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AN ANALYSIS OF THE RESERVE MARKET: INTERPRETING VECTOR AUTOREGRESSIONS USING A THEORETICAL MODEL

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This paper studies the reserve market by interpreting vector autoregression (VAR) using an optimizing equilibrium model. A theoretical model of loan and reserve markets, where banks solve a dynamic maximization problem and the Federal Reserve Board controls the supply of reserves, is constructed and solved numerically. A trivariate VAR summarizes the dynamics of the federal funds rate, the one-month commercial paper rate, and real nonborrowed reserves from 1984:3 to 1996:1. In the spirit of Gallant and Tauchen (1996), the scores of the estimated VAR are used as orthogonality conditions for Generalized Methods of Moments to calibrate the model. It is shown that, since 1984, the reserve supply shocks dominate reserve demand shocks in the residual of the federal funds rate equation in the VAR. I conclude that the residual offers a reasonable proxy for an exogenous shock to monetary policy.

JEL Classification: E52, E43

I. INTRODUCTION

The purpose of this paper is to identify the effect of monetary policy from an analysis of the supply and demand for reserves. A number of different variables have been proposed as indicators of monetary policy. For example, Bernanke and Blinder (1992) use innovations in the federal funds rate, while Strongin (1995) employs the mix of borrowed and nonborrowed reserves. Implicitly, these authors are assuming that most of the variation of the variable is due to shocks to reserve supply rather than shocks to the reserve demand. Bernanke and Blinder (1992) argued that the reserve supply curve is quite flat so that reserve demand

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shocks cannot change the federal funds rate. However, if changes in the funds rate target are closely correlated with reserve demand shocks, then movements in the funds rate could still be caused by reserve demand shocks even though the supply curve is flat. Strongin (1995) argued that the mix variable defined as nonborrowed reserves divided by total reserves is a reasonable choice for the indicator of monetary policy, based on institutional details of the reserve market. However, if monetary policy responds to reserve demand shocks, then the mix variable could be contaminated with the demand shocks. These kinds of difficulties motivate interpreting an empirical model such as a vector autoregression (VAR), on the basis of a theoretical model which embodies a fully articulated account of the dynamic interaction between reserve suppliers and demanders.

Theoretical models which deal with the interplay between monetary policy and banks' reserve management based on dynamic optimization are difficult to find. In the banking models of dynamic rational expectations equilibrium, the target of monetary policy, i.e., the reserve supply rule, is exogenously given. For example, Cosimano (1987) took the targets for the monetary base, nonborrowed reserves, and demand deposits as exogenous. Cosimano and Huyck (1989) assumed that the reserve target is unrelated to innovations in the demand deposit market. General equilibrium models abstract from the market for reserves, acting as if money itself rather than bank reserves is directly injected into economy; see for example Christiano and Eichenbaum (1995), Fuerst (1992), Carlstrom and Fuerest (1995), and Coleman (1996).

This paper constructs a model which has two markets: the federal funds and the loan markets, and two types of players: commercial banks and the monetary authority. Banks buy and sell federal funds in the funds market, and make loans given loan demand in the loan market, maximizing their profit by dynamic optimization. The Federal Reserve System supplies real nonborrowed reserves (RNR) to banks. A shift of the loan demand curve causes the reserve demand curve to shift in the same direction. A shift of the reserve supply curve causes the loan supply curve to shift in the same direction. The Fed employs a federal funds rate target to stabilize the volume of loans. If loan demand is high, the Fed increases the funds rate target, and if the loan demand is low, the Fed decreases the target. Banks create demand deposits by making loans, simultaneously increasing their assets and liabilities. However, their profit-maximization problem is not different from other financial intermediaries' since they maximize their profit by equating the marginal benefit of one unit of reserves with that of one unit of loans (Tobin, 1963; Fama, 1986).

Structural parameters are inferred by matching the model's predictions to estimates from a VAR. This can be viewed both as a way to calibrate the model and as a framework for interpreting the VAR. In the real business cycle literature, it is typical that first moments are matched for calibration, while second moments are used for specification tests. This paper uses a similar approach, except that the moments to be matched are the orthogonality conditions associated with estimating a VAR. It further offers a new approach to interpreting a VAR. A VAR expresses the relation among variables by projecting current variables on the past variables; the meaning of this projection can be difficult to interpret in the context of economic agents' optimizing behavior. However, by comparing the VAR estimation results with data generated from the theoretical model and with real data, a VAR can be interpreted in the context of banks' dynamic optimization. Parameters are estimated in the spirit of Gallant and Tauchen (1996). They suggest using the scores of an empirical model for the orthogonality conditions of generalized method of moments (GMM) estimation of the structural parameters of a theoretical model. Here the VAR is the empirical model and the optimizing model of the two markets is the theoretical model.

The main result of this paper is that, for data from 1984:3 to 1996:1, the reserve supply shock dominates the reserve demand shock in the residual of the funds rate equation in the VAR. That is, the impulse-response function for a shock to the orthogonalized residual of the funds rate in the estimated VAR is very similar to that for a shock to the funds rate target in the theoretical model. And the exogenous shock to monetary policy explains the 65% of the variations in the target, and the remaining 35% of the variations come from endogenous response of monetary policy to the loan demand shock or reserve demand shock. This finding provides support for using the funds rate innovation in the VAR as an indicator of monetary policy, as in Bernanke and Blinder (1992).

The paper proceeds as follows: Section 2 presents the theoretical description of bank behavior and monetary policy. Section 3 fits a VAR to real data, and presents statistics that this paper seeks to interpret using the structural model. Section 4 explains how to calibrate the theoretical model. Section 5 shows how the predictions of the theoretical model can be matched to the facts. Section 6 shows the estimates of the parameters in the theoretical model, and offers interpretations. Section 7 concludes the paper.

Ⅱ. MODEL

There are two banks in this model, but it could be easily extended to include more banks.

(1) An Individual Bank's Problem

Banks have an infinite horizon consisting of discrete periods. Each period consists of two stages. In the first stage of a period, banks do not know their current demand deposits, the funds rate, or the funds rate target. Banks choose the volume of loans given the loan rate and expectations of the unknown variables. In the second stage, banks learn the current demand deposits and funds rate, and then trade Federal funds. The private sector is assumed to prefer demand deposits to cash, so all cash is deposited in the bank at the end of the period.[1] Therefore Real Nonborrowed Reserves(RNR hereafter) do not change unless the Fed changes them.

Bank profits are given as follows:

$$\pi_{it} = r_{L,t} L_{it} + r_{L,t-1} L_{it-1} - 0.5\delta(L_{it} - L_{it-2})^2 \\ + (\alpha R_{it} - 0.5\gamma_1 R_{it}^2) + (r_{F,t} F_{it} - 0.5\gamma_2 F_{it}^2) - \beta(R_{it} - 0.1D_{it})^{-1}$$

where i is the index for the bank, L_{it} is the volume of two-period loans initiated by bank i at the first stage of period t, F_{it} is the volume of Federal funds lent by bank i at the second stage of period t, R_{it} represents the volume of reserves held by bank i at the end of period t, $r_{L,t}$ is the loan rate, $r_{F,t}$ represents the funds rate, and D_{it} is the volume of demand deposits of bank i at the end of period t. All the parameters are positive. The third term of the profit function is the cost of the loan adjustment. The magnitude of $L_{it} - L_{it-2}$ is the volume of loans adjusted, because the volumes of loans in the previous and present periods are $L_{it-1} + L_{it-2}$ and $L_{it} + L_{it-1}$ respectively. The adjustment cost is quadratic, which implies an increasing marginal adjustment cost, assumed to be symmetric for simplicity. This cost occurs because of the decision making about which projects to fund. The term $\alpha R_{it} - 0.5\gamma_1 R_{it}^2$ expresses the benefit of holding reserves. Banks need reserves to function efficiently (Hamilton, 1996a), even though the reserves do not contribute to profit directly. The marginal benefit of reserves, $\alpha - \gamma_1 R_{it}$, decreases as the reserves held by bank i increase. The term, $r_{F,t} F_{it} - 0.5\gamma_2 F_{it}^2$, denotes the benefit of Federal funds lending, or the cost of Federal funds borrowing. When bank i lends Federal funds ($F_{it} > 0$), the marginal benefit, $r_{F,t} - \gamma_2 F_{it}$, decreases as F_{it} increases. When bank i borrows Federal funds ($F_{it} < 0$), the marginal cost, $r_{F,t} - \gamma_2 F_{it}$, increases as F_{it} decreases. In the absence of any friction in the Fed funds market, γ_2 would be zero. The required reserve ratio is assumed to be 10 %. Therefore $R_{it} - 0.1D_{it}$ represents excess reserves. The last term generates a demand for excess reserves; banks would experience infinite loss unless excess reserves are positive[2].

The balance sheet constraint is as follows:

$$L_{it} + L_{it-1} + R_{it} + F_{it} = D_{it}$$

When bank i chooses reserves in the second stage, the loans, L_{it} and L_{it-1}, are

[1] Typically in the banking literature, banks buy demand deposits from the private sector by paying interest. This model abstracted from the market for the demand deposits, since the shock to loan demand is the primary source of the disturbance to private financial markets.

[2] The functional form of the last term is not used in research papers that try to find any analytical solutions. Instead, they generally use the quadratic functional form for convenience. However, this functional form helps to reach a numerical solution by preventing the search process for the numerical solution away from the corner (zero excess reserve).

predetermined and D_{it} is regarded as exogenously given. Bank i can choose its optimal reserves, R_{it}, only by trading Federal funds, F_{it}.

Bank i maximizes the expected present value of the profit flow, $E_2\sum_{t=2}^{\infty}\tau_t\pi_{it}$ where $\tau_t=\Pi_{m=1}^{t-1}\frac{1}{1+r_{L,m}}$, and E_t is the expectation operator conditional on information available at time t. Let's solve this problem backward by considering the second stage first. Bank i solves the following problem:

$$
\begin{aligned}
Max_{F_{it}}\pi_{it} = Max_{F_{it}}\{&(a\,R_{it}-0.5\gamma_1 R_{it}^2)+(r_{F,t}F_{it}-0.5\gamma_2 F_{it}^2)\\
&-\beta(R_{it}-0.1\bar{D}_{it})^{-1}\}\\
\text{subject to } &\bar{L}_{it}+\bar{L}_{it-1}+R_{it}+F_{it}=\bar{D}_{it},\\
&R_{it}>0.1\bar{D}_{it}
\end{aligned}
$$

where an upper bar means that the variable is predetermined. That is, loans and demand deposits are predetermined. The first order conditions are as follows:

$$
\begin{aligned}
&a-\gamma_1 R_{it}+\beta(R_{it}-0.1\bar{D}_{it})^{-2}\\
&\qquad=r_{F,t}-\gamma_2(\bar{D}_{it}-L_{it}-L_{it-1}-\bar{R}_{it}),\quad i=1,2,
\end{aligned} \tag{1}
$$

$$
R_{1t}+R_{2t}=\bar{R}_t. \tag{2}
$$

When bank i borrows Federal funds, the left hand side of equation (1) is the marginal benefit of reserves, and the right is the marginal cost. When bank i lends, the left hand side is the marginal cost of lending a unit of reserves, and the right hand side is the marginal benefit. The left hand side of equation (2) is the reserve demand of banks, and the right is the reserve supply of the Fed.

