THE TEMPORAL SINGLE-SYSTEM INTERPRETATION OF MARX'S VALUE THEORY: A CRITICAL APPRAISAL

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This paper examines the temporal single-system interpretation (TSSI) of Marx's value theory. The main criticisms of the principal propositions of the TSSI are summarized in three points. (1) While the TSSI may be a superior interpretation because, unlike other interpretations, it replicates many of Capital's results, it fails to eradicate the problems related to value magnitude. (2) The context of historical cost cannot be maintained within the discrete time setting usually used in the TSSI. Furthermore, it can be shown that the TSSIs logic is inconsistent in the framework of continuous time. (3) The TSSI does not fully succeed in refuting the redundancy critique of Marx's value theory more effectively than the standard interpretation.

I. INTRODUCTION

A recent development in Marx's value theory is the emergence of the temporalist interpretation, also known as the non-dualist interpretation or the theory of sequential values. In current literature, however, it is generally dubbed the 'Temporal Single-System Interpretation' (TSSI), especially in the papers presented at the conferences of the *International Working Group on Value Theory*.[1]

In the TSSI, value and price are defined in the form of first-order difference equations as follows:

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[1] See Freeman and Carchedi eds., 1996; Kliman, 1997; Ramos, 1997; Kliman and McGlone, 1999; Freeman, 1999.

$$\lambda_{t+1} = p_t A + l \tag{1}$$

$$p_{t+1} = p_t A + l + g_t \tag{2}$$

where λ, p, A and l denote the value vector, the price vector, the physical-technological input coefficient matrix (non-singular and indecomposable) and the labor-input vector, respectively. The subscript denotes time defined discretely, and g represents price-value difference.[2]

The main propositions of the TSSI can be summarized as follows:

(1) The value of capital advanced depends on the prices, not the values, of the inputs. This is the 'single' nature of the interpretation in the sense that value and price are regarded as mutually penetrating parts of a single system, not as two independent and distinct systems.

(2) Value magnitude is determined within historical time in the sense that the inputs are valuated at the current market prices of the purchasing moment.[3] In other words, value is defined diachronically, not synchronically. This is the 'temporal' nature of the interpretation, according to which the concept of simultaneous equilibrium is negated.

(3) All the results described in Marx's *Capital* can be replicated. For example, the two aggregate value-price equalities in the transformation procedure ('total value = total price' and 'total surplus value = total profit') and the tendency of the rate of profit to fall hold in Marx's original form.

As TSSI adherents maintain that their theory is an interpretation of Marx's own theory, not a new theory or approach, it is important to examine textual evidence to decide whether TSSI is faithful to Marx's conception. However, at least up to now, questions such as this have tended to result in a never-ending cycle of (re)citations of the original texts.[4] The same passage of *Capital* has even been used to support entirely different interpretations.[5]

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[2] Since TSS theorists typically emphasize the continuous change in technical conditions, A and l should also be time-indexed. However, for analytical simplicity, they usually start with equations (1) and (2). See Kliman and McGlone, 1999. On the other hand, g also represents the profit-surplus value difference.

[3] In earlier versions of the TSSI, the concept of price of production was also used to valuate the inputs in order to facilitate comparison with other interpretations. See, for example, Kliman and McGlone, 1988. On the other hand, Moseley (1999) tried to show that Kliman and McGlone (1988)'s concept of prices of production is different from Marx's concept.

[4] I do not deny the necessity and importance of the exegetical work itself. For example, Ramos (1999) is a promising work, which presents some new evidences from Marx's drafts.

[5] One of the most notorious examples is the following.

