Foreign Direct Investment and Transfer Pricing*

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Abstract

This paper analyzes an MNE’s incentives to manipulate an internal transfer price to take advantage of tax differences across countries. We first consider a monopoly case and derives conditions under which FDI takes place and show that tax-induced FDI can entail inefficient internal production. With imperfect competition we show that the internal transfer price has additional strategic effects that further strengthen incentives to inflate the transfer price at the expense of the rival firm’s profits. The tax-induced FDI by the MNE has spillover effects that reduce tax revenues from other domestic firms as well as the MNE. We also explore implications of the arm’s length principle and import tariffs to mitigate this problem.

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Key Words: FDI, Transfer Pricing, Arm’s Length Principle

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1 Introduction

This paper analyzes an MNE’s incentives to manipulate an internal transfer price to take advantage of tax differences across countries. We first consider a monopoly case and derives conditions under which FDI takes place and show that tax-induced FDI can entail inefficient internal production. With imperfect competition we show that the internal transfer price has additional strategic effects that further strengthen incentives to inflate the transfer price at the expense of the rival firm’s profits. The tax-induced FDI by the MNE has spillover effects that reduce tax revenues from other domestic firms as well as the MNE. We also explore implications of the arm’s length principle and import tariffs to mitigate this problem.

It has been well documented that MNEs engage in tax manipulation to reduce their tax obligations by shifting their profits from high tax countries to low tax jurisdictions (see Hines and Rice, 1994 and Bauer and Langenmayr, 2013). For instance, inspections by the Vietnamese tax authorities found that "the most common trick played by FDI enterprises to evade taxes was hiking up prices of input materials and lowering export prices to make losses or reduce profits in books."\(^1\) In addition, Egger et al. (2010) find that an average subsidiary of a multinational corporation pays about 32% less tax in a high tax country than a similar domestically-owned firms.

To analyze tax-induced FDI and its welfare implications, we consider a very stylized simple set-up of two countries with different corporate tax rates. To fix the scenario, we first consider a setting in which the monopolistic final good producer is located in Home country \((H)\) with a higher tax rate whereas its input can be more cheaply produced in Foreign country \((F)\) with a lower tax rate. For instance, the input is labor-intensive and country \(F\) has a lower wage. Alternatively, the necessary input is a natural resource that is available only in country \(F\). In this scenario, the input is needed to be procured from \(F\), but there are two ways to do it. It can be outsourced from outside firms in \(F\), or can be produced internally with FDI. Not surprisingly, we show that FDI can be used even if it is less efficient in producing the input because it can be used as a vehicle to lessen its tax burden with an inflated internal price when \(F\) has a lower tax rate.

If there is no government oversight on internal exchanges within the firm, the MNE will shift all profits to the country with a lower tax rate via transfer price. Governments thus impose transfer pricing rules (TPRs) to control tax manipulation. The standard practice

\(^1\)http://vietnamlawmagazine.vn/transfer-pricing-unbridled-at-fdi-enterprises-4608.html
is to stipulate that internal transfer prices follow the so-called "Arm’s Length Principle" (ALP), which requires intrafirm transfer prices to meet the arm’s length standard, that is, the transfer price should not deviate from the price two independent firms would trade at. Currently, the ALP is the international transfer pricing principle that OECD member countries have agreed should be used for tax purposes by MNE groups and tax administrations.

The basic approach of the ALP is that the members of an MNE group should be treated "as operating as separate entities rather than as inseparable parts of a single unified business" and the controlled internal transfer price should mimic the market price that would be obtained in comparable uncontrolled transactions at arm’s length. This kind of “comparability analysis”, is at the heart of the application of the arm’s length principle. For instance, the 2010 OECD Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations 2010 states that the comparable uncontrolled price (CUP) method

"compares the price charged for property or services transferred in a controlled transaction to the price charged for property or services transferred in a comparable uncontrolled transaction in comparable circumstances. If there is any difference between the two prices, this may indicate that the conditions of the commercial and financial relations of the associated enterprises are not arm’s length, and that the price in the uncontrolled transaction may need to be substituted for the price in the controlled transaction." (p. 63)

As the CUP method is the most direct and reliable, it is the preferred method to apply the ALP. In practice, however, it may be difficult to find a transaction between independent enterprises that is similar enough. This would be particularly so in the monopoly context where the required input is demanded only by the monopolist and there is no comparable input market available. In such a case, there are other methods suggested to apply the ALP. In our theoretical set-up, we assume that the "cost plus method" which mandates that the transfer price should reflect the production cost of the input internally transacted.\footnote{Other methods suggested include the resale price method, the transactional net margin method, and the transactional profit split method. See OECD (2010) for more details.} However, the true production cost is typically non-observable to tax authorities and hard to ascertain. As a result, it can be manipulated at certain costs.
To analyze incentives to engage in FDI and how the internal price is determined when the cost plus method is used due to the absence of comparable transactions in the market, we introduce "concealment costs." More specifically, when an MNE’s internal price deviates from its true marginal cost in the presence of the ALP with the cost plus method, there are costs to avoid such institutional constraints on the internal transfer price. These costs can be literally concealment costs to keep two separate books or can reflect expected punishment for the deviation as in Kant (1988). The MNE thus trade-offs potential tax benefits against concealment costs in its choice of the optimal transfer price. We show that the optimal transfer price is equivalent to the minimization of what we call "virtual marginal cost" and this characterization provides a very simple condition for the optimality of FDI vis-a-vis outsourcing if the concealment cost is linear in the quantity of inputs internally transferred.

If concealment costs are convex in the quantity of inputs internally transferred, there may be incentives for the MNE to engage in dual sourcing, that is, part of the required input is internally produced with FDI whereas the rest is outsourced. Outsourcing, however, creates a benchmark transaction against the internal transfer can be compared. As a result, a dual sourcing strategy may provide the tax authority with the ability to identify a comparable market price and adopt the CUP method as an application of the ALP. In such a case, we demonstrate the imposition of the CUP method with dual sourcing can have unintended consequences and detrimental effects if it triggers the MNE’s sourcing decision from dual sourcing to internal sourcing only.

We also analyze import tariff as a countermeasure against potential tax shifting. Even though import tariff can be completely offset the incentives to engage in inflated transfer price for tax manipulation purposes, we show that the optimal import tariff may be set to allow tax manipulation to some extent. The reason is that the tax manipulation by the MNE leads to more production in the domestic market which can alleviate allocative inefficiency due to monopoly power.

We then extend our analysis to oligopolistic market structure in the domestic market. As the MNE has incentives to produce more for profit shifting motives, it can have strategic effects vis-a-vis its rival firms in the final produce market. As a result, the rival firms reduce its outputs and their profits suffer. This implies that tax-induced FDI by the MNE has spillover effects that reduce tax revenues from other domestic firms as well as the MNE. We also consider implications of ALP when the input supplier in country F is monopolistic. If the input purchased by the rival firm is considered as a comparable input
used by the MNE, then the price set by the foreign supplier can affect the internal price of the MNE with ALP. Thus, the imposition of ALP can have implications of strategic price setting of the monopolistic supplier of input in $F$.

Horst (1971) initiated the theory of multinational firms in the presence of different tariff and tax rates across countries and explores the profit-maximizing strategy for a monopolistic firm selling to two national markets, that is, how much to be produced in each country and what would be the optimal transfer price for goods exported from parent to subsidiary. Horst (1971) and subsequent papers (such as Batra and Hadar (1979) and Itagaki (1979, 1981)) show that MNE’s optimum price would be either the highest or the lowest possible allowed by the limits of government rules and regulations, depending on tax and tariff schedules across countries.\footnote{In contrast to Horst (1971) who assumes that output decisions are centralized, Bond (1980) considers a situation in which decision making is decentralized. He shows that the optimal transfer prices trade off the gain from tax avoidance against the efficiency losses associated from resource misallocation.} Kant (1988) shows how an interior transfer price can be derived endogenously in the presence of so-called "concealment cost."

Bond also analyzes optimal transfer pricing when branches of a vertically integrated enterprise are located in multiple jurisdictions with different tax rates. As in Hirshleifer (1956), he assumes that decision-making across branches is decentralized and the transfer prices in his model is chosen to align the production decisions of the various divisions. In our paper, we assume that the decision making process is centralized.

Kato and Okoshi (2017) consider the optimal location of production facilities in the presence of tax differences across countries and how the ALP principle can impact the location choice.

Samuelson (1982) is the first one to point out that for an MNE subject to the ALP principle, the arm’s length reference price itself can be partially determined by the firm’s activities. In a similar vein, Gresik and Osmundsen (2008) consider transfer pricing in a vertically integrated industries in the absence of transactions between independent entities. More specifically, they examine the implications of arm’s length principle as a transfer price regulation when all firms are vertically integrated and the only source of comparable data may be from transactions between affiliated firms. In our framework with imperfect competition in both the upstream and downstream markets, the reference price for an MNE is determined by an outsider, but the outsider firm recognizes the strategic effects of its price decision on its input demand via the transfer price of the MNE. It is shown that the outsider has incentives to set a lower price compared to the
case of no linkage via the transfer price.

The rest of the paper is organized in the following way. Section 2 introduces the basic set-up of the monopoly model with FDI and transfer pricing. We first analyze the optimal transfer price with the concealment cost and the incentives to engage in FDI due to tax differential between the source and destination countries. We then explore implications of such FDI for the efficiency of global sourcing and identify the wedge between the efficient outcome and the market equilibrium and how such decision can be influenced by the imposition of ALP. Section 3 considers import tariffs as a countermeasure against profit-shifting. Section 4 extends the analysis to a duopoly setting to explore implications of strategic interactions in the final good market. We show that profit-shifting strategy of the multinational firm has further consequences for tax revenues from other firms due to strategic effects. We also show how the market price can be endogenized with the imposition of ALP. Section 5 concludes the paper.

