

## CUSTOMER POACHING IN VERTICAL DIFFERENTIATION

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*We analyze the effects of poaching in vertical differentiation to compare with that in horizontal differentiation as in Fudenberg and Tirole (1999). We find second-period prices are lower than the static equilibrium prices as in FT. However, the poacher's price of the high quality product in the second-period is strictly higher than the price of the low quality firm unlike FT. We show that the firm of the low quality product sets the price in the first-period above the static level. The same result as this in FT is only applied to the low quality product in vertical differentiation. We also illustrate that the firm of the high quality product may set the second-period price above, or below, or equal to the price in a static case. The possibility that the poaching price in the first-period may be lower than the static price is similar to the standard switching-cost models. This paper is in the vein that the horizontal differentiation model is a special case of the vertical differentiation one. (Gremer and Thisse, 1991)*

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### I. INTRODUCTION

In many markets of homogeneous and differentiated products, firms have strong incentive to poach the rival firm's customers. To switch to the other firm, customers are usually induced by various tools such as the repeat-purchase discounts, one-time bonuses and so on. To poach the customers of the opposing

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firm's, a firm should know the past history of potential customers' buying patterns.

In the market of the mobile phone service, one service provider may try to poach some of the potential customers of their competitors' by exempting the fixed fee for those who once paid it to the rival firm while no exemption is made for the new customers to this market. Despite that no fixed fee is paid to the switched firm, switching customers must bear switching costs such as the cost of informing his or her new phone numbers to others.

When consumers switch to the other firm, switching costs incur. Before the emergence of the "poaching" theory, many papers have analyzed the effects of switching costs. Introducing three different kinds of switching costs (transaction costs, learning costs, and artificial or contractual costs), Klemperer (1987a) shows that switching costs make demand more inelastic and so reduce rivalry.<sup>1</sup> He also shows that firms set the price lower than that in static level in the first-period, then, raise their prices in the second-period.

Carminal and Matutes (1990) consider i.i.d. preferences to analyze the market with artificial switching costs and show that the first-period price can exceed the second-period price. Chen (1997) considers a market with real costs from switching, but firms that can artificially reduce the switching costs borne by consumers. He shows that market shares do not affect equilibrium prices and that firms set price below static levels in the first-period, and then raise their prices in the second-period. Kim and Koh (forthcoming) consider a firm that removes strategically switching costs created artificially by its rivals such as firm's honoring its rivals' discount coupons. Using a location model with both newcomers and old consumers, they show that the firm with a lower market share honors the rival's coupon until its market share reaches up to one half. They also show that the small firm's incentive to honor the rival's coupons is positively related with the proportion of new consumers, and that its incentive to honor them becomes stronger with its market share. They also provided a result that once the smaller firm honors the coupons by the larger firm, the larger firm benefits more than the smaller firm.

Recently, Fudenberg and Tirole (2000) (FT, hereafter) introduce the conceptual issues, and the nature of equilibrium of the "poaching" theory. They consider a simple Hotelling model of duopoly with horizontal differentiation. Consumers, being assumed to be fully rational, take the second-period poaching into account when making their first-period decisions. Under the scheme that firms lack commitment power under short-term contracts, they show that first-period demand becomes less elastic than it would be if price discrimination were banned, and first-period prices set to be higher than those of the static model. They, then, consider the effects of binding long-term contracts in the first-period.

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<sup>1</sup> The first two switching costs are called "real" and the third "artificial" switching costs. Also, refer to the Klemperer's many other works (for example, 1987b and 1995) on switching costs.

FT (2000) cited above is closely related with their previous article Upgrades, Trade-ins, and Buybacks (1998). FT (1998) studies monopolist's price discrimination for two-period with quality discrimination. FT (1998) considers three different information structures (identified, semi-anonymous, and anonymous), and two sorts of goods (those with an active secondhand market and anonymous consumers, and those with no secondhand market and consumers who purchased the old good to qualify for a discount on the new one). Even though this paper is similar with FT (1998) in that price discrimination is observationally based and two-period model. This paper is a duopoly one with non-durable goods of different quality (or vertical differentiation) while FT (1998) is a monopoly model with durable goods. Levinthal and Purohit (1989) consider monopolist's inter-temporal price discrimination for durable goods in the anonymous case. They find that the firm's optimal policy is to phase out sales of the old product for the modest product upgrading, while a buy-back is optimal for large upgrading. Levinthal and Purohit (1989) differ from FT (1998) in a way that an increase in the new generation's output has an equal or unequal impact on the rental prices of the two products. Mussa and Rosen (1978) consider monopoly pricing, to compare pricing under competition, on a quality-differentiated spectrum of goods with a one-period model. They show that the monopolist enlarges the quality spectrum and uses lower-quality goods as a market-segmentation technique. They also illustrate that for any quality level ( $q^h > 0$ ) sold both by the monopolist and under competition, the monopoly price is larger than the competitive price. (p. 315)

