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LOCATION DIFFERENTIATION OF HOUSING MARKET IN THE PRESENCE OF A UNANIMOUS CHARACTERISTIC *

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This paper examines how the presence of a unanimous characteristic affects housing locations. An example of the unanimous characteristic, considered in the paper, is the regional educational environment. Our findings are as follows. First, if the relative attraction of the unanimous characteristic is over a critical threshold, maximal differentiation no longer holds. Second, as the transportation cost is lowered, agglomeration forces are intensified—both in social optimum and competitive outcome. Third, since allowing competitive suppliers to determine their locations results in suboptimal outcomes, the government may need to intervene with a subsidy-tax scheme. However, the marginal benefit of the government intervention would decrease with improvement in transportation system.

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I. INTRODUCTION

Educational environment, such as the quality of the local school district, is an important factor in searching for a new residential development project. The demand for houses is high, since house and prices are generally dependent on the quality of educational environment, more houses tend to be constructed in better school districts, which results in housing concentration in those areas. This paper develops a model that explains the role of educational environment in housing markets and suggests some policy implications to improve social welfare.

The hedonic pricing method is typically used to estimate the value of environmental amenities that affect the prices of marketed goods such as housing. According to Brasington and Hite (2005), the price of a house represents the sum of expenditures on a number of bundled housing characteristics, each of which has its own implicit price. These housing characteristics include structural attributes, such as the number of rooms and the square footage of the house. Expenditures on other less tangible characteristics, including educational and cultural environment, also contribute to the price of a house. Shyn, Ha and Koo (2006), who analyzed the housing prices in Korea, found that the average housing price of a district in Seoul is proportional to the admission rate of top universities.[1] With the data of the city, Reading in England, Cheshire and Sheppard (2004b) showed that housing markets capitalize on the value of local public goods such as school quality.[2] When the access to high quality educational services is primarily determined by housing location, as in Korea, school quality is clearly reflected in housing prices.

In general, the educational characteristic is not only exogenously given at least in the short-run but also unanimously ranked by the people. We define the characteristic that satisfies this condition as a “unanimous characteristic.” Assuming the presence of a unanimous characteristic, this paper presents a simple model of location differentiation based on the

[1] The Gangnam area in Seoul leads Korea with the highest housing prices. One of the main reasons why households prefer this area is the superior education cluster in this area, which comprises not only high quality public schools but also prominent private educational facilities.

[2] They also found that the variation of capitalized price depends on the elasticity of the supply of school quality in the local market.

hedonic hypothesis that goods are valued for their utility-bearing characteristics.

The purpose of this paper is to develop a new housing location differentiation model by introducing a unanimous characteristic and explain the relationship between education quality and housing location. We examine why households tend to concentrate in a large metropolitan area, although the transportation cost is getting lowered due to the increasing availability of high-speed transportation infrastructure. In addition, we investigate whether this phenomenon is desirable from a social point-of-view.

This paper develops a two-period model, which is a variant of the model developed by D'Aspremont, Gabszewicz and Thisse (1979, hereafter DGT), who modified the classic Hotelling (1929) model by introducing quadratic transportation costs to make profit functions continuous and well behaved.[3] Contrary to the standard model of DGT, which considers one characteristic, we assume that each supplier provides households with two characteristics, such as, 'educational characteristic' and 'all other characteristics'. A specific district (e.g., Gangnam area in Seoul, Korea) is assumed to be unanimously preferred because of its superior educational environment. Furthermore, we assume that households have a unanimous preference for all feasible locations based on their educational environment. The educational characteristic with this property is called the "unanimous characteristic" in our model. However, households have heterogeneous tastes with regard to other characteristics, which might include their working places, the vicinity of their relatives, natural environment, etc. We refer to the other characteristics as the "personal characteristic."

To explain the choice of housing location and its implications, our model combines the two kinds of differentiations in industrial organization literature: the vertical differentiation (unanimous characteristic) and the horizontal differentiation (personal characteristic). In our model, the two characteristics are measured in one dimension instead of two, because in our model, once a housing location is chosen,

[3] D'Aspremont, Gabszewicz and Thisse (1979) showed that a firm's profit function in the Hotelling's model is both discontinuous and non-quasi concave, and leads to no pure strategy equilibrium.

the unanimous characteristic is exogenously given to the specific location. Thus, the choice for the two characteristics is not separable to each individual household.

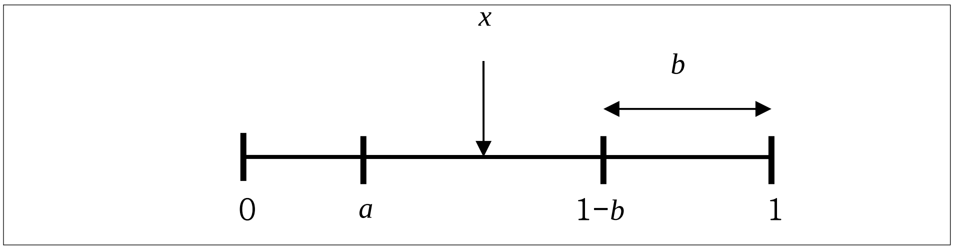
The purpose of this paper is to develop a new model of housing location differentiation by introducing a unanimous characteristic and to provide a more realistic explanation on housing locations. In Hotelling's model, the two housing districts are located in close distance, while in DGT model the two districts are located in maximal distance. Our model suggests that the location differentiation should be in between the two extremes. We also provide a new view on the role of improvement in transportation system and differentiation of educational quality in deciding housing location. Our model adopts a hedonic method in the analysis of housing demand, as in Cheshire and Sheppard (2004b) and Brasington and Hite (2005), but instead of simply including relevant variables in housing price equations, as in the previous studies, we incorporate unanimous and personal characteristics explicitly in the household utility function. Thus, we are able to provide a theoretical basis for the hedonic pricing method and determine housing locations and prices at the same time. Moreover, the household utility function in the model allows us to determine social optimum housing locations. Thus, we can derive useful policy implications on residential development projects to improve social welfare.