Let R_{1t}^*, R_{2t}^* and $r_{F,t}^*$ be the solutions of the above three equations. Then the funds rate is the following function of RNR, individual loans and reserves:

$$
r_{F,t}^*=a-0.5\gamma_1\bar{R}_t+0.5\beta\sum_{i=1}^{2}(R_{it}^*-0.1D_{it})^{-2} \tag{3}
$$

This equation says that the federal funds rate is determined by the sum of the marginal benefit of the reserves and that of excess reserves. If bank i has a very small amount of excess reserves, then the funds rate will be very high, because of the high marginal benefit of excess reserves, $\beta(R_{it}^*-0.1D_{it})^{-2}$. If the banks have made too many loans, demand deposits will be high, because of the balance sheet constraint. That will increase required reserves, put a lot of reserve pressure on banks, and drive up the funds rate by increasing the marginal benefit of excess reserves. If the Fed decreases RNR, then the funds rate goes up since the marginal benefit of reserves will go up.

In the first stage, banks solve the following dynamic optimization problem:

$$V(X_t) = \max_{L_{it}} \left\{ E_t \pi_{it} + \frac{1}{1+r_{L,t}} E_t V(X_{t+1}) \right\},$$

where $V(X_t) = \sum_{j=t}^{\infty} \tau_j \pi_j$, $X_t = \{L_{1t-1}, L_{1t-2}, L_{2t-1}, L_{2t-2}, \vartheta_t, \xi_{t-1}, \eta_{t-1}\}$. The terms ϑ_t, ξ_{t-1}, and η_{t-1} represent shocks to reserve supply or demand that will be explained shortly. Let $R^*_{it}(Z_t)$, $i=1,2$, be the solution in the second stage, where $Z_t = \{L_{1t}, L_{1t-1}, L_{2t-1}, D_{1t}, D_{2t}, \xi_t, \eta_t\}$. Then, the first order conditions are as follows due to the envelope theorem:

$$\begin{aligned} &r_{L,t} - \delta(L_{it} - L_{it-2}) + \frac{r_{L,t}}{1+r_{L,t}} + \frac{\delta}{1+r_{L,t}} E_t\left(\frac{L_{i,t+2} - L_{i,t}}{1+r_{L,t+1}}\right) \\ &= E_t r_{F,t} - \gamma_2 E_t(D_{it} - L_{it} - L_{it-1} - R^*_{it}(Z_t)) \\ &\quad + \frac{1}{1+r_{L,t}}[E_t r_{F,t+1} - \gamma_2 E_t(D_{it+1} - L_{it+1} - L_{it} - R^*_{it+1}(Z_{t+1}))] \end{aligned} \tag{4}$$

This condition is an arbitrage condition between the loan and funds market. The left hand side of equation (4) is the marginal benefit of increasing loans, and the right is the marginal cost of borrowing Federal funds.

(2) Loan Demand and Demand Deposits

Banks supply loans to the private sector, and loan demand is given exogenously as follows:

$$\begin{aligned} &r_{L,t} = \omega \times \exp(\vartheta_t) \times \left(\frac{L^d_t}{L}\right)^{-\frac{1}{\phi}} \\ &\vartheta_t = \rho_1 \vartheta_{t-1} \times \varepsilon_{1t} \\ &L^d_t = L_{1t} + L_{1t-1} + L_{2t} + L_{2,t-1} \end{aligned} \tag{5}$$

where all the parameters are positive and $\varepsilon_{1t} \sim$ i.i.d. $N(0, \rho^2_{\varepsilon 1})$. The term ϑ_t is the loan demand shock. The third equation of (5) is the equilibrium condition of the loan market. That is, the left hand side is the loan demand and its right is the loan supply.

The individual bank regards the volume of loans it creates as having a negligible effect on its own or total demand deposits, and thus regards these as exogenously given. In other words, we model the market structure of the banking sector as perfect competition, despite the fact that there are only two banks in this model. The demand deposit of an individual bank is determined by a random variable, η_t; specifically

$$\begin{aligned} &D_t = R_t + L_t + L_{t-1}, \\ &L_t = L_{1t} + L_{2t}, \\ &D_{1t} = D_t \eta_t, \end{aligned}$$

$$D_{2t} = D_t(1-\eta_t),$$
$$\eta_t = 0.5 \times (1-\rho_3) + \rho_3\eta_{t-1} + \varepsilon_{3t},$$

where D_{1t} and D_{2t} are the demand deposits of bank 1 and bank 2 respectively, η_t is the demand deposit split ratio, and ρ_3 makes η_{t-1} informative about η_t which is not known until the end of a period. If $\rho_3 = 0$, then expectation of the demand deposit by bank 1 is the same as that by bank 2, and the funds trading between two banks does not occur. ε_{3t} follows an i.i.d. censored $N(0, \sigma_{\varepsilon3}^2)$.[3]

(3) Monetary Policy and Measurement Errors

We model the Fed as determining a funds rate target and an allowable band around this target, forcing the funds rate to be in the target band. For example, if the funds rate is less than the lower bound of the band, then the Fed increases the funds rate by decreasing RNR. The stochastic process for the target and the target band are as follows:

$$r_{F,t}^t = \omega \times \exp(\xi_t),\ \xi_t = \rho_2\xi_{t-1} + \varepsilon_{2t},$$
$$B^* = \left[r_{F,t}^t - \frac{tb}{2},\quad r_{F,t}^t + \frac{tb}{2} \right] \tag{6}$$

and

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} = N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\varepsilon1}^2 & \sigma_{\varepsilon1,\varepsilon2}\,\sigma_{\varepsilon1}\,\sigma_{\varepsilon2} & 0 \\ \sigma_{\varepsilon1,\varepsilon2}\,\sigma_{\varepsilon_1}\,\sigma_{\varepsilon2} & \sigma_{\varepsilon2}^2 & 0 \\ 0 & 0 & \sigma_{\varepsilon3}^2 \end{pmatrix} \right),$$

where $r_{F,t}^t$ is the target, and tb is the target band width.

One of the goals of monetary policy in this model is to stabilize fluctuations of the loan volume and funds rate. When banks make too many loans and the demand for RNR is therefore high, the funds rate can shoot up due to the two banks' marginal benefits of excess reserves, $\beta\sum_{i=1}(R_{it} - 0.1D_{it})^{-2}$ in equation (3). If the loan demand is low and thus the demand of RNR is low, then the two interest rates do not go down as much as loan demand, because the funds rate is always larger than $\alpha - 0.5\gamma_1\bar{R}_t$ in equation (3) and $\bar{R}_t$ is fixed unless the Fed changes it. Therefore the Fed decreases the target to prevent banks from decreasing loans too much when the demand is low. The Fed increases the target to prevent banks from making too many loans when the loan demand is high. This policy is called a policy of leaning against the wind, and is parameterized with the correlation coefficient, $\rho_{\varepsilon1,\varepsilon2}$, which is expected to be

[3] In the numerical experiment, the random numbers for ε_{3t} are drawn from $N(0, \sigma_{\varepsilon3}^2)$, using *ex post* boundedness from the *finite* drawings.

positive, between innovations of the loan demand shock and the target shock.

The reserves have measurement errors, and so they are reported with noise, $R_t = R_t + \varepsilon_{4t}$, where ε_{4t} follows a censored normal distribution.[4] In other words, ε_{4t} can be interpreted as the variation of nonborrowed reserves which is independent of the variation of the funds rate.

Ⅲ. DATA DESCRIPTION

(a) Data

The corresponding data for the funds rate, loan rate and RNR in the model are the funds rate, one-month commercial paper rate(CP rate), and real nonborrowed reserves from 1984:3 to 1996:1. The reason why CP rate is chosen is that commercial paper is a substitute for bank loans to issuers and bankers, and the issuers of commercial paper are financial and industrial companies.

Data for the funds rate and the CP rate were taken from the Federal Reserve Bulletin from March 2nd, 1984 to February 2nd, 1996. These figures represent averages of the daily values over the two-week reserve maintenance period. The two interest rates are not converted into real terms, because it is difficult to find any reliable measure of inflation expectations. Moreover, the inflation rate was stable (Gordon and Leeper, 1994) over this period. The biweekly nonborrowed reserves, which are adjusted for reserve requirement changes, are from Table 4 of Reserves of Depository Institutions of Federal Reserve Board, and represent Monday figures at the end of the reserve computation period. Nonborrowed reserves were divided by the consumer price index to be expressed in real terms. The monthly data for the consumer price index are from Citibase Economic Database, and interpolated into biweekly data.

The second column of Table I shows the averages, standard deviations and correlations of the data. The two interest rates are highly positively correlated with each other, and highly negatively correlated with RNR. Figure 1 shows these relationships. The differences of the two interest rates are positively correlated, and not significantly correlated with the difference of RNR.

(b) Summarizing the data with a VAR

The dynamics of the variables are summarized by a VAR(2), the order of which was determined by the Schwarz Criterion. All of the variables are demeaned. Table IIa shows the estimation result of the trivariate VAR(2). The residual of the funds rate is highly correlated with that of the CP rate, their correlation coefficient being 0.655. The covariances between the two interest rates and RNR are not significantly different from zero, which is the reason why RNR is always the last in the ordering of variables in the VAR.

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[4] In the numerical experiment, the random numbers for ε_{4t} are drawn from $N(0, \sigma^2_{\varepsilon 4})$, using expost boundedness from the *finite* drawings.

