manager randomly for promotion. If there are no such managers, the firm hires a CEO from an external labor market.*

This promotion rule can be understood as a situation where a firm sets up a certain requirement such that it considers managers who satisfy the requirement as candidates for internal

[16] Agent 0 represents CEO.

[17] This is not so much restrictive. With **Assumption 1**, this condition embraces any concave function $s(e)$ with $s'(e) > 0$.

promotion. Note that this promotion rule is different from the rule considered in Lazear and Rosen (1981), where they use the following one:[18]

(**Promotion Rule 2**) *The firm promotes the best manager, whose outcome could be good ($\mathcal{G}$) or bad ($\mathcal{B}$). If more than one agent makes the best outcome, the firm chooses one of them randomly.*

The difference between two promotion rules is the possibility of external hiring. When all managers' outcomes are bad, the firm appoints its CEO from an external labor market under promotion rule 1 while it still uses the internal labor market under promotion rule 2.[19] I show that promotion rule 1 could be preferred by firms to promotion rule 2 if they do not know a managers' ability and want to promote a more talented candidate in section 9.

While the CEO's job can also have a good or bad outcome, the effect on the output can be different from that of managers. For brevity, I denote CEO's good and bad outcomes by $\mathcal{G}_C$ and $\mathcal{B}_C$, respectively, with manager's outcomes denoted by $\mathcal{G}_M$ and $\mathcal{B}_M$. Also, the marginal productivity of the CEO's effort depends on the firm's operational structure. As the benchmark model, I study a firm where the CEO's task and managers' jobs are independent of each other. That is, CEO and managers are substitutable in terms of the firm's profit. I relax this assumption later on.

2.2 Contracts

In order to make the problem tractable, I assume that contracts only depend on agent's positions regardless of time as well as internal and external hiring.

Assumption 2. *The firm offers contracts based on agents' positions.*

This assumption implies that a firm offers the same contract to a future CEO as the current one. That is, in every period, the firm offers a contract (e_C, W_C^G, W_C^B) to the CEO choosing

[18] Strictly speaking, there is a difference. Lazear and Rosen (1981) consider a continuous outcome space. This yields no ties with probability 1.

[19] As Kale, Reis and Venkateswaran (2009) show, the firm recruits its CEO from outside as well although internal promotion is more common.

each component in order to maximize its profit. In the contract, e_C represents the firm's recommended effort level while W_C^O is the wage if the CEO's output turns out to be $O \in \{G,\ B\}$. On the other hand, the firm provides a contract $(e_{Mi}, W_{Mi}^G, W_{Mi}^B)$ to manager i, where the role of each component is the same as that of the CEO contract. Note that the firm does not need to specify the prize for the winner of promotion since **Assumption 2** allows current managers to know what they will get if promoted to CEO in the next period. This is in line with the tournament literature in the sense that firms can commit to the prize for the winner. For simplicity, I focus on a symmetric equilibrium in this paper. That is, the firm requires the same effort e_M from every manager. Therefore, the firm offers the same contract (e_M, W_M^G, W_M^B) to all managers. After I solve this simple benchmark model, I extend the model by including job security issues, complex operational structure, and heterogeneous managers.

3 The Effect of Promotion Incentive on Agents' Wages

In this section, before I move to a general problem, I study how promotion incentive affects agents' compensation. Since each output is binary, the two types of agents' ex-ante utility can be described by the following two terms for each agent:

$$\text{Manager} \begin{cases} u(W_M^G) + P(\mathbf{e}_{-M})\mathcal{V} & \text{when } X = \mathcal{G}_M \\ u(W_M^B) & \text{when } X = \mathcal{B}_M \end{cases} \quad \text{and CEO} \begin{cases} u(W_C^G) & \text{when } X = \mathcal{G}_C \\ u(W_C^B) & \text{when } X = \mathcal{B}_C \end{cases},$$

where $P(\mathbf{e}_{-M})$ represents the probability that a manager is promoted to CEO when he gets a good outcome and other managers' effort levels are $\mathbf{e}_{-M}$. $\mathcal{V}$ is the CEO's expected utility, which the manager will get if promoted in the next period. Note that the value of $\mathcal{V}$ determines the power of promotion incentive since this is the benefit the winner of the tournament enjoys.

How does this promotion incentive affect the incentive based on absolute performance? Note that for agents' incentives, the difference between two utility levels is all that matters. Hence, when the firm requires a certain effort from its executives, the firm chooses wages fixing the value of these two gaps at some positive values: 1) $\left[u(W_M^G) + P(\mathbf{e}_{-M})\mathcal{V}\right] - u(W_M^B)$ and 2) $u(W_C^G) - u(W_C^B)$.

From the manager's perspective, raising the promotion incentives makes the firm reduce incentives based on pay for performance, that is, the gap between W_M^G and W_M^B. This leads to a decrease in managers' compensation. However, for the CEO incentives, higher promotion incentives yield a bigger gap between W_C^G and W_C^B if the CEO is risk-averse. This makes the compensation for the CEO rise.

Therefore, there is a tension between incentives for managers and CEO as well as a trade-off between the two types of agents' wages: higher promotion incentives make it more difficult for the firm to incentivize its CEO but easier for managers. This feature is captured by the movement of the gap between wages associated with a good and bad outcome. It is worth mentioning that this incentive trade-off rises because the CEO is risk-averse. If the CEO is risk-neutral, the wage gap between W_C^G and W_C^B is always a constant unless the firm requires a different effort level.

Hence, adjusting promotion incentives affects two types of agents' absolute performance based incentive schemes in different ways. In the following sections, I analyze how the firm optimally sets up the level of promotion incentives and how this decision changes when the firm's contracting environment alters.

4 Formulation of the Firm's Problem

In this section, I explicitly state the firm's problem and simplify it. First, I consider the agents' problem. The CEO's utility maximization problem is straightforward. For a given compensation scheme (W_C^G, W_C^B), the CEO chooses an effort level e_C maximizing her expected utility

$$s(e_C)u(W_C^G) + (1 - s(e_C))u(W_C^B) - g(e_C).$$

The strict convexity of $g(\cdot)$ guarantees a unique solution to the CEO's problem. Notice that this problem does not depend on other agents' effort choices.

On the other hand, the managers' problem depends on other managers' effort choices $\mathbf{e}_{-M}$ in the same cohort. Since I consider a symmetric equilibrium, it is enough to focus on $\mathbf{e}_{-M}$

such that $\mathbf{e}_{-M} = (e_{-M}, e_{-M}, \ldots, e_{-M}) \in [0,1]^{N-1}$. Therefore, for a compensation scheme $(W_C^G, W_C^B, W_M^G, W_M^B)$, and an effort level of $\mathbf{e}_{-M}$, the managers' problem can be rewritten as

$$\max_{e_M} s(e_M)u(W_M^G) + (1 - s(e_M))u(W_M^B) - g(e_M) + s(e_M)P(\mathbf{e}_{-M})V_C,$$

where

$$P(\mathbf{e}_{-M}) = \frac{1 - (1 - s(e_{-M}))^N}{Ns(e_{-M})}$$

represents the conditional probability that a manager will be promoted to the next period's CEO when he achieves a good outcome and other managers' effort level $\mathbf{e}_{-M}$ is given.[20] Also,

$$V_C = s(e_C)u(W_C^G) + (1 - s(e_C))u(W_C^B) - g(e_C)$$

is the expected utility a manager will get if he is promoted to CEO. Recall that managers know the contract they will get if promoted under **Assumption 2**.

Under the fixed employment structure, **Assumption 2**, and the symmetric equilibrium condition, the firm's objective is to offer contracts (e_C, W_C^G, W_C^B) and (e_M, W_M^G, W_M^B) to CEO and managers that maximize its profit under the incentive compatibility and individual rationality constraints. Mathematically, the problem is:

$$\max_{\{(e_C, W_C^G, W_C^B), (e_M, W_M^G, W_M^B)\}} \sum_{t=1}^{\infty} \delta^{t-1} \Big\{ s(e_C)(\mathcal{G}_C - W_C^G) + (1 - s(e_C))(\mathcal{B}_C - W_C^B) + N\left[s(e_M)(\mathcal{G}_M - W_M^G) + (1 - s(e_M))(\mathcal{B}_M - W_M^B)\right] \Big\}$$

subject to

$$s(e_C)u(W_C^G) + (1 - s(e_C))u(W_C^B) - g(e_C) \geq \underline{U}_C \ (IR_C) \tag{1}$$

$$s(e_M)u(W_M^G) + (1 - s(e_M))u(W_M^B) - g(e_M) + s(e_M)P(\mathbf{e}_{-M})V_C \geq \underline{U}_M \ (IR_M) \tag{2}$$

$$e_C \in \arg\max_{\hat{e}} s(\hat{e})u(W_C^G) + (1 - s(\hat{e}))u(W_C^B) - g(\hat{e}) \ (IC_C) \tag{3}$$

$$e_M \in \arg\max_{\hat{e}} s(\hat{e})u(W_M^G) + (1 - s(\hat{e}))u(W_M^B) - g(\hat{e}) + s(e_M)P(\mathbf{e}_{-M})V_C \ (IC_M). \tag{4}$$

[20] The derivation of this equation is found in the Appendix.

Since contracts are not time-dependent, the above problem is equivalent to solve the following problem:

$$\max_{\{(e_C, W_C^G, W_C^B), (e_M, W_M^G, W_M^B)\}} s(e_C)(\mathcal{G}_C - W_C^G) + (1 - s(e_C))(\mathcal{B}_C - W_C^B)$$
$$+ N\left[s(e_M)(\mathcal{G}_M - W_M^G) + (1 - s(e_M))(\mathcal{B}_M - W_M^B)\right]$$
$$\text{subject to the above four constraints, (1), (2), (3), and (4).}$$

From now on, I call this the firm's problem. It is worth mentioning the role of **Assumption 2** in the simplification of the firm's problem. This allows one to focus on a repeating part by restricting the firm to offer the same contract to the CEO. In section 7, I investigate a more complex model with dynamics in a simplified form with the same argument.

4.1 The basic characterization of the firm's problem

In this section, I analyze some preliminary features of the firm's problem before examining its main properties.

First of all, by a similar argument with Grossman and Hart (1983), I can show that the firm's problem has a solution.

Lemma 1. *There exists a solution to the firm's problem.*

Proof. All proofs are presented in the Appendix. □

In order to analyze agents' incentive compatibility constraint, I can apply the first-order approach from Rogerson (1985*b*). The two first-order conditions yield the following results:

$$u(W_C^G) = u(W_C^B) + \frac{g'(e_C)}{h'(e_C)}, \text{ and}$$
$$u(W_M^G) = u(W_M^B) + \frac{g'(e_M)}{h'(e_M)} - P(\mathbf{e}_{-M})V_C.$$

Since the firm can control (W_M^G, W_M^B) without affecting the CEO's problem, the manager's individual rationality constraint must bind to maximize its profit. In particular, the firm can decrease W_M^B such that the managers' incentive compatibility constraint is satisfied at the given effort level e_M when W_M^G decreases for a given fixed value V_C. This means that the firm can increase its profit if the managers' individual rationality constraint does not bind.

Lemma 2. *Managers' individual rationality constraint binds.*

However, the same logic cannot be applied to the CEO's individual rationality constraint since adjusting W_C^G and W_C^B inevitably affects V_C, which directly enters into the managers' problem. Reducing a CEO's expected utility yields a higher W_M^G for given e_M and W_M^B, which could decrease the firm's profit. This observation makes it difficult to analyze the properties of the firm's problem. In the next section, I consider a modified method circumventing this obstacle.

4.2 CEO's individual rationality constraint

In this section, I examine a problem where the CEO's individual rationality constraint binds at a certain value in order to indirectly solve the firm's problem. In particular, I consider the following problem:

$$F(\mathcal{V}) \equiv \max_{\{(e_C,W_C^G,W_C^B),(e_M,W_M^G,W_M^B)\}} s(e_C)(\mathcal{G}_C - W_C^G) + (1-s(e_C))(\mathcal{B}_C - W_C^B)$$
$$+ N\left[s(e_M)(\mathcal{G}_M - W_M^G) + (1-s(e_M))(\mathcal{B}_M - W_M^B)\right]$$

subject to

$$s(e_C)u(W_C^G) + (1-s(e_C))u(W_C^B) - g(e_C) = \mathcal{V}\ (IR_C) \quad (5)$$

$$s(e_M)u(W_M^G) + (1-s(e_M))u(W_M^B) - g(e_M) + s(e_M)P(\mathbf{e}_{-M})\mathcal{V} = \underline{U}_M\ (IR_M) \quad (6)$$

$$e_C \in \arg\max_{\hat{e}} s(\hat{e})u(W_C^G) + (1-s(\hat{e}))u(W_C^B) - g(\hat{e})\ (IC_C) \quad (7)$$

$$e_M \in \arg\max_{\hat{e}} s(\hat{e})u(W_M^G) + (1-s(\hat{e}))u(W_M^B) - g(\hat{e}) + s(e_M)P(\mathbf{e}_{-M})\mathcal{V}\ (IC_M) \quad (8)$$

for $\mathcal{V} \in [0,\infty)$. The difference from the original firm's problem is that the CEO's individual rationality constraint binds at a positive value $\mathcal{V}$. Since both individual rationality constraints are binding now, the compensation scheme $(W_C^G, W_C^B, W_M^G, W_M^B)$ can be expressed by functions of (e_C, e_M):

$$
\begin{aligned}
u(W_C^G) &= \mathcal{V} + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{h'(e_C)} \\
u(W_C^B) &= \mathcal{V} + g(e_C) - s(e_C)\frac{g'(e_C)}{h'(e_C)} \\
u(W_M^G) &= \underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{h'(e_M)} - P(\mathbf{e}_{-M})\mathcal{V} \\
u(W_M^B) &= \underline{U}_M + g(e_M) - s(e_M)\frac{g'(e_M)}{h'(e_M)}.
\end{aligned}
$$

These equations characterize the compensation scheme for a given effort level (e_C, e_M) and a parameter $\mathcal{V}$. However, the solution to this modified problem is not necessarily the same as that of the firm's original problem. One special case when the two problems have the same solution is $\mathcal{V} = \underline{U}_C$ and the CEO's individual rationality constraint binds.

In order to find the optimal compensation schemes, I exploit the modified problem. Suppose that $F(\cdot)$ is a strictly quasi-concave function. Then, I can solve the original problem indirectly using $F(\cdot)$. In particular, consider a maximization problem:

$$\max_{\mathcal{V}} F(\mathcal{V}).$$

Let $\mathcal{V}^*$ denotes $\inf \{\arg\max_{\mathcal{V}} F(\mathcal{V})\}$.[21] Then, the strict quasi-concavity ensures that the CEO's individual rationality constraint binds and the firm's expected profit is equal to $F(\underline{U}_C)$ if $\mathcal{V}^* \leq \underline{U}_C$. On the other hand, the CEO's individual rationality constraint does not bind and the firm's expected profit is equal to $F(\mathcal{V}^*)$ if $\mathcal{V}^* > \underline{U}_C$. Before analyzing this modified problem, I impose some parametric assumptions on $\underline{U}_M$.

Assumption 3. *When $N = 1$ and agents are risk-averse, $\underline{U}_M$ is such that*

$$\mathcal{V}^* = \inf \left\{\arg\max_{\mathcal{V}} F(\mathcal{V})\right\} > 0.$$ [22]

Under this assumption, I can exclude a corner solution implying that promotion is strictly beneficial for a manager if a firm hires only one manager. Note that the manager who is pro-

[21] From now on, I use the superscript star to denote the optimal variable.

[22] Since $\frac{\partial \mathcal{V}^*}{\partial \underline{U}_M} > 0$ and there exists $\underline{U}_M^*$ such that $F'(\mathcal{V}|\underline{U}_M^*)|_{\mathcal{V}=0} > 0$, there is $\underline{U}_M$ satisfying the condition.

moted to CEO enjoys the expected utility $\mathcal{V}^*$ during the second period. Hence, I can interpret this value as the promotion incentive from the manager's point of view.[23]

5 The Effect of Promotion Incentive on Agents' compensation when the Effort is Observable

Before analyzing the firm's problem with incomplete information, I first study the solution without any agency problem. In this section, I focus on risk-averse agents and use **Promotion rule 2** not **Promotion rule 1** for simplicity.[24] Then, the firm's modified problem is reduced to:

$$\max_{e_C, e_M, W_C^G, W_C^B, W_M^G, W_M^B} s(e_C)(\mathcal{G}_C - W_C^G) + (1 - s(e_C))(\mathcal{B}_C - W_C^B) + N(s(e_M)(\mathcal{G}_M - W_M^G) + (1 - s(e_M))(\mathcal{B}_M - W_M^B))$$

subject to

$$s(e_C)u(W_C^G) + (1 - s(e_C))u(W_C^B) - g(e_C) = \mathcal{V} \quad (IR_C)$$
$$s(e_M)u(W_M^G) + (1 - s(e_M))u(W_M^B) - g(e_M) + \frac{1}{N}\mathcal{V} = \underline{U}_M \quad (IR_M).$$

Since there is no agency problem, incentive compatibility constraints are dropped. Moreover, because the agents are risk-averse while the firm is risk-neutral, the two individual rationality constraints can be rewritten as:[25]

$$u(W_C) - g(e_C) = \mathcal{V} \quad (IR_C)$$
$$u(W_M) - g(e_M) + \frac{1}{N}\mathcal{V} = \underline{U}_M \quad (IR_M),$$

[23]More formally, the promotion incentive is $\max\{\mathcal{V}^* - \underline{U}_C, 0\}$ since the firm has to offer $\underline{U}_C$ in order to hire a CEO. However, I simply consider $\mathcal{V}^*$ as the promotion incentive since $\underline{U}_C$ is a constant.

[24]There are two reasons. First of all, **Promotion rule 2** simplifies the analysis if there is no agency problem. Also, if there is no agency problem, the argument in Section 9 does not work since the adverse selection issue is also related to incomplete information.

[25]This can be easily checked using Jensen's inequality.

where the fixed payments (W_C, W_M) are paid to the CEO and managers when they exert the required effort levels (e_C, e_M), respectively.

Therefore, for given effort levels (e_C, e_M), the optimal wage levels are determined by

$$u(W_C) = \mathcal{V} + g(e_C), \text{ and}$$
$$u(W_M) = \underline{U}_M + g(e_M) - \frac{1}{N}\mathcal{V}.$$

Also, the following two first order conditions characterize the optimal effort choice levels by the firm:[26]

$$\beta(\mathcal{G}_C - \mathcal{B}_C) = \frac{g'(e_C)}{u'(W_C)}, \text{ and}$$
$$\beta(\mathcal{G}_M - \mathcal{B}_M) = \frac{g'(e_M)}{u'(W_M)}.$$

Recall that a strict quasi-concavity of $F(\cdot)$ allows the use of the modified method in order to solve the firm's original problem. The following lemma confirms that this is the case when there is no information friction between the firm and agents.

Lemma 3. *$F(\mathcal{V})$ is strictly concave.*

Now, I analyze the property of the firm's problem using the modified one. The main concern is the CEO's expected utility level $\mathcal{V}^*$.

Proposition 1. *If agents are risk-averse, $\mathcal{V}^*$ strictly increases as N increases.*

In other words, it is optimal for the firm to offer a contract providing a higher expected utility to the CEO if the internal pool of candidates for promotion is getting bigger. Combining this result with **Lemma 3** yields the following result.

Corollary 1. *CEO's expected utility is an increasing function in N.*

This implies that a CEO can obtain utility beyond her reservation value if a firm is big enough.[27] Although this does not mean that the firm provides a higher compensation to the CEO, this is true when there is no agency problem.

[26]The second order conditions can be checked easily.

[27]Here, the size of a firm is determined by the number of managers or candidates for internal promotion.

Corollary 2. *The optimal CEO compensation W_C^* is an increasing function in N.*

However, the following proposition illustrates that there are two restrictions on the CEO's compensation level when agents' effort levels are contractible.

Proposition 2. *If every agent is risk-averse, $W_C^* = W_M^*$. Moreover, if $\mathcal{G}_C - \mathcal{B}_C = \mathcal{G}_M - \mathcal{B}_M$, then $\mathcal{V}^*$ is equal to $\frac{N}{N+1}\underline{U}_M$.*

The first restriction is that CEO and managers receive exactly the same level of compensation when there is no information asymmetry. This is true since the firm wants to reduce the total pay by equally distributing remuneration. According to this result, if there is no informational friction, there is no wage gap between CEO and managers.

Moreover, if CEO and managers have the same marginal productivity[28], then CEO's expected utility is strictly bounded by the manager's reservation value. Recall that the firm offers the utility $\mathcal{V}^*$ to its CEO only when $\mathcal{V}^* \geq \underline{U}_C$. In other words, if CEO's reservation value is greater than managers', the CEO's individual rationality constraint always binds regardless of the number of managers. This implies that the CEO's reservation value should be less than managers' to obtain both the wage gap and a positive relationship between CEO compensation and the firm size, In the next section, I consider the asymmetric information case.

6 The Effect of Promotion on CEO Compensation under Moral Hazard

6.1 Preliminary - no promotion possibility

In this section, I briefly illustrate several properties of the solution to the firm's problem when there is no promotion possibility. Comparing the optimal contract with and without promotion possibility can clarify the effect of promotion incentives on CEO compensation.[29] If there is no promotion possibility, the firm's problem turns into a simple moral hazard problem and the

[28]In this paper, agents' marginal productivity is measured by $\beta(\mathcal{G}_i - \mathcal{B}_i)$, where $i \in \{C,\ M\}$.

[29]The problem in this section is exactly the same as Grossman and Hart (1983).

CEO's individual rationality constraint must bind.[30] Binding individual rationality constraint yields a limitation in explaining the trend in CEO compensation. If changes in the firm's contracting environment make it require higher effort from the CEO, the firm provides higher expected pay to its CEO and the ratio of the fixed pay to the incentive pay falls. That is to say, the theory predicts the fixed pay and the incentive pay should move in the opposite directions. In practice, however, the base salary, i.e., fixed pay for CEO has not fallen enough while the incentive pay has risen. In the following section, I illustrate how this prediction changes if a firm uses the two incentive schemes, absolute performance based and relative performance based compensation, together.

6.2 Risk-neutral agents

First, I consider risk-neutral agents. The following proposition says that it is optimal for the firm not to offer a contract providing a higher utility than CEO's reservation value to her if all agents are risk-neutral.

Proposition 3. *If every agent is risk-neutral, the CEO's individual rationality constraint binds. Specifically, the firm's profit is a strictly decreasing function in* $\mathcal{V}$.

This implies that promotion incentive scheme is dominated by pay for performance if agents are risk-neutral. According to Lazear and Rosen (1981), a compensation scheme based on relative ranking achieves the first best allocation if agents are risk-neutral, which is also true for the incentive scheme based on absolute performance. Since the promotion rule considered in this paper has a penalty[31] compared to the tournament structure in Lazear and Rosen (1981), the incentive based on promotion is less efficient than absolute performance-based incentives.[32] I analyze how this observation changes if agents are risk-averse as follows.

[30] This is true with any finite number of possible outcomes when CEO's utility function is additively separable.

[31] There is a possibility of external hiring.

[32] If I adopt **Promotion rule 2**, the two schemes are perfect substitutes. However, under **Promotion rule 2**, pay for performance scheme still dominates promotion incentive scheme if managers discount the future.

6.3 The first stage

In this section, I consider a simple situation where the firm's desired effort levels are given exogenously. That is, the firm requires fixed effort levels $(e_C, e_M) \in (0,1) \times (0,1)$ from the CEO and managers, respectively. First of all, the following lemma guarantees that I can use the modified method to solve the firm's problem. Moreover, the strict concavity ensures that the solution of the firm's problem is unique.

Lemma 4. *If agents are risk-averse, $F(\mathcal{V})$ is strictly concave.*

The next question is how the CEO's expected utility changes when there is a competition among managers for promotion. First, I consider the effect of the number of managers on the CEO's expected utility, which determines the power of promotion incentives. This is important because the number of candidates for promotion might be a good proxy of the firm size. For example, Zabojnik and Bernhardt (2001) use the number of competitors for a promotion as the measure of the firm size. From now on, I use the number of managers N to refer the size of a firm.

Proposition 4. *If agents are risk-averse, $\mathcal{V}^*$ strictly increases as N increases.*

Again, this proposition and the previous lemma lead to the following result.

Corollary 3. *CEO's expected utility rises as N increases.*

Hence, the firm raises the promotion incentive, although the power of it decreases as the competition gets severe. From this result, one can confirm that the firm wants to divide payments between absolute and relative performance based compensation depending on the size of internal labor market. In other words, firms effectively employ a hybrid incentive scheme not just one of the two schemes. Moreover, if the CEO's required effort level does not change as the number of candidates varies, the rise in CEO's expected utility yields a higher expected compensation to CEO.

Corollary 4. *The expected compensation to CEO increases as N grows.*

Therefore, it is optimal for a firm to provide a higher compensation to the CEO when there are more candidates for internal promotion. Therefore, the increase in promotion incentives might play a significant role in the rise of CEO compensation. When it comes to the wage inequality between CEO and managers, I can assert that the size of the firm has a positive correlation with the expected wage gap between CEO and managers as well as CEO compensation if agents have the log utility function.

Corollary 5. *Suppose that agents have the log utility function. Then, the expected compensation gap between CEO and managers widens as N increases.*

Then, can the CEO compensation still have the upper bound of **Proposition 2**?[33] This question is important since if the compensation has the same upper bound, the CEO compensation is always lower than managers' one when the firm requires the same effort level. However, the following proposition indicates that this is not the case if there is a moral hazard problem.

Proposition 5. *When $0 < e_C \leq e_M < 1$, there is $\widehat{N}$ such that $\mathcal{V}^* > \underline{U}_M$ if $N > \widehat{N}$.*

Figure 1 numerically illustrates previous results. In this example, the CEO's individual rationality constraint binds when $\underline{U}_C = 3$, which is the same as the manager's reservation value in the example, and $N = 1, 2$, or 3. This means that the CEO's compensation level remains at the same level even when the firm size increases. Namely, the size of the firm and CEO compensation do not have a strictly monotonic relationship. This feature makes this model different from Gabaix and Landier (2008), where CEO payment monotonically moves with changes in the size of the firm.[34]

By comparing this result with **Proposition 2**, I derive two channels to explain why the firm increases promotion incentives as N grows. The first channel comes from the difference between two reservation values $\underline{U}_C$ and $\underline{U}_M$. If $\underline{U}_C$ is less than $\underline{U}_M$, by offering a higher compensation to the CEO than to the managers, the firm can reduce overall payouts to agents. The other channel is to reduce the gap between W_M^G and W_M^B by increasing the CEO's expected utility. Note that the inefficiency of information asymmetry arises when agents are risk-averse if the firm does not use internal promotion schemes. This is because the wage gap incurs an

[33] If I consider **Promotion rule 1** under the first-best case, it can be shown that $\mathcal{V}^*$ is strictly less than $\frac{N}{N+1}\underline{U}_M$.

[34] Gabaix and Landier (2008) use earnings of firms as a proxy of the size of a firm. Analyzing the relationship between the number of candidates and other measures of the firm size such as earnings, market values, and sales will be a topic for future research.

additional cost to the firm. Hence, the firm can partly remove the inefficiency by increasing promotion incentives, which reduces the wage gap. This effect gets stronger as N grows because promotion incentives affects N managers' wage gaps.

