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UNCERTAIN LIFETIMES AND THE IMPLICATIONS OF SOCIAL SECURITY IN AN ENDOGENOUS COMMUNICATION MODEL

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This paper examines the implications of a social security program in the presence of private information on survival probabilities for the Pareto efficiency of real allocations when the private annuity market is characterized by endogenous communication. It turns out that the implications of a social security system are significantly changed if the private annuity market is characterized by endogenous communication: first, a mandatory social security type program cannot yield Pareto superior allocations; second, while the publicly provided annuity program is a perfect substitute for the pooling contract from the view point of all the agents, it is different, and exists simultaneously with, privately supplied supplementary annuities for the high-risk groups; third, the implications for aggregate private savings are different depending on the level of the public program. It is noteworthy that there exist no Pareto improving roles for government policies in our endogenous information-sharing set-up despite the fact that the high-risk group exerts a negative externality on the low-risk group as in the other models of private information.

JEL Classification: H55, G22
Keywords: Endogenous Communication, Social Security, Uncertain Lifetimes

I. INTRODUCTION

This paper examines the implications of social security in the presence of private information on survival probabilities for the Pareto efficiency of real allocations when the private annuity market is characterized by endogenous information-sharing.

The effects of social security in the presence of uncertain lifetimes have been

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studied by Sheshinski and Weiss (1981) and Abel (1985) in models in which there is no private annuity market. However, if a competitive annuity market were introduced into these models, social security would then have no effect because the rate of return on private annuities would be the same as the rate of return implicit in actuarially fair fully funded social security; thus consumers would exactly offset the effects of social security by adjusting their purchases of private annuities.

In papers by Eckstein, Eichenbaum and Peled (1985) and Abel (1986, 1988), they introduced a private market for annuities and demonstrated that with private information on the survival probabilities, social security does have real effects on the allocations. The important common implications of private information in their studies can be summarized as follows: first, a mandatory social security system may yield a Pareto superior allocation; second, the publicly provided annuity program is different from, and exists simultaneously with, privately supplied annuities which satisfy residual demands for annuities; and third, because public and private annuities offer different rates of return they are not perceived of as perfect substitutes by individuals. Consequently, both aggregate private savings and rates of return are affected by the level of the public program.

Both studies, however, were based on the rather ad hoc assumptions on the customer-information-sharing among annuity companies; Eckstein, Eichenbaum and Peled (1985) assumed-following Rothschild and Stiglitz (1976) and Wilson (1977)-that annuity companies always share their customer information fully, while Abel's works (1986,1988) were based on the assumption that annuity companies never share customer information with one another. We argue, however, that, as in Jaynes' (1978) and Hellwig' (1988) model of the insurance market, it is always profitable for some (but not all) annuity companies to guarantee that they will never divulge the names of their customers to other companies. This point is incorporated into our model in deriving an equilibrium in which one group of annuity firms chooses to disclose information on their customers while the other group does not. And we examine the effects of social security when the annuity market is characterized by this sort of endogenous communication. It turns out that the implications of a social security system are significantly changed: first, a mandatory social security type program cannot yield Pareto superior allocations[1]; second, while the publicly provided annuity program is a perfect substitute for the pooling contract from the viewpoint of all the agents, it is different, and exists simultaneously with, privately supplied supplementary annuities for the high-risk groups; and third, the implications for aggregate private savings differ depending on the level of the public program. It is noteworthy that there exists no Pareto improving roles for government policies in our endogenous

[1] Although Abel(1986, 1988) didn't explicitly analyze the welfare implications of the social security system, we can show that this proposition still holds within our framework with no communication assumption. For the proof, see Appendix A.

information-sharing set-up despite the fact that the high-risk group exerts a negative externality on the low-risk group like in the other models of private information.

The plan of the paper is as follows. The next section presents a basic model and characterizes the complete annuity market. In section 3, we introduce the private information and explain incentives for communication and finally characterize the equilibrium in the annuity market with endogenous information-sharing. Section 4 analyzes the effects of a mandatory social security program and section 5 presents concluding remarks.

