

**Combining sign and long-run parametric restrictions in a weak instrument case:
Monetary policy and exchange rates.**

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Abstract

In a SVAR for four small open economies, sign restrictions together with parametric restrictions are used to separate the shocks using the instrumental variable (IV) method of Ouliaris and Pagan (2016). The long-run restriction we utilise is that the monetary policy shock has a zero long-run effect on the real exchange rate. Because there is a 'weak' instrument problem associated with the long-run restriction, the IV method breaks down. This paper develops a procedure in conjunction with their method which circumvents this problem. When this procedure is implemented, we find that none of the accepted responses for Australia and Canada show an exchange rate puzzle. For New Zealand fewer than five percent show an exchange rate puzzle, while for the United Kingdom it is substantially more. In Australia, Canada and New Zealand, there is a systematic monetary policy response to a real exchange rate shock but not in the United Kingdom.

Key Words: sign restrictions, long-run parametric restrictions, weak instruments, long-run and contemporaneous impact matrices, exchange rate puzzles, monetary policy reaction

JEL Classification: C32, C36, C51, F41

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1. Introduction

In a recent contribution, Ouliaris and Pagan (2016) show how to implement sign and parametric restrictions in a structural equation system. The complete system of equations can be estimated by maximum likelihood methods or by instrumental variable (IV) methods, which may be easier to use in some software packages. In the case of a long-run parametric restriction, the IV approach may break down due to the problem of a 'weak' instrument, which occurs when the correlation between the variable and its instrument is very low so that the variable's coefficient is imprecisely estimated. The consequence is that the long-run parametric restriction may not be evident in the accepted responses from the estimated model, even though the long-run restriction is part of the structural model by design.

In this paper, we first show that this occurs in a SVAR for four small open economies: Australia, Canada, New Zealand and the United Kingdom. The monetary policy shock is identified by both sign restrictions and a zero long-run parametric restriction. The parametric restriction is that the monetary policy shock has a zero long-run effect on the real exchange rate. This restriction is built into the specification of the SVAR and shows up in the structural equation for the real exchange rate where the contemporaneous change (and not the contemporaneous level) of the interest rate appears as a right-hand side variable. The instrument for the contemporaneous change in the interest rate is the lagged level of the interest rate. This instrument is 'weak' because the interest rate is a highly persistent variable so that it has a low correlation with its first difference. As a consequence, a large number of the exchange rate responses to the monetary policy shock in the sign restrictions methodology do not converge to zero as required by the long-run parametric restriction so the IV approach breaks down.

This paper develops an alternative method which avoids the weak instrument problem. In this method, the structural equation for the real exchange rate has the level of the contemporaneous interest rate as a right-hand side variable and an estimate of its coefficient is derived from an expression that enforces the long-run zero restriction. This expression is obtained from the relationship between the long-run impact matrices of the structural and reduced-form shocks and involves the contemporaneous structural impact matrix in the Ouliaris and Pagan sign restrictions framework. An estimate for this interest rate coefficient is obtained from the expression on each 'draw' in sign restrictions. Using this method, we find that the response of the real exchange rate to the monetary policy shock converges to zero on each 'draw'. We report the results from this estimated SVAR as it circumvents the 'weak' instrument problem.

The SVAR consists of the five-variables that were utilised by Bjørnland (2009) to investigate the relationship between monetary policy and the real exchange rate in several small economies. The structural shocks in Bjørnland's model were identified by recursive zero restrictions on their contemporaneous impacts and by the restriction that the monetary policy shock has a zero long-run effect on the real exchange rate. In Bjørnland's model, the imposition of the long-run restriction means that it is not necessary to restrict the contemporaneous response of the exchange rate to a monetary policy shock nor the contemporaneous response of the interest rate to an exchange rate shock. That is the case

in our model where it is not necessary to sign restrict these responses and that is the advantage of utilising the long-run restriction. The advantage of utilising sign restrictions is that a range of response outcomes can be established from the sets of accepted responses. Because each accepted response to a shock is equally valid under sign restrictions, it is the range of accepted responses to the shock which is of interest. Full parametric identification of the shocks as in Bjørnland's model, produces a single set of impulse response functions.

The main empirical findings of the paper are as follows. First, for Australia and Canada, all of the accepted responses show an impact appreciation in response to a monetary policy shock which raises the interest rate. None of the accepted responses show an 'exchange rate puzzle', which is said to occur if there is instead a depreciation on impact (e.g. Eichenbaum and Evans (1995), Kim and Roubini (2000), Scholl and Uhlig (2008)). For New Zealand, only a very small fraction of accepted responses (about 4 percent) show an exchange rate puzzle while for the United Kingdom it is one-fifth. Second, in response to an exchange rate shock that depreciates the currency, all of the accepted responses in Canada and New Zealand, and nearly all in Australia, show a rise in the interest rate on impact while for the United Kingdom, one-fifth show a fall in the interest rate. The monetary authority in Australia, Canada and New Zealand appears to respond systematically to an exchange rate shock but that does not appear to be the case for the United Kingdom where one-fifth of the impact responses have the interest rate moving in the counter direction.

The paper has the following structure. Section 2 describes the structural model and how the zero long-run restriction is imposed on it. Section 3 applies the Ouliaris and Pagan (2016) sign restrictions method to the SVAR and describes the IV estimation of the model on each 'draw'. Section 4 presents the sign restrictions that are applied to the impulse responses. Section 5 shows that there is a 'weak' instrument problem in the SVARs. Section 6 develops the procedure which is used to circumvent the 'weak' instrument problem in the framework of the sign restriction methodology of Ouliaris and Pagan (2016). This procedure is used to generate the results which are reported in Section 7. Lastly, Section 8 concludes.

2. The structural model

Following Bjørnland (2009), the variables that enter the structural model are a country's log real GDP (y_t), inflation (π_t), the interest rate (i_t), the trade-weighted foreign interest rate (i_t^*), and the first difference of the log of the trade-weighted real exchange rate (Δq_t).

Inflation is the change in the consumer price index from the same quarter of the previous year, measured as percent per annum. The domestic interest rate is the 3-month rate and the foreign interest rate is a trade-weighted average of the 3-month rates for the country's major trading partners, expressed as percent per annum. The real exchange rate is defined as the number of 'domestic' goods per unit of the 'foreign' good so that a decrease in its value represents a real appreciation of the domestic country's real exchange rate. The log of the real exchange rate is treated as the only I(1) variable in the system and thus appears in first difference form. Data on the five variables is taken from Fisher and Huh (2016) for four small open economies; Australia, Canada, New Zealand and the United Kingdom. The data is quarterly for the period 1994:Q1 to 2014:Q1.