   As the price of production of a commodity can diverge from its value, so the cost price of a commodity, in which the price of production of other commodities is involved, can also stand above or below the portion of its total value that is formed by the value of the means of production going into it. It is necessary to bear in mind this modified significance of the cost price, and therefore to bear in mind too that if the cost price of

This paper, therefore, will try to evaluate the TSSI in a more indirect way. The guidelines used in this evaluation are presented below in order to reveal the focus of this paper as clearly as possible.[6]

(1) Insofar as we accept its definition of value, the TSSI is a superior interpretation because it replicates many of *Capital's* results whereas other interpretations cannot. With regard to the value magnitude in the TSSI, however, some problems still remain.

(2) As the TSSI abandons the synchronization principle contrary to the accepted view, the consistency of its time concept should be tested. The context of historical cost cannot be maintained within the discrete time setting usually used in the TSSI. As the TSS value is defined as something cumulated from the past, we have somewhat perverse implications. Namely, in some cases, value magnitude can diverge to infinity despite the decrease in labor input. Furthermore, it can be shown that the TSSIs internal logic is inconsistent in the framework of continuous time.

(3) The TSSI does not fully succeed in refuting the redundancy critique of Marx's value theory (e.g. Steedman, 1977) more effectively than the standard interpretation. Especially, as the TSSI holds that the so-called redundancy results from the fallacy of simultaneous valuation, the comparison must be made between simultaneous system and temporal system.

These points will be incorporated into the discussions below. Section II examines the problems pertaining to magnitude.[7] This is related to the single system nature of the TSSI. The main thrust of the historical time concept is presented in section III, which examines the temporal system nature of the TSSI. Section IV is devoted to comparing the TSSI to the standard, simultaneous approach as a means of refuting the redundancy critique of Marx's value theory. This is the problem pertaining to determination. Section V will conclude.

II. THE SINGLE SYSTEM NATURE

The TSS theorists classify various interpretations of Marx's value theory according to two criteria: (i) dual or single system, that is, whether value and

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a commodity is equated with the value of the means of production used up in producing it, it is always possible to go wrong. Our present investigation does not require us to go into further detail on this point (Marx, 1981: 265).

Whereas Shaikh (1977: 130-1) quotes this passage to support his iterative transformation procedure and Duménil (1983-4: 448) considers it as the stepping stone of his 'New Interpretation', Kliman and McGlone (1999: 39) use the passage to derive the single system nature of value and price. Namely, almost all interpretations of Marx's labor theory of value have been derived from the same passage.

[6] Note that these do not necessarily correspond one to one to the above three points of the TSSI.

[7] The expression, 'pertaining to magnitude/determination', is borrowed from Kliman and McGlone (1999).

price are regarded as a dual system of two independent systems or a single system of mutually penetrating parts and (ii) simultaneous or temporal system, that is, whether inputs and outputs are simultaneously valuated in the equilibrium framework or temporally valuated by introducing historical time. This section is concerned with the first classification.

2.1. Dimensionality problem

The dimensionality problem discussed below is specific to single system interpretation in general, not only to the TSSI. Therefore, we start with the value equation defined in the single system.[8]

$$\lambda = pA + l \tag{3}$$

This means that the values of constant inputs should be defined as the prices at which they were purchased. Once equation (3) is admitted, the aggregate value of output becomes equal to the aggregate price of output.[9] At first sight, however, equation (3) juxtaposes value and price terms. Therefore, value represents a bizarre hybrid of labor time and monetary unit.[10]

Without doubt, the TSS theorists translated money units into labor-time units by means of 'value of money' or 'monetary expression of labor time' (Ramos, 1997; Kliman, 1999), originating from Duménil-Foley. Duménil-Foley, actually, presented the so-called 'New Interpretation' arguing that the value of variable capital depends on the price, not the value, of subsistence (Duménil, 1980; Foley, 1982). Therefore, the single system nature of the TSSI means that the approach of the 'New Interpretation' should also be applied to the constant capital.

Insofar as the variable portion of capital is concerned, the following logic was developed to validate this procedure.