Ⅱ. THE BASIC MODEL[2]

2.1. Description

The economy is a variant of Samuelson's (1958) pure exchange overlapping generations model. At each period t, the population consists of old members of generation $t-1$ who all die at the end of that period, and young members of generation t. Each generation t is partitioned into two distinct groups, A and B, whose relative size is fixed at t, so that for each agent of type A, there are Θ agents of type B, $\Theta>0$. Members of each group live at most two periods, the first of which they survive with certainty. Death can occur at the beginning of the second period with probability $(1-p_i)$, $0<p_i<1$, $i=A, B$. It is assumed that $p_A<p_B$. With a continuum of agents, each of whom correctly perceives his death to occur with probability $(1-p_i)$, a proportion $(1-p_i)$ of group i passes away after living only one period. There is no aggregate uncertainty implied by agents' random lifetime.

There is a single non-storable and non-producible consumption good in this economy. Each young agent is endowed at birth with W units of the good. Generations are of equal size so that each member of generation t is viewed as giving birth to one identical agent before the uncertainty about his continued life is resolved. Preferences over lifetime consumption (C_{i1}, C_{i2}) of a representative member of group i are given by the expected utility function

$$V_i(C_{i1}, C_{i2})=U(C_{i1})+p_iU(C_{i2}), \quad i=A, B. \tag{1}$$

with $U'>0, U''<0$, $U'(C)\to\infty$ and $U'(C)\to 0$ as $C\to\infty$.[3]

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[2] The model in this section heavily draws on Eckstein, Eichenbaum and Peled (1985) and Eichenbaum and Peled (1987).

[3] As pointed out in Eckstein, Eichenbaum and Peled (1985), this time-separable specification of preferences is consistent with Yaari (1965) and Barro and Friedman (1977), among others, who adopt such a specification in order to parameterize over lifetime consumption bundles in the face of uncertain lifetimes.

We assume the existence of a safe asset whose gross real rate of return is unity. Agents allocate consumption intertemporally by purchasing annuities which are supplied by competitive firms that are specializing in holding the safe asset.

We model an annuity bond at period t as a claim to a certain quantity of the consumption good at period $t+1$ which is payable only if the original purchaser of the annuity is alive. Normalizing the purchasing price of a period t annuity to one unit of the good at t, the annuity's rate of return, $R_i(t)$, represents the intertemporal terms of trade faced by its buyer; i.e., $R_i(t)$ is the real payoff, at $t+1$, to an annuity purchased at time t by a member of group i contingent on his being alive at $t+1$. Let $D_i(t)$ denote the utility maximizing purchase of such annuities.

It is easy to show that a Pareto optimal allocation in the economy has the following property:

$$\frac{U'(C_{A1})}{U'(C_{A2})} = \frac{U'(C_{B1})}{U'C_{B2}} \tag{2}$$

$$C_{A1} + \Theta C_{B1} + P_A C_{A2} + \Theta P_B C_{B2} = W(1+\Theta) \tag{3}$$

(2) comes from the first-order condition for the central planner's problem and (3) is the feasibility condition.

Given heterogeneity with respect to survival probabilities, (2) implies that optimal allocations have the property that ex-ante marginal rates of substitution are not equalized across members of different groups, i.e.,

$$\frac{U'(C_{A1})}{P_A U'(C_{A2})} \neq \frac{U'(C_{B1})}{P_B U'(C_{B2})}$$

In a competitive equilibrium, agents equate their expected intertemporal marginal rate of substitution to the rate of return on savings that they face. Consequently, the competitive equilibrium will be (full information) Pareto optimal because all agents face actuarially fair rates of return $(1/p_i)$ for agents of type i.

2.2. Full Information Equilibrium

Following Rothschild and Stiglitz (1976) and Jaynes (1978), we define an equilibrium in this market as a set of contracts such that when agents maximize utility, (1) there is no contract in the equilibrium set that makes negative expected profits, and (2) there is no contract outside the equilibrium set that, if offered, will make a nonnegative profit. The absence of aggregate uncertainty implies zero profits in the equilibrium.

