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MONETARY POLICY RULES AND THE FORWARD DISCOUNT BIAS

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This is an attempt to explain forward discount bias in the foreign exchange market as influenced by monetary policy rules. A government response function to external shocks is combined with the monetary model for exchange rate determination, and the risk premium is assumed to follow a first order autoregressive process. The forward discount bias is more probable when the government is concerned with interest rate or money supply stability than when monetary policy focuses on the stabilization of foreign exchange rate or price level. These results are consistent with the experiences of ERM in the past – where forward discount bias is not found – and the survey results of Froot and Thaler (1990).

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I. INTRODUCTION

Recently interest parity (UIP) has been an important building block for many exchange rate models along with purchasing power parity.

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Uncovered interest rate parity, or the assumption that the forward exchange rate equals the expected future value of spot exchange rate, has been subject to rigorous testing. Since expectations for exchange rates are not directly observable, researchers have searched for indirect evidence under the assumption that expectations are formed rationally so that UIP would imply that forward rates are unbiased predictors of future spot rates. This would call for $\alpha = 0$ and $\beta = 1$ in the following equation.

$$s_{t+1} - s_t = \alpha + \beta(f_{t,t+1} - s_t) + \varepsilon_{t+1}, \qquad (1)$$

where s_t and s_{t+1} are the log of spot exchange rates at period *t* and *t*+1, respectively, and $f_{t,t+1}$ is forward exchange rate at period *t* for contracts maturing at period *t*+1; ε_{t+1} is an error term with a mean of zero.

The literature indicates two common interpretations of the empirical results: reported forward rate is not an unbiased predictor of future spot rates and the change in the future exchange rate is often negatively related to the forward discount. This is referred to as forward discount bias or the forward discount puzzle. Some authors argue that the rejection of the unbiasedness hypothesis is evidence of a time-varying risk premium. Others, assuming that investors are risk-neutral, claim that there is a failure of rational expectations. Lewis (1995) and Engel (1996) give comprehensive surveys in this area. Recently, Verschoor and Wolff (2001) report that the bias is not found clearly in the Scandinavian exchange market with significant irrationality and time-varying risk premia. Since there is still a lack of consensus on the reasons for forward discount bias, this paper focuses on the role of government. Boyer and Adams (1988) and McCallum (1994), in their exogenous risk premium models, ask whether exchange rates could react endogenously in such a way as to produce a negative β coefficient. They claim that there are plausible scenarios in which an exogenous risk premium is negatively correlated with expected depreciation; therefore a negative β coefficient can arise in estimates of equation (1).

Boyer and Adams suggest that in general, the negative correlation between the forward premium and exchange rate depreciation arises if demand for money is interest rate elastic when the money supply is a function of interest differential. McCallum (1994), in a simple two-equation model on UIP and government response function for interest rate differential, notes that the forward discount bias is the result of monetary policy. In a related study, Anker (1999) suggests that central banks react to exogenous shocks, identifying parameter values in the reaction function with certain policy goals.

A few empirical studies attempt to explain the government role in forward discount.[1] It is suggested that sterilized intervention can affect the risk premium through a portfolio-balance channel. Baillie and Osterberg (1997), in a two-country inter-temporal asset-pricing model, arrive at a similar conclusion. From an analysis of ERM (Exchange Rate Mechanism), Flood and Rose (1996) find that forward discount bias does not appear to characterize the target-zone exchange-rate data set.

This paper explains government behavior in the forward discount bias, claiming that the bias depends on the type of money supply policy rule that the central bank adopts. First, the estimate of β coefficient in equation (1) is less than one under any circumstances, which is consistent with the survey results of Froot and Thaler (1990). Second, the bias is more probable when the monetary authority focuses on interest rate smoothing or money targeting rather than price or exchange rate stabilization. These results overcome some potential problems found in McCallum (1994). According to his model, β coefficient will be either positive or negative infinity when the central bank exclusively reacts to the interest rate spread of previous period, or the β coefficient will be always positive when the risk premium is very highly autoregressive. In addition, this variation explains the forward bias better than Anker (1999). In his paper, a negative probability limit for the β coefficient is difficult to obtain.

