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THE INDUSTRY EQUILIBRIUM OF THE LABOR-MANAGED FIRM IN MONOPOLISTIC COMPETITION UNDER DEMAND UNCERTAINTY

SUYEOL RYU*

This study contrasts the comparative statics results with the Mai(1993) ones in a perfectly competitive labor-managed(LM) industry and the Ishii(1991) ones in a monopolistically competitive profit-maximizing(PM) industry. First, some results in a perfectly competitive LM industry do not generally carry over to a monopolistically competitive LM industry. One of difference between the results in two industries comes from the fact that depends on whether any firm in the industry may affect the market price or not. Second, this study also shows that some results in a monopolistically competitive PM industry do not generally carry over to a LM industry.

JEL Classification: D0, D80, D81
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I. INTRODUCTION

There were various forms of labor management after World War II and the Yugoslav economy was subject to workers' management. As a modern form, Kibbutzim comprise about four percent of the Israeli economy. The labor-managed firms are applied to professional services such as law and medicine. Professional services firms are generally constituted as partnership rather than joint-stock companies.

The theory of labor-managed(hereafter LM) firm, which maximizes profit per worker, has attracted economist's attention under perfect competition. There has been a large volume of papers regarding the firm under certainty. In an interesting paper, Paroush and Kahana(hereafter P-K)(1980) have analyzed the short-run

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* Research Fellow, Busan Development Institute(BDI), Nulwon Bldg., 825-3 Bumil-Dong, Dong-Gu, Busan 601-720, Korea. The author is grateful to two anonymous referees for valuable comments and suggestions.

Phone: 051-640-2025, Fax: 051-637-6295, E-Mail: ryusuyeo@bdi.re.kr

behavior of a competitive LM firm under price uncertainty and shown that the comparative statics results are quite different from the one in the profit-maximizing(hereafter PM) firm as obtained by Sandmo(1971). While Applebaum and Katz(hereafter A-K)(1986) have dealt with the long-run industry equilibrium of a competitive PM firm where the output of individual firms and the number of firms in the industry are both endogenously determined under demand uncertainty, Mai(1993) has analyzed the long-run industry equilibrium of a competitive LM firm under demand uncertainty and contrasted his comparative statics results with P-K and A-K ones. He showed that the long-run response of the LM firm to a change in wage rate was in opposite direction to a short-run response and that the output behavior of a competitive LM firm was not identical to a competitive PM firm, which was contrary to the assertion obtained by Ward(1958) and Vanek(1970) under certainty. In practice, the finite number of firms might be inconsistent with a perfectly competitive industry as pointed out by Fama and Laffer(1972).

While the literature on the purely competitive situation under demand uncertainty is extensive, a little attention has been paid to the formal analysis of imperfect market structures under demand uncertainty. Recently Ishii(1991) has established a model of industry equilibrium under monopolistic competition with n Cournot PM firms and contrasted his comparative statics results with the A-K ones. He has shown that some results do not always equal the A-K ones even if the inverse demand function of the industry is linear as a special case. However, how a LM firm operating under demand uncertainty would response to changes in some parameters in the market equilibrium of a monopolistically competitive industry seems to have been neglected in the literature. Therefore, it may be worthwhile to make a detailed comparative statics analysis.

The purpose of this paper is to present a model of monopolistically competitive industry with LM firms under demand uncertainty. First, we conduct the comparative statics analysis of a LM economy, in which firms face a downward sloping demand curve characterized by a random shift parameter and in which the number of firms in the industry is endogenously determined by free entry and exit and then to investigate the extent to which the results replicate those of a capital economy. Second, we examine how the results obtained by Mai under perfect competition are modified in a monopolistically competitive industry and contrast the results in this paper with the Ishii ones.

This paper considers the fact that the firm faces a downward sloping demand curve because of its monopolistic power and emphasizes that the comparative statics results heavily depend on the shape of the demand function.

II. THE FRAMEWORK OF INDUSTRY EQUILIBRIUM FOR A LM ECONOMY

Consider a monopolistically competitive industry of n Cournot LM firms, each of which produces a homogenous product $q_i(i=1, 2, 3, \cdots, n)$ with a single input labor L_i, together with certain unspecified fixed inputs. The production function is assumed to exhibit non-increasing returns to labor input, i.e.,

$$q=f(L), \qquad \text{where } f'>0 \text{ and } f''\leq 0 \tag{1}$$

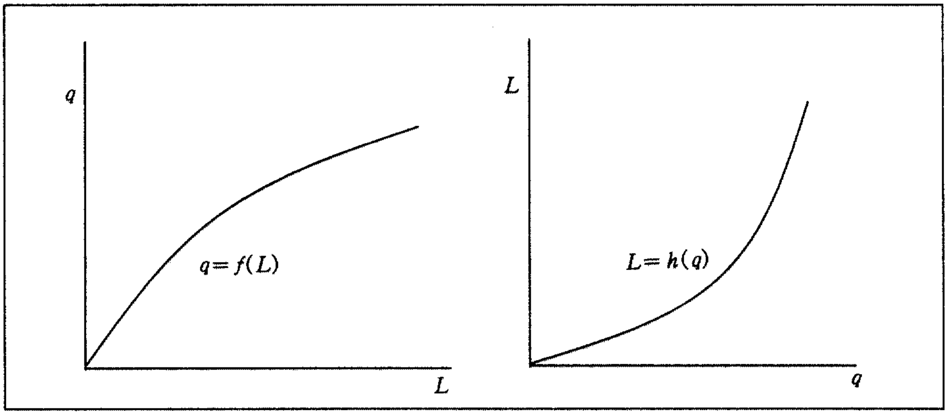
which means that the amount of labor can be described as a function of output, i.e.,

$$L=h(q), \qquad \text{where } h(0)>0,\ h'>0 \text{ and } h''\geq 0. \tag{2}$$

Figure 1 shows the relationship between the equation (1) and (2).