The foreign interest rate shock (denoted FI) is identified under four contemporaneous zero restrictions. The other orthogonal shocks in the structural model are obtained by imposing a long-run parametric restriction together with sign restrictions to separate the shocks as

either an aggregate supply (AS), an aggregate demand (AD), a monetary policy (MP) or a real exchange rate (RX) shock. The parametric restriction is that the monetary policy shock has a zero long-run effect on the real exchange rate. We write the structural equations with one lag (i.e. for a SVAR(1)), although in the actual estimation, each equation has two lags and includes both a constant and time trend, following Bjørnland; generalisation of the discussion below to this case is straightforward. The structural model is:

$$i_t^* = a_{12}^0 y_t + a_{13}^0 \pi_t + a_{14}^0 i_t + a_{15}^0 \Delta q_t + a_{11}^1 i_{t-1}^* + a_{12}^1 y_{t-1} + a_{13}^1 \pi_{t-1} + a_{14}^1 i_{t-1} + a_{15}^1 \Delta q_{t-1} + \varepsilon_{1t} \quad (1)$$

$$y_t = a_{21}^0 i_t^* + a_{23}^0 \pi_t + a_{24}^0 i_t + a_{25}^0 \Delta q_t + a_{21}^1 i_{t-1}^* + a_{22}^1 y_{t-1} + a_{23}^1 \pi_{t-1} + a_{24}^1 i_{t-1} + a_{25}^1 \Delta q_{t-1} + \varepsilon_{2t} \quad (2)$$

$$\pi_t = a_{31}^0 i_t^* + a_{32}^0 y_t + a_{34}^0 i_t + a_{35}^0 \Delta q_t + a_{31}^1 i_{t-1}^* + a_{32}^1 y_{t-1} + a_{33}^1 \pi_{t-1} + a_{34}^1 i_{t-1} + a_{35}^1 \Delta q_{t-1} + \varepsilon_{3t} \quad (3)$$

$$i_t = a_{41}^0 i_t^* + a_{42}^0 y_t + a_{43}^0 \pi_t + a_{45}^0 \Delta q_t + a_{41}^1 i_{t-1}^* + a_{42}^1 y_{t-1} + a_{43}^1 \pi_{t-1} + a_{44}^1 i_{t-1} + a_{45}^1 \Delta q_{t-1} + \varepsilon_{4t} \quad (4)$$

$$\Delta q_t = a_{51}^0 i_t^* + a_{52}^0 y_t + a_{53}^0 \pi_t + a_{54}^0 i_t + a_{51}^1 i_{t-1}^* + a_{52}^1 y_{t-1} + a_{53}^1 \pi_{t-1} + a_{54}^1 i_{t-1} + a_{55}^1 \Delta q_{t-1} + \varepsilon_{5t} \quad (5)$$

We treat the foreign interest rate as exogenous in the equations for GDP, inflation, the interest rate and the real exchange rate by imposing the exclusion restrictions $a_{12}^0 = 0$, $a_{13}^0 = 0$, $a_{14}^0 = 0$ and $a_{15}^0 = 0$ on Eq. (1) which becomes:

$$i_t^* = a_{11}^1 i_{t-1}^* + a_{12}^1 y_{t-1} + a_{13}^1 \pi_{t-1} + a_{14}^1 i_{t-1} + a_{15}^1 \Delta q_{t-1} + \varepsilon_{1t} \quad (6)$$

These restrictions isolate ε_{1t} as the foreign interest rate shock and can be shown to imply that the other shocks, i.e. the AS, AD, MP and RX shocks, do not have a contemporaneous effect on the foreign interest rate (see Fn. 4 below).

Following Fisher, Huh and Pagan (2016), the long-run zero parametric restriction is imposed by using the method of Shapiro and Watson (1988) of replacing Eq. (5) with:

$$\Delta q_t = a_{51}^0 i_t^* + a_{52}^0 y_t + a_{53}^0 \pi_t + a_{54}^0 \Delta i_t + a_{51}^1 i_{t-1}^* + a_{52}^1 y_{t-1} + a_{53}^1 \pi_{t-1} + a_{55}^1 \Delta q_{t-1} + \varepsilon_{5t} \quad (7)$$

This follows because the long-run restriction is $a_{54}^0 + a_{54}^1 = 0$, and imposing it on Eq. (5) produces Eq. (7), where the contemporaneous change in the interest rate and not its level now appears as a right-hand side variable. The orthogonal shock in Eq. (4), ε_{4t} , will have a zero long-run effect on the real exchange rate. It is the only candidate for the monetary policy shock and it is subject to sign restrictions as are the other shocks, apart from the foreign interest rate shock which is isolated by the contemporaneous restrictions alone.

3. Generating impulse responses for sign restrictions

In this section, we describe the Ouliaris and Pagan (2016) method for generating the large sets of impulse response functions to be judged against the sign restrictions. Consider the structural system given by Eqs. (6), (2), (3), (4) and (7). Define the matrix of contemporaneous coefficients on the variables as:

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\bar{a}_{21}^0 & 1 & -\bar{a}_{23}^0 & -\bar{a}_{24}^0 & -\bar{a}_{25}^0 \\ -\bar{a}_{31}^0 & -\bar{a}_{32}^0 & 1 & -\bar{a}_{34}^0 & -\bar{a}_{35}^0 \\ -\bar{a}_{41}^0 & -\bar{a}_{42}^0 & -\bar{a}_{43}^0 & 1 & -\bar{a}_{45}^0 \\ -\bar{a}_{51}^0 & -\bar{a}_{52}^0 & -\bar{a}_{53}^0 & -\bar{a}_{54}^0 & 1 \end{bmatrix} \quad (8)$$

where the “-” above a coefficient indicates that a value is assigned to it. In their procedure, the value assigned to each of these coefficients is generated as follows:

$$\begin{aligned} \bar{a}_{23}^0 &= \frac{\theta_1}{1 - \text{abs}(\theta_1)} & \bar{a}_{24}^0 &= \frac{\theta_2}{1 - \text{abs}(\theta_2)} & \bar{a}_{25}^0 &= \frac{\theta_3}{1 - \text{abs}(\theta_3)} \\ \bar{a}_{34}^0 &= \frac{\theta_4}{1 - \text{abs}(\theta_4)} & \bar{a}_{35}^0 &= \frac{\theta_5}{1 - \text{abs}(\theta_5)} \end{aligned} \quad (9)$$

where abs denotes absolute value and the θ_i , $i = 1, \dots, 5$ are drawn from a uniform probability density function. For a given draw of the five θ_i coefficients, the instrumental variable estimation of the system proceeds in the following steps:

- (i) Estimate Eq. (6) by regressing i_t^* on the first lag of the variables and compute $\hat{\varepsilon}_{1t}$.
- (ii) Estimate Eq. (2) by regressing $y_t - \bar{a}_{23}^0 \pi_t - \bar{a}_{24}^0 i_t - \bar{a}_{25}^0 \Delta q_t$ on i_t^* and the first lag of the variables using $\hat{\varepsilon}_{1t}$ as the instrument for i_t^* and compute $\hat{\varepsilon}_{2t}$.
- (iii) Estimate Eq. (3) by regressing $\pi_t - \bar{a}_{34}^0 i_t - \bar{a}_{35}^0 \Delta q_t$ on the remaining right-hand side variables using $\hat{\varepsilon}_{1t}$ as the instrument for i_t^* and $\hat{\varepsilon}_{2t}$ as the instrument for y_t . Compute $\hat{\varepsilon}_{3t}$.
- (iv) Estimate Eq. (7) by regressing Δq_t on the remaining right-hand side variables by using $\hat{\varepsilon}_{1t}$ as the instrument for i_t^* , $\hat{\varepsilon}_{2t}$ as the instrument for y_t , $\hat{\varepsilon}_{3t}$ as the instrument for π_t and i_{t-1} as the instrument for Δi_t . Compute $\hat{\varepsilon}_{5t}$.
- (v) Estimate Eq. (4) by regressing i_t on the variables using $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$, $\hat{\varepsilon}_{3t}$ and $\hat{\varepsilon}_{5t}$ as the instruments for i_t^* , y_t , π_t and Δq_t , respectively. Compute $\hat{\varepsilon}_{4t}$.