This paper is motivated to study the equilibrium and effects of poaching in vertical differentiation while most studies on the switching-cost and poaching models are performed in horizontal differentiation. We introduce the high and low quality products for vertical differentiation. One firm produces the high quality product, and the other firm the low quality product. The quality level is exogenously given. In the first-period, consumers buy one unit of one product out of two qualities based on their preference. In the second-period, poaching occurs such that each firm offers the second-period price to its own past customers as well as to the rival firm's buyers. This paper follows the similar steps as done in FT (2000), and compares the results in vertical differentiation with those in FT (2000) only under short-term contracts.

While FT's results are symmetry in equilibrium prices and the size of switchers from the first-period to the next, this paper illustrates asymmetry in many aspects. FT show that the demand becomes less elastic than when price discrimination is banned or in the static case, thus, making the first-period equilibrium price higher than that in the static model, and the second-period price lower. This result can be compared with the one of the typical switching-cost models that firms price below the static levels in the first-period, then raise the price in the second-period. We show in this paper that the first-period equilibrium price of the low quality product is above the price in

the static case under vertical differentiation, then below it in the second-period, which resembles the FT's. The first-period equilibrium price of the high quality product, however, may be above, below, or equal to the price in the static case. The second-period prices of both of the high and low quality products are lower than those in the static model.

Section II introduces the basic model for vertical differentiation, and derives static equilibrium for a benchmark. In section III, we derive the equilibrium in the first- and second-period to compare it with FT's results. Concluding remarks follow in section IV

## II. MODEL

FT consider a simple Hotelling model of duopoly with horizontal differentiation. Two firms, A and B, produce goods A and B, respectively, with same constant cost of  $C$ . Each period consumers buy either a unit of good A or B, and the market is assumed covered. Consumer preferences are indexed by  $\theta \in [\underline{\theta}, \bar{\theta}]$ , where  $\underline{\theta} = -\bar{\theta} < 0$ , and  $\theta$  is a measure of the consumer's relative preferences for B over A. There is a known cumulative distribution function  $F$  over  $\theta$ . Customer's utility is  $u_t(\theta) = v - \theta/2 - p_t$ , if he buys good A, and  $u_t(\theta) = v + \theta/2 - p_t$ , if he buys good B for a given price of  $p_t$ , at date  $t$ . The discount factor  $\delta$  is assumed to be identical to all the agents.

In the first period, each firm offers a single first-period price  $p_a$  and  $p_b$ . Consumers with  $\theta$  greater than some cutoff  $\theta^* = p_b - p_a$  buy from firm B, and others buy from firm A. Customer poaching occurs only in the second period. Firm A (B) can offer the second-period price of  $P_a(P_b)$  to its own past customers, and  $P_A(P_B)$  to those who purchase from the rival.

Below is introduced a model for vertical differentiation. We assume there are two firms producing two products of different qualities. Quality is exogenously given and is indexed as  $S_i$ ,  $i = a, b$ .  $S_a < S_b$  is assumed.<sup>2</sup> Firm A (B) produces low (high) quality goods at the same constant cost of  $c$ . In this section, we introduce a static model as a benchmark case. Firm B is thus the top quality firm. Prices offered by firms A and B are  $P_a$ , and  $P_b$ , respectively.

Consumers buy either a unit of good A or B. Consumer preferences are as follows:  $U(\theta) = \theta S_b - P_b$  when consuming a product of quality  $S_b$  at a price  $P_b$ ,  $U(\theta) = \theta S_a - P_a$ , when consuming a product of quality  $S_a$  at a price  $P_a$  and his utility is zero when not buying either of the products.<sup>3</sup> The population of consumers is described by the parameter  $\theta$ , which has distribution function  $F$

<sup>2</sup> In modeling, the introduction of the quality variable in the vertical differentiation is one of the most distinguishing points from the horizontal differentiation.

<sup>3</sup> See Tirole (1988) for a detail.

over  $\theta$ . (See FT (1999)) for details on the distribution function,  $F$ ) In this paper, the parameter is assumed to be uniformly distributed between  $\underline{\theta}$  and  $\bar{\theta}$  with  $0 < \underline{\theta} < \bar{\theta}$ .