We investigate how a unanimous characteristic affects suppliers' location decisions in the first period and their price choices in the second period. In the following section, we outline the model. In Section Ⅲ, we look at the location choices that would be made if a social planner were to choose those locations. The bulk of the analysis is presented in Section Ⅳ, where we describe the market equilibrium for two competing suppliers and compare the market outcome with the socially optimal one. Section Ⅴ presents conclusions and suggests directions for further extension of the model.

II. THE MODEL

Feasible housing districts are assumed to be uniformly distributed with Density 1 along the interval $[0,1]$. In order to satisfy housing demand from households, two housing districts are scheduled. They are denoted as Service A and Service B. Service A's location is *a* and Service B's location is 1-*b* on the interval $[0,1]$, where $1-b \geq a$ and $0 \leq a, b \leq 1$. Two suppliers are planning to choose one out of the two locations. The unit cost of housing service is assumed to be "0" for simplicity. Each supplier provides a housing service that contains two characteristics, that is, the unanimous characteristic (characteristic *u*) and the personal characteristic (characteristic *p*). There is a continuum of feasible districts in the interval $[0,1]$, which provides the two characteristics.

[Figure 1] Housing Locations



Household's willingness to pay for each characteristic is as follows. All households are willing to pay the most for the characteristic *u* (from the right-end of the line "1"), but they want to pay the least for the characteristic *u* (from the left-end of the line "0"). As households move leftward from the right-end, the resulting utility loss is assumed to be proportional to the square of the distance between them. Therefore, if a consumer chooses a location a, his utility loss from not consuming his favorite characteristic u on the right-end of the service is given by $s(1-a)^2$, where s is a positive proportional constant.

Contrary to the characteristic *u*, the households' preferences for the characteristic *p* are heterogeneous. Each household envisions its own district that provides the most preferable characteristic *p*. One household's district, including the most preferable *p*, is different from those of others. Thus, we can obtain one-to-one mapping from households to districts, including their most preferable characteristic *p*. The household that has x as its most preferred characteristic *p* is willing to pay less for a service on a site that is different from x, which is proportional to the square of the distance between them. Therefore, if a household chooses district a, its utility loss is given by $t(x-a)^2$, where t is a positive proportional constant.

Next, we define the household's utility. Households have unit demands (i.e., each buys one or zero unit of the house). If a household whose most preferred personal characteristic *p*, located at x, chooses *a* (Service A) for its housing location, the household's willingness to pay for the housing service is given by:

$$U(x,a) = \overline{S} - t(x-a)^2 - s(1-a)^2,$$

where $\overline{S}$ is the ideal surplus the household obtains from the service, which is composed of its favorite characteristics.[4] For later use, $t(x-a)^2 + s(1-a)^2$ is denoted by $TC(x,a)$ and called the "social generalized cost" of the household whose most preferred personal characteristic *p* is located at x and chooses Service A. We assume that the ideal surplus a household obtains from the housing service, $\overline{S}$, is sufficiently large so that all households purchase one house (i.e., the market is covered).

III. SOCIALLY OPTIMAL OUTCOME

In deriving the social optimum, a social planner would choose Service A's location (a) and Service B's location (1-b) that maximize social welfare, *W*.

$$W \equiv \int_0^{\hat{x}} U(x,a)dx + \int_{\hat{x}}^1 U(x,b)dx,$$

where $\hat{x}$ is derived from $U(\hat{x},a) = U(\hat{x},b)$, and $1-b \geq a$ and $0 \leq a,\ b \leq 1$.

Since $\overline{S}$ is sufficiently large, the problem of maximizing W is

[4] We build the model based on demand side of the housing services by introducing the two characteristics of households. The housing suppliers in the model just respond to the behavior of households by maximizing their profits. Considering the cost function of suppliers and externalities of the development project would make the model more realistic, but we focus on demand side to make the model as simple as possible while highlighting the role of unanimous characteristic in housing market.

equivalent to that of minimizing sum of households' social generalized costs (TC's) with respect to a and b. The generalized cost of the household, whose most preferred personal characteristic is located at x and choice of housing location is either A or B, is given by the following equations.

$$f(x;\, a) = t\{(1-a)^2\nu + (x-a)^2\},$$
$$g(x;b) = t\{b^2\nu + (x-(1-b))^2\},$$

where $\nu = \dfrac{s}{t}$ which indicates attractiveness of characteristic *u* relative to characteristic *p*.