Having estimated the system, the next step is to expand terms involving the first difference of the real exchange rate (and the interest rate in Eq. 7) so that all variables in the estimated system appear in levels. The full set of impulse responses to one standard error orthogonal shocks are calculated and judged for either retention or rejection by the sign restrictions. The process is then repeated for another draw of the θ_i parameters. Once a predetermined number of sets of responses are accepted by the sign restrictions, no further draws are made and the process ends.

4. Sign restrictions on the impulse responses

The sign restrictions to be applied to the impulse responses are shown in Table 1 where “ \geq ” indicates a non-negative response (i.e. the variable does not fall in response to the shock), “ \leq ” indicates a non-positive response (i.e. the variable does not rise in response to the shock) and “UR” indicates an unrestricted response. They are applied to the responses

which occur on impact and in the subsequent quarter.¹ No sign restrictions are applied to the responses to the foreign interest rate shock since it is isolated by the contemporaneous zero restrictions alone.

The only candidate for the monetary policy shock is ε_{4t} as it is the only shock which is restricted to have a zero long-run impact on the real exchange rate. This is indicated as LR=0 in Table 1. However, for it to be a monetary policy shock, its responses must satisfy the sign restrictions shown in the table, which rule out “price” and “output” puzzles. The sign restrictions along with the long-run parametric restriction uniquely separate the AS, AD, MP and RX shocks. It is not necessary to sign restrict the contemporaneous response of the exchange rate to the monetary policy shock so an “exchange rate” puzzle can emerge, nor the contemporaneous interest rate response to the exchange rate shock. In the absence of the long-run parametric restriction, it would be necessary to impose a further sign restriction to separate the MP shock from the RX shock. For example, Fisher and Huh (2016) impose the restriction that the interest rate cannot fall in response to an RX shock which depreciates the home currency, while Bjørnland and Halvorsen (2014) require the exchange rate to appreciate following a positive monetary policy shock (i.e. they rule out an “exchange rate” puzzle).

On a successful draw, there are $3!=6$ possible attributions that can be given to the set of structural shocks $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t}, \varepsilon_{5t})$, since ε_{1t} and ε_{4t} can only be the foreign interest rate (FI) shock and the monetary policy shock, respectively. They are: (FI, AS, AD, MP, RX), (FI, AS, RX, MP, AD), (FI, AD, AS, MP, RX), (FI, AD, RX, MP, AS), (FI, RX, AS, MP, AD) and (FI, RX, AD, MP, AS). A draw is successful if the set of impulse responses to all the shocks satisfy the sign restrictions for one of these attributions. If they do not satisfy the sign restrictions for any of these attributions, the draw is unsuccessful and all the impulse responses are discarded.

In sign restrictions, the accepted responses are arranged into ascending order *at each horizon* and summary measures are then calculated. In this paper, we will focus on the maximum and minimum response at each horizon, which provides the range of accepted responses. Following the literature, we also report the median, 16th and 84th percentile responses and the mid-range response, calculated as the sum of the maximum and minimum responses divided by two. While reported, these summary measures are not specifically discussed as we focus on the range of accepted responses. We note that all of these responses (including the maximum and minimum responses) at a given horizon and across horizons are almost certainly from different models i.e. they are from impulse responses that are generated from different draws of the θ_i parameters.²

In sign restrictions, it is the range of accepted responses to a shock which is of interest because all of the accepted responses are equally valid as they are observationally equivalent. Accordingly, we focus on the range of accepted responses to the MP and RX shocks in this paper and, in particular, on two statistics; the maximum and minimum

¹ The sign restrictions methodology was introduced by Faust (1998), Canova and De Nicoló (2002) and Uhlig(2005). Scholl and Uhlig (2008) is an application of Uhlig’s (2005) approach to monetary policy and exchange rates.

² We also report, though not discuss, the median target and mid-range target responses which are calculated using the metric suggested by Fry and Pagan (2011). Each target response comes from a single model which corresponds to a specific draw of the θ_i parameters.

response at each horizon. However, Baumeister and Hamilton (2015) have shown that statistics which summarise the accepted responses, like the median or some other percentile response or, for that matter, the maximum or minimum response, are not unique because they depend on the method which generates the accepted responses; in our case, on the method used to generate the \bar{a}_{ij}^0 coefficients. In a simulated demand and supply model, Ouliaris and Pagan (2016) show that the maximum and minimum of impulse responses are less affected by the generation method than the median of the impulse responses and conclude that “It is really the range of responses that one can get which is of ultimate interest” (Ouliaris and Pagan, 2016, p. 620).

5. The weak instrument problem

We generated the sets of impulse responses using the procedure of Section 3 and judged each set for either acceptance or rejection by the sign restrictions. Figure 1 reports the summary measures of the responses of the real exchange rate to the monetary policy shock from the 1,000 accepted responses to all the shocks for the case of Canada. It is clear that many of the accepted responses of the real exchange rate to the MP shock do not converge to zero since the summary measures do not show a zero response at long horizons. The maximum response converges to a long-run depreciation of slightly over 2 percent while the minimum response converges to a long-run appreciation of between 1 and 2 percent.³ Although the SVAR is designed so that the long-run response of the real exchange rate to the MP shock will be zero, that does not show up in a large number of the accepted responses for Canada, nor is it evident in the accepted responses of the other three countries (not shown).

The reason why the long-run restriction is not evident in the accepted responses for each country is due to the use of a weak instrument in the estimation of Eq. (7). Fry and Pagan (2005) show that the lagged level of a variable will be a weak instrument for its contemporaneous first difference when the variable itself is highly persistent. Specifically, they show that if the first-order autocorrelation coefficient of a variable (X_t) is in excess of 0.82 in a sample of 100 observations, the correlation between the instrument (X_{t-1}) and the variable it is instrumenting (ΔX_t) will be less than 0.3 in absolute value, so the instrument is weak. In the estimation of Eq. (7), i_{t-1} is used as the instrument for Δi_t , but the interest rate is a highly persistent variable rendering the instrument weak. Table 2 shows that the interest rate in each country is a highly persistent variable (the first order autocorrelation coefficients are all above 0.9) and the correlation between the first difference and lagged level of the interest rate is very low for each country. The strongest correlation is only -0.15 , which is for Canada, while the weakest correlation is -0.03 , which is for the country with the most persistent interest rate, namely, the United Kingdom.