In the pooling equilibria, $R_A(t) = R_B(t) = R(t)$ and profits at period $t+1$ are given by

$$D_A(t+1) + \Theta D_B(t+1) - R(t)[P_A D_A(t) + \Theta P_B D_B(t)] = 0 \tag{5}$$

In a stationary pooling equilibrium, $D_A(t) = D_A = D_B(t) = D_B$ and $R(t) = R$ for all t. Substituting these identities into (5), we obtain the expression for the economy-wide rate of return.

$$R_0 = \frac{1+\Theta}{P_A + \Theta P_B} \tag{6}$$

In the full information economy, the pooling contract can never be an equilibrium contract because at this rate, given by (6), positive profits can be made by selling annuities to only group A. Hence, the equilibrium will necessarily be a separating one with each contract netting zero profits, so the equilibrium rates of return are given by $R_A = 1/p_A$ and $R_B = 1/p_B$. Note that $R_A = 1/p_A > R_0 > R_B = 1/p_B$ hold. These are precisely the intertemporal rates of return which induce an equilibrium in which (4) is satisfied. To see this, note that the problem of the representative young agent of group i of generation t is:

$$\begin{aligned} &\max\ U(C_{i1}) + P_i U(C_{i2}) \\ &s.t.\ C_{i1} = W - D_i(t) \\ &\qquad C_{i2} = R_i(t) D_i(t) \end{aligned}$$

Necessary and sufficient conditions for the solution of this problem are given by

$$\frac{U'(C_{i1})}{P_i U'(C_{i2})} = R_i(t), \quad i = A, B \tag{7}$$

These conditions imply (4) when $R_i(t) = 1/p_i$ for $i = A, B$. Notice that (7) implies that

$$C_{i1} = C_{i2} = \frac{W}{1+P_i} \tag{8}$$

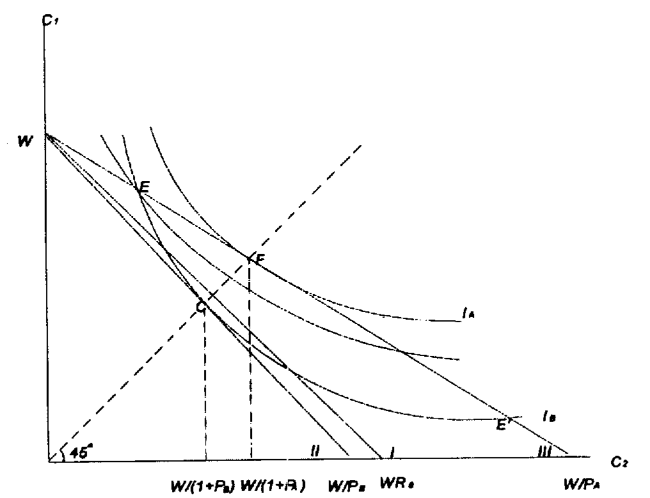
for all agents who live for two periods. This full information equilibrium can be represented by two points C and F in figure 1. Thus, in the absence of private information, competitive annuity markets discriminate between groups in an actuarially fair way, and thereby induce risk-sharing within each specific group.

Ⅲ. THE COMPETITIVE ANNUITY MARKETS IN THE PRESENCE OF PRIVATE INFORMATION

Now we are ready to introduce private information into our economy. We assume that agents know their own survival probabilities as well as Θ, the number of B type agents per agent of type A. Similarly, the government knows the value of p_A, p_B and Θ. However, neither the government nor the annuity companies know whether any particular individual belongs to group A or B. This information structure gives rise to a classic adverse selection problem. The high-risk consumers from the view point of annuity companies are those consumers with a high survival probability i.e. group B agents. These consumers will demand more annuities than the consumers with low survival probabilities, and they will be more likely to survive and receive annuity payments.

In an annuity market with asymmetric information, the set of contracts purchased by a consumer contains information about his survival probability. Annuity companies would like to exploit this information. To do so, a given annuity company must have an access to information about its customers' annuity purchases from other companies. This requires some degree of cooperation from the other annuity companies. Depending on the assumptions about the degree of cooperation, there have been three types of models: the models with No Communication, the models with Full Communication and the models with Endogenous Communication.