2 The Monopoly Model of FDI and Transfer Pricing

2.1 The Basic Set-up

There are two countries, Home and Foreign, with different tax rates with $t$ and $\tilde{t}$, respectively. We assume a monopolistic final good producer. We assume that Home (denoted as $H$) is a high tax rate country in which the headquarter that produces final goods is immobile and tied to consumer markets in $H$ while inputs are more cheaply produced in Foreign (denoted as $F$) with a lower tax rate, that is, $t > \tilde{t}$. The monopolist located in $H$ have two possible ways to procure its essential input from $F$. There is a competitive open market from which the input can be procured at the price of $\varpi$ (later we consider the external source with market power and endogenize $\varpi$). Alternatively, it can be an MNE by setting up its own input production plant in $F$ with FDI. We assume a constant returns technology. In such a case, its input production cost is given by $c$. The MNE can choose an internal transfer price ($\gamma$) when its foreign subsidiary supply its input to the headquarter firm that produces the final good. Without any tax rate differential between the two countries, the MNE’s optimal internal transaction price for the input $\gamma$ is simply its marginal production cost of $c$ in order to eliminate any double marginalization problem. However, with different tax rates between $H$ and $F$, the MNE can choose an internal transfer price ($\gamma$) as a mechanism to shift profits to minimize its tax burden. In
particular, the monopolistic MNE solves the following problem

$$\max_q \tilde{\Pi} = (1 - t) \pi(q; \gamma) + (1 - \bar{t}) (\gamma - c)q$$  \hspace{.5cm} (1)$$

where $\pi(q; \gamma) = [P(q) - \gamma]q$ and $P(q)$ is the downward sloping inverse demand function facing the monopolist. Figure 1 describes the basic set-up.

Note that the objective function of the monopolist can be rewritten as

$$\tilde{\Pi} = (1 - t)[P(q) - \theta]q,$$

where

$$\theta = \frac{(1 - \bar{t})c - (t - \bar{t})\gamma}{1 - t}$$  \hspace{.5cm} (2)$$

That is, the MNE facing different tax rates across countries behaves as if its marginal production cost were $\theta$, which can be considered as the MNE’s virtual MC of production \textit{adjusted for transfer price induced by differential tax rates across countries}. As the MNE’s profit decreases in $\theta$, the monopolist’s optimal choice of $\gamma$ is equivalent to the choice of $\gamma$ that minimizes $\theta$. Note that $\theta$ is decreasing in its internal transfer price $\gamma$ because it can be used as a vehicle to shift profit from the high tax country $H$ to the low tax rate.
country $F$. As pointed out by Horst (1971), it immediately follows that the optimal choice is to set $\gamma$ as high as possible potentially subject to the constraint that the downstream headquarter profit cannot be negative. Otherwise, this implies that $H$ country make up any losses incurred by the headquarter with a subsidy up to the rate of $t$.\(^4\)

In reality, however, there are restrictions that would prevent the choice from being a corner solution and limit the MNE’s profit shifting motives. We consider two such mechanisms and analyze how they affect the MNE’s behavior and resource allocations. The first one is concealment costs. We analyze concealment costs and ALP, respectively, in subsections 2.2 and 2.3.

### 2.2 Profit-Shifting Transfer Pricing with Concealment Costs

As shown in the previous subsection, without any external or regulatory restriction on the transfer price, all profits would be shifted towards to a lower tax country with FDI being used as a vehicle. However, this type of behavior can be a violation of tax laws. We thus explore implications of institutional constraints on the internal transfer price. To this end, we assume that a deviation of an MNE’s internal price from its true marginal cost entails costs of $\Psi(\gamma - c, q)$. This could be interpreted as concealment costs or can reflect expected punishment for the deviation as in Kant (1988). For analytical tractability, we assume the concealment costs are separable in the deviation of the internal price from its true MC and the amount of inputs transferred, that is, $\Psi(\gamma - c, q) = \phi(\gamma - c)\mu(q)$. We make the following assumption about the concealment costs.

**Assumption 1.** $\phi' > 0$, $\phi'' > 0$ for $\gamma - c > 0$ and $\mu' > 0$, $\mu'' \geq 0$ for $q > 0$ with

$$\phi(0) = \mu(0) = \phi'(0) = \mu'(0) = 0$$

Assumption 1 states that concealment costs increases with the transfer price’s deviation from its true cost and the amount of inputs transferred. In addition, concealment costs are convex in the degree of deviations with the usual Inada conditions.

With this concealment cost constraint being into account, the monopolist solves the following problem

\(^4\)If we consider a dynamic model, the headquarter’s loss may be used as tax offset against future profits. However, it cannot be used as a tax offset if the headquarters make losses all the time.
\[ \begin{align*}
Max_q \Pi &= (1 - t) [P(q) - \gamma]q + (1 - \tilde{t})(\gamma - c)q - \Psi(\gamma - c, q) \\
&= \tilde{\Pi} - \Psi(\gamma - c, q)
\end{align*} \] (3)

The first order condition for this is given by
\[ \frac{\partial \Pi}{\partial q} = (1 - t) \frac{\partial \pi(q; \gamma)}{\partial q} + (1 - \tilde{t})(\gamma - c) - \Psi_q(\gamma - c, q) = 0 \] (5)

Let \( \hat{q}^m(\gamma) \) be the solution to (2), that is,
\[ \hat{q}^m(\gamma) = \arg \max \Pi \]

Note that \( \hat{q}^m(c) = q^m(c) \).

### 2.2.1 Optimal Transfer Price with Concealment Costs

If we further assume that the concealment costs are proportional to the MNE’s output, that is, \( \Phi(\gamma - c, q) = \phi(\gamma - c)q \) with \( \phi' > 0, \phi'' > 0 \), and \( \phi'(0) = 0 \), as in Egger and Seidel (2013), we have a very clean characterization concerning the MNE’s optimal transfer price and its sourcing decision. With linear concealment costs in the output, we can write down the monopolistic MNE’s profit function as
\[ \begin{align*}
\Pi &= (1 - t)[P(q) - \gamma]q + (1 - \tilde{t})(\gamma - c)q - \phi(\gamma - c)q \\
&= (1 - t)[P(q) - \xi]q,
\end{align*} \] (6) (7)

where
\[ \xi = \gamma - \frac{(1 - \tilde{t})(\gamma - c)}{1 - t} + \frac{\phi(\gamma - c)}{1 - t} = \frac{(1 - \tilde{t})c - (t - \tilde{t})\gamma + \phi(\gamma - c)}{1 - t} \]
\[ = \theta + \frac{\phi(\gamma - c)}{1 - t}, \]

where \( \theta = \frac{(1 - \tilde{t})c - (t - \tilde{t})\gamma}{1 - t} \). Thus, the optimal choice of the transfer price \( \gamma^* \) is equivalent to minimize the MNE’s "virtual marginal cost" of \( \xi \), which is adjusted for transfer price.
induced by differential tax rates and concealment costs, and implicitly defined by

\[ t - \tilde{t} = \phi'(\gamma - c) \]

The optimal \( \gamma^* \) thus can be derived as

\[ \gamma^* = c + \phi'^{-1}(t - \tilde{t}) > c \]

For instance, if we assume \( \phi(\gamma - c) = \frac{k}{2}(\gamma - c)^2 \), where a higher \( k \) represents better institutional monitoring which make it more costly for the MNEs to engage in profit shifting. Then, we have \( \gamma^* = c + \frac{t - \tilde{t}}{k} \). The optimal choice of the transfer price is consistent with empirical findings. For instance, Clausing (2003) shows that as the counter-party tax rates are lower, US intrafirm import prices are higher (note that we have different predictions without concealment costs).

### 2.2.2 FDI vs. Outsourcing

Let \( \xi^* \) be the minimized virtual MC with the choice of optimal transfer price \( \gamma^* \). Then, the MNE’s profit from FDI can be written as

\[ \Pi^{FDI} = (1 - t)[P(q) - \xi^*]q, \]

whereas the monopolist’s profit from simply outsourcing can be written as

\[ \Pi^{OS} = (1 - t)[P(q) - \omega]q \]

Thus, the monopolist’s sourcing decision boils down a simple comparison of \( \xi^* \) and \( \omega \); FDI takes place if and only if \( \xi^* < \omega \).

**Lemma 1.** \( \xi^* < c \)

**Proof.** Note that \( \xi^* \) can be written as

\[ \xi^* = c - \frac{(t - \tilde{t})(\gamma^* - c) - \phi(\gamma^* - c)}{1 - t} \]
Figure 2: Globally Efficient Sourcing vs. MNE’s Sourcing Decision

Using the first order condition that defines $\gamma^*$, we find that

$$(t - \tilde{t})(\gamma^* - c) + \phi(\gamma^* - c) = (\gamma^* - c) \left[ \phi'(\gamma^* - c) - \frac{\phi'(\gamma^* - c)}{(\gamma^* - c)} \right] > 0$$

Therefore, $\xi^* < c$. □

Lemma 1 implies that the MNE’s profit is global profit is higher due to tax manipulation compared to the case where the firm transfers its input at its marginal cost $c$. The MNE’s profit is as if its cost were the virtual cost of $\xi^*$ which is lower than its true marginal cost of $c$. This, in turn, implies that the MNE’s sourcing decision can inefficient from the global production efficiency viewpoint (see Figure 2). Profit shifting motives due to tax differences across countries create a wedge of $\lambda(=\frac{(t-\tilde{t})(\gamma^* - c) - \phi'(\gamma^* - c)}{1-t}) > 0)$, which distorts the MNE’s sourcing decision.