Demand for good  $i$  is derived by the set of consumers who maximize utility when buying product  $i$ . Given  $P_a$ , and  $P_b$ , the marginal consumer who is indifferent between consuming either of the two products is denoted by  $\theta^*(P_a, P_b)$  such that

$$\theta^*(P_a, P_b)S_a - P_a = \theta^*(P_a, P_b)S_b - P_b. \tag{1}$$

Those consumers with  $\theta > \theta^* = \frac{P_b - P_a}{S_b - S_a}$  buys the product from firm B, and consumers with types smaller than  $\theta^*$  purchase the product from firm A. Accordingly, demand for good  $i$  is derived as follows:<sup>4</sup>

The demand for good A,  $D_A^i$ , becomes  $\frac{(P_a - P_b)\Delta S - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$ , and the demand for good B,  $D_B^i$ ,  $1 - \frac{(P_a - P_b)\Delta S - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$ . The Bertrand-Nash static equilibrium in this model is given by:  $P_a^s = \frac{\bar{\theta} - 2\underline{\theta}}{3\Delta S} + c$ , and  $P_b^s = \frac{2\bar{\theta} - \underline{\theta}}{3\Delta S} + c$ , where  $\Delta S = \frac{1}{S_b - S_a}$ . For the market to be covered and positive sales by two firms,  $0 < 2\underline{\theta} < \bar{\theta}$  should be satisfied.<sup>5</sup>

The following lemma can be established:

**Lemma:** The static equilibrium obtained in FT is a special case of vertical differentiation in this paper. In particular, the static price of the top quality firm,  $P_b^s$ , is higher than that ( $P_a^s$ ) of the low quality firm.

**Proof:** When  $S_a = -1/2$  and  $S_a = +1/2$ , it makes  $\Delta S = 1$ , and the utility function in our model is virtually same as that in FT. When  $\Delta S = 1$  and  $\bar{\theta} = -\underline{\theta}$  are substituted into  $P_a^s$  and  $P_b^s$ , the static equilibrium in this paper becomes that for the horizontal differentiation in FT such as  $P^A = P^B = \bar{\theta} + c$ . Otherwise, the static equilibrium for vertical differentiation in our model is different from that for horizontal differentiation in FT. It is immediate  $P_b^s - P_a^s = \frac{\bar{\theta} + \underline{\theta}}{3\Delta S} > 0$  for  $\Delta S > 0$   $0 < 2\underline{\theta} < \bar{\theta}$  in this paper.

This lemma supports the main result of Cremer and Thisse (1991) that every model belonging to a very large class of Hotelling-type models is actually a

<sup>4</sup> The market is assumed to be covered.

<sup>5</sup> An anonymous referee points out that this condition is required for two firms to be active in price game.

special case of a vertical product differentiation model.

### III. ANALYSIS

With the introduction of the quality variable for the vertical differentiation, we now introduce price discrimination in the second-period as in FT. The simple poaching model considers the case where firms observe the first-period behaviors of consumers, but only short-term contracts are allowed, and firms cannot commit themselves in the first-period to behavior in the second period in this paper.

Prices offered by firms A and B are  $P_a$  and  $P_b$ , respectively in the first period. As in the static equilibrium, consumers with  $\theta > (<) \theta^*$  would buy good B (A) in the first-period. In the second period, firms A and B charge  $P_a$ , and  $P_b$  to their own past customers, and  $P_A$  and  $P_B$  to the past customers of their rival. Marginal cost is assumed to keep the same in both periods. The firms maximize the expected discounted profits in two periods, with common discount factor,  $\delta$  ( $< 1$ ). We solve this problem with the backward induction.

#### 1. Second-Period equilibrium

In the first-period, customers to the left of  $\theta^*$  will be in the firm A's turf, and those to the right of  $\theta^*$  will be in the firm B's turf. In the second-period both firms try to make some of their rival's first period buyers switch to their product from the opposing firm's. Types in  $[\underline{\theta}, \theta^A]$  are the first-period customers to firm A and are still loyal to it even in the second period. Types in  $[\theta^A, \theta^*]$  are the first-period customers to firm A and are the ones who are willing to switch to firm B in the second period. By the same token, types in  $[\theta^B, \bar{\theta}]$  are the first-period customers to firm B and are still loyal to it even in the second period. Types in  $[\theta^*, \theta^B]$  are the first-period customers to firm B and are the ones who like to switch to firm A in the second period. The solution concept for the second-period is as follows: we have simultaneous game between firms A and B in A's turf and between both firms in the B's turf. In other words, we divide the first-period market into two in order to analyze them separately.

