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FISCAL POLICY EFFECTS ON CONSUMPTION AND CURRENT ACCOUNT BY USING A DISCONTINUOUS TIME HORIZON MODEL IN KOREA

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This paper has investigated the responses of consumption and current account in Korea, to the fiscal policies using a discontinuous time horizon model in which the fiscal policy effects can be different according to whether economic agents have the finite or infinite time horizons.

This paper uses the Korean quarterly data for the period 1976:I -1997:IV to test the theoretical model empirically. The empirical results show that private consumption and current account are not significantly affected by government debt in contrast to conventional Keyensian macroeconomic model. Ricardian equivalence proposition with the infinite horizons seems to be more persuasive to explain the fluctuation of consumption and current account in Korea.

JEL Classification: F31, F32
Keywords: Discontinuous Time Horizon, Fiscal Policy, REP.

I. INTRODUCTION

The conventional Keyensian macroeconomic analysis tells us that the government deficit and debt affect the economy. That is, the domestic consumption decreases and the current account is worse.

Barro (1974, 1990), however, argued that households need not view government deficit as the net wealth using the intertemporal budget constraint in the context of Ricardian Equivalence Proposition (REP). Thus the conventional Keyensian macroeconomic view has been challenged by Evans (1985, 1986, 1988, 1990).

There is no convincing reason for the wealth effect of government debt because the future tax liabilities implicit in debt financing are foreseen by

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forward-looking economic agents and because intergenerational transfers and bequest mechanism are available to intertemporal optimizing economic agents. As a consequence, changes in the relative amounts of tax and debt finance for given amount of government spending would have no effect on the private consumption and interest rate.

A major purpose of this paper is to study the effects of fiscal policies on the consumption and current account using a somewhat modified intertemporal general equilibrium model. Accordingly, the undertaking in this paper is especially of interest in view of following considerations.

First, this paper develops the stochastic intertemporal general equilibrium model of consumption and current account of balance of payments.

Second, this paper allows for the channel of infinite life time horizon that may rise to deviation from conventional macroeconomic policy implication[1]. And this paper extends REP to the open economy version.

Third, since it has been a hot issue whether conventional macroeconomic policy is effective or not in that context, it will be important to test the implication of REP in Korea.

To develop and extend Blanchard(1985), Frenkel and Razin(1987), Obstfeld (1989), and Evans (1990), Obstfeld and Rogoff(1996), and Betts and Devereux (1996), this paper is based on an intertemporal general equilibrium model of fiscal policy on private consumption and current account.

The rest of this paper is organized as follows. As a variant of Blanchard model(1985), section II lays out the stochastic discontinuous intertemporal optimizing model economy and the specification of the model. Section III shows the comparative statics of government debt and government consumption. Section IV explores the empirical econometric models and specification. Section V describes the data in the empirical analysis, investigates the unit root and generalized cointegration of economic variables, and presents generalized method of moment (GMM) tests on the implications derived from the theoretical models. Section VI presents the conclusion.

II. INTERTEMPORAL OPTIMIZING MODEL

This section develops an intertemporal optimizing model similar to that of Frenkel and Razin(1987) who develop an international version of Blanchard's (1985) uncertain life-time model with a fixed labor force and fixed supplies of output. The focus of Frenkel and Razin model(1987) is on the dynamics of real exchange rate. This paper modifies and extends Evans model (1988, 1990) to real consumption and current account using stochastic real wage income.

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[1] See Obstfeld and Rogoff(1996), Betts and Devereux(1996), and Eichenbaum and Evans(1995).

2.1. Private Agent Consumption

The representative household of age a attempts to maximize the time-separable utility function which is also separable from consumption else, and which is assumed to reside within the constant relative aversion family. We assume that the household maximizes the expected value of the integral of discounted future instantaneous utility over the life-time horizon which is computed on the basis of his or her probability of survival. With the constant instantaneous probability of survival $\delta(\leq\delta\leq 1)$, the representative household maximizes the following utility function which consist of exportable good (C_x) and importable good (C_M):

$$V_t = E_t \sum_{i=0}^{\infty} \beta^i U(C_{a,Xt+i}, C_{a,Mt+i}) = \sum_{i=0}^{\infty} (\delta\beta)^i \left[\frac{C_{a,t+i}^{1-\theta}}{1-\theta} \right], \quad \theta \neq 1, \tag{1}$$

where $U(C_{a,Xt+i}, C_{a,Mt+i}) = \alpha \log C_{a,Xt} + (1-\alpha) \log C_{a,Mt}, \quad \theta = 1$