**Proposition 1.** *(Inefficiency of Internal Sourcing)* With tax differentials across countries, there can be excessive FDI. The global efficiency requires that FDI takes place if $c < \omega$ whereas FDI takes place in equilibrium if $\xi^* < \omega$. Thus, if $c \in (\omega, \omega + \lambda)$, where $\lambda = \frac{(t-\tilde{t})(\gamma^* - c) - \phi'(\gamma^* - c)}{1-t} > 0$, there is an inefficient FDI.
2.2.3 Parametric Example

As shown above, with $\phi(\gamma - c) = \frac{k}{2}(\gamma - c)^2$, we have

$$\gamma^* = c + \frac{t - \tilde{t}}{k}$$

By plugging back this into $\xi$, we can easily verify that

$$\xi^* = c - \frac{(t - \tilde{t})^2}{2k(1 - t)}$$

This implies that FDI takes place if and only if

$$c < \varpi + \frac{(t - \tilde{t})^2}{2k(1 - t)}$$

That is, unless the MNE’s internal production cost does not exceed the open market price by $\frac{(t - \tilde{t})^2}{2k(1 - t)}$, FDI takes place. In particular, if $c \in (\varpi, \varpi + \frac{(t - \tilde{t})^2}{2k(1 - t)})$, FDI is inefficient, but still optimal from the perspective of the MNE due to tax manipulation via transfer price.

2.3 Non-Linear Concealment Costs and Dual Sourcing

With the concealment costs linear in the amount internally transferred (with an inflated price) $q$, the MNE will procure its input only from a single source (i.e., either all from the internal source or all from the open market). However, if the concealment costs are convex in $q$, the MNE may source its inputs from the internal and external sources. To see this, let us assume that $\Psi(\gamma - c, q) = \phi(\gamma - c)\mu(q)$ with $\mu'$ and $\mu'' > 0$.

$$\Pi = (1 - t)[P(q) - \gamma]q + (1 - \tilde{t})(\gamma - c)q - \phi(\gamma - c)\mu(q)$$

$$= (1 - t) \left( [P(q) - \frac{(1 - \tilde{t})c - (t - \tilde{t})\gamma}{1 - t}]q - \frac{\phi(\gamma - c)}{1 - t} \mu(q) \right)$$

Thus, given $\gamma$, the virtual marginal cost $\xi$ from internal sourcing via FDI is not constant can be expressed as

$$\xi(q; \gamma) = \frac{[(1 - \tilde{t})c - (t - \tilde{t})\gamma] + \phi(\gamma - c)\mu'(q)}{1 - t}$$
This also implies that depending on the production quantity, the optimal transfer price changes. For a given quantity \( q \), the transfer price that minimizes the total production cost \( [(1 - \tilde{t})c - (t - \tilde{t})\gamma]q + \phi(\gamma - c)\mu(q) \) is given by the following first order condition

\[
(t - \tilde{t})q = \phi'(\gamma - c)\mu(q)
\]

By totally differentiating the condition above, we can easily verify that the optimal internal price \( \gamma^*(q) \) is decreasing in \( q \):

\[
(t - \tilde{t})dq = \phi''(\gamma - c)\mu(q)d\gamma + \phi'(\gamma - c)\mu'(q)dq
\]

Thus, we have

\[
\frac{d\gamma}{dq} = \frac{[(t - \tilde{t}) - \phi'(\gamma - c)\mu'(q)]}{\phi''(\gamma - c)\mu(q)} < 0
\]

because \( \phi'(\gamma - c)\mu'(q) > \phi'(\gamma - c)\frac{\mu(q)}{q} = (t - \tilde{t}) \) by the convexity of \( \phi \) and the first order condition for \( \gamma \)

Let \( q_I \) and \( q_O \) denote the amount of inputs from internal (i.e., FDI) and outside sources, respectively. Then, the fully optimal sourcing decision can be derived from the following optimization program

\[
\text{Min}_{q_I, q_O, \gamma} \left[ \frac{(1 - \tilde{t})c - (t - \tilde{t})\gamma}{1 - t}q_I + \phi(\gamma - c)\mu(q_I) + \omega q_O \right]
\]

subject to

\[
q_I + q_O = q
\]
\[
q_I, q_O \geq 0
\]

The Lagrangian for this problem can be written as

\[
\mathcal{L} = \left[ \frac{(1 - \tilde{t})c - (t - \tilde{t})\gamma}{1 - t}q_I + \phi(\gamma - c)\mu(q_I) \right] + \omega q_O + \eta[q - (q_I + q_O)],
\]

where \( \eta \) is the Lagrangian multiplier associated with the constraint \( q_I + q_O = q \).

The first order conditions can be written as
\[
\frac{\partial L}{\partial q_I} = \xi(q_I; \gamma) - \eta \geq 0, \quad \frac{\partial L}{\partial q_I} q_I = 0
\]
\[
\frac{\partial L}{\partial q_O} = \varpi - \eta \geq 0, \quad \frac{\partial L}{\partial q_O} q_O = 0
\]
\[
\frac{\partial L}{\partial \gamma} = -(t - \tilde{t})q + \phi'(\gamma - c)\mu(q) = 0
\]

There are two types of solutions to this problem.

(i) Dual Sourcing with \( q_I > 0, q_O > 0 \).

In this solution, \( \xi(q_I; \gamma) = \varpi \). Let \( \hat{q} \) be the unique output level such that ... This would be the case when \( q > \hat{q} \). Then, the amount of internal sourcing is given by \( q_I = \hat{q} \), and the rest is outsourced, that is, \( q_O = (q - \hat{q}) \).

(ii) Internal Sourcing Only with \( q_O = 0 \).

In this case, we have \( \xi(q_I; \gamma) = \eta < \varpi \). This would be the case when \( q < \hat{q} \).

\[
(t - \tilde{t})q = \phi'(\gamma - c)\mu(q)
\]
\[
\xi(q; \gamma) = \frac{[(1 - \tilde{t})c - (t - \tilde{t})\gamma] + \phi(\gamma - c)\mu'(q)}{1 - t} = \varpi
\]

Let solution to this be \( \hat{q} \). Then, up to \( \hat{q} \) internal production and beyond which outsourcing. So if \( MR(\hat{q}) > \varpi \), the dual sourcing. If not, then only single sourcing. That is the MNE solves

\[
\min_{q_I, \gamma} \frac{[(1 - \tilde{t})c - (t - \tilde{t})\gamma]q_I + \phi(\gamma - c)\mu(q_I)}{1 - t}
\]

which defines \( \gamma(q) \). Thus, the cost function up to \( \hat{q} \) is given by

\[
C(q) = \frac{[(1 - \tilde{t})c - (t - \tilde{t})\gamma(q)]q + \phi(\gamma(q) - c)\mu(q)}{1 - t}
\]

\[
C'(q) = \frac{\partial C}{\partial q} + \frac{\partial C}{\partial \gamma} \frac{\partial \gamma}{\partial q} = \frac{[(1 - \tilde{t})c - (t - \tilde{t})\gamma(q)] + \phi(\gamma(q) - c)\mu'(q)}{1 - t} = \xi(q; \gamma) < \varpi,
\]

which is described by \( \xi \) curve in Figure 3.
As Figure 3 illustrates, with convex concealment costs, there will be internal sourcing only with small market demand, but as market size grows, the MNE relies on dual sourcing. Note that in our model, we abstract away from fixed costs of setting up a subsidiary by FDI. If there are any fixed costs associated with FDI, then our model would predict that for a very small market size, the sourcing will be done by pure outsourcing, but once the market size grows enough to justify fixed set-up costs, then the monopolist will switch to internal sourcing, and the market size becomes sufficiently large, it will also use outside sourcing. That is, the use of outsourcing is not monotonic with the market size if there are fixed costs of FDI.

Let us work with a parametric example of $\Psi(\gamma - c, q) = k\phi(\gamma - c)\mu(q)$, where $\phi(\gamma -$
c) = (γ - c)α with α > 1, and μ(q) = qβ, that is, Ψ(γ - c, q) = k(γ - c)αqβ. Thus, φ'(γ - c) = αk(γ - c)α-1. As a result, the optimal γ and q satisfies

\[(t - \tilde{t})q = k\alpha(\gamma - c)^{α-1}q^β\]
\[[ (1 - \tilde{t})c - (t - \tilde{t})γ] + k\beta(\gamma - c)^αq^{β-1} = (1 - t)ω\]

From the first equation, we have γ(q) = c + \(\frac{t - \tilde{t}}{k\alpha}\) \(\frac{α}{α-1}\) q \(\frac{β-1}{α-1}\). By substituting this for γ in the second equation, we have

\[(1 - t)c - (t - \tilde{t})\left(\frac{t - \tilde{t}}{k\alpha}\right)^{\frac{α}{α-1}} q^{\frac{β-1}{α-1}} + kβ\left[\left(\frac{t - \tilde{t}}{k\alpha}\right)^{\frac{α}{α-1}} q^{\frac{β-1}{α-1}}\right]^{α} q^{β-1} = (1 - t)ω\]

\[\Rightarrow \]
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\[\Rightarrow \]

Thus, q* is given by

\[q^{\frac{β-1}{α-1}} = \frac{Ω}{(1 - t)(c - ω)},\]

where Ω = \(\left[\frac{α-β}{α}(ak)^{-\frac{1}{α-1}} (t - \tilde{t})^{\frac{α}{α-1}}\right]\)

\[\Rightarrow \]
\[\Rightarrow \]
\[\Rightarrow \]
\[\Rightarrow \]
\[\Rightarrow \]
\[\Rightarrow \]

\[\hat{q} = \left[\frac{Ω}{(1 - t)(c - ω)}\right]^{\frac{α}{β-1}} = \left[\frac{α - β}{α(1 - t)(c - ω)}\right]^{\frac{α}{β-1}} (ak)^{-\frac{1}{β-1}} (t - \tilde{t})^{\frac{α}{β-1}}\]

To illustrate the idea, let us assume that P(q) = A - q, where A represents the market size. Then, MR = A - 2q and the MR curve intersects with ω (the outsourcing MC) at q = \(\frac{A - ω}{2}\). Thus, dual sourcing takes place if and only if \(\frac{A - ω}{2} > \hat{q}\), i.e.,

\[A > 2\hat{q} + ω = 2\left[\frac{α - β}{α(1 - t)(c - ω)}\right]^{\frac{α}{β-1}} (ak)^{-\frac{1}{β-1}} (t - \tilde{t})^{\frac{α}{β-1}} + ω\]

2.4 Dual Sourcing and Invocation of the CUP Method
In the previous section, we analyzed the MNE’s sourcing behavior in the presence of concealment costs. The basic premise of the analysis was that for the monopoly case we have considered the applicability of the ALP with the CUP method can be limited if there is only one firm that produces the product and there are no similar transactions that can be observed and used as a benchmark. This is especially so when all input acquisitions are done internally via FDI. Even if an alternative input is available at the price of $\bar{\omega}$, the MNE may argue that the input available in the open market is not suitable for specific purposes of the MNE and the unavailability of suitable input is the reason for FDI and internal sourcing to begin with. However, such an argument loses appeal once the MNE engages in dual sourcing and acquires some of their input requirements from outsourcing because it is an implicit admission that the open market input is suitable for its final product. This implies that dual sourcing may entail a risk that it may induce the government to adopt the CUP method instead of the cost plus method.