[Table I] The Basic Statistics

variables	real data	generated data
mean of ff	6.53	6.70(0.155)
mean of cp	6.56	6.70(0.155)
mean of RNR	32.63	30.10(0.374)
mean of RNR	0.033	0.000(0.00044)
s.d. of ff	2.19	1.040(0.132)
s.d. of ff	2.08	1.032(0.132)
s.d. of RNR	5.30	2.146(0.271)
s.d. of $cp-ff$	0.24	0.201(0.0055)
$\rho_{ff,cp}$	0.994	0.980(0.0044)
$\rho_{ff,RNR}$	-0.826	-0.776(0.052)
$\rho_{cp,RNR}$	-0.812	-0.760(0.0540)
mean of Δff	-0.013	0.000(0.000)
mean of Δcp	-0.013	0.000(0.000)
mean ΔRNR	0.037	0.000(0.0012)
s.d. of Δff	0.22	0.196(0.0152)
s.d. of Δcp	0.23	0.225(0.00721)
s.d. ΔRNR	0.612	0.625(0.0183)
$\rho_{\Delta ff,\Delta cp}$	0.62	0.590(0.016)
$\rho_{\Delta ff,\Delta RNR}$	-0.030	-0.203(0.050)
$\rho_{\Delta cp,\Delta RNR}$	-0.045	-0.139(0.0363)

Note: 1) The numbers in parentheses are standard deviations from 20 estimates from 20 data sets, each of which has 2000 observations.

2) ff is the federal funds rate, and ck is the commercial paper rate.

[Table IIa] A VAR(2) Estimated from Real Data

$$\begin{bmatrix} ff_t \\ \\ cp_t \\ \\ RNR_t \end{bmatrix} = \begin{bmatrix} \underset{(0.0762)}{0.774} & \underset{(0.0718)}{0.354} & \underset{(0.0206)}{-0.0481} \\ \\ \underset{(0.0780)}{0.0825} & \underset{(0.0735)}{1.022} & \underset{(0.0210)}{-0.0617} \\ \\ \underset{(0.206)}{0.0881} & \underset{(0.194)}{0.00634} & \underset{(0.0558)}{0.726} \end{bmatrix} \begin{bmatrix} ff_{t-1} \\ \\ cp_{t-1} \\ \\ RNR_{t-1} \end{bmatrix}$$

$$+\begin{bmatrix} 0.0905 & -0.210 & 0.0553 \\ (0.075) & (0.073) & (0.028) \\ 0.269 & -0.377 & 0.0767 \\ (0.0768) & (0.0749) & 0.0767 \\ -0.0388 & -0.191 & 0.217 \\ (0.203) & (0.198) & (0.0542) \end{bmatrix} \begin{bmatrix} ff_{t-2} \\ cp_{t-2} \\ RNR_{t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix},$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.0422 & 0.0284 & -0.0104 \\ (0.00356) & & \\ 0.0284 & 0.0499 & -0.0105 \\ (0.0029) & (0.00498) & \\ -0.0104 & -0.0105 & 0.310 \\ (0.0131) & (0.0155) & (0.1922) \end{bmatrix} \right)$$

Note: The numbers in the parentheses are standard errors.

[Table IIb] A VAR(2) Estimated from Generated Data:

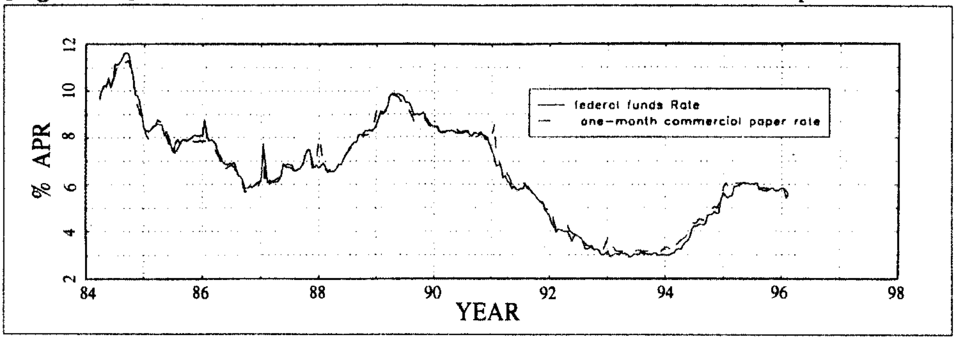
$$\begin{bmatrix} ff_t \\ cp_t \\ RNR_t \end{bmatrix} = \begin{bmatrix} 0.824 & 0.172 & -0.0272 \\ (0.065) & (0.036) & (0.0078) \\ 0.308 & 0.683 & -0.0228 \\ (0.0483) & (0.0484) & (0.0103) \\ -0.437 & -0.0856 & 0.624 \\ (0.128) & (0.0898) & (0.0185) \end{bmatrix} \begin{bmatrix} ff_{t-1} \\ cp_{t-1} \\ RNR_{t-1} \end{bmatrix}$$

$$+\begin{bmatrix} 0.515 & -0.0888 & 0.0188 \\ (0.057) & (0.0339) & (0.0079) \\ 0.0240 & -0.0465 & 0.0182 \\ (0.0461) & (0.0335) & (0.0102) \\ 0.296 & 0.132 & 0.307 \\ (0.111) & (0.080) & (0.0207) \end{bmatrix} \begin{bmatrix} ff_{t-2} \\ cp_{t-2} \\ RNR_{t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.0371 & 0.0271 & -0.0279 \\ (0.0057) & & \\ 0.0271 & 0.0457 & -0.022 \\ (0.0035) & (0.00314) & \\ -0.0279 & -0.022 & 0.338 \\ (0.00462) & (0.0043) & (0.0205) \end{bmatrix} \right)$$

Note: The numbers in the parentheses are the standard deviations from 20 coefficients or covariance estimates from 20 sets of data.

[Figure 1a] The Fecleral Funds Rate and one-Morth Eommercial Paper Rate



[Figure 1b] Real nonborrowed Reserves

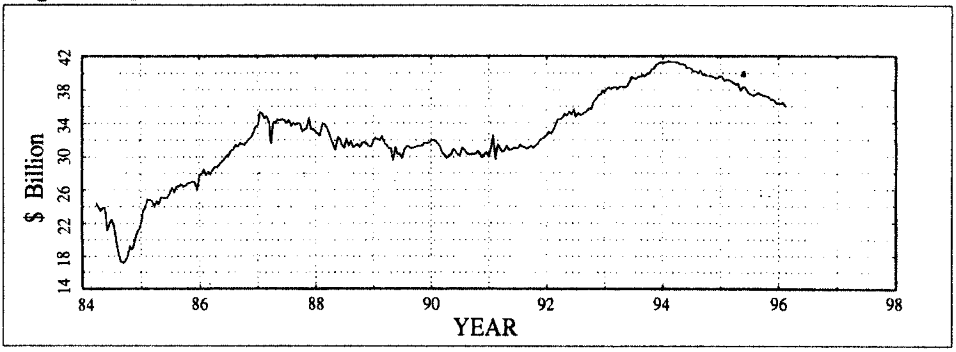
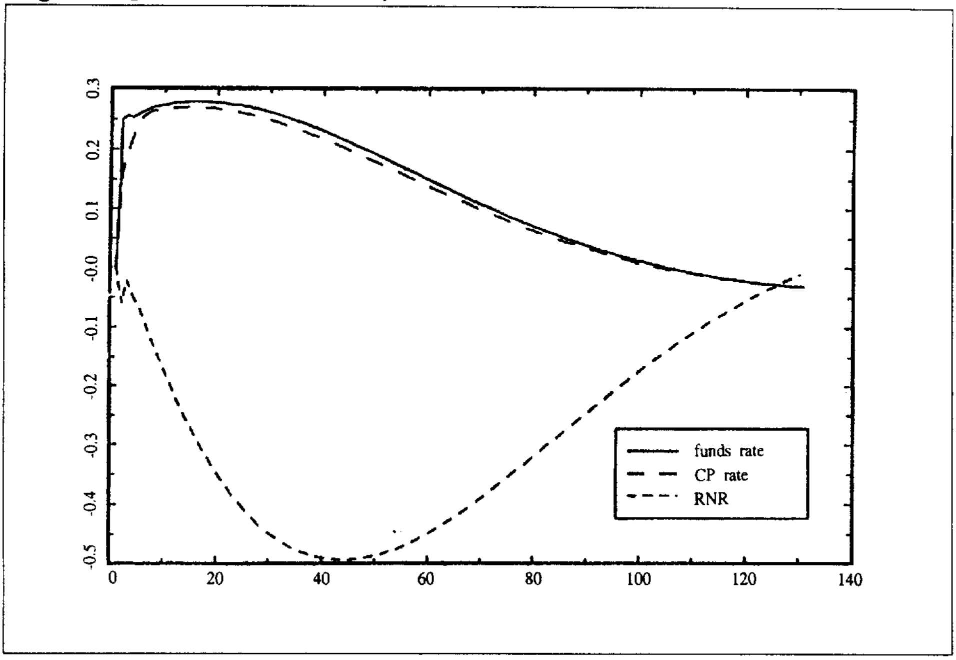


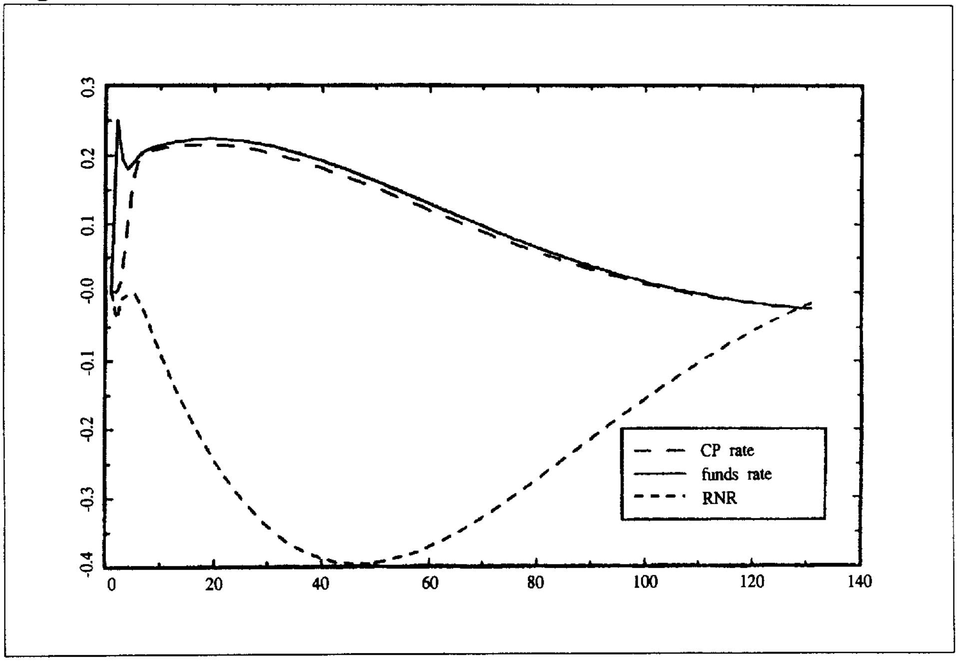
Figure 2a displays impulse-response functions for the effect of a shock of 25 basis points (bp) to the orthogonalized residual of the funds rate when the funds rate is placed first; in Figure 2b, the funds rate is placed second. In Figure 2a, the two interest rates go up by roughly 25 bp at the beginning, and then go down very slowly. RNR decreases slowly by $0.5 Billion for about one and a half years, and then goes up. In Figure 2b, the CP rate goes up about 20 bp, and goes down with the funds rate. RNR goes down slowly by less than $0.4 Billion for one and a half years, and then goes up. If the shock to the funds rate were caused by a shock to reserve demand, then one would expect RNR to go up rather than down. Therefore the observation is consistent with the argument that the shock comes from reserve supply.