The most preferred personal characteristic of a household that is indifferent between the two housing services is located at $\hat{x} = \hat{x}(a,b)$, where $\hat{x}$ is given by equating the social generalized costs, that is, $f(\hat{x};\, a) = g(\hat{x};b)$.[5] Thus, $\hat{x} = \dfrac{1-b+a}{2} - \dfrac{1-a+b}{2}\nu$, and $\dfrac{\partial\hat{x}}{\partial a} = \dfrac{1+\nu}{2} = -\dfrac{\partial\hat{x}}{\partial b}$. Then, TC for the whole society is given by:

$$TC = t\int_0^{\hat{x}}\{(1-a)^2\nu + (x-a)^2\}dx + t\int_{\hat{x}}^1\{b^2\nu + (x-(1-b))^2\}dx .$$

We obtain the necessary conditions for this problem as follows.

$$\frac{\partial TC}{\partial a} = t\left[f(\hat{x})(\frac{1+\nu}{2}) - \int_0^{\hat{x}}\{2(1-a)\nu + 2(x-a)\}dx - g(\hat{x})(\frac{1+\nu}{2})\right] = 0,$$
$$\frac{\partial TC}{\partial b} = t\left[-f(\hat{x})(\frac{1+\nu}{2}) + \int_{\hat{x}}^1[2b\nu + 2\{x-(1-b)\}]dx + g(\hat{x})(\frac{1+\nu}{2})\right] = 0.$$

Using the definition of $\hat{x}$, we have:

[5] Since the ideal surplus ($\bar{S}$) is constant, $\hat{x}$ can be derived either from $U(\hat{x},a) = U(\hat{x},b)$ or from $f(\hat{x};\, a) = g(\hat{x};b)$.

$$\frac{\partial TC}{\partial a} = t\left[-2(1-a)\hat{x}\nu - \hat{x}^2 + 2a\hat{x}\right] = 0\,, \tag{1}$$

$$\frac{\partial TC}{\partial b} = t\left[2b(1-\hat{x})\nu + 1 - \hat{x}^2 - 2(1-b)(1-\hat{x})\right] = 0\,. \tag{2}$$

In order to obtain the socially optimal solution, we investigate the following three cases.

[Case 1] $\hat{x} \neq 0$ and $\hat{x} \neq 1$
Dividing Equation (1) by $\hat{x}$ and Equation (2) by $(1-\hat{x})$, we obtain the following conditions.

$$-2(1-a)\nu - \hat{x} + 2a = 0\,,$$
$$2b\nu + 1 + \hat{x} - 2(1-b) = 0\,.$$

Using these conditions and the definition of $\hat{x}$, we obtain the following solutions:

$$a^s = \frac{1+4\nu}{4(1+\nu)}\,,\quad b^s = \frac{1}{4(1+\nu)}\,,\quad TC(a^s, b^s) = t\frac{1+16\nu}{48(1+\nu)}\,,\ \text{and}\ \ \hat{x} = \frac{1}{2}\,.$$

[Case 2] $\hat{x} = 0$
For the case of $\hat{x} = 0$, Service A is not provided. From the definition of *TC*, $TC' = t\int_0^1 \{b^2\nu + (x-(1-b))^2\}dx$. In this case, the objective function is minimized at $b' = \dfrac{1}{2(\nu+1)}$ and its value is $TC' = t\dfrac{1+4\nu}{12(1+\nu)}$.

[Case 3] $\hat{x} = 1$
For the case of $\hat{x} = 1$, Service B is not provided. From the definition of *TC*, it is easy to show that this problem has the same optimal value of [Case 2] (i.e., $TC' = t\dfrac{1+4\nu}{12(1+\nu)}$). Since $TC(a^s, b^s) < TC'$ for all $\nu \in [0, \infty)$, it is always socially optimal to provide two housing services for consumers. Hence, we have the following proposition as a global

optimal solution.

Proposition 1. In response to the change in ν, the social optimum locations are given by:

$$a^s = \frac{1+4\nu}{4(1+\nu)} \text{ and } b^s = \frac{1}{4(1+\nu)}.$$

Using the Proposition 1, we investigate the change of Service A's optimal location a^s and Service B's optimal location $1-b^s$ with the variation in ν. We know that $\frac{da^s}{d\nu} = \frac{3}{4(1+\nu)^2} > 0$ and $\frac{d(1-b^s)}{d\nu} = \frac{1}{4(1+\nu)^2} > 0$ for all $\nu \geq 0$. Therefore, the higher the relative attraction of characteristic *u* (i.e., ν), the closer location *a* and location *b* move toward the right-end side. Table 1 illustrates this result with several numbers of ν.

[Table 1] Housing Location in Social Optimum

	$\nu = 0$	$\nu = 1$	$\nu = 10$	$\nu = \infty$
a^s	0.25	0.625	0.9318	1
$1-b^s$	0.75	0.875	0.9773	1
$(1-b^s)-(a^s)$	0.5	0.25	0.0455	0

Next, the location differentiation between the two housing services is given by:

$$(1-b^s)-a^s = \frac{1}{2(1+\nu)}.$$

This implies that as the relative attraction of characteristic *u* (ν) increases, the socially optimal differentiation between the two services decreases (see Table 1). As ν goes to infinity, the location differentiation disappears and both housing services are provided at the right-end district. On the contrary, when $\nu = 0$, our social optimum results are the same to the DGT model. When $\nu = 0$, the two service locations become $\frac{1}{4}$ and $\frac{3}{4}$, and the market share of each service is $\frac{1}{2}$.

This result answers the question raised by Fujita and Thisse (2003) as to whether agglomeration forces would vanish if increasing availability of a high-speed transportation infrastructure lowers the transportation cost to a sufficiently low level. In our model, if a new high-speed transportation infrastructure reduces transportation cost by decreasing the proportional constant for personal characteristic, *t*, the relative attraction of characteristic, *u* (i.e., ν) increases. With a smaller transportation cost related to the personal characteristic, the two welfare-maximizing housing services should be located in the districts which have higher ranks in unanimous characteristic. If a country has a stable urban hierarchy with respect to a unanimous characteristic, such as the quality of educational environment as in Korea, we expect that the lower the transportation cost is, the more households concentrate in higher ranked districts in social optimum.