If dual sourcing invokes the use of the CUP method as an application of the ALP, the MNE thus has two choices. One is to engage in internal sourcing only to avoid the CUP method. The other is to do outsourcing in which case the CUP method will be imposed and the internal price should be set at $\bar{\omega}$ if internal sourcing is also used.

**Proposition 2.** If dual sourcing triggers the CUP method, the monopolistic firm never engages in (meaningful) dual sourcing.

**Proof.** Suppose that the monopolistic firm engages in dual sourcing with $q_I > 0$ and $q_O > 0$, where $q = q_I + q_O$. Then, its internal price should be $\gamma = \bar{\omega}$. Thus, the monopolist’s profit is with dual sourcing under ALP is given by

$$
\Pi^D = (1-t)[P(q_I + q_O) - \bar{\omega})(q_I + q_O) + (1-\tilde{t})(\bar{\omega} - c)q_I
$$

$$
= (1-t)[P(q) - \bar{\omega}]q + (1-\tilde{t})(\bar{\omega} - c)q_I
$$

We consider two cases depending on the MNE’s true marginal cost of production via FDI exceeds the market price or not.

(i) $c > \bar{\omega}$

In this case, the foreign subsidiary that produces internally makes loss due to ALP. Thus, the profit from dual sourcing is less than the profit under outsourcing only which
Figure 4: Equilibrium Change with ALP

is given by

$$\Pi^{OS} = (1 - t)[P(q) - \bar{\omega}]q$$
$$> (1 - t)[P(q) - \bar{\omega}]q + (1 - \bar{t})(\bar{\omega} - c)q_I = \Pi^D$$

(ii) $c < \bar{\omega}$

In this case, essentially internal sourcing only will dominate dual sourcing because $\Pi^D$ is maximized when $q_I$ is as close as $q$ with a minimal $q_O$. □

The imposition of ALP thus may have different effects depending on the situations. If $c < \bar{\omega}$, it will have desirable effects [explain]. However, if $c > \bar{\omega}$, the monopolistic firm may engage in internal sourcing only with the imposition of ALP (see Figure 4)

3 Import Tariff as a Countermeasure against Profit-Shifting

We consider a specific industry in which the MNE is operating. Implicitly we assume that the overall corporate tax rate is determined by factors beyond the specific industry we consider. The overall corporate tax rate thus cannot be tailored for this particular industry
and is considered exogenously given. However, in face of profit-shifting incentives of the MNE, the government can impose industry-specific ad-valorem import tariffs to eliminate such incentives. We analyze how import tariffs can be adopted as a countermeasure against profit-shifting.

Let \( \tau_m \) denote ad-valorem import tariff imposed by country \( H \) where the MNE is located. Now the MNE’s problem with FDI can be written as

\[
\hat{\Pi} = (1 - t)[P(q) - (1 + \tau_m)\gamma]q + (1 - \tilde{t})(\gamma - c)q - \phi(\gamma - c)q
\]  \hspace{1cm} (8)

\[
\hat{\Pi} = (1 - t)[P(q) - \hat{\xi}]q
\]  \hspace{1cm} (9)

where

\[
\hat{\xi} = \tau_m \gamma + \frac{(1 - \tilde{t})c - (t - \tilde{t})\gamma + \phi(\gamma - c)}{1 - t}
\]  \hspace{1cm} (10)

with \( \xi = \frac{(1 - \bar{t})c - (t - \bar{t})\gamma + \phi(\gamma - c)}{1 - t} = c - \frac{(t - \tilde{t})(\gamma - c) - \phi(\gamma - c)}{1 - t} \)

In the presence of import tariffs, the optimal choice of the transfer price \( \hat{\gamma}^* \) is equivalent to minimize the MNE’s "virtual marginal cost cum tariffs" of \( \hat{\xi} \), and implicitly defined by

\[
(t - \bar{t}) - \tau_m(1 - t) = \phi'(\hat{\gamma} - c)
\]

By totally differentiating the first order condition above, we have

\[-(1 - t)d\tau_m = \phi''(\hat{\gamma}^* - c)d\hat{\gamma}^*
\]

Thus, we have

\[
\frac{\partial \hat{\gamma}^*}{\partial \tau_m} = -\frac{(1 - t)}{\phi''(\hat{\gamma}^* - c)} < 0
\]

indicating that the incentives to inflate the internal price by the MNE can be mitigated by the imposition of import tariffs. Note that the optimal \( \hat{\gamma}^* \) chosen by the MNE can be written as

\[
\hat{\gamma}^* = c + \phi'^{-1}((t - \bar{t}) - \tau_m(1 - t))
\]

\(^{5}\) We denote all variables in the presence of import tariffs with a hat (\(\hat{\cdot}\))
This implies that import tariffs of \( \tau_m = \tau_m(= \frac{t}{1-t}) \) can be used to completely offset any incentives for profit-shifting. In addition, with \( \tau_m = \tau_m, \tilde{\xi} = c \) and the MNE will be engaged in FDI only when its internal production is more efficient than the open market. However, consumer welfare goes down compared to the case of no import tariffs. Thus, the optimal import tariffs can be lower than \( \tau_m \), the import tariff that eliminates any incentives for profit-shifting as shown below.

### 3.1 Optimal Import Tariffs

Let us analyze the government’s optimal choice of import tariffs given \((t, \tilde{t})\) when it maximizes domestic social welfare, which is defined as

\[
W = \text{Producer Surplus} + \text{Consumer Surplus} + \text{Tax Revenue}.
\]

We consider import tariffs as a second-best policy when the transfer price and output choices are left to the firm. Let \( \tilde{\xi}^* \) be the minimized virtual MC with the choice of optimal transfer price \( \hat{\gamma}^* \), that is,

\[
\tilde{\xi}^* = \tau_m \hat{\gamma}^* + \frac{(1 - \tilde{t})c - (t - \tilde{t})\hat{\gamma}^* + \phi(\hat{\gamma}^* - c)}{1 - t}
\]

Let the corresponding output level be \( q(\tilde{\xi}^*) \). Then, social welfare with FDI can be written as

\[
W = (1 - t)[P(q) - \tilde{\xi}^*]q + \int_0^q P(x)dx - P(q)q + [t[P(q) - (1 + \tau_m)\gamma]q + \tau_m \gamma q]
\]

\[
= (1 - t)[P(q) - (1 + \tau_m)\gamma]q + (1 - \tilde{t})(\gamma - c)q - \phi(\gamma - c)q
\]

\[
+ \int_0^q P(x)dx - P(q)q + [t[P(q) - (1 + \tau_m)\gamma]q + \tau_m \gamma q]
\]

\[
= \int_0^q (P(x) - \tilde{\xi}^{SP}) dx
\]

where \( \tilde{\xi}^{SP} = c + \tilde{t}(\hat{\gamma}^* - c) + \phi(\hat{\gamma}^* - c) \) and represents the marginal cost of FDI production from the perspective of the social planner of H country. It consists of the physical production cost of \( c \), tax transfer to the host country, and any concealment costs incurred by the MNE. Note that the production level by the MNE is determined by its perceived MC of \( \hat{\xi}^* \), not the social planner’s \( \tilde{\xi}^{SP} \). This implies that the choice of \( \tau_m \) that minimizes
\( \hat{\xi}^{SP} \) is not necessarily the optimal import tariff.