Next, it is here assumed that all firms are identical and symmetrical in the sense that they have the same cost structures, the same demand function, and the same monopolistic power. Following Appelbaum and Katz(1986), it is assumed that the industry demand is stochastic and given by

[Figure 1]



$$P=g(Q)+\gamma\varepsilon^{1}\ , \tag{3}$$

where $Q=\sum_{i=1}^{n} q_i$ is the total industry output, P is the output price for all firms with the characteristics of $P'=g'(Q)=\frac{\partial g}{\partial Q}<0$ and $P''=g''(Q)=\frac{\partial^2 g}{\partial Q^2}\ \substack{\leq \\ >}\ 0$[2]

[1] For more general types of changes in risk, Meyer and Ormiston(1989) defined a simple increase in risk which includes the Sandmo linear transformation as a special case and Ormiston (1992) introduced a more general First-degree stochastic dominant (FSD) improvement. Analyses concerning these general types of changes in risk are beyond the scope of this paper.

[2] See Denike and Parr(1970), and Greenhut, Hwang and Ohta(1975) for market demand curves which are convex to origin, and Dastidar(1996) for market demand curves which are concave to origin.

which is an important role in signing the comparative statics results, which depend on whether the industry demand curve is linear, concave, or convex to origin, ε is a random variable such that $E(\varepsilon)=0$, $E(\varepsilon^2)=1$, and $Prob\left(\varepsilon > -\frac{g(Q)}{\gamma}\right)=1$ and γ is a shift parameter such that $\gamma>0$. Given that all firms are identical and symmetrical, we have

$$Q=\sum_{i=1}^{n} q_i = nq. \tag{4}$$

It is assumed that the number of firms in the monopolistically competitive industry is small enough so that every firm may affect the market price unlike in a perfectly competitive industry. Thus, let each firm's profit be formulated as

$$\pi = P(Q)q - wh(q) - k, \tag{5}$$

where k is fixed costs and w is the competitive wage rate per labor unit in the rest of economy.

Then the profit of firm per unit of labor[3] can be defined by $z=\frac{\pi}{h(q)}$.

The firm is assumed to maximize the expected utility of its profit per labor.

$$E[u(z)] = E\left[u\left\{\frac{P(Q)q - wh(q) - k}{h(q)}\right\}\right], \tag{6}$$

where $u(z)$ is a von Neumann-Morgenstern utility function with $u'(z)>0$ and $u''(z)<0$ which implies that the firm is risk averse. Thus, the first-order condition of a Cournot firm in a monopolistically competitive industry is given by

$$E\left[u'(z)\left\{\frac{(P'q+P-wh')h-(Pq-wh-k)h'}{h^2}\right\}\right]=0. \tag{7}$$

Rewriting (7) gives

$$E\left[u'(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}\right]=0, \tag{8}$$

where $P'<0$ and the second-order condition is given by

[3] Parouch and Kahana(1980), Hill and Waterson(1983), Mai(1993), Okuguchi(1993) and others use the profit per unit labor to analyze the labor-managed firm. On the other hand, Hawawini and Michel(1983), Ireland(1987), Haruna(1988) and others use the (net) income or dividend per worker.

$$E\left[u'(z)\left\{\frac{(P'(h-qh')+P(h'-h'-qh'')+P''qh+P'h+P'qh'+kh'')h^2}{h^4}\right\}\right]$$

$$-E\left[u'(z)\left\{\frac{2hh'(P(h-qh')+P'qh+kh')}{h^4}\right\}\right]$$

$$+E\left[u''(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}^2\right]<0, \tag{9}$$

where the second term of equation (9) is zero by the first-order condition. Rewriting (9) gives

$$E\left[u'(z)\left\{\frac{(P''q+2P')h-(Pq-k)h''}{h^2}\right\}\right]$$

$$+E\left[u''(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}^2\right]<0. \tag{10}$$

Note that in equations (8) and (10), we assumed that $\frac{\partial P}{\partial q}=P'$ and $\frac{\partial^2 P}{\partial q^2}=P''$, since in the Cournot industry each firm maximizes its output as if all other firms would not react to its own action.

Looking at (10), the second-order condition is not always satisfied. Since z is non-negative for a labor-managed firm to be viable, the term $Pq-k$ is positive. There exists the possibility that the first term of left-hand side of (10) is positive and larger than the second term that is negative. Therefore, we assume that first, the industry demand function is linear or concave to the origin. In this case the second-order condition is always satisfied. Second, if the industry demand function is convex to the origin, the second-order condition is not always satisfied. Thus we assume

$$(P''q+2P')h-(Pq-k)h''<0 \quad \text{for } q>0. \tag{11}$$

Then the second-order condition is always satisfied for two cases, and thus the optimal output of the firm is uniquely determined by solving (8), given the value of n .