IV. COMPETITIVE EQUILIBRIUM

We assume a two-stage game in which the suppliers choose their locations simultaneously at first, and then, choose prices simultaneously at given locations. Each supplier understands how its choice of position affects not only its demand functions, but also the intensity of price competition. We solve the problem by backward induction. At first, the price competition is examined, and then, the housing location is determined. Note that the competitive equilibrium in our model is different from the equilibrium in perfect competition. The competition in our model is between the two suppliers, so there is a room for government intervention to correct the market inefficiency caused by imperfect competition. The issue of government intervention will be discussed at the end of this section.

4.1. Price Competition

We take the housing locations as given and look for the Nash equilibrium in determining the market prices. We assume that Supplier 1, who provides Service A, is located at district $a \geq 0$, and Supplier 2, who provides Service B, is located at district $1-b$, where $b \geq 0$ and, without loss of generality, $1-a-b>0$ (Supplier 1 is to the left of Supplier 2).[6] The suppliers choose their prices p_1 and p_2 that maximize their profits. A household who has its most preferred personal characteristic at $\hat{x}$ is indifferent between the two services, and $\hat{x}$ is given by the following equality.

$$\bar{S} - (1-a)^2 s - (a-\hat{x})^2 t - p_1 = \bar{S} - b^2 s - (1-b-\hat{x})^2 t - p_2 \text{, or}$$
$$(1-a)^2 s + (a-\hat{x})^2 t + p_1 = b^2 s + (1-b-\hat{x})^2 t + p_2 .$$

Next, we define the "private generalized cost" as the household's "social generalized cost," plus the service price. Equality in the above equation implies that the household with its most preferred personal characteristic at $\hat{x}$ has a private generalized cost of consuming Service A which is equal to the private generalized cost of consuming Service B.

Assuming that $1-b-a>0$, we have:

$$\hat{x} = \frac{1-b+a}{2} - \frac{1+b-a}{2}v + \frac{p_2 - p_1}{2(1-b-a)t},$$

where $\hat{x}$ shows Supplier 1's market share. Since the ideal surplus $\bar{S}$ is large enough to buy a house for all households, the suppliers' respective demands are:

$$D_1(p_1, p_2) = \hat{x}, \tag{3}$$

[6] For the case of $1-a-b=0$, it is easy to show that there is no equilibrium in the first period. The reason is that at the same locations, both prices of housing services A and B should be zero because of Bertrand competition. In this case, if one supplier chooses a different location, he could earn a positive profit. Thus the supplier deviates.

$$D_2(p_1, p_2) = 1 - \hat{x}\,. \tag{4}$$

Thus, supplier i's profit is $\Pi_i = p_i D_i(p_1, p_2)$, where $i = 1,2$.
We have the following first order necessary conditions of Suppliers 1 and 2, respectively.[7]

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{1-b+a}{2} - \frac{1+b-a}{2}v + \frac{p_2 - p_1}{2(1-b-a)t} - \frac{p_1}{2(1-b-a)t} = 0\,, \tag{5}$$

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{1+b-a}{2} + \frac{1+b-a}{2}v - \frac{p_2 - p_1}{2(1-b-a)t} - \frac{p_2}{2(1-b-a)t} = 0\,. \tag{6}$$

From Equations (5) and (6), we obtain the Nash equilibrium as follows.

$$p_1^c(a,b) = \frac{1}{3}(1-b-a)t\{(3-b+a)-(1+b-a)v\}\,, \tag{7}$$

$$p_2^c(a,b) = \frac{1}{3}(1-b-a)t\{(3+b-a)+(1+b-a)v\}\,. \tag{8}$$

Therefore, we obtain the following demands of housing services:

$$D_1(a,b) = \hat{x} = \frac{1-b+a}{2} - \frac{1+b-a}{2}v + \frac{1}{3}\{b-a+(1+b-a)v\}\,, \tag{9}$$

$$D_2(a,b) = 1-\hat{x} = \frac{1+b-a}{2} + \frac{1+b-a}{2}v - \frac{1}{3}\{b-a+(1+b-a)v\}\,. \tag{10}$$

4.2. Location Choice

As mentioned earlier, each supplier must anticipate how its choice of location affects not only its demand for housing but also the intensity of price competition. Here, we use the reduced form profit functions as:

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[7] It is easy to show that second order sufficient conditions are satisfied.

$$\Pi_i = p_i^c D_i(a, b, p_1^c, p_2^c), \quad i = 1,2,$$

where D_i's are given by Equations (3) and (4).

First, we investigate Supplier 2's optimal location (1-*b*).[8] To find it, we compute the reduced form profit function $\Pi_i(a,b)$ explicitly by using Equations (5) through (8) and solve for the Nash equilibrium. Using the envelope theorem, we have the following equation:

$$\frac{d\Pi_2}{db} = p_2\left(\frac{\partial D_2}{\partial b} + \frac{\partial D_2}{\partial p_1}\frac{\partial p_1}{\partial b}\right).$$

When $p_2 > 0$, then the sign of $\frac{d\Pi_2}{db}$ is determined by $\left(\frac{\partial D_2}{\partial b} + \frac{\partial D_2}{\partial p_1}\frac{\partial p_1}{\partial b}\right)$. By changing b, Supplier 2 has a direct effect on its demand ($\frac{\partial D_2}{\partial b}$), which is called the 'market share effect' and given by:

$$\frac{\partial D_2}{\partial b} = \frac{3 - a - 5b + \nu(1 - a - 5b)}{6(1 - a - b)}.$$