To be more precise, the marginal effect of import tariff on social welfare can be written as follows:

\[
\frac{dW}{d\tau_m} = \frac{d}{d\tau_m} \left[ \int_0^{q(\hat{\xi}(\hat{\gamma}, \tau_m))} P(x) - \hat{\xi}^{SP}(\hat{\gamma}) \, dx \right]
\]

\[
= \left[ P(q(\hat{\xi}(\hat{\gamma}, \tau_m))) - \hat{\xi}^{SP}(\hat{\gamma}) \right] \frac{dq}{d\xi} \left[ \frac{\partial \hat{\xi}^*}{\partial \tau_m} + \frac{\partial \hat{\gamma}^*}{\partial \tau_m} \right] - \int_0^{q(\hat{\xi}(\hat{\gamma}))} \frac{d\hat{\xi}^{SP}(\hat{\gamma})}{d\hat{\gamma}} \frac{\partial \hat{\gamma}^*}{\partial \tau_m} \, dx
\]

\[
= \left[ P(q(\hat{\xi}(\hat{\gamma}, \tau_m))) - \hat{\xi}^{SP}(\hat{\gamma}) \right] \frac{dq}{d\xi} \frac{\partial \hat{\xi}^*}{\partial \tau_m} - q(\hat{\xi}(\hat{\gamma})) \frac{\partial \xi^{SP}(\hat{\gamma})}{\partial \tau_m} = 0,
\]

where

\[
\frac{\partial \hat{\xi}^*}{\partial \hat{\gamma}} = \tau_m - \frac{(t - \tilde{t})}{1 - t} + \frac{\phi'(\hat{\gamma} - c)}{1 - t} = 0 \text{ by the envelope theorem}
\]

and

\[
\frac{\partial \hat{\xi}^{SP}(\hat{\gamma})}{\partial \hat{\gamma}} = \tilde{t} + \phi'(\hat{\gamma} - c) > 0, \quad \frac{dq}{d\xi} < 0, \quad \frac{\partial \hat{\xi}^*}{\partial \tau_m} = \hat{\gamma}^*, \quad \frac{\partial \hat{\gamma}^*}{\partial \tau_m} = -\frac{(1 - t)}{\phi''(\hat{\gamma}^* - c)} < 0
\]

In a more compact form, we have

\[
\frac{dW}{d\tau_m} = \left[ P(q(\tau_m)) - \hat{\xi}^{SP}(\tau_m) \right] \frac{d\tau_m}{d\tau_m} \left( \frac{d\tau_m}{d\tau_m}(+) \right) - q(\tau_m) \frac{d\hat{\xi}^{SP}}{d\tau_m} \left( \frac{d\tau_m}{d\tau_m}(-) \right)
\]

The first term on the RHS represents the negative effect on consumer welfare as the imposition of import tariffs increases the MNE’s virtual MC which induces the firm to reduce outputs in the domestic market. The second term on the RHS is the positive effect of reducing tax shifting to country \( F \) and concealment costs.

**Lemma 2.** The optimal import tariff is never higher than \( \tau_m = \frac{t - \tilde{t}}{1 - t} \).

**Proof.** See the Appendix. ■

We thus consider only import tariff \( \tau_m \leq \tau_m \). Let us define the wedge between the social planner’s perceived MC and the MNE’s perceived MC as \( \delta = \hat{\xi}(\gamma(\tau_m)) - \hat{\xi}^{SP}(\gamma(\tau_m)) \).

20
Then, we have
\[ \tilde{\xi}(\gamma(\tau_m)) - \tilde{\xi}^{SP}(\gamma(\tau_m)) = \tau_m \gamma + \frac{t}{1-t} \left[ \phi(\gamma - c) - (1 - \tilde{\gamma})(\gamma - c) \right], \]
where \( \tilde{\gamma}^* = c + \phi'^{-1}(t - \tilde{t}) - \tau_m(1 - t) \).
Since \( \gamma = c \) when \( \tau_m = \frac{t - \tilde{t}}{1 - \tilde{t}} \), we have
\[ \delta|_{\tau_m=t-\tilde{t}/1-\tilde{t}} = \left. \tilde{\xi}(\gamma) - \tilde{\xi}^{SP}(\gamma) \right|_{\tau_m=t-\tilde{t}/1-\tilde{t}} = \frac{t - \tilde{t}}{1 - \tilde{t}} c > 0 \]

We can also easily verify that
\[ \delta|_{\tau_m=0} = \left. \tilde{\xi}(\gamma) - \tilde{\xi}^{SP}(\gamma) \right|_{\tau_m=0} = \frac{t}{1-t} \left[ \phi(\gamma - c) - (1 - \tilde{\gamma})(\gamma - c) \right] |_{\tau_m=0} < 0 \]

To see this, note that at \( \gamma \) satisfies the first order condition \( (t - \tilde{t}) = \phi'(\gamma - c) \). Since \( \phi \) is convex, we have
\[ \frac{\phi(\gamma - c)}{\gamma - c} < \phi'(\gamma - c) = (t - \tilde{t}) < 1 - t \]
Thus, \( (1 - \tilde{t})(\gamma - c) > \phi(\gamma - c) \). We also know that
\[ \frac{d}{d\tau_m} \left[ \tilde{\xi}(\gamma(\tau_m)) - \tilde{\xi}^{SP}(\gamma(\tau_m)) \right] = \gamma - \left[ \tilde{\gamma} + \phi'(\gamma - c) \right] \frac{\partial \tilde{\gamma}}{\partial \tau_m} > 0 \]
because \( \phi'(\gamma - c) > 0 \) and \( \frac{\partial \tilde{\gamma}}{\partial \tau_m} < 0 \). Thus, we have the following lemma.

**Lemma 3.** There is a unique \( \tau_m^o \in (0, \frac{t - \tilde{t}}{1 - \tilde{t}}) \) such that
\[ \begin{cases} \tilde{\xi}(\gamma) > \tilde{\xi}^{SP}(\gamma) \text{ if } \tau_m < \tau_m^o \\ \tilde{\xi}(\gamma) = \tilde{\xi}^{SP}(\gamma) \text{ if } \tau_m = \tau_m^o \\ \tilde{\xi}(\gamma) < \tilde{\xi}^{SP}(\gamma) \text{ if } \tau_m > \tau_m^o \end{cases} \]

A sufficient condition for the optimal import tariff to be less than \( \tau_m \) is
\[ \frac{dW}{d\tau_m} |_{\tau_m=\tau_m} = \left. \frac{d}{d\tau_m} \left[ \int_0^{\tilde{\theta}(\gamma)} \frac{P(x) - \tilde{\xi}^{SP}(\gamma)}{\tilde{\xi}} \right] dx \right|_{\tau_m=\frac{t - \tilde{t}}{1 - \tilde{t}}} < 0 \]
The following proposition provides a sufficient condition for this.
Proposition 3. Let \( \rho = P' \frac{d\xi}{d\tau_m} \) denote the cost-price pass-through rate for the monopolist.

\[
\frac{dW}{d\tau_m} \bigg|_{\tau_m = \tau_m} < 0 \quad \text{if} \quad \rho > \left. \frac{\frac{d\xi^{SP}}{d\tau_m}}{\frac{d\xi}{d\tau_m}} \right|_{\tau_m = \frac{t - \gamma}{1 - t}}.
\]

Proof. We have

\[
\frac{dW}{d\tau_m} = \left[ P(q(\tau_m)) - \xi^{SP}(\tau_m) \right] \frac{dq(\tau_m)}{d\tau_m} - q(\tau_m) \frac{d\xi^{SP}}{d\tau_m}
\]

\[
= \left[ P(q(\tau_m)) - \hat{\xi}^{*} + (\hat{\xi}^{*} - \xi^{SP}(\tau_m)) \right] \frac{dq(\tau_m)}{d\tau_m} - q(\tau_m) \frac{d\xi^{SP}}{d\tau_m}
\]

By Lemma 3, we know \( \hat{\xi}^{*} - \xi^{SP} > 0 \) at \( \tau_m = \tau_m \). In addition, we know that

\[
P(q(\tau_m)) - \hat{\xi}^{*} = -P'q
\]

by the first order condition for the MNE’s profit maximization and \( \frac{d\xi^{SP}}{d\tau_m} < 0 \). As a result, we have

\[
\frac{dW}{d\tau_m} \bigg|_{\tau_m = \tau_m} < -P'q \left. \frac{dq(\tau_m)}{d\tau_m} - q(\tau_m) \frac{d\xi^{SP}}{d\tau_m} \right|_{\tau_m = \frac{t - \gamma}{1 - t}}
\]

\[
= -q \left[ P' \left. \frac{dq}{d\xi} \frac{d\xi}{d\tau_m} + \frac{d\xi^{SP}}{d\tau_m} \right|_{\tau_m = \frac{t - \gamma}{1 - t}} \right]
\]

Therefore,

\[
\frac{dW}{d\tau_m} \bigg|_{\tau_m = \tau_m} < -q \left[ P' \left. \frac{dq}{d\xi} \frac{d\xi}{d\tau_m} + \frac{d\xi^{SP}}{d\tau_m} \right|_{\tau_m = \frac{t - \gamma}{1 - t}} \right] < 0 \quad \text{if} \quad \rho > \left. \frac{\frac{d\xi^{SP}}{d\tau_m}}{\frac{d\xi}{d\tau_m}} \right|_{\tau_m = \frac{t - \gamma}{1 - t}}.
\]

Thus, we can conclude that the optimal import tariff \( \tau^{*} < \tau_m = \frac{t - \gamma}{1 - t} \), that is, the optimal import tariffs mitigates incentives to engage in tax manipulation via transfer price, but does not completely eliminate it, if \( \rho > \frac{\frac{d\xi^{SP}}{d\tau_m}}{\frac{d\xi}{d\tau_m}} \bigg|_{\tau_m = \tau_m} \). This is because the transfer price induces the MNE produces more, which can enhance consumer welfare. For instance, this condition is satisfied if \( \tilde{t} \) is sufficiently small because \( \left. \frac{d\xi^{SP}}{d\tau_m} \right|_{\tau_m = \tau_m} = \frac{\tilde{t} - (1-t)}{\phi (\xi^{*} - c)} \approx 0 \) and
\[ \frac{dg}{d\tau_m} \bigg|_{\tau_m=\overline{\tau}_m} = c. \]

In particular, if we assume quadratic concealment costs and linear demand of \( P = A - q \), the condition can be written as

So far, our discussion of the optimal ad valorem import tariff was on the premise that the MNE is engaged in FDI. However, if the optimal \( \tau_m^* \) derived above is sufficiently large, we may have a situation in which \( \hat{\xi}(\tau_m = 0) < \varpi \), but \( \hat{\xi}(\tau_m = \tau_m^*) > \varpi(1 + \tau_m^*) \). In this case, there is a unique import tariff level \( \tau_m \in (0, \tau_m^*) \) such that \( \hat{\xi}(\tau_m) = \varpi(1 + \tau_m) \).