> ...to regard labour-power as a commodity whose money value requires transformation into a price of production, and whose input cost structure likewise requires transformation, is doubly misconceived : first, because labour-power is not a commodity which is produced; and second, because

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[8] As the input prices and output values are determined at the same time, equation (3) is a single-system without a temporal nature. The TSS theorists refer to this interpretation as a simultaneous single-system. Wolff, Roberts and Callari (1982), Lee (1993) and Moseley (1993) belong to the group advocating this interpretation.

[9] The transformation problem handed down from Bortkiewicz was due to the fact that the values of the constant and variable inputs are not equal to their prices. Therefore, if the two are equal by definition, as in equation (3), the 'transformation problem' cannot exist.

[10] If a is measured in kilograms, b in meters and c in dollars, the expression '$a = b + c$' is meaningless (Okishio, 1993 : 11).

there is no average rate of profit to be earned in the (re)production of people, indeed no value and surplus-value involved at all. To consider otherwise loses the distinction between labour and labour-power, in effect treating capitalism as a slave mode of production (Mohun, 1994: 401).

Mohun's argument is in the same vein as the Duménil-Foley tradition. One can legitimatize this position from the perspective against the single system interpretation as follows.

Value and price are two distinct systems that measure the same physical system by two different principles. In the cases of ordinary commodities, the quantitative weights in the two systems are different, unless the organic composition of capital in the sector concerned is equal to the social average. On the other hand, the hypothetical sector that 'produces' labor-power cannot be conceptualized as a production combining constant and variable inputs. No doubt, the rate of profit cannot be applicable to this sector.[11] In other words, labor-power itself must be regarded as a 'net product', and its value magnitude is proportional to its price.[12] Therefore, the value of labor-power is determined as the quantity of abstract labor proportional to money wage.

It is not certain, however, if this same process can be applied to the constant portion of capital. At least up to now, the TSSI's textual evidence does not seem to be decisive enough to refute the accepted view which holds that the values of constant inputs must be transformed into their prices of production. As the following statement shows, Marx does not think that the values and the production prices of commodities can be equal in general.

Commodities produced by capital II [capital of higher composition] thus have a value less than their price of production, and those produced by capital III [capital of lower composition] have a price of production less than their value. Only for capitals such as I, in branches of production whose composition chanced to coincide with the social average, would the value and the price of production be the same (Marx, 1981: 264).

As it is obvious that the elements of constant capital are traded as commodities, the relation between their values and prices of production must follow the above rule. In order for TSSI not to violate the rule, there are only two alternatives: to assume that the organic compositions of capital in the sectors producing constant inputs happen to be equal to the social average, or to assume that the value of constant capital depends on the prices of the means of

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[11] Without doubt, there are many for-profit institutions for training and education. However, in Marx's terminology, at least, the ability of 'simple labor' itself is not (re)produced in the capitalist way.

[12] Remember that it lies at the heart of Duménil-Foley's 'New Interpretation' to apply the total value-price equivalence to the net product, not gross product, of the economy.

production. The former is an unrealistic condition, and the latter, upon which the TSSI is actually premised, does not belong to the realm of logical proof. Rather, the latter assumption is a definition problem of how to define the term 'value', it is a position that cannot be argued on the basis of logic, but only postulated.

2.2. Price-value and profit-surplus value differences

The TSS theorists argue that Marx's insight, that price-value difference stems only from the profit-surplus value difference, can only be preserved in the single-system interpretation. This is also related to the single nature of the TSSI. However, we can show that the price-value difference can also be reduced to profit-surplus value difference in the dual system interpretation.

Under the condition, $\lim A^n = 0$,[13] we can get the following result from (1) and (2).

$$\lambda_{t+1} = p_t A + l$$

$$= (p_{t-1} A + l + g_{t-1})A + l$$

$$= \sum_{k=1}^{\infty} g_{t-k} A^k + l(I + A + A^2 + \ldots\ldots) + P_{t-k} A^{k+1}$$

$$= l(I-A)^{-1} + \sum_{k=1}^{\infty} g_{t-k} A^k \tag{4}$$

The TSS value at period $t+1$, therefore, is equal to the vertically integrated labor plus power series of input coefficient matrices.