Any effect of changes in *b* on Supplier 2's profit is also channeled through Supplier 1's price choice. This indirect effect is called the "price effect," which is given by:

$$\frac{\partial D_2}{\partial p_1}\frac{\partial p_1}{\partial b} = \frac{-2 + b + b\nu}{3(1 - b - a)}.$$

Thus, total effect of b on Π_2 is the sum of the market share effect and the price effect. By a simple manipulation, we obtain the following result:

$$\frac{d\Pi_2}{db} = p_2^c\left(\frac{\partial D_2}{\partial b} + \frac{\partial D_2}{\partial p_1}\frac{\partial p_1}{\partial b}\right) = p_2\left[\frac{-1 - a - 3b + \nu(1 - a - 3b)}{6(1 - a - b)}\right]. \qquad (11)$$

[8] Supplier 1's optimal location can also be derived with the same procedure.

Similarly, for supplier 1, we have:

$$\frac{d\Pi_1}{da} = p_1^c\left(\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right) = p_1\left[\frac{-1-3a-b+\nu(3-3a-b)}{6(1-a-b)}\right]. \qquad (12)$$

By assuming $1-a-b>0$, we know that $\text{sgn}[\frac{d\Pi_1}{da}] = \text{sgn}[-1-3a-b+\nu(3-3a-b)]$. In order to obtain a competitive equilibrium, we investigate the following three cases.

[Case 1] $b^c = 1$
If $b^c = 1$, then $1-b^c = 0$. By assuming $1-b^c > a^c$, we obtain $0 > a^c$. This contradicts the assumption of $a \geq 0$.

[Case 2] $b^c = 0$
If $b^c = 0$, we have $\text{sgn}[\frac{d\Pi_1}{da}] = \text{sgn}[\frac{-1-3a+\nu(3-3a)}{6(1-a)}] = \text{sgn}[\frac{3\nu-1-3(1+\nu)a}{6(1-a)}]$. The sign depends on the value of a as in the following two sub-cases.
[Case 2.1] $b^c = 0$ and $\frac{3\nu-1}{3(1+\nu)} > a$
If $\frac{3\nu-1}{3(1+\nu)} > a$, then $\frac{d\Pi_1}{da} > 0$. Hence, we have $a^c = 1$. This contradicts the assumption of $1-b^c > a^c$.
[Case 2.1] $b^c = 0$ and $\frac{3\nu-1}{3(1+\nu)} < a$
If $\frac{3\nu-1}{3(1+\nu)} < a$, then $\frac{d\Pi_1}{da} < 0$. Hence we have $a^c = 0$. Therefore, it is necessary that $\nu < \frac{1}{3}$. We have a solution in this case. If $\frac{3\nu-1}{3(1+\nu)} = a$, then $\frac{d\Pi_1}{da} = 0$. Hence, if $\nu \geq \frac{1}{3}$, we have the solution $a^c = \frac{3\nu-1}{3(1+\nu)}$. The second order condition $\frac{d^2\Pi_1}{da^2} < 0$ is also satisfied at this point.

[Case 3] $0 < b^c < 1$

If $0 < b^c < 1$, then $\frac{d\Pi_2}{db} = 0$. Thus, from Equation (12) we have the following equation.

$$b = -\frac{1}{3(1+\nu)}[(1+a) - \nu(1-a)]. \tag{13}$$

However, since $b > 0$, for this case to hold, it is necessary that $\nu > \frac{1+a}{1-a}(\geq 1)$. We divide this case into the following two sub-cases.

[Case 3.1] $0 < b^c < 1$ and $a^c = 0$

If $a^c = 0$, from Equation (13), we have $b = -\frac{1-\nu}{3(1+\nu)}$. However,

$$\begin{aligned}
&\mathrm{sgn}[\frac{d\Pi_1}{da}] = \mathrm{sgn}[-1-3a-b+\nu(3-3a-b)] \\
&= \mathrm{sgn}[-1+\frac{1-\nu}{3(1+\nu)}+\nu\{3+\frac{1-\nu}{3(1+\nu)}\}] \\
&= \mathrm{sgn}[8\nu^2+6\nu-2].
\end{aligned}$$

Since $\nu > 1$, $8\nu^2 + 6\nu - 2 > 12$. We have $\mathrm{sgn}[\frac{d\Pi_1}{da}] > 0$, which contradicts the necessary condition for maximization.

[Case 3.2] $0 < b^c < 1$ and $0 < a^c < 1$

If $0 < a^c < 1$, $\frac{d\Pi_1}{da} = 0$. From Equation (12), $-1-3a-b+\nu(3-3a-b) = 0$ and from Equation (11), $-1-a-3b+\nu(1-a-3b) = 0$. Thus we have $a = -\frac{1-4\nu}{4(1+\nu)}$ and $b = -\frac{1}{4(1+\nu)}$. However, for all ν, $b < 0$. Therefore, we have a contradiction. The Proposition 2 summarizes the above findings.

Proposition 2. In response to the change in ν, the optimal housing locations for the suppliers are as follows:

If $0 \le \nu \le \frac{1}{3}$, then $a^c = 0$, $1-b^c = 1$.

If $\nu > \frac{1}{3}$, then $a^c = \frac{3\nu - 1}{3(1+\nu)}$, $1-b^c = 1$.