##### 1) Game in the A's turf

This sub-section analyses the game in A's turf, the next sub-section the game in the B's turf. For the old customers purchased in the first period, firm A offers  $P_a$ , and firm B  $P_b$ . Then among old customers in A's turf (i.e. in  $[\underline{\theta}, \theta^*]$ ), those with  $\theta < \theta^A = \frac{P_b - P_a}{S_b - S_a}$  are the loyalists to the firm A in the second-period, while those with  $\theta > \theta^A$  are the switchers to firm B in the second-period. The profit maximization in the second-period in the A's turf is as

follows:

$$\text{Max}_{P_a} \Pi_{2A}^A = (P_a - c) \frac{(P_B - P_a) \Delta S - \theta}{\theta - \theta} \tag{2}$$

$$\text{Max}_{P_b} \Pi_{2A}^B = (P_B - c) \frac{\theta^* - (P_B - P_a) \Delta S}{\theta - \theta}, \tag{3}$$

where  $\Delta S = \frac{1}{S_b - S_a}$ .

Assuming an interior solution, the Nash-equilibrium is  $P_a = \frac{\theta^* - 2\theta}{3\Delta S} + c$  and  $P_B = \frac{2\theta^* - \theta}{3\Delta S} + c$ .

Proposition 1: With an interior solution in the A's turf,

- 1) Both firms' second-period prices are lower than they are in static equilibrium prices in vertical differentiation.
- 2) The poacher's price is strictly higher than the incumbent's.
- 3) The incumbent's (i.e. firm A's) market share becomes smaller than the poacher's (i.e. firm B's).

The proof is in the appendix.

We compare these results with the FT's. The result of 1) in the Proposition 1 is similar to the FT's, while it is reverse of those in models with switching costs.<sup>6</sup> One reason that both firms' second-period prices are lower in vertical differentiation is that the differentiation in consumer preference in the second period becomes smaller than that in the static model. Consumer preference in the static model is uniformly distributed on  $\underline{\theta}$  and  $\bar{\theta}$ . Consumer preference in the second period in vertical differentiation is uniformly distributed on  $\underline{\theta}$  and  $\theta^*$  in the firm A's turf, and on  $\theta^*$  and  $\bar{\theta}$  in the firm B's turf, where  $\theta^*$  exists somewhere between  $\underline{\theta}$  and  $\bar{\theta}$ . Since each firm, which competes in each firm's turf, faces a market less vertically differentiated than that in the static model, market price in the second period becomes lower than that in the static model.<sup>7</sup>

The result of 2) above is reverse of that in FT. This is a bit surprising result because in general the poacher sets his price lower than the incumbent's. The producer of the high quality good (firm B), who sets higher price, however, could poach some of those buyers of the low quality product, whose tastes are close to the high quality. That is, even though they were firm A's customers for the low quality product at the low price in the first-period, some customers

<sup>6</sup> Refer to Klemperer (1987a) for example.

<sup>7</sup> I am grateful to an anonymous referee who provides this interpretation.

who have cherished for the high quality product now shy away from the good that they purchased before. They switch to the high quality product at the higher price offered by firm B in the second-period. They know that this price is higher than the low quality product they purchased before, but that it is lower than the price offered to those who used this high quality product before. Those consumers who evaluate switching to make them better-off actually switch the brand from firm A to B.

Although FT claim that the incumbent market share is strictly larger than the poacher, the former in this model becomes smaller than the latter as shown in Proof for 3) in appendix. In the second period, increasing number of consumers who once purchased before now churn to the high quality product, making the low quality firm's (the incumbent's) market share fall. We provide more clear-cut picture on churning in 2) of Proposition 5.

2) Game in the B's turf

Since the solution concept in this sub-section is exactly same as in the game in the A's turf, we introduce here the set-up of the objective functions. For the old customers purchased in the first period, firm B offers  $P_\beta$  and firm A  $P_A$ . Then among old customers in B's turf (i.e. in  $[\theta^*, \theta]$ ), those with  $\theta > \theta^B = \frac{P_\beta - P_A}{S_b - S_a}$  are the loyalists to the firm B in the second-period while those with  $\theta^* < \theta < \theta^B$  are the switchers to firm A in the second-period. The profit maximization in the second-period in the B's turf is as follows:

$$Max_{P_A} \Pi_{2B}^A = (P_A - c) \frac{(P_\beta - P_A)\Delta S - \theta^*}{\bar{\theta} - \underline{\theta}} \tag{4}$$

$$Max_{P_\beta} \Pi_{2B}^B = (P_\beta - c) \frac{\bar{\theta} - (P_\beta - P_A)\Delta S}{\theta - \bar{\theta}} \tag{5}$$

Assuming an interior solution, the Nash-equilibrium is  $P_A = \frac{\bar{\theta} - 2\theta^*}{3\Delta S} + c$  and  $P_\beta = \frac{2\bar{\theta} - \theta^*}{3\Delta S} + c$ .