As $l(I-A)^{-1}$ is merely the value definition in the dual system interpretation,[14]

$$p_{t+1} - \lambda^* = (p_{t+1} - \lambda_{t+1}) + (\lambda_{t+1} - \lambda^*)$$

$$= g_t + \sum_{k=1}^{\infty} g_{t-k} A^k$$

$$= g_t + g_{t-1} A + g_{t-2} A^2 + \ldots\ldots \tag{5}$$

Equation (5) shows that, in the dual system interpretation, the price-value difference can also be represented as a polynomial function in which the coefficients are expressed in terms of profit-surplus value differences at different periods $(g_t\text{'s})$. Furthermore, A^k becomes smaller as k increases because the

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[13] If r_M is the eigenvalue of A which is maximum in the modulus, $|r_M| < 1$ is this condition. For proof of this property, see Pasinetti, 1977: 264-5.

[14] Here λ^* denotes the value in the dual system. Note that time index is not needed because the dual system is generally accompanied by simultaneous conception.

elements in each column of matrix A are non-negative numbers adding up to less than 1 in general. Namely, the effect of profit-surplus value differences in the more distant past is smaller than those in the less distant past.

Whereas the dual system cannot replicate Marx's text in the original form, the intuition that price-value difference results from the redistribution of surplus value in the form of profit is maintained, although in a much more complicated form. The TSS theorists seem to believe that this intuition can be maintained if and only if the price-value difference and the profit-surplus value difference are equal.[15] However, this is not so. In order to reach this intuition, it is sufficient to show that some functional relation exists between the price-value difference and the profit-surplus value difference. Without doubt, as Steedman (1977) showed, profit can be negative while surplus value is positive (and vice-versa) in dual system interpretations, especially in the case of joint production. However, while it is one thing to assert that the TSSI can evade this paradox, shutting down the theoretical possibility of any dual system interpretation altogether is an entirely different matter. For example, 'New Interpretation', a dual system theory is also a good candidate for the (re)formulation of Marx's value theory.[16]

III. THE TEMPORAL SYSTEM NATURE

3.1. Destruction paradox

Adherents of the TSSI argue that the single system nature does not sufficiently maintain Marx's results on the dynamics of capitalism, especially the theory on the tendency of the profit rate to fall and argue that the temporal system nature is additionally needed. Therefore, they define value and price as equations [1] and [2]. The TSS theorists vindicate these formulations by defining value within historical time. However, these equations have some perverse implications because value is defined as something cumulated from the past. To see this point more clearly, suppose the economy with a single good whose technology is as follows:

a units of goods + l units of labor-input $->$ l unit of good

Then, equation (1) is transformed into:[17]

$$\lambda_{t+1} = ap_t + l = a\lambda_t + l \qquad\qquad (1)'$$

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[15] "...the dual-system premise implies a more elemental and general critique of Marx's value theory: price-value and profit-surplus value differences cannot be identical"(Kliman and McGlone, 1999: 37).

[16] This does not necessarily mean that the present author is either a supporter of the 'New Interpretation' or an opponent of the TSSI.

[17] As there is only one commodity, value is equal to price, i.e. $p_t = \lambda_t$.