The remainder of the paper is organized as follows. Section 2 introduces the model and derives its equilibrium exchange rate. In Section 3, we look into the effects of several policy rules of money supply on the sign and the size of the probability limit of β under a special stationary assumption for money supply, and Section 4 summarizes.

[1] See Maynard *et al*. (2001).

II. EXCHANGE RATE DETERMINATION MODEL

This section presents a model and discusses how we determine the equilibrium exchange rate and interest rate. The probability limit of β is then introduced and the terms needed in the calculation of the probability limit are derived from the equilibrium exchange rate and the equilibrium interest rate.

1. Model

In a two-country framework, a response function of the central bank is added to a model of exchange rate determination, in which the risk premium follows a first order autoregressive process. The model is as follows:

$$s_t = p_t - p_t^* + q_t , \quad q_t \sim (0, \sigma_q^2) \tag{2}$$

$$i_t = i_t^* + E_t s_{t+1} - s_t + rp_t \tag{3}$$

$$rp_t = \rho rp_{t-1} + u_t , \quad 0 < \rho < 1 , \quad u_t \sim (0, \sigma_u^2) \tag{4}$$

$$m_t - p_t = -ai_t \qquad a > 0 \tag{5}$$

$$m_t = \alpha_m m_{t-1} + \alpha_q q_t + \alpha_u u_t \tag{6}$$

where s_t is the logarithm of the spot exchange rate at time *t*, and p_t and p_t^* are the logarithms of domestic and foreign price levels, respectively. q_t, the shock to real exchange rate, is an independent white noise process.[2] i_t and i_t^* are interest rates on domestic and foreign deposits, respectively, with equivalent risk and maturity. m_t represents logarithm of domestic money supply. α_m, α_q and α_u are monetary policy reaction coefficients. rp_t is risk premium and u_t is a white noise risk premium shock. Without the assumption of risk neutrality but with

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[2] p_t^* and i_t^* are constants set to zero.

conditional log normality, the difference between forward exchange rate and expected spot exchange rate is risk premium and Jensen's inequality terms. That is to say, UIP may not hold when we incorporate the Jensen's inequality terms even if the investors are risk-neutral. Since it is argued that the inequality is small empirically, we exclude it for the sake of the simplicity.

In the model, equation (2) is a purchasing power parity equation with stationary deviations. Equation (3) is an uncovered interest rate parity equation, and equation (4) shows that the risk premium follows a first order autoregressive process, as is Svensson (2000). McCallum (1994) suggests persistence in the risk premium, in the sense of positive serial correlation, and Wolff (2000) shows that the risk premium follows either an AR (1) or an ARMA (1,1) process. Equation (5) represents the money market equilibrium condition, in which a is interest rate elasticity of money demand. Equations (2), (3) and (5) constitute a simple version of the standard monetary model for exchange rate determination. Equation (6) describes the central bank's reaction function to exogenous shocks to the system, such as real exchange rate shocks and risk premium shocks.

2. Equilibrium Exchange Rate and Interest Rate

The reduced form of the exchange rate equation from the model is:

$$s_t = \frac{a}{1+a} E_t s_{t+1} + \frac{a\rho}{1+a} rp_{t-1} + \frac{1}{1+a}\left\{\alpha_m m_{t-1} + (1+\alpha_q) q_t + (a+\alpha_u) u_t\right\} \tag{7}$$

The relevant state variables for s_t are rp_{t-1}, m_{t-1}, q_t, and u_t. To obtain the linear solution, we conjecture that, by the method of undetermined coefficients, it is of the following form:

$$s_t = \phi_m m_{t-1} + \phi_{rp} rp_{t-1} + \phi_q q_t + \phi_u u_t \tag{8}$$

Updating and taking the expectation of equation (8), and using equations

(5) and (6), we get equation (9):

$$E_t s_{t+1} = \phi_m m_t + \phi_{rp} rp_t = \phi_m \alpha_m m_{t-1} + \rho \phi_{rp} rp_{t-1} + (\phi_m \alpha_u + \phi_{rp}) u_t + \phi_m \alpha_q q_t \tag{9}$$

Equation (9) indicates that the expected spot exchange rate is a linear function of money supply and risk premium for this period. This is, in turn, determined by money supply and risk premium of the last period and white noise terms for this period.