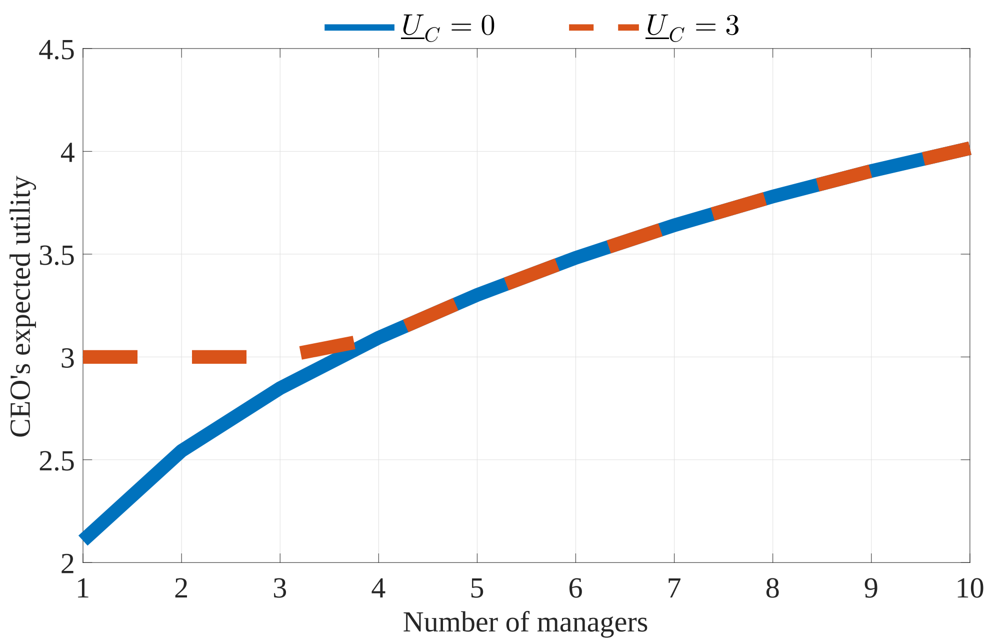

Figure 1: CEO's expected utility for $\underline{U}_C = 0$ (solid line) or $\underline{U}_C = 3$ (dashed line) when $\underline{U}_M = 3$. Other parameters are $\mathcal{G}_C = \mathcal{G}_M = 1,000$, $\mathcal{B}_C = \mathcal{B}_M = 0$, $\alpha = 0.05$, $\beta = 0.45$, $e_C = 0.35$, $e_M = 0.43$, $u(x) = \log(x)$ and $g(a) = -\frac{1}{a-1} - a - 1$.

As the next step, I examine how the level of promotion incentives behaves according to the adjustment of required effort levels. The following proposition shows how this changes.

Proposition 6. *Suppose that* $\frac{u''(x)}{u'(x)^3}$ *is a decreasing function in* x*. If agents are risk-averse,* $\mathcal{V}^*$ *strictly decreases as* e_C *increases while* $\mathcal{V}^*$ *strictly increases when* e_M *increases.*

One interesting result is that the firm provides a higher expected utility to the CEO when it requires less effort from her. This makes sense when CEO compensation is related to her incentives as well as promotion incentives for managers. This result gives an important implication about promotion incentives. Recall that the degree of promotion incentives is determined by the level of $\mathcal{V}^*$. Therefore, if the firm requires a high (low) effort from the CEO, it reduces (increases) the promotion incentives. This implies that if a change in contracting environments yields a growth in the CEO's marginal productivity, then the firm decreases the CEO's expected utility reducing promotion incentives.

The sufficient condition regarding agents' utility functions for **Proposition 6** holds for all CARA utility functions and CRRA utility functions with a coefficient of relative risk aversion higher than one half.

When the firm uses relative as well as absolute performance based incentive schemes, the gap between the managers' wage for a good and a bad performance could shrink to zero.[35] In other words, it can be optimal for a firm to focus on promotion incentives instead of performance tied compensation.

Proposition 7. *Assume that* $\lim_{x\to\infty} u'(x) = 0$. *Then, for a given* (N, e_C, e_M)*, there is* $\widehat{\underline{U}}_M$ *such that*

$$(W_M^G)^* \leq (W_M^B)^*$$

if $\underline{U}_M \geq \widehat{\underline{U}}_M$.

Therefore, if people do not consider the possibility of promotion when they analyze executives' compensations they can misinterpret the incentives behind them. That is, although managers' compensation does not depend on their output, their wages have already reflected incentives through the promotion possibility. **Figure 2** illustrates this result. As the managers' reservation value increases from 8 to 9, the situation of $(W_M^G)^* \leq (W_M^B)^*$ happens when $N < 4$. It is worth mentioning that this result does not mean that managers have less utility when they make a good outcome rather than a bad outcome. If managers achieve a good outcome, they obtain utility through two channels: 1) utility from the first period's wage, W_M^G, in the first period and 2) utility from the promotion possibility, $P(\mathbf{e}_{-M})\mathcal{V}$, in the second period. On the other hand, a bad outcome only gives them the wage in the first period, W_M^B. For the sum of expected utilities, the firm has to offer a higher utility for a good outcome than a bad outcome in order to induce a positive effort from managers. That is, $u(W_M^G) + P(\mathbf{e}_{-M})\mathcal{V}$ must be greater than $u(W_M^B)$.

[35] Similar result holds under the Promotion Rule 2. The difference is that $W_g^* > W_b^*$ always holds when $N = 1$.

The previous result illustrates that promotion incentive can dominate the incentive associated with pay for performance. The following result shows that the opposite situation can also happen.[36]

Corollary 6. *For a given (N, e_C, e_M), there is $\widetilde{\underline{U}}_M$ such that*

$$\mathcal{V}^* = 0$$

if $\underline{U}_M \leq \widetilde{\underline{U}}_M$.

That is, the pay for performance incentive scheme might dominate the promotion based scheme. In this case, there is no incentive for the firm to introduce a hierarchical structure. One can interpret the previous two results in two ways. First, if a managers' reservation value is really high (low) and the CEO's individual rationality constraint does not bind, a firm should focus on the promotion (pay for performance) incentive scheme. Also, if the CEO's reservation value is really high, the firm would prefer the promotion based incentive scheme to the pay for performance incentive scheme.

6.3.1 The effect of internal competition on firm's profit per agent

Up to this point, I focus on how the two incentive schemes interact in a firm. Here, I turn to another question: why does the firm use a hierarchical structure based on the competition among managers by paying higher compensation to the CEO? In order to answer this question, I focus on the firm's profit per agent measured by

$$\Pi(\mathcal{V}^*|N) \equiv \frac{1}{N+1} F(\mathcal{V}^*|N).$$

In order to concentrate on the effect of competition between managers, I assume that $\mathcal{G} \equiv \mathcal{G}_C = \mathcal{G}_M$ and $\mathcal{B} \equiv \mathcal{B}_C = \mathcal{B}_M$. In particular, I try to answer the question: can firms increase the profit per agent by adopting a hierarchical structure? The following result shows what the firm can do by introducing internal competition between managers.

[36]For this result, I drop **Assumption 3**. That is, I allow corner solutions.

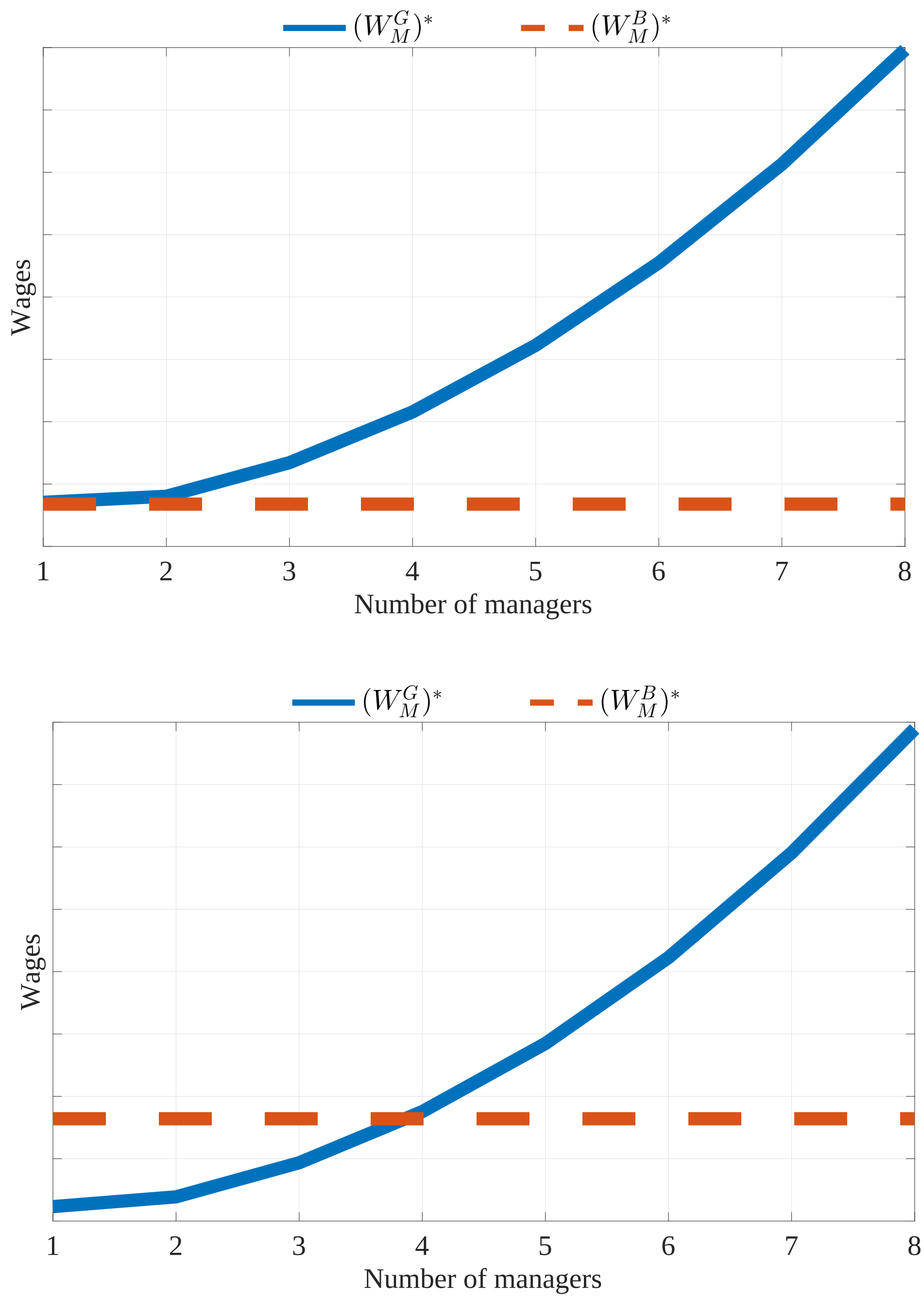

Figure 2: Managers' wage levels for $\underline{U}_M = 8$ (up) and $\underline{U}_M = 9$ (down) when $\underline{U}_C = 0$. Other parameters are the same as those in **Figure 1**.

Proposition 8. *If* $1 > e_M > e_C > 0$*, there is* $(\mathcal{G}^*, \mathcal{B}^*)$ *such that for* $\mathcal{G} - \mathcal{B} \geq \mathcal{G}^* - \mathcal{B}^*$*,* $\Pi(\mathcal{V}^*|N) > \Pi(\mathcal{V}^*|1)$ *for a given* N*. Moreover, for a given* $\mathcal{G} - \mathcal{B} \leq \overline{O}$*, there is* $\widehat{N}$ *such that* $\Pi(\mathcal{V}^*|N) < \Pi(\mathcal{V}^*|1)$ *if* $N > \widehat{N}$*, where* $\overline{O}$ *is derived in the Appendix.*

Moreover, for a given $\mathcal{G} - \mathcal{B} \leq \overline{O}$*, there is* $\widehat{N}$ *such that* $\Pi(\mathcal{V}^*|N) < \Pi(\mathcal{V}^*|1)$ *if* $N > \widehat{N}$ *and* $e_M \in (e_C, \bar{e}_M]$*, where* $\overline{O}$ *and* $\bar{e}_M$ *are derived in the Appendix.*

Figure 3 illustrates this result graphically. Under the given parameters, the profit per agent is maximized when $N = 2$ if $\underline{U}_C = 0$ and when $N = 3$ if $\underline{U}_C = 3$. The latter case means that the firm can increase its profit by hiring three managers and one CEO with internal competition compared to the situation where there are two lower-level managers and two upper-level managers without any internal competition. In the second scenario, there is no competition for promotion among managers, but the decision is only determined by their own performances. These two scenarios are represented by $N = 3$ case and two $N = 1$ structures.

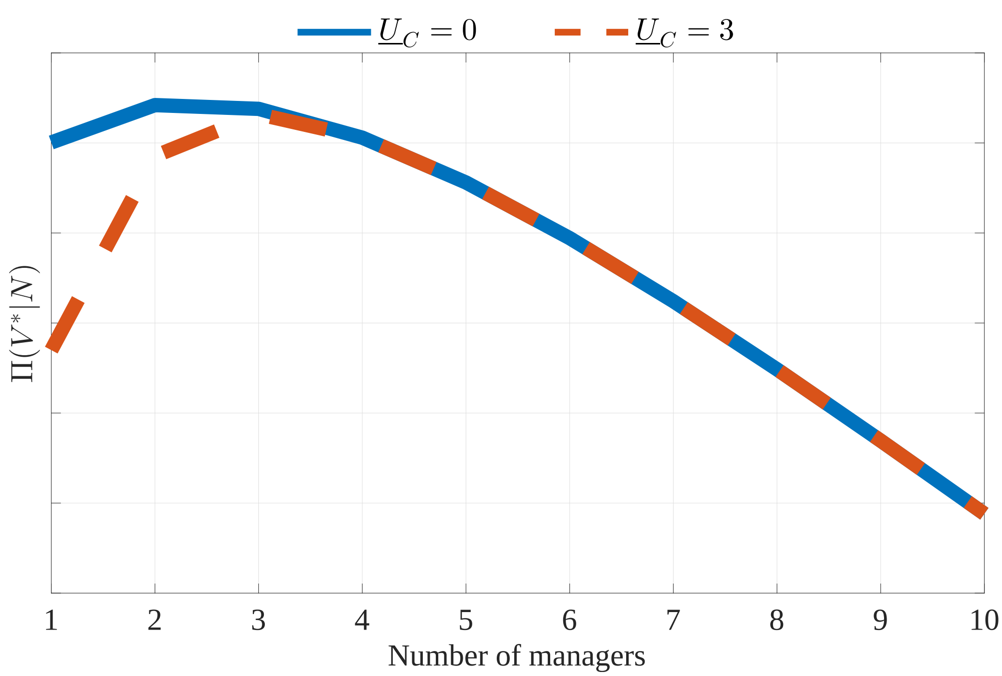

Figure 3: Firm's profit per agent for $\underline{U}_C = 0$ (solid line) or $\underline{U}_C = 3$ (dashed line) when $\underline{U}_M = 3$. Other parameters are the same as those in **Figure 1**.

Moreover, the following result extends **Proposition 8** with the log utility function and $e_C = e_M = e$. The difference is that the previous result comes from the managers' higher contribution to the firm's output than the CEO's. However, the following proposition is related to the reduction in managers' wages. That is, as the number of managers increases, the firm reduces W_M^G by raising promotion incentives.

Proposition 9. *Suppose that agents have the log utility function,* $u(x) = \log(x)$*. Also, assume that* $\lim_{e\to 1} g'(e) = \infty$*. Then, for a given* $N \geq 2$*, there is* $(\underline{U}_M^*, \underline{U}_M^{**}, e^*)$ *such that* $\Pi(\mathcal{V}^*|N) > \Pi(\mathcal{V}^*|1)$ *if* $e > e^*$ *and* $\underline{U}_M \in [\underline{U}_M^*, \underline{U}_M^{**})$*, where* $\underline{U}_M^* < \underline{U}_M^{**}$*. Moreover, for a fixed* $(\underline{U}_M, e)$*, there is* $N^* \geq 1$ *such that* $\Pi(\mathcal{V}^*|N) < \Pi(\mathcal{V}^*|1)$ *if* $N \geq N^*$*.*

Both **Proposition 8** and **9** illustrate that the profit per agent decreases beyond some point of N. In other words, the relationship between the profit per agent and the number of managers is not monotonic.

6.4 The second stage

In this section, I relax the condition that the effort levels are determined exogenously. In order to make the analysis more tractable, I impose the following assumption.

Assumption 4. *The agents' cost function* $g(\cdot)$ *satisfies* $g'''(\cdot) \geq 0$ *with* $\lim_{e\to 1} g'(\cdot) = \infty$*. Also, the two parameters in the probability of good outcome* $s(e) = \alpha + \beta e$ *satisfies* $\alpha + \beta \leq \frac{1}{2}$*.* $u''(x)/u'(x)^3$ *is a decreasing function in* x*.*

This assumption provides a sufficient condition for concavity of the firm's objective function with respect to agents' effort level for a given $\mathcal{V}$. This allows one to focus on the first order condition with respect to agent's effort when solving the problem. Intuitively, these conditions together make the firm's payout more rapidly increase as the firm requires a higher effort from an agent. It is worth mentioning the condition that $\alpha + \beta \leq \frac{1}{2}$. Different from Lazear and Rosen (1981), the agent's output has only two possible outcomes, good or bad. Under this binary output process, change of its mean by controlling the agent's effort inevitably leads to change of the variance. Moreover this variance is equal to zero if $s(e)$ is zero or one, which eliminates the moral hazard problem since the realized output perfectly reveals the agent's exerted effort. In particular, a probability around $s(e) = 1$ could be problematic since the variance of the output decreases as $s(e)$ increases. This yields a tension between a higher wage and a lower agency problem around $s(e)$ equal to one. By restricting the two parameters in the given way, I can constrain the variance and the mean of the output to have a positive relationship. However, this assumption can be relaxed if the cost function $g(e)$ is sufficiently convex. This condition makes the growth of the wage component dominate the decrease of the agency problem when the firm requires a higher effort even if this reduces the variance of the output.

6.4.1 Optimal CEO effort choice

I relax the condition of exogenous effort choice in this section: I allow the firm to optimally choose the CEO's effort level in order to maximize its profit, still fixing the managers' effort level. I assume that the function $F(\mathcal{V})$ satisfies the strict quasi-concavity condition in order to solve the firm's original problem using the modified one.[37]

Assumption 5. *If agents are risk-averse, $F(\mathcal{V})$ is strictly quasi-concave.*

The following result confirms that the CEO's expected utility is still an increasing function in the size of the firm. Moreover, the CEO's optimal effort level decreases as the firm size increases.

Proposition 10. *When agents are risk-averse, $\mathcal{V}^*$ is a strictly increasing function in N while e_C^* is a strictly decreasing function in N.*

Intuitively, under the operational structure characterized by agents' independent outputs, the relative importance of the CEO's output, compared to the managers' total output, decreases as the number of managers increases. Therefore, the firm uses the CEO's position as a bonus rather than an output source. For the purpose of reducing the cost of increasing promotion incentives, the firm decreases the CEO's effort level.

It is important to note that a higher promotion incentive does not directly imply a higher expected compensation in this case. The movements of the optimal effort level and the promotion incentive predict the change of expected compensation in the opposite directions when the firm size grows. **Figure 4** and **Figure 5** illustrate an example for given parameters to see

[37]Numerically, this condition is investigated, and I confirm that the condition holds generally.

how CEO compensation changes as the firm size grows. **Figure 4** shows that the relationship between CEO effort level and the ratio of CEO's fixed pay to incentive pay remains the same as the prediction without promotion possibility: the ratio increases as the firm requires a lower effort level from the CEO. Also, a moral hazard model without promotion possibility predicts lower compensation and incentive pay when a firm requires a lesser effort level. However, the two variables in **Figure 5** present the opposite results. This result demonstrates that the trade-off between the required effort level and compensation could be reversed if people take into account the presence of internal labor markets. As a special case, if agents have the log utility function, the following corollary shows that CEO's incentive pay rises as the firm size increases although it requires lower effort from the CEO.

Corollary 7. *Suppose that agents have the log utility function,* $u(x) = log(x)$*. Then, CEO's incentive pay measured by* $(W_C^G)^* - (W_C^B)^*$ *rises as* N *increases.*

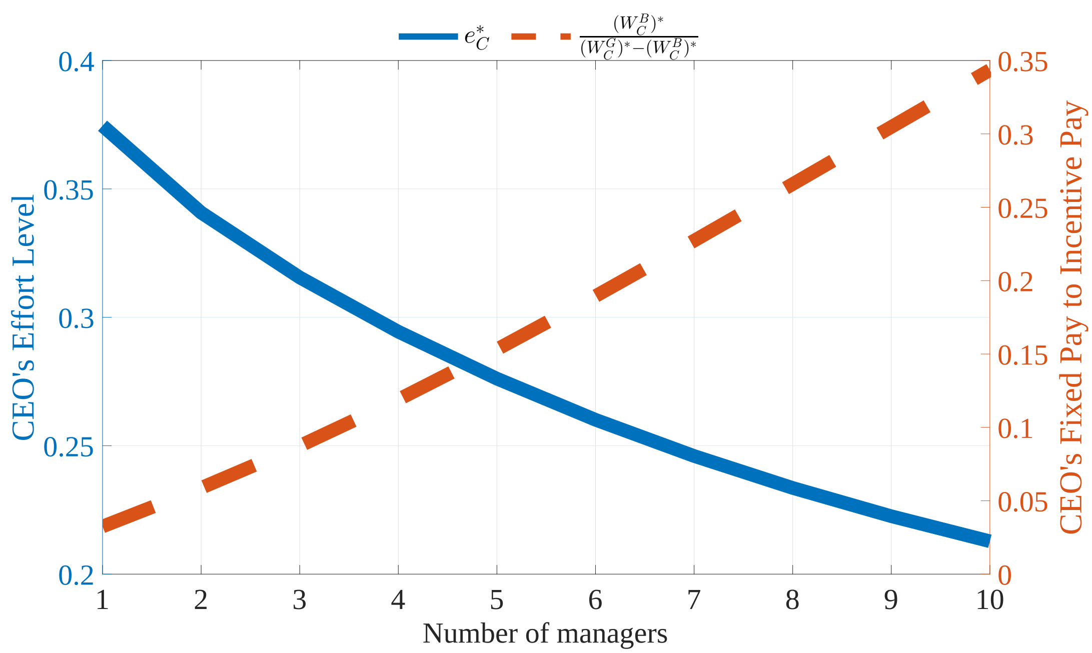

Figure 4: CEO effort level (solid line) and the ratio of an CEO's fixed pay relative to incentive pay (dashed line) for $\underline{U}_C = 0$ and $\underline{U}_M = 3$. Other parameters are the same as those in **Figure 1** except e_C.

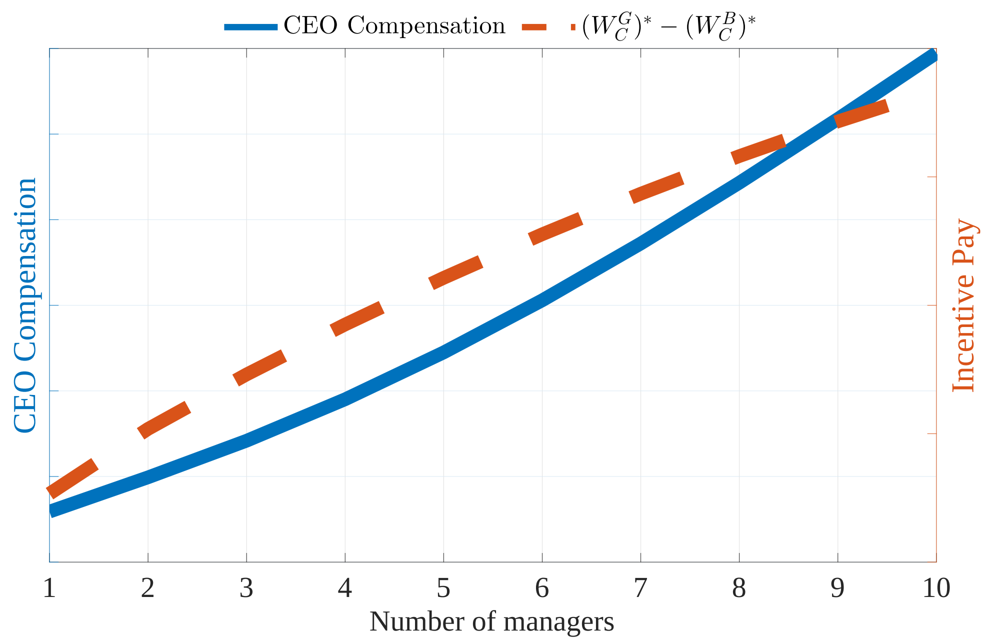

Figure 5: CEO compensation (solid line) and the level of incentive pay (dashed line) for $\underline{U}_C = 0$ and $\underline{U}_M = 3$. Other parameters are the same as those in **Figure** 4.

6.4.2 General case

In this section, I mainly deal with the numerical examples illustrating that the previous results still hold when the firm chooses both effort levels e_C and e_M optimally. In addition, I analytically show that $e_C^* < e_M^*$ could be a consequence of the firm's optimal decision.[38] Again, I assume that the function $F(\mathcal{V})$ satisfies the desired condition.[39]

Assumption 6. *If agents are risk-averse, $F(\mathcal{V})$ is strictly quasi-concave.*

First, **Proposition** 11 shows that the firm can optimally require a higher effort from managers than that of the CEO. **Figure** 6 illustrates this numerically.

Proposition 11. *Suppose that $\underline{U}_M = 0$, then there exists $\widehat{N}$ such that $e_M^* > e_C^*$ if $N > \widehat{N}$.*

This implies that it can be optimal for a firm to provide an expected utility to the CEO beyond her and managers' reservation values since **Proposition** 5 only requires $e_C \leq e_M$. Also, the firm can raise its profit per agent by adopting the internal labor market based on

[38]Recall that **Proposition** 5 and 8 depend on this feature.

[39]The condition is checked numerically.

competition between managers as the same reason. **Figure 7** and **8** confirm these observations. In particular, in this example, CEO's expected utility is higher than managers' reservation value if N is greater than or equal to three. Also, **Figure 8** shows that the firm's profit per agent is maximized when the number of managers is equal to three.

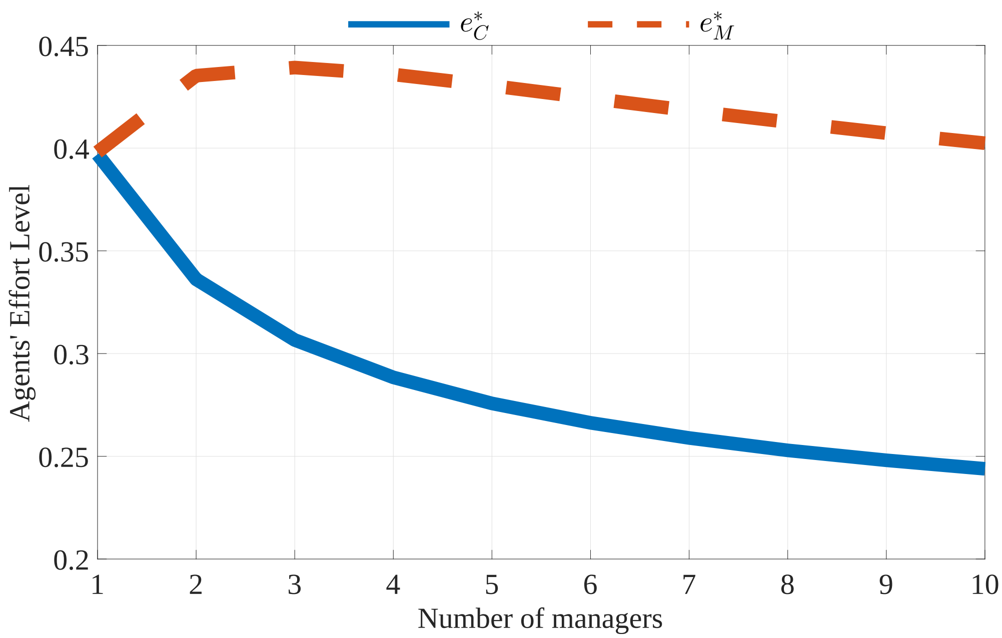

Figure 6: CEO effort level (solid line) and Manager's effort level (dashed line) for $\underline{U}_C = 0$ and $\underline{U}_M = 3$. Other parameters are the same as those in **Figure 1** except e_C and e_M.

Furthermore, **Figure 9** demonstrates that $(W_M^B)^*$ could be greater than $(W_M^G)^*$ as $\underline{U}_M$ increases from 6 to 7. In particular, $(W_M^B)^*$ is greater than $(W_M^G)^*$ if $N \in [5, 54]$. That is, the firm can optimally reduce the dependency of managers' compensation on their performance.