Figure 3a displays impulse-response functions for the effect of a shock of 25 bp to the orthogonalized residual of the CP rate when the CP rate is placed first; in Figure 3b, the CP rate is placed second. In Figure 3a, the funds rate goes up by 22 bp, and goes down with the CP rate. RNR goes down slowly by about $0.4 Billion for one and a half years, and then goes up. In Figure 3b, the funds rate goes up by 10 bp, and then goes down with the CP rate. RNR goes down slowly by $0.15 Billion for one and a half years. The funds rate and RNR change only half as much in Figure 3b as in Figure 3a. If the shock to

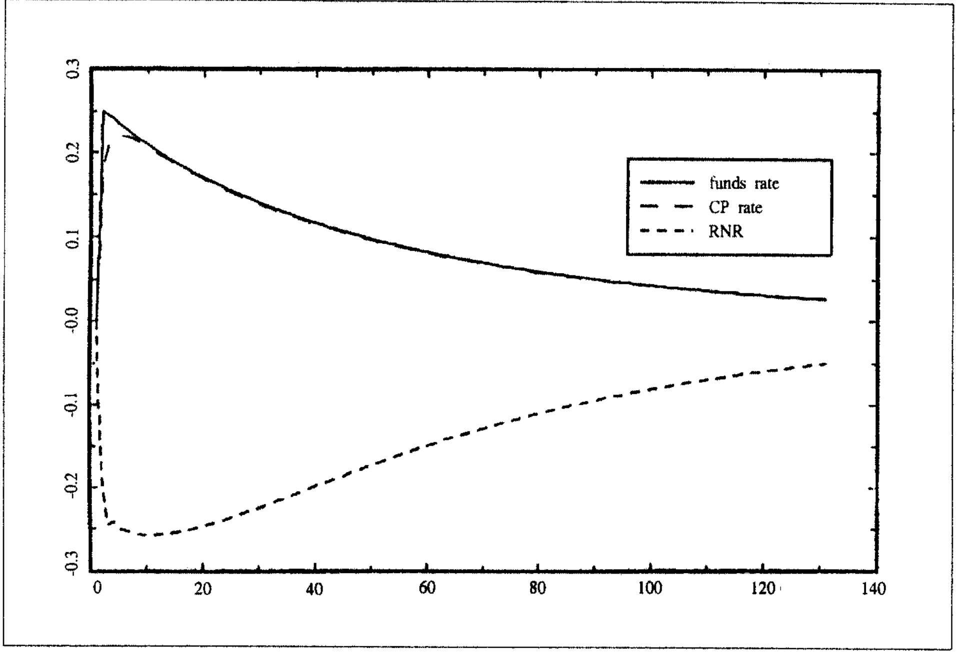
[Figure 2a] Shock in funds rate, Order: funds rate-CP rate-RNR



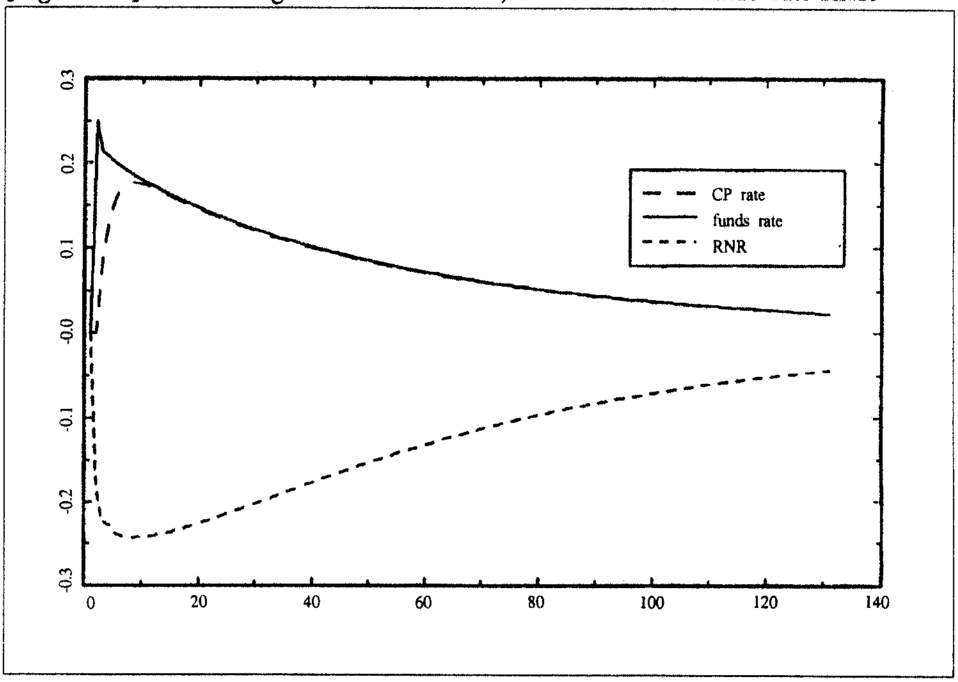
[Figure 2b] Shock in funds rate, Order: CP rate-funds rate-RNR



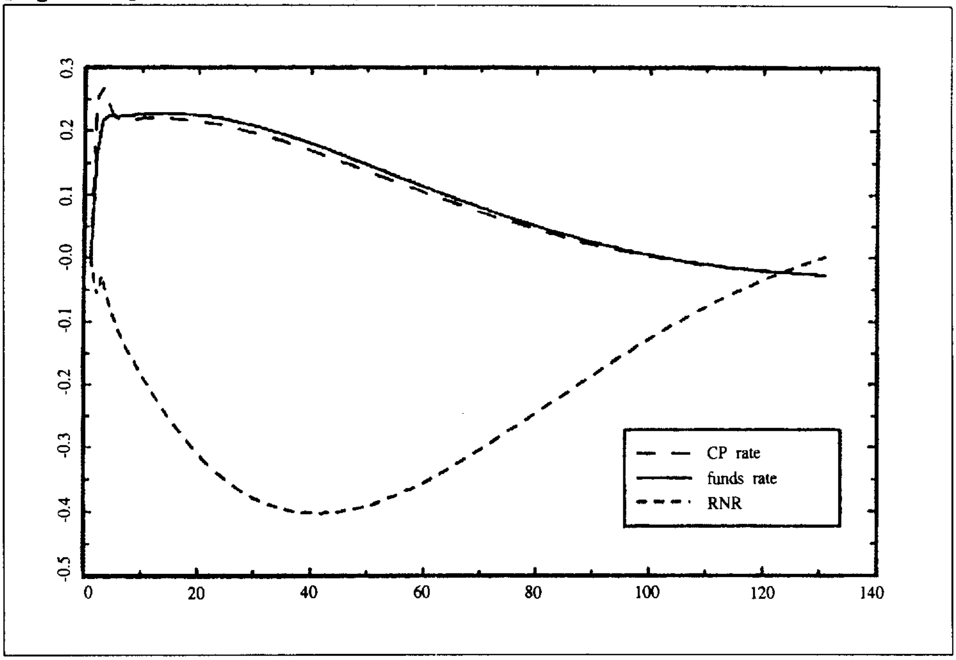
[Figure 2c] Shock in generated funds rate, Order: funds rate-CP rate-RNR



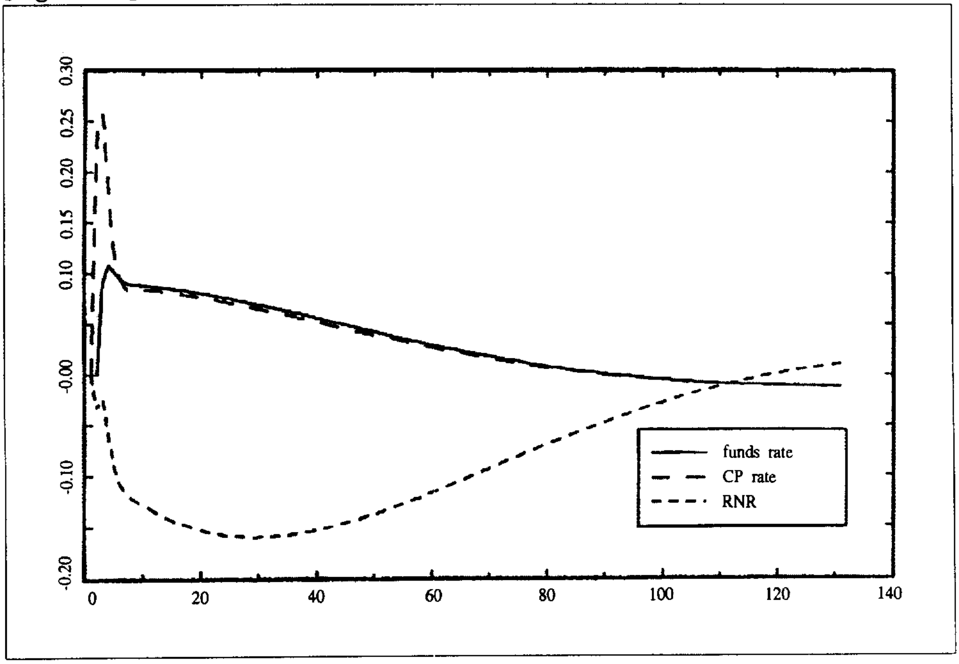
[Figure 2d] Shock in generated funds rate, Order: CP rate-funds rate-RNR