The solutions for ϕ-coefficients are obtained by inserting equations (8) and (9) into equation (7). At this point, it is worthy to introduce a special case of stationary money supply. Imposing the restriction $\alpha_m = 0$ leads to the following ϕ-coefficients:

$$\phi_m = 0, \ \phi_{rp} = \frac{a\rho}{1+a-a\rho}, \ \phi_u = \frac{\alpha_u(1+a-a\rho)+a(1+a)}{(1+a)(1+a-a\rho)}, \quad \phi_q = \frac{1+\alpha_q}{1+a} \tag{10}$$

Therefore, the solution for the exchange rate is:

$$s_t = \frac{a\rho}{1+a-a\rho} rp_{t-1} + \frac{\alpha_u(1+a-a\rho)+a(1+a)}{(1+a)(1+a-a\rho)} u_t + \frac{1+\alpha_q}{1+a} q_t \tag{11}$$

Equation (11) shows that exchange rate for the current period is the sum of the risk premium of the previous period and two white noise terms. By updating and then taking expectation of equation (11) and using equation (4), $rp_t - rp_{t-1} = (\rho - 1) rp_{t-1} + u_t$, we find expected depreciation:

$$E_t(s_{t+1} - s_t) = \frac{a\rho(\rho-1)}{1+a-a\rho} rp_{t-1} + \frac{(\rho-1)a(1+a) - \alpha_u(1+a-a\rho)}{(1+a)(1+a-a\rho)} u_t - \frac{1+\alpha_q}{1+a} q_t \tag{12}$$

From equation (3) and equation (12), we derive the equilibrium interest rate as follows:

$$i_t = \frac{\rho}{1+a-a\rho} rp_{t-1} + \frac{a+1-\alpha_u(1+a-a\rho)}{(1+a)(1+a-a\rho)} u_t - \frac{1+\alpha_q}{1+a} q_t \tag{13}$$

3. Probability Limit of β Coefficient

In Fama (1984), the probability limit of $\hat{\beta}$ in the standard equation for testing the unbiasedness of forward exchange rate (equation (1)) is

$$\begin{aligned} p\lim\hat{\beta} &= \frac{Cov(s_{t+1}-s_t, f_{t,t+1}-s_t)}{Var(f_{t,t+1}-s_t)} \\ &= \frac{Var(E_t(s_{t+1}-s_t)) + Cov(rp_t, E_t(s_{t+1}-s_t))}{Var(rp_t) + Var(E_t(s_{t+1}-s_t)) + 2Cov(rp_t, E_t(s_{t+1}-s_t))} \end{aligned} \tag{14}$$

From the equation and the empirical evidence, he concludes that the covariance between expected depreciation and risk premium is negative and the variance of risk premium is larger than the variance of expected depreciation. More specifically:

$$Var(rp_t) > \left|Cov(rp_t, E_t(s_{t+1}-s_t)\right| > Var(E_t(s_{t+1}-s_t)) \tag{15}$$

From equation (12), the variance of expected depreciation is:

$$\begin{aligned} &Var(E_t(s_{t+1}-s_t)) = \\ &\left[\frac{a^2\rho^2(1-\rho)}{(1+a-a\rho)^2(1+\rho)} + \left\{\frac{a(1+a)(\rho-1)-\alpha_u(1+a-a\rho)}{(1+a)(1+a-a\rho)}\right\}^2\right]\sigma_u^2 \\ &+\left(\frac{1+\alpha_q}{1+a}\right)^2\sigma_q^2 \end{aligned} \tag{16}$$

Since $Cov(aX+bY, cX+dY) = acVar(X) + bdVar(Y)$ when X and Y are independent random variables, the covariance between expected depreciation and risk premium is:

$$Cov(E_t(s_{t+1} - s_t), rp_t) = \frac{-a(1+a) - \alpha_u(1+a-a\rho)(1+\rho)}{(1+a)(1+\rho)(1+a-a\rho)}\sigma_u^2 \qquad (17)$$

When α_u is greater than $\dfrac{-a(1+a)}{(1+a-a\rho)(1+\rho)}$, the covariance is negative, which is in line with literature frequently reporting a negative β. Now that we have the explicit forms of the terms in equation (14) – variance of expected depreciation, variance of risk premium, and the covariance of risk premium and expected depreciation – we can calculate the probability limit of $\hat{\beta}$. The parameter values for α_m, α_u, and α_q will be determined according to a variety of monetary policy rules.