7 The Effect of Job Security on Promotion Incentives

As the first extension of the benchmark model, I analyze the effect of employment structure on promotion incentives in this section. In particular, I focus on how the level of promotion incentives changes according to the firm's policy on the executives' job security. This question is important since issues related to executives' job security have received significant attention from both researchers and practitioners. For example, Jenter and Kanaan (2015) illustrate that

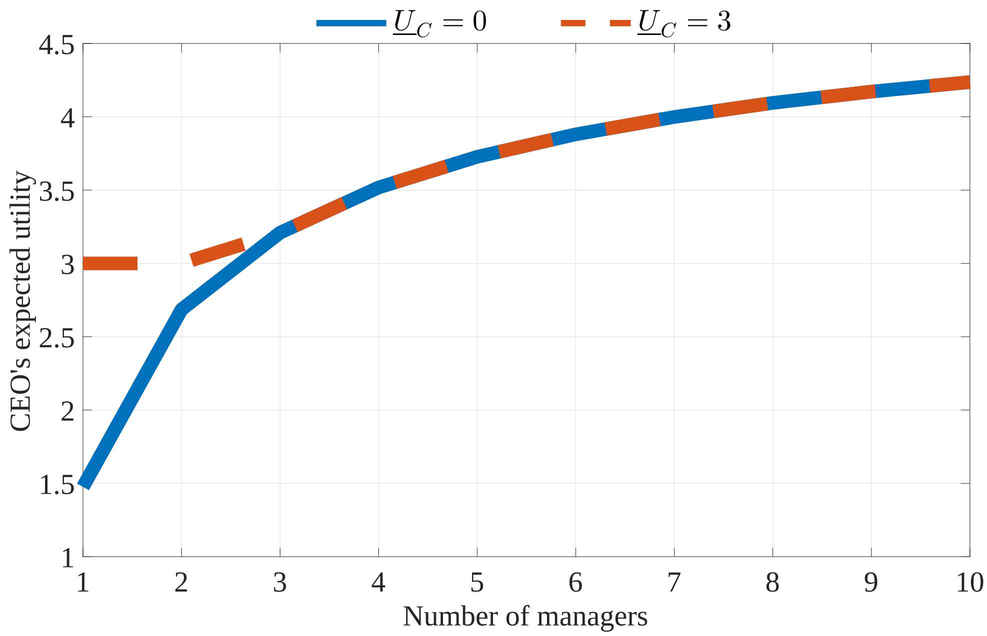

Figure 7: CEO's expected utility for $\underline{U}_C = 0$ (solid line) or $\underline{U}_C = 3$ (dashed line) when $\underline{U}_M = 3$. Other parameters are the same as those in **Figure** 7.

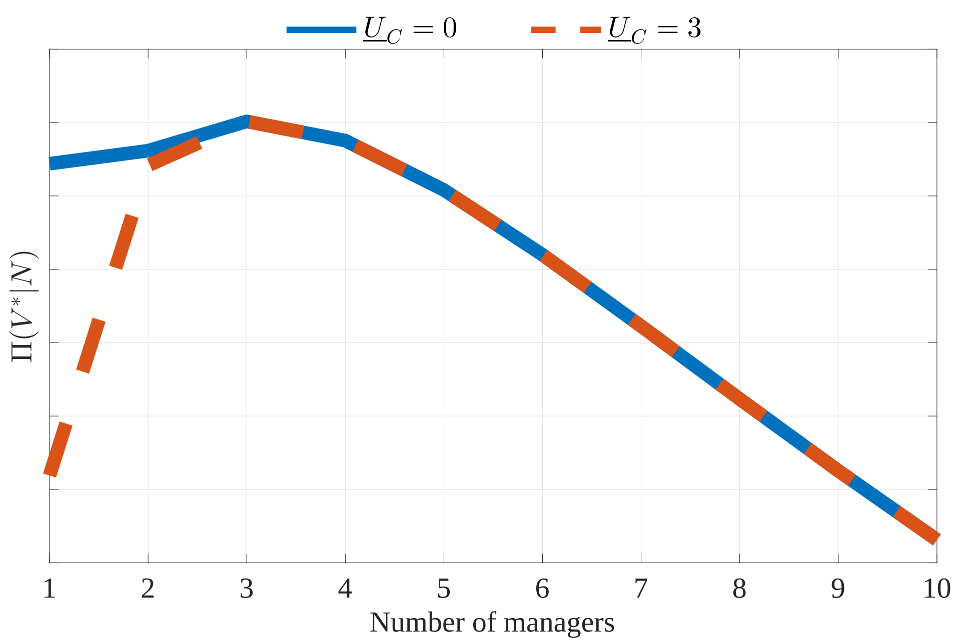

Figure 8: Firm's profit per agent when $\underline{U}_C = 0$ (solid line) or 3 (dashed line) when $\underline{U}_M = 3$. Other parameters are the same as those in **Figure** 7.

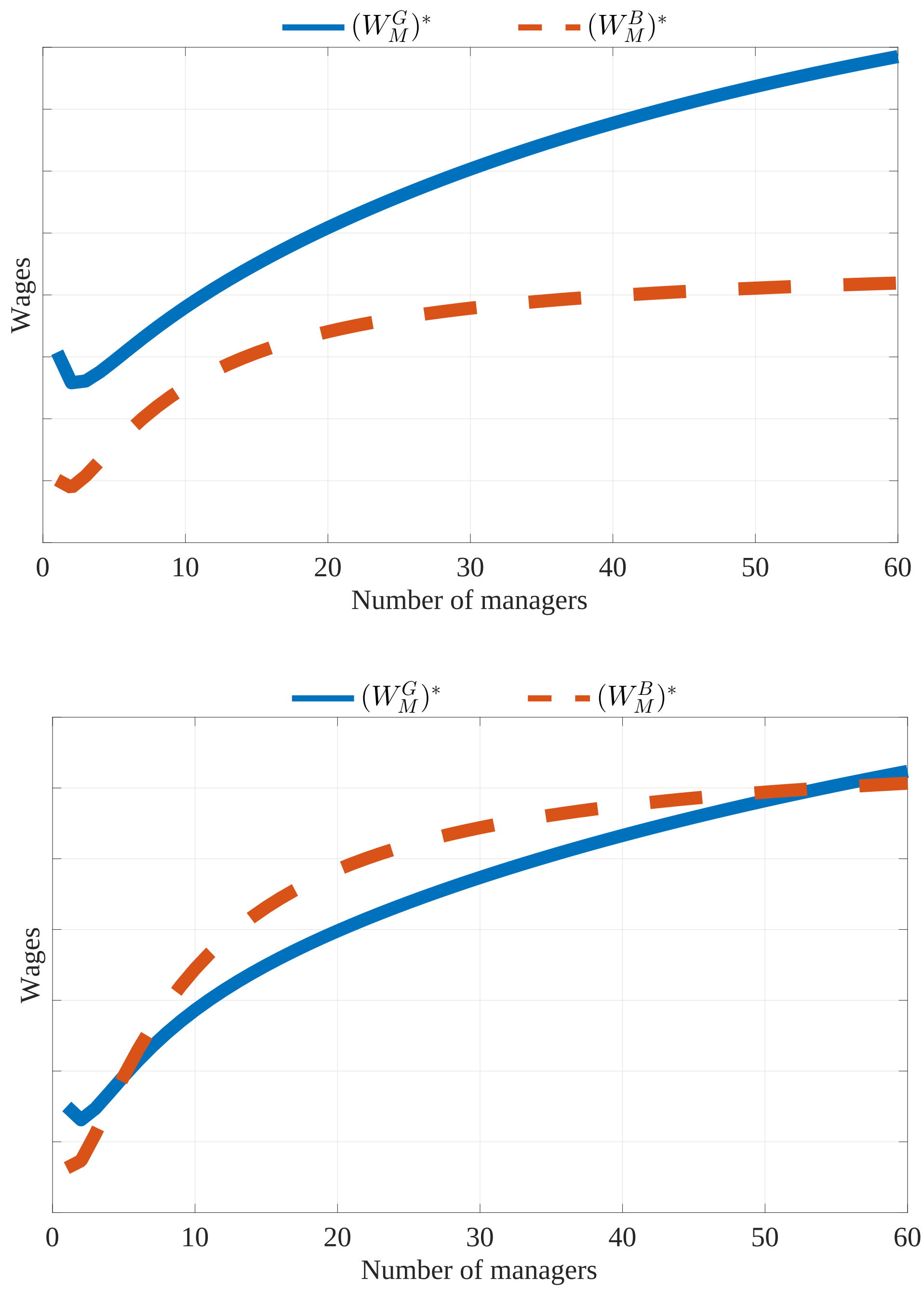

Figure 9: Managers' wage levels for $\underline{U}_M = 6$ (up) and $\underline{U}_M = 7$ (down) when $\underline{U}_C = 0$. Other parameters are the same as those in **Figure** 7.

bad performance related to negative shocks causes a forced CEO turnover. Hence, in order to understand executives' compensation more clearly, it is valuable to examine the effect of executives' job security on their compensation. Throughout the section, I treat agents' effort levels as exogenously given.

7.1 The effect of managers job security on promotion incentives

So far I assume that managers leave the firm if they are not promoted to CEO although they perform well in their position. In this section, I consider a slightly different employment structure. Namely, even if managers fail to be promoted to CEO, they can stay in the firm one more period as senior managers if their performance is good.[40] However, the senior managers do not have any chance for promotion. That is, senior managers retire regardless of their outcomes. Except this strengthened job security for managers, every employment structure is the same as the benchmark model. That is, the firm still hires N managers from an external labor market every period, and uses the same promotion rule. Then, does the firm have an incentive to provide more utility to the CEO than other successful managers? That is, can promotion incentives still contribute to the increase in CEO compensation?

Denote the senior manager's compensation by (W_M^{GG}, W_M^{GB}), which are tied to a good and bad outcome, respectively. Note that only managers with a good outcome remain in the firm as senior managers. Also, I use the term e_{M2} to distinguish the senior manager's effort from the (young) manager's effort, e_{M1}. The difference from the benchmark model is that successful managers obtain the expected utility,

$$s(e_{M2})u(W_M^{GG}) + (1 - s(e_{M2}))u(W_M^{GB}) - g(e_{M2}),$$

though they are not picked as the CEO.[41] I denote this utility by U_{M2} for brevity. Then, I show that managers' compensation scheme satisfies

[40] Since managers do not have any chance of promotion if they face a bad outcome in the first period under promotion rule 1, it does not affect the result in terms of promotion incentives whether managers can stay in the firm after a bad outcome.

[41] The detail of the firm's problem can be found in Appendix B.1.

$$u(W_M^G) = \underline{U}_M + g(e_{M1}) + (1 - s(e_{M1}))\frac{g'(e_{M1})}{\beta} - P(\mathbf{e}_{-M})\mathcal{V} - (1 - P(\mathbf{e}_{-M}))U_{M2}$$
$$u(W_M^B) = \underline{U}_M + g(e_{M1}) - s(e_{M1})\frac{g'(e_{M1})}{\beta}$$
$$u(W_M^{GG}) = U_{M2} + g(e_{M2}) + (1 - s(e_{M2}))\frac{g'(e_{M2})}{\beta}$$
$$u(W_M^{GB}) = U_{M2} + g(e_{M2}) - s(e_{M2})\frac{g'(e_{M2})}{\beta}.$$

The first equation illustrates that the manager's wage tied to good outcome depends on the senior manager's expected utility, U_{M2} as well as the promotion incentive, $\mathcal{V}$. Moreover, this wage and the senior manager's compensation satisfy the following relationship according to Rogerson (1985*a*)

$$\frac{1}{u'(W_M^G)} = \delta\left[\frac{s(e_{M2})}{u'(W_M^{GG})} + \frac{1 - s(e_{M2})}{u'(W_M^{GB})}\right]. \tag{9}$$

For this problem, the main concern is the difference between the CEO's expected utility ($\mathcal{V}^*$) and senior managers' expected utility (U_{M2}^*). If senior managers have higher expected utility than the CEO, promotion incentives do not play any effective role as an incentive tool. In this case, managers do not want to be promoted to the CEO. Hence, from now on, I add one more constraint in the firm's problem, call this promotion constraint.

$$\mathcal{V} \geq U_{M2} \qquad \text{(Promotion Constraint)}$$

The following result indicates when the promotion constraint does not bind.

Proposition 12. *Assume that $\frac{u''(x)}{u'(x)^3}$ is a decreasing function in x. Then, if δ is sufficiently large*[42]*, there is $(\hat{e}_C, \widehat{N})$ such that $\mathcal{V}^* > U_{M2}^*$ when $e_C \leq \hat{e}_C$ and $N \geq \widehat{N}$, where $\hat{e}_C < e_{M2}$. Moreover, $\mathcal{V}^*$ is an increasing function in N when $\mathcal{V}^* \geq U_{M2}^*$.*

Therefore, the firm can still optimally provide its CEO a compensation beyond her reservation value even if managers have strong job security. That is, **Proposition 4** still holds under this extension. This means that promotion incentives contribute the rise in CEO compensation when the size of the firm is large and the firm requires less effort from CEO than managers, even if managers have job security.

[42] The condition for δ can be found in the proof.

7.2 The effect of CEO job security on promotion incentives

In the benchmark model, a CEO leaves the firm after one period regardless of her performance. In this section, I assume differently that a CEO will stay and work one more period if she performs well in the first period. In other words, the CEO will be sacked as punishment for bad performance. Regarding managers, I maintain the same structure as the benchmark model. That is, only the manager promoted to the CEO remains in the firm and works one more period. In this section, I slightly modify **Assumption 2**.

Assumption 7. *The firm offers contracts based on agents' positions and the seniority of its CEO. That is, it offers three (possibly) different types of contracts: 1) CEO, 2) managers with new CEO, and 3) managers with CEO close to retirement.*

Under this assumption, the firm can offer different contracts to managers according to the length of the CEO's remaining term. Therefore, the extension allows one to examine managers' cohort effects as well as the effect of the CEO's job security. [43] Similar with section 4, I reduce the firm's problem to a more tractable form under **Assumption 7**[44].

The following proposition tells how the level of promotion incentives changes according to the change of CEO job security. More formally, call the case where a CEO is sacked if her performance is bad in the first period "unguaranteed job security". On the contrary, name it "guaranteed job security" if a CEO is not fired regardless of the first period's performance. By comparing these situations, one can see when the firm puts more weight on promotion incentives. In particular, I focus on the level of promotion incentives by fixing the required effort at the same level for both cases.

Proposition 13. *Assume that* $\lim_{e\to 1} g'(e) = \infty$ *and* $\lim_{x\to\infty} u'(x) = 0$*. Also, suppose that the firm requires an effort level* e_C *anf* e_M *from the CEO and managers, respectively, in every period. Then, there is* $e_C^* \in (0,1)$ *such that* $\mathcal{V}^*(e_C) \geq \widehat{\mathcal{V}}^*(e_C)$ *if* $e_C \in [e_C^*, 1)$ *and* δ *is sufficiently high*[45]*, where* $\mathcal{V}^*(e_C)$ *and* $\widehat{\mathcal{V}}^*(e_C)$ *are the optimal level of promotion incentives for*

[43] By the cohort effects, I mean that a cohort who earn more on entry maintains its advantage through time according to Baker, Gibbs and Holmstrom (1994).

[44] The detail of the firm's problem can be found in Appendix B.2.

[45] The condition for δ can be found in the proof.

unguaranteed and guaranteed job security cases, respectively, for a given e_C*. Moreover, for the sufficiently high* δ*, if* $\widehat{\mathcal{V}}^*(0) > 0$*, there is an interval* $[e_C^*, \bar{e}_C]$*, where* $\bar{e}_C \in (e_C^*, 1)$*, such that* $\mathcal{V}^*(e_C) > \widehat{\mathcal{V}}^*(e_C)$ *for* e_C *in the interval.*

It is important to know that the CEO's individual rationality constraint binds in both cases if one does not consider promotion possibilities. However, taking into account promotion possibilities gives a different answer. That is, it shows that a CEO can receive a favorable contract when her job security is less guaranteed. This result is also related to the tension between CEO incentives and managers' incentives. If the firm does not guarantee CEO job security, it gives more promotion chances to its managers. On the other hand, raising promotion incentive is more costly to the firm under the guaranteed situation than the other case since this make the firm need to pay more in the second period. Hence, when the firm requires a higher effort from the CEO, it wants to emphasize more the incentive for the CEO under the guaranteed situation. This emphasis yields fewer promotion incentives than the unguaranteed case.

Moreover, it can be shown that $W_{M1}^G > W_{M2}^G$ if $e_{M1} = e_{M2}$. Hence, managers' wage can exhibit a gap between cohorts although their abilities and required effort are the same. The reason is straightforward. The promotion possibility of managers relies on the timing they enter the firm: managers working with a new CEO expect less to be promoted than managers below a CEO who is about to retire, other things being equal. This affects managers' wages in an unambiguous way.

Now, I extend the above model allowing managers stay in the firm one more period if their performance is good although they are not promoted to the CEO. Then, do the managers who start a career with a new CEO still earn more money than managers starting with CEO close to retirement the second period? The following result shows that this is the case if the promotion is desirable for managers.

Proposition 14. *Suppose that the firm requires the same effort* $(e_{M11}, e_{M12}) = (e_{M21}, e_{M22})$*, where* e_{Mij} *is the required effort level in the period* j *from managers with a new CEO* ($i = 1$)*, or managers with CEO close to retirement* ($i = 2$)*. Then,* $(U_{M1}^2)^* > (U_{M2}^2)^*$ *if* $\mathcal{V}^* > (U_{M1}^2)^*$*. This also implies that* $(W_{M1}^G)^* > (W_{M2}^G)^*$*.*

Hence, this model can also predict a cohort effect. That is, the expected wage level of managers who have a lesser chance of promotion can be larger than that of managers having better chance in their whole careers. The condition that being the CEO is strictly beneficial than staying in the manager position is closely related to **Proposition 12**. Although I do not formally analyze the condition, if the firm does not require a high effort from the CEO in her second period, **Proposition 12** might hold since the firm optimally distributes the cost of raising promotion incentives.

8 Complementary Tasks: Multiplication Specification

I study how the firm's operational structure affects the firm's optimal contract in this section. Until now, I assume that a CEO's contribution to the firm's profit is independent of managers' outputs. As the second extension of the benchmark model, I investigate how the previous results change if there is a dependency between them. In particular, I consider the following firm's output structure:

$$[s(e_C)\mathcal{G}_C + (1 - s(e_C))\mathcal{B}_C]\, E\left[f\left(\sum_{i=1}^{N} X_i\right)\right],$$

where the CEO's task has a multiplication effect on the managers' aggregated output through the function $f(\cdot)$. For the function $f(\cdot)$, I impose some assumptions.

Assumption 8. *The function* $f(\cdot)$ *satisfies* $f(\cdot) > 0$*,* $f'(\cdot) > 0$*, and* $\lim_{x\to\infty} f'(x) > 0$*.*

The first two conditions are quite general. The last condition rules out the case when the size effect disappears since the firm size N only affects the firm's output through the function $f(\cdot)$. For example, a linear function taking positive numbers on its domain satisfies all requirements. Also, I adopt **Assumption 4** for the same reason as mentioned above. Note that the argument of the function $f(\cdot)$ is $[i\mathcal{G}_M + (N - i)\mathcal{B}_M]$, $i = 0, \ldots, N$. In order to assure that the argument is a strictly increasing function in the number of managers, I assume that $\mathcal{B}_M > 0$. Under these assumptions, I analyze the following problem with an exogenously given managers' effort level e_M like in Section 6.4.1.

$$\max_{\mathcal{V}\in[0,\infty)} F(\mathcal{V}), \text{ where}$$

$$F(\mathcal{V}) = \max_{\{(e_C,W_C^G,W_C^B),(W_M^G,W_M^B)\}} [s(e_C)\mathcal{G}_C + (1-s(e_C))\mathcal{B}_C]\, E\left[f\left(\sum_{i=1}^{N} X_i\right)\right] - s(e_C)W_C^G - (1-s(e_C))W_C^B - N[s(e_M)W_M^G + (1-s(e_M))W_M^B]$$

subject to four constraints, (5), (6), (7), (8).

The distinction between the independent and the multiplication specification is whether the marginal productivity of the CEO changes according to the size of the firm. Since the multiplication specification assumes an increasing productivity, the CEO's task is more crucial to the output when the firm is larger. This change of the CEO's role yields a different result from **Proposition 10**.

Proposition 15. *Assume that* $\lim_{x\to\infty} u'(x) = 0$. *Then, there is* $\widehat{N}$ *such that* $\mathcal{V}^*$ *is a decreasing function in* N *while* e_C^* *is a strictly increasing function in* N. *Also, if* $\mathcal{V}^* > 0$, $\mathcal{V}^*$ *strictly decreases as* N *grows.*

Recall that **Proposition 10** says that the promotion incentive increases as the firm size grows, and the firm requires less effort from the CEO. However, **Proposition 15** reveals that the exactly opposite situation happens if the size of the firm is sufficiently large, and the marginal contribution of the CEO's effort increases with the firm size. **Figure 10** and **11** show numerical examples under the multiplication specification. **Figure 10** illustrates a non-monotonic relationship between the promotion incentive and the number of managers. Especially, if CEO's reservation value is equal to one, which is exactly the same as managers' reservation value in the example, the CEO's individual rationality constraint binds when N is less than 7 or greater than 22. Also, **Figure 11** shows that the CEO's effort level increases as N grows. Intuitively, the firm wants the CEO to exert more effort since her marginal productivity grows as the number of managers increases. However, the concavity of the CEO's utility function makes it very costly for the firm to induce her to exert more effort especially when e_C is high. Hence, the firm puts more emphasis on the CEO's absolute performance-based compensation not the promotion incentive for managers. Namely, by reducing the promotion incentive, the firm can increase the CEO's effort level at a lower cost.

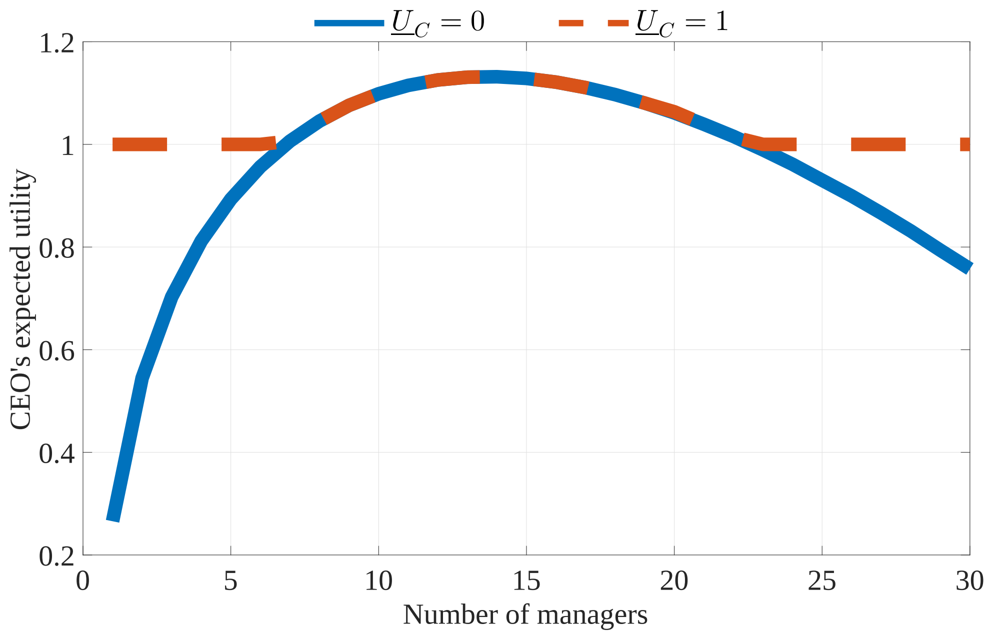

Figure 10: CEO's expected utility under a multiplication specfication for $\underline{U}_C = 0$ (solid line) or $\underline{U}_C = 1$ (dashed line) when $\underline{U}_M = 1$. Other parameters are $\mathcal{G}_C = 1.7$, $\mathcal{B}_C = 0.7$, $\mathcal{G}_M = 35$, $\mathcal{B}_M = 5$, $f(x) = x$, $\alpha = 0.1$, $\beta = 0.4$, $e_M = 0.22$, $u(x) = \log(x)$ and $g(a) = -\frac{1}{a-1} - a - 1$.

9 Comparing Two Promotion Rules

In this section, I explain when promotion rule 1 can be preferred to promotion rule 2 by the firm. Consider the situation where managers' abilities are private information to them. Recall that a manager's effort affects the probability of good outcome by the function $s(e) = \alpha + \beta e$. In the following model, managers' ability is determined by β. Specifically, assume that β can be one of two values, $\bar{\beta}$ and $\underline{\beta}$, with the probability $q \in (0, 1)$ and $1 - q$, respectively. Also, assume that the two values satisfy $\bar{\beta} > \underline{\beta} > 0$ and $\bar{\beta} \leq \frac{1}{2} - \alpha$. Call the agent with $\beta = \bar{\beta}$ by high type (H) and the agent with $\beta = \underline{\beta}$ by low type (L). For brevity, I denote the probability of good performance for a given e by $s_H(e)$ and $s_L(e)$ according to manager's type. If the firm hires the CEO from the external labor market, they also have the same prior probability about

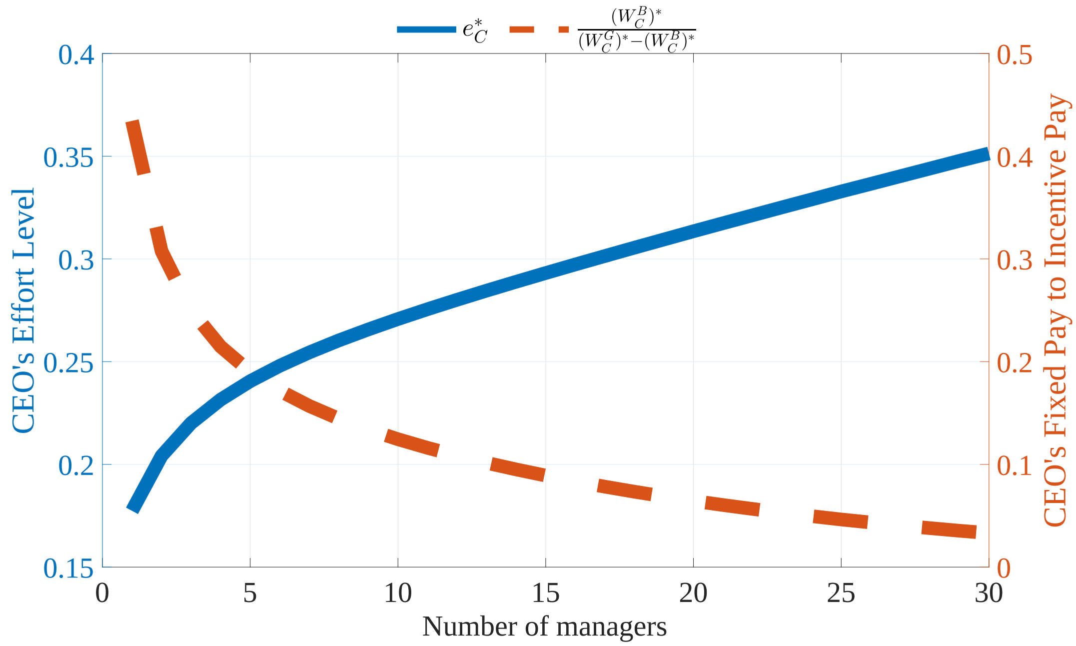

Figure 11: Under a multiplication specification, CEO effort level (solid line) and the ratio of an CEO's fixed pay relative to incentive pay (dashed line) for $\underline{U}_C = 0$ and $\underline{U}_M = 1$. Other parameters are the same as those in **Figure 10**.

types of agents. That is, the firm's prior probability that a CEO from the external labor market is high type is q. In order to focus on the effect of promotion rules, I additionally assume that N is equal to 2 in this section. Then, the firm's problem is the following:

$$\max_{(W_C),(e_L,e_H,W_M^G,W_M^B)} \gamma E[\beta|e_H,e_L] - W_C + E[\Pi_M|(e_H,e_L,W_M^G,W_M^B)]$$

subject to

$$u(W_C) \geq \underline{U}_C \ (IR_C)$$

$$s_H(e_H)[u(W_M^G) + P(\mathbf{e}_{-M})V_C] + (1 - s_H(e_H))[u(W_M^B) + R(\mathbf{e}_{-M})V_C] - g(e_H) \geq \underline{U}_M \ (IR_H)$$

$$s_L(e_L)[u(W_M^G) + P(\mathbf{e}_{-M})V_C] + (1 - s_L(e_L))[u(W_M^B) + R(\mathbf{e}_{-M})V_C] - g(e_L) \geq \underline{U}_M \ (IR_L)$$

$$e_H \in \arg\max_{\hat{e}} s_H(\hat{e})[u(W_M^G) + P(\mathbf{e}_{-M})V_C] + (1 - s_H(\hat{e}))[u(W_M^B) + R(\mathbf{e}_{-M})V_C] - g(\hat{e}) \ (IC_H)$$

$$e_L \in \arg\max_{\hat{e}} s_L(\hat{e})[u(W_M^G) + P(\mathbf{e}_{-M})V_C] + (1 - s_L(\hat{e}))[u(W_M^B) + R(\mathbf{e}_{-M})V_C] - g(e_L) \ (IC_L),$$

where $E[\Pi_M|(e_H, e_L, W_M^G, W_M^B)]$ represents the expected profit from two managers[46], and $E[\beta|e_H, e_L]$ is the expected β when the firm requires e_H and e_L from high-type and low-type managers, respectively. γ determines the firm's benefit of hiring high-type CEO. More formally, γ is the firm's marginal benefit from higher expected CEO's β. Also, the managers' promotion probability when their outcome is good ($P(\mathbf{e}_{-M})$) and bad ($R(\mathbf{e}_{-M})$) are

$$P(\mathbf{e}_{-M}) = \frac{2 - s_H(e_H)q - s_L(e_L)(1-q)}{2}, \text{ and}$$

$$R(\mathbf{e}_{-M}) = \begin{cases} 0 & \text{under promotion rule 1} \\ \frac{1 - s_H(e_H)q - s_L(e_L)(1-q)}{2} & \text{under promotion rule 2} \end{cases}.$$

Here, I only consider a pooling equilibrium so that the firm provides the same contract regardless of managers' types.[47] In this problem, moral hazard aspects related to the CEO are ignored. However, a hired CEO's ability, which determines her productivity, depends on the promotion rule and managers' compensation. Specifically, the choice of promotion rule determines the functional form of $E[\beta|e_H, e_L]$ while the compensation affects managers' effort choice.[48] Moreover, I simply assume that agents' cost function is a quadratic function, that is, $g(e) = \frac{\kappa e^2}{2}$ with $\kappa > 0$.