The Proposition 2 implies that if $0 \le \nu \le \frac{1}{3}$, the equilibrium has two suppliers locating at the two extremes of the interval. In regard to this, DGT (1979) showed that the equilibrium has two suppliers locating at the two extremes of the interval, which is called maximal differentiation.[9] In that case, as noted by Tirole (1988), each supplier locates far from its rival in order not to trigger a lower price from its rival, and thus price competition is minimized. We showed that if ν is not greater than $\frac{1}{3}$, maximal differentiation holds. However, when ν increases over that point, Supplier 1 begins to move to the right. This means that the higher the relative attraction of character *u* is, the lower the difference between the housing locations of Supplier 1 and Supplier 2. In this case, maximal differentiation does not hold. Table 2 shows a numerical example with various ν's. As ν increases from $\frac{1}{3}$ to 1, the distance between the two locations drops from 1 to 0.5. It becomes very small as ν grows to a large number.

[Table 2] Housing Location in Competitive Equilibrium

	$\nu = 0$	$\nu = 1/3$	$\nu = 1$	$\nu = 10$	$\nu = \infty$
a^c	0	0	0.5	0.8788	1
$1-b^c$	1	1	1	1	1
$(1-b^c)-(a^c)$	1	1	0.5	0.1212	0

We can consider agglomeration of housing development in two aspects. One aspect of agglomeration is measured by the difference between the two housing locations as shown in Table 2. As the relative attraction of

9 If $\nu = 0$, our model is equivalent to that of DGT (1979).

the unanimous characteristic (ν) increases, the two housing locations move closer to each other, which implies intensified agglomeration. Another aspect of agglomeration is represented by the location of housing Service B, which is always located at the right-end of the available regions. Regardless of the variations in ν, every one supplier in a pair of two housing suppliers considered in our model chooses the right-end location. It will lead to an increased agglomeration at the right-end of the region.

Now, we compare the differentiation of competitive equilibrium to that of a socially optimal outcome. For the location of service A, we have the following results.

$$\text{If } \frac{1}{3} \geq \nu \geq 0, \text{ then } a^s - a^c = \frac{1+4\nu}{4(1+\nu)} > 0,$$
$$\text{If } \nu > \frac{1}{3}, \text{ then } a^s - a^c = \frac{7}{12(1+\nu)} > 0.$$

This implies that the location of Service A at the competition equilibrium is farther away from the right-end service than that of Service A at the socially optimal outcome. For the location of Service B, it can be shown that:

$$(1-b^c)-(1-b^s) = \frac{1}{4(1+\nu)} > 0 \quad \text{for all } \nu \in [0,\infty).$$

The location of Service B in competitive equilibrium is at the right of the socially optimal outcome. Competition tends to induce the suppliers to be more differentiated in housing location than in a socially optimal outcome. However, note that as ν goes to infinity, the difference between social optimum location and competitive equilibrium location becomes negligible. Table 3 provides a numerical example with various ν's. Both $a^c - a^s$ and $(1-b^c)-(1-b^s)$ are zero when $\nu = \infty$.

[Table 3] Location Differentiations between Competitive Equilibrium and Social Optimum

	$\nu = 0$	$\nu = 1/3$	$\nu = 1$	$\nu = 10$	$\nu = \infty$
a^c	0	0	0.5	0.8788	1
a^s	0.25	0.4375	0.625	0.9318	1
$a^c - a^s$	-0.25	-0.4375	-0.125	-0.053	0
$1-b^c$	1	1	1	1	1
$1-b^s$	0.75	0.8125	0.875	0.9773	1
$(1-b^c)-(1-b^s)$	0.25	0.1875	0.125	0.0227	0

4.3. Price and Profit in Competitive Equilibrium

In this section, we substitute the profit maximization locations derived from the previous section into Equations (7) and (8) to find the suppliers' optimal prices in competition. We also determine suppliers' profits using Equations (7) ~ (10). From a simple operation, we can derive the following results. In response to the change in ν, the optimal prices and profits are as follows:

If $0 \le \nu \le \frac{1}{3}$, then $p_1^c = \frac{t(3-\nu)}{3}$, $p_2^c = \frac{t(3+\nu)}{3}$,
$\Pi_1^c = \frac{t(3-\nu)^2}{18}$, $\Pi_2^c = \frac{t(3+\nu)^2}{18}$.
If $\nu > \frac{1}{3}$, then $p_1^c = \frac{32t}{27(1+\nu)}$, $p_2^c = \frac{40t}{27(1+\nu)}$,
$\Pi_1^c = \frac{4}{9} p_1^c$, $\Pi_2^c = \frac{5}{9} p_2^c$.

It can be shown that for all ν, $p_2^c - p_1^c \ge 0$ and $\Pi_2^c - \Pi_1^c \ge 0$. This implies that Supplier 2 (the more attractive supplier) sells at a higher price than Supplier 1 (the less attractive supplier), and the more attractive supplier earns higher profit than the less attractive supplier.

Numerical examples for various ν's are shown in Table 4. In the case of $0 \le \nu \le \frac{1}{3}$, $p_2^c - p_1^c$ and $\Pi_2^c - \Pi_1^c$ are increasing in ν. However, in the case of $\nu > \frac{1}{3}$, $p_2^c - p_1^c$ and $\Pi_2^c - \Pi_1^c$ are decreasing in ν. Note that for all $i = 1, 2$, $\lim_{\nu\to\infty} p_i^c = \lim_{\nu\to\infty} \Pi_i^c = 0$. Moreover, $\lim_{\nu\to\infty}(p_2^c - p_1^c) = \lim_{\nu\to\infty}(\Pi_2^c - \Pi_1^c) = 0$. This means that if the relative attraction of the unanimous characteristic increases to infinity, then all prices, profits, and differences between them drop to zero. This happens because the greater the relative attraction of characteristic, *u*, is, the closer Supplier 1 moves its service location toward the right-end location. This intensifies the competition between the two housing suppliers. Therefore, the greater the attraction of characteristic, *u*, becomes, the smaller the difference between the prices and profits of the two suppliers is realized.