[Figure 1] An Equilibrium in the Annuity Market with Full Information and Asymmetric Information



(NC) In the models with no communication, it is assumed that the annuity companies know how much annuity he has sold to the individual, but he does not know how much the individual has bought from others; annuity companies cannot determine whether an individual consumer holds annuities from other annuity companies. In the context of a competitive annuity market with many firms, this assumption is equivalent to precluding communication among firms. Annuity companies establish a price[4] of insurance and allow their customers to buy as much or as little annuities as they want at that price. Since contract purchases are the only potential signal of individual's risk characteristics, this model allows no possibility that the annuity companies can distinguish among different types of consumers. Consumers with different risk characteristics will be charged the same price. Examples of this type of model are Pauly (1974) and Johnson (1978).

(FC) The models with full communication assume that each annuity company has complete knowledge about the total amount of coverage its buyers purchase. This means that firms have power to limit consumer purchases to a single contract. It is obvious that inexpensive enforcement of such an information structure necessarily requires explicit cooperation among all firms. In other words, all the firms communicate with each other regarding their customers' purchases, enabling themselves to observe the purchasing behavior of each customer. Examples of this type of model are Rothschild and Stiglitz (1976) and Wilson (1977).

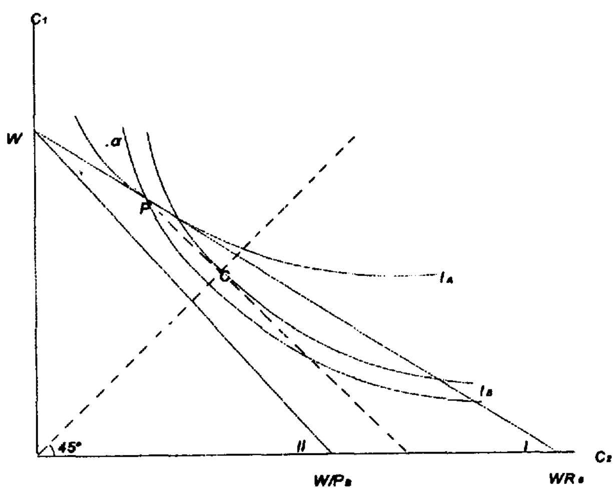
(EC) The major departures of the model with endogenous communication from the previous two types of models is to argue that the assumption that firms can costlessly monitor the total insurance coverage obtained by buyers is untenable. Jaynes (1978) and Hellwig(1988) showed that it is always profitable for some firms to guarantee their customers that they will not divulge the names of their clients to other firms unlike in the full communication model, and that a group of firms (all of whom offer pooling contracts) has positive incentive to trade information exclusively among themselves unlike in the no communication model. Thus, contrary to the full communication model, consumers are allowed to contract with multiple insurance companies.

Jaynes (1976) and Hellwig(1988) suggest that, if there is an equilibrium, it would be supported by two groups of annuity companies, one sharing customer identity information and the other refusing to divulge it. Since a separating equilibrium is not possible, the equilibrium should involve some degree of pooling. Cosider the situation depicted in figure 2. On the pooling line I the contract (D^P, R_0) maximizes the utility of type A agents. If the contracts on this line are the only contracts offerd, all type A agents will also demand D^P. All type B agents would prefer more coverage at the given price and would

[4] In the context of insurance market, the price of insurance can be defined as the cost per unit coverage. It corresponds to $1/R$ in our setting.

like to purchase from many of firms. However, these firms have a profit incentive to prevent this and no incentive to do otherwise. Therefore, type B agents will also demand D^P. Both types must be less than fully insured. Each would now be willing to purchase additional coverage at the respective actuarially fair price. But for type A agents this is infeasible. The reason is that if the contracts on the line III (in figure 1) are offered, then this will attract type B agents as well as type A agents making the contract unprofitable from the firms' point of view. And yet, there is an incentive for firms to offer the set of contracts on line II. Then, type B agents will buy the contract $(D^C - D^P,\ 1/p_B)$. That is, there is an incentive for some firms to offer additional insurance coverage along the line II, with the promise that they will not divulge the names of their customers to those firms offering the pooling contract $(D^P,\ R_0)$. This enables type B consumers to purchase the two contracts $(D^P,\ R_0)$ and $(D^C - D^P,\ 1/P_B)$ and attain the point C.[5] Thus all the consumers purchase the pooling contract $(D^P,\ R_0)$ and type B consumers also purchase a supplementary contract $(D^C - D^P,\ 1/P_B)$. Thus we have established the following:

[Figure 2] A Market Equilibrium in an Endogenous Communication Model



[5] This is not saying that type B agents always want to and can obtain full insurance (point C).