The following result illustrates that the firm will prefer the promotion rule 1 to the promotion rule 2 if the benefit from hiring a high type CEO is big enough.

Proposition 16. *There is $\hat{\gamma} > 0$ such that the profit under promotion rule 1 is strictly greater than that under promotion rule 2 if $\gamma \geq \hat{\gamma}$.*

That is, if the firm emphasize the role of internal promotion as a screening device, promotion rule 1 will be optimally chosen rather than promotion 2.

[46]The exact expression can be found in Appendix B.3.

[47]The argument in this section still holds even when one considers a separating equilibrium if the firm determines its promotion decision based on managers' outputs not revealed types.

[48]The functional form can be found in Appendix B.3.

10 Discussion

In this section, I connect the previous results with two lines of literature: 1) CEO compensation, and 2) Empirical studies in tournament literature.

10.1 Implications for the trend in executive compensation

CEO compensation and the wage gap between CEO and other managers have received extensive attention from both researchers and media. Since the influential work by Gabaix and Landier (2008),[49] researchers have focused on the size of the firm in order to explain the rise of CEO compensation. However, the relationship is not monotonic from a long-term perspective as Frydman and Saks (2010) and Frydman and Jenter (2010) illustrate. Here, I connect my model with the non-monotonic relationship between the two factors.

First, I illustrate that the promotion incentive channel predicts that CEO compensation increases non-monotonically as the size of the firm, measured by the number of internal candidates for the CEO position, grows. **Corollary 4** and **Figure 1** demonstrate this relationship. In **Figure 1**, when the CEO' reservation value is equal to the managers', the CEO's expected utility and compensation remains a constant up to the point that N is equal to 3 although the firm size increases since the CEO's individual rationality constraint binds. However, beyond this point, the CEO's expected utility and the size of the firm show a monotonic relationship. This rise of CEO's expected utility leads to the increase of CEO's compensation according to **Corollary 4** if her effort level remains a constant. Moreover, **Figure 5** shows that this can also be the case when the firm optimally choose the CEO's effort level.

Also, my model suggests that the rise of promotion incentives might play a key role in the growth of the wage gap between CEO and managers during the past 30 years but not before 1980. Therefore, similar with the previous argument, the relationship between the wage gap and the firm size is not monotonic since the CEO's compensation does not change if her individual rationality constraint binds. However, if the number of managers is high enough

[49] See also Tervio (2008).

such that the CEO's individual rationality constraint does not bind, the wage gap might start to widen as **Figure 12** illustrates.[50]

It is worth mentioning that the rise of promotion incentives can also affect the CEO's pay for performance incentives. Basically, the CEO's incentive payment, such as stock options and restricted stock, has a positive correlation with the required effort level. However, if researchers take into account promotion incentives, this relationship can be reversed. **Figures 4** and **5** show this possibility. In this example, the CEO's incentive payment expressed by $(W_C^G)^* - (W_C^B)^*$ increases although the firm requires less effort from her as the size of the firm grows.

10.2 Implications for empirical research on tournament theory

Since the seminal work of Lazear and Rosen (1981), an extensive empirical literature studies the implications of tournament theory. Executive compensation is one of the most studied applications in order to test the implications. In these applications, researchers have used the wage gap between CEO and the next level executives as the measure of promotion incentives based on Lazear and Rosen (1981).[51] However, to the best of my knowledge, the interaction between CEO incentives, which depend on the role of CEO in a firm, and promotion incentives has not been considered. The interaction, however, should be useful in understanding the cause of the wage gap in a firm as well as CEO compensation. For example, the Securities and Exchange Commission announced that it adopted a rule that required a public company to disclose the ratio of its CEO to the median compensation of its employees. Hence, if researchers misinterpret the meaning of the ratio, it could lead to a distortion in the wage structure or incentives.

In this section, I compare **Proposition 10** and **15**, and derive an important implication regarding a measure of promotion incentives. The important message is that it can be misleading to use the wage gap between CEO and managers as the measure of promotion incentives. The

[50] When $\underline{U}_C = \underline{U}_M = 3$, the wage gap reduces up to N is equal to 3, which is the smallest N that the CEO's individual rationality constraint does not bind, in this example.

[51] Lazear and Oyer (2012) and Waldman (2012) provide reviews on the literature.

reason for this is that two factors, high-powered incentives for CEO and strong promotion incentives, both should result in a high CEO compensation and a large wage gap when other conditions remain the same. Moreover, when the CEO is risk-averse, the two factors are negatively correlated according to **Proposition 6**. Therefore, researchers need to disentangle these effects in order to measure promotion incentives properly.

One possible way of identifying promotion incentives is to see CEO's base salary (W_C^B in my model). To increase CEO's effort level, the base salary should decrease because of moral hazard problem. Thus, the fall of base salary is related to the demand for higher effort from CEO. On the other hand, when a firm wants to increase promotion incentives for the lower-rung executives or employees, the CEO's base salary should increase. In particular, **Proposition 10** predicts that enhancing promotion incentives leads to the increase in CEO's base salary.

Unfortunately, this identification strategy based on base salary may not work when a firm is small. Under the multiplication specification, for instance, a CEO's effort level and promotional incentives can move in such a way that they affect the base salary in the opposite directions. Despite this limited applicability, I expect that the CEO's base salary can still be useful for identifying promotion incentives for two reasons. First, this undesired situation happens when the size of the firm is small, which makes the problem less of a concern. For small firms, the CEO's individual rationality constraint binds or promotion incentives may dominate the effect of high-powered incentives for CEO. In the former case, the growth in the firm size does not yield the increase in promotion incentives and base salary. Therefore, the base salary can weakly identify promotion incentives. In the latter case, as the firm size grows, I expect that the promotion incentive will be the major driving force of the base salary movement. Second and more importantly, the dataset that economists mostly use consists of large firms. For example, *ExecuComp* dataset only contains public firms. For firms large enough, I expect that promotion incentives move in the opposite direction of CEO effort level based on **Proposition 15**.

As an example of this prediction, **Figure 12** and **13** illustrate two numerical examples based on given parameters. As these two figures show, the expected wage gap measured by:

$$E[W_C] - E[W_M] = [s(e_C)W_C^G + (1 - s(e_C))W_C^B] - [s(e_M)W_M^G + (1 - s(e_M))W_M^B]$$

grows as the number of managers increases regardless of the direction of promotion incentives. On the other hand, CEO's base salary, overall, captures the change of promotion incentives. This is especially true when the number of managers is large enough as the theory predicts. The future research is to study this relationship under a more complex operational structure. However, unless promotion incentives and the CEO's required effort level move in opposite directions, researchers can exploit the CEO's base salary in order to identify promotion incentives. Moreover, this property is generally applicable to other problems if the organizer of a contest needs to provide incentives to the winner of the contest.

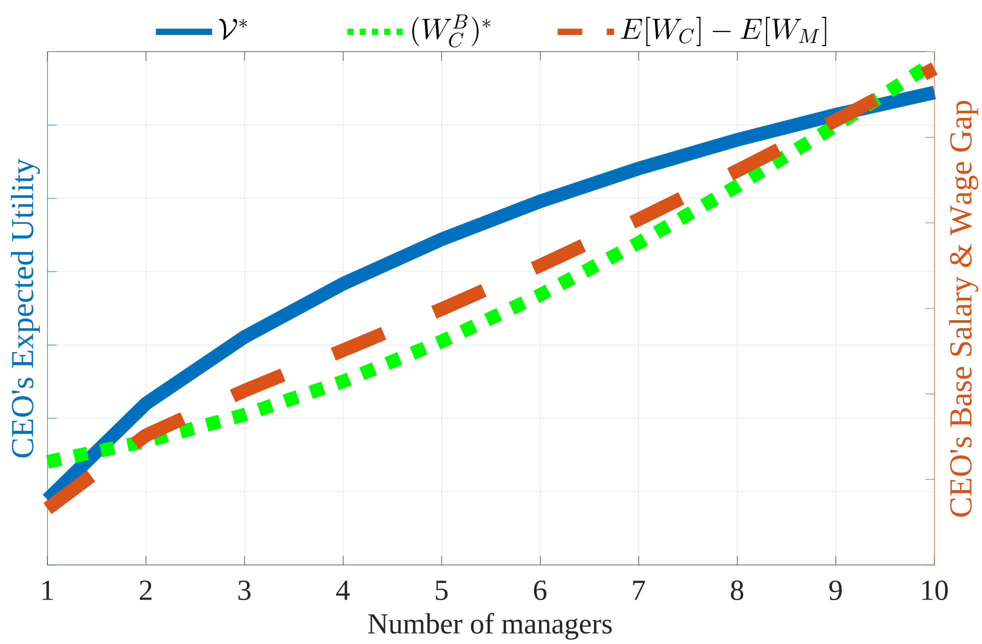

Figure 12: CEO's expected utility (solid line), CEO's base salary (dotted line), and Wage gap between CEO and manager (dashed line) under the independent specification for $\underline{U}_C = 0$. Other parameters are the same as those in **Figure 4**.

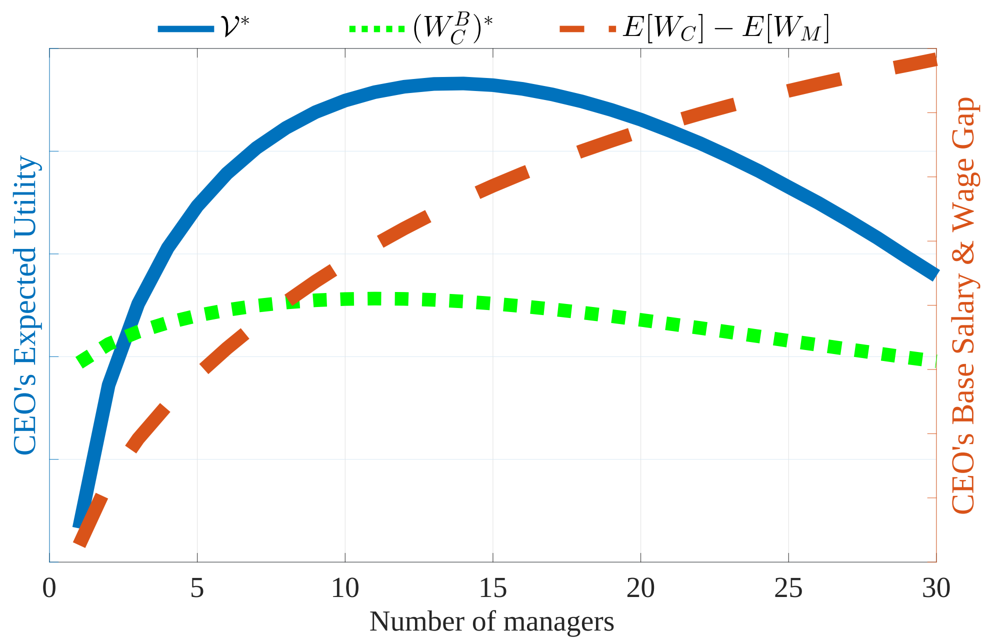

Figure 13: CEO's expected utility (solid line), CEO's base salary (dotted line), and Wage gap between CEO and manager (dashed line) under a multiplication specification for $\underline{U}_C = 0$. Other parameters are the same as those in **Figure 10**.

11 Conclusion

I examine the effect of a managers' possibility of promotion on executive compensation. Using a theoretical model which incorporates both relative and absolute performance based compensation, I find that it is optimal for a firm to provide a compensation to its CEO higher than her reservation value in order to incentivize lower-rung executives. In particular, this is true if agents are risk-averse, the CEO's marginal productivity is independent of firm size, and there are enough number of competitors for promotion. Therefore, the promotion possibilities yield the growth of CEO compensation as well as create a wage gap between the CEO and other executives. Moreover, promotion incentives reduce the dependence of managers' compensation on their performance. As a justification for the hybrid incentive scheme, I examine the effect of it on a firm's profit per agent. The result illustrates that firms can increase their profit per agent by introducing a promotion structure based on competition among managers. In addition, the relation between the CEO's task and the managers' outputs significantly affects the CEO's compensation. In particular, if there is a complementarity between them, the contribution of promotion incentives to CEO compensation can vanish. These results give valuable implications for CEO compensation. Researchers need to take into account the promotion possibility besides performance based compensation when they analyze CEO compensation. Also, interpreting the role of the CEO in a firm is of importance when one tries to answer why CEO compensation has risen.

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Appendix to "High CEO Compensation: Incentives for CEO or Managers?"

Myungkoo Song

June 14, 2017

This online appedix contains all proof of the paper, "High CEO Compensation: Incentives for CEO or Managers?" and details of the firm's problem in Section 7.1, 7.2 and 9.

Appendix A Proofs

A.1 Derivation of $P(\mathbf{e}_{-Mi})$

Note that

$$\begin{aligned} P(\mathbf{e}_{-Mi}) &= \sum_{j=0}^{N-1} \frac{1}{j+1} \binom{N-1}{j} s(e_{-Mi})^j (1-s(e_{-Mi}))^{N-1-j} \\ &= \frac{1}{Ns(e_{-Mi})} \sum_{j=0}^{N-1} \binom{N}{j+1} s(e_{-Mi})^{j+1} (1-s(e_{-Mi}))^{N-1-j} \\ &= \frac{1}{Ns(e_{-Mi})} \left(\sum_{s=0}^{N} \binom{N}{s} s(e_{-Mi})^s (1-s(e_{-Mi}))^{N-s} - (1-s(e_{-Mi}))^N \right) \\ &= \frac{1-(1-s(e_{-Mi}))^N}{Ns(e_{-Mi})}. \end{aligned}$$

A.2 Proof of Lemma 1

Denote the firm's profit when the CEO's IR condition binds as $\underline{\Pi}$. Then, the firm's optimal profit must be greater than or equal to $\underline{\Pi}$.

First, for an action $(e_C^*, e_M^*) \in [0,1]\times[0,1]$, I show that there exists a solution $(W_C^G, W_C^B, W_M^G, W_M^B)$ to the firm's problem. Note that when the CEO's IR condition binds the firm's problem is reduced to the case of Grossman and Hart (1983). Hence, a solution exists to this restricted problem. Now, I show that I can artificially bound the constraint set of $(W_C^G, W_C^B, W_M^G, W_M^B)$. They are bounded below by two IR conditions. Moreover, they are also bounded above since the firm's optimal profit is lower than $\underline{\Pi}$ and the firm's profit is a strictly decreasing function in all four components in (W_C^G, W_C^B, W_g, W_b) without a lower bound. Also, the constraint set is closed according to two IC and two IR conditions. Hence, there exists a solution by the Extreme value theorem. The remaining proof exactly follows the proof in Grossman and Hart (1983).

A.3 Proof of Lemma 2

Suppose this is not true. That is,

$$s(e_M)u(W_M^G) + (1 - s(e_M))u(W_M^B) - g(e_M) + s(e_M)P(\mathbf{e}_{-M})V_C > \underline{U}_M.$$

Then, choosing new wage scheme $(\widehat{W}_M^G, \widehat{W}_M^B) = (W_M^G - \epsilon_1, W_M^B - \epsilon_2)$, where $\epsilon_1 > 0$ and $\epsilon_2 > 0$, satisfying

$$u(W_M^G) - u(W_M^B) = u(\widehat{W}_M^G) - u(\widehat{W}_M^B) \text{ and}$$
$$s(e_M)u(\widehat{W}_M^G) + (1 - s(e_M))u(\widehat{W}_M^B) - g(e_M) + P(\mathbf{e}_{-M})V_C \geq \underline{U}_M$$

gives a higher profit to the firm without affecting other constraints. Hence, the wage scheme (W_M^G, W_M^B) is not optimal.

A.4 Proof of Lemma 3

Note that

$$\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}} = -\frac{1}{u'(W_C)} + \frac{1}{u'(W_M)}$$

using the envelope theorem.

Differentiating this with respect to $\mathcal{V}$ gives

$$\begin{aligned}
\frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V}^2} &= \frac{u''(W_C)}{u'(W_C)^3} + \frac{1}{N}\frac{u''(W_M)}{u'(W_M)^3} + \frac{u''(W_C)}{u'(W_C)^3} g'(e_C^*)\frac{\partial e_C^*}{\partial \mathcal{V}} - \frac{u''(W_M)}{u'(W_M)^3} g'(e_M^*)\frac{\partial e_M^*}{\partial \mathcal{V}} \\
&= \frac{u''(W_C)}{u'(W_C)^3} - \frac{u''(W_C)}{u'(W_C)^3} g'(e_C^*) \frac{\frac{u''(W_C)}{u'(W_C)^2} g'(e_C^*)}{\frac{u''(W_C)}{u'(W_C)^2} g'(e_C^*)^2 - g''(e_C^*)} \\
&\quad + \frac{1}{N}\frac{u''(W_M)}{u'(W_M)^3} - \frac{u''(W_M)}{u'(W_M)^3} g'(e_M^*) \frac{\frac{1}{N}\frac{u''(W_M)}{u'(W_M)^2} g'(e_M^*)}{\frac{u''(W_M)}{u'(W_M)^2} g'(e_M^*)^2 - g''(e_M^*)} \\
&= \frac{u''(W_C)}{u'(W_C)^3}\left(1 - \frac{\frac{u''(W_C)}{u'(W_C)^2} g'(e_C^*)^2}{\frac{u''(W_C)}{u'(W_C)^2} g'(e_C^*)^2 - g''(e_C^*)}\right) \\
&\quad + \frac{1}{N}\frac{u''(W_M)}{u'(W_M)^3}\left(1 - \frac{\frac{u''(W_M)}{u'(W_M)^2} g'(e_M^*)^2}{\frac{u''(W_M)}{u'(W_M)^2} g'(e_M^*)^2 - g''(e_M^*)}\right) \\
&< 0.
\end{aligned}$$

That is, $F(\mathcal{V})$ is a strictly concave function.

A.5 Proof of Proposition 1

It is enough to show that

$$\frac{\partial^2 F(\mathcal{V})}{\partial N \partial \mathcal{V}} > 0.$$

Notice that

$$\begin{aligned}
\frac{\partial^2 F(\mathcal{V})}{\partial N \partial \mathcal{V}} &= -\frac{1}{N^2}\frac{u''(W_M)}{u'(W_M)^3}\mathcal{V} - \frac{u''(W_M)}{u'(W_M)^3} g'(e_M^*)\left(\frac{-\beta(\mathcal{G}_M - \mathcal{B}_M)\frac{1}{N^2}\frac{u''(W_M)}{u'(W_M)}}{\beta^2(\mathcal{G}_M - \mathcal{B}_M)^2 u''(W_M) - g''(e_M^*)}\right)\mathcal{V} \\
&= -\frac{1}{N^2}\frac{u''(W_M)}{u'(W_M)^3}\left(1 - \frac{\beta(\mathcal{G}_M - \mathcal{B}_M)\frac{u''(W_M)}{u'(W_M)} g'(e_M^*)}{\beta^2(\mathcal{G}_M - \mathcal{B}_M)^2 u''(W_M) - g''(e_M^*)}\right)\mathcal{V} \\
&= -\frac{1}{N^2}\frac{u''(W_M)}{u'(W_M)^3}\left(1 - \frac{\beta^2(\mathcal{G}_M - \mathcal{B}_M)^2 u''(W_M)}{\beta^2(\mathcal{G}_M - \mathcal{B}_M)^2 u''(W_M) - g''(e_M^*)}\right)\mathcal{V} \\
&> 0.
\end{aligned}$$

A.6 Proof of Corollary 2

Suppose this is not the case. From the first order condition

$$\beta(\mathcal{G}_C - \mathcal{B}_c) = \frac{g'(e_C)}{u'(W_C)},$$

it can be shown that e_C^* and W_C^* move in the opposite direction since the left hand side is a constant. Therefore, e_C^* should increase if W_C^* decreases. Since $\mathcal{V}^*$ increases as N increases, W_C^* must increase according to

$$u(W_C^*) = \mathcal{V}^* + g(e_C^*).$$

This contradicts the premise that W_C^* decreases. Hence,

$$\frac{\partial W_C^*}{\partial N} > 0.$$

A.7 Proof of Proposition 2

Recall that the first order condition is

$$-\frac{1}{u'(W_C^*)} + \frac{1}{u'(W_M^*)} = 0.$$

Hence, W_C^* should be the same as W_M^*.

If $\mathcal{G}_C - \mathcal{B}_C = \mathcal{G}_M - \mathcal{B}_M$, using the previous result and two first order conditions, it can be shown that

$$g'(e_C^*) = g'(e_M^*).$$

That is, $e_C^* = e_M^*$.

Also, this result and the two individual rationality constraints imply that

$$u(W_M) - g(e_M) + \frac{1}{N}\mathcal{V}^* = \mathcal{V}^* + \frac{1}{N}\mathcal{V}^* = \underline{U}_M.$$

Hence,

$$\mathcal{V}^* = \frac{N}{N+1}\underline{U}_M.$$

A.8 Proof of Proposition 3

Note that when agents are risk-neutral

$$\begin{aligned}\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}} &= -s(e_C) - (1 - s(e_C)) + N * s(e_M)P(\mathbf{e}_{-M}) \\ &= -1 + (1 - (1 - s(e_M))^N) \\ &< 0\end{aligned}$$

using the envelope theorem.[52] Hence, the firm's profit decreases as the level of $\mathcal{V}$ increases.

A.9 Proof of Lemma 4

Using the envelope theorem,

$$\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}} = -\frac{s(e_C)}{u'(W_C^G)} - \frac{1 - s(e_C)}{u'(W_C^B)} + Ns(e_M)P(\mathbf{e}_{-M})\frac{1}{u'(W_M^G)}.$$

Differentiating this with respect to $\mathcal{V}$ gives

$$\frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V}^2} = s(e_C)\frac{u''(W_C^G)}{u'(W_C^G)^3} + (1 - s(e_C))\frac{u''(W_C^B)}{u'(W_C^B)^3} + Ns(e_M)P(\mathbf{e}_{-M})^2\frac{u''(W_M^G)}{u'(W_M^G)^3} < 0.$$

A.10 Proof of Proposition 4

It is enough to show that

$$\frac{\partial^2 F(\mathcal{V})}{\partial N \partial \mathcal{V}} > 0.$$

Note that

$$\begin{aligned}\frac{\partial^2 F(\mathcal{V})}{\partial N \partial \mathcal{V}} &= -(1 - s(e_M))^N \log(1 - s(e_M))\frac{1}{u'(W_M^G)} \\ &\quad + [1 - (1 - s(e_M))^N]\frac{\partial P(\mathbf{e}_{-M})}{\partial N}\mathcal{V}\frac{u''(W_M^G)}{u'(W_M^G)^3} > 0,\end{aligned}$$

where

$$\begin{aligned}\frac{\partial P(\mathbf{e}_{-M})}{\partial N} &= \frac{1}{s(e_M)N^2}\left[-\log(1 - s(e_M))(1 - s(e_M))^N N - 1 + (1 - s(e_M))^N\right] \\ &< 0\end{aligned}$$

since $k(s) \equiv -\log(1-s)(1-s)^N N - 1 + (1-s)^N$ is equal to zero when $s = 0$ and $k'(s) < 0$. Here, I use the condition that $\mathcal{V} \geq 0$.

[52] In equilibrium, $s(e_M)$ is equal to $s(e_{-M})$ since I am considering a symmetric equilibrium.

A.11 Proof of Corollary 4

Denote the expected compensation to CEO by $E[W_C]$,

$$E[W_C] = s(e_C)W_C^G + (1 - s(e_C))W_C^B.$$

Since $\frac{\partial \mathcal{V}^*}{\partial N} > 0$, it is enough to show that

$$\frac{\partial E[W_C]}{\partial \mathcal{V}} = \frac{s(e_C)}{u'(W_C^G)} + \frac{1 - s(e_C)}{u'(W_C^B)} > 0.$$

A.12 Proof of Corollary 5

I need to show that the wage gap

$$\left[s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^B)^*\right] - \left[s(e_M)(W_M^G)^* + (1 - s(e_M))(W_M^B)^*\right]$$

widens as N increases. Since $(1 - s(e_M))(W_M^B)^*$ has a fixed value regardless of the number of managers, it is enough to show that

$$\left[s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^B)^*\right] - s(e_M)(W_M^G)^*$$

is an increasing function in N. When agents have the log utility function, the first order condition with respect to $\mathcal{V}$ is

$$s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^G)^* = (1 - (1 - s(e_M))^N)(W_M^G)^*.$$

Since the left hand side of the equation is a strictly increasing function in $\mathcal{V}$ and $\frac{\partial \mathcal{V}^*}{\partial N} > 0$, this side strictly increases as N increases. Hence,

$$\begin{aligned}\frac{\partial}{\partial N}(1 - (1 - s(e_M))^N)(W_M^G)^* = &- \log(1 - s(e_M))(1 - s(e_M))^N(W_M^G)^* \\ &+ (1 - (1 - s(e_M))^N)\frac{\partial (W_M^G)^*}{\partial N} > 0.\end{aligned}$$

Since

$$\left[s(e_C)(W_C^G)^* + (1 - s(e_C))(W_C^B)^*\right] - s(e_M)(W_M^G)^* = (1 - s(e_M) - (1 - s(e_M))^N)(W_M^G)^* \quad (10)$$

$$
\begin{aligned}
&\frac{\partial}{\partial N}\left\{\left[s(e_C)(W_C^G)^* + (1-s(e_C))(W_C^B)^*\right] - s(e_M)(W_M^G)^*\right\} = \\
&\quad -\log(1-s(e_M))(1-s(e_M))^N (W_M^G)^* + (1-s(e_M) - (1-s(e_M))^N)\frac{\partial (W_M^G)^*}{\partial N} \\
&> -\log(1-s(e_M))(1-s(e_M))^N (W_M^G)^* + (1-s(e_M) - (1-s(e_M))^N)\cdot \\
&\quad \frac{\log(1-s(e_M))(1-s(e_M))^N (W_M^G)^*}{1-(1-s(e_M))^N} \\
&= -\frac{s(e_M)}{1-(1-s(e_M))^N}\log(1-s(e_M))(1-s(e_M))^N (W_M^G)^* \\
&> 0
\end{aligned}
$$

when $N > 1$. Also, when $N = 1$, the wage gap is equal to zero according to 10. On the other hand, the gap has a positive value when $N = 2$ since $(W_M^G)^* > 0$. Therefore, the expected compensation gap is a strictly increasing function in N.