Solving (1)' with an initial value $p_0 = \lambda_0$, value magnitude in period t is determined explicitly:

$$\lambda_t = [p_0 - l/(1-a)]a^t + l/(1-a) = p_0 a^t + l(1-a^t)/(1-a) \tag{6}$$

In the simultaneous system interpretation, the so-called 'productivity condition' $(a < 1)$ must be satisfied. If not, λ will be negative, and so, meaningless. In the TSSI, however, 'productivity condition' is not needed for the positivity of value. As $\lambda_t = p_{t-1} A + l$, λ_t is always positive insofar as $p_{t-1} > 0$. The TSS theorists seem to believe that this attests to the superiority of their interpretation over the standard one with special regard to the case of joint production (Kliman and McGlone, 1999: 45-8). However, whereas this aspect contributes to eliminating the logical inconsistency, it introduces another difficulty. It nullifies one of the key predictions of Marx's labor theory of value, that value magnitude decreases, *ceteris paribus*, as the quantity of labor input decreases. For example, even in the case of 'destruction', in which more than one unit of commodity is needed to produce only one unit of the same commodity, value magnitude is positive in the TSSI. As can be known from (6), λ will diverge to infinity in this case.

A similar problem can also be demonstrated in the dynamic context. For example, Kliman (1999) notes that a simultaneous system approach is problematic in the sense that the monetary expression of labor time simultaneously defined, denoted by τ, can be negative. Whereas negative τ means negative profit with positive τ surplus labor, positive τ guarantees positive (negative) profit with positive (negative) surplus labor. Therefore, the logical superiority of the TSSI allegedly lies in the fact that τ-series are necessarily positive insofar as the initial value of τ is positive. Furthermore, he defines the rate of profit in the following two ways: π_N and π_R, which represent nominal and real profit respectively. Here x denotes total output and i, the inflation rate of the monetary expression of labor time, is defined as $(\tau_{t+1} - \tau_t)/\tau_t$ where subscript denotes period.[18]

$$\pi_N = p_{t+1} x - p_t Ax - w_t lx \tag{7}$$

$$\pi_R = [1/(1+i)]p_{t+1}x - p_t Ax - w_t lx \tag{8}$$

However, as known from equation (9) derivable from the definition of i,[19]

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[18] For the TSS theorists, the distinction between nominal and real profit is indispensable for maintaining the law of the tendency of the profit rate to fall. They argue that despite the increase in the nominal rate of profit, the real rate of profit declines with the introduction of the capital-using technological progress. On the other hand, Ramos (1997) uses the concept of 'rate of profit in labor time' instead of the real rate of profit in the same context as Kliman (1999).

τ_N will diverge to infinity with N unless i is negative or zero. This means that price can diverge to infinity even with the infinitesimal quantity of labor input.

$$\tau_N = \tau_0 (1 + i)^N \tag{9}$$

Therefore, we can conclude that the cumulatively defined TSS value results in an unavoidable paradox.

3.1.A. Productivity paradox

Recently, Duménil and Lévy(2000) presented a critique of Freeman(1996), a representative of the TSSI. Because their criticism is related to the problem of divergent value mentioned above, it is worth examining this problem further here.

Now, suppose a single-good economy. Assume that the amount of physical input is maintained, $a_t = a$, and that of labor input reduced at each period: $l_t = a + \beta \gamma^t$ where α, $\beta > 0$, $0 < \gamma < 1$. Subscript here denotes period. Then, in this situation, the sequential value or the TSS value is:[20]

$$\lambda_t = a/(1 - a) + [p_0 - a/(1 - a) + \beta \gamma/(a - \gamma)]a^t - \beta \gamma^{t+1}/(a - \gamma)$$

Therefore, λ_t always increases with time as long as the following condition is satisfied:[21]

$$a > \gamma \text{ and } p_0 < a/(1 - a) - \beta \gamma/(a - \gamma)$$

As $\beta > 0$, the following is the necessary condition for 'productivity paradox':

$$a > \gamma \text{ and } p_0 < a/(1 - a) = \lim \lambda_t$$

An implication of the above necessary condition is as follows: when the initial price, p_0, is less than the long-run equilibrium value, $a/(1 - a)$, the TSSI can lead to a paradoxical result. This situation is one of merely excess

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[19] As $i = (\tau_{t+1} - \tau_t)/\tau_t, \tau_{t+1} = (1 = i)\tau_t$. Therefore, $\tau_N = (1 = i)^N \tau_0$ where τ_0 denotes initial value of τ.