[Table 4] Price and Profit in Competitive Equilibrium

Panel A. With a constant *t* (Cost from the Personal Characteristic)

		$\nu = 0$	$\nu = 1/3$	$\nu = 1$	$\nu = 10$	$\nu = \infty$
$t = 1$ or $\nu = s$	p_1^c	1	0.8889	0.5926	0.1077	0
	p_2^c	1	1.1111	0.7407	0.1347	0
	$p_2^c - p_1^c$	0	0.2222	0.1481	0.0269	0
	Π_1^c	0.5	0.3951	0.2634	0.0479	0
	Π_2^c	0.5	0.6173	0.4115	0.0748	0
	$\Pi_2^c - \Pi_1^c$	0	0.2222	0.1481	0.0269	0

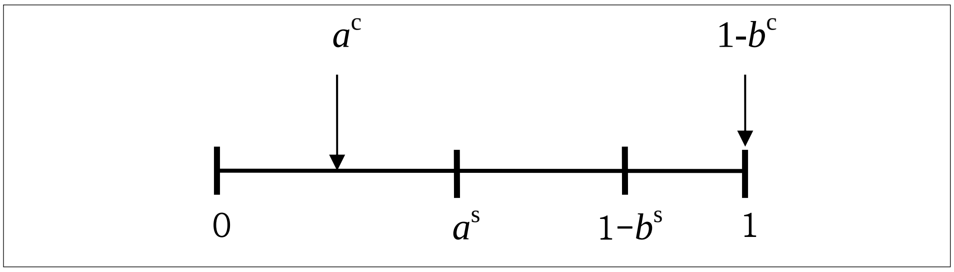
Panel B. With a constant *s* (Cost from the Unanimous Characteristic)

		$\nu = 0$	$\nu = 1/3$	$\nu = 1$	$\nu = 10$	$\nu = \infty$
$s = 1$ or $\nu = \frac{1}{t}$	p_1^c	∞	2.2667	0.5926	0.0108	0
	p_2^c	∞	3.3333	0.7407	0.0135	0
	$p_2^c - p_1^c$	0	0.6667	0.1481	0.0027	0
	Π_1^c	∞	1.1851	0.2634	0.0048	0
	Π_2^c	∞	1.8519	0.4115	0.0275	0
	$\Pi_2^c - \Pi_1^c$	0	0.6667	0.1481	0.0027	0

4.4. Government Intervention

When the competitive market results in housing locations different from social optimum, the government may intervene the housing market to improve social welfare. The government can adopt a direct requirement, like a housing development permit for each specific locations or an indirect provision of a subsidy-tax scheme to induce the housing suppliers to choose socially optimal locations. Since the result from the permit requirement is trivial, we consider the case of subsidy-tax scheme. The subsidy-tax scheme is designed to balance the government budget[10].

[Figure 2] Housing Locations in Competitive Equilibrium and Social Optimum



From the analysis in the previous sections, we know that one housing location in competitive market is farther away from the other location than in social optimum as shown in Figure 2. Thus, we need to find the subsidies for distance which give incentives for the suppliers to locate socially optimal districts. The Supplier 1 who provides housing service at a receives subsidy az_1. The Supplier 2 who provides housing service at $(1-b)$ receives subsidy bz_2. The reduced form profit functions for each supplier become:

$$\tilde{\Pi}_1 = p_1^c D_1(a,b,p_1^c,p_2^c) + az_1,$$
$$\tilde{\Pi}_2 = p_2^c D_2(a,b,p_1^c,p_2^c) + bz_2,$$

where D_i's are given by Equations (3) and (4). Then, the location choices are determined by:

[10] Balancing the government budget implies that the total amount of subsidy is equal to the total amount of tax revenue. If there were any net tax revenue from the intervention, the government may use the collected revenue to improve transportation infrastructure or education system, which would results in changes of parameter values in the model. However, the government intervention considered in the model does not produce any net tax revenue, so the effect of government intervention on housing market other than housing location should be negligible.

$$\begin{aligned}\frac{\partial\tilde{\Pi}_1}{\partial a} &= p_1^c\left(\frac{\partial D_1}{\partial a}+\frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right)+z_1\\ &= p_1^c\left[\frac{-1-3a-b+\nu(3-3a-b)}{6(1-a-b)}\right]+z_1=0\,,\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{d\tilde{\Pi}_2}{db} &= p_2^c\left(\frac{\partial D_2}{\partial b}+\frac{\partial D_2}{\partial p_1}\frac{\partial p_1}{\partial b}\right)+z_2\\ &= p_2^c\left[\frac{-1-a-3b+\nu(1-a-3b)}{6(1-a-b)}\right]+z_2=0\,.\end{aligned} \tag{15}$$

We need to find the subsidies which give incentives for suppliers to choose the socially optimal locations. Since $p_1^c(a^s,b^s)=p_2^c(a^s,b^s)=\frac{1}{2(1+\nu)}t$ from Equations (7) and (8), we find $z_1(a^s,b^s)=\frac{t}{3}$ and $z_2(a^s,b^s)=\frac{t}{3}$. Thus, the subsidies to suppliers 1 and 2 are $\frac{t}{3}a$ and $\frac{t}{3}b$, respectively. Next, to make government budget balanced, we need to find a tax scheme that does not alter suppliers' location choice. Total tax revenue should be the sum of subsidies (Z) which is $\mathrm{Z}\equiv a^s z_1+b^s z_2=(a^s+b^s)\frac{3}{t}=\frac{t(1+2\nu)}{6(1+\nu)}$. Thus, the government can impose fixed fees to the suppliers (f_1 and f_2) as follows:

$$f_1=(1-\delta)\mathrm{Z} \quad \text{and} \quad f_2=\delta\mathrm{Z}\,, \tag{16}$$

where $\delta=\frac{1}{2}$ for $\nu\le\frac{3}{2}$, and $\delta=\frac{2}{1+2\nu}$ for $\nu>\frac{3}{2}$.