III. MONETARY POLICY RULES AND BIAS

Three basic strategies can be envisaged for the choice of a monetary policy regime to anchor inflation.[3] The first regime is interest rate smoothing. Some researchers such as Bernanke and Mishkin (1992) note that the interest-rate stability is an independent objective of monetary policy in many cases. Second, under exchange rate targeting, the nominal anchor is a desired exchange rate. The third regime is price level stabilization. The fourth is money supply targeting, in which the policy rule relies on a pre-committed path for the money supply to anchor inflation. The money supply rules and respective effects on the predicted forward discount bias are discussed in the following sub-sections.

1. Interest Rate Smoothing

Most central banks use an interest rate as their instrument of choice, and in countries with interest-rate targets, policymakers make efforts to smooth interest rates. In this subsection, the central bank is assumed to be exclusively concerned about interest rate stability. In other words, the loss function of the central bank consists of interest volatility only. The appropriate reaction parameters to minimize the interest rate volatility are:

[3] Refer to Corbo (2000) for more details.

$$\alpha_m = 0\,,\ \ \alpha_u = \frac{1+a}{1+a-a\rho}\,,\ \ \alpha_q = -1 \tag{18}$$

With the parameters, spot exchange rate and interest rate are:

$$s_t = \frac{a\rho}{1+a-a\rho}rp_{t-1} + \frac{1+a}{1+a-a\rho}u_t \tag{19}$$

$$i_t = \frac{\rho}{1+a-a\rho}rp_{t-1} \tag{20}$$

The variances of spot exchange rate and interest rate are, respectively:

$$Var\,(s_t) = \frac{(a+1)^2 - \rho^2(2a+1)}{(1-\rho^2)(1+a-a\rho)^2}\sigma_u^2 \tag{21}$$

$$Var\,(i_t) = \frac{\rho^2}{(1-\rho^2)(1+a-a\rho)^2}\sigma_u^2 \tag{22}$$

Obviously, the variance of interest rate in equation (22) is smaller than the variance of interest rate (equation (13)) without the reaction of the central bank aiming at smoothing of interest rate. With interest rate smoothing the interest rate, variance is proportional to the variance of the risk premium, σ_u^2. Accordingly, the interest rate variance will be zero when risk premium is white noise (when $\rho = 0$).

The monetary authority responds to positive shocks in the risk premium by raising the money stock, as can be seen in equations (6) and (18) with $\alpha_u > 0$. Accordingly, the spot exchange rate (and the price level) increases when $u_t > 0$, as seen in equation (19). With a positive shock to risk premium, Δu_t, the spot exchange rate increases by $\frac{1+a}{1+a-a\rho}\Delta u_t$. Since the expected spot exchange rate rises, but by less than the increase in the spot exchange rate, expected appreciation occurs rather than expected depreciation $(\frac{dE_t(s_{t+1}-s_t)}{du_t} = -1)$. Accordingly, the covariance between risk premium and expected depreciation has a negative sign, distinguishing the autoregressive-process risk-premium model from a white noise risk-premium model in which expected spot exchange rate change is always zero. We can check this by equation (23), derived by plugging equation (18) into equation (17):

$$Cov(E_t(s_{t+1}-s_t), rp_t) = \frac{-(1+a+\rho)}{(1+\rho)(1+a-a\rho)}\sigma_u^2 \tag{23}$$

Next, let us look at the variance of expected depreciation with an interest rate smoothing rule. Inserting equation (18) into equation (16), we get the variance of expected depreciation:

$$Var(E_t(s_{t+1}-s_t)) = \left\{\frac{a^2\rho^2(1-\rho)}{(1+\rho)(1+a-a\rho)^2}+1\right\}\sigma_u^2 \tag{24}$$

It is straightforward to check that the variance of expected depreciation in equation (24) is smaller than the variance of risk premium $(\sigma_u^2/(1-\rho^2))$. In Anker (1999), however, variance of expected depreciation is greater than the variance of risk premium and the covariance between the risk premium and expected depreciation is positive. This violates the necessary condition for probability limit of $\hat{\beta}$ to be negative, which was suggested by Fama (1984).