Moreover, the number of firms in the monopolistically competitive industry is determined by free entry and exit, such that in equilibrium the expected utility of being in the industry is equal to the utility of some benchmark activity R. Thus the industry equilibrium condition of a monopolistically competitive industry is given by

$$E\left[u\left\{\frac{P(Q)q-wh(q)-k}{h(q)}\right\}\right]-R=0. \tag{12}$$

Hence, the system of equation (8) and (12) can be used to solve for the market equilibrium of a monopolistically competitive industry under demand uncertainty, which is defined by the pair (q, n).

To examine the stability of the system, we make usual assumption about the response of the firms and the industry when out of equilibrium. We assume that when $E[u(z)]$ is larger (smaller) than R, some firms enter(leave) the industry and that when $E\left[u'(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}\right]$ is positive(negative), firms increase(decrease) their output. Thus the dynamics of the system is written as

$$\frac{dq}{dt}=\lambda_1 E\left[u'(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}\right], \tag{13a}$$

$$\frac{dn}{dt}=\lambda_2\left\{E\left[u\left\{\frac{P(Q)q-wh(q)-k}{h(q)}\right\}\right]-R\right\}, \tag{13b}$$

where λ_1 and λ_2 are positive adjustment parameters.

The system is locally stable[4] if trace $[D]<0$ and $|D|>0$ hold, where

$$[D]=\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

$$D_{11}=E\left[u'(z)\left\{\frac{(P''q+2P')h-(Pq-k)h''}{h^2}\right\}\right]$$

$$+E\left[u''(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}^2\right]<0 \text{ from (10)},$$

$$D_{12}=E\left[u'(z)\left\{\frac{P''q^2h+P'q(h-qh')}{h^2}\right\}\right]$$

$$+E\left[u''(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^3}\right\}P'q^2\right],$$

$D_{21}=0$ from(8), and

$$D_{22}=E\left[u'(z)\frac{P'q^2}{h}\right]<0 \quad \text{from } P'<0 \text{ and } u'(z)>0 .$$

After a simple calculation,

$$\text{trace }[D]=D_{11}+D_{22}<0, \tag{13c}$$

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[4] For the stability of the system, see Appelbaum and Lim(1982), Appelbaum and Katz(1986) and Ishii(1991).

and $|D| = D_{11} D_{12} > 0$ (13d)

hold locally in a neighborhood of the equilibrium point. Thus the comparative statics results are meaningful at least in the neighborhood of the industry equilibrium point.

III. COMPARATIVE STATICS ANALYSIS

The market equilibrium of a monopolistically competitive industry under demand uncertainty depends on all parameters, which are included in equation (8) and (12). In this section, we analyze the effects of changes in main parameters such as fixed costs, mean demand, demand uncertainty, wage rate and the reservation utility on the output of individual firms (q), on the number of firms in the industry (n), on industry output (Q), and on the expected market price ($E[P]$).

3-1 A Change in Fixed Costs

We investigate the effects of a change in fixed costs on industry equilibrium under demand uncertainty.

Proposition 1:
(a) If the industry demand curve is concave to the origin ($P''<0$), an increase in fixed costs increases the output of individual firms and reduces the number of firms in the industry, but the effects of a change in fixed costs on industry output and on the expected market price are ambiguous.
(b) If the industry demand curve is linear ($P''=0$), the comparative statics results of a change in fixed costs are the same as (a).
(c) If the industry demand curve is convex to the origin ($P''>0$), an increase in fixed costs reduces the number of firms in the industry, but the effects of a change in fixed costs on the output of individual firms, on industry output, and on the expected market price are ambiguous.
The above results are independent of absolute and/or relative risk aversion.

Proof: To conduct the comparative statics analysis, we totally differentiate equation (8) and (12) with respect to k.

$$\begin{bmatrix} D_{11} & D_{12} \\ 0 & D_{22} \end{bmatrix} \begin{bmatrix} \partial q/\partial k \\ \partial n/\partial k \end{bmatrix} = \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} \tag{14}$$

where $A_1 = -E\left[u'(z)\frac{h'}{h^2}\right] + E\left[u''(z)\left\{\frac{P(h-qh') + P'qh + kh'}{h^3}\right\}\right]$, and

$$B_1 = E\left[u'(z)\frac{1}{h}\right].$$

Thus, (14) gives

$$\frac{\partial q}{\partial k} = -\frac{(P''q+P')qh[Eu'(z)]^2}{h^3|D|}.$$

if $P'' \leq 0$, $\partial q/\partial k$ is positive.
if $P'' > 0$, $\partial q/\partial k$ is ambiguous.

$$\frac{\partial n}{\partial k} = \frac{D_{11}Eu'(z)}{h|D|} < 0. \tag{16}$$

The sign of (16) is always negative and does not depend on the shape of the demand function. Next, the effect of a change in fixed costs on industry output $(Q = nq)$ gives

$$\frac{\partial Q}{\partial k} = n\frac{\partial q}{\partial k} + q\frac{\partial n}{\partial k}. \tag{17}$$

From (15) and (16), the sign of (17) is ambiguous. Finally, the expected market price is defined by $E[P] = g(Q)$. Therefore,

$$\frac{\partial E[P]}{\partial k} = g'\frac{\partial Q}{\partial k} = P'\frac{\partial Q}{\partial k}. \tag{18}$$

From (17), the effect of a change in fixed costs on the expected market price is ambiguous.