A.13 Proof of Proposition 5

First, I consider a case when $e_C = e_M$.

Note that

$$
F(\underline{U}_M|N) = -\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + \frac{1-(1-s(e_M))^N}{u'(W_M^G)},
$$

where

$$
\begin{aligned}
u(W_C^G) &= \underline{U}_M + g(e_C) + (1-s(e_C))\frac{g'(e_C)}{h'(e_C)}, \\
u(W_C^B) &= \underline{U}_M + g(e_C) - s(e_C)\frac{g'(e_C)}{h'(e_C)}, \text{ and} \\
u(W_M^G) &= \underline{U}_M + g(e_M) + (1-s(e_M))\frac{g'(e_M)}{h'(e_M)} - P(\mathbf{e}_{-M})\underline{U}_M.
\end{aligned}
$$

Denote the difference between $1/u'(W_C^G)$ and $1/u'(W_C^B)$ by $\mathcal{D}$.

For given $(1-s(e_C))\mathcal{D} > \epsilon > 0$, there is $\widehat{N}$ such that

$$
\frac{1}{u'(W_C^G)} - \frac{1-(1-s(e_M))^N}{u'(W_G)} < \epsilon
$$

when $N \geq \widehat{N}$ since $P(\mathbf{e}_{-M}) \to 0$ and $(1 - s(e_M))^N \to 0$ as $N \to \infty$. Therefore, when $N \geq \widehat{N}$,

$$\begin{aligned} F(\underline{U}_M|N) &= -\frac{s(e_C)}{u'(W_C^G)} - \frac{1 - s(e_C)}{u'(W_C^B)} + \frac{1 - (1 - s(e_M))^N}{u'(W_g)} \\ &> -\frac{s(e_C)}{u'(W_C^G)} - \frac{1 - s(e_C)}{u'(W_C^B)} + \frac{1}{u'(W_C^G)} - \epsilon \\ &= (1 - s(e_C)) \left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)} \right) - \epsilon \\ &> 0. \end{aligned}$$

Since $F(\mathcal{V}|N)$ is a strictly concave in $\mathcal{V}$, $\mathcal{V}^* > \underline{U}_M$ when $N \geq \widehat{N}$.

Second, I show that there is N^* such that $\mathcal{V}^* > \underline{U}_M$ if $N > N^*$ when $0 < e_C < e_M < 1$.

There are two possibilities;

$$\frac{s(e_M)}{u'(W_M^G)} \geq \frac{1}{u'(W_C^G)} \text{ or } \frac{s(e_M)}{u'(W_M^G)} < \frac{1}{u'(W_C^G)}$$

when $\mathcal{V} = \underline{U}_M$ and $N = 1$.

1. $\left(\frac{s(e_M)}{u'(W_M^G)} \geq \frac{1}{u'(W_C^G)} \right)$

   This condition implies that

$$\begin{aligned} F(\underline{U}_M|1) &= -\frac{s(e_C)}{u'(W_C^G)} - \frac{1 - s(e_C)}{u'(W_C^B)} + \frac{s(e_M)}{u'(W_M^G)} \\ &\geq (1 - s(e_C)) \left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)} \right) \\ &> 0. \end{aligned}$$

   Since $\frac{\partial \mathcal{V}^*}{\partial N} > 0$, $\mathcal{V}^* > \underline{U}_M$ for every N.

2. $\left(\frac{s(e_M)}{u'(W_M^G)} < \frac{1}{u'(W_C^G)} \right)$

   Again, denote the difference between $1/u'(W_C^G)$ and $1/u'(W_C^B)$ by $\mathcal{D}$. Then, for given $(1 - s(e_C))\mathcal{D} > \epsilon$, there exists $\widehat{N}$ such that

$$0 \leq \frac{1}{u'(W_C^G)} - \frac{1 - (1 - s(e_M))^N}{u'(W_M^G)} < \epsilon.$$

Therefore, when $N \geq \widehat{N}$,

$$\begin{aligned} F(\underline{U}_M|N) &= -\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + \frac{1-(1-s(e_M))^N}{u'(W_M^G)} \\ &> -\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + \frac{1}{u'(W_C^G)} - \epsilon \\ &= (1-s(e_C))\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right) - \epsilon \\ &> 0. \end{aligned}$$

Since $F(\mathcal{V}|N)$ is a strictly concave in $\mathcal{V}$, $\mathcal{V}^* > \underline{U}_M$ when $N \geq \widehat{N}$.

A.14 Proof of Proposition 6

Under the given assumption, it can be shown that

$$\begin{aligned} \frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V} \partial e_C} &= -\beta\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right) + s(e_C)(1-s(e_C))\frac{g''(e_C)}{\beta}\left(\frac{u''(W_C^G)}{u'(W_C^G)^3} - \frac{u''(W_C^B)}{u'(W_C^B)^3}\right) < 0, \\ \frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V} \partial e_M} &= N\beta\frac{(1-s(e_M))^{N-1}}{u'(W_g)} \\ &\quad - [1-(1-s(e_M))^N]\frac{u''(W_M^G)}{u'(W_M^G)^3}\left((1-s(e_M))\frac{g''(e_M)}{\beta} - \frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\mathcal{V}\right) > 0. \end{aligned}$$

A.15 Proof of Proposition 7

First, I show that $\mathcal{V}^*$ increases as $\underline{U}_M$ increases. From the first order condition with respect to $\mathcal{V}^*$:

$$-\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + \frac{1-(1-s(e_M))^N}{u'(W_M^G)} = 0,$$

$$\frac{\partial \mathcal{V}^*}{\partial \underline{U}_M} = \frac{[1-(1-s(e_M))^N]\frac{u''(W_M^G)}{u'(W_M^G)^3}}{s(e_C)\frac{u''(W_C^G)}{u'(W_C^G)^3} + (1-s(e_C))\frac{u''(W_C^B)}{u'(W_C^B)^3} + Ns(e_M)P(\mathbf{e}_{-M})^2\frac{u''(W_M^G)}{u'(W_M^G)^3}} > 0.$$

Note that for a given (N, e_M), $(W_M^G)^* = (W_M^B)^*$ if

$$\mathcal{V}^* = \frac{g'(e_M)}{\beta P(\mathbf{e}_{-M})}$$

since $u(W_M^G) - u(W_M^B) = \frac{g'(e_M)}{\beta} - P(\mathbf{e}_{-M})\mathcal{V}$. For given (N, e_C, e_M), denote $\mathcal{V}$ satisfying $(W_M^G)^* = (W_M^B)^*$ by $\widehat{\mathcal{V}}$. That is,

$$\widehat{\mathcal{V}} = \frac{g'(e_M)}{\beta P(\mathbf{e}_{-M})} > 0.$$

Then,

$$\left.\frac{\partial F(\mathcal{V}|\underline{U}_M)}{\partial \mathcal{V}}\right|_{\mathcal{V}=\widehat{\mathcal{V}}} = -\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + \frac{1-(1-s(e_M))^N}{u'(W_M^G)}$$

There are two possible cases, $\left.\frac{\partial F(\mathcal{V}|\underline{U}_M)}{\partial \mathcal{V}}\right|_{\mathcal{V}=\widehat{\mathcal{V}}} \geq 0$ and $\left.\frac{\partial F(\mathcal{V}|\underline{U}_M)}{\partial \mathcal{V}}\right|_{\mathcal{V}=\widehat{\mathcal{V}}} < 0$. If $\left.\frac{\partial F(\mathcal{V}|\underline{U}_M)}{\partial \mathcal{V}}\right|_{\mathcal{V}=\widehat{\mathcal{V}}} \geq 0$, then $\mathcal{V}^* > \widehat{\mathcal{V}}$. This implies that $(W_M^G)^* \leq (W_M^B)^*$. Suppose that $\left.\frac{\partial F(\mathcal{V}|\underline{U}_M)}{\partial \mathcal{V}}\right|_{\mathcal{V}=\widehat{\mathcal{V}}} < 0$. Since $\mathcal{V}$ is fixed at $\widehat{\mathcal{V}}$,

$$\lim_{\underline{U}_M \to \infty} \frac{1-(1-s(e_M))^N}{u'(W_M^G)} = \infty.$$

Hence, there is $0 < \underline{U}_M^* < \infty$ such that

$$\left.\frac{\partial F(\mathcal{V}|\underline{U}_M)}{\partial \mathcal{V}}\right|_{\mathcal{V}=\widehat{\mathcal{V}}} = 0.$$

Since $\frac{\partial \mathcal{V}^*}{\partial \underline{U}_M} > 0$, $(W_M^G)^* \leq (W_M^B)^*$ if $\underline{U}_M \geq \underline{U}_M^*$.

A.16 Proof of Corollary 6

It is enough to show that there is $\underline{U}_M$ such that $W_C^B \geq W_M^G$ when $\mathcal{V} = 0$ since this implies that

$$\left.\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}}\right|_{\mathcal{V}=0} < 0.$$

Note that

$$u(W_C^B) - u(W_M^G) = g(e_C) - s(e_C)\frac{g'(e_C)}{\beta} - \underline{U}_M - g(e_M) - (1-s(e_M))\frac{g'(e_M)}{\beta}$$

when $\mathcal{V} = 0$. Hence, if

$$\underline{U}_M \leq g(e_C) - s(e_C)\frac{g'(e_C)}{\beta} - g(e_M) - (1-s(e_M))\frac{g'(e_M)}{\beta},$$

$W_C^B \geq W_M^G$.

Since $\frac{\partial \mathcal{V}^*}{\partial \underline{U}_M} > 0$ and there is $\widetilde{\underline{U}}_M$ such that $\mathcal{V}^* > 0$ according to **Proposition 7**, there exists $\widetilde{\underline{U}}_M$ such that $\mathcal{V}^* = 0$ with $\left.\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}}\right|_{\mathcal{V}=\mathcal{V}^*} = 0$. Also, if $\underline{U}_M$ is less than $\widetilde{\underline{U}}_M$, the solution is $\mathcal{V}^* = 0$ with $\left.\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}}\right|_{\mathcal{V}=\mathcal{V}^*} < 0$.

A.17 Proof of Proposition 8

Before I prove the proposition, I show that $\mathcal{V}^*$ is bounded for every N. The first order condition with respect to $\mathcal{V}$ implies that $(W_M^G)^* > (W_C^B)^*$ for every N. Therefore,

$$\underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta} - P(\mathbf{e}_{-M})\mathcal{V}^* \geq \mathcal{V}^* + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta}.$$

Since $0 \leq P(\mathbf{e}_{-M}) \leq 1$,

$$\underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta} - g(e_C) + s(e_C)\frac{g'(e_C)}{\beta} > \mathcal{V}^*,$$

where the left hand side does not depend on N.

First, I denote the optimal compensations by $(W_C^G(N), W_C^B(N), W_M^G(N), W_M^B)$ for a given N.[53]

Then, the difference between the two profit per agent is

$$\begin{aligned}2(N+1)(\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)) &= (N-1)(s(e_M) - s(e_C))(\mathcal{G} - \mathcal{B}) \\ &\quad - 2W_C(N) + (N+1)W_C(1) - 2NW_M(N) + (N+1)W_M(1),\end{aligned}$$

where

$$\begin{aligned}W_C(N) &= s(e_C)W_C^G(N) + (1 - s(e_C))W_C^B(N) \\ W_M(N) &= s(e_M)W_M^G(N) + (1 - s(e_M))W_M^B(N).\end{aligned}$$

Since $\mathcal{V}^*$ is bounded, optimal compensations $(W_C(N), W_M(N), W_C(1), W_M(1))$ are also bounded. Also, they are not depend on $\mathcal{G}$ and $\mathcal{B}$. Hence, there is $\mathcal{G}^* - \mathcal{B}^*$ such that the difference has a positive value for a given N since $s(e_M) > s(e_C)$.

For the second part, I impose a restriction on e_M. Namely, for a given e_C. e_M satisfies the condition that

$$\mathcal{V}^*(e_C, e_M|N = 1) + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta} \geq 0,$$

where $\mathcal{V}^*(e_C, e_M|N = 1)$ is the optimal $\mathcal{V}$ when $N = 1$ for a given (e_C, e_M). Since $\mathcal{V}^*(e_C, e_M|N = 1) + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta} > 0$ if $e_M = e_C$, there is $\bar{e}_M$ such that $\mathcal{V}^*(e_C, e_M|N = 1) + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta} \geq 0$ if $e_M \in (e_C, \bar{e}_M]$.

[53] Note that W_M^B does not depend on the number of candidates.

Note that

$$\begin{aligned}
\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) &= \frac{N-1}{2(N+1)}(s(e_M) - s(e_C))(\mathcal{G} - \mathcal{B}) \\
&\quad - \frac{1}{2}s(e_M)(W_M^G(N) - W_M^G(1)) \\
&\quad + \frac{1}{2(N+1)}\left[-(N-1)W_M(N) - 2W_C(N) + (N+1)W_C(1)\right] \\
&< \frac{N-1}{2(N+1)}(s(e_M) - s(e_C))(\mathcal{G} - \mathcal{B}) \\
&\quad - \frac{1}{2}s(e_M)(W_M^G(N) - W_M^G(1)) \\
&\quad + \frac{1}{2(N+1)}\left[-(N-1)W_M(N) - 2W_C(N) + (N-1)W_M(1) + 2W_C(N)\right] \\
&= \frac{N-1}{2(N+1)}(s(e_M) - s(e_C))(\mathcal{G} - \mathcal{B}) \\
&\quad - \frac{N}{N+1}s(e_M)(W_M^G(N) - W_M^G(1)).
\end{aligned}$$

The inequality holds since $W_C(N) > W_C(1)$ and $W_M(1) > W_C(1)$. Note that $W_C(N) > W_C(1)$ is true because $\frac{\partial \mathcal{V}^*}{\partial N} > 0$. On the other hand, the condition imposed on e_M guarantees that $W_M(1) > W_C(1)$.[54] Here, I show that $W_M(1) > W_C(1)$ if $e_M \in (e_C, \bar{e}_M]$. The first order condition with respect to $\mathcal{V}$ when $N = 1$ is

$$\frac{s(e_C)}{u'(W_C^G(1))} + \frac{1 - s(e_C)}{u'(W_C^B(1))} = \frac{s(e_M)}{u'(W_M^G(1))},$$

which implies that $u(W_M^G(1)) > u(W_C^G(1))$. Therefore,

$$\underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta} > 2\mathcal{V}^* + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{\beta}.$$

This inequality and the condition on e_M imply that

$$\begin{aligned}
u(W_M^B) &= \underline{U}_M + g(e_M) - s(e_M)\frac{g'(e_M)}{\beta} \\
&> \mathcal{V}^* + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta} + \mathcal{V}^* + \frac{g'(e_C)}{\beta} - \frac{g'(e_M)}{\beta} \\
&\geq \mathcal{V}^* + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta} \\
&= u(W_C^B(1)).
\end{aligned}$$

Therefore, $W_M(1) > W_C(1)$.

Hence, if

$$\mathcal{G} - \mathcal{B} < \frac{2N}{N-1}\frac{s(e_M)}{s(e_M) - s(e_C)}\left[W_M^G(N) - W_M^G(1)\right],$$

[54] If agents have the log utility function, the condition is not needed.

$$\Pi(\mathcal{V}^*|N) < \Pi(\mathcal{V}^*|1).$$

Notice that there is $\widehat{N}$ such that $W_M^G(N) > W_M^G(1)$ if $N > \widehat{N}$ since $P(\mathbf{e}_{-M})$ converges to zero as N approaches infinity and $\mathcal{V}^*$ is bounded. Let $\overline{O}$ denote

$$\inf_{N\in[\widehat{N},\infty)} \frac{2N}{N-1}\frac{s(e_M)}{s(e_M)-s(e_C)}\left[W_M^G(N) - W_M^G(1)\right].$$

Then, $\Pi(\mathcal{V}^*|N) < \Pi(\mathcal{V}^*|1)$ if $N > \widehat{N}$ and $\mathcal{G} - \mathcal{B} \leq \overline{O}$.

A.18 Proof of Proposition 9

Proof. First, note that

$$\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) = \frac{(N+1)W_C(1) - 2W_C(N) + (N+1)W_M(1) - 2NW_M(N)}{2(N+1)},$$

where $W_C(k)$ and $W_M(k)$ represent CEO's and managers' expected compensation when the firm hires k managers[55], respectively. When agents have the log utility function, the first order condition with respect to $\mathcal{V}$ is

$$s(e)W_C^G(N) + (1-s(e))W_C^B(N) = (1-(1-s(e))^N)W_M^G(N).$$

Using this condition, it can be shown that

$$\begin{aligned}\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) &= s(e)[W_M^G(1) - W_M^G(N)] \\ &\quad - \frac{(1-s(e))(1-(1-s(e))^{N-1})W_M^G(N)}{N+1} - \frac{N-1}{2(N+1)}(1-s(e))W_M^B.\end{aligned}$$

This indicates that $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) < 0$ if $W_M^G(N) > W_M^G(1)$.

[55] These are defined in A.17.

Also, when $u(x) = \log(x)$ and $e_C = e_M = e$, $\mathcal{V}(N)$ is[56]

$$\mathcal{V}^*(N) = -\frac{1}{1+P(\mathbf{e}_{-M})} \cdot \log\left[\frac{s(e_C)\exp\left[g(e_C) + (1-s(e_C))\frac{g'(e_C)}{\beta}\right] + (1-s(e_C))\exp\left[g(e_C) - s(e_C)\frac{g'(e_C)}{\beta}\right]}{(1-(1-s(e_M))^N)\exp\left[\underline{U}_M + g(e_M) + (1-s(e_M))\frac{g'(e_M)}{\beta}\right]}\right]$$

$$= -\frac{1}{1+P(\mathbf{e}_{-M})}\log\left[\frac{s(e) + (1-s(e))\exp\left[-\frac{g'(e)}{\beta}\right]}{(1-(1-s(e))^N)\exp\left[\underline{U}_M\right]}\right]$$

$$= \frac{1}{1+P(\mathbf{e}_{-M})}\underline{U}_M + \frac{1}{1+P(\mathbf{e}_{-M})}\log\left[\frac{(1-(1-s(e))^N)}{s(e) + (1-s(e))\exp\left[-\frac{g'(e)}{\beta}\right]}\right].$$

Note that when $\underline{U}_M = -\log\left[\frac{s(e)}{s(e)+(1-s(e))\exp\left[-\frac{g'(e)}{\beta}\right]}\right]$, $\mathcal{V}^*(1) = 0$, and

$$\mathcal{V}^*(N) = \frac{1}{1+P(\mathbf{e}_{-M})}\log\left[\frac{(1-(1-s(a))^N)}{s(a)}\right].$$

Therefore, $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)$ is greater than 0 if

$$s(e) - \left[\frac{Ns(e) + 1 - (1-s(a))^N}{N+1}\right]\exp[-P(\mathbf{e}_{-M}|N)\mathcal{V}^*(N)] - \frac{N-1}{2(N+1)}(1-s(e))\exp\left(-\frac{g'(e)}{\beta}\right) > 0.$$

This is equivalent to

$$1 > \frac{N}{N+1}\frac{1+P(\mathbf{e}_{-M}|N)}{[NP(\mathbf{e}_{-M}|N)]^{\frac{P(\mathbf{e}_{-M}|N)}{1+P(\mathbf{e}_{-M}|N)}}} + \frac{N-1}{2(N+1)}\frac{1-s(e)}{s(e)}\exp\left(-\frac{g'(a)}{\beta}\right).$$

For a fixed N, the first term on the right hand side is increasing in the agents' effort level e since

$$\frac{\partial}{\partial e}\left(\frac{1+P(\mathbf{e}_{-M}|N)}{[NP(\mathbf{e}_{-M}|N)]^{\frac{P(\mathbf{e}_{-M}|N)}{1+P(\mathbf{e}_{-M}|N)}}}\right) = -\frac{\log[NP(\mathbf{e}_{-M}|N)]}{(1+P(\mathbf{e}_{-M}|N))[NP(\mathbf{e}_{-M}|N)]^{\frac{P(\mathbf{e}_{-M}|N)}{1+P(\mathbf{e}_{-M}|N)}}}\frac{\partial P(\mathbf{e}_{-M}|N)}{\partial e} > 0.$$

Also, the first term on the right hand side is bounded above by 1. On the other hand, the second term on the right hand side is decreasing in a and converges to zero as a goes to 1.[57] Therefore, there is

[56] In this proof, I explicitly indicate the dependency of the variable on N.

[57] This result relies on the condition that $\lim_{e\to 1} g'(e) = \infty$.

$e^*(N) \in (0, 1)$ such that

$$1 > \frac{N}{N+1} \frac{1 + P(\mathbf{e}_{-M}|N)}{[NP(\mathbf{e}_{-M}|N)]^{\frac{P(\mathbf{e}_{-M}|N)}{1+P(\mathbf{e}_{-M}|N)}}} + \frac{N-1}{2(N+1)} \frac{1-s(e)}{s(e)} \exp\left(-\frac{g'(e)}{\beta}\right)$$

holds if $e \geq e^*(N)$. That is, $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)$ is greater than zero.

However, for fixed e and N, there is $\underline{U}_M^*$ such that $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) < 0$ if $\underline{U}_M > \underline{U}_M^*(N)$. Notice that

$$W_M^G(1) - W_M^G(N) = \exp\left[g(e) + (1 - s(e))\frac{g'(e)}{\beta}\right] [\exp[-\mathcal{V}^*(1)] - \exp[-P(\mathbf{e}_{-M}|N)\mathcal{V}^*(N)]],$$

which is less than zero if $\mathcal{V}^*(1) > P(\mathbf{e}_{-M}|N)\mathcal{V}^*(N)$. The difference between these two terms is

$$\mathcal{V}^*(1) - P(\mathbf{e}_{-M}|N)\mathcal{V}^*(N) = \left(\frac{1}{2} - \frac{1}{1 + P(\mathbf{e}_{-M}|N)}\right) \underline{U}_M + R(N, e), \tag{11}$$

where $R(N, e)$ is a constant only depending on N and e not $\underline{U}_M$. Since $P(\mathbf{e}_{-M}|N)$ is strictly less than 1 when $N \geq 2$, there is $\underline{U}_M^*(N)$ such that $W_M^G(1) < W_M^G(N)$ if $\underline{U}_M > \underline{U}_M^*(N)$. This implies that $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) < 0$ when $\underline{U}_M > \underline{U}_M^*(N)$.

Now, I show that $\frac{\partial^2}{\partial(\underline{U}_M)^2} [\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)] < 0$ if $\frac{\partial}{\partial \underline{U}_M} [\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)] \leq 0$.

Suppose that

$$\frac{\partial}{\partial \underline{U}_M} [\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)] = \frac{s(e)}{2} W_M^G(1) - \frac{N}{N+1} s(e) W_M^G(N) - \frac{N-1}{2(N+1)} (1 - s(e)) W_M^B$$

has a negative value. Then,

$$\begin{aligned}
\frac{\partial^2}{\partial(\underline{U}_M)^2} [\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)] &= \frac{s(e)}{4} W_M^G(1) - \frac{N}{N+1} \frac{s(e)}{1 + P(\mathbf{e}_{-M}|N)} W_M^G(N) \\
&\quad - \frac{N-1}{2(N+1)} (1 - s(a)) W_M^B \\
&< \frac{s(e)}{4} W_M^G(1) - \frac{N}{N+1} \frac{s(e)}{2} W_M^G(N) \\
&\quad - \frac{N-1}{4(N+1)} (1 - s(a)) W_M^B \\
&= \frac{1}{2} \left[\frac{s(e)}{2} W_M^G(1) - \frac{N}{N+1} s(e) W_M^G(N) - \frac{N-1}{2(N+1)} (1 - s(e)) W_M^B\right] \\
&= \frac{1}{2} \frac{\partial}{\partial \underline{U}_M} [\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)] \leq 0.
\end{aligned}$$

When $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) > 0$ as $\underline{U}_M = -\log\left[\frac{s(a)}{s(a) + (1 - s(a)) \exp\left[-\frac{g'(a)}{\beta}\right]}\right] \equiv \underline{U}_M^0$, there are two possible cases.

1. $\left(\frac{\partial}{\partial \underline{U}_M}\left[\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)\right] \leq 0\right)$

   Since there is $\underline{U}_M^*(N)$ such that $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) < 0$ when $\underline{U}_M > \underline{U}_M^*(N)$, there is a unique $\widehat{\underline{U}}_M(N)$ such that $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) > 0$ if $\underline{U}_M \in \left[\underline{U}_M^0, \widehat{\underline{U}}_M(N)\right)$.

2. $\left(\frac{\partial}{\partial \underline{U}_M}\left[\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)\right] > 0\right)$.

   The condition that $\frac{\partial^2}{\partial (\underline{U}_M)^2}\left[\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)\right] < 0$ if $\frac{\partial}{\partial \underline{U}_M}\left[\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)\right] = 0$ implies that there is a unique $\underline{U}_M$ such that $\frac{\partial}{\partial \underline{U}_M}\left[\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1)\right] = 0$. Hence, there is a unique $\widehat{\underline{U}}_M(N)$ such that $\Pi(\mathcal{V}^*|N) - \Pi(\mathcal{V}^*|1) > 0$ if $\underline{U}_M \in \left[\underline{U}_M^0, \widehat{\underline{U}}_M(N)\right)$.

Lastly, I show that there is N^* such that $W_M^G(1) < W_M^G(N)$ if $N \geq N^*$ for a given $(\underline{U}_M, e)$. This implies that $\Pi(\mathcal{V}^*|N) < \Pi(\mathcal{V}^*|1) > 0$. In the equation (11), $R(N,e)$ is equal to

$$R(N,e) = \frac{1}{2}\log\left(\frac{s(e)}{s(e) + (1-s(e))\exp\left[-\frac{g'(a)}{\beta}\right]}\right) - \frac{P(\mathbf{e}_{-M}|N)}{1+P(\mathbf{e}_{-M}|N)}\log\left(\frac{1-(1-s(e))^N}{s(e)+(1-s(e))\exp\left[-\frac{g'(e)}{\beta}\right]}\right).$$

Since $\left(\frac{1}{2} - \frac{1}{1+P(\mathbf{e}_{-M}|N)}\right)\underline{U}_M > 0$, it is enough to show that $R(N,e) > 0$ if $N \geq N^*$. The first term of $R(N,e)$ does not depend on N and has a strictly positive number. Denote this number by C. On the other hand, the second term is always less than

$$\frac{P(\mathbf{e}_{-M}|N)}{1+P(\mathbf{e}_{-M}|N)}\log\left(\frac{1}{s(e)+(1-s(e))\exp\left[-\frac{g'(a)}{\beta}\right]}\right), \tag{12}$$

which is strictly decreasing function in N and converges to zero. Hence, there is N^* such that (12) is less than C if $N \geq N^*$. This implies that $R(N,e) > 0$.