From the equations (23) and (24), and using $Var(rp_t) = \frac{\sigma_u^2}{1-\rho^2}$, we get the probability limit of $\hat{\beta}$:

$$p\lim\hat{\beta} = -a(1-\rho) \tag{25}$$

This probability limit is always negative and the absolute size is larger with higher interest rate elasticity of money demand and lower autoregressive coefficient of risk premium.

2. Exchange Rate Targeting

Now suppose the authority's loss function only consists of exchange rate variability. The behavior of the central banks in the ERM (Exchange Rate Mechanism) in the past may be the result of this type of loss function. The exchange rate of the member countries were allowed to vary in a fixed small band and their policy priority was the stabilization of exchange rates.

To minimize exchange rate volatility, the central bank will choose the following policy parameters:

$$\alpha_m = 0\,, \;\; \alpha_u = \frac{-a(1+a)}{1+a-a\rho}\,, \;\; \alpha_q = -1 \tag{26}$$

With these policy parameters, we get a new exchange rate and interest rate:

$$s_t = \frac{a\rho}{1+a-a\rho} rp_{t-1}\,, \tag{27}$$

$$i_t = \frac{\rho}{1+a-a\rho} rp_{t-1} + \frac{1+a}{1+a-a\rho} u_t\,. \tag{28}$$

As expected, a trade-off is observed here. Compared to the variances of exchange rate and interest rate in subsection A, the variance of exchange rate is smaller while the variance of interest rate is larger with exchange rate stabilization.

The variance of expected depreciation is:

$$Var(E_t(s_{t+1} - s_t)) = \frac{2a^2\rho^2}{(1+\rho)(1+a-a\rho)^2} \sigma_u^2\,. \tag{29}$$

The variance is smaller than the variance of risk premium, and this is consistent with the empirical findings of Fama (1984). The difference from Fama's findings are that the covariance between expected depreciation and risk premium is positive as long as the central bank focuses on exchange rate stabilization. Namely:

$$Cov(E_t(s_{t+1} - s_t), rp_t) = \frac{a\rho}{(1+\rho)(1+a-a\rho)}\sigma_u^2 > 0. \tag{30}$$

A positive shock to the risk premium raises the interest rate and decreases money demand in equations (3) and (5). To prevent the price level and spot exchange rate from escalating, the central bank tightens the money stock, which is reflected in the negative sign of α_u in equation (26). With this exchange rate stabilizing money tightening, the coefficient of u_t in equation (11) becomes zero so that the u_t term does not appear in equation (27). This means that spot exchange rate is not affected by the positive shock in u_t, while the expected exchange rate rises by $\frac{a\rho}{1+a-a\rho}\Delta u_t$. Therefore, expected depreciation has a positive sign, and hence the covariance between risk premium and expected depreciation is positive, which implies that the probability limit of β is always positive. Obviously, the covariance is zero when the risk premium is white noise, since expected depreciation is zero under the assumption.

Combining the equations (29) and (30):

$$p\lim\hat{\beta} = \frac{2a\rho(1+a)(1-\rho)}{(1-\rho^2)a^2 + 2(1-\rho^2)a + 1}, \tag{31}$$

which is positive but less than 1 since the denominator is larger than the numerator by $(1+a-a\rho)^2$.

3. Price Level Stabilization

Under price level stabilization, the central bank will be concerned about the stability of the price level.[4] From equations (2) and (11), the

[4] The Reserve Bank of New Zealand Act of 1989 told the Central Bank “to formulate and implement monetary policy directed to the economic objective of achieving and maintaining stability in the general level of prices.”

price level is expressed as

$$p_t = s_t - q_t = \frac{a\rho}{1+a-a\rho} rp_{t-1} + \frac{\alpha_u(1+a-a\rho)+a(1+a)}{(1+a)(1+a-a\rho)} u_t + \frac{\alpha_q - a}{1+a} q_t \,. \tag{32}$$