Q.E.D.

The results of (b) and (c) are similar to those in its capitalist twin as obtained by Ishii(1991) who assumes that the industry demand curve is downward sloping and convex to the origin. However, the above results should be contrasted with Mai's results that in a perfectly competitive industry equilibrium for a LM economy, an increase in fixed costs reduces industry output and the number of firms in the industry, but increases the output of individual firms remaining in the industry. First, when the industry demand curve is linear or concave to the origin ($P'' \leq 0$), we get $\partial q/\partial k > 0$ and $\partial n/\partial k < 0$ from (15) and (16) which are similar to the results shown by Mai. That is, an increase in fixed costs increases the output of individual firms and reduces the number of firms in the industry. However, the signs of $\partial Q/\partial k$ and $\partial E[P]/\partial k$ are still ambiguous. Second, when the industry demand curve is convex to the origin ($P'' > 0$), we obtain $\partial n/\partial k < 0$ from (15) which is similar to the result shown by Mai. But the signs of $\partial q/\partial k$, $\partial Q/\partial k$, and $\partial E[P]/\partial k$ are still ambiguous.

3-2 A Spread-Preserving Increase in Demand

By redefining the industry demand as $P=g(Q)+\gamma\varepsilon+\alpha$[5] in which α is a certain shift parameter, we analyze the effects of a spread-preserving change in random demand on industry equilibrium.

Proposition 2:
(a) If the industry demand curve is concave to the origin $(P''<0)$, a spread-preserving increase in random demand raises the number of firms in the industry and the expected market price, but reduces the output of individual firms. The effect of a spread-preserving increase in random demand on industry output is still ambiguous.
(b) If the industry demand curve is linear $(P''=0)$, a spread-preserving increase in random demand leaves the output of individual firms and the expected market price unchanged, and increases the number of firms in the industry and industry output.
(c) If the industry demand curve is convex to the origin $(P''>0)$, a spread-preserving increase in random demand raises the output of individual firms, the number of firms in the industry and industry output, but reduces the expected market price.
The above results are independent of absolute and/or relative risk aversion.

Proof: Let $P=g(Q)+\gamma\varepsilon+\alpha$.
To conduct the comparative statics analysis, we totally differentiate equation (8) and (12) with respect to α.

$$\begin{bmatrix} D_{11} & D_{12} \\ 0 & D_{22} \end{bmatrix} \begin{bmatrix} \partial q/\partial \alpha \\ \partial n/\partial \alpha \end{bmatrix} = \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} \tag{19}$$

where $A_2=-E\left[u'(z)\frac{(h-qh')}{h^2}\right]-E\left[u''(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}\right]\frac{q}{h}$,

$B_2=-E\left[u'(z)\frac{q}{h}\right]$, and the results are evaluated at $\alpha=0$.

Thus, (19) gives

[5] A referee suggests that this industry demand function can be generalized such as $P=(Q,\ \varepsilon)$ including additive or multiplicative cases, where $\partial P/\partial\varepsilon>0$ and $\partial(\partial P/\partial Q)/\partial\varepsilon>0$. However, this functional form is beyond the scope of this paper.

$$\frac{\partial q}{\partial \alpha} = \frac{P''q^3[Eu'(z)]^2}{h^2|D|} \tag{20}$$

If $P''>0$, $\partial q/\partial \alpha$ is positive.
If $P''=0$, $\partial q/\partial \alpha$ is zero.
If $P''<0$, $\partial q/\partial \alpha$ is negative.

$$\frac{\partial n}{\partial \alpha} = -\frac{qD_{11}Eu'(z)}{h|D|} > 0 \tag{21}$$

The sign of (21) is always positive and does not depend on the shape of the demand function. Next, the effect of a spread-preserving change in random demand on industry output $(Q=nq)$ gives

$$\frac{\partial Q}{\partial \alpha} = n\frac{\partial q}{\partial \alpha} + q\frac{\partial n}{\partial \alpha}. \tag{22}$$

From (20) and (21), the sign of (22) is positive if $P''\geq 0$, but ambiguous if $P''<0$. Finally, the expected market price is defined by $E[P]=g(Q)+\alpha$. Thus

$$\begin{aligned}\frac{\partial E[P]}{\partial \alpha} &= P'\frac{\partial Q}{\partial \alpha} + 1\\ &= P'\left(n\frac{\partial q}{\partial \alpha} + q\frac{\partial n}{\partial \alpha}\right) + 1\\ &= P'n\frac{\partial q}{\partial \alpha} \quad \text{because} \quad P'q\frac{\partial n}{\partial \alpha} = -1\\ &= \frac{P'P''nq^3[Eu'(z)]^2}{h^2|D|}. \end{aligned} \tag{23}$$

If $P''>0$, $\partial E[P]/\partial \alpha$ is negative.
If $P''=0$, $\partial E[P]/\partial \alpha$ is zero.
If $P''<0$, $\partial E[P]/\partial \alpha$ is positive. Q.E.D.