[20] Duménil & Lévy presented the following solution: $\lambda_t = a/(1 - a) + [p_0 - a/(1 - a) + \beta/(a - \gamma)]a^t - \beta \gamma^t/(a - \gamma)$ However, this is an incorrect solution.

[21] Duménil and Lévy called this phenomenon 'productivity paradox'. However, the possibility that this condition holds is greater than in the case of the incorrect solution of Duménil and Lévy. See Duménil and Lévy, 2000: 195.

supply. Namely, the cumulative character of the TSS value cannot rightly explain the situation in which the market price of the constant input is abnormally low, e.g. when there is a large amount of stock because of excess supply. This is the very situation that the following statement by Marx indicates.

> [In] the case with a fall in the price of raw material......the commodities on the market, articles in preparation and stocks of raw material *are all devalued*......The present argument is just as valid if prices rise or fall *not as a result of fluctuations in value*...(Marx, 1981 : 208. Italics added).

This passage shows us two things. First, changes in the prices of the constant inputs are not always relevant to the changes in the values of the constant inputs. Marx seems to distinguish between price changes with value change and price changes without value change. Obviously, this is inconsistent with the TSSI, which states that all price changes at the point of purchase entail value changes, at least, in the case of constant inputs. Secondly, even if such a definition of the TSSI is accepted, the abnormally low price of raw materials will ultimately devalue the values of constant inputs in Marx's case. Therefore, a 'productivity paradox' will not arise in Marx's own system.

3.2. Discrete vs. continuous time

According to equation (1) of the TSSI, the values of the constant inputs should be calculated by their historical costs, not current costs. However, the context of historical versus current cost, which had been implicitly accepted, came to be negated in the following discussion.[22]

> ...it is a complete misnomer to treat the distinction between the above [sequential valuation-Rieu] and equilibrium valuations as a distinction between 'historical' and 'current' cost. The value transferred to the prod-uct is not given by the magnitude of capital when purchased; it is given by the magnitude of this capital when it is used. This is its 'current' cost. The equilibrium determination substitutes a completely different notion, redefining the word 'current' to mean 'future'; it says that the value transferred by the cotton is given by what the cotton *will* cost when it has been produced using a technology that does not exist at the time it is used (Freeman, 1999: 10. Italics in original).

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[22] As Kliman and McGlone (1999: 34) admit, the TSSI is not a homogeneous body of work. Therefore, the time concept on which the TSSI is premised might differ between theorists. For example, contrary to the other TSS theorists on the inter-temporal setting, Freeman (1996; 1999) has consistently been working on a continuous time framework. So the context emphasizing the distinction between historical versus replacement cost (Laibman, 2001) cannot be imputed to Freeman's work.

In a nutshell, the TSSIs conception of time is alleged to be premised upon the correct interpretation of 'current cost', not upon the (re)definition of value by historical cost. However, Freeman's statement can be refuted from the perspective of the standard interpretation of Marx's value theory. Insofar as not assuming an instantaneous production, it will necessarily take some time to produce commodities. In fact, the time-consuming characteristic of production is one of the main themes of the second volume of Marx's *Capital*. Therefore, the state of technology by the time production is completed must be considered in determining the value magnitude at the present point of time. This idea is not incompatible with the definition of value of commodity as the socially necessary time to produce the commodity at the particular point in time. Assume that the same commodity is produced by two technologies, A and B. Then, the social value of this commodity is determined by the weighted average of the two individual values, respectively produced by A and B. If the distribution of technologies is expected to change in favor of technology A rather than technology B in the near future, the former has a more decisive effect on determining the value magnitude of this commodity than the latter. In this sense, contrary to Freeman's argument, the value transferred by cotton is affected by what cotton will cost in the near future. Therefore, one can say that Marx's value concept dialectically integrates past and future into the present.