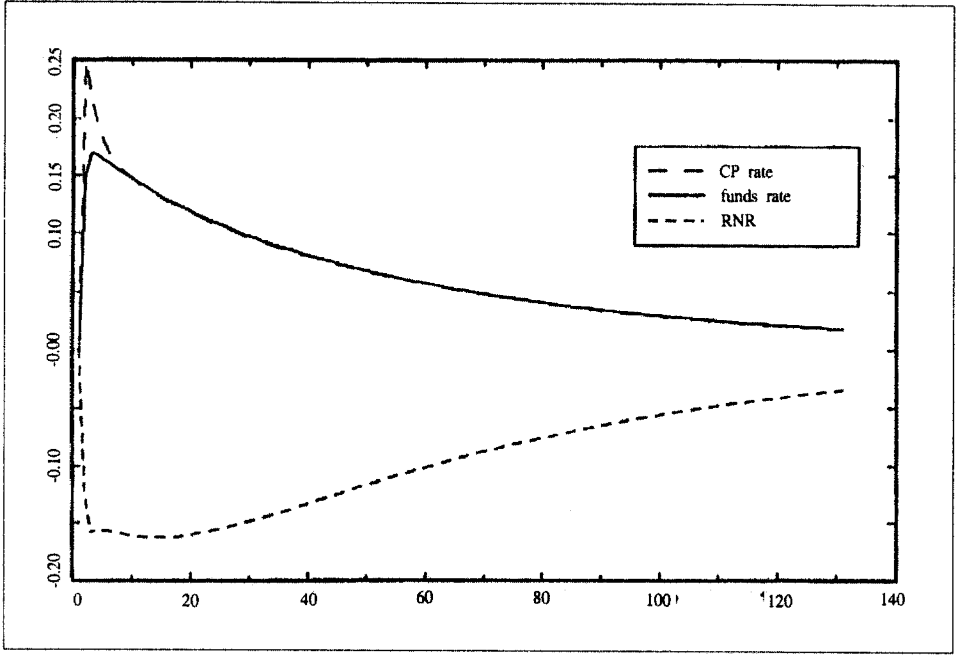
[Figure 3a] Shock in CP rate, Order: CP rate-funds rate-RNR



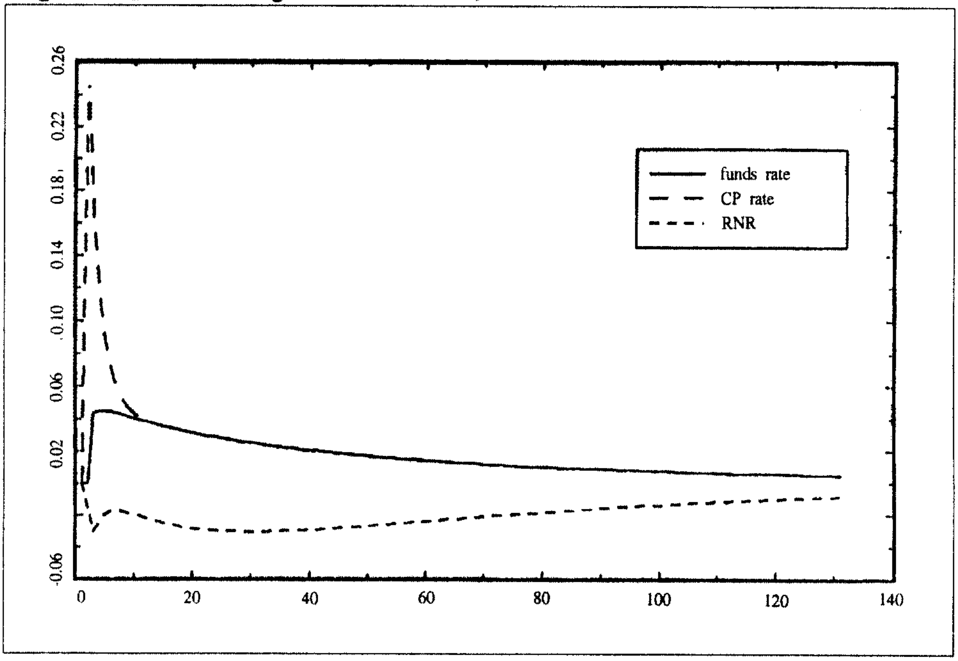
[Figure 3b] Shock in CP rate, Order: CP rate-funds rate-RNR



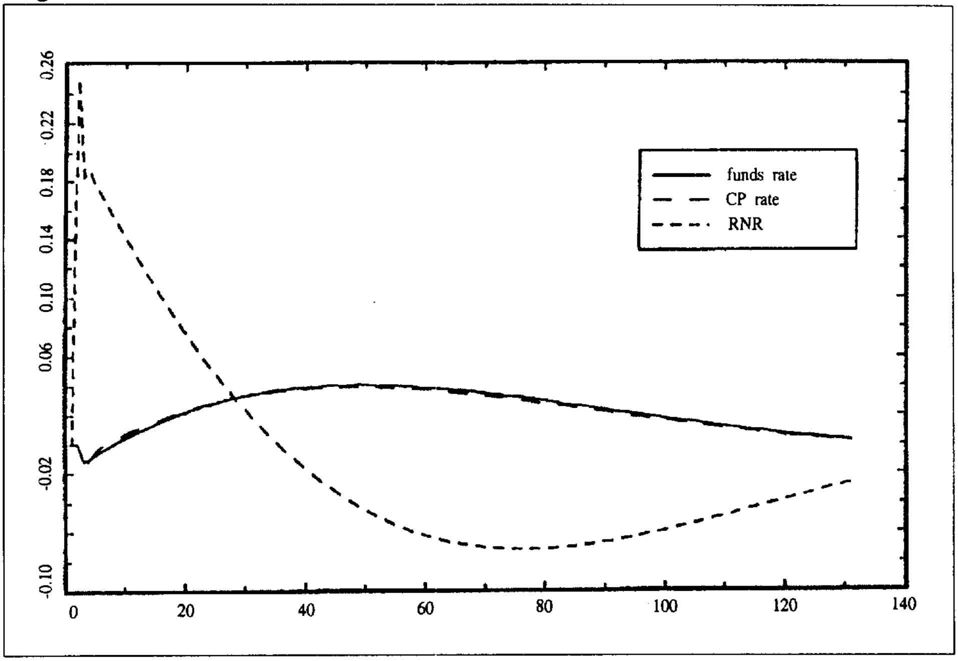
[Figure 3c] Shock in generated CP rate, Order: CP rate-funds rate-RNR



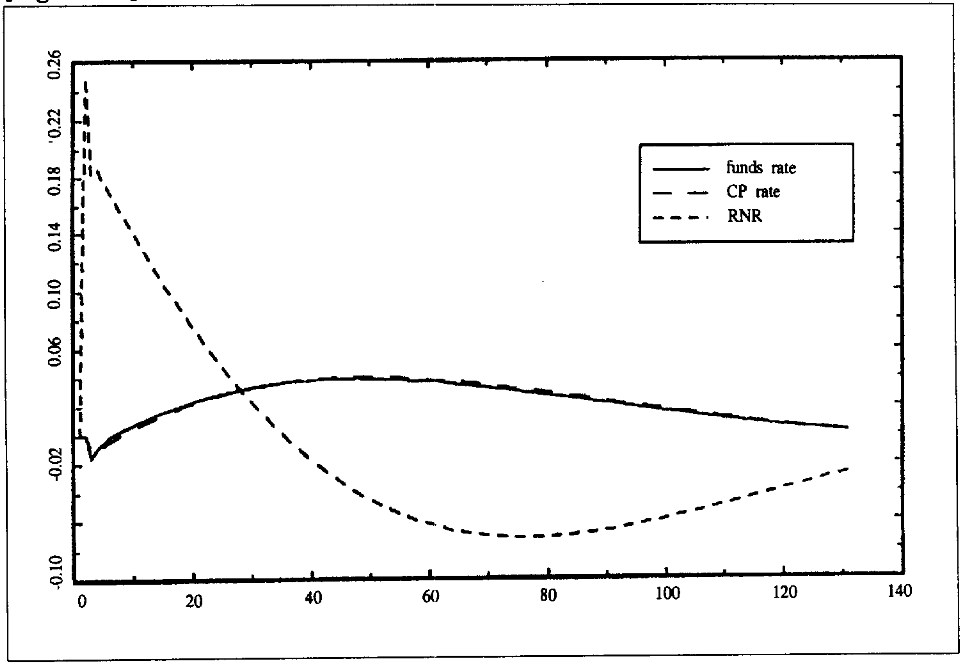
[Figure 3d] Shock in generated CP rate, Order: CP rate-funds rate-RNR



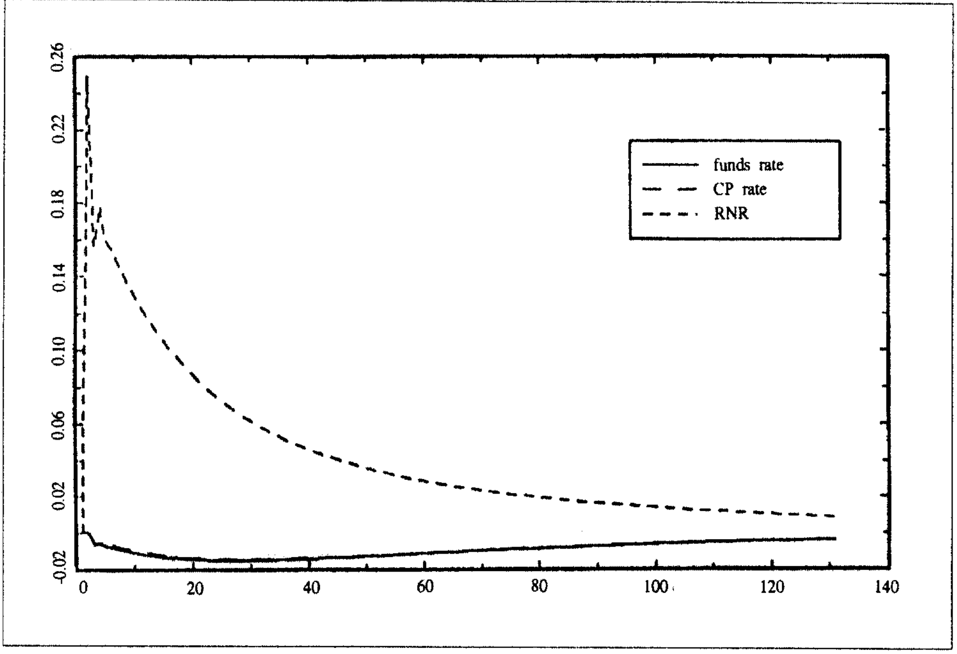
[Figure 4a] Shock in RNR, Order: funds rate-CP rate-RNR