The results of (b) and (c) are similar to those in its capitalist twin as obtained by Ishii. However, the result concerning the individual firm's output should be contrasted with the Mai's result that a spread-preserving change in random demand leaves the output of individual firms unchanged in a perfectly competitive industry. The difference between Mai's result and that in this paper stems from the fact that any firm in a perfectly competitive industry does not affect the market price, but individual firms in a monopolistically competitive industry do affect the market price. The result about the individual firm's output depends on the shape of the demand function, i.e., whether the industry demand curve is concave or convex to the origin. However, if the demand function is linear, even in monopolistically competitive industry model we get the same results as those in a perfectly competitive industry model.

3-3 The Marginal Impact of Uncertainty

Proposition 3:
(a) If the industry demand curve is concave to the origin ($P''<0$), a mean-preserving increase in demand uncertainty increases the output of individual firms and reduces the number of firms in the industry, but the effects of a mean-preserving change in demand uncertainty on industry output and on the expected market price appear to be ambiguous in general.
(b) If the industry demand curve is linear ($P''=0$), the comparative statics results of a mean-preserving change in demand uncertainty are the same as (a).
(c) If the industry demand curve is convex to the origin ($P''>0$), a mean-preserving increase in demand uncertainty decreases the number of firms in the industry, but the effects of a mean-preserving change in demand uncertainty on the output of individual firms, on industry output and on the expected market price appear to be ambiguous in general.
The above results are independent of absolute and/or relative risk aversion.

Proof: Totally differentiating (8) and (10) with respect to γ, we get

$$\begin{bmatrix} D_{11} & D_{12} \\ 0 & D_{22} \end{bmatrix} \begin{bmatrix} \partial q/\partial \gamma \\ \partial n/\partial \gamma \end{bmatrix} = \begin{bmatrix} A_3 \\ B_3 \end{bmatrix} \tag{24}$$

where $A_3 = -E\left[u'(z)\varepsilon\frac{(h-qh')}{h^2}\right] - E\left[u''(z)\varepsilon\frac{q\{P(h-qh')+P'qh+kh'\}}{h^3}\right]$, and
$B_3 = -E\left[u'(z)\varepsilon\frac{q}{h}\right]$.

Thus, from (24), $|D|>0$, $D_{11}<0$ and (A.1)-(A.3) in the Appendix, we obtain

$$\frac{\partial q}{\partial \gamma} = \frac{E[u'(z)\varepsilon]E[u'(z)P''q^3]}{h^2|D|}$$
$$-\frac{P'q^3E[u'(z)]E[u''(z)\{P(h-qh')+P'qh+kh'\}^2]}{h^4\gamma(h-qh')|D|}. \tag{25}$$

If $P''>0$, $\partial q/\partial\gamma$ is ambiguous.
If $P''\leq 0$, $\partial q/\partial\gamma$ is positive.

$$\frac{\partial n}{\partial \gamma} = -\frac{D_{11}E[u'(z)\varepsilon]}{h|D|} < 0. \tag{26}$$

The sign of (26) is always negative and does not depend on the shape of the demand function. Next, the effect of a mean-preserving change in demand uncertainty on industry output ($Q = nq$) gives

$$\frac{\partial Q}{\partial \gamma} = n\frac{\partial q}{\partial \gamma} + q\frac{\partial n}{\partial \gamma}. \tag{27}$$

Form (25) and (26), the sign of (27) is ambiguous. Finally, the expected market price is defined by $E[P] = g(Q)$. Therefore,

$$\frac{\partial E[P]}{\partial \gamma} = P'\frac{\partial Q}{\partial \gamma}. \tag{28}$$

From (27), the effect of a mean-preserving change in demand on the expected market price is ambiguous. Q.E.D.

First, the result should be contrasted with Ishii's result that a mean-preserving increase in demand uncertainty reduces the output of individual firms, the number of firms in the industry, and industry output, but raises the expected market price. This paper shows that its effects on industry output and the expected market price are ambiguous, but its effect on the output of individual firms depends on the shape of the demand function, i.e., if $P''\leq 0$, $\partial q/\partial \gamma$ is positive and if $P''>0$, $\partial q/\partial \gamma$ is ambiguous. Second, the result should be contrasted with Mai's result that a mean-preserving increase in demand uncertainty raises the output of individual firms, but reduces industry output and the number of firms in the industry. If the industry demand function is linear or concave to the origin ($P''\leq 0$), we obtain $\partial q/\partial \gamma > 0$ and $\partial n/\partial \gamma < 0$ from (25) and (26) which are similar to the results shown by Mai. However, the sign of $\partial Q/\partial \gamma$ and $\partial E[P]/\partial \gamma$ are ambiguous. If the industry demand function is convex to the origin ($P''>0$), we get $\partial n/\partial \gamma<0$ from (26) which is similar to the result shown by Mai. But the signs of $\partial q/\partial \gamma$, $\partial Q/\partial \gamma$ and $\partial E[P]/\partial \gamma$ are ambiguous.