For each country, the weak instrument problem arises in the estimation of Eq. (7), with the consequence that the coefficient a_{54}^0 is imprecisely estimated on each draw. This shows up with the failure of the long-run restriction to be evident in many of the accepted responses.

³ Recall that a decrease in the real exchange rate (a negative response) corresponds to a real appreciation while an increase (a positive response) corresponds to a real depreciation.

We now provide a method to obtain a precise estimate of the coefficient a_{54}^0 on each draw, which avoids the weak instrument problem.

6. A method that circumvents the weak instrument problem

To begin, the reduced-form VAR is $z_t = B_1 z_{t-1} + e_t$, where e_t are the VAR errors. The method begins from the relationship between the long-run impact matrix of the structural shocks and of the VAR errors. It is:

$$C(1)A_0 = B(1) \quad (10)$$

where $C(1)$ is the long-run impact matrix of the structural shocks, $B(1)$ is the long-run impact matrix of the VAR errors and A_0 is given by Eq. (8). The element in the fifth row and fourth column of $C(1)$ is set to zero (i.e. $c_{54} = 0$) by the restriction that ε_{4t} (i.e. the MP shock) has a zero long-run effect on the real exchange rate. We estimate the VAR and obtain estimates of the VAR errors \hat{e}_t and an estimate of $B(1)$.

We seek to find an expression for a_{54}^0 . By Eq. (10), the inner product of the fifth row of $C(1)$ with the fifth column of A_0 is equal to the element in the fifth row and fifth column of $B(1)$. This equation can be written as:

$$c_{55} = b_{55} + \bar{a}_{25}^0 c_{52} + \bar{a}_{35}^0 c_{53} \quad (11)$$

Similarly, the inner product of the fifth row of $C(1)$ with the fourth column of A_0 is equal to element b_{54} and this equation can be written as:

$$a_{54}^0 = -(1/c_{55})(b_{54} + \bar{a}_{24}^0 c_{52} + \bar{a}_{34}^0 c_{53}) \quad (12)$$

Substitute Eq. (11) into Eq. (12) to obtain:

$$a_{54}^0 = -\frac{(b_{54} + \bar{a}_{24}^0 c_{52} + \bar{a}_{34}^0 c_{53})}{(b_{55} + \bar{a}_{25}^0 c_{52} + \bar{a}_{35}^0 c_{53})} \quad (13)$$

As indicated earlier, we can obtain estimates for the elements in $B(1)$ so we have an estimate for b_{54} and b_{55} to use in Eq. (13). We also have values for the generated coefficients, the \bar{a}_{ij}^0 in Eq. (13), on each draw. Then we can estimate a_{54}^0 from Eq. (13) once estimates of c_{52} and c_{53} are found.

Denote the elements of A_0^{-1} as a_0^{ij} . It follows from Eq. (10) that:

$$(c_{52} \quad c_{53}) = (b_{51} \quad b_{52} \quad b_{53} \quad b_{54} \quad b_{55}) \begin{bmatrix} a_0^{12} & a_0^{13} \\ a_0^{22} & a_0^{23} \\ a_0^{32} & a_0^{33} \\ a_0^{42} & a_0^{43} \\ a_0^{52} & a_0^{53} \end{bmatrix} \quad (14)$$

We need to find estimates of the elements in the second and third columns of A_0^{-1} on each 'draw', in order to find estimates of c_{52} and c_{53} .

Recall that we estimate Eq. (6) of the structural model first as in Step (i) and obtain $\hat{\varepsilon}_{1t}$. We then estimate Eqs. (2) and (3) as in Steps (ii) and (iii) and obtain the estimated structural shocks, $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{3t}$. By the relationship between the reduced-form (VAR) errors and the structural shocks, given by $e_t = A_0^{-1}\varepsilon_t$, it follows that:

$$e_{it} = a_0^{i2}\varepsilon_{2t} + \eta_{it}, \quad \eta_{it} = a_0^{i1}\varepsilon_{1t} + a_0^{i3}\varepsilon_{3t} + a_0^{i4}\varepsilon_{4t} + a_0^{i5}\varepsilon_{5t}, \quad i = 1, 2, 3, 4, 5. \quad (15)$$

and

$$e_{it} = a_0^{i3}\varepsilon_{3t} + v_{it}, \quad v_{it} = a_0^{i1}\varepsilon_{1t} + a_0^{i2}\varepsilon_{2t} + a_0^{i4}\varepsilon_{4t} + a_0^{i5}\varepsilon_{5t}, \quad i = 1, 2, 3, 4, 5. \quad (16)$$

Because the structural shocks are orthogonal, η_{it} is uncorrelated with ε_{2t} and v_{it} is uncorrelated with ε_{3t} . We can obtain a consistent estimate of the coefficients in the second column of A_0^{-1} by an OLS regression of \hat{e}_{it} on $\hat{\varepsilon}_{2t}$ for $i = 1, 2, 3, 4, 5$, respectively, and similarly for the coefficients in the third column of A_0^{-1} by an OLS regression of \hat{e}_{it} on $\hat{\varepsilon}_{3t}$ for $i = 1, 2, 3, 4, 5$, respectively.⁴ We then obtain an estimate of c_{52} and c_{53} from Eq. (14), which are used in Eq. (13) to form the estimate of a_{54}^0 , which is denoted \hat{a}_{54}^0 .

We then estimate Eq. (5) by regressing $\Delta q_t - \hat{a}_{54}^0 i_t$ on the right-hand side variables using $\hat{\varepsilon}_{1t}$ as the instrument for i_t^* , $\hat{\varepsilon}_{2t}$ as the instrument for y_t , and $\hat{\varepsilon}_{3t}$ as the instrument for π_t , and compute $\hat{\varepsilon}_{5t}$. Finally, we estimate Eq. (4) by regressing i_t on the variables using $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$, $\hat{\varepsilon}_{3t}$ and $\hat{\varepsilon}_{5t}$ as the instruments for i_t^* , y_t , π_t and Δq_t , respectively, and compute $\hat{\varepsilon}_{4t}$.

We close this section by noting that the expression for a_{54}^0 in Eq. (13) can be specialised to the case of full parametric identification of the shocks, which would comprise a further five contemporaneous zero recursive restrictions, bringing the total number of parametric restrictions (including the long-run restriction) to ten. The further contemporaneous restrictions are that a_{23}^0 , a_{24}^0 , a_{25}^0 , a_{34}^0 and a_{35}^0 are all zero. In this case, the expression in Eq. (13) reduces to $a_{54}^0 = -(b_{54} / b_{55})$, which is analogous to the expression derived as Eq. (9) in Fry and Pagan (2005, p. 10) and to the expression implied by the long-run restriction in Gospodinov (2010, p. 4), and in Gospodinov, Maynard and Pesavento (2011, p. 459).

We now present the results for the structural model estimated this way.