E_t is the expectation operator conditional on informational available in the period t. V is the expected utility, U is a function that is increasing and concave in its two arguments. $C(=C_X^{\alpha} C_M^{1-\alpha})$ consists of the exportable goods and importable goods. The household has the infinite horizon if $\delta=1$ and the household has the finite horizon if $0<\delta<1$ as in Blanchard model(1985). θ denotes the reciprocal of intertemporal elasticity of substitution(σ). α is a share parameter measuring the degree to which the consumption of exportable goods contributes to the household utility. $\beta(>0)$ is the constant subjective discount factor.

$Z_{a,t}$ denotes the total physical consumption in terms of exportable good which consists of the exportable good$(C_{a,Xt})$ and importable good$(C_{a,Mt})$ of a household of age a in the period t.

$$Z_{a,t} = C_{a,Xt} + P_t C_{a,Mt}, \tag{2}$$

where P_t is the price of importable good in terms of exportable good so that $(1/P_t)$ is defined as the terms of trade.

If there exists an efficient and competitive annuity market with free entry here, the household of age a maximizes the equation (1) subject to the following individual constraint in the period t.

$$Z_{a,t} + A_{a,t} = W_{a,t} - T_{a,t} + (1/\delta)(1+\gamma_t) A_{a-1,t-1}, \tag{3}$$

where $A_{a,t}$ denotes the end of period bond purchases issues by home and foreign countries with the real interest rate γ_t which is determined in the world market, and at which households can freely borrow and lend in a small country.

Assume that $W_{a,t}$ denotes the stochastic real wage income measured in exportable good. $T_{a,t}$ denotes the real nondistortionary lump-sum tax of the household of age a at time t. All assets and taxes as well as interest rates are measured in terms of exportable good.

Equation (3) states the sum of the real value of the household's current consumption and bonds equals to the sum of the real value of current after tax income and the gross real return from last period's bond purchases under the perfect asset market.[2] The household maximizes the equation (1) subject to the equation (3). Then we characterize the solution using the maximum principle. The optimal conditions for the private agent's problem are obtained from the Bellman equation principle. Thus we can derive the first order conditions:

$$\frac{(1-\alpha)C_{a,Xt}}{\alpha C_{a,Nt}} = P_t, \tag{4a}$$

$$\left[\frac{C_{a,Xt+1}}{C_{a,Xt}}\right] = \beta(1+r_{t+1}), \tag{4b}$$

$$\left[\frac{C_{t+1}}{C_t}\right]^{\theta} = \beta(1+r_{t+1})\left(\frac{P_{t+1}}{P_t}\right)^{\alpha-1}. \tag{4c}$$

The equation (4a) implies that the household chooses the exportable and importable goods to equate the relative price and the marginal rate of substitution, and equations (4b) and (4c) ensure that household adjusts his consumption profile so that the marginal rate of substitution between current and future consumption is equal to the subjectively discounted return to bond.

Now consider the derivation of aggregate consumption. Blanchard (1985) simplified that age distribution of population is constant over time and population is normalized to 1 so that at any time period t there are $(1-\delta)\delta^a$ cohorts of age a. Since the aggregate size of population is

$$(1-\delta)\sum_{a=0}^{\infty}\delta^a = 1,$$

the aggregate consumption in terms of exportable good at time t is

$$Z_t = (1-\delta)\sum_{a=0}^{\infty}\delta^a(C_{a,Xt} + P_tC_{a,Mt}). \tag{5}$$

Aggregating equation (3), the aggregate budget constraint of households can be

[2] Assume that the asset in the period 0 is fixed.