Proposition 2: with an interior solution in the B's turf,

- 1) Both firms' second-period prices are lower than they are in static equilibrium prices in vertical differentiation.
- 2) The poacher's price is strictly lower than the incumbent's.
- 3) The incumbent's (i.e. firm B's) market share becomes larger than the poacher's (i.e. firm A's).

The proof is omitted.

We get the similar results for the B's turf as in the A's turf except the

result of 2) in Proposition 1. The result of 2) above is similar to FT's. Some of customers who purchased the high quality product in the first-period, but who were close to the low quality like to switch to the low quality at the lower price of the poacher in the second-period. The result of 3) implies that the low quality firm does not attract many users of the past high quality product.

We also find other result as follows:

Proposition 3: Each firm sets the second-period price lower to the past customers of the opposing firm than to its own customers (i.e.  $P_\beta > P_B$ , and  $P_a > P_A$ ) if  $\frac{\bar{\theta} + 2\theta}{3} < \theta^* < \frac{2\bar{\theta} + \theta}{3}$ .

The proof is omitted. This result is same as FT's.

## II. FIRST-PERIOD EQUILIBRIUM

As FT assume that "firms have no commitment power," firms take into account in setting first-period equilibrium the fact that the first-period outcomes affect second-period equilibrium. Prices offered by firms A and B in the first period are  $P_a$  and  $P_b$ , respectively. Given  $P_a$  and  $P_b$ , the marginal consumer who is indifferent between consuming either of the two products is denoted by such that  $\theta^* = \left(\frac{1}{3+\delta}\right)(\delta(\bar{\theta} + \theta) + 3\Delta S(P_b - P_a))$ , where  $\Delta S = \frac{1}{S_b - S_a}$ . Since  $\frac{\partial \theta^*}{\partial P_a} = \frac{-3}{3+\delta}$  in FT, the first period demand is less elastic than in the static case. In this paper, however, we have  $\left|\frac{\partial \theta^*}{\partial P_i}\right| = \frac{3}{3+\delta} \Delta S$ , for  $i = a, b$ . Since  $\Delta S = \frac{1}{S_b - S_a}$ , and  $S_a < S_b$ , we may have  $\Delta S$  is greater than, smaller than, or equal to one, which may result in the first-period demand even more elastic (not less elastic).

Firm A's (B's) discounted profit to be maximized in the first-period with no-commitment power,  $\Pi^A(\Pi^B)$  becomes as follows:

$$\begin{aligned} \text{Max}_{P_a} \Pi^A(P_a, P_b) = & (P_a - c)F(\theta^*(P_a, P_b)) + \delta[(P_a(\theta^*(P_a, P_b)) - c) \\ & F(\theta^A(\theta^*(P_a, P_b))) + (P_A(\theta^*(P_a, P_b)) - c) \\ & (F(\theta^B(\theta^*(P_a, P_b))) - F(\theta^*(P_a, P_b)))] \text{, and} \end{aligned} \tag{6}$$

$$\begin{aligned} \text{Max}_{P_b} \Pi^B(P_a, P_b) = & (P_b - c)(1 - F(\theta^*(P_a, P_b))) + \delta[(P_b(\theta^*(P_a, P_b)) - c) \\ & (1 - F(\theta^B(\theta^*(P_a, P_b)))) + (P_B(\theta^*(P_a, P_b)) - c) \\ & (F(\theta^*(P_a, P_b)) - F(\theta^A(\theta^*(P_a, P_b))))] \end{aligned} \tag{7}$$

In equation (6), the first term is a profit earned by the firm A in the first-period, the first term in the bracket the second-period profit from the old customers of its own ( $\Pi_{2A}^A$ ), and the second term in the bracket the

second-period profit from the customer switching from firm B ( $\Pi_{2B}^A$ ). Terms in equation (7) have a similar interpretation as in equation (6). The first term is a profit earned by the firm B in the first-period, the first term in the bracket shows the second-period profit from the old customers of its own ( $\Pi_{2B}^B$ ), and the second term in the bracket is the second-period profit from the customer switching from the firm B ( $\Pi_{2A}^B$ ). Solution for equations (6) and (7) gives the equilibrium prices of  $P_a^*$  and  $P_b^*$ , and we have  $P_a^* < P_b^*$ .<sup>8</sup> The second-order condition for the system is satisfied for  $0 < \delta < 1$ .<sup>9</sup>

This paper shows an interesting result as follows.