In this case, the optimal tariff is \( \overline{\tau}_m \). The government in country \( H \) then sets an import tariff at the rate of \( \min[\tau_m^*, \overline{\tau}_m] \) because \( H \) country prefers outsourcing at \( \tau_m = \overline{\tau}_m \), that is, \( \hat{\xi}(\tau_m = \tau_m^*) = c(1 + \tau_m^*) > \varpi(1 + \tau_m^*) \).

To see this, note that at \( \tau_m = \overline{\tau}_m \), the MNE produces the same quantity and thus CS is the same. With outsourcing, welfare can be written as

\[ \hat{W}^{OS} = \int_0^q [P(x) - \varpi] \, dx \]
What we need to show is $\hat{\xi}(\tau_m) > \omega$.

\[
(1 + \tau_m) \left[ \hat{\xi}^{SP}(\tau_m) - \omega \right] = (1 + \tau_m) \left[ c + \bar{t}(\hat{\gamma}^* - c) + \phi(\hat{\gamma}^* - c) \right] \\
- \left[ \tau_m \hat{\gamma}^* + c + \frac{-(t - \bar{t}) \cdot (\hat{\gamma}^* - c) + \phi(\hat{\gamma}^* - c)}{1 - t} \right] \\
> (1 + \tau_m) \left[ c + \bar{t}(\hat{\gamma}^* - c) + \phi(\hat{\gamma}^* - c) \right] \\
- \left[ \tau_m \hat{\gamma}^* + c + \frac{-(t - \bar{t}) \cdot (\hat{\gamma}^* - c) + (t - \bar{t}) \cdot (\hat{\gamma}^* - c)}{1 - t} \right] \\
= -\tau_m (\hat{\gamma}^* - c) + \left[ (1 + \tau_m) - \frac{1}{1 - t} \right] [\bar{t}(\hat{\gamma}^* - c) + \phi(\hat{\gamma}^* - c)] \\
+ \frac{t}{1 - t} (\hat{\gamma}^* - c) \\
= \left[ \frac{(t - \bar{t})}{1 - t} - \tau_m \right] \cdot (\hat{\gamma}^* - c) + (1 + \tau_m) \bar{t}(\hat{\gamma}^* - c) \\
+ \left[ (1 + \tau_m) - \frac{1}{1 - t} \right] \phi(\hat{\gamma}^* - c) \\
> \left[ \frac{(t - \bar{t})}{1 - t} - \tau_m \right] \cdot (\hat{\gamma}^* - c) + (1 + \tau_m) \bar{t}(\hat{\gamma}^* - c) \\
+ \left[ (1 + \tau_m) - \frac{1}{1 - t} \right] (t - \bar{t}) \cdot (\hat{\gamma}^* - c) \\
= (1 + \tau_m) t - \tau_m > 0 \text{ because } \tau_m < \frac{t - \bar{t}}{1 - t} < \frac{t}{1 - t}
\]

3.1.1 Parametric Example

As shown above, with $\phi(\gamma - c) = \frac{k}{2}(\gamma - c)^2$, we have

$$\hat{\gamma}^* = c + \frac{(t - \bar{t}) - \tau_m (1 - t)}{k}$$

By plugging back this into $\hat{\xi}$, we can easily verify that
\[
\hat{\xi}^* = \left( c + \frac{(t - \bar{t}) - \tau_m(1-t)}{k} \right) + \frac{(1 - \bar{t})c - (t - \bar{t})\left[ c + \frac{(t-\bar{t}) - \tau_m(1-t)}{k} \right] + \frac{k}{2}\left( \frac{(t-\bar{t}) - \tau_m(1-t)}{k} \right)^2}{1 - t}
\]

From outsourcing, the MNE’s cost becomes \( \omega(1 + \tau_m) \). So given \( \tau_m \), the MNE engages in FDI if and only if

\[
c(1 + \tau_m) - \frac{[(t - \bar{t}) - \tau_m(1-t)]^2}{2k(1-t)} < \omega(1 + \tau_m)
\]

or

\[
c < \omega + \tilde{\lambda},
\]

where \( \tilde{\lambda} = \frac{[(t - \bar{t}) - \tau_m(1-t)]^2}{2k(1-t)(1+\tau_m)} \). Note that \( \tilde{\lambda} \) is decreasing in \( \tau_m \) and \( \tilde{\lambda}|_{\tau_m=0} = \lambda = \frac{(t-\bar{t})^2}{2k(1-t)} \) and \( \tilde{\lambda}|_{\tau_m=\tau_m} = 0 \).

4 The Duopoly Model with Strategic Interactions

In this section we consider a duopoly model in which an MNE competes with another firm in the domestic market in order to explore implications of strategic effects. The set-up is otherwise the same as in the monopoly model. More specifically, two final good producers, firm 1 and firm 2, compete in \( H \). Firm 2 is a domestic firm and simply procures its input from F with an exogenously given market price \( \tilde{\gamma} \) (later we extend the model to endogenize \( \tilde{\gamma} \)). Firm 1 has two choices as before. It can procure its input from \( F \) like firm 2. Or it can be an MNE by setting up its own input production plant in \( F \) with FDI. In such a case, its input production cost is given by \( c \). In this case, the MNE can choose an internal transfer price (\( \gamma \)) when its foreign subsidiary supply its input to the headquarter firm that produces the final good. Figure 4 describes the duopoly model.

In the monopoly case, we assumed that the CUP method is not applicable because there is no comparable downstream firm and the input market simply does not exist in the case of FDI (unless the MNE is engaged in dual sourcing that also relies on outside suppliers). As a result, the ALP was based on the cost plus method and the MNE
was assumed to operate with concealment costs when its transfer price deviates from its marginal cost. In the case of duopoly, the applicability of the CUP method depends on whether the transactions between the rival downstream firm and its input suppliers can be regarded as "externally comparable" to internal transactions of the MNE (OECD 2010, p. 71). We present two sets of results depending on the comparability of the external transactions. First, we consider a scenario in which the external transactions are not considered as comparable. This would be the case if the two downstream firms produce differentiated products and use very different types of inputs. Then, the ALP should be based on the cost-plus method and the MNE operates with concealment costs. In contrast, if the external transactions are considered as comparable, then the MNE is constrained to use the comparable market price as the internal transfer price.

4.1 Profit Shifting in Duopoly Model with Concealment Costs

We first analyze the case in which the transactions between the rival downstream firm and its input suppliers are not comparable to the internal transactions of the MNE. In this case, the MNE’s behavior can be described with the presence of concealment costs for transfer price that deviates from its true marginal cost, as in the previous section. The case of comparable external transactions is analyzed in section 4.2.

To focus on implications of strategic interactions for the MNE’s behavior, we assume concealment costs that is linear in output, that is, $\Phi(\gamma - c, q) = \phi(\gamma - c)q$ with $\phi' > 0$. 

Figure 5: Duopoly Model with Strategic Interactions
Firm 1 solves the following problem

\[
\max_{q_1} \Pi_1 = (1 - t) \pi_1(q_1, q_2; \gamma) + (1 - \tilde{t})(\gamma - c)q_1 - \phi(\gamma - c)q_1
\]

where \(\pi_1(q_1, q_2; \gamma) = [P(q_1, q_2) - \gamma]q_1\). Once again, by collecting terms with \(q_1\), we can rewrite it as

\[
\Pi_1 = (1 - t)[P(q_1, q_2) - \xi]q_1,
\]

where

\[
\xi = \frac{(1 - \tilde{t})c - (t - \tilde{t})\gamma + \phi(\gamma - c)}{1 - t}
\]

The first order condition for firm 1 is given by

\[
\frac{1}{1 - t} \frac{\partial \Pi_1}{\partial q_1} = \frac{\partial \pi_1(q_1, q_2; \xi)}{\partial q_1} = 0
\]

(12)

Firm 2 similarly makes its decision on \(q_2\) to solve the following problem:

\[
\max_{q_2} \pi_2(q_1, q_2; \omega) = [P(q_1, q_2) - \omega]q_2
\]

\[
\Rightarrow \quad \frac{\partial \pi_2(q_1, q_2; \omega)}{\partial q_2} = 0
\]

(14)

Eq (12) and (14) implicitly define reaction functions for firm 1 and firm 2 respectively. The equilibrium quantities for each firm, \(q_1^*(\gamma)\) and \(q_2^*(\gamma)\), given the transfer price \(\gamma\) are at the intersection of these two reaction functions.

Assume \(\left| \frac{\partial^2 \pi_i}{\partial q_i^2} \right| > \left| \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} \right|\), where \(i = 1, 2\) and \(j \neq i\).

**Lemma 4.** Let \(\gamma^*\) be the unique \(\gamma\) that minimizes the virtual cost of \(\xi\) such that \((t - \tilde{t}) = \phi'(\gamma^* - c)\). Then we have the following comparative statics results: (i) if \(\gamma < \gamma^*\), \(\frac{dq_1^*}{d\gamma} > 0\) and \(\frac{dq_2^*}{d\gamma} < 0\), (ii) if \(\gamma > \gamma^*\), \(\frac{dq_1^*}{d\gamma} < 0\) and \(\frac{dq_2^*}{d\gamma} > 0\), and (iii) if \(\gamma = \gamma^*\), \(\frac{dq_1^*}{d\gamma} = \frac{dq_2^*}{d\gamma} = 0\).