The policy parameters that minimize the variance of the price level are:

$$\alpha_m = 0\,,\ \ \alpha_u = \frac{-a(1+a)}{1+a-a\rho}\,,\ \ \alpha_q = a\,. \tag{33}$$

In the price stabilization, the policy parameter for u_t in risk premium, α_u, is the same as that in the exchange rate stabilization policy, while the policy parameter for real exchange rate shock differs. With those policy parameters, we get price level, variance of expected depreciation, and covariance between expected depreciation and risk premium:

$$p_t = \frac{a\rho}{1+a-a\rho} rp_{t-1}\,, \tag{34}$$

$$Var(E_t(s_{t+1}-s_t)) = \frac{2a^2\rho^2}{(1+\rho)(1+a-a\rho)^2}\sigma_u^2 + \sigma_q^2\,, \tag{35}$$

$$Cov(E_t(s_{t+1}-s_t), rp_t) = \frac{a\rho}{(1+\rho)(1+a-a\rho)}\sigma_u^2\,. \tag{36}$$

The positive covariance between expected depreciation and risk premium is the same as that in exchange rate stabilization. When there is a positive shock in the risk premium, the interest rate increases in equation (3). In an attempt to prevent the price level from changing in equation (5), the central bank reduces the money supply in equation (5), which is reflected in the negative policy parameter above. In this case, the exchange rate does not change in equation (11) since the second term becomes zero with the policy while the expected exchange rate increases.

From equation (11), $Var(s_t)=\frac{a^2\rho^2}{(1-\rho^2)(1+a-a\rho)^2}\sigma_u^2+\sigma_q^2$, which is greater than the variance of exchange rate in subsection B by σ_q^2. This implies that the price level stabilization is achieved partly at the cost of real exchange rate stability. Now the probability limit of $\hat{\beta}$ is obtained as follows:

$$p\lim\hat{\beta}=\frac{2a\rho(1+a)(1-\rho)\sigma_u^2+\sigma_q^2}{\{(1-\rho^2)a^2+2(1-\rho^2)a+1\}\sigma_u^2+\sigma_q^2}. \tag{37}$$

Comparing equation (37) with (31), we observe that the only difference between the two is the variance of real exchange rate shock (σ_q^2) in equation (37) (both in the numerator and in the denominator). This means that the probability limit in (37) is closer to one than the probability limit of β with exchange rate stabilization.

Svensson(2000) suggests that monetary authority's inflation targeting may result in large foreign exchange risk premium, which can create deviations in UIP and potentially explain the 'bias'[5]. By including the exchange rate in the discussion of inflation targeting he explains that monetary policy affects domestic currency prices of imported goods. And imported good prices enter the consumer price index(CPI). Therefore, government's efforts to control CPI will affect the risk premium as well as exchange rate. The results show inflation targeting is different from price level stabilization with regard to the forward bias. The objective of monetary policy is stabilizing inflation around the inflation target rather than minimizing the variance of the price level. As a matter of fact the forward discount bias can be found in the tests for the countries which have adopted inflation targeting. In other words, the forward discount bias is probable when inflation targeting is pursued, but is unlikely when price level stabilization is pursued.

[5] Inflation targeting is an increasingly popular money supply policy rule. It has been introduced in Australia (Sep. 1993), Canada (Feb. 1991), Israel (Mar. 1991), Sweden (Jan. 1993) and the UK (Oct. 1992).

4. Money Supply Targeting

Monetarists advocate steady growth of the money supply. If a central bank is full of monetarists, monetary authority will be concerned about the stability of money supply. Now the monetary reaction function in equation (6) will be restricted to $\alpha_m = 1+g$ (g is money supply growth rate), $\alpha_u = 0$, $\alpha_q = 0$. This means that shocks to the economy are ignored under this policy rule. Using the same method as above, we derive the solution under this targeting:

$$\phi_m = \frac{1+g}{1-ag}, \quad \phi_{rp} = \frac{a\rho}{1+a-a\rho}, \quad \phi_u = \frac{a}{1+a-a\rho}, \quad \phi_q = \frac{1}{1+a}. \tag{38}$$