The proportion of tax burden on Supplier 2, δ, is held to 1/2 as long as ν is less than or equal to 3/2, but it decreases as ν rises beyond this level. The variation of relative tax burden between the two suppliers is introduced in this scheme to prevent the profit of any supplier from being negative.[11]

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[11] Since the sum of profits at the social optimum location before imposing the subsidy-tax scheme, $t/(2(1+\nu))$, is positive, the government can always find a budget balancing subsidy-tax

Based on the subsidy-tax scheme, the government can improve social welfare. However, the effectiveness of the government intervention depends on the quality of information known to the government. If the information on the utility of the unanimous characteristic is not accurate, and the information asymmetric problem between the government and households are severe, government intervention would not improve social welfare.[12] Moreover, as the rapid development of the transportation system which lowers t and raises ν in our model, the difference of housing locations between competitive equilibrium and social optimum decreases (see Table 3). In this case, the need for government intervention to housing market decreases in that the marginal benefit of the intervention policy could be very small.[13]

V. CONCLUSION

This paper has examined how the presence of a unanimous characteristic affects suppliers' locations in the housing market. By adding a unanimous characteristic to the typical horizontal differentiation model of industrial organization, we can explain the agglomeration phenomenon in metropolitan areas and evaluate it with social welfare perspectives. Our findings are summarized as follows.

First, the maximal differentiation, suggested by DGT (1979), no longer holds in the housing market when a unanimous characteristic is

scheme that leads to social optimum with non-negative profits for the suppliers. As an example different from Equation (16), if the government wants to equalize the profits of the two suppliers after the subsidy-tax scheme, the government can impose fixed fees such as $f_1 = t(1+4\nu)/(12(1+\nu))$ and $f_2 = t/(12(1+\nu))$.

[12] Note that because of the simple setup of our model, the actual government policy suggested by the model should be implemented with great caution.

[13] The improvement in social welfare by moving from the competitive equilibrium to the social optimum ($W^s - W^c$) is getting smaller as ν rises beyond 1/3 (see the Table below). This implies that the marginal benefit of government intervention could be very small when the value of ν is very high.

	ν=0	ν=1/3	ν=1	ν=10	$\nu=\infty$
$a^c - a^s$	-0.2500	-0.4375	-0.1250	-0.0530	0.0000
$(1-b^c)-(1-b^s)$	0.2500	0.1875	0.1250	0.0227	0.0000
$W^s - W^c$	0.0625	0.1233	0.0822	0.0149	0.0000

considered. If the relative attraction of the unanimous characteristic is smaller than a critical threshold, the housing suppliers follow the maximal differentiation. However, the greater the attraction is over the threshold, the lower the differentiation between the hosing service locations.

Second, as the transportation cost decreases, agglomeration forces are intensified—not only in a socially optimal outcome but also in a competitive outcome. In the competitive market, if the transportation cost is lowered over a critical threshold ($\nu = \frac{1}{3}$), agglomeration is achieved both by housing services concentrating in the highest ranked district and by housing services moving into the higher ranked district. On the contrary, in the socially optimal outcome, agglomeration is achieved through both housing suppliers' moving into high ranked districts.

Third, competition tends to induce housing suppliers to choose more distant location than in a socially optimal outcome. In this case, the government needs to intervene with subsidy-tax scheme to make the competitive equilibrium be equal to the social optimum. The success of government intervention depends on the accuracy of government information. Note that the need for intervention decreases with the increased relative attraction of the unanimous characteristic.

By introducing a unanimous characteristic, we provide an explanation for the agglomeration phenomenon observed in large metropolitan areas, such as New York, Paris, Tokyo, and Seoul. Rapid transportation system is usually considered as a major source for regional diversification of housing development, but our model with a unanimous characteristic suggests that the reduced transportation costs can lead to more concentration in housing location. In Seoul area, for example, educational environment is considered as a representative unanimous characteristic causing high housing concentration.

Our model also suggests that greater agglomeration can be desirable for social welfare when the transportation cost is lowered. In this case, government policies for regional diversification in housing development may not improve social welfare. The regional diversification policy can result in a worse outcome, especially if the difference in the value of unanimous characteristic between areas is getting larger (e.g., educational inequality between areas becomes more severe).

Our model can be extended in several ways. Considering endogeneity between educational quality and housing development would improve the reality of the model. In our model, we assumed that the regions have the stability of hierarchy with respect to a unanimous characteristic. However, the educational characteristic may be endogenous, where its level depends on the investment of educational infrastructure. In this case, we need to analyze the desirability of agglomeration, including more active supply-side policy instruments, to overcome inefficient path dependence. Another related issue is the congestion problem followed by an increase in agglomeration. Thus, we need to consider congestion, or spatial spillover from agglomeration, for a comprehensive understanding of housing location. Finally, empirical studies based on our model would allow us to see how well the model fits the real world situation.

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