**Proposition 4:** The firm of low quality product (the firm A's) sets the price in the first-period above the static level, while that of the high quality product (the firm B's) may set the first-period price above, below, or equal to the price in a static case.

**Proof:**<sup>10</sup> We have from the solutions that  $P_a^* - P_a^s = \sigma \theta \left( \frac{\bar{\theta}}{\theta} - \frac{1}{\phi} \right)$ , and  $P_b^* - P_b^s = \tau \theta \left( \frac{\bar{\theta}}{\theta} - \phi \right)$ , where  $\sigma = \frac{10\delta(\delta-2)}{3(-27+11\delta)} > 0$ , and  $\tau = \frac{\delta(\delta-7)}{3(-27+11\delta)} > 0$  for  $0 < \delta < 1$ .  $\phi = \frac{10(\delta-2)}{\delta-7}$  is monotonically decreasing between about  $\frac{5}{3}$  and  $\frac{20}{7}$  (more specifically,  $2 \leq \phi < \frac{20}{7}$  for  $0 < \delta \leq \frac{3}{4}$  and  $\frac{5}{3} < \phi \leq 2$  for  $\frac{3}{4} \leq \delta < 1$ ), and  $1/\phi$  is monotonically increasing between about  $\frac{7}{20}$  and  $\frac{3}{5}$  for  $0 < \delta < 1$ . For firms to be active in price game, we have a required condition that  $0 < 2\theta \leq \bar{\theta}$ , which implies that  $2 \leq \frac{\bar{\theta}}{\theta}$ . Since  $\frac{7}{20} < \frac{1}{\phi} < \frac{3}{5}$ , we have  $P_a^* - P_a^s > 0$ . Thus, the first-period price of the low quality product is higher than that in the static case. However, the sign of  $P_b^* - P_b^s$  is not unambiguously determined. When  $\frac{5}{3} < \phi < 2$  for  $\frac{3}{4} \leq \delta < 1$ ,  $P_b^* - P_b^s = \tau \theta \left( \frac{\bar{\theta}}{\theta} - \phi \right) > 0$  because it requires a condition of  $\frac{\bar{\theta}}{\theta} \geq 2$ . When  $2 \leq \phi < \frac{20}{7}$ , the sign of  $P_b^* - P_b^s$  may be positive, negative, or equal to zero with a condition of  $\frac{\bar{\theta}}{\theta} \geq 2$ .<sup>11</sup> If  $\frac{\bar{\theta}}{\theta} \geq \frac{20}{7}$ ,  $P_b^* - P_b^s = \tau \theta \left( \frac{\bar{\theta}}{\theta} - \phi \right) > 0$  for  $\frac{5}{3} < \phi < \frac{20}{7}$ . We summarize the sign of  $P_i^* - P_i^s (i = a, b)$  as follows:

<sup>8</sup> See the appendix.

<sup>9</sup> See the appendix.

<sup>10</sup> I thank to an anonymous referee's criticisms on an earlier version, which prompted me to have more complete proof to Proposition 4.

<sup>11</sup> For example, suppose  $\bar{\theta} = 5$  and  $\theta = 2$ , satisfying the required condition of  $0 < 2\theta \leq \bar{\theta}$ . If  $\phi = 2.5$ ,  $P_b^* - P_b^s = 0$ . If  $\phi = 2.7$ ,  $P_b^* - P_b^s < 0$ .

$$\begin{array}{lll}
 & 2 \leq \frac{\bar{\theta}}{\theta} \leq \frac{20}{7} & \frac{\bar{\theta}}{\theta} \geq \frac{20}{7} \\
 \frac{7}{20} < \frac{1}{\phi} < \frac{3}{5} & P_a^* - P_a^s > 0 & P_a^* - P_a^s > 0 \\
 \frac{5}{3} < \phi < 2 & P_b^* - P_b^s > 0 & P_b^* - P_b^s > 0 \\
 2 \leq \phi < \frac{20}{7} & P_b^* - P_b^s \leq 0 & P_b^* - P_b^s > 0
 \end{array}$$

This is one of surprising results in this paper compared with the poaching and the standard switching-cost models. In Proposition 2 in FT (1999), they show that the equilibrium price of the first-period is symmetric in which  $a = b = (1 + \delta/3)\theta + c$ , as opposed to  $P^A = P^B = \theta + c$  in the static case (or when price discrimination is banned). Thus, the first-period equilibrium price is above that in the static case. FT assert that poaching prices are above the static levels in the first period and below them in the second, while in the typical switching cost theory, prices are undercut static levels in the first period and then the second-period prices are raised.