Proposition 1 If an equilibrium exists,[6] it is a pooling equilibrium where all the agents buy the contract (D^P, R_0) and the type B agents buy a supplementary contract $(D^S, 1/p_B)$ where D^P maximizes $[U(W-D)+p_A U(R_0 D)]$ and D^S maximizes $[(U(W-D^P-D)+p_B U(R_0 D^P+(1/p_B D))]$.

Note that, with private information regarding survival probabilities, the presence of high-risk individuals (type B) exerts a negative externality on other agents. In the private information equilibrium described above, while group B is better off than it would be in the absence of private information, group A is worse off. Given these negative externalities, we get a surprising result in the next section that there exist no Pareto improving policies which the government can undertake.

IV. THE EFFECTS OF A MANDATORY SOCIAL SECURITY PROGRAM

We are ready to introduce a mandatory social security system to our endogenously communicating economy. The government requires all agents to purchase a given amount, X, of publicly provided annuities that pay a gross real rate of return of R_0, the economy-wide actuarially fair rate of return as defined in (6). Private markets for residual demand for annuities are allocated to operate in an unfettered way. The public program breaks even on the whole, and can be viewed as shifting the individual endowment vector from $(W, 0)$ to $(W-X, R_0 X)$.

With the introduction of publicly provided annuities X, we can rewrite an indirect utility function

$$V_i(D_i; R, X) = U(W-X-D_i) + p_i U(RoX + RD_i), \; i = A, B \tag{9'}$$

The first-order condition for an interior optimum is

$$U'(W-X-D_i) = RP_i U'(R_0 X + RD_i)$$

As we know from the discussions in section 3, group A agents will buy a pooling contract (D^P, R_0), and group B agents will buy the same pooling contract and a supplementary annuity contract $(D^s, 1/p_B)$ in the absence of a social security program. Since the government-provided annuities whose rate of

[6] Jaynes (1978) showed that an insurance market with asymmetric information always has a Nash-Cournot type equilibrium if insurance companies are free to choose whether to share information about their customers or not. The endogenous treatment of interfirm communication would invalidate the conclusion of Rothschild and Stiglitz (1976) and Wilson (1977) that such a market may not have an equilibrium. But Hellwig (1988) argued that it is not the endogenous treatment of interfirm communication but the sequential specification of the firm behavior which allows each firm to react to the other firms' contract offer that solves the existence problem.

return is R_0 are perfect substitutes for the pooling contract, the following relationship holds.

$$D^P(X) = D^P(0) - X \tag{10}$$

where $D^P(X) = \text{argmax}\ [U(W - X - D^P) + p_A U(R_0 X + R_0 D^P)]$

Depending on the level of publicly provided annuities, we will consider three cases:

Case I: $X \leq D^P(0)$

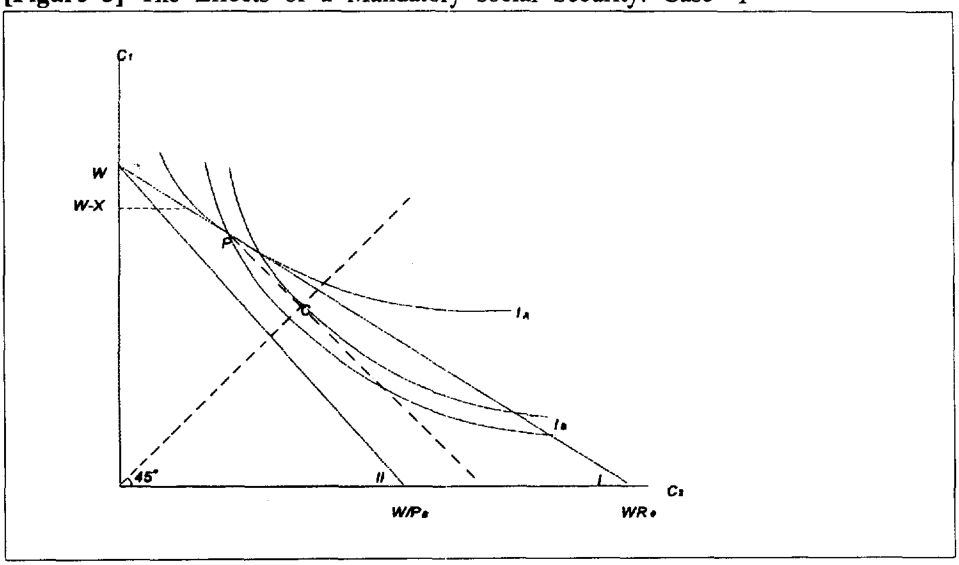
If the size of publicly provided annuities is smaller than the type A agents' demand for private annuity in the absence of a social security system, it is obvious that the following relationships hold:

$$\begin{aligned} D_A &= D^P(X) = D^P(0) - X \\ D_B &= D^P(X) + D^S = D^P(0) - X + D^S \end{aligned} \tag{11}$$

$$\text{Where}\ \ D^S = \underset{D}{argmax}\Big[U(W - X - D^P(X) - D) + p_B U\Big(R_0 X + R_0 D^P(X) + \frac{1}{P_B} D\Big)\Big]$$

Note that D^S is invariant with respect to X since

[Figure 3] The Effects of a Mandatory social Security: Case I



$$D^S = \underset{D}{argmax}\left[U(W-X-D^P(X)-D)+p_B U\left(R_0X+R_0D^P(X)+\frac{1}{p_B}D\right)\right]$$
$$= \underset{D}{argmax}\left[U(W-D^P(0)-D)+p_B U(R_0D^P(0)+\frac{1}{p_B})\right]$$

It follows that the consumption of each type does not depend on X;

$$C_{A1}(X) = W-X-D_A = W-D^P(0)$$
$$C_{B1}(X) = W-X-D_B = W-D^P(0)-D^S$$
$$C_{A2}(X) = R_0X+R_0D^P(X) = R_0D^P(0)$$
$$C_{B2}(X) = R_0X+R_0D^P(X)+\frac{1}{p_B}D^S = R_0D^P(0)+\frac{1}{p_B}D^S$$

As long as $X \leq D^P(0)$, the publicly provided annuities are a perfect substitute for the privately provided pooling contract, and thus aggregate savings by young agents do not change. Obviously, in this case, there is no Pareto improving role for a mandatory non-discriminatory social security system. The intuition behind this result may be described as follows: By assumption, the government does not have any informational advantage over the private sector so that the only measures it possesses to affect real allocation is its ability to force the pooling contract on all consumers. In the present model with endogenous communication, the private annuity market supports the pooling contract. The size of the pooling contract offered by firms will adjust so that the total size of the two pooling contracts, governmental and private, remains constant.

Case II: $D^P(0) < X \leq [D^P(0)+D^S]$

If the level of a social security tax is in between the demand for private annuities of type A agents and type B agents in the absence of government intervention, then the following relationships hold.