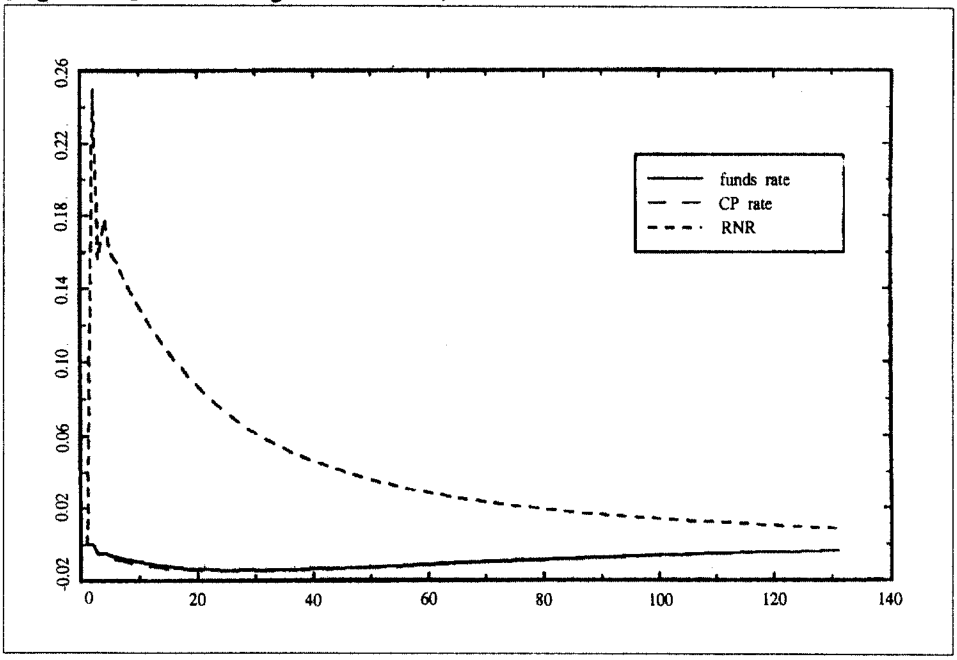
[Figure 4b] Shock in RNR, Order: funds rate-CP rate-RNR



[Figure 4c] Shock in generated RNR, Order: funds rate-CP rate-RNR



[Figure 4d] Shock in generated RNR, Order: funds rate-CP rate-RNR



the CP rate comes from loan demand, then RNR would be expected to go up rather than down, because the shock to loan demand will increase reserve demand. However, the shock cannot be interpreted as coming from the reserve supply shock, because if it did, the funds rate will change more than the CP rate. This observation suggests an interaction of loan demand and monetary policy. In Figures 4a and 4b, a shock of $0.25 Billion is given to the orthogonalized residual of RNR. The two interest rates fall very slightly initially, and later go up. After two years, two interest rates go back down, and RNR comes back up.

Ⅳ. CALIBRATION

The calibration involves two numerical problems. The first is to solve the model numerically given the values for the structural parameters. The second is to determine which values for the structural parameters to use. The first represents a mapping from the parameter space to a realization for a possible time path of $r_{L,t}$, $r_{F,t}$ and $\tilde{R}_t$ for the model economy. One method for getting a numerical solution for a nonlinear dynamic model is to find a fixed point of policy functions by iteration, and to stop iterating when the updates of the functions are smaller than a convergence criterion. This convergence criterion makes the mapping discontinuous. Therefore it is difficult to use a gradient-search method for the second numerical problem because of the discontinuity of the mapping from structural parameters to sample realizations. This paper employs the simplex method for numerical optimization because gradients are not used in the method.

The estimates are also functions of the random numbers, i.e. ε_{1t}, ε_{2t}, ε_{3t} and ε_{4t}, that are used to generate values for $r_{L,t}$, $r_{F,t}$ and $\tilde{R}_t$ from the model. They will be different if the seeds for ε_{1t}, ε_{2t}, ε_{3t} and ε_{4t} are different. The strategy used in this study is as follows. First, we obtained ball-park values for the parameters given a fixed set of random numbers, using the simplex method. Second, given the values for the parameters, 20 additional sets of realizations were generated to check the average impulse-response functions across data sets. Subsection (a) explains the method of finding a numerical solution to the model based on the parameterized expectations of Den Haan and Marcet (1993). Subsection (b) describes the criterion function suggested by Gallant and Tauchen (1996).

(a) Generating Numerical Solutions to the Theoretical Model

Summing equation (4) across two banks yields

$$r_{L,t}-\frac{\delta}{2}(L_t-L_{t-2})+\frac{r_{L,t}}{1+r_{L,t}}+\frac{\delta}{2(1+r_{L,t})}E_t\left[\frac{L_{t+2}-L_t}{1+r_{L,t+1}}\right] \tag{7}$$
$$=E_t r_{F,t}+\frac{E_t r_{F,t+1}}{1+r_{L,t}}$$

To solve for L_t, the terms $E_t\left[\frac{L_{t+2}}{1+r_{L,t+1}}\right], E_t\left[\frac{1}{1+r_{L,t+1}}\right]$, $E_t r_{F,t}$, and $E_t r_{t+1}$ in equation (7) have to be approximated. To solve for L_{1t}, the terms $E_t L_{1t+1}$, $E_t\left[\frac{L_{1t+2}}{1+r_{L,t+1}}\right]$, $E_t R_{1t}$, $E_t R_{1+1}$, $E_t D_{1t+1}$ and $E_t D_{1t+1}$ in equation (4) have to be approximated. A first-order exponential polynomial in the state variables is used to approximate the conditional expectations:

$$Y_t^j = E_t Y_t^j + u_t = \exp(\mu_t^j) + u_t, \qquad \text{where}$$

$$\mu_t^j = \tau_{j1} + \tau_{j2}\log L_{1t-1} + \tau_{j3}\log L_{1t-2} + \tau_{j4}\log L_{2t-1} + \tau_{j5}\log L_{2t-2} + \tau_{j6}\theta_t + \tau_{j7}\xi_{t-1} + \tau_{j8}\log\eta_{t-1},$$

$$\{Y_t^j\}_{j=1}^{10} = \left\{\frac{L_{t+2}}{1+r_{L,t+1}},\ \frac{1}{1+r_{Lt+1}},\ r_{F,t},\ r_{F,t+1},\ L_{1t+1},\ \frac{L_{1t+2}}{1+r_{L_{t+1}}},\ R_{1t},\ R_{1t+1},\ D_{1t},\ D_{1t+1}\right\}.$$

Given initial values for $\{\tau_{jk}\}_{j=1,\cdots,10,k=1,\cdots,8}$, the solutions of $\{L_{it}, r_{L,t}, R_{it}, D_{it}, F_{it}, r_{F,t}\}$ are generated. With these solutions, $\{\tau_{jk}\}_{j=1,\cdots,10,k=1,\cdots,8}$ are estimated using nonlinear least squares, which minimizes $\frac{1}{T}\sum_{t=1}^{T}[Y_{jt}-\exp(\mu_t^j)]^2$. Iterations of this procedure continue until the magnitude of update of the estimates of $\{\tau_{jk}\}_{j=1,\cdots,10,k=1,\cdots,8}$ is less than a tolerance level. That is, $\sum_{k=1}^{8}\sum_{j=1}^{10}\left|\frac{\tau_{jk}^{m+1}-\tau_{jk}^{m}}{\tau_{jk}^{m}}\right| < 0.02$.[5]

(b) The Criterion Function

The empirical model employed to determine the values of the structural parameters is a trivariate VAR(1) with Normal innovations. Associated with this Normal VAR(1) is a likelihood function, and the derivative of the log likelihood with respect to the vector of parameters of the VAR is known as the score. The score is a function of the VAR parameters and of the data. If the data were really generated from the process modeled by the VAR, and if the score were evaluated at the true value for the VAR parameters, then the expected value of the score, where this expectation is with respect to alternative possible realizations of the data, would equal zero.

Now, suppose we generate the data not from a VAR but from a structural model, namely, from the theoretical model described in Section 2. These generated data are implicitly a function of the values for the structural parameters we used in the theoretical model. We can evaluate the scores for a

[5] The sum of the absolute percentage changes of 80 parameter estimates should be less than 2 %. The index for iterations is m.

VAR, using data generated by the model but with VAR parameters estimated from the actual data. If the theoretical model is consistent with the data as summarized by the VAR, then these generated scores should be zero on average. In this way, the simulated VAR scores can be used as orthogonality conditions for the structural parameters.

Employing the covariance matrix of the scores as the efficient weighting matrix, the criterion function is as follows:

$$\hat{\Psi}_n = \operatorname{argmin}_{\Psi\in R}\, m_n'(\Psi, \tilde{\theta}_n)(\hat{I}_n)^{-1} m_n(\Psi, \tilde{\theta}_n), \quad \text{where}$$

$$m_n(\Psi, \tilde{\theta}_n) = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\partial}{\partial\theta}\right)\ln f(\tilde{y}_t \mid \tilde{x}_t, \tilde{\theta}_n)$$

$$\tilde{\theta}_n = \arg\max_{\theta\in\Theta}\frac{1}{n}\sum_{t=1}^{n}\ln f(y_t \mid x_t, \theta),$$

$$\tilde{I}_n = \sum_{v=-[n^{1/5}]}^{v=+[n^{1/5}]} w\left(\frac{v}{n^{1/5}}\right) S_{n,v}$$

$$w(x) = \begin{cases} 1-6|x|^2+6|x|^3 & \text{if } 0\le |x|\le \frac{1}{2} \\ 2(1-|x|)^3 & \text{if } \frac{1}{2}\le |x|\le 1 \end{cases}$$

$$S_{n,v} = \begin{cases} \frac{1}{n}\sum_{t=1+v}^{n}\left[\frac{\partial}{\partial\theta}\ln f(y_t \mid x_t, \tilde{\theta}_n)\right]\left[\frac{\partial}{\partial\theta}\ln f(y_{t-v} \mid x_{t-v}, \tilde{\theta}_n)\right] & \text{if } v\ge 0 \\ (S_{n,-v})' & \text{if } v<0 \end{cases}$$

and $\{f(x_1 \mid \theta),\ f(y_t \mid x_t, \theta)\}_{t=2}^{\infty}$ is the sequence of densities of the trivariate VAR with two interest rates and RNR, $\{y_t, x_t\}$ are real data, and $\{\tilde{y}_t, \tilde{x}_t\}$ are generated data.[6] The sample size T for the random numbers was taken to be 2000.