7. Results

As indicated earlier, each equation of the SVAR has a constant and a time trend and the sample is 1994:Q1 to 2014:Q1. In the actual estimation, the lag length of the SVAR was

⁴ We note that the estimates for a_0^{12} and a_0^{13} in these regression will be close to zero because the exclusion restrictions $a_{12}^0 = 0$, $a_{13}^0 = 0$, $a_{14}^0 = 0$ and $a_{15}^0 = 0$ on the foreign interest rate equation can be shown to imply $a_0^{12} = 0$, $a_0^{13} = 0$, $a_0^{14} = 0$ and $a_0^{15} = 0$.

selected by the AIC criterion and it selected two lags for every country. The econometric procedure continues to 'draw' until 1,000 sets of impulse responses are retained i.e. that satisfy the sign restrictions. Recall that the initial shocks are one-standard error in size and that the sign restrictions are applied to the impact and subsequent quarter responses. The success rates (the 1,000 acceptances divided by the number of draws that were required to obtain them) are 0.03% for Australia, 0.26% for Canada, 0.11% for New Zealand and 0.30% for the United Kingdom.

Our discussion will focus on the responses to the monetary policy and real exchange rate shocks from amongst the accepted responses to all of the shocks. While we report several summary response measures in the figures, we will focus on the maximum and minimum response at each horizon as they provide the range of accepted responses. As indicated earlier, the maximum response at each horizon will likely come from a different model i.e. from a different draw of the five θ_i coefficients, and that is similarly the case for the minimum response at each horizon.

7.1. Responses to the monetary policy shock

Fig. 2 shows the responses of the domestic variables to a monetary policy shock which raises the interest rate. For Australia and Canada, all of the accepted responses show an appreciation of the real exchange rate on impact, a further appreciation the next quarter, and a subsequent depreciation to its initial level. For New Zealand, 96.9% of the accepted responses show an impact appreciation while for the United Kingdom it is 80%. On the basis of the accepted responses, there is no evidence of an exchange rate puzzle for Australia and Canada, and negligible evidence for New Zealand. The United Kingdom is the exception where one-fifth of the accepted responses show an exchange rate puzzle. The absence of an exchange rate puzzle in Australia and Canada, and its near absence in New Zealand, is a strong result, given that the SVAR is agnostic about whether or not there is an exchange rate puzzle i.e. the SVAR does not sign restrict the response of the real exchange rate to the MP shock. At long horizons, all of the accepted exchange rate responses for each country converge to zero under our method as required by the long-run parametric restriction.

Bjørnland (2009) identified the shocks in her SVAR by parametric restrictions alone, which were comprised of zero recursive contemporaneous restrictions and the zero long-run restriction we utilize. In her SVAR, which is also agnostic about an exchange rate puzzle, the real exchange rate appreciated on impact in Australia, Canada and New Zealand, following a positive monetary policy shock. An advantage of combining sign restrictions with the long-run parametric restriction is that a range can be established from the accepted responses, all of which are equally valid. For Australia, the impact appreciation ranges from 1 to 1.5 percent, and for Canada from 0.25 to slightly more than 1 percent. For New Zealand, the range is for a depreciation of 0.5 percent (though less than four percent of responses show a depreciation) to an appreciation of 1.5 percent. The range is similar for the United Kingdom, although the proportion of responses showing an impact depreciation is much larger at 20 percent. Note, it can clearly be seen in Fig. 2 that the accepted responses satisfy the sign restrictions to the MP shock: output and inflation fall and the interest rises, in the current and subsequent quarter. At long horizons, the responses of GDP, inflation and the interest rate converge to zero because they are treated as I(0) variables.

7.2. Responses to the real exchange rate shock

Fig. 3 shows the responses of the domestic variables to an exchange rate shock which depreciates the real value of the domestic currency. Recall that the responses of the interest rate are not sign restricted to this shock. For Canada and New Zealand, all of the interest rate responses are positive on impact in response to this shock, and for Australia, nearly all of them (98.6%) are. The United Kingdom is again the exception with only 78.9% of responses showing an increase in the interest rate on impact. For Canada and New Zealand, and almost always for Australia as well, the monetary authorities respond systematically to an exchange rate shock that depreciates the currency by increasing the domestic interest rate on impact. For Canada the interest rate increases by between 0.1 and 0.3 percentage points and for New Zealand the increase ranges from near zero to 0.3 percentage points. For Australia, the increase can be nearly 0.2 percentage points. In the case of the United Kingdom, it is less certain that the monetary authority responds systematically to an exchange rate shock as about one-fifth of accepted responses have the interest rate moving in the counter direction on impact.

In the study of Bjørnland and Halvorsen (2014), an exchange rate shock which depreciates the currency results in the median response of the interest rate rising on impact in Canada and New Zealand, while for the United Kingdom it rises negligibly and for Australia it falls. They conclude that monetary policy responds systematically to an exchange rate shocks in Canada and New Zealand but not in Australia and the United Kingdom.⁵ We conclude similarly, except for Australia where we find that 98.6% of interest rate responses to the exchange rate shock have the same sign on impact, (positive to a depreciating shock), indicating a systematic monetary policy response in Australia.

7.3. Responses to the AS and AD shocks

Here we provide only a brief summary of what the accepted responses to these shocks show. The accepted responses of the interest rate to the AS shock range over the zero-axis at the impact and subsequent horizons in each country and they all converge to zero since the interest rate is treated as an $I(0)$ variable. The accepted responses of the real exchange rate to the AS shock range over the zero-axis at almost all horizons for Australia, Canada and New Zealand. For the United Kingdom, the range is for only a depreciation at medium and long horizons. Recall that both interest rate and real exchange rate responses are not sign restricted in this case. The wide range of responses indicates that there is considerable model uncertainty with respect to the direction of movement of the interest rate and the real exchange rate in response to the AS shock in each country. In response to the AD shock, the range of accepted responses shows that the real exchange rate appreciates over all horizons, except in the United Kingdom, where the range covers the zero axis at long horizons.

8. Conclusion

This paper separates the shocks in a SVAR for four small open economies by combining sign restrictions with parametric restrictions using the recently developed framework of Ouliaris and Pagan (2016). We utilise their IV approach and find that the instrument associated with the estimation of the coefficient associated with the long-run restriction is 'weak' so that

⁵ In their SVAR, the response of the interest rate to an exchange rate shock is not sign restricted but the response of the exchange rate to the monetary policy shock is whereas in the SVAR of Fisher and Huh (2016) it is the converse. The SVAR of the present paper sign restricts neither.

the coefficient is imprecisely estimated. This shows up as a serious problem because many of the real exchange rate responses to the monetary policy shock converge in the long-run to values considerably away from zero, when they should converge to zero by design of the SVAR. As a demonstration, this was specifically shown for the case of Canada. We provide a method to obtain a direct estimate of the coefficient associated with the long-run restriction on each 'draw' in the Ouliaris and Pagan framework that avoids this problem. When this method is applied, the accepted responses of the real exchange rate to the MP shock all converge to zero.