**Proof.** See the Appendix. ■

Lemma 2 implies that with concealments costs, the strategic effects of an increase in the transfer price depends on the relative magnitudes of the tax differential and the deviation of the transfer price from the MNE’s true marginal costs. Note that \(\frac{dq_1}{d\gamma} = \frac{dq_2}{d\gamma} = 0\) if
\( t = \tilde{t} \). That is, the transfer price is irrelevant for downstream competition if there is no tax differential between H and F. As the MNE behaves as if its MC were \( \xi \) which is the weighted average of its true marginal cost \( c \) and the internal transfer price, with the weights determined by the tax rates. The higher the transfer price, the MNE behaves as if its MC were lower and compete more aggressively in the downstream market and this effect is larger with the tax rate differential \((t - \tilde{t})\). Note that \( \psi = c \) if \( t = \tilde{t} \) or \( \gamma = c \). This implies that both differential tax rates and inflated internal transfer price are required for strategic effects on the downstream market.

Now let us analyze the optimal choice of the MNE’s transfer price \( \gamma \). To this end, we analyze the effects of \( \gamma \) on the overall net profits of the MNE.

\[
\frac{d\Pi_1}{d\gamma} = \left[(1-t)\frac{d\pi_1}{d\gamma} + (1-\tilde{t})q_1\right] + (1-t)\frac{\partial\pi_1}{\partial q_2}\frac{dq_2}{d\gamma} - \phi'(\gamma - c)q_1
\]

\[
= \left[(t-\tilde{t}) - \phi'(\gamma - c)\right]q_1 + (1-t)\frac{\partial\pi_1}{\partial q_2}\frac{dq_2}{d\gamma} = 0
\]

By lemma 2, we know that the direct tax effect and the strategic effect move in the same direction. That is,

\[
\frac{d\Pi_1}{d\gamma} = \begin{cases} 
> 0 & \text{if } \gamma < \gamma^* \\
= 0 & \text{if } \gamma = \gamma^* \\
< 0 & \text{if } \gamma > \gamma^*
\end{cases}
\]

Thus, the optimal transfer price is given by \( \gamma^* \) as in the monopoly case.

As expected, the MNE’s tax manipulation strategy reduces its tax obligation in H (high tax rate country). In fact, in our model, its internal transfer price is artificially jacked up to the extent to shift all its profit at the downstream stage to the upstream subsidiary which resides in F (low tax rate country). As a result, the tax revenue from the MNE is zero because all profits are shifted to a subsidiary that is located in a lower tax country. In addition, we uncover additional tax revenue loss from other firms in the presence of imperfect competition due to strategic effects. However, it is not the end of the story; there is a collateral damage due to spillover effects. The aggressive behavior of the MNE with the tax-induced transfer price also reduces the rival firm’s profits. Thus, the tax revenue from the other firm that is not engaged in tax manipulation also decreases
(even though consumer surplus increases).

It is also worthwhile to point out the crucial difference between the strategic effects driven by tax differences in our model and strategic transfer pricing in the IO and management literature (see Alles and Datar (1998)). The basic premise of strategic transfer pricing in oligopoly models is to assume decentralized decision making and each division maximizes its own profits, rather than the overall profits of the firm. Otherwise, the optimal decisions will be based on true marginal costs and the transfer prices would not matter and does not generate any strategic effects because internal transfer prices are just transfers among divisions within the firm and cancel out each other from the perspective of firm’s overall profits. Only when the decision of each division is driven by its own profits, transfer price can have any meaningful effects. In contrast, our model assumes centralized decision making. If the decision is not centralized, when the transfer price is inflated to reduce the tax burden, the strategic effects will go the other way around.

4.2 Arm’s Length Principle with the CUP Method

We now consider a scenario in which the transactions between the rival downstream firm and its input suppliers can be considered comparable. In this case, the ALP can be applied as a requirement that the transfer price be equal to similar input price in the market, which is the input price paid by firm 2.\(^6\) If the input market for firm 2 is perfectly competitive with the price of \(\omega\) and the same input can be used for firm 1, the analysis is trivial. As its transfer price is constrained to be at \(\omega\) with the CUP method, it will engage in FDI if and only if FDI is efficient from the global efficiency point, that is, \(c < \omega\). In this case, profit-shifting will take place to some extent, but it is limited by the competitive market price \(\omega\). If \(c > \omega\), there is no inefficient FDI for profit shifting purpose. However, if we assume that the input available in the open market is supplied by a firm with market power, we can restore inefficient FDI for tax manipulation.

To see this, we now endogenize \(\bar{\gamma}\) by assuming that \(F\) input supplier has market power. More specifically, let us assume that \(F\) input supplier is a monopolist and sets the input

\(^6\)Gresik and Osmundsen (2008) analyzes the arm’s length principle when all firms are vertically integrated and comparable but independent transactions on which the application of the arm’s length principle can be based is not available. Such issue does not arise in our model because the rival downstream firm acquires its input from an independent source.
price $\tilde{\gamma}$ given its marginal cost of $\tilde{c}$.

$$\max_{\tilde{\gamma}} (\tilde{\gamma} - \tilde{c}) q_2^{\text{CUP}}(\tilde{\gamma})$$

Let $q(x, y)$ denote the equilibrium output level for a downstream firm when its input cost is $x$ while the rival firm’s cost is given by $y$. Let us assume that firm 1 engages in FDI. Then,

$$q_2^{\text{AL}}(\tilde{\gamma}) = q(\tilde{\gamma}, \xi^{\text{CUP}}),$$

where

$$\xi^{\text{CUP}} = \frac{(1 - \tilde{t})c - (t - \tilde{t})\tilde{\gamma}}{1 - t}$$

The first order condition on $\tilde{\gamma}$ is given by

$$q(\tilde{\gamma}, \xi^{\text{CUP}}) + (\tilde{\gamma} - \tilde{c}) \left[ \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} \frac{\partial \xi^{\text{CUP}}}{\partial \tilde{\gamma}} \right] = 0$$

Thus, the optimal $\tilde{\gamma}^*$ is implicitly defined by

$$\tilde{\gamma}^* = \tilde{c} - \frac{q(\tilde{\gamma}^*, \xi^{\text{CUP}})}{\frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} \frac{\partial \psi^{\text{AL}}}{\partial \tilde{\gamma}}} > \tilde{c}$$

With the CUP applied as the ALP, upstream firm $F$’s input price $\tilde{\gamma}$ influences the MNE’s transfer price and indirectly affects MNE’s competitive behavior in the downstream market via its effect on $\xi^{\text{CUP}}$. Since $\frac{\partial \psi^{\text{AL}}}{\partial \tilde{\gamma}} = \frac{-(t - \tilde{t})\tilde{\gamma}}{1 - t} < 0$, a higher input price to firm 2 reduces firm 1’s virtual cost $\psi^{\text{AL}}$ and indirectly reduces firm 2’s output via strategic effects. Thus, under the CUP, upstream firm $F$ charges a lower input price compared to the case of its absence if firm 1 produces internally with FDI.

We now consider firm 1’s FDI decision. Let us assume that the monopolistic input supplier cannot price discriminate between two downstream firms and charge an input price of $\tilde{\gamma}$. Then, firm 1 engages in FDI if and only if

$$\xi^{\text{CUP}} = \frac{(1 - \tilde{t})c - (t - \tilde{t})\tilde{\gamma}}{1 - t} \leq \tilde{\gamma},$$

which is equivalent to $c \leq \tilde{\gamma}$. This implies that if the monopolist charges an input price higher than $c$, its demand is $q(\tilde{\gamma}, \psi^{\text{AL}})$ whereas if it charges an input price less than $c$, its demand is $2q(\tilde{\gamma}, \tilde{\gamma})$. It is immediate that firm 1 always engages in FDI if its production
cost is lower than the input supplier, i.e., \( c \leq \tilde{c} \). We thus focus on the case where \( c > \tilde{c} \), that is, firm 1’s internal production is less efficient than the input supplier. In this case, the input monopolist’s profit can be written as

\[
\pi^m = \begin{cases} 
\pi^m_2 = (\tilde{\gamma} - \tilde{c}) q(\tilde{\gamma}, \xi^{CUP}) & \text{if } \tilde{\gamma} > c \\
\pi^m_b = (\tilde{\gamma} - \tilde{c}) 2q(\tilde{\gamma}, \tilde{\gamma}) & \text{if } \tilde{\gamma} \leq c 
\end{cases}
\]

Note that \( \xi^{CUP} = c \) when \( \tilde{\gamma} = c \). This implies that the monopolist’s profit jumps down discretely (more precisely, halves down) as its price is increased from \( c \) by \( \epsilon \) when \( \tilde{c} < c \).

For analytical simplicity, let us assume that both \( \pi^m_2 = (\tilde{\gamma} - \tilde{c}) q(\tilde{\gamma}, \psi^{AL}) \) and \( \pi^m_b = (\tilde{\gamma} - \tilde{c}) 2q(\tilde{\gamma}, \tilde{\gamma}) \) are concave in \( \tilde{\gamma} \). To reduce the number of cases to consider, we assume that the gap between \( \tilde{c} \) and \( c \) is not too large. More specifically, we assume

\[
\frac{\partial \pi^m_b}{\partial \tilde{\gamma}} \bigg|_{\tilde{\gamma} = c} = 2 \left[ q(c, c) + (c - \tilde{c}) \left( \frac{\partial q(c, c)}{\partial x} + \frac{\partial q(c, c)}{\partial y} \right) \right] > 0
\]

that is,

\[
(c - \tilde{c}) < \frac{q(c, c)}{\left| \frac{\partial q(c, c)}{\partial x} + \frac{\partial q(c, c)}{\partial y} \right|}
\]

It can also be easily verified that this condition also implies that \( \tilde{\gamma}^*(\tilde{c}) > c \). Otherwise, the outside monopolistic input supplier’s cost is drastically lower than that of the MNE and FDI would never be a viable strategy. In such a case, the monopolistic input supplier can either to set the priced at \( \tilde{\gamma}^m = \tilde{\gamma}^*(\tilde{c}) \) and sell only to firm 2 or set the price at \( \tilde{\gamma}^m = c \) and sell to both firms. That is,

\[
\pi^m = \max[(c - \tilde{c}) 2q(c, c), (\tilde{\gamma}^* - \tilde{c}) q(\tilde{\gamma}^*, \xi^{CUP})]
\]

It is clear that selling only to firm 2 at the price of \( \tilde{\gamma}^*(\tilde{c}) \) is a better option than selling at the price of \( c \) if \( \tilde{c} \) is very close to \( c \). However, if \( \tilde{c} \) is sufficiently lower than \( c \), it may be optimal to set the input price at \( c \), as the next lemma shows.