3-4 An Increase in Wage Rate

Proposition 4:

(a) If the industry demand curve is concave to the origin ($P''<0$), an increase in wage rate reduces the number of firms in the industry, but the effects of a change in wage rate on the output of individual firms, on industry output, and on the expected market price are ambiguous.

(b) If the industry demand curve is linear ($P''=0$), the comparative statics results of a change in wage rate are the same as (c).

(c) If the industry demand curve is convex to the origin ($P''>0$), an increase in wage rate reduces the output of individual firms, the number of firms in the industry, and industry output, but raises the expected market price.
The above results are independent of absolute and/or relative risk aversion.

Proof: Totally differentiating (8) and (10) with respect to w, we get

$$\begin{bmatrix} D_{11} & D_{12} \\ 0 & D_{22} \end{bmatrix} \begin{bmatrix} \partial q/\partial w \\ \partial n/\partial w \end{bmatrix} = \begin{bmatrix} A_4 \\ B_4 \end{bmatrix} \tag{29}$$

where $$A_4 = E\left[u'(z)\left\{\frac{P(h-qh')+P'qh+kh'}{h^2}\right\}\right], \text{ and}$$

$$B_4 = E[u'(z)].$$

Thus, (29) gives

$$\frac{\partial q}{\partial w} = -\frac{[Eu'(z)]^2\{P''q^2h+P'q(h-qh')\}}{h^2|D|}. \tag{30}$$

If $P''\geq 0$, $\partial q/\partial w$ is positive.
If $P''<0$, $\partial q/\partial w$ is ambiguous.

$$\frac{\partial n}{\partial w} = \frac{D_{11}Eu'(z)}{|D|} < 0. \tag{31}$$

The sign of (31) is always negative and does not depend on the shape of the demand function. Next, the effect of a change in wage rate on industry output ($Q=nq$) gives

$$\frac{\partial Q}{\partial w} = n\frac{\partial q}{\partial w} + q\frac{\partial n}{\partial w}. \tag{32}$$

From (30) and (31), the sign of (32) is negative if $P''\geq 0$, but ambiguous if $P''<0$. Finally, the effect of a change in wage rate on the expected market price gives

$$\frac{\partial E[P]}{\partial w} = P'\frac{\partial Q}{\partial w}. \tag{33}$$

If $P''\geq 0$, $\partial E[P]/\partial w$ is positive.
If $P''<0$, $\partial E[P]/\partial w$ is ambiguous. Q.E.D.

The result should be contrasted with Mai's result that an increase in wage rate reduces the output of individual firms and industry output, but its effect on the number of firms in the industry is ambiguous. This paper shows that its effects on the output of individual firms, on industry output, and on the expected market price depend on the shape of the demand function, but its effect on the number of firms in the industry is judged definitely to be negative in a monopolistically competitive industry regardless of the shape of the demand function.

3-5 An Increase in the Reservation Utility

Proposition 5:
(a) When the industry curve is concave to the origin ($P''<0$), an increase in the reservation utility reduces the number of firms in the industry, but its effects on the output of individual firms, on industry output, and on the expected market price are ambiguous even if non-increasing absolute and/or non-decreasing relative risk aversion are assumed.
(b) When the industry demand curve is linear ($P''=0$), the comparative statics results of a change in the reservation utility are the same as (c).
(c) When the industry demand curve is convex to the origin ($P''>0$), an increase in the reservation utility reduces the output of individual firms and industry output, but raises the expected market price if absolute risk aversion is non-increasing. It reduces the number of firms in the industry regardless of absolute and/or relative risk aversion.

Proof: Totally differentiating (8) and (10) with respect to R, we get

$$\begin{bmatrix} D_{11} & D_{12} \\ 0 & D_{22} \end{bmatrix} \begin{bmatrix} \partial q/\partial R \\ \partial n/\partial R \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{34}$$

Thus, (34) gives

$$\begin{aligned} \frac{\partial q}{\partial R} &= -\frac{D_{12}}{|D|} \\ &= -\frac{E[u'(z)\{P''qh+P'q(h-qh')\}]}{h^2|D|} \\ &\quad + \frac{P'q^2E[R_A u'(z)\{P(h-qh')+P'qh+kh'\}]}{h^3|D|} \end{aligned} \tag{35}$$

where $R_A(=-u''(z)/u'(z))$ is the measure of absolute risk aversion.
The sign of $\partial q/\partial R$ is negative if $P''\geq 0$ and absolute risk aversion is

non-increasing. The sign of $\partial q/\partial R$ is ambiguous if $P''<0$.

$$\frac{\partial n}{\partial R}=\frac{D_{11}}{|D|}<0. \tag{36}$$

The sign of (36) is always negative regardless of absolute and/or relative risk aversion and does not depend on the shape of the demand function. Next, the effect of a change in the reservation utility on industry output $(Q=nq)$ gives

$$\frac{\partial Q}{\partial R}=n\frac{\partial q}{\partial R}+q\frac{\partial n}{\partial R}. \tag{37}$$

From (35) and (36), the sign of (37) is negative if $P''\geq 0$ and absolute risk aversion is non-increasing, but ambiguous if $P''<0$. Finally, the effect of a change in the reservation utility on the expected market price gives

$$\frac{\partial E[P]}{\partial R}=P'\frac{\partial Q}{\partial R}. \tag{38}$$

The sign of (38) is positive if $P''\geq 0$ and absolute risk aversion is non-increasing, but ambiguous if $P''<0$. Q.E.D.