There are two main findings from the SVARs. First, for Australia and Canada there is no evidence for an exchange rate puzzle and for New Zealand an exchange rate puzzle only emerges in four percent of accepted responses. The United Kingdom is the exception with twenty percent of accepted responses showing an exchange rate puzzle. Second, the monetary authority in Australia, Canada and New Zealand appears to systematically respond to a real exchange rate shock. For Canada and New Zealand, all of the accepted responses, and almost all for Australia, show an impact rise in the interest rate following an exchange rate shock which depreciates the real value of the currency. The United Kingdom is again the exception where it appears that the monetary authority does not respond systematically to an exchange rate shock as twenty percent of the accepted responses have the interest rate moving in the counter direction on impact.

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Table 1. Sign restrictions

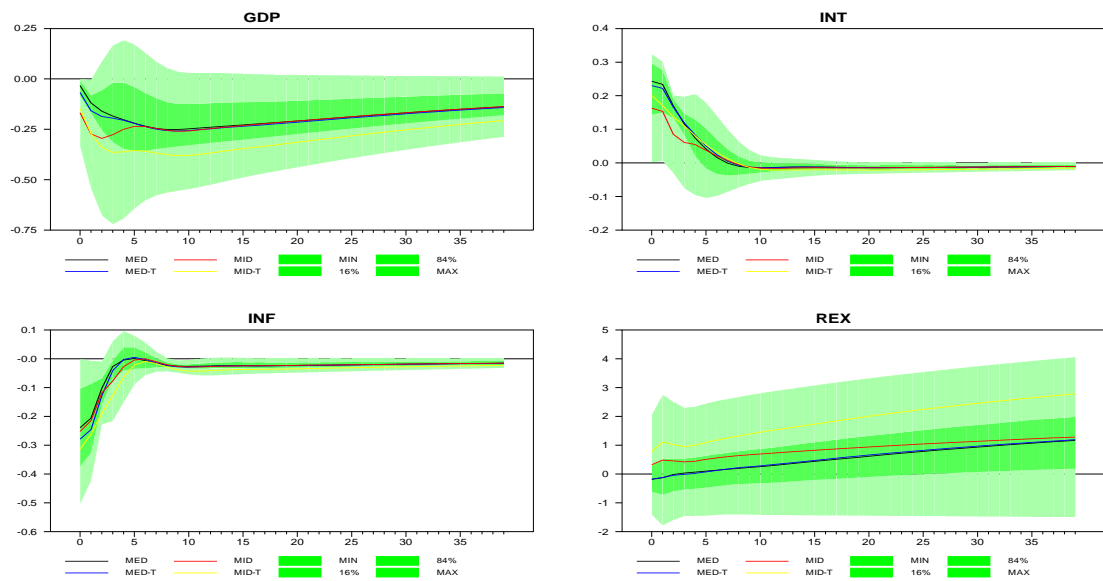
Shock\Variable	GDP	Inflation	Interest Rate	Real Exchange Rate
AS	≥ 0	≤ 0	UR	UR
AD	≥ 0	≥ 0	≥ 0	≤ 0
MP	≤ 0	≤ 0	≥ 0	UR, LR=0
RX	≥ 0	≥ 0	UR	≥ 0

Notes: AS denotes an aggregate supply shock, AD an aggregate demand shock, MP a monetary policy shock and RX a real exchange rate shock. The designation " \geq " indicates a non-negative response so that the variable does not fall in response to the shock while " \leq " indicates a non-positive response so that the variable does not rise in response to the shock. UR denotes an unrestricted response of the variable to the shock. The sign restrictions are imposed on the impact response and on the response for the following quarter. LR=0 is the parametric restriction that the MP shock has a zero impact on the real exchange rate in the long-run.

Table 2. The weak instrument problem

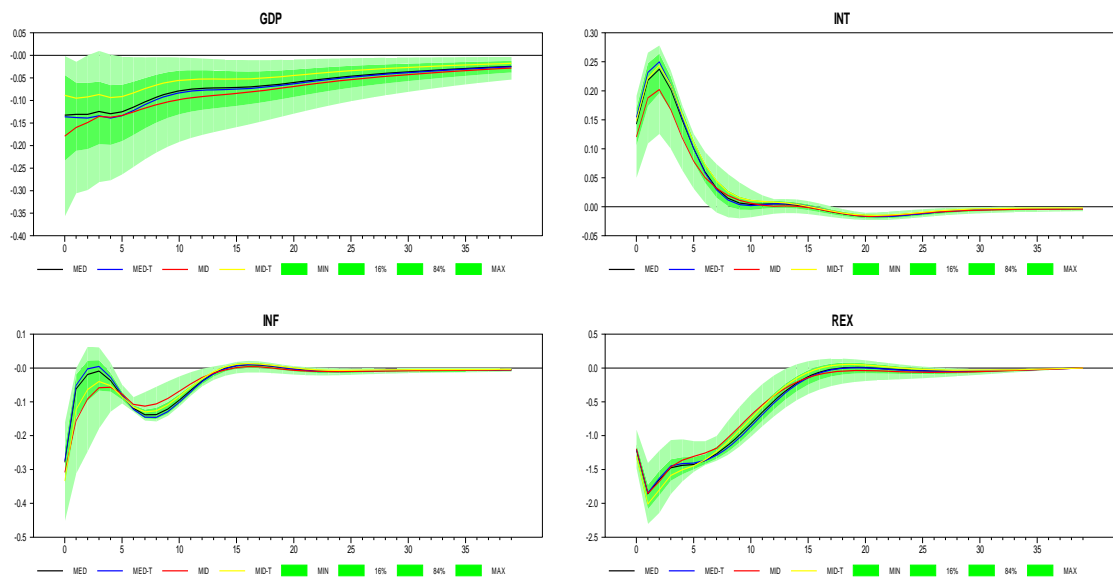
Statistic\Country	Australia	Canada	New Zealand	United Kingdom
$AR(1)$	0.937	0.960	0.957	0.980
$corr(\Delta i_t, i_{t-1})$	-0.125	-0.154	-0.134	-0.034

Notes: $AR(1)$ is the first-order autocorrelation coefficient of the interest rate and $corr(\Delta i_t, i_{t-1})$ is the correlation between the first difference of the interest rate and its level lagged one quarter. The sample is 1994:Q1–2014:Q1 for each country.