**Lemma 5.** Suppose that \( \tilde{c} < c \). Then, there is a critical level of \( \tilde{\gamma}^* \) (which is strictly lower than \( c \)) such that the optimal price for the monopolistic input supplier \( \tilde{\gamma}^m \) is given by

\[
\tilde{\gamma}^m = \begin{cases} 
c & \text{if } \tilde{c} < \tilde{\gamma}^* \\
\tilde{\gamma}^*(\tilde{c}) & \text{if } \tilde{c} \geq \tilde{\gamma}^*
\end{cases}
\]
There is inefficient FDI if \( \bar{c} \in (\bar{c}^*, c) \).

**Proof.** Let \( \Delta(\bar{c}) = (c - \bar{c})2q(c, c)-(\bar{\gamma}^* - \bar{c})q(\bar{\gamma}^*, \xi^{CUP}) \). We know \( \Delta(\bar{c} = c) = -(\bar{\gamma}^* - \bar{c})q(\bar{\gamma}^*, \xi^{CUP}) < 0 \). In addition,

\[
\frac{\partial \Delta(\bar{c})}{\partial c} = -2q(c, c) + \xi^{CUP}(\bar{\gamma}^*, \psi^{AL})
\]

Since \( \bar{\gamma}^* > c \) and \( \psi^{AL} < c \), we have \( q(\bar{\gamma}^*, \xi^{CUP}) < q(c, c) \). Thus, \( \frac{\partial \Delta(\bar{c})}{\partial c} < -2q(c, c) + q(c, c) < 0 \). Therefore, there can be a critical value of \( \bar{c}^* \) such that the statement in the Lemma is true. If \( \Delta(\bar{c} = 0) < 0 \), the input monopolist always sell only to downstream firm 2 and we can take \( \bar{c}^* = 0 \). ■

**Lemma 6.** Let \( \bar{\gamma} \) be the optimal price for the input monopolist if the MNE supplies its input at its marginal cost \( c \) without inflated transfer price. Under ALP with the CUP method, the monopolistic input supplier charges a price \( \bar{\gamma}^* < \bar{\gamma} \) if \( \frac{\partial^2 q(x,y)}{\partial x \partial y} \geq 0 \)

**Proof.** By definition, \( \bar{\gamma} \) satisfies the following first order condition

\[
q(\bar{\gamma}, c) + (\bar{\gamma} - \bar{c}) \left[ \frac{\partial q(\bar{\gamma}, c)}{\partial x} \right] = 0
\]

If we evaluate the first order condition under the ALP at \( \bar{\gamma} = \bar{\gamma} \), we have

\[
q(\bar{\gamma}, \xi^{CUP}) + (\bar{\gamma} - \bar{c}) \left[ \frac{\partial q(\bar{\gamma}, \psi^{AL})}{\partial x} + \frac{\partial q}{\partial y} \frac{\partial \xi^{CUP}}{\partial \bar{\gamma}} \right] < q(\bar{\gamma}, c) + (\bar{\gamma} - \bar{c}) \left[ \frac{\partial q(\bar{\gamma}, c)}{\partial x} \right] = 0
\]

because \( \xi^{CUP} < c \), which implies that \( q(\bar{\gamma}, \xi^{CUP}) < q(\bar{\gamma}, c) \) and \( \frac{\partial q(\bar{\gamma}, \psi^{AL})}{\partial x} < \frac{\partial q(\bar{\gamma}, c)}{\partial x} < 0 \). As a result, we have \( \bar{\gamma}^* < \bar{\gamma} \). ■

**Linear Demand Example:**

We can easily verify that

\[
q_2^{CUP}(\bar{\gamma}) = \frac{a - 2\bar{\gamma} + \xi^{AL} \bar{\gamma}}{3} = \frac{a - 2\bar{\gamma} + \frac{(1-t)c-(t-1)\bar{\gamma}}{1-t}}{3}
\]

The optimal choice of F input supplier and the subsequent internal transfer price due to the arm’s length principle is given by

\[
\bar{\gamma}^{CUP} = \frac{a + (1 + \lambda)c + (2 + \lambda)\bar{c}}{4 + \lambda}
\]
where $\lambda = \frac{(t-b)}{1-t}$.

In contrast, if the MNE transfers its input at its marginal cost of $c$, we have

$$\overline{\gamma} = \frac{a + c + 2\tilde{c}}{4} > \tilde{\gamma}^{AL}$$

It can also be easily verified that this gap between $\overline{\gamma}$ and $\tilde{\gamma}^{AL}$ increases in $\lambda$.

## 5 Concluding Remarks

We have analyzed an MNE’s incentives to manipulate an internal transfer price to take advantage of tax differences across countries. Our analysis of the monopoly case derives conditions under which FDI takes place and shows that tax-induced FDI can entail inefficient internal production. We also analyzed implications of ALP and it can have the opposite effect to the one intended if it induces the MNE’s sourcing decisions from dual sourcing to internal sourcing only to avoid the application of ALP. With imperfect competition we show that the internal transfer price has additional strategic effects that further strengthen incentives to inflate the transfer price at the expense of the rival firm’s profits. The tax-induced FDI by the MNE has spillover effects that reduce tax revenues from other domestic firms as well as the MNE.

We have analyzed FDI decision of only one firm in isolation in the oligopoly case assuming that other firms engage in outsourcing. However, each firm’s FDI decision may also depend on other firm’s FDI decision. This is an area of future research.
References


Appendix

Proof of Lemma 2:

By totally differentiating these two equilibrium conditions, we have

$$
\frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1^2} dq_1 + \frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial q_2} dq_2 + \frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial \xi} \frac{d\xi}{d\gamma} d\gamma = 0
$$

$$
\frac{\partial^2 \pi_2(q_1, q_2)}{\partial q_1 \partial q_2} dq_1 + \frac{\partial^2 \pi_2(q_1, q)}{\partial q_2^2} dq_2 + 0 d\gamma = 0
$$

To conduct a comparative static analysis on \( \gamma \), we can rewrite the equations above as

$$
\begin{bmatrix}
\frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1^2} & \frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial q_2} & \frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial \xi}
\end{bmatrix}
\begin{bmatrix}
\frac{dq_1}{d\gamma}
\frac{dq_2}{d\gamma}
\frac{d\xi}{d\gamma}
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial^2 \pi_2(q_1, q_2)}{\partial q_1 \partial q_2} & \frac{\partial^2 \pi_2(q_1, q)}{\partial q_2^2} & 0
\end{bmatrix}
\begin{bmatrix}
\frac{dq_1}{d\gamma}
\frac{dq_2}{d\gamma}
0
\end{bmatrix}
$$

Note that \( \frac{\partial \pi_1(q_1, q_2; \xi)}{\partial \xi} = -q_1 \) by the envelope theorem. This implies that \( \frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial \xi} = -1 \).

By applying the Cramer’s rule, we have

$$
\begin{align*}
\frac{dq_1}{d\gamma} &= \frac{D d\xi}{D} = \frac{D}{D} \frac{d\xi}{d\gamma} = \frac{\frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial q_2} \frac{dq_1}{d\gamma} + \frac{\partial^2 \pi_2(q_1, q)}{\partial q_2^2} \frac{dq_2}{d\gamma} + \frac{\partial \pi_1(q_1, q_2; \xi)}{\partial q_1} \frac{d\xi}{d\gamma}}{rac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial q_2} \frac{dq_1}{d\gamma} + \frac{\partial^2 \pi_2(q_1, q)}{\partial q_2^2} \frac{dq_2}{d\gamma} + \frac{\partial \pi_1(q_1, q_2; \xi)}{\partial q_1}}
\end{align*}
$$

$$
\begin{align*}
\frac{dq_2}{d\gamma} &= \frac{D d\xi}{D} = \frac{D}{D} \frac{d\xi}{d\gamma} = \frac{\frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial q_2} \frac{dq_1}{d\gamma} + \frac{\partial^2 \pi_2(q_1, q)}{\partial q_2^2} \frac{dq_2}{d\gamma} + \frac{\partial \pi_1(q_1, q_2; \xi)}{\partial q_1} \frac{d\xi}{d\gamma}}{rac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial q_2} \frac{dq_1}{d\gamma} + \frac{\partial^2 \pi_2(q_1, q)}{\partial q_2^2} \frac{dq_2}{d\gamma} + \frac{\partial \pi_1(q_1, q_2; \xi)}{\partial q_1}}
\end{align*}
$$

where \( D = \left[ \frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1^2} \right] - \left[ \frac{\partial^2 \pi_1(q_1, q_2; \xi)}{\partial q_1 \partial q_2} \frac{\partial^2 \pi_2(q_1, q)}{\partial q_2^2} \right] > 0 \). Thus, the sign of \( \frac{dq_1}{d\gamma} \) is the opposite of the sign of \( \frac{d\xi}{d\gamma} \) whereas the sign of \( \frac{dq_2}{d\gamma} \) is the same as the sign of \( \frac{d\xi}{d\gamma} \). We have the desired result because we have

$$
\frac{d\xi}{d\gamma} = \begin{cases} 
< 0 \text{ if } \gamma < \gamma^* \\
= 0 \text{ if } \gamma = \gamma^* \\
> 0 \text{ if } \gamma > \gamma^*
\end{cases}
$$