Once again the result should be contrasted with Mai's result that an increase in the reservation utility reduces the output of individual firms if absolute risk aversion is non-increasing, and also reduces industry output regardless of absolute and/or relative risk aversion, but its effect on the number of firms in the industry is ambiguous. There are some differences between Mai's result and that in this paper. First, we can definitely sign its effect on the number of firms in the industry regardless of the shape of the demand function and absolute and/or relative risk aversion, but he could not. Second, we need two assumptions such as $P''\geq 0$ and non-increasing absolute risk aversion to sign its effect on industry output, but he signed it without them.

Ⅳ. CONCLUSION

This paper has taken into consideration a monopolistically competitive LM industry which characterizes the fact that risk averse Cournot firms face a downward sloping demand curve and the number of firms in the industry is determined by free entry and exit. We have analyzed the effects of changes in some main parameters on industry equilibrium under demand uncertainty. The shape of the industry demand function is a very important role in signing the comparative statics results.

As a result, this paper has contrasted the results with the Mai ones in a perfectly competitive LM industry and the Ishii ones in a monopolistically competitive PM industry. First, some results in a perfectly competitive LM industry do not generally carry over to a monopolistically competitive LM industry. One of differences between the results in two industries comes from the fact that depends on whether any LM firm in the industry may affect the market price or not. That is, it depends on the market structure where LM firms operate: any LM firm in a perfectly competitive industry does not affect the market price, but individual LM firms in a monopolistically competitive industry do affect the market price. It is shown that a spread-preserving change in demand uncertainty keeps the output of individual LM firms unchanged in a perfectly competitive industry, but its result in this paper depends on the shape of the demand function. Furthermore, we have shown that some results in this paper do not always equal the Mai ones even if the industry demand function is linear as a special case. However, the effects of a mean-preserving increase in demand uncertainty on the output of individual LM firms and the number of LM firms are similar in two different industries if the industry demand function is linear or concave to the origin.

Second, it has also shown that some results in a monopolistically competitive PM industry do not generally carry over to a LM industry. The difference between Ishii's result and that in this paper stems from the fact that a firm operates in the same industry, but its structure is different. Contrasting to a long-run monopolistically competitive PM industry, LM firms respond to the effect of demand uncertainty on the output of individual firms in an opposite way to PM firms where the demand function is linear. That is, the PM firm responds to a mean-preserving increase in random demand by reducing optimum output so that $\partial q/\partial \gamma<0$. On the other hand, the LM firm has a positive response so that $\partial q/\partial \gamma>0$. However, LM firms respond to a spread-preserving increase in demand uncertainty in a similar way to PM firms because both firms have the same market structure.

APPENDIX

This appendix provides some calculation used in proving the proposition 3.

A. Since $\varepsilon=(P-E[P])/\gamma$ holds from (3), we get the following equations.

$$\begin{aligned}
E[u'(z)\varepsilon] &= E[u'(z)(P-E[P])]/\gamma \\
&= E[u'(z)(P-E[P])(h-qh')]/\gamma(h-qh') \\
&= E[u'(z)\{P(h-qh')+P'qh+kh' \\
&\qquad -(E[P](h-qh')+P'qh+kh')\}]/\gamma(h-qh') \\
&= -\{E[P](h-qh')+P'qh+kh'\}E[u'(z)]/\gamma(h-qh') \text{ from (8).} \qquad \text{(A.1)}
\end{aligned}$$

$$\begin{aligned}
&E[u''(z)\varepsilon\{P(h-qh')+P'qh+kh'\}] \\
&\quad = E[u''(z)(P-E[P])(h-qh')\{P(h-qh')+P'qh+kh'\}]/\gamma(h-qh') \\
&\quad = E[u''(z)\{P(h-qh')+P'qh+kh'-(E[P](h-qh')+P'qh+kh')\} \\
&\qquad\qquad \times\{P(h-qh')+P'qh+kh'\}]/\gamma(h-qh') \\
&= \{E[u''(z)\{P(h-qh')+P'qh+kh'\}^2]-(E[P](h-qh')+P'qh+kh') \\
&\qquad\qquad \times E[u''(z)(P(h-qh')+P'qh+kh')]\}/\gamma(h-qh'). \qquad \text{(A.2)}
\end{aligned}$$

B. First-order condition (8) becomes

$$\begin{aligned}
0 &= E[u'(z)\{P(h-qh')+P'qh+kh'\}] \quad \text{since} \quad h\geq 0 \\
&= E[P(h-qh')+P'qh+kh']E[u'(z)]+Cov(\gamma\varepsilon(h-qh'),u'(z))
\end{aligned}$$

where $Cov(\gamma\varepsilon(h-qh'), u'(z))$ is a covariance between $\gamma\varepsilon(h-qh')$ and $u'(z)$. Since $\partial u'(z)/\partial\varepsilon=\gamma q u''(z)/h<0$ and $\partial\gamma\varepsilon(h-qh')/\partial\varepsilon=\gamma(h-qh')<0$ imply

$Cov(\gamma\varepsilon(h-qh'),u'(z))>0$, we get $E[P(h-qh')+P'qh+kh']<0$. From this result and (A.1), we obtain

$$E[u'(z)\varepsilon]<0. \qquad \text{(A.3)}$$

REFERENCES

Appelbaum, E. and C. Lim, (1982), "Long Run Industry Equilibrium with Uncertainty," *Economics Letters*, 9, 139-145.