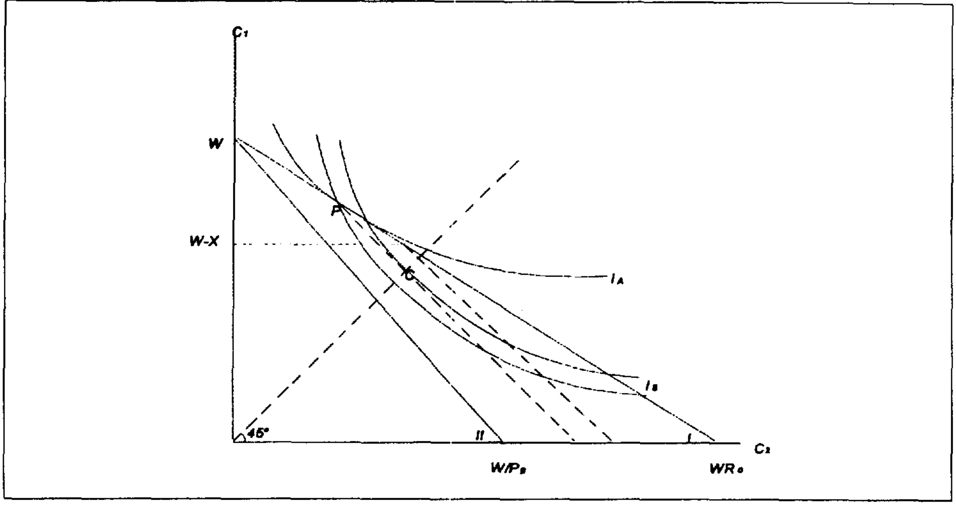
$$D_A = X$$
$$D_B = X + D^S \tag{12}$$
$$\text{where } D^S = \underset{D}{argmax}\left[U(W-X-D)+p_B U\left(R_0X+\frac{1}{p_B}D\right)\right]$$

Unlike in case I, D^S is negatively related to X. (In fact $(\partial D^S/\partial X) < -1$. We can prove this by differentiating the first-order condition of (12) and by using the fact $(1/p_B) < R_0$).

While the first-period consumption of type A agents decrease, those of type B agents increase as a result of the introduction of a social security system.

$$C_{A1}(X) = W-X$$

[Figure 4] The Effects of a Mandatory social Security: Case Ⅱ



$$C_{B1}(X) = W - X - D^S \quad (13)$$
$$C_{A2}(X) = R_0 X$$
$$C_{B2}(X) = R_0 X + \frac{1}{p_B} D^S$$

In this case, the young members of group B decrease their total savings relative to the situation in the absence of a social security system while the converse holds for group A. The impact on aggregate private savings by young agents cannot be determined in the absence of further restrictions on the model. Note that the introduction of a mandatory social security system improves the welfare of type B agents at the expenses of type A consumers. (For a proof, see appendix B.) The intuition behind this result may be described as follows: The introduction of a social security system will decrease the first-period consumption of type A agents and increase the first-period consumption of type B agents (see (13)). By forcing the mandatory saving which is greater than the optimal level of voluntary saving $D^P(0)$, the government is reducing the welfare of type A agents. On the contrary, type B agents can satisfy their relatively higher demand for insurance with a lower price. Thus, the introduction of a social security system is simply a subsidy for high risk individuals at the expenses of low risk individuals, and therefore no Pareto-improvement is possible.

Case Ⅲ: $X > [D_P(0) + D^S]$

It is trivial to see that the introduction of a mandatory social security system will hurt both type A agents and type B agents.

To summarize our discussions so far, we can conclude that if the private annuities market is characterized by endogenous communication: first, a mandatory social security type program cannot yield Pareto superior allocations; second, while the publicly provided annuity program is a perfect substitute for the pooling contract from the view point of all the agents, it is different from, and exists simultaneously with, privately supplied supplementary annuities for type B agents; third, implications for the aggregate private savings are different depending on the level of the public program.

V. CONCLUSION

In this paper, we have analyzed the effects of social security system, when there is a longevity uncertainty and when the private annuity market is characterized by endogenous communication. The private annuity market equilibrium generated by endogenous communication has the property that all the contracts purchased except one (that of the highest-risk type) involve pooling of different risk types. The rate of return from the equilibrium contract pooling all the risk types is equal to the rate of return from mandatory social security. This equality of the rates of return gives no power to the government to Pareto-improve the market allocations. These results are quite different from the ones obtained in full communication or no communication model. Thus, we need to know how the private annuity companies communicate among themselves in order to analyze the effects of social security. This conforms to the general idea that the effects of government interventions in private markets differ depending on how private agents behave.