V. COMPARISON BETWEEN REAL AND GENERATED DATA

The impulse-response function for the real and generated data are similar in their patterns of ups and downs, but not in terms of magnitude and timing[7]. Figures 2c and 2d are the counterparts of Figures 2a and 2b respectively, which means that they are different only because the former uses real data and the latter generated data. Comparing Figure 2a with Figure 2c and Figure 2b with Figure 2d, the similarity is that the CP rate closely follows the funds rate, and RNR keeps decreasing for the first half year. Their main difference is that the two interest rates stay above 20 bp more than a year in Figure 2a and 2b, but

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[6] In this paper, $x_t = (y_{t-1}, y_{t-2}, \cdots, y_{t-p})$, where p is the number of lags, $y_t = \{ff_t, cp_t, \pi_t\}$, where ff_t is the federal funds rate, cp_t is the one-month commericial paper rate, π_t is the inflation rate. $f(\cdot \mid \cdot)$ is the trivariate normal density.

[7] The statistical significance of the closeness of the impulse response functions between real and generated data cannot be provided in this paper. The reason is that the confidence intervals of the parameter estimates are not calculated because of the large computational burden. Due to this burden, simplex method was chosen over other optimization modules using gradients.

not Figure 2c and 2d. Comparing Figures 2c and 2d, the ordering of variables does not make a big difference as in Figure 2a and 2b; therefore the contemporaneous correlation between the innovations is not important in explaining the effect of the shock to the residual of the funds rate. The reason is that the forecasting power of the funds rate to the CP rate comes from the Fed's targeting and the banks' arbitrage through time[8].

Figures 3c and 3d are the counterparts of Figures 3a and 3b respectively. The comparisons between Figures 3a and 3c are almost the same as those between Figures 2a and 2c. In Figure 3d, the funds rate does not increase as much as the CP rate at the beginning, as in Figure 3b. RNR goes down less than half as much in Figure 3b as in Figure 3a, and so does RNR in Figure 3d when compared to Figure 3c. Since the innovations in the CP rate come from loan demand shocks, we might expect that RNR would go up because of the positive shock to loan demand. That is, if loan demand goes up, then the loan rate and volume increase, reserve demand is supposed to go up, and so RNR goes up. The reason why RNR goes down instead of going up while reserve demand goes up is the "leaning against the wind" policy implicit in the generated data. The monetary authority increases the funds rate target in reaction to the positive loan demand shock, which decreases RNR.

Figures 4c and 4d are the counterparts of Figures 4a and 4b respectively. In Figures 4c and 4d, the two interest rates do not go above zero, but they do in Figures 4a and 4b. In terms of change, RNR and the two interest rates move in roughly opposite directions.

The third column of Table I shows the averages, standard deviations and correlations of the generated funds rate, CP rate and RNR. The correlations among the level and differenced variables, $\rho_{ff,cp}$, $\rho_{ff,RNR}$, $\rho_{cp,RNR}$, $\rho_{\Delta ff,\Delta cp}$ in Table I, are matched well, and so are the standard deviations, i.e. s.d. of $cp-ff$ in Table I, of the spread between the funds rate and the CP rate. Since the structural parameters are not chosen to match these moments, their similarity is encouraging. The standard deviations of the generated data are much less than those of real data. To generate the large standard deviations, the target shock should be highly persistent, which would be inconsistent with the relatively quick mean reversion of the funds rate in the impulse response in Figure 2a.

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[8] The marginal condition, equation (4), says that the marginal benefit of making one additional unit of loan is equal to the marginal cost of borrowing that of federal funds. In this equation, the loan rate, $r_{L,t}$ is the function of the expectations, that is,

$E_t\left(\frac{L_{i,t+2}-L_{i,t}}{1+r_{L,t+1}}\right)$, $E_t r_{F,t}$, $E_t(D_{it}-L_{it}-L_{it-1}-R^*_{it}(Z_t))$, $E_t(D_{it+1}-L_{it+1}-L_{it}-R^*_{it+1}(Z_{t+1}))$,

$E_t r_{F,t+1}$. These expectations are the functions of the state variables,
$X_t=\{L_{1t-1}, L_{1t-2}, L_{2t-1}, L_{2t-2}, L_{2t-1}, \theta_t, \xi_{t-1}, \eta_{t-1}\}$. Here ξ_{t-1} is the monetary policy shock which determines the lagged federal funds rate target; $r^t_{F,t-1}=\omega\times\exp(\xi_{t-1})$. The lagged funds rate contains information about the lagged target. Therefore the lagged funds rate has forecasting power for the loan rate.

The funds rate in this paper is regarded as a biweekly interest rate and the basis of the one-month loan rate. The expectations hypothesis of the term structure of interest rates holds that the long-term interest rate is the weighted average of short-term interest rates, e.g., $r_{L,t}=\frac{r_{F,t}+E_t r_{F,t+1}}{2}$. Assuming that the expectation of the future funds rate is rational, then $E_t r_{F,t+1}=r_{F,t+1}+\varepsilon_{t+1}$, where ε_{t+1} is orthogonal to the information set up to period t. Combining the two equations and rearranging as in Fama (1986) produces

$$r_{L,t}-\frac{r_{F,t}+r_{F,t+1}}{2}=k_0+k_1(r_{L,t}-r_{F,t})+u_{1,t} \tag{8}$$

$$\frac{r_{F,t+1}-r_{F,t}}{2}=\lambda_0+\lambda_1(r_{L,t}-r_{F,t})+u_{2t} \tag{9}$$

where k_1 and λ_1 should be zero and unity respectively under the traditional rational expectations hypothesis of the term structure. Table III shows the estimation results of (8) and (9) with real and generated data. The estimates with real data in the second column are far away from the predicted values of the hypothesis. The estimates of k_0, k_1, λ_0 and λ_1, and the standard errors with generated data well approximate the real ones, however. This similarity between the estimates from the real and generated data increases credit to the theoretical model of this paper.

The reason for this close approximation can be explained by comparing the Fama equations with the first order conditions of the model. Rearranging equation (7) produces

$$\begin{aligned} r_{L,t}-\frac{(1+r_{L,t})E_t r_{F,t}+E_t r_{F,t+1}}{2+r_{L,t}} \\ =0.5\delta\frac{1+r_{L,t}}{2+r_{L,t}}(L_t-L_{t-2})-0.5\delta\frac{1}{2+r_{L,t}}E_t\left(\frac{L_{t+2}-L_t}{1+r_{L,t+1}}\right) \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{E_t(r_{F,t+1}-r_{F,t})}{2}=\frac{2+r_{L,t}}{2}(r_{L,t}-E_r r_{F,t}) \\ -\frac{\delta(1+r_{L,t})}{4}(L_t-L_{t-2})+\frac{\delta}{4}E_t\left[\frac{L_{t+2}-L_t}{1+r_{L,t+1}}\right] \end{aligned} \tag{11}$$

If $\delta=0$ and $E_t r_{F,t}=r_{F,t}$, then (10) and (11) are close to (8) and (9) when $k_0=k_1=0$ and $\lambda_0=0$, $\lambda_1=1$ because $r_{L,t}$ is between 0.2 and zero in real and generated data; therefore, $\frac{1+r_{L,t}}{2+r_{L,t}}$, $\frac{1}{2+r_{L,t}}$ are not far away from 0.5 and $\frac{2+r_{L,t}}{2}$ is not far away from 1. The parameter δ represents the cost of moving funds between the loan market and the funds market. The reason why $E_t r_{F,t}\neq r_{F,t}$ is that banks cannot know $r_{F,t}$ when they make decisions about

[Table III] Evidence on the Term Structure of Interest Rates

parameter	real data	generated data
k_0	0.0109 (.00624)	0.000 (0.00029)
k_1	0.876 (0.0251)	0.936 (0.014)
σ_{u1}	0.108	0.0972(0.0074)
λ_0	-0.0109 (0.0062)	0.000 (0.00030)
λ_1	0.123 (0.0250)	0.0638(0.0148)
σ_{u2}	0.108	0.0972(0.0074)

Note: The numbers in parentheses in the second column are standard errors, those in the third column are the standard deviations of 20 estimates from 20 data sets.

[Table IVa] Calibrated Structural Parameters (taken as given and not estimated)

Parameter	Values	Note
ω	0.06545	mean of the loan rate, the funds rate target (6.545 % APR)
α	0.12	marginal benefit when $R_t = 0$ (12 % APR)
tb	0.004	width of target band (40 basis points) : $[r^t_{F,t} - 0.5tb,\ r^t_{F,t} + 0.5tb]$
ρ_3	0.6	AR(1) coefficient of the demand deposit split ratio : $\eta_t = 0.5 \times (1 - \rho_3) + \rho_3 \eta_{t-1} + \varepsilon_{3t}$
$\sigma_{\varepsilon 3}$	0.002	Standard Deviation of : $\eta_t = 0.5 \times (1 - \rho_3) + \rho_3 \eta_{t-1} + \varepsilon_{3t}$
γ_2	0.0025	transaction cost of federal funds trading (25 basis points)

the level of loans. Therefore the traditional hypothesis approximately holds when there are no friction between the short and long term security markets.