Appelbaum, E. and E. Katz, (1986), "Measures of Risk Aversion and Comparative Statics of Industry Equilibrium," *American Economic Review*, 76, 524-529.

Askildsen, J. E. and N. J. Ireland, (1993), "Human Capital, Property Rights, and Labor Managed Firms," *Oxford Economic Papers*, 45, 229-242.

Baron, D., (1970), "Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition," *International Economic Review*, 11, 463-480.

Dastidar, K. G., (1996), "Quantity versus Price in a Homogeneous Product Duopoly," *Bulletin of Economic Research*, 48, 83-91.

Denidke, K. G. and J. B. Parr, (1970), "Production in Space, Spatial Competition, and Restricted Entry," *Journal of Regional Science*, 10, 49-63.

Domar, E., (1966), "The Soviet Collective Firm as a Producer Cooperative," *American Economics Review*, 56, 734-757.

Fama, E. F. and A. B. Laffer, (1972), "The Number of Firms and Competition," *American Economic Review*, 62, 670-640.

Fedar, G., R. Just and A. Schmitz, (1977), "Storage with Price Uncertainty in International Trade," *International Economic Review*, 18, 553-568.

Greenhut, M. L., M. Hwang and H. Ohta, (1975), "Observations on the Shape and Relevance of the Spatial Demand Function," *Econometrica*, 43, 669-682.

Haruna, S., (1988), "Industry Equilibrium with Uncertainty and Labor-Managed Firms," *Economics Letters*, 26, 83-88.

Hawawini, G. and P. A. Michel, (1983), "The Effect of Production Uncertainty on the Labor-Managed Firm," *Journal of Comparative Economics*, 7, 25-42.

Hill, M. and M. Waterson, (1983), "Labor-Managed Cournot Oligopoly and Industry Output," *Journal of Comparative Economics*, 7, 43-51.

Ireland, N. J., (1987), "The Economic Analysis of Labour-Managed Firms," *Bulletin of Economic Research*, 39, 249-272.

Ishii, Y., (1989), "Measures of Risk Aversion and Comparative Statics of Industry Equilibrium : Correction," *American Economic Review*, 79, 285-286.

Ishii, Y., (1991), "On the Theory of Monopolistic Competition under Demand Uncertainty," *Journal of Economics*, 54, 21-32.

Katz, E., (1981), "A Note on a Comparative Statics Theorem for Choice under Risk," *Journal of Economic Theory*, 25, 318-319.

Katz, E., J. Paroush and N. Kahana, (1983), "Price Uncertainty and the Price Discriminating Firm in International Trade," *International Economic Review*, 23, 389-400.

Kraus, M., (1979), "A Comparative Statics Theorem for Choice under Risk," *Journal of Economic Theory*, 21, 510-517.

Leland, H. E., (1972), "Theory of the Firm Facing Uncertain Demand," *American*

Economic Review, 62, 278-291.

Mai C. C., (1993), "The Long-Run Industry Equilibrium of the Labor-Managed Firm under Price Uncertainty," *Rivista Internazionale di Scienze Economiche e Commerciali*, 40, 501-511.

Meade J.E., (1974), "Labor-Managed Firms in Conditions of Imperfect Competition," *Economic Journal*, 84, 817-824.

Meyer, J. and M. B. Ormiston, (1989), "Deterministic Transformations of Random Variables and the Comparative Statics of Risk," *Journal of Risk and Uncertainty*, 2, 179-188.

Neumann, J. von and O. Morgenstern, (1947), *Theory of Games and Economic Behavior,* 2nd ed.. Princeton, N.J., Princeton University Press.

Okuguchi K., (1993), "Comparative Statics for Profit-Maximizing and Labor-managed Cournot Oligopolies," *Managerial and Decision Economics*, 14, 433-444.

Ormiston, M. B., (1992), "First and Second Degree Transformations and Comparative Statics under Uncertainty," *International Economic Review*, 33, 33-44.

Paroush J. and N. Kahana, (1980), "Price Uncertainty and the Cooperative Firm," *American Economic Review*, 70, 212-216.

Pratt, J., (1964), "Risk Aversion in the Small and in the Large, Econometrica, 32, 122-136.

Sandmo A., (1971), "On the Theory of the Competitive Firm under Price Uncertainty," *American Economic Review*, 61, 65-73.

Vanek J., (1970), *The General Theory of Labor-Managed Market* Economies, Ithaca: Cornell University Press.

Ward B., (1958), "The Firm in Illyria: Market Syndicalism," *American Economic Review*, 48, 566-589.