Caution should be used in relying on these results. Although the model used in this analysis extends our understanding on the effects of a mandatory social security system, the results should not be taken literally since the model is built on the assumption that annuities play a very important role as a means of saving. As a matter of fact, well-developed markets for life annuities do exist in the United States. However, the puzzle is that so few people choose to use them.[7] Several potential answers to this puzzle have been suggested, but further research is needed: (1) Most people save not to smooth consumption patterns over one's lifetime but, instead, to leave bequests to their heirs (Friedman and Warshawsky (1990)); (2) Most individuals automatically receive life annuities from Social Security and, for a significant fraction of the labor force, employer-sponsored pension plans(Poterba(1997) and Brown(1999)); (3) People shun individual annuities because they are not priced "fairly" in the actuarial sense (Friedman and Warshasky (1988, 1990)); (4) Many people avoid buying

[7] The Retirement History Survey indicates that only 2 percent of the elderly population own individual annuities of any sort. See Friedman and Sjorgren (1988) and Friedman and Warshawsky (1990) for a detail.

annuities from fear of the consequences of catastrophic illness; (5) The individual life annuities available in U.S. markets guarantee specified nominal (rather than real) payments; (6) Risk-sharing is done between husband and wife as well as among family members more generally (Kotlikoff and Spivak (1981)).

Appendix A

In this appendix, we show that a mandatory social security system may yield a Pareto superior allocation if annuity companies are assumed not to share their customer identity information. Since contract purchases are the only potential signal of individuals' risk characteristics, this assumption allows no possibility that annuity companies may distinguish among different types of consumers. Consumers with different risk characteristics will be charged the same price. The price can be determined by the arbitrage condition.

Note that the total quantity of annuities supplied to the whole consumers is $[\Theta D_A+(1-\Theta)D_B]$, while the expected return is $R[\Theta p_A D_A+(1-\Theta) p_B D_B]$. Thus, the expected rate of return is

$$Z(R, X)=R[\Theta p_A D_A(R, X)+(1-\Theta)p_B D_B(R, X)] \\ /[\Theta D_A(R, X)+(1-\Theta)D_B(R, X)]$$

We can define the equilibrium rates of return, R^*, which satisfy the following arbitrage condition; $Z(R^*, X)=1$. The utility level in the equilibrium can be represented by

$$V_i=U(W-X-D_i(R^*, X))+p_i U(R_0 X+R^* D_i(R^*, X))$$
$$dV_i/dX=-U'(W-X-D_i)+p_i[R_0+(dR^*/dX)D_i]U'(R_0 X+R^* D_i)$$
$$=p_i[-R^*+R_0+(dR^*/dX)D_i]U'(R_0 X+R^* D_i)$$

Thus, as long as $[R_0-R^*]>-(dR^*/dX)D_i$, $dV_i/dX>0$ (Q.E.D.)

Appendix B

In this appendix, we prove $dV_A/dX<0$ and $dB_B/dX>0$ in case II using the envelope theorem. The utility level in the equilibrium can be represented by $V_i(X)$.

$$V_A(X)=U(W-X)+p_A U(R_0 X)$$
$$dV_A/dX=-U'(C_1(X))+p_A R_0 U'(C_2(X))$$

Since the optimal level of private annuity demand is equal to zero in this case, i.e. $D^P(X)=0$, the following holds from the first-order condition of the problem (9).

$$-U'(W-X-D^P(X))+R_0 p_A U'(R_0 X+RD^P(X))|_{D=0}$$
$$=-U'(W-X)+R_0 p_A U'(R_0 X)$$
$$=-U'(C_1(X))+p_A R_0 U'(C_2(X))\leq 0.$$

Thus, $dV_A/dX\leq 0$.

$$V_B(X)=U(W-X-D^S(X))+p_B U(R_0 X+(1/p_B)D^S(X))$$
$$dV_B/dX=-(1+\partial D^S/\partial X)U'(W-X-D^S(X))$$
$$+(R_0+(1/p_B)\partial D^S/\partial X)p_B U'(R_0 X+(1/p_B)D^S(X))$$
$$=(p_B R_0-1)U(C_1(X))>0. \quad \text{(from (6))} \qquad \text{(Q.E.D.)}$$

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