In vertical differentiation in this paper, it is mixed and conditional. The poaching price of the low quality product is always higher than the static level. However, the first-period equilibrium price of the high quality product can be higher than (lower than, or equal to) the static price. The plausibility that the poaching price of the high quality product in the first period may be lower than the price in a static model is a new finding in this paper, and this is similar to the traditional switching-cost model. Given domain of  $\phi$  (and of  $1/\phi$ ),  $\bar{\theta}$  is never smaller than  $\theta/\phi$ , i.e.  $P_a^*$  is never smaller than  $P_a^s$ . This does imply that the first-period price of the low quality product is always higher than that in the static case. For the high quality product, the poaching price can be higher than, lower than, or equal to the static price. A result of FT that poaching price is higher than that in the static case is only a special case, which is applied only to the low quality product. In vertical differentiation model, poaching equilibrium prices are turned out to be of variety as in this paper.

FT shows 1/3 of all consumers switch brands from one period to the next, and this does not depend upon the value of the discount rate. Different results on the size of switching entail in this paper as follows:

Proposition 5:

- 1) It is undetermined in this model what proportion of consumers switch brands from the first-period to the second-period albeit a positive proportion of consumer switches.
- 2) In the second-period, the size of switchers from firm A to B is larger than those from firm B to A.

Proof: For 1), let  $X = \theta^* - \theta^A$  be those consumers who purchased a good from firm A in the first period, but switched the brand to firm B in the second

period. Let  $Y = \theta^B - \theta^*$  be those consumers who purchased a good from firm B in the first period, but switched to firm A in the second period. We have  $\theta^A = (\theta^* + \underline{\theta})/3$ , and  $\theta^B = (\bar{\theta} + \theta^*)/3$  from section III, and  $\theta^* = \left(\frac{2\delta - 9}{11\delta - 27}\right) (\bar{\theta} + \underline{\theta})$ . Then we have  $X = \frac{2(2\delta - 9)\bar{\theta} + (9 - 7\delta)\underline{\theta}}{3(11\delta - 27)}$ , and  $Y = \frac{-(9 - 7\delta)\bar{\theta} - 2(2\delta - 9)\underline{\theta}}{3(11\delta - 27)}$ . If  $\underline{\theta} = -\bar{\theta}$ ,  $X = Y = 1/3$  as in FT. If not, the size of X and Y is hard to say although X and Y is positive. For 2), it is immediate that  $X - Y = \left(\frac{-(\delta + 3)}{11\delta - 27}\right) (\bar{\theta} + \underline{\theta}) > 0$  for  $0 < \delta < 1$ .

It is well known that switching does not actually occur in many standard switching-cost models. Poaching makes switching actually occur in this paper as in FT. The size of switching is symmetric in FT. This paper, however, shows that is not necessary and that the size of switchers could be asymmetric. The FT's result that one third of consumers is switching from one period to the next is only a special case in this paper.

#### IV. CONCLUDING REMARKS

This paper analyzes the effects of poaching under the short-term contract a la Fudenberg and Tirole (2000). Our purpose is to compare the equilibrium and effects of poaching in vertical differentiation with horizontal differentiation. The analysis in this paper is limited to short-term contracts where firms lack commitment power.

We show that the static equilibrium obtained in FT is a special case of vertical differentiation in this paper. We find asymmetry in equilibrium unlike FT's. The second-period prices of both firms' are lower than they are in static equilibrium in vertical differentiation as in FT. However, in the second-period the poacher's price of the high quality product is strictly higher than the price of the incumbent of the low quality firm, and the poacher's price of the low quality product is strictly lower than the price of the incumbent of the high quality product. This is a different result from the FT's showing that the poacher's price is strictly lower than the incumbent's. We also obtain that each firm sets the second-period price lower to the past customers of the opposing firm than to its own customers. We show in this paper that the firm of the low quality product (the firm A's) sets the price in the first-period above the static level. The same result in FT is only applied to the low quality product in vertical differentiation. We illustrate that the firm of the high quality product (the firm B's) may set the second-period price above, below, or equal to the price in a static case. The possibility that the first-period price of the high quality product may be lower than the static price is different from FT's, but similar to the standard switching-cost models. The model of vertical differentiation in this paper comprises results of both poaching and the standard switching-cost theories all in horizontal differentiation.