□

A.19 Proof of Proposition 10

Note that

$$\begin{aligned}
\frac{\partial^2 F}{\partial \mathcal{V}\partial N} &= -(1-s(e_M))^N \log(1-s(e_M))\frac{1}{u'(W_g)} > 0, \\
\frac{\partial^2 F}{\partial(-e_C)\partial N} &= 0, \text{ and} \\
\frac{\partial^2 F}{\partial \mathcal{V}\partial(-e_C)} &= s(e_C)\frac{\partial^2 W_C^G}{\partial \mathcal{V}\partial e_C} + (1-s(e_C))\frac{\partial^2 W_C^B}{\partial \mathcal{V}\partial e_C} + \beta\left(\frac{\partial W_C^G}{\partial \mathcal{V}} - \frac{\partial W_C^B}{\partial \mathcal{V}}\right) \\
&= -s(e_C)(1-s(e_C))\frac{g''(e_C)}{\beta}\left(\frac{u''(W_C^G)}{u'(W_C^G)^3} - \frac{u''(W_C^B)}{u'(W_C^B)^3}\right) + \beta\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right).
\end{aligned}$$

This result shows that $\frac{\partial \mathcal{V}^*}{\partial N} \geq 0$ and $\frac{\partial e_C^*}{\partial N} \leq 0$ according to Milgrom and Shannon (1994). First, consider e_C^* as a function of $\mathcal{V}$. Then, under the condition that $u''(x)/u'(x)^3$ is a decreasing function in x,

$$\begin{aligned}
\frac{\partial^2 F}{\partial \mathcal{V}\partial N} =& \left[-\beta\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right) + s(e_C^*)\frac{u''(W_C^G)}{u'(W_C^G)^2}\frac{\partial W_C^G}{\partial e_C^*} + (1-s(e_C^*))\frac{u''(W_C^B)}{u'(W_C^B)^2}\frac{\partial W_C^B}{\partial e_C^*}\right]\frac{\partial e_C^*}{\partial N} \\
&- (1-s(e_M))^N \log(1-s(e_M))\frac{1}{u'(W_g)} \\
&> 0
\end{aligned}$$

since $\frac{\partial e_C^*}{\partial N} \leq 0$. This implies that $\frac{\partial \mathcal{V}^*}{\partial N} > 0$. Now, consider the first order condition with respect to e_C:

$$\begin{aligned}
\beta(\mathcal{G}-\mathcal{B}) &= \beta(W_C^G - W_C^B) + s(e_C^*)\frac{\partial W_C^G}{\partial e_C} + (1-s(e_C))\frac{\partial W_C^B}{\partial e_C} \\
&= \beta(W_C^G - W_C^B) + s(e_C^*)(1-s(e_C^*))\frac{g''(e_C^*)}{\beta}\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right).
\end{aligned}$$

The right hand side of the equation is a strictly increasing function in e_C^* if $g'''(e_C) \geq 0$ and a strictly decreasing function in $\mathcal{V}$. Hence, $\frac{\partial \mathcal{V}^*}{\partial N} > 0$ indicates that $\frac{\partial e_C^*}{\partial N} < 0$.

A.20 Proof of Corollary 7

Note that

$$\begin{aligned}
\frac{\partial}{\partial N}[(W_C^G)^* - (W_C^B)^*] &= \frac{\partial \mathcal{V}}{\partial N}\left[\frac{1}{u'((W_C^G)^*)} - \frac{1}{u'((W_C^B)^*)} + \left(\frac{1-s(e_C^*)}{u'((W_C^G)^*} + \frac{s(e_C^*)}{u'((W_C^B)^*)}\right)\frac{g''(e_C^*)}{\beta}\frac{\partial e_C^*}{\partial \mathcal{V}}\right] \\
&= \frac{\partial \mathcal{V}}{\partial N}\left[(W_C^G)^* - (W_C^B)^* + \left((1-s(e_C^*))(W_C^G)^* + s(e_C^*)(W_C^B)^*\right)\frac{g''(e_C^*)}{\beta}\frac{\partial e_C^*}{\partial \mathcal{V}}\right]
\end{aligned}$$

when agents have the log utility function. Also, it can be shown that

$$\frac{\partial e_C^*}{\partial \mathcal{V}} = -\frac{\left(\beta + s(e_C^*)(1-s(e_C^*))\frac{g''(e_C^*)}{\beta}\right)((W_C^G)^* - (W_C^B)^*)}{D_1 + D_2},$$

where

$$D_1 = \left(\beta(1-2s(e_C^*))\frac{g''(e_C^*)}{\beta} + s(e_C^*)(1-s(e_C^*))\frac{g'''(e_C^*)}{\beta}\right)((W_C^G)^* - (W_C^B)^*)$$
$$D_2 = \left(\beta + s(e_C^*)(1-s(e_C^*))\frac{g''(e_C^*)}{\beta}\right)\frac{g''(e_C^*)}{\beta}((1-s(e_C^*))(W_C^G)^* + s(e_C^*)(W_C^B)^*).$$

Hence,

$$\frac{\partial}{\partial N}[(W_C^G)^* - (W_C^B)^*] = \frac{(W_C^G)^* - (W_C^B)^*}{D_1 + D_2} D_1 > 0.$$

A.21 Proof of Proposition 11

Note that when $\mathcal{V} = 0$, $e_C^* = e_M^*$ by two first order conditions. Also, this implies that $W_C^G = W_M^G$. Therefore,

$$\left.\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}}\right|_{\mathcal{V}=0} = -\frac{s(e_C^*)}{u'(W_C^G)} - \frac{1-s(e_C^*)}{u'(W_C^B)} + \frac{1-(1-s(e_M^*))^N}{u'(W_M^G)}$$
$$= (1-s(e_C^*))\left(\frac{1-(1-s(e_C^*))^{N-1}}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right).$$

Since $W_C^G > W_C^B$ and $(1-s(e_C^*))^{N-1} \to 0$ as $N \to \infty$, there is $\widehat{N}$ such that $\left.\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}}\right|_{\mathcal{V}=0} > 0$ if $N > \widehat{N}$. This implies that $\mathcal{V}^* > \underline{U}_M = 0$ when $N > \widehat{N}$. As a next step, I show that the promotion incentive is bounded regardless of the firm size.

Claim 1. *The optimal promotion incentive $\mathcal{V}^*$ is bounded for any N.*

Proof. First, fix $(e_M) \in (0,1)$ for a given N. Denote the optimal $\mathcal{V}$ by $\mathcal{V}^*(e_C)$ for a given e_C. Recall that $\frac{\partial \mathcal{V}^*(e_C)}{\partial e_C} < 0$. Therefore,

$$\mathcal{V}^*(e_C) \leq \mathcal{V}^*(0).$$

When $e_C = 0$, the first order condition with respect to $\mathcal{V}$ is

$$-\frac{1}{u'(W_C^F)} + \frac{1-(1-s(e))^N}{u'(W_M^G)} = 0,$$

where $u(W_C^F) = \mathcal{V}^*(0)$. It can be easily shown that

$$\mathcal{V}^*(0) = u(W_C^F) < u(W_M^G) = \underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta} - P(\mathbf{e}_{-M})\mathcal{V}^*(0),$$

implying that

$$\mathcal{V}^*(0) < \frac{1}{1 + P(\mathbf{e}_{-M})}\left[\underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta}\right].$$

Notice that this bound does not depend on e_C. Now, suppose that

$$\mathcal{V} = \frac{1}{1 + P(\mathbf{e}_{-M})}\left[\underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta}\right].$$

Then, the manager's wage for good performance satisfies

$$\begin{aligned} u(W_M^G) &= \frac{1}{1 + P(\mathbf{e}_{-M})}\left[\underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta}\right] \\ &\geq \frac{1}{2}\left[\underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta}\right]. \end{aligned}$$

The bound for $u(W_M^G)$ does not depend on N. This result means that the firm has to pay the wage satisfying the lower bound if it requires an effort level e_M form its managers. Since the wage approaches infinity as e_M converges to one, there is $\bar{e}_M < 1$ such that $e_C^* \leq \bar{e}_M$ regardless of N and e_C. Hence, $\mathcal{V}^*$ is bounded by $\frac{1}{1+P(\bar{\mathbf{e}}_{-M})}\left[\underline{U}_M + g(\bar{e}_M) + (1 - s(\bar{e}_M))\frac{g'(\bar{e}_M)}{\beta}\right]$, which is less than $\left[\underline{U}_M + g(\bar{e}_M) + (1 - s(\bar{e}_M))\frac{g'(\bar{e}_M)}{\beta}\right]$. I denote this bound by $\bar{\mathcal{V}}$. This upper bound does not depend on N.

□

Based on this result, I show the following result.

Claim 2. *There is $\bar{N}$ such that $(W_M^G)^* > (W_M^B)^*$ if $N > \bar{N}$.*

Proof. Recall that

$$(W_M^G)^* \leq (W_M^B)^*$$

if and only if

$$\frac{g'(e_M^*)}{\beta} \leq P(\mathbf{e}_{-M})\mathcal{V}^*.$$

Since $e_M^* \geq \underline{e}_M$, where $\underline{e}_M$ is the firm's optimal effort choice when $\mathcal{V} = 0$, and $P(\mathbf{e}_{-M}) \to 0$ as $N \to \infty$, there exists $\bar{N}$ such that

$$\frac{g'(e_M^*)}{\beta} \geq \frac{g'(\underline{e}_M)}{\beta} > P(\underline{\mathbf{e}}_{-M})\bar{\mathcal{V}} \geq P(\mathbf{e}_{-M})\bar{\mathcal{V}}$$

if $N > \bar{N}$. Therefore, $(W_M^G)* > (W_M^B)*$ if $N > \bar{N}$. □

Now, I show that $e_M^* > e_C^*$ if $N > N^* \equiv \max\{\widehat{N}, \bar{N}\}$.

Note that two optimal effort levels e_C^* and e_M^* are decided by two first order conditions for a given $\mathcal{V}$:

$$\begin{aligned}\beta(\mathcal{G}-\mathcal{B}) &= \beta(W_C^G - W_C^B) + s(e_C^*)\frac{\partial W_C^G}{\partial e_C} + (1-s(e_C))\frac{\partial W_C^B}{\partial e_C} \\ &= \beta(W_C^G - W_C^B) + s(e_C^*)(1-s(e_C^*))\frac{g''(e_C^*)}{\beta}\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right), \text{ and} \qquad (13)\end{aligned}$$

$$\begin{aligned}\beta(\mathcal{G}-\mathcal{B}) &= \beta(W_M^G - W_M^B) + s(e_M^*)\frac{\partial W_M^G}{\partial e_M} + (1-s(e_M^*))\frac{\partial W_M^B}{\partial e_M} \\ &= \beta(W_M^G - W_M^B) + s(e_M^*)(1-s(e_M^*))\frac{g''(e_M^*)}{\beta}\left(\frac{1}{u'(W_M^G)} - \frac{1}{u'(W_M^B)}\right) \\ &\quad - s(e_M^*)\frac{\partial P(\mathbf{e}_{-M}^*)}{\partial e_M}\frac{1}{u'(W_M^G)}\mathcal{V}. \qquad (14)\end{aligned}$$

If $\mathcal{V}$ is equal to $\underline{U}_M = 0$, two conditions yield $e_C^* = e_M^*$. The right hand side of (13) is a strictly increasing function in $\mathcal{V}$ while that of (14) is a strictly decreasing function in $\mathcal{V}$ since

$$\begin{aligned}\frac{\partial J_C(\mathcal{V}, e_C)}{\partial \mathcal{V}} &= \beta\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right) - s(e_C)(1-s(e_C))\frac{g''(e_C)}{\beta}\left(\frac{u''(W_C^G)}{u'(W_C^G)^3} - \frac{u''(W_C^B)}{u'(W_C^B)^3}\right) > 0, \\ \frac{\partial J_M(\mathcal{V}, e_M)}{\partial \mathcal{V}} &= -\beta P(\mathbf{e}_{-M})\frac{1}{u'(W_M^G)} + s(e_M)(1-s(e_M))\frac{g''(e_M)}{\beta}P(\mathbf{e}_{-M})\frac{u''(W_M^G)}{u'(W_M^G)^3} \\ &\quad - s(e_M)\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\frac{1}{u'(W_M^G)} - s(e_M)\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}P(\mathbf{e}_{-M})\frac{u''(W_M^G)}{u'(W_M^G)^3}\mathcal{V} \\ &= s(e_M)(1-s(e_M))\frac{g''(e_M)}{\beta}P(\mathbf{e}_{-M})\frac{u''(W_M^G)}{u'(W_M^G)^3} - \beta(1-s(e_M))^{N-1}\frac{1}{u'(W_M^G)} \\ &\quad - s(e_M)\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}P(\mathbf{e}_{-M})\frac{u''(W_M^G)}{u'(W_M^G)^3}\mathcal{V} < 0,\end{aligned}$$

where

$$\begin{aligned}J_C(\mathcal{V}, e_C) &= \beta(W_C^G - W_C^B) + s(e_C)(1-s(e_C))\frac{g''(e_C)}{\beta}\left(\frac{1}{u'(W_C^G)} - \frac{1}{u'(W_C^B)}\right), \text{ and} \\ J_M(\mathcal{V}, e_M) &= \beta(W_M^G - W_M^B) + s(e_M)(1-s(e_M))\frac{g''(e_M)}{\beta}\left(\frac{1}{u'(W_M^G)} - \frac{1}{u'(W_M^B)}\right) \\ &\quad - s(e_M)\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\frac{1}{u'(W_M^G)}\mathcal{V}, \text{and} \\ \frac{\partial P(\mathbf{e}_{-M})}{\partial e_M} &= \frac{\beta}{s(e_M)}[(1-s(e_M))^{N-1} - P(\mathbf{e}_{-M})] < 0.\end{aligned}$$

In addition, the following results

$$\frac{\partial J_C(\mathcal{V},e_C)}{\partial e_C} = 2g''(e_C)\left(\frac{1-s(e_C)}{u'(W_C^G)}+\frac{s(e_C)}{u'(W_C^B)}\right) - g''(e_C)\left(\frac{s(e_C)}{u'(W_C^G)}+\frac{1-s(e_C)}{u'(W_C^B)}\right)$$
$$- s(e_C)(1-s(e_C))\frac{g''(e_C)^2}{\beta^2}\left[(1-s(e_C))\frac{u''(W_C^G)}{u'(W_C^B)^3}+s(e_C)\frac{u''(W_C^B)}{u'(W_C^B)^3}\right]$$
$$+ s(e_C)(1-s(e_C))\frac{g'''(e_C)}{\beta}\left(\frac{1}{u'(W_C^G)}-\frac{1}{u'(W_C^B)}\right) > 0,$$

$$\frac{\partial J_M(\mathcal{V},e_M)}{\partial e_M} = \left.\frac{\partial J_C(\mathcal{V},e_C)}{\partial e_C}\right|_{e_C=e_M} - 2\beta\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\frac{1}{u'(W_M^G)}\mathcal{V}$$
$$+\left[2s(e_M)(1-s(e_M))\frac{g''(e_M)}{\beta}\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\mathcal{V} - s(e_M)\left(\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\right)^2\mathcal{V}^2\right]\frac{u''(W_M^G)}{u'(W_M^G)^3}$$
$$- s(e_M)\frac{\partial^2 P(\mathbf{e}_{-M})}{\partial (e_M)^2}\frac{1}{u'(W_M^G)}\mathcal{V}$$
$$= \left.\frac{\partial J_C(\mathcal{V},e_C)}{\partial e_C}\right|_{e_C=e_M} + \beta^2(N-1)(1-s(e_M))^{N-2}\frac{1}{u'(W_M^G)}\mathcal{V}$$
$$+\left[2s(e_M)(1-s(e_M))\frac{g''(e_M)}{\beta}\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\mathcal{V} - s(e_M)\left(\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M}\right)^2\mathcal{V}^2\right]\frac{u''(W_M^G)}{u'(W_M^G)^3}$$
$$> 0,$$

where

$$\frac{\partial^2 P(\mathbf{e}_{-M})}{\partial (e_M)^2} = \frac{\beta^2}{Ns(e_M)^3}\left[-N(N-1)s(e_M)^2(1-s(e_M))^{N-2}\right.$$
$$\left.+2-2(1-s(e_M))^{N-1}-2(N-1)s(e_M)(1-s(e_M))^{N-1}\right]$$
$$= -\frac{\beta^2}{s(e_M)}(N-1)(1-s(e_M))^{N-2} - 2\frac{\beta}{s(e_M)}\frac{\partial P(\mathbf{e}_{-M})}{\partial e_M} \geq 0,$$

validate the first order approach.

Hence, $e_C^* < e_M^*$ since $\mathcal{V}^* > \underline{U}_M = 0$ when $N > \widehat{N}$.

A.22 Proof of Proposition 12

For a given $(\mathcal{V}, N)$, U_{M2}^* is determined by the equation (9). I denote this by $U_{M2}^*(\mathcal{V}, N)$ to explicitly express the dependency. The first order condition with respect to $\mathcal{V}$ and the equation (9) imply that $\mathcal{V}^*$ and $U_{M2}^*(\mathcal{V}^*, N)$ satisfy

$$\frac{s(e_C)}{u'((W_C^G)^*)}+\frac{1-s(e_C)}{u'((W_C^B)^*)}+\frac{(1-s(e_{M1}))^N}{u'((W_M^G)^*)} = \delta\left[\frac{s(e_{M2})}{u'((W_M^{GG})^*)}+\frac{1-s(e_{M2})}{u'((W_M^{GB})^*)}\right],$$

where the first order condition with respect to $\mathcal{V}$ is

$$\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}} = -\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + Ns(e_{M1})\left[\frac{1}{u'(W_M^G)}\left(P(\mathbf{e}_{-M}) + (1-P(\mathbf{e}_{-M}))\frac{\partial U_{M2}(\mathcal{V})}{\partial \mathcal{V}}\right)\right]$$
$$- \delta\left(Ns(e_{M1}) - 1 + (1-s(e_{M1}))^N\right)\left(\frac{s(e_{M2})}{u'(W_M^{GG})} + \frac{1-s(e_{M2})}{u'(W_M^{GB})}\right)\frac{\partial U_{M2}(\mathcal{V})}{\partial \mathcal{V}}$$
$$= -\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + \frac{1-(1-s(e_{M1}))^N}{u'(W_M^G)}$$

by (9).

Note that $\frac{(1-s(e_{M1}))^N}{u'(W_M^G)}$ approaches zero as N goes to infinity. Also, when $e_C = e_{M2}$, $\mathcal{V} = U_{M2}$, and $\delta = 1$, the condition is equal to

$$\frac{s(e_C)}{u'(W_C^G)} + \frac{1-s(e_C)}{u'(W_C^B)} = \frac{s(e_{M2})}{u'(W_M^{GG})} + \frac{1-s(e_{M2})}{u'(W_M^{GB})}.$$

Since the left hand side of this equation is a strictly increasing function in e_C and $\mathcal{V}$, there is $\widehat{N}$, $\hat{e}_C < e_{M2}$ such that

$$\frac{s(\hat{e}_C)}{u'(W_C^G)} + \frac{1-s(\hat{e}_C)}{u'(W_C^B)} + \frac{(1-s(e_{M1}))^{\widehat{N}}}{u'(W_M^G)} < \delta\left[\frac{s(e_{M2})}{u'(W_M^{GG})} + \frac{1-s(e_{M2})}{u'(W_M^{GB})}\right]$$

when $\mathcal{V} = \underline{U}_M$ for a sufficiently large δ. This implies that for given $(\delta, e_C, e_{M1}, e_{M2}, N)$, where $e_C \leq \hat{e}_C < e_{M2}$ and $N \geq \widehat{N}$, $\mathcal{V}^* > U_{M2}^*(\mathcal{V}^*, N)$.

Now, I show that $\mathcal{V}^*$ is an increasing function in $N > \widehat{N}$ when $\mathcal{V}^* \geq U_{M2}^*(\mathcal{V}^*, \widehat{N})$.

First, note that for a given $\mathcal{V}$ and N,

$$\frac{\partial U_{M2}^*(\mathcal{V}, N)}{\partial N} = -\frac{\frac{u''(W_M^G)}{u'(W_M^G)^3}\frac{\partial P(\mathbf{e}_{-M1})}{\partial N}(\mathcal{V} - U_{M2}^*(\mathcal{V}, N))}{\frac{u''(W_M^G)}{u'(W_M^G)^3}(1-P(\mathbf{e}_{-M1})) + \delta\left[s(e_{M2})\frac{u''(W_M^{GG})}{u'(W_M^{GG})^3} + (1-s(e_{M2}))\frac{u''(W_M^{GB})}{u'(W_M^{GB})^3}\right]}.$$

Therefore,

$$\frac{\partial^2 F(\mathcal{V})}{\partial N \partial \mathcal{V}} = -(1-s(e_{M1}))^N \log(1-s(e_{M1}))\frac{1}{u'(W_M^G)}$$
$$+ [1-(1-s(e_{M1}))^N]\frac{u''(W_M^G)}{u'(W_M^G)^3}\cdot$$
$$\left[\frac{\partial P(\mathbf{e}_{-M1})}{\partial N}(\mathcal{V}^* - U_{M2}^*) + (1-P(\mathbf{e}_{-M1}))\frac{\partial U_{M2}^*(\mathcal{V}, N)}{\partial N}\right]$$
$$= -(1-s(e_{M1}))^N \log(1-s(e_{M1}))\frac{1}{u'(W_M^G)}$$
$$- [1-(1-s(e_{M1}))^N]\left[s(e_{M2})\frac{u''(W_M^{GG})}{u'(W_M^{GG})^3} + (1-s(e_{M2}))\frac{u''(W_M^{GB})}{u'(W_M^{GB})^3}\right] > 0.$$

Moreover,

$$
\begin{aligned}
\frac{\partial^2 F(\mathcal{V})}{\partial \mathcal{V}^2} &= s(e_C)\frac{u''(W_C^G)}{u'(W_C^G)^3} + (1-s(e_C))\frac{u''(W_C^B)}{u'(W_C^B)^3} \\
&\quad + (1-(1-s(e_{M1}))^N)\frac{u''(W_M^G)}{u'(W_M^G)^3}\left[P(\mathbf{e}_{-M1}) + (1-P(\mathbf{e}_{-M1}))\frac{\partial U_{M2}^*(\mathcal{V},N)}{\partial \mathcal{V}}\right] \\
&= s(e_C)\frac{u''(W_C^G)}{u'(W_C^G)^3} + (1-s(e_C))\frac{u''(W_C^B)}{u'(W_C^B)^3} \\
&\quad + (1-(1-s(e_{M1}))^N)\frac{u''(W_M^G)}{u'(W_M^G)^3} \\
&\quad \cdot \left[P(\mathbf{e}_{-M1})\frac{\delta\left[s(e_{M2})\frac{u''(W_M^{GG})}{u'(W_M^{GG})^3} + (1-s(e_{M2}))\frac{u''(W_M^{GB})}{u'(W_M^{GB})^3}\right]}{(1-P(\mathbf{e}_{-M1}))\frac{u''(W_M^G)}{u'(W_M^G)^3} + \delta\left[s(e_{M2})\frac{u''(W_M^{GG})}{u'(W_M^{GG})^3} + (1-s(e_{M2}))\frac{u''(W_M^{GB})}{u'(W_M^{GB})^3}\right]}\right] \\
&< 0,
\end{aligned}
$$

where I exploit

$$
\frac{\partial U_{M2}^*(\mathcal{V},N)}{\partial \mathcal{V}} = -\frac{P(\mathbf{e}_{-M1})\frac{u''(W_M^G)}{u'(W_M^G)^3}}{(1-P(\mathbf{e}_{-M1}))\frac{u''(W_M^G)}{u'(W_M^G)^3} + \delta\left[s(e_{M2})\frac{u''(W_M^{GG})}{u'(W_M^{GG})^3} + (1-s(e_{M2}))\frac{u''(W_M^{GB})}{u'(W_M^{GB})^3}\right]}
$$

using the implicit function theorem.

A.23 Derivation of the Firm's Problem in Section 7.2

The firm's problem can be written as

$$
\max_{C,M_1,M_2} E_0\left[\sum_{t=1}^{\infty}\delta^{t-1}\mathcal{P}_t(C,M_1,M_2|\mathcal{H}_{t-1})\right]
$$

subject to $(IR_C),\ (IC_C),\ (IR_{M1}),\ (IC_{M1}),\ (IR_{M2}),\ (IC_{M2}),$

where

$$
\mathcal{P}_1(C,M_1,M_2|\mathcal{H}_0) = \mathcal{P}_C(C) + \mathcal{P}_M(M_1)
$$

$$
\mathcal{P}_t(C,M_1,M_2|\mathcal{H}_{t-1}) = \begin{cases} \mathcal{P}_C(C) + N\mathcal{P}_M(M_1) & \text{if } \mathcal{H}_{t-1}\in\mathcal{S}_1 \\ N\mathcal{P}_M(M_2) & \text{if } \mathcal{H}_{t-1}\in\mathcal{S}_2 \end{cases}
$$

with

$$\begin{aligned}\mathcal{P}_C(C) &= s(e_{C1})(\mathcal{G}_C - W_C^G) + (1 - s(e_{C1}))(\mathcal{B}_C - W_C^B) \\ &\quad + \delta s(e_{C1})[s(e_{C2})(\mathcal{G}_C - W_C^{GG}) + (1 - s(e_{C2}))(\mathcal{B}_C - W_C^{GB})] \\ \mathcal{P}_M(M_i) &= s(e_{Mi})(\mathcal{G}_M - W_{Mi}^G) + (1 - s(e_{Mi}))(\mathcal{B}_M - W_{Mi}^B) \\ C &= (W_C^G, W_C^B, W_C^{GG}, W_C^{GB}) \\ M_i &= (W_{Mi}^G, W_{Mi}^B).\end{aligned}$$

Also, $\mathcal{H}_t$ denotes the CEO's seniority and outcome at time t. Hence,

$$\mathcal{H}_t \in \{(C_1, \mathcal{G}_C),\ (C_1, \mathcal{B}_C),\ (C_2, \mathcal{G}_C),\ (C_2, \mathcal{B}_C)\},$$

where C_i is equal to C_1 (C_2) if the CEO is her first period (second period) in the position. For brevity, I use two terms, $\mathcal{S}_1$ and $\mathcal{S}_2$, in order to represent

$$\begin{aligned}\mathcal{S}_1 &= \{(C_1, \mathcal{B}_C),\ (C_2, \mathcal{G}_C),\ (C_2, \mathcal{B}_C)\} \\ \mathcal{S}_2 &= \{(C_1, \mathcal{G}_C)\},\end{aligned}$$

respectively.