Therefore, the equilibrium exchange rate and interest rate are, respectively:

$$s_t = \frac{1+g}{1-ag} m_{t-1} + \frac{a\rho}{1+a-a\rho} rp_{t-1} + \frac{a}{1+a-a\rho} u_t + \frac{1}{1+a} q_t \tag{39}$$

$$i_t = \frac{g(1+g)}{1-ag} m_{t-1} + \frac{\rho}{1+a-a\rho} rp_{t-1} + \frac{1}{1+a-a\rho} u_t - \frac{1}{1+a} q_t \tag{40}$$

Under the money supply targeting, however, neither spot exchange rate nor interest rate is a function of policy reaction parameters α_u or α_q (actually, both α_u and α_q are zero). Since $Var(m_{t-1}) = (1+g)^{2(t-1)} Var(m_0) = 0$, the variance of expected depreciation and the covariance between expected depreciation and risk premium with the monetary policy are:

$$Var(E_t(s_{t+1} - s_t)) = \frac{a^2(1-\rho)}{(1+a-a\rho)^2(1+\rho)} \sigma_u^2 + \left(\frac{1}{1+a}\right)^2 \sigma_q^2 \tag{41}$$

$$Cov(E_t(s_{t+1} - s_t), rp_t) = \frac{-a}{(1+\rho)(1+a-a\rho)} \sigma_u^2 . \tag{42}$$

When a positive risk premium shock occurs, the exchange rate and the expected exchange rate rise by $\frac{a}{1+a-a\rho}$ and $\frac{a\rho}{1+a-a\rho}$ respectively, so that the covariance between expected exchange rate and risk premium is less than zero.

Plugging equations (41) and (42) into (14), we get the following probability limit:

$$p\lim\hat{\beta} = \frac{\frac{-a}{(1+\rho)(1+a-a\rho)^2}\sigma_u^2 + \left(\frac{1}{1+a}\right)^2\sigma_q^2}{\frac{1}{(1+a-a\rho)^2(1-\rho^2)}\sigma_u^2 + \left(\frac{1}{1+a}\right)^2\sigma_q^2}. \tag{43}$$

Negative probability limit is not inevitable in equation (43). However, it will be less than zero if the variance of risk premium is large enough compared to the variance of real exchange rate shock. Interestingly, the probability limit is exactly the same as that under interest rate smoothing when real exchange rate shock is constant.

IV. SUMMARY

Let us try to order the monetary policy rules with regard to the magnitude of the probability limit of β coefficient or, more generally, the forward discount bias. We have observed that the probability limit of $\hat{\beta}$ for price-level stabilization with stationary monetary policy (equation (37)) is larger than $\hat{\beta}$ for exchange rate targeting with stationary monetary policy (equation (31)). In addition, the probability limit of $\hat{\beta}$ for money supply stabilization (equation (43)) is larger than the probability limit for interest rate smoothing (equation (25)). Note also that $\hat{\beta}$ for money supply targeting is smaller than $\hat{\beta}$ for exchange rate stabilization when a certain condition is satisfied.[6] We then we get the following order of monetary policy rules in terms of the size of the probability limit of $\hat{\beta}$:

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[6] The condition holds as long as $\sigma_u^2 > 0.36\sigma_q^2$ when a = 1 and $\rho = 0.5$, for example.

Interest Rate Smoothing < Money Supply Targeting
< Exchange Rate Targeting < Price Level Stabilizing

This means that the forward discount bias is most probable when interest rate smoothing is pursued under a stationary money supply and least likely when the central bank focuses on the stabilization of price under a stationary money supply.

These results might be interpreted as evidence that the interest rate or money supply has been the target of monetary policy for many central banks. If exchange rate or price level is the major policy target, then we may not observe the forward discount bias under the stationary money supply assumption. These results support the empirical study of Flood and Rose (1996) in which, using ERM (Exchange Rate Mechanism) data for 1981-1984, they report that forward discount bias is not clear in the fixed exchange rate system. In their empirical work for ERM, β is estimated to be positive (0.58) and significantly greater than zero, even though the estimate is significantly less than one. In addition, these results are consistent with the survey of Froot and Thaler (1990) in that the coefficient is less than one in any case. The little difference between exchange rate stabilization and price level stabilization is associated with the fact that the purchasing power parity is a building block in the model.

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