VI. INERPRETATION OF THE STRUCTURAL MODEL

(a) The Structural Parameters

Table IVa shows the structural parameters, i.e., the parameters in the model of Section 2, of which values are not estimated but given *a priori*. The parameter ω used in equation (6) determines the mean of the federal funds rate target. Its value is chosen to match the average of the funds rate. The parameter α is the marginal benefit of holding an infinitesimal amount of reserves. Its value is set so as to match the average level of RNR. The

[Table IVb] Estimated Values of Structural Parameters

parameter	estimated values	Note
ρ_1	0.982	AR(1) Coefficient for loan demand shocks: $\theta_t = \rho_1 \theta_{t-1} + \varepsilon_{1t}$
$\sigma_{\varepsilon 1}$	0.044	Standard Deviation for loan demand innovation, : ε_{1t} : $\theta_t = \rho_1 \theta_{t-1} + \varepsilon_{1t}$
ϕ	0.196	Elasticity of loan demand : $r_{L,t} = \omega \times \exp(\theta_t) \times \left(\frac{L_t^d}{L}\right)^{-\frac{1}{\phi}}$
ρ_2	0.979	AR(1) Coefficient of the target shock: $\xi_t = \rho_2 \xi_{t-1} + \xi_{2t}$
$\sigma_{\varepsilon 2}$	0.0324	Standard Deviation of the target innovation: $\xi_t = \rho_2 \xi_{t-1} + \xi_{2t}$
δ	0.00706 (70.6 bp)	Parameter for the loan adjustment cost: $\delta(L_{it} - L_{it-2})^2$
$\rho_{\varepsilon 1, \varepsilon 2}$	0.592	Correlation between the two innovations, ε_{1t} and ε_{2t}
γ_1	0.00395	$r_{F,t} = \alpha - 0.5\gamma_1 \bar{R}_t + 0.5\beta \sum_{i=1}^{2} (R_{it}^* - 0.1D_{it})^{-2}$
$\sigma_{\varepsilon 4}$	0.00039	$R_t = R_t + \varepsilon_{4t}$
β	0.00112 (11.2 bp)	$r_{F,t} = \alpha - 0.5\gamma_1 \bar{R}_t + 0.5\beta \sum_{i=1}^{2} (R_{it}^* - 0.1D_{it})^{-2}$
L	251.4 ($ Billion)	$r_{L,t} = \omega \times \exp(\theta_t) \times \left(\frac{L_t^d}{L}\right)^{-\frac{1}{\phi}}$

parameter, *tb*, which is the width of the target band, is set at 0.004, i.e., 40 bp, which is consistent with the ability of the lagged funds rate to forecast the CP rate. The last three parameters are set so as to match the standard deviations of the differenced loan rate, funds rate, and RNR.

Table IVb shows the parameter estimates. The loan demand shock, θ_t, and the funds rate target shock, ξ_t, are highly persistent since $\hat{\rho}_1$ and $\hat{\rho}_2$ are 0.982 and 0.979 respectively. The elasticity of loan demand, i.e., the slope ϕ of the log of the loan demand curve, is 0.196. If the loan rate changes by 5% from 6.5% APR to 6.82% APR, then the loan demand decreases by around 1%. The magnitude of δ, 0.00706, means that it costs \$135,769 [9] to increase the loan by \$1 Billion. The magnitude of $\rho_{\varepsilon 1, \varepsilon 2}$, 0.592, means that 35% of the variation of the target innovations comes from the endogenous response of monetary policy to loan demand innovations. The remaining 65% of the variation comes from the exogenous shock to monetary policy. The magnitude of $\hat{\gamma}_1$, 0.00395, means that if the total reserves increase by \$1 Billion with the excess reserves of each bank fixed, then the funds rate increases by around 20 bp from equation (3). The magnitude of β, 0.00112, means that when each bank has excess reserves of \$1 Billion, then the portion of the funds rate explained by the marginal benefit of the excess reserves is 11.2 bp. The standard deviation of the noise in the reserves is \$0.39 Billion, which is around one one-hundredth of the total reserves.

(b) The Impulse Response Analysis with the Model

Consider the following experiment to estimate the dynamic effect of shocks to reserve supply on the two interest rates. For each set of data, the innovations of the target shock changes as follows:

$$\tilde{\varepsilon}_{2t}^{i} = \varepsilon_{2t}^{i} + q_t^i, \ t = 1000$$
$$\tilde{\varepsilon}_{2t}^{i} = \varepsilon_{2t}^{i} \ \ otherwise,$$

where $i = 1, 2, \cdots, 20$, and q_t^i is the magnitude which causes the federal funds rate target to change by 25 bp. As a result, the target changes by 25 bp independently of the shocks to the loan demand. Let $\{r_{L,t}^i, \ r_{F,t}^i, \ L_t^i + L_{t-1}^i, \ R_t^i\}_{t=2}^{2000}$ be the solution with $\{\Sigma_{2t}^i\}_{t=1}^{2000}$, and $\{\tilde{r}_{L,t}^i, \ \tilde{r}_{F,t}^i, \ \tilde{L}_t^i + \tilde{L}_t^i + \tilde{L}_{t-1}^i, \ \tilde{R}_t^i\}_{t=2}^{2000}$ with $\{\tilde{\varepsilon}_{2t}^i\}_{t=1}^{2000}$. The differences between them are calculated, and averaged across 20 data sets. Most differences occur between $t = 1000$ and $t = 1150$.

Figure 5 shows the differences. The funds rate increases by 13 bp in the first period, which is less than the target change because the funds rate is allowed to be different from the target within the target band. After seven fortnights, the funds rate increases to about 18.7 bp. It becomes around 15 bp after one year, and 10 bp after two years. The CP rate does not increase in the first period because the target change does not affect the CP rate contemporaneously. In the second period it becomes 6.6 bp, and keeps increasing to 18.6 bp for seven fortnights. It declines to around 15 bp after one year, and to 9 bp after two

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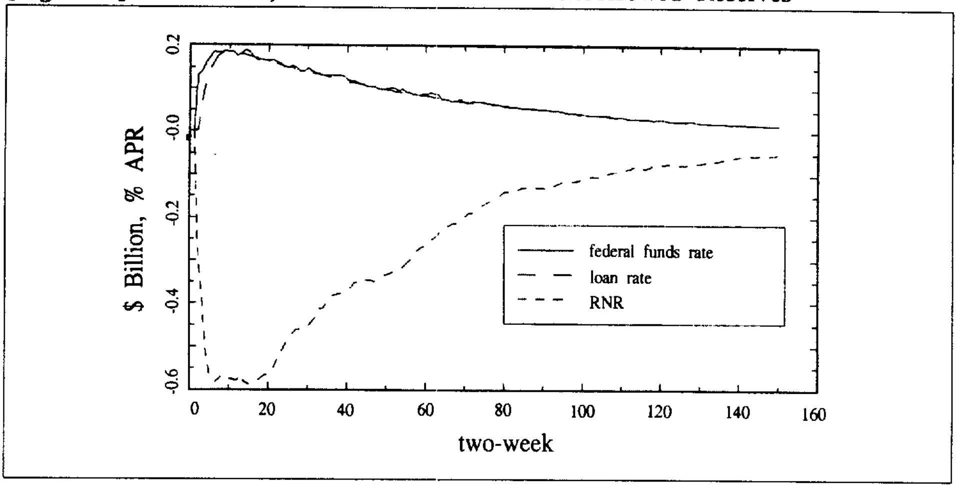
[9] $135,769 \approx 0.5 \times \frac{0.00706}{26} \times (\$1\,Billion)^2$. The number of weeks in one year is regarded as 26. The simulation was done in annual terms.

[Figure 5] Funds Rate, Loan Rate and Real Nonborrowed Reserves



years. RNR decreases by $0.28 billion in the first period, and becomes -$0.57 billion after seven fortnights. In the first period, the funds rate increases by 13 bp and RNR decreases by $0.28 billion. For seven fortnights, the funds rate increases by 5.7 bp, while RNR decreased by $0.30 Billion. That is, in the first period $0.28 billion decrease of RNR increased the funds rate by 13 bp, while in the subsequent seven fortnights $0.30 billion decrease of RNR increased the funds rate just 5.7 bp, which is less than the half of the first period's change, i.e. 13bp. The reason for this asymmetry lies in the dynamics between the volumes of loans and required reserves. When the funds rate increases, the cost of making loans increases and thus the volume of loans decreases, which induces required reserves to decrease and thus funds rate to go down. To keep the funds rate persistently higher than before, the Fed keeps draining reserves.

Figure 5 shows the dynamic effect of open market operations on the two interest rates. The effect does not depend on *ad hoc* identifying assumptions in a VAR, but on the identifying assumptions for describing the behavior of the economic agents, the plausibility of which is checked in the process of matching the statistics between real and generated data. Figure 5 shows the same ups and downs of variables as those in Figures 2a and 2b, where a shock is given to the orthogonalized residual of the funds rate. The CP rate increases, following the funds rate, and RNR goes down for the first ten periods. Figure 5 supports the argument that Figures 2a and 2b show the effect of open market operations on the two interest rates while banks maximize their profit by adjusting their portfolio.

Ⅶ. CONCLUSION

This paper took two steps to analyze the reserve market. The first was to estimate a VAR with the two interest rates and real nonborrowed reserves. The second was to interpret the estimation result in the context of the dynamic optimization of banks and the endogenous response of the monetary authority to loan demand. The marginal benefit of making one more unit of loans is the same as the marginal cost of borrowing one more unit of federal funds; this explains the power of the lagged funds rate to forecast the one-month commercial paper rate. The Fed increases the target when loan demand is high, and decreases the target when it is low; this explains the power of the lagged one-month commercial paper rate to forecast the funds rate.

By calibrating the model to match the VAR estimation results of real and generated data, the dynamic effect of open market operations on the two interest rates was investigated in the context of banks' dynamic optimization. If an exogenous shock is given to the target *ceteris paribus*, then the funds rate is forced to follow the target because of the Fed's policy actions, and the CP rate follows the funds rate by banks' arbitrage between the two markets. The impulse-response functions for the effect of a shock to the orthogonalized residual of the funds rate using the estimated VAR are very similar to the impulse-response functions for the effect of a shock to the federal funds rate target, which is a true structural shock. And it was shown in section 6 that the 65% of the variation in the target comes from the exogenous shock to monetary policy, and its 35% of the variation comes from endogenous response of monetary policy to loan demand innovations. Therefore this experiment supports the argument that reserve supply shocks dominate reserve demand shocks in the residual of the funds rate equation in the VAR from 1984:3 to 1996:1. I conclude that the residual offers a reasonable proxy for an exogenous shock to monetary policy.

One of the preconditions for the validity of this result is the rule of the policy, which is to stabilize the volume of loans. The stable volume of loans brings the stable transaction services in this model. Therefore, the monetary policy does not instigate any expectations of inflation or deflation, but mitigates them with the rule. This can be a justification for the assertion that the model in this paper, which is a real model, can explain the effect of open market operations. One of the future tasks is to expand this partial equilibrium model to a general equilibrium model, and to consider the inflationary effects of monetary policy.

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