**Appendix**

Proof of Proposition 1

Proof for 1)

From equations (2) and (3) we get  $P_a = \frac{\theta^* - 2\theta}{3\Delta S} + c$  and  $P_B = \frac{2\theta^* - \theta}{3\Delta S} + c$ . Meanwhile, the prices in the static equilibrium are  $P_a^s = \frac{\bar{\theta} - 2\theta}{3\Delta S} + c$  and  $P_b^s = \frac{2\bar{\theta} - \theta}{3\Delta S} + c$ . Thus,  $P_a - P_a^s < 0$ , and  $P_B - P_b^s < 0$ .

Proof for 2)

It is immediate that  $P_B - P_a = \frac{\theta^* + \theta}{3\Delta S} > 0$ .

Proof for 3)

From equations (2) and (3), sales by firm A ( $Q_{2A}^A$ ) and sales by firm B ( $Q_{2A}^B$ ) in the A's turf become  $Q_{2A}^A = \frac{(\theta^* - 2\theta)}{3(\bar{\theta} - \theta)}$ , and  $Q_{2A}^B = \frac{(2\theta^* - \theta)}{3(\bar{\theta} - \theta)}$ . Thus, we have the sign of  $Q_{2A}^A - Q_{2A}^B = \frac{-(\theta^* + \theta)}{3(\bar{\theta} - \theta)} < 0$ .

Solutions for  $P_a^*$  and  $P_b^*$ :

Equations (6) and (7) become as follows:

$$\begin{aligned} \Pi^A &= (P_a - c) \left( \frac{\theta^* - \theta}{\bar{\theta} - \theta} \right) + \frac{\delta((\theta^* - 2\theta)^2 + (\bar{\theta} - 2\theta^*)^2)}{9(\bar{\theta} - \theta)\Delta S} \quad \text{and} \\ \Pi^B &= (P_b - c) \left( \frac{\bar{\theta} - \theta^*}{\bar{\theta} - \theta} \right) + \frac{\delta((2\bar{\theta} - \theta^*)^2 + (2\bar{\theta}^* - \theta)^2)}{9(\bar{\theta} - \theta)\Delta S}, \quad \text{where} \\ \theta^* &= \frac{\delta(\bar{\theta} + \theta) + 3\Delta S(P_b - P_a)}{3 + \delta}. \end{aligned}$$

Solving the first-order conditions,  $\frac{\partial \Pi^A}{\partial P_a} = 0$  and  $\frac{\partial \Pi^B}{\partial P_b} = 0$ , gives us:

$$\begin{aligned} P_a^* &= \frac{E}{3(27 - 11\delta)\Delta S} + c, \quad \text{and} \quad P_b^* = \frac{G}{3(27 - 11\delta)\Delta S} + c, \quad \text{where} \\ E &= 6\delta(2 - \delta)(\bar{\theta} + \theta) + 2(9 - 2\delta)(\delta\bar{\theta} - 3\theta) + (9 - 7\delta)(3\bar{\theta} - \delta\theta) \quad \text{and} \\ G &= -6\delta(2 - \delta)(\bar{\theta} + \theta) + (9 - 7\delta)(\delta\bar{\theta} - 3\theta) + 2(9 - 2\delta)(3\bar{\theta} - \delta\theta). \end{aligned}$$

$$P_a^* - P_b^* = \left( \frac{(3\delta^2 - 8\delta + 9)}{(11\delta - 27)\Delta S} \right) (\bar{\theta} + \theta) < 0 \quad \text{since numerator is positive and denomi-}$$

nator is negative for  $0 < \delta < 1$ .

The second-order condition:

For the stability of the system, we show the negative-definite in Hessian matrix of the profit functions of the system in the first-period. Total differentiation of first-order conditions of the discounted first-period profit functions of the firms provide Hessian matrix as follows:

$$\begin{bmatrix} \Pi_{aa}^A & \Pi_{ab}^A \\ \Pi_{ba}^B & \Pi_{bb}^B \end{bmatrix}, \text{ where } \Pi_{ij}^t = \frac{\partial(\Pi^t)}{\partial i \partial j} \text{ for } t = A, B \text{ and } i, j = P_a, P_b.$$

$$\Pi_{aa}^A = \Pi_{bb}^B = 2(-9 + 2\delta) < 0, \quad \text{and} \quad \Pi_{aa}^A \Pi_{bb}^B - \Pi_{ab}^A \Pi_{ba}^B = 3(27 - 1\delta)(3 + \delta) > 0 \quad \text{for} \\ 0 < \delta < 1.$$

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