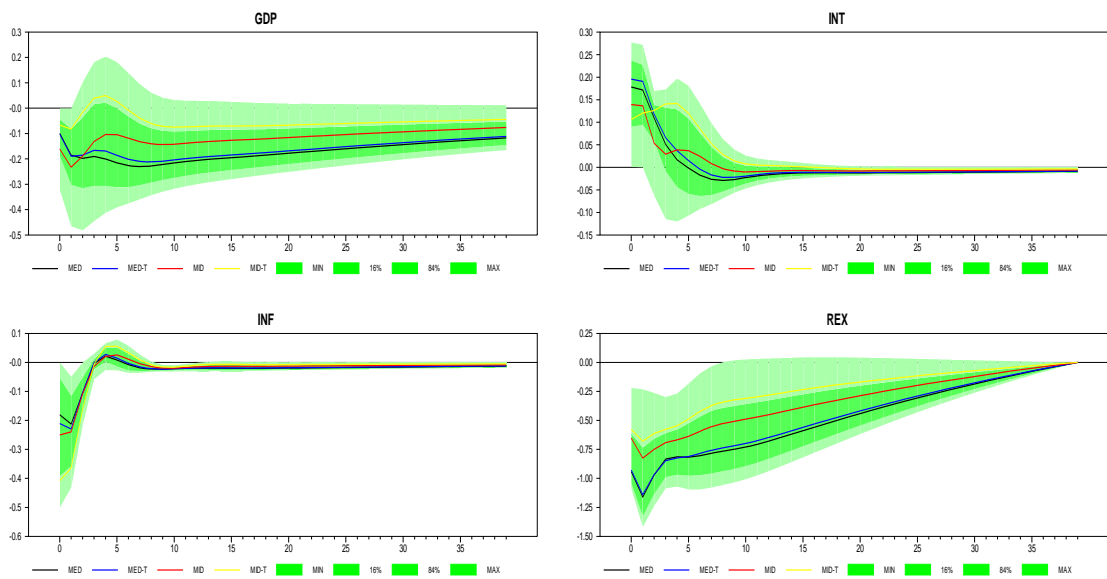


Canada: Weak instrument case

Fig. 1. Response of variables to the monetary policy shock under the weak instrument:
Canada