Then,

$$\begin{aligned}E_0\left[\sum_{t=1}^{\infty}\delta^{t-1}\mathcal{P}_t(C, M_1, M_2|\mathcal{H}_{t-1})\right] &= \mathcal{P}_C(C) + N\mathcal{P}_M(M_1) \\ &\quad + \delta s(e_{C1})\left\{N\mathcal{P}_M(M_2) + \delta E_0\left[\sum_{t=1}^{\infty}\delta^{t-1}\mathcal{P}_t(C, M_1, M_2|\mathcal{H}_{t-1})\right]\right\} \\ &\quad + \delta(1 - s(e_{C1}))E_0\left[\sum_{t=1}^{\infty}\delta^{t-1}\mathcal{P}_t(C, M_1, M_2|\mathcal{H}_{t-1})\right],\end{aligned}$$

where I exploit

$$E_0\left[\sum_{t=1}^{\infty}\delta^{t-1}\mathcal{P}_t(C, M_1, M_2|\mathcal{H}_{t-1})\right] = E_s\left[\sum_{t=s+1}^{\infty}\delta^{t-(s+1)}\mathcal{P}_t(C, M_1, M_2|\mathcal{H}_{t-1})\right]$$

if $\mathcal{H}_s \in \mathcal{S}_1$. Therefore,

$$E_0\left[\sum_{t=1}^{\infty}\delta^{t-1}\mathcal{P}_t(C, M_1, M_2|\mathcal{H}_{t-1})\right] = \frac{1}{(1-\delta)(1+\delta s(e_{C1}))}\left[\mathcal{P}_C(C) + N\mathcal{P}_M(M_1) + \delta s(e_{C1})N\mathcal{P}_M(M_2)\right].$$

Since I treat e_{C1} as an exogenous variable, the firm's problem is to choose (C, M_1, M_2) maximizing

$$\mathcal{P}_C(C) + N\mathcal{P}_M(M_1) + \delta s(e_{C1})N\mathcal{P}_M(M_2).$$

A.24 Proof of Proposition 13

The firm's problem for guaranteed situation is to choose $\widehat{\mathcal{V}} \in [0, \infty)$ maximizing $\widehat{F}(\widehat{\mathcal{V}})$ defined by[58]

$$\begin{aligned}
\widehat{F}(\widehat{\mathcal{V}}) \equiv \max_{\widehat{\mathcal{A}}} \; & s(e_C)(\mathcal{G}_C - \widehat{W}_C^G) + (1 - s(e_C))(\mathcal{B}_C - \widehat{W}_C^B) \\
& + \delta s(e_C)[s(e_C)(\mathcal{G}_C - \widehat{W}_C^{GG}) + (1 - s(e_C))(\mathcal{B}_C - \widehat{W}_C^{GB})] \\
& + \delta(1 - s(e_C))[s(e_C)(\mathcal{G}_C - \widehat{W}_C^{BG}) + (1 - s(e_C))(\mathcal{B}_C - \widehat{W}_C^{BB})] \\
& + N\left[s(e_M)(\mathcal{G}_M - \widehat{W}_{M1}^G) + (1 - s(e_M))(\mathcal{B}_M - \widehat{W}_{M1}^B)\right] \\
& + \delta N\left[s(e_M)(\mathcal{G}_M - \widehat{W}_{M2}^G) + (1 - s(e_M))(\mathcal{B}_M - \widehat{W}_{M2}^B)\right]
\end{aligned}$$

subject to

$$\begin{aligned}
u(\widehat{W}_C^G) &= \mathcal{V} + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{\beta} - V_2^G, \\
u(\widehat{W}_C^B) &= \mathcal{V} + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta} - V_2^B, \\
u(\widehat{W}_C^{GG}) &= V_2^G + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{\beta}, \\
u(\widehat{W}_C^{GB}) &= V_2^G + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta}, \\
u(\widehat{W}_C^{BG}) &= V_2^B + g(e_C) + (1 - s(e_C))\frac{g'(e_C)}{\beta}, \\
u(\widehat{W}_C^{BB}) &= V_2^B + g(e_C) - s(e_C)\frac{g'(e_C)}{\beta}, \\
u(\widehat{W}_{M1}^G) &= \underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta}, \\
u(\widehat{W}_{M1}^B) &= \underline{U}_M + g(e_M) - s(e_M)\frac{g'(e_M)}{\beta}, \\
u(\widehat{W}_{M2}^G) &= \underline{U}_M + g(e_M) + (1 - s((e_M))\frac{g'((e_M)}{\beta} - P(\mathbf{e}_{-M})\mathcal{V}, \text{ and} \\
u(\widehat{W}_{M2}^B) &= \underline{U}_M + g(e_M) - s(e_M)\frac{g'(e_M)}{\beta},
\end{aligned}$$

[58] I use the hat notation to indicate guaranteed job security case.

where

$$\begin{aligned}\widehat{\mathcal{A}} &= \{(\widehat{W}_C^G, \widehat{W}_C^B, \widehat{W}_C^{GG}, \widehat{W}_C^{GB}, \widehat{W}_C^{BG}, \widehat{W}_C^{BB}), (\widehat{W}_{M1}^G, \widehat{W}_{M2}^B), (\widehat{W}_{M2}^G, \widehat{W}_{M2}^B)\},\\ V_2^G &= s(e_C)u(\widehat{W}_C^{GG}) + (1-s(e_C))u(\widehat{W}_C^{GB}) - g(e_C), \text{ and}\\ V_2^B &= s(e_C)u(\widehat{W}_C^{BG}) + (1-s(e_C))u(\widehat{W}_C^{BB}) - g(e_C).\end{aligned}$$

Then, the first order condition with respect to $\mathcal{V}$ for unguaranteed situation is

$$-\frac{s(e_C)}{u'(W_C^G)} - \frac{1-s(e_C)}{u'(W_C^B)} + \delta(1-s(e_C))\frac{1-(1-s(e_M))^N}{u'(W_{M1}^G)} + \delta s(e_C)\frac{1-(1-s(e_M))^N}{u'(W_{M2}^G)} = 0.$$

On the other hand, the condition for guaranteed case is

$$-\frac{s(e_C)}{u'(\widehat{W}_C^G)} - \frac{1-s(e_C)}{u'(\widehat{W}_C^B)} + \delta\frac{1-(1-s(e_M))^N}{u'(\widehat{W}_{M2}^G)} = 0.$$

First, I show that there is $\delta^* \in (0,1)$ such that $(V_2^B)^* < \widehat{\mathcal{V}}$ for a given $\widehat{\mathcal{V}} \in [0,\infty)$. Note that $\widehat{\mathcal{V}}$ and $(V_2^B)^*$ satisfy

$$\frac{1}{u'(\widehat{W}_C^B)} - \delta\left[\frac{s(e_C)}{u'(\widehat{W}_C^{BG})} + \frac{1-s(e_C)}{u'(\widehat{W}_C^{BB})}\right] = 0.$$

Suppose that $\delta = 1$. Then, the equation cannot hold if $\widehat{\mathcal{V}} \leq (V_2^B)^*$ since this inequality implies that $(\widehat{W}_C^{BG})^* > (\widehat{W}_C^{BB})^* \geq (\widehat{W}_C^G)^*$. Here, the first inequality holds since $e_C > 0$ and the last inequality holds as a strict inequality unless $\mathcal{V} = (V_2^B)^* = 0$. Since this is true for all $\widehat{\mathcal{V}} \in [0,\infty)$, there is $\delta^* \in (0,1)$ such that $(V_2^B)^* < \widehat{\mathcal{V}}$.

There are two possible cases when $e_C = 0$: 1) $\widehat{\mathcal{V}}^*(0) > 0$, and 2) $\widehat{\mathcal{V}}^*(0) = 0$, where $\widehat{\mathcal{V}}^*(e_C)$ is the optimal promotion incentive for a given CEO's effort level e_C. First, I show that $\frac{\partial (V_2^B)^*}{\partial \widehat{\mathcal{V}}} > 0$ and $\frac{\partial \widehat{\mathcal{V}}^*(e_C)}{\partial e_C} < 0$ when $\widehat{\mathcal{V}}^*(e_C) > 0$ for $e_C \in (0,1)$. Notice that, by the implicit function theorem,

$$\frac{\partial (V_2^B)^*}{\partial \widehat{\mathcal{V}}} = \frac{\frac{u''(\widehat{W}_C^B)}{u'(\widehat{W}_C^B)^3}}{\frac{u''(\widehat{W}_C^B)}{u'(\widehat{W}_C^B)^3} + \delta\left[s(e_C)\frac{u''(\widehat{W}_C^{BG})}{u'(\widehat{W}_C^{BG})^3} + (1-s(e_C))\frac{u''(\widehat{W}_C^{BB})}{u'(\widehat{W}_C^{BB})^3}\right]} > 0.$$

Also, this means that $\frac{\partial (V_2^B)^*}{\partial \widehat{\mathcal{V}}}$ is less than 1. Likewise, $0 < \frac{\partial (V_2^G)^*}{\partial \widehat{\mathcal{V}}} < 1$. Hence,

$$\frac{\partial \widehat{\mathcal{V}}^*(e_C)}{\partial e_C} = -\frac{\frac{\partial^2 \widehat{F}(\widehat{\mathcal{V}}^*)}{\partial e_C \partial \widehat{\mathcal{V}}}}{\frac{\partial^2 \widehat{F}(\widehat{\mathcal{V}}^*)}{\partial \widehat{\mathcal{V}}^2}} < 0,$$

where

$$\frac{\partial^2 \widehat{F}(\widehat{\mathcal{V}}^*)}{\partial e_C \partial \widehat{\mathcal{V}}} = -\beta \left[\frac{1}{u'((\widehat{W}_C^G)^*)} - \frac{1}{u'((\widehat{W}_C^B)^*)} \right] + s(e_C) \frac{u''((\widehat{W}_C^G)^*)}{u'((\widehat{W}_C^G)^*)^3} \left[(1 - s(e_C)) \frac{g''(e_C)}{\beta} - \frac{\partial (V_2^G)^*}{\partial e_C} \right]$$

$$+ (1 - s(e_C)) \frac{u''((\widehat{W}_C^B)^*)}{u'((\widehat{W}_C^B)^*)^3} \left[-s(e_C) \frac{g''(e_C)}{\beta} - \frac{\partial (V_2^B)^*}{\partial e_C} \right]$$

$$= \beta \left[\frac{1}{u'((\widehat{W}_C^G)^*)} - \frac{1}{u'((\widehat{W}_C^B)^*)} \right]$$

$$- s(e_C) \frac{u''((\widehat{W}_C^G)^*)}{u'((\widehat{W}_C^G)^*)^3} \cdot$$

$$\left[\frac{\delta \left[\beta \left(\frac{1}{u'((\widehat{W}_C^{GG})^*)} - \frac{1}{u'((\widehat{W}_C^{GB})^*)} \right) - s(e_c)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left(\frac{u''((\widehat{W}_C^{GG})^*)}{u'((\widehat{W}_C^{GG})^*)^3} - \frac{u''((\widehat{W}_C^{GB})^*)}{u'((\widehat{W}_C^{GB})^*)^3} \right) \right]}{\frac{u''((\widehat{W}_C^G)^*)}{u'((\widehat{W}_C^G)^*)^3} + \delta \left[s(e_C) \frac{u''((\widehat{W}_C^{GG})^*)}{u'((\widehat{W}_C^{GG})^*)^3} + (1 - s(e_C)) \frac{u''((\widehat{W}_C^{GB})^*)}{u'((\widehat{W}_C^{GB})^*)^3} \right]} \right]$$

$$- (1 - s(e_C)) \frac{u''((\widehat{W}_C^B)^*)}{u'((\widehat{W}_C^B)^*)^3} \cdot$$

$$\left[\frac{\delta \left[\beta \left(\frac{1}{u'((\widehat{W}_C^{BG})^*)} - \frac{1}{u'((\widehat{W}_C^{BB})^*)} \right) - s(e_c)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \left(\frac{u''((\widehat{W}_C^{BG})^*)}{u'((\widehat{W}_C^{BG})^*)^3} - \frac{u''((\widehat{W}_C^{BB})^*)}{u'((\widehat{W}_C^{BB})^*)^3} \right) \right]}{\frac{u''((\widehat{W}_C^B)^*)}{u'((\widehat{W}_C^B)^*)^3} + \delta \left[s(e_C) \frac{u''((\widehat{W}_C^{BG})^*)}{u'((\widehat{W}_C^{BG})^*)^3} + (1 - s(e_C)) \frac{u''((\widehat{W}_C^{BB})^*)}{u'((\widehat{W}_C^{BB})^*)^3} \right]} \right]$$

$$+ \delta s(e_C)(1 - s(e_C)) \frac{g''(e_C)}{\beta} \cdot$$

$$\frac{1}{\left(\frac{u''((\widehat{W}_C^G)^*)}{u'((\widehat{W}_C^G)^*)^3} + \delta \left[s(e_C) \frac{u''((\widehat{W}_C^{GG})^*)}{u'((\widehat{W}_C^{GG})^*)^3} + (1 - s(e_C)) \frac{u''((\widehat{W}_C^{GB})^*)}{u'((\widehat{W}_C^{GB})^*)^3} \right] \right)} \cdot$$

$$\frac{1}{\left(\frac{u''((\widehat{W}_C^B)^*)}{u'((\widehat{W}_C^B)^*)^3} + \delta \left[s(e_C) \frac{u''((\widehat{W}_C^{BG})^*)}{u'((\widehat{W}_C^{BG})^*)^3} + (1 - s(e_C)) \frac{u''((\widehat{W}_C^{BB})^*)}{u'((\widehat{W}_C^{BB})^*)^3} \right] \right)} \cdot$$

$$\left\{ \frac{u''((\widehat{W}_C^G)^*)}{u'((\widehat{W}_C^G)^*)^3} \frac{u''((\widehat{W}_C^B)^*)}{u'((\widehat{W}_C^B)^*)^3} \left[s(e_C) \left(\frac{u''((\widehat{W}_C^{GG})^*)}{u'((\widehat{W}_C^{GG})^*)^3} - \frac{u''((\widehat{W}_C^{BG})^*)}{u'((\widehat{W}_C^{BG})^*)^3} \right) \right. \right.$$

$$\left. + (1 - s(e_C)) \left(\frac{u''((\widehat{W}_C^{GB})^*)}{u'((\widehat{W}_C^{GB})^*)^3} - \frac{u''((\widehat{W}_C^{BB})^*)}{u'((\widehat{W}_C^{BB})^*)^3} \right) \right]$$

$$+ \delta \left(s(e_C) \frac{u''((\widehat{W}_C^{GG})^*)}{u'((\widehat{W}_C^{GG})^*)^3} + (1 - s(e_C)) \frac{u''((\widehat{W}_C^{GB})^*)}{u'((\widehat{W}_C^{GB})^*)^3} \right) \cdot$$

$$\left. \left(s(e_C) \frac{u''((\widehat{W}_C^{BG})^*)}{u'((\widehat{W}_C^{BG})^*)^3} + (1 - s(e_C)) \frac{u''((\widehat{W}_C^{BB})^*)}{u'((\widehat{W}_C^{BB})^*)^3} \right) \left(\frac{u''((\widehat{W}_C^G)^*)}{u'((\widehat{W}_C^G)^*)^3} - \frac{u''((\widehat{W}_C^B)^*)}{u'((\widehat{W}_C^B)^*)^3} \right) \right\}$$

$$< 0,$$

and

$$\frac{\partial^2 \widehat{F}(\widehat{\mathcal{V}}^*)}{\partial \widehat{\mathcal{V}}^2} = s(e_C) \frac{u''((\widehat{W}_C^G)^*)}{u'((\widehat{W}_C^G)^*)^3} \left[1 - \frac{\partial (V_2^G)^*}{\partial \widehat{\mathcal{V}}} \right] + (1 - s(e_C)) \frac{u''((\widehat{W}_C^B)^*)}{u'((\widehat{W}_C^B)^*)^3} \left[1 - \frac{\partial (V_2^B)^*}{\partial \widehat{\mathcal{V}}} \right]$$

$$+ \delta (1 - (1 - s(e_M))^N) P(\mathbf{e}_{-M}) \frac{u''((\widehat{W}_{M2}^G)^*)}{u'((\widehat{W}_{M2}^G)^*)^3} < 0,$$

where I use the following two results

$$\frac{\partial (V_2^G)^*}{\partial e_C} = \frac{1}{\frac{u''(\widehat{W}_C^G)}{u'(\widehat{W}_C^G)^3} + \delta\left[s(e_C)\frac{u''(\widehat{W}_C^{GG})}{u'(\widehat{W}_C^{GG})^3} + (1-s(e_C))\frac{u''(\widehat{W}_C^{GB})}{u'(\widehat{W}_C^{GB})^3}\right]}\Bigg\{(1-s(e_C))\frac{g''(e_C)}{\beta}\frac{u''(\widehat{W}_C^G)}{u'(\widehat{W}_C^G)^3}$$
$$+\delta\Bigg[\beta\left(\frac{1}{u'(\widehat{W}_C^{GG})} - \frac{1}{u'(\widehat{W}_C^{GB})}\right)$$
$$-s(e_c)(1-s(e_C))\frac{g''(e_C)}{\beta}\left(\frac{u''(\widehat{W}_C^{GG})}{u'(\widehat{W}_C^{GG})^3} - \frac{u''(\widehat{W}_C^{GB})}{u'(\widehat{W}_C^{GB})^3}\right)\Bigg]\Bigg\}, \text{ and}$$
$$\frac{\partial (V_2^B)^*}{\partial e_C} = \frac{1}{\frac{u''(\widehat{W}_C^B)}{u'(\widehat{W}_C^B)^3} + \delta\left[s(e_C)\frac{u''(\widehat{W}_C^{BG})}{u'(\widehat{W}_C^{BG})^3} + (1-s(e_C))\frac{u''(\widehat{W}_C^{BB})}{u'(\widehat{W}_C^{BB})^3}\right]}\Bigg\{-s(e_C)\frac{g''(e_C)}{\beta}\frac{u''(\widehat{W}_C^B)}{u'(\widehat{W}_C^B)^3}$$
$$+\delta\Bigg[\beta\left(\frac{1}{u'(\widehat{W}_C^{BG})} - \frac{1}{u'(\widehat{W}_C^{BB})}\right)$$
$$-s(e_c)(1-s(e_C))\frac{g''(e_C)}{\beta}\left(\frac{u''(\widehat{W}_C^{BG})}{u'(\widehat{W}_C^{BG})^3} - \frac{u''(\widehat{W}_C^{BB})}{u'(\widehat{W}_C^{BB})^3}\right)\Bigg]\Bigg\}$$

based on the implicit function theorem.

For the previous results, I exploit the condition $(\widehat{W}_C^G)^* > (\widehat{W}_C^B)^*$, $(\widehat{W}_C^{GG})^*) > (\widehat{W}_C^{BG})^*$, and $(\widehat{W}_C^{GB})^*) > (\widehat{W}_C^{BB})^*$, which all hold since $(V_2^G)^* > (V_2^B)^*$. These imply that

$$\frac{1}{u'((\widehat{W}_C^G)^*)} = \delta\left[\frac{s(e_C)}{u'((\widehat{W}_C^{GG})^*)} + \frac{1-s(e_C)}{u'((\widehat{W}_C^{GB})^*)}\right] > \delta\left[\frac{s(e_C)}{u'((\widehat{W}_C^{BG})^*)} + \frac{1-s(e_C)}{u'((\widehat{W}_C^{BB})^*)}\right] = \frac{1}{u'((\widehat{W}_C^B)^*)}.$$

Here, I show that why the condition, $(V_2^G)^* > (V_2^B)^*$, holds. Suppose $(V_2^G)^* \leq (V_2^B)^*$. Then $(\widehat{W}_C^G)^* \leq (\widehat{W}_C^B)^*$ according to the same logic above. Note that $u((\widehat{W}_C^G)^*) + (V_2^G)^*$ must be strictly greater than $u((\widehat{W}_C^B)^*) + (V_2^B)^*$ in order to induce managers to exert a positive effort. However, two conditions, $(V_2^G)^* \leq (V_2^B)^*$ and $(\widehat{W}_C^G)^*) \leq (\widehat{W}_C^B)^*$, yield $u((\widehat{W}_C^G)^*) + (V_2^G)^* \leq u((\widehat{W}_C^B)^*) + (V_2^B)^*$. Therefore, $(V_2^G)^*$ must be strictly greater than $(V_2^B)^*$.

The next step is to show that there is $\bar{e}_C \in (0,1)$ such that $\widehat{\mathcal{V}}^*(e_C) = 0$ if $e_C \in [\bar{e}_C, 1)$ and $\widehat{\mathcal{V}}^*(e_C) > 0$ if $e_C \in [0, \bar{e}_C)$. Since $\frac{\partial \widehat{\mathcal{V}}^*(e_C)}{\partial e_C} < 0$ when $\widehat{\mathcal{V}}^*(e_C) > 0$, it is enough to show that there is $\bar{e}_C$ such that $\widehat{\mathcal{V}}^*(e_C) = 0$. Recall that

$$\frac{\partial \widehat{F}(\widehat{\mathcal{V}})}{\partial \widehat{\mathcal{V}}} = -\frac{s(e_C)}{u'(\widehat{W}_C^G)} - \frac{1-s(e_C)}{u'(\widehat{W}_C^B)} + \delta\frac{1-(1-s(e_M))^N}{u'(\widehat{W}_{M2}^G)}.$$

When $\widehat{\mathcal{V}} = 0$, the last term is a positive constant regardless of the value of e_C. On the other hand, from the condition

$$\frac{1}{u'(\widehat{W}_C^G)} - \delta\left[\frac{s(e_C)}{u'(\widehat{W}_C^{GG})} + \frac{1-s(e_C)}{u'(\widehat{W}_C^{GB})}\right] = 0$$

it can be shown that the first term approaches negative infinity as e_C approaches one because $(V_2^G)^* \geq \frac{\widehat{\mathcal{V}}}{2} = 0$ and $u((W_C^{GG})^*)$ approaches positive infinity as e_C converges to one.

Hence, there is $\bar{e}_C$ supporting the optimal choice of zero promotion incentive. This result yields that there is $\hat{e}_C \in [0, \bar{e}_C)$ such that $(V_2^B)^* \leq 0$ if $e_C \in [\hat{e}_C, 1)$ since $(V_2^B)^* < \widehat{\mathcal{V}}^*(e_C)$.

The remaining proof is to show that $\mathcal{V}^*(e_C) \geq \widehat{\mathcal{V}}^*(e_C)$ when $e_C \in [\hat{e}_C, 1)$. Notice that, when $\mathcal{V} = \widehat{\mathcal{V}} \in [0, \widehat{\mathcal{V}}^*(e_C)]$ for $e_C \in [\hat{e}_C, 1)$,

$$\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}} > \frac{\partial \widehat{F}(\widehat{\mathcal{V}})}{\partial \widehat{\mathcal{V}}}$$

since $(W_C^G)^* = (\widehat{W}_C^G)^*$, $(W_C^B)^* < (\widehat{W}_C^G)^*$, and $(W_{M1}^G)^* > (\widehat{W}_{M2}^G)^*$. Hence, $\mathcal{V}^*(e_C) \geq \widehat{\mathcal{V}}^*(e_C)$ when $e_C \in [\hat{e}_C, 1)$. Moreover, when $e_C \in [\hat{e}_C, \bar{e}_C]$, $\mathcal{V}^*(e_C) > \widehat{\mathcal{V}}^*(e_C)$ since $\frac{\partial \widehat{F}(\widehat{\mathcal{V}})}{\partial \widehat{\mathcal{V}}}\Big|_{\widehat{\mathcal{V}} = \widehat{\mathcal{V}}^*(e_C)} = 0$.

Consider the second case, $\widehat{\mathcal{V}}^*(0) = 0$. In this case, $\widehat{\mathcal{V}}^*(e_C) = 0$ for every $e_C \in (0, 1)$. Hence, $\mathcal{V}^*(e_C) \geq \widehat{\mathcal{V}}^*(e_C)$ regardless of the value of $e_C \in (0, 1)$.

A.25 Proof of Proposition 14

For brevity, I denote $e_{M11} = e_{M21}$ by e_{M1} and $e_{M12} = e_{M22}$ by e_{M2}. Suppose that $(U_{M1}^2)^* \leq (U_{M2}^2)^*$. Note that, for a given $\mathcal{V}$, the expected utility for the second period, U_{Mi}^2, is determined according to the equation

$$\frac{1}{u'(W_{Mi}^G)} = \delta\left[\frac{s(e_{Mi2})}{u'(W_{Mi}^{GG})} + \frac{1-s(e_{Mi2})}{u'(W_{Mi}^{GB})}\right],$$

$i = 1,\ 2$. This equation and the condition that $(U_{M1}^2)^* \leq (U_{M2}^2)^*$ imply that $(W_{M1}^G)^* \leq (W_{M2}^G)^*$. In order for the inequality to hold, the following must hold:

$$(1-s(e_{C1}))P(\mathbf{e}_{-M1})\mathcal{V} + (1-(1-s(e_{C1}))P(\mathbf{e}_{-M1}))(U_{M1}^2)^* \geq P(\mathbf{e}_{-M1})\mathcal{V} + (1-P(\mathbf{e}_{-M1}))(U_{M2}^2)^*,$$

which implies that $(1 - P(\mathbf{e}_{-M1}))((U_{M1}^2)^* - (U_{M2}^2)^*) + s(e_{C1})P(\mathbf{e}_{-M1})(U_{M1}^2)^* \geq s(e_C)P(\mathbf{e}_{-M})\mathcal{V}$. Then, $(U_{M1}^2)^*$ must be greater than $\mathcal{V}$ since $(U_{M1}^2)^* \leq (U_{M2}^2)^*$. This contradicts to the given condition. Hence, $(U_{M1}^2)^* > (U_{M2}^2)^*$ if $\mathcal{V}^* > (U_{M1}^2)^*$. Moreover, the difference between $u(W_{M1}^G)$ and $u(W_{M2}^G)$

is

$$u(W_{M1}^G) - u(W_{M2}^G) = s(e_{C1})P(\mathbf{e}_{-M})\mathcal{V} - \left[(1 - P(\mathbf{e}_{-M}))(U_{M1}^2 - U_{M2}^2) + s(e_{C1})P(\mathbf{e}_{-M1})U_{M1}^2\right],$$

which has a positive value when $\mathcal{V}^* > (U_{M1}^2)^* > (U_{M2}^2)^*$. That is, $(W_{M1}^G)^* > (W_{M2}^G)^*$.

A.26 Proof of Proposition 15

First, I show that there is a constant $\widehat{N}$ such that $\mathcal{V}^*(N+1) - \mathcal{V}^*(N) \leq 0$ if $N > \widehat{N}$.

For a given $\mathcal{V}$ and N, $e_C(N)$ is determined by

$$\begin{aligned} E\left[f\left(\sum_{i=1}^{N} X_i\right)\right]\beta(\mathcal{G}_C - \mathcal{B}_C) &= \beta(W_C^G(N) - W_C^B(N)) \\ &+ s(e_C(N))(1 - s(e_C(N)))\frac{g''(e_C(N))}{\beta}\left[\frac{1}{u'(W_C^G(N))} - \frac{1}{u'(W_C^B(N))}\right]. \end{aligned} \tag{15}$$

Claim 3. *There is* $\mathcal{M}$ *such that*

$$E\left[f\left(\sum_{i=1}^{N+1} X_i\right)\right] - E\left[f\left(\sum_{i=1}^{N} X_i\right)\right] > \mathcal{M}$$

for every N.

Proof. For brevity, denote

$$E\left[f\left(\sum_{i=1}^{N} X_i\right)\right] = \sum_{i=0}^{N}\binom{N}{i}s(e_M)^i(1 - s(e_M))^{N-i}f(i\mathcal{G}_M + (N-i)\mathcal{B}_M)$$

by $I(N)$. Also, I denote

$$\min_i \left[f(i\mathcal{G}_M + (N+1-i)\mathcal{B}_M) - f(i\mathcal{G} + (N-i)\mathcal{B}_M)\right]$$

by $\underline{f}$. Note that there is $\mathcal{M}_f > 0$ such that $\underline{f} \geq \mathcal{M}_f$ for every N since $f'(x) > 0$ for every $x \geq 0$.

Then,

$$
\begin{aligned}
I(N+1)-I(N) &= \sum_{i=0}^{N}\binom{N}{i}s(e_M)^i(1-s(e_M))^{N-i}\cdot \\
&\left[\frac{N+1}{N+1-i}(1-s(e_M))f(i\mathcal{G}_M+(N+1-i)\mathcal{B}_M)-f(i\mathcal{G}_M+(N-i)\mathcal{B}_M)\right] \\
&+s(e_M)^{N+1}f((N+1)\mathcal{G}_M) \\
&\geq \sum_{i=0}^{N}\binom{N}{i}s(e_M)^i(1-s(e_M))^{N-i}\cdot \\
&\left[\frac{N+1}{N+1-i}(1-s(e_M))-1\right]f(i\mathcal{G}_M+(N-i)\mathcal{B}_M) \\
&+s(e_M)^{N+1}f((N+1)\mathcal{G}_M)+\underline{f}.
\end{aligned}
$$

Denote $\lceil (N+1)s(e_M)\rceil$ by $\hat{s}$. Then,

$$
\left[\frac{N+1}{N+1-i}(1-s(e_M))-1\right]\begin{cases} >0 & \text{if } i\geq \hat{s} \\ \leq 0 \text{ otherwise.} & \end{cases}
$$

There are two possible cases.