To follow the internal logic of the TSS theorists, let us now examine whether the TSSI is consistent with the above argument by introducing continuous time. Assume that the price of constant input changes at the instantaneous rate of σ from t_0 to t_1. In the case of technological progress in the sector producing the means of production, σ will be less than 0. And, we can safely assume that from t_0 to t_1 is one period without losing generality. The physical quantity of constant input is assumed to remain unchanged as q_0. And $p(t)$, p_0 denotes, respectively, its price at t and initial price at t_0 Then,

$$p(t) = p_0 e^{\sigma t} \tag{10}$$

If the TSSI does not argue for historical cost, prices (therefore, values) of constant inputs should be calculated as follows.

$$\int_{t_0}^{t_1} p_0 q_0 e^{\sigma t} dt$$

$$= p_0 q_0 \int_{t_0}^{t_1} e^{\sigma t} dt = p_0 q_0 [(1/\sigma)e^{\sigma t}]_{t_0}^{t_1} dt$$

$$= (1/\sigma) p_0 q_0 (e^{\sigma t_1} - e^{\sigma t_0}) \tag{11}$$

However, in the TSSI, values of constant inputs are actually calculated by the cost actually incurred at the time of their purchase as follows.

$$p(t_0)q_0 = q_0\, q_0\, e^{\sigma t 0} \tag{12}$$

To find the condition for which the TSSI's calculation method is validated, assuming (11) is equal to (12), the following equation can be used.

$$(1/\sigma)p_0 q_0 (e^{\sigma t 1} - e^{\sigma t 0}) = p_0 q_0 e^{\sigma t 0} \tag{13}$$

From (13), we can get the following result.

$$(1+\sigma)e^{\sigma t 0} = p_0 q_0 e^{\sigma t 1} \tag{14}$$

Therefore, unless (14) is satisfied, the calculation method of the TSSI is incorrect even according to its internal logic. Equation (14) can hold only when σ is 0. Namely, if the TSSI rests upon the concept of current cost, no technological progress in the sector producing the means of production can be posited. This is clearly a paradoxical result.

This paradoxical result can be explained more intuitively using the equation [1] of the TSSI in a discrete time setting.

$$\lambda_{t+1} = p_t A + l \tag{1}$$

The physical elements represented by matrix A were purchased at p_t at the end of period t (or in the beginning of period $t+1$). At the end of period $t+1$, however, when value magnitude is calculated, the elements can no longer be purchased at p_t unless p_t is equal to p_{t+1}. In other words, p_t is not an actual price, but only the price in the account book. Therefore, to correctly find the 'current cost', we must calculate them by $(1+\rho)p_t$, not p_t, where ρ represents time factor depending upon the growth rate of prices and labor productivity. So value equation (1) should be changed into (1)″.

$$\lambda_{t+1} = (1+\rho)p_t A + l \tag{1″}$$

From (1) and (1)″, we can confirm the result already obtained, that the TSSI implicitly presupposes the condition, $\rho=0$. Furthermore, λ_t can diverge to infinity if ρ is greater than a certain value.

In a conclusion, this paradox must be removed if the TSSI can survive the critique that its main results specifically depend on the discretization of time or it ignores the problem of heterogeneous vintages of the constant inputs.

IV. REDUNDANCY CRITIQUE

This section aims to evaluate the TSSI by refuting the critique that Marx's value theory is not needed to know prices. The TSSI alleges the refutability of this redundancy critique by arguing that because only relative values matter in the simultaneous system interpretation, the extraction of labor turns out to be redundant to its value rate of profit. For example, as proportionate changes in living labor requirements have no effect on the rate of profit in the simultaneous system (Kliman, 1999), it is argued that the temporal nature of the TSSI should be introduced.