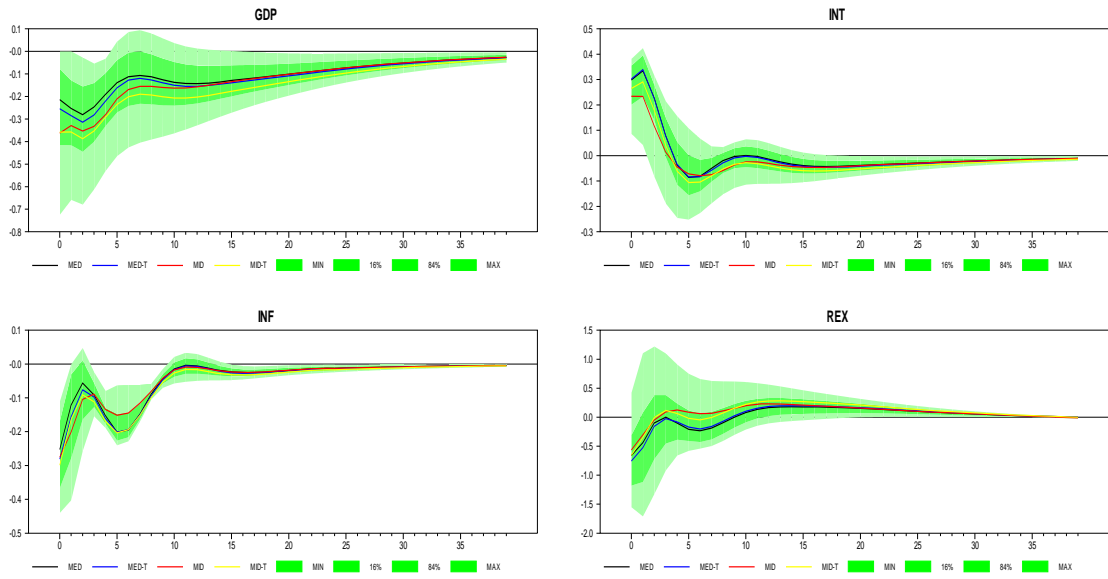


(a) Australia

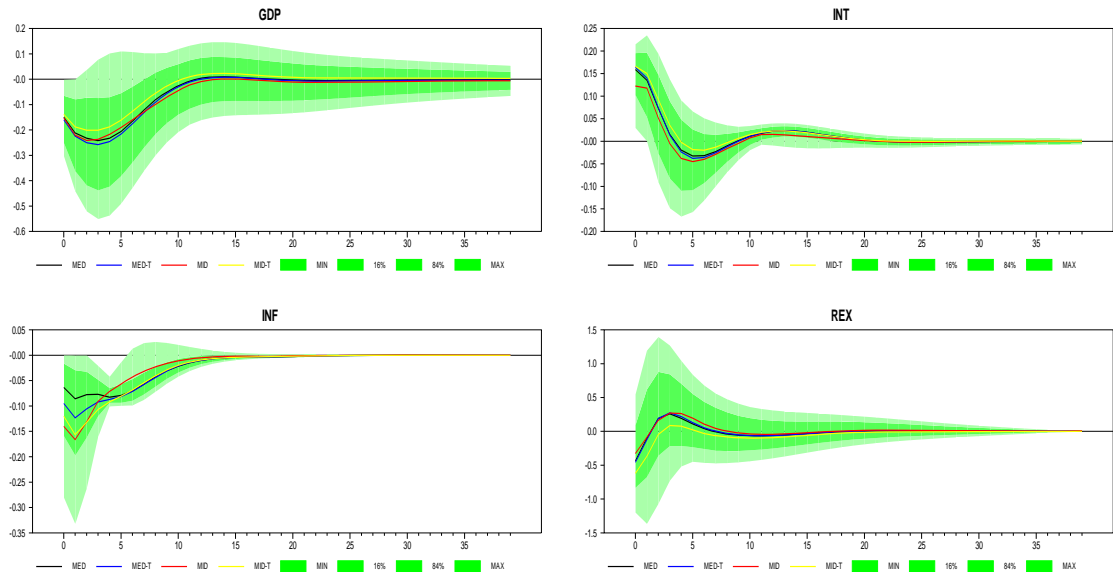


(b) Canada

Fig. 2. Response of variables to monetary policy shock

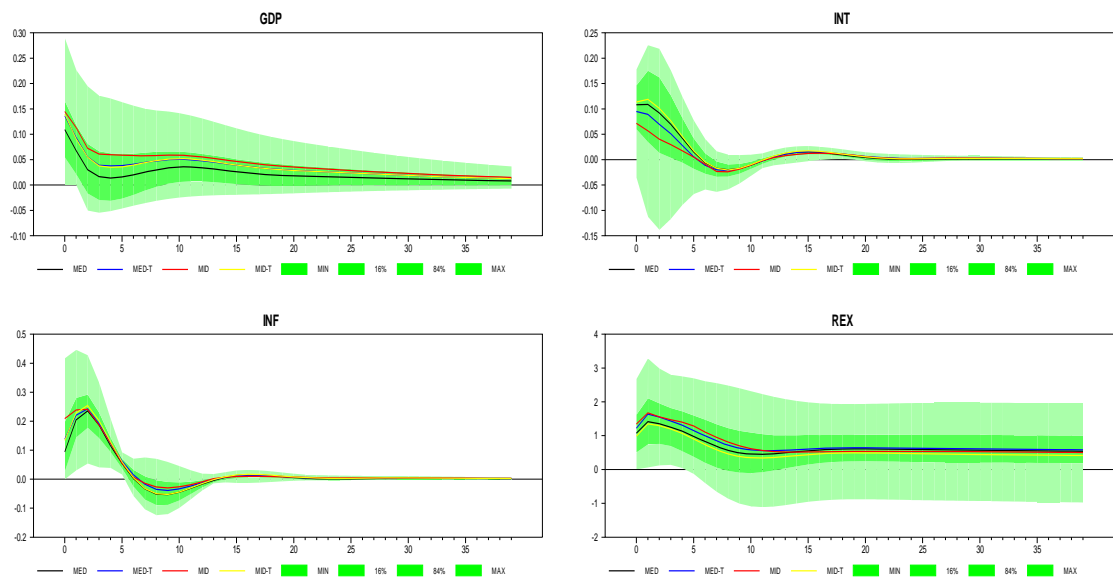


(c) New Zealand

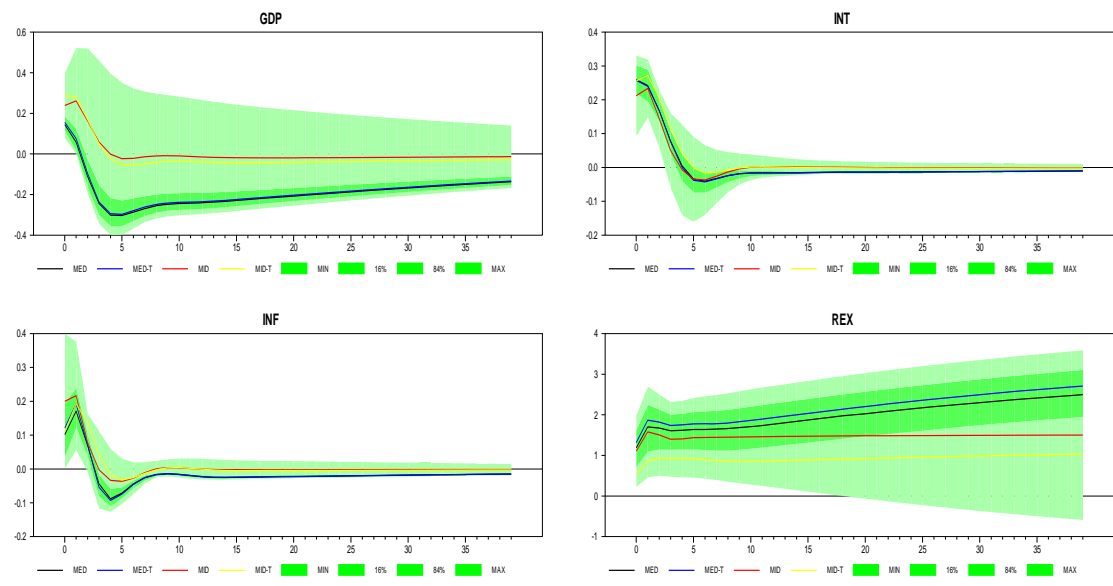


(d) United Kingdom

Fig. 2 (continued). Response of variables to monetary policy shock

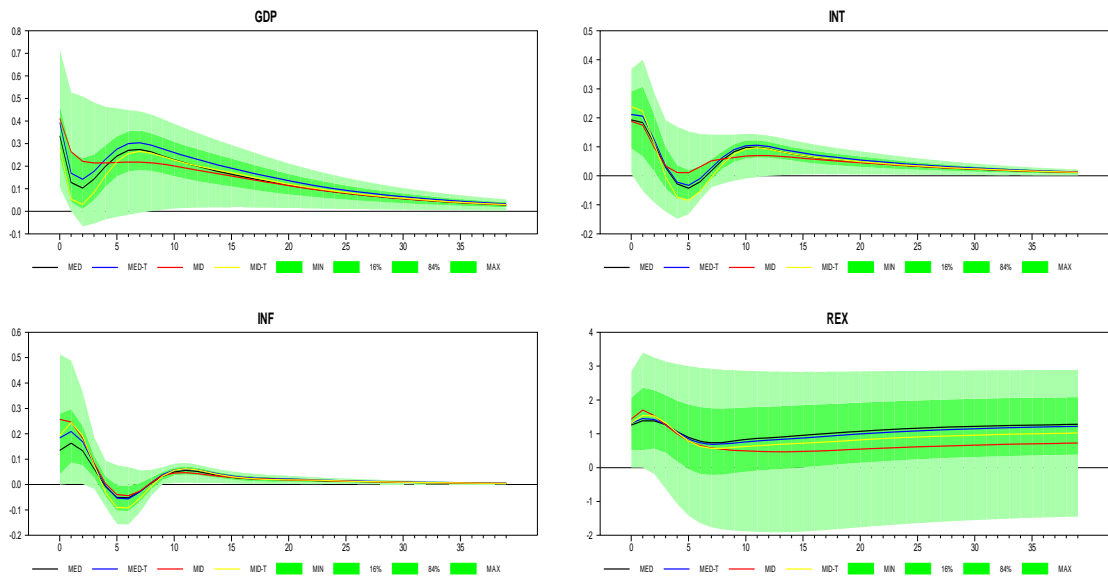


(a) Australia

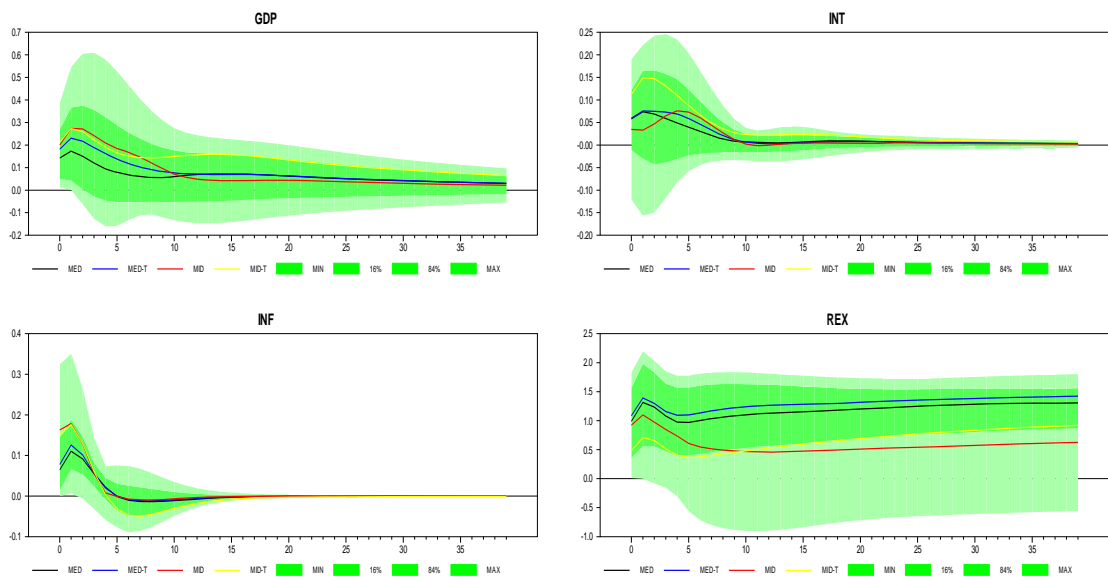


(b) Canada

Fig. 3. Response of variables to real exchange rate shock



(c) New Zealand



(d) United Kingdom

Fig. 3 (continued). Response of variables to real exchange rate shock