1. $(\hat{s}=N+1)$

   Then,

$$
\begin{aligned}
I(N+h)-I(h) &\geq \sum_{i=0}^{N}\binom{N}{i}s(e_M)^i(1-s(e_M))^{N-i}\cdot \\
&\left[\frac{N+1}{N+1-i}(1-s(e_M))-1\right]f(i\mathcal{G}_M+(N-i)\mathcal{B}_M) \\
&+s(e_M)^{N+1}f(N\mathcal{G}_M) \\
&+s(e_M)^{N+1}\left[f((N+1)\mathcal{G}_M)-f(N\mathcal{G}_M)\right]+\underline{f} \\
&>s(e_M)^{N+1}[f((N+h)\mathcal{G}_M)-f(N\mathcal{G}_M)]+\underline{f},
\end{aligned}
$$

   where the last inequality holds since

$$
\sum_{i=0}^{N}\binom{N}{i}s(e_M)^i(1-s(e_M))^{N-i}\left|\left[\frac{(N+h)!(N-i)!}{N!(N+h-i)!}(1-s(e_M))^h-1\right]\right|=s(e_M)^{N+1}.
$$

2. $(\hat{s}<N+1)$

First, notice that if $\frac{N}{N+1} > s(e_M)$, then $\hat{s} < N+1$. That is, if N is sufficiently large, $\hat{s} < N+1$. In this case,

$$\begin{aligned}
I(N+1) - I(N) \geq & \sum_{i=0}^{\hat{s}-1} \binom{N}{i} s(e_M)^i (1-s(e_M))^{N-i} \cdot \\
& \left[\frac{N+1}{N+1-i}(1-s(e_M)) - 1\right] f(i\mathcal{G}_M + (N-i)\mathcal{B}_M) \\
& + \sum_{i=\hat{s}}^{N} \binom{N}{i} s(e_M)^i (1-s(e_M))^{N-i} \cdot \\
& \left[\frac{N+1}{N+1-i}(1-s(e_M)) - 1\right] f(i\mathcal{G}_M + (N-i)\mathcal{B}_M) \\
& + s(e_M)^{N+1} f(N\mathcal{G}_M) + \underline{f} \\
& > \sum_{i=\hat{s}}^{N} \binom{N}{i} s(e_M)^i (1-s(e_M))^{N-i} \left[\frac{N+1}{N+1-i}(1-s(e_M)) - 1\right] \cdot \\
& [f(\hat{s}\mathcal{G}_M + (N-\hat{s})\mathcal{B}_M) - f((\hat{s}-1)\mathcal{G}_M + (N-\hat{s}+1)\mathcal{B}_M)] \\
& + s(e_M)^{N+1} f((N+1)\mathcal{G}_M) + \underline{f} \\
& > \sum_{i=\hat{s}}^{N} \binom{N}{i} s(e_M)^i (1-s(e_M))^{N-i} \left[\frac{N+1}{N+1-i}(1-s(e_M)) - 1\right] \cdot \\
& [f(\hat{s}\mathcal{G}_M + (N-\hat{s})\mathcal{B}_M) - f((\hat{s}-1)\mathcal{G}_M + (N-\hat{s}+1)\mathcal{B}_M)] + \underline{f} \\
& > \underline{f}
\end{aligned}$$

Hence

$$E\left[f\left(\sum_{i=1}^{N+1} X_i\right)\right] - E\left[f\left(\sum_{i=1}^{N} X_i\right)\right] > \underline{f} \geq \mathcal{M}_f > 0.$$

□

This result means that $e_C(N+1) > e_C(N)$ if $\mathcal{V}$ is fixed.

Now, I show that there is $\widetilde{N}$ such that $\frac{\partial F(\mathcal{V})}{\partial \mathcal{V}} < 0$ for every $\mathcal{V} \in [0, \bar{\mathcal{V}}]$. Note that

$$\begin{aligned}
\frac{\partial F(\mathcal{V}|N)}{\partial \mathcal{V}} &= -\frac{s(e_C(\mathcal{V}, N))}{u'(W_C^G(\mathcal{V}, e_C(\mathcal{V}, N))} - \frac{1 - s(e_C(\mathcal{V}, N))}{u'(W_C^B(\mathcal{V}, e_C(\mathcal{V}, N))} + \frac{1-(1-s(e_M))^N}{u'(W_M^G)} \\
&\leq -\frac{s(e_C(\bar{\mathcal{V}}, N))}{u'(W_C^G(0, e_C(\bar{\mathcal{V}}, N))} - \frac{1 - s(e_C(\bar{\mathcal{V}}, N))}{u'(W_C^B(0, e_C(\bar{\mathcal{V}}, N))} + \frac{1-(1-s(e_M))^N}{u'(W_M^G)},
\end{aligned}$$

where

$$W_C^G(\mathcal{V}_1, e_C(\mathcal{V}_2, N)) = \mathcal{V}_1 + g(e_C(\mathcal{V}_2, N)) + (1 - s(e_C(\mathcal{V}_2, N)))\frac{g'(e_C(\mathcal{V}_2, N))}{\beta}$$
$$W_C^B(\mathcal{V}_1, e_C(\mathcal{V}_2, N)) = \mathcal{V}_1 + g(e_C(\mathcal{V}_2, N)) - s(e_C(\mathcal{V}_2, N))\frac{g'(e_C(\mathcal{V}_2, N))}{\beta}$$

and $e_C(\mathcal{V}_2, N)$ satisfies

$$E\left[f\left(\sum_{i=1}^{N} X_i\right)\right]\beta(\mathcal{G}_C - \mathcal{B}_C) = \beta(W_C^G(\mathcal{V}_2, e_C(\mathcal{V}_2, N)) - W_C^B(\mathcal{V}_2, e_C(\mathcal{V}_2, N)) + s(e_C(\mathcal{V}_2, N))(1 - s(e_C(\mathcal{V}_2, N)))\frac{g''(e_C(\mathcal{V}_2, N))}{\beta} \cdot \left[\frac{1}{u'(W_C^G(\mathcal{V}_2, e_C(\mathcal{V}_2, N)))} - \frac{1}{u'(W_C^B(\mathcal{V}_2, e_C(\mathcal{V}_2, N)))}\right]$$

since

$$\frac{\partial}{\partial \mathcal{V}_1}\left[\frac{s(e_C(\mathcal{V}_2, N))}{u'(W_C^G(\mathcal{V}_1, e_C(\mathcal{V}_2, N))} + \frac{1 - s(e_C(\mathcal{V}_2, N))}{u'(W_C^B(\mathcal{V}_1, e_C(\mathcal{V}_2, N))}\right] > 0,$$
$$\frac{\partial}{\partial \mathcal{V}_2}\left[\frac{s(e_C(\mathcal{V}_2, N))}{u'(W_C^G(\mathcal{V}_1, e_C(\mathcal{V}_2, N))} + \frac{1 - s(e_C(\mathcal{V}_2, N))}{u'(W_C^B(\mathcal{V}_1, e_C(\mathcal{V}_2, N))}\right] < 0.$$

Since $\mathcal{V}$ is bounded $\frac{1-(1-s(e_M))^N}{u'(W_M^G)} \leq \frac{1}{u'(\underline{W}_M^G)}$, where $\underline{W}_M^G$ satisfies

$$u(\underline{W}_M^G) = \underline{U}_M + g(e_M) + (1 - s(e_M))\frac{g'(e_M)}{\beta}.$$

Moreover, there is $\hat{e}_C \in (0, 1)$ such that

$$\frac{s(e_C)}{u'(W_C^G(0, e_C))} + \frac{1 - s(e_C)}{u'(W_C^B(0, e_C)} > \frac{1}{u'(\underline{W}_M^G)}$$

if $e_C \geq \hat{e}_C$ since $\lim_{e_C \to 1} W_C^G(0, e_C) = \infty$. Since there is N_1 such that $e_C(\bar{\mathcal{V}}, N) \geq \hat{e}_C$ if $N \geq N_1$,

$$\begin{aligned}
\frac{\partial F(\mathcal{V}|N)}{\partial \mathcal{V}} &\leq -\frac{s(e_C(\bar{\mathcal{V}}, N))}{u'(W_C^G(0, e_C(\bar{\mathcal{V}}, N))} - \frac{1 - s(e_C(\bar{\mathcal{V}}, N))}{u'(W_C^B(0, e_C(\bar{\mathcal{V}}, N))} + \frac{1 - (1 - s(e_M))^N}{u'(W_M^G)} \\
&< -\frac{s(e_C(\bar{\mathcal{V}}, N))}{u'(W_C^G(0, e_C(\bar{\mathcal{V}}, N))} - \frac{1 - s(e_C(\bar{\mathcal{V}}, N))}{u'(W_C^B(0, e_C(\bar{\mathcal{V}}, N))} + \frac{1}{u'(\underline{W}_M^G)} \\
&< 0
\end{aligned}$$

when $N \geq N_1$.

Hence, $\mathcal{V}^* = 0$ if $N \geq N_1$. Also, this implies that there is $N_2 < N_1$ such that $\mathcal{V}^*(N+1) - \mathcal{V}^*(N) < 0$ when $N \in [N_2, N_1 - 1]$ if there is $N^* < N_1$ such that $\mathcal{V}^*(N^*) > 0$.

Moreover, $e_C^*(N+1) - e_C^*(N) > 0$ when $N \geq N_2$ since

$$\frac{\partial E(\mathcal{V}, e_C)}{\partial \mathcal{V}} > 0 \text{ and } \frac{\partial E(\mathcal{V}, e_C)}{\partial e_C} > 0,$$

where

$$\begin{aligned} E(\mathcal{V}, e_C) \equiv\ & \beta(W_C^G(\mathcal{V}, e_C) - W_C^B(\mathcal{V}, e_C)) \\ & + s(e_C)(1 - s(e_C))\frac{g''(e_C)}{\beta}\left[\frac{1}{u'(W_C^G(\mathcal{V}, e_C))} - \frac{1}{u'(W_C^B(\mathcal{V}, e_C))}\right] \end{aligned}$$

comes from the right hand side of (15).

A.27 Proof of Proposition 16

First, note that constraints regarding managers give the following results:

$$\begin{aligned} u(W_C) &= \mathcal{V} \\ u(W_M^G) &= \underline{U}_M + g(e_L) + (1 - s_L(e_L))\frac{g'(e_L)}{\underline{\beta}} - P(\mathbf{e}_{-M})\mathcal{V}, \\ u(W_M^B) &= \underline{U}_M + g(e_L) - s_L(e_L)\frac{g'(e_L)}{\underline{\beta}} - R(\mathbf{e}_{-M})\mathcal{V}, \text{ and} \\ e_H &= e_L\frac{\bar{\beta}}{\underline{\beta}} \end{aligned}$$

for a given $(e_L, \mathcal{V})$. From now on, I use subscript 1 for promotion rule 1 and subscript 2 for promotion 2 in order to distinguish two problems. For a given e_{L1} and e_{L2}, denote the optimal $\mathcal{V}$ by $\mathcal{V}_1^*(e_{L1})$ under promotion rule 1 and $\mathcal{V}_2^*(e_{L1})$ under promotion rule 2, Also, denote the firm's objective function under promotion rule 1 and promotion rule 2 by $F_1(e_{L1}|\gamma)$ and $F_2(e_{L2}|\gamma)$, respectively, for a given γ. That is,

$$\begin{aligned} F_1(e_L|\gamma) = \gamma\left[(\bar{\beta} - \underline{\beta})H_1(e_L) + \underline{\beta}\right] - W_{C1} &+ 2[qs_H(e_H) + (1-q)s_L(e_L)](\mathcal{G} - W_{M1}^G) \\ &+ 2[1 - qs_H(e_H) - (1-q)s_L(e_L)](\mathcal{B} - W_{M1}^B), \text{ and} \\ F_2(e_L|\gamma) = \gamma\left[(\bar{\beta} - \underline{\beta})H_2(e_L) + \underline{\beta}\right] - W_{C2} &+ 2[qs_H(e_H) + (1-q)s_L(e_L)](\mathcal{G} - W_{M2}^G) \\ &+ 2[1 - qs_H(e_H) - (1-q)s_L(e_L)](\mathcal{B} - W_{M2}^B), \end{aligned}$$

where

$$\begin{aligned} H_1(e_{L1}) = &\frac{qs_H(e_{H1})}{qs_H(e_{H1}) + (1-q)s_L(e_{L1})}\left[2(qs_H(e_{H1}) + (1-q)s_L(e_{L1})) - (qs_H(e_{H1}) + (1-q)s_L(e_{L1}))^2\right] \\ &+ q\left[1 - 2(qs_H(e_{H1}) + (1-q)s_L(e_{L1})) + (qs_H(e_{H1}) + (1-q)s_L(e_{L1}))^2\right], \text{ and} \\ H_2(e_{L2}) = &\ q + q(1-q)(s_H(e_{H2}) - s_L(e_{L2})). \end{aligned}$$

Then, the firm's problem is to choose e_L in order to maximize its objective function. Notice that, for a given e_{Lj}, $\mathcal{V}_j^*(e_{Lj})$, $j = 1$ and 2, satisfies

$$\begin{aligned}
\frac{\partial F_1(e_{L1}|\gamma)}{\partial \mathcal{V}_1} &= -\frac{1}{u'(W_{C1})} + \frac{2(qs_H(e_{H1}) + (1-q)s_L(e_{L1}))}{u'(W_{M1}^G)} P(\mathbf{e}_{-M1}) = 0, \text{ and} \\
\frac{\partial F_2(e_{L2}|\gamma)}{\partial \mathcal{V}_2} &= -\frac{1}{u'(W_{C2})} + \frac{2(qs_H(e_{H2}) + (1-q)s_L(e_{L2}))}{u'(W_{M2}^G)} P(\mathbf{e}_{-M2}) \\
&\quad + \frac{2(1 - qs_H(e_{H2}) - (1-q)s_L(e_{L2}))}{u'(W_{M2}^B)} R(\mathbf{e}_{-M2}) = 0.
\end{aligned}$$

Also, the first order conditions with respect to e_{Lj} are

$$\begin{aligned}
\frac{\partial F_1(e_{L1}|\gamma)}{\partial e_{L1}} &= \gamma(\bar{\beta} - \underline{\beta})\frac{\partial H_1(e_{L1})}{\partial e_{L1}} + 2\left(q\frac{\bar{\beta}^2}{\underline{\beta}} + (1-q)\underline{\beta}\right)[\mathcal{G} - \mathcal{B} - (W_{M1}^G - W_{M1}^B)] \\
&\quad - 2\frac{\kappa}{\underline{\beta}}\left[(1 - s_L(e_{L1}))\frac{qs_H(e_{H1}) + (1-q)s_L(e_{L1})}{u'(W_{M1}^G)} - s_L(e_{L1})\frac{1 - qs_H(e_{H1}) - (1-q)s_L(e_{L1})}{u'(W_{M1}^B)}\right] \\
&\quad + 2\frac{qs_H(e_{H1}) + (1-q)s_L(e_{L1})}{u'(W_{M1}^G)}\frac{\partial P(\mathbf{e}_{-M1})}{\partial e_{L1}}\mathcal{V}_1^*(e_{L1}), \text{ and} \\
\frac{\partial F_2(e_{L2}|\gamma)}{\partial e_{L2}} &= \gamma(\bar{\beta} - \underline{\beta})\frac{\partial H_2(e_{L2})}{\partial e_{L2}} + 2\left(q\frac{\bar{\beta}^2}{\underline{\beta}} + (1-q)\underline{\beta}\right)[\mathcal{G} - \mathcal{B} - (W_{M2}^G - W_{M2}^B)] \\
&\quad - 2\frac{\kappa}{\underline{\beta}}\left[(1 - s_L(e_{L2}))\frac{qs_H(e_{H2}) + (1-q)s_L(e_{L2})}{u'(W_{M2}^G)} - s_L(e_{L2})\frac{1 - qs_H(e_{H2}) - (1-q)s_L(e_{L2})}{u'(W_{M2}^B)}\right] \\
&\quad + 2\frac{qs_H(e_{H2}) + (1-q)s_L(e_{L2})}{u'(W_{M2}^G)}\frac{\partial P(\mathbf{e}_{-M2})}{\partial e_{L2}}\mathcal{V}_2^*(e_{L2}) \\
&\quad + 2\frac{1 - qs_H(e_{H2}) - (1-q)s_L(e_{L2})}{u'(W_{M2}^B)}\frac{\partial R(\mathbf{e}_{-M2})}{\partial e_{L2}}\mathcal{V}_2^*(e_{L2}).
\end{aligned}$$

According to Milgrom and Shannon (1994), $\frac{\partial e_{L1}^*}{\partial \gamma} \geq 0$ and $\frac{\partial e_{L1}^*}{\partial \gamma} \geq 0$ since

$$\begin{aligned}
\frac{\partial^2 F_1(e_{L1}|\gamma)}{\partial e_{L1}\partial\gamma} &= (\bar{\beta} - \underline{\beta})\frac{\partial H_1(e_{L1})}{\partial e_{L1}} \\
&= 2(\bar{\beta} - \underline{\beta})q(1-q))\left(\frac{\bar{\beta}^2}{\underline{\beta}} - \underline{\beta}\right)[1 - qs_H(e_{H1}) - (1-q)s_L(e_{L1})] > 0, \text{ and} \\
\frac{\partial^2 F_2(e_{L2}|\gamma)}{\partial e_{L2}\partial\gamma} &= (\bar{\beta} - \underline{\beta})\frac{\partial H_2(e_{L2})}{\partial e_{L2}} \\
&= (\bar{\beta} - \underline{\beta})q(1-q)\left(\frac{\bar{\beta}^2}{\underline{\beta}} - \underline{\beta}\right) > 0.
\end{aligned}$$

Moreover, these inequalities imply that $\frac{\partial e_{L1}}{\partial \gamma} > 0$ and $\frac{\partial e_{L1}}{\partial \gamma} > 0$ if $e_{L1}^* \in \left(0, \frac{\beta}{\bar{\bar{\beta}}}\right)$ and $e_{L2}^* \in \left(0, \frac{\beta}{\bar{\bar{\beta}}}\right)$, respectively, according to Edlin and Shannon (1998).

Also, there is γ_1^* such that $e_{L1}^* = \frac{\underline{\beta}}{\bar{\beta}}$, which means that $e_{H1} = 1$, if $\gamma \geq \gamma_1^*$ since $\frac{\partial F_1(e_L|\gamma)}{\partial e_L}\Big|_{e_{L1}=1}$ is a strictly increasing function in γ and $\lim_{\gamma\to\infty} \frac{\partial F_1(e_L|\gamma)}{\partial e_L}\Big|_{e_{L1}=1} = \infty$.

Moreover, by the envelope theorem,

$$\frac{\partial F_1(e_{L1}^*|\gamma)}{\partial \gamma} = (\bar{\beta} - \underline{\beta})H_1(e_{L1}^*) + \underline{\beta}, \text{ and } \frac{\partial F_2(e_{L2}^*|\gamma)}{\partial \gamma} = (\bar{\beta} - \underline{\beta})H_2(e_{L2}^*) + \underline{\beta}.$$

Since $H_1(e_{L1}) > H_2(e_{L1})$ when $e_{L1} = e_{L2}$, there is $\hat{e}_{L1} \in \left(0, \frac{\underline{\beta}}{\bar{\beta}}\right)$ such that $H_1(\hat{e}_{L1}) > H_2\left(\frac{\underline{\beta}}{\bar{\beta}}\right)$. Hence, there is $\hat{\gamma}$ such that $F_1(e_{L1}^*|\gamma) > F_2(e_{L2}^*|\gamma)$ if $\gamma \geq \hat{\gamma}$.

Now, I show that $F_1(e_{L1}^*|\gamma) < F_2(e_{L2}^*|\gamma)$ when $\gamma = 0$. This is true since

$$F_2(e_{L2}^*|\gamma) \geq F_2(e_{L1}^*|\gamma) > F_1(e_{L1}^*|\gamma).$$

The second inequality holds since $(W_{M1}^G)^* = W_{M2}^G$ and $(W_{M1}^B)^* > W_{M2}^B$ if $(e_{L2}, \mathcal{V}_2) = (e_{L1}^*, \mathcal{V}_1^*)$. Hence, $\hat{\gamma} > 0$.

Appendix B Firm's Problems in Detail

B.1 The Firm's Problem in Section 7.1

Under this extension, the firm's problem is to choose $\mathcal{V} \in [0, \infty)$ maximizing $F(\mathcal{V})$ defined by

$$\begin{aligned} F(\mathcal{V}) \equiv \max_{\{(W_C^G, W_C^B), (W_M^G, W_M^B), (W_M^{GG}, W_M^{GB}),\}} & s(e_C)(\mathcal{G}_C - W_C^G) + (1 - s(e_C))(\mathcal{B}_C - W_C^B) \\ & + N\left[s(e_{M1})(\mathcal{G}_M - W_M^G) + (1 - s(e_{M1}))(\mathcal{B}_M - W_M^B)\right] \\ & + \delta(Ns(e_{M1}) - 1 + (1 - s(e_{M1}))^N)[s(e_{M2})(\mathcal{G}_M - W_M^{GG}) + (1 - s(e_{M2}))(\mathcal{B}_M - W_M^{GB})] \end{aligned}$$

subject to

$$E[\mathcal{U}(W_C^G, W_C^B, e_{C1})] = \mathcal{V} \ (IR_C),$$

$$E[\mathcal{U}(W_M^G, W_M^B, e_{M1})] + s(e_{M1})\{P(\mathbf{e}_{-M1})\mathcal{V} + (1 - P(\mathbf{e}_{-M1}))E[\mathcal{U}(W_M^{GG}, W_M^{GB}, e_{M2})]\} = \underline{U}_M \ (IR_M),$$

$$e_C \in \arg\max_{\hat{e}} E[\mathcal{U}(W_C^G, W_C^B, \hat{e}] \ (IC_C),$$

$$e_{M1} \in \arg\max_{\hat{e}} E[\mathcal{U}(W_M^G, W_M^B, \hat{e})] + s(\hat{e})\{P(\mathbf{e}_{-M1})\mathcal{V} + (1 - P(\mathbf{e}_{-M1}))E[\mathcal{U}(W_M^{GG}, W_M^{GB}, e_{M2})]\} \ (IC_{M1}),$$

$$e_{M2} \in \arg\max_{\hat{e}} E[\mathcal{U}(W_M^{GG}, W_M^{GB}, \hat{e})] \ (IC_{M2}),$$

where

$$E[\mathcal{U}(W^G, W^B, e)] = s(e)u(W^G) + (1 - s(e))u(W^B) - g(e).$$

Note that the expected number of senior managers in the second period is

$$\begin{aligned}\sum_{k=1}^{N} \binom{N}{k} s(e_{M1})^k (1 - s(e_{M1}))^{N-k}(k-1) &= \sum_{k=0}^{N} \binom{N}{k} k s(e_{M1})^k (1 - s(e_{M1}))^{N-k} \\ &\quad - \sum_{k=0}^{N} \binom{N}{k} s(e_{M1})^k (1 - s(e_{M1}))^{N-k} + (1 - s(e_{M1}))^N \\ &= N s(e_{M1}) - 1 + (1 - s(e_{M1}))^N.\end{aligned}$$

B.2 The Firm's Problem in Section 7.2

The objective of the firm is to choose $\mathcal{V} \in [0, \infty)$ maximizing $F(\mathcal{V})$ defined by

$$\begin{aligned}F(\mathcal{V}) \equiv \max_{\mathcal{A}} \; & s(e_{C1})(\mathcal{G}_C - W_C^G) + (1 - s(e_{C1}))(\mathcal{B}_C - W_C^B) \\ & + \delta s(e_{C1})[s(e_{C2})(\mathcal{G}_C - W_C^{GG}) + (1 - s(e_{C2}))(\mathcal{B}_C - W_C^{GB})] \\ & + N\left[s(e_{M1})(\mathcal{G}_M - W_{M1}^G) + (1 - s(e_{M1}))(\mathcal{B}_M - W_{M1}^B)\right] \\ & + \delta N s(e_{C1})\left[s(e_{M2})(\mathcal{G}_M - W_{M2}^G) + (1 - s(e_{M2}))(\mathcal{B}_M - W_{M2}^B)\right]\end{aligned}$$

subject to

$$\begin{aligned}E[\mathcal{U}(W_C^G, W_C^B, e_{C1})] + s(e_{C1})E[\mathcal{U}(W_C^{GG}, W_C^{GB}, e_{C2})] &= \mathcal{V} \;\; (IR_C) \\ E[\mathcal{U}(W_{M1}^G, W_{M1}^B, e_{M1})] + s(e_{M1})(1 - s(e_{C1}))P(\mathbf{e}_{-M1})\mathcal{V} &= \underline{U}_M \;\; (IR_{M1}), \\ E[\mathcal{U}(W_{M2}^G, W_{M2}^B, e_{M2})] + s(e_{M2})P(\mathbf{e}_{-M2})\mathcal{V} &= \underline{U}_M \;\; (IR_{M2}) \\ e_{C1} \in \arg\max_{\hat{e}} E[\mathcal{U}(W_C^G, W_C^B, \hat{e}] + s(\hat{e})E[\mathcal{U}(W_C^{GG}, W_C^{GB}, e_{C2})] & \;\; (IC_{C1}) \\ e_{C2} \in \arg\max_{\hat{e}} E[\mathcal{U}(W_C^{GG}, W_C^{GB}, \hat{e})] & \;\; (IC_{C2}) \\ e_{M1} \in \arg\max_{\hat{e}} E[\mathcal{U}(W_{M1}^G, W_{M1}^B, \hat{e})] + s(\hat{e})(1 - s(e_{C1}))P(\mathbf{e}_{-M1})\mathcal{V} & \;\; (IC_{M1}), \\ e_{M2} \in \arg\max_{\hat{e}} E[\mathcal{U}(W_{M2}^G, W_{M2}^B, \hat{e})] + s(\hat{e})P(\mathbf{e}_{-M2})\mathcal{V} & \;\; (IC_{M2}),\end{aligned}$$

where

$$\mathcal{A} = \{(W_C^G, W_C^B, W_C^{GG}, W_C^{GB}), (W_{M1}^G, W_{M2}^B), (W_{M2}^G, W_{M2}^B)\}, \text{ and}$$
$$E[\mathcal{U}(W^G, W_B, e)] = s(e)u(W^G) + (1 - s(e))u(W^B) - g(e).$$

When the CEO's individual rationality constraint binds at $\mathcal{V}$, the compensation scheme $(W_C^G, W_C^B, W_C^{GG}, W_C^{GB})$ for the CEO satisfies

$$u(W_C^G) = \mathcal{V} + g(e_{C1}) + (1 - s(e_{C1}))\frac{g'(e_{C1})}{\beta} - V_2,$$
$$u(W_C^B) = \mathcal{V} + g(e_{C1}) - s(e_{C1})\frac{g'(e_{C1})}{\beta},$$
$$u(W_C^{GG}) = V_2 + g(e_{C2}) + (1 - s(e_{C2}))\frac{g'(e_{C2})}{\beta},$$
$$u(W_C^{GB}) = V_2 + g(e_{C2}) - s(e_{C2})\frac{g'(e_{C2})}{\beta},$$

where

$$V_2 = s(e_{C2})u(W_C^{GG}) + (1 - s(e_{C2}))u(W_C^{GB}) - g(e_{C2})$$

is the successful CEO's expected utility in the second period.

On the other hand, the compensation schemes for managers are characterized by

$$u(W_{M1}^G) = \underline{U}_M + g(e_{M1}) + (1 - s(e_{M1}))\frac{g'(e_{M1})}{\beta} - (1 - s(e_{C1}))P(\mathbf{e}_{-M1})\mathcal{V},$$
$$u(W_{M1}^B) = \underline{U}_M + g(e_{M1}) - s(e_{M1})\frac{g'(e_{M1})}{\beta},$$
$$u(W_{M2}^G) = \underline{U}_M + g(e_{M2}) + (1 - s((e_{M2}))\frac{g'((e_{M2})}{\beta} - P(\mathbf{e}_{-M2})\mathcal{V}, \text{ and}$$
$$u(W_{M2}^B) = \underline{U}_M + g(e_{M2}) - s(e_{M2})\frac{g'((e_{M2})}{\beta}.$$

B.3 The Firm's Problem in Section 9

The expected profit from two managers is

$$\begin{aligned}E[\Pi_M|(e_H, e_L, W_M^G, W_M^B)] &= q^2 2[s_H(e_H)(\mathcal{G}_M - W_M^G) + (1 - s_H(e_H))(\mathcal{B}_M - W_M^B)] \\ &+ 2q(1-q)[s_H(e_H)(\mathcal{G}_M - W_M^G) + (1 - s_H(e_H))(\mathcal{B}_M - W_M^B) \\ &+ s_L(e_L)(\mathcal{G}_M - W_M^G) + (1 - s_L(e_L))(\mathcal{B}_M - W_M^B)] \\ &+ (1-q)^2 2[s_L(e_L)(\mathcal{G}_M - W_M^G) + (1 - s_L(e_L))(\mathcal{B}_M - W_M^B)].\end{aligned}$$

Also, the choice of promotion rule determines the expected β when the firm requires e_H and e_L from high-type and low-type managers according to 1) when the firm uses promotion rule 1, $E[\beta|e_H, e_L]$ is equal to

$$E^{P1}[\beta|e_H, e_L] = \frac{qs_H(e_H)}{qs_H(e_H) + (1-q)s_L(e_L)} \left[2(qs_H(e_H) + (1-q)s_L(e_L)) - (qs_H(e_H) + (1-q)s_L(e_L))^2\right] + q\left[1 - 2(qs_H(e_H) + (1-q)s_L(e_L)) + (qs_H(e_H) + (1-q)s_L(e_L))^2\right],$$

2) while the expectation has the following value

$$E^{P2}[\beta|e_H, e_L] = q + q(1-q)(s_H(e_H) - s_L(e_L))$$

if the firm adopts promotion rule 2.