However, it can be shown that the TSSI can no more refute the redundancy critique than the simultaneous single-system interpretation can. In other words, the temporal system nature of the TSSI does not contribute to the refutation of the redundancy critique.

Suppose the simultaneous single-system as follows:

$$\lambda_t = p_t A + l \tag{15}$$

$$p_t = p_t A + l + g_t \tag{16}$$

From (15) and (16), we can get the following results.[23]

$$\Delta \lambda_t = \Delta p_t A$$
$$\Delta p_t = \varepsilon_t (I - A)^{-1} \qquad \text{where} \qquad \varepsilon_t = g_t - g_{t-1}$$

Therefore,

$$\Delta \lambda_t = \varepsilon_t (I - A)^{-1} A = \varepsilon_t (I + A + A^2 + \ldots) A = \varepsilon_t \sum_{k=1}^{\infty} A^k \tag{17}$$

On the other hand, applying the same manipulation to the TSSI equations (1) and (2),

$$\Delta \lambda_t = \Delta p_{t-1} A$$
$$\Delta p_t = \Delta p_{t-1} A + \varepsilon_{t-1}$$

Therefore,

$$\Delta \lambda_t = (\Delta p_{t-2} A + \varepsilon_{t-2}) A$$

$$= \Delta p_{t-2} A^2 + \varepsilon_{t-2} A = (\Delta p_{t-3} A + \varepsilon_{t-3}) A^2 + \varepsilon_{t-2} A$$

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[23] Without doubt, the most important cause of changes in value would be change in labor-input vector l. However, the following discussion presupposes the constant quantity of labor input, because we want to compare the simultaneous single-system interpretation and the TSSI in their usual formulations. See note 2.

$$= \ldots = \sum_{k=1}^{\infty} \varepsilon_{t-k-1} A^k \qquad\qquad (17)'$$

Comparing (17) with (17)', the changes in value magnitude from period $t-1$ to period t can be represented by the weighted sum of physical data, A^k, in both interpretations. Only the magnitude of the weight differs. In the TSSI, the given technical data and historical profile of past prices are sufficient in determining the absolute magnitude of value. Therefore, the redundancy critique cannot be avoided in the sense that only physical data are needed to know prices. The TSS theorists would undoubtedly argue that value is needed to know prices. However, this is possible only because they define the value at period t as the price at period $t-1$. Here, too, the problem is solved through the definition itself, not by logical deduction. However, the TSS theorists would object to the above procedure, arguing that introducing change in past prices could not be valid because prices are unchangeable, given data. However, the derivation of (17) and (17)' is from a purely mathematical procedure, and one cannot deny the logical results obtained from them.[24]

V. CONCLUSION

The TSSI has the potential to be a superior interpretation in the sense that it can replicate many of Marx's results conventionally negated by other interpretations. However, the TSS theorists changed the definition of value itself. Despite their attempts to present textual evidence, there are still many compelling arguments against them with different readings of Marx's texts. At least at this stage of the controversy, the procedure of applying 'monetary expression of labor time' to the constant inputs and regarding this as the value of constant inputs may well be taken as a new paradigm or approach, rather than an interpretation loyal to Marx. At any rate, for the TSSI to be a new paradigm or interpretation, the consistency of its time concept must be established. As simply introducing the inter-temporal setting is not sufficient for showing Marx's dynamic context, the framework of continuous time should be pursued further. This paper has tried to note the inconsistency of the time concept in the TSSI. Last but not least, the TSSI's refutation of the redundancy critique of Marx's value theory still seems unsatisfactory. At the least, this paper has shown that the TSSI does not refute the redundancy critique more effectively than the simultaneous single-system interpretation. We have tried to show these points by distinguishing the single nature and the temporal nature of the TSSI.

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[24] After reading an earlier version of the present paper, Andrew Kliman argued that this result would hold only if physical coefficients never changed. It might be so. However, if the TSSI wants to be a general theory, it should also be applicable to the case of constant coefficients.

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