$$
\begin{aligned}
(1-\delta) \sum_{a=0}^{\infty} \delta^{a}[Z_{a,t}+A_{a,t}] &= (1-\delta) \sum_{a=0}^{\infty} \delta^{a}[W_{a,t}-T_{a,t}] \\
&\quad + [(1-\delta)/\delta](1+\gamma_{t}) \sum_{a=1}^{\infty} \delta^{a} A_{a-1,t-1} \\
&= (1-\delta) \sum_{a=0}^{\infty} \delta^{a}[W_{a,t}-T_{a,t}] + (1+\gamma_{t})(1-\delta) \sum_{a=0}^{\infty} \delta^{a} A_{a-1,t-1}
\end{aligned}
$$

$$
Z_{t}+A_{t}=W_{t}-T_{t}+(1+r_{t})A_{t-1}, \tag{6}
$$

where Z_t, A_t, W_t, and T_t denote the aggregate value of individuals at time t, respectively. Equation (6) states the sum of the real value of the aggregate current consumption and bonds equals to the sum of the real value of aggregate current after tax income and the gross real return from last period's bond purchases.

2.2. Government

The government purchases exportable goods (G_X) and importable goods (G_M). Thus at time t total government consumption in terms of exportable goods (G_t) is

$$
G_{t}=G_{Xt}+P_{t} G_{Mt}. \tag{7}
$$

Assume that public spending is exhaustive consumption only so that the government purchases do not affect the marginal rate of substitution of private consumption. The financing of these purchases as well as payment of interest on the government bond is achieved by levying the lump-sum taxes and issuing new government bonds to its residents only. Then the government's budget constraint can be written as

$$
G_{t}+(1+r_{t})D_{t-1}=D_{t}+T_{t}, \tag{8}
$$

where D_t denotes the government's bond in terms of exportable good issued by the home government. Equation (8) states the sum of the real value of good purchases and the real interest payment from last period's debt equals to the sum of government's current tax revenue and the current debt.

2.3. Economy-wide Budget constraint and Current Account

Now from equation (6) and (8), we can derive the nation-wide intertemporal budget constraint (see the appendix to derive this)

$$\sum_{i=0}^{\infty} \delta^i R_{it} Z_{t+i} = \sum_{i=0}^{\infty} \delta^i R_{it} \left[W_{t+i} - G_{t+i} \right] + (1+\gamma_t)(A_{t-1} - D_{t-1}) \tag{9}$$

$$\text{where } R_{it} = \frac{1}{\prod_{j=1}^{i} (1+\gamma_{t+j})}, \quad i>0, \quad R_{it} = 1, \quad i=0, \tag{9'}$$

under the transversality condition, as $T \to \infty$ $\lim_{T\to\infty} R_{it} A_{t+T} = 0$.

Equation (9) sets the present discounted value of real consumption equals to the present discounted value of real income less government consumption plus the gross real return from the economy's last period asset. The transversality condition rules out the possibility that the economy can attain unbounded consumption by borrowing and meeting all interest payments through further borrowing(a Ponzi game).

The current account of balance of payments(CA) is defined as the difference between economy's income and absorption so that it can be written as

$$CA_t = W_t + r_t(A_{t-1} - D_{t-1}) - Z_t - G_t . \tag{10}$$

Ⅲ. COMPARATIVE STATICS OF GOVERNMENT DEBT AND GOVERNMENT CONSUMPTION

From the first order conditions, we can derive

$$Z_{a,t} = P_t C_{a,Mt} / (1-\alpha). \tag{11}$$

From the equations (1) and (4b), we have

$$C_{a,Mt} = \left[(1-\alpha) / \alpha P_t \right]^{\alpha} C_{a,t}. \tag{12}$$

Now substituting equations (12) into (11) and aggregating this, the total consumption in terms of exportable good can be derived as

$$Z_t = \left[P_t / (1-\alpha) \right]^{1-\alpha} \alpha^{-\alpha} C_t. \tag{13}$$

From the equations (11), (12), and (13), the aggregate consumption of exportable good is written as(see the appendix the derivation of equations (14))

$$Z_t = (1/H_i) \left[\sum_{i=0} \delta^i R_{it} \{ W_{t+i} - G_{t+i} \} + (1+r_t)(A_{t-1} - D_{t-1}) \right] \tag{14}$$

where

$$H_i = \sum_{i=0}^{\infty} (\delta\beta)^i.$$

Now using the equations (6) and (14), we can have(see the appendix the derivation of equations (15))

$$Z_t = \left(\sum_{i=0}^{\infty} \delta^i \beta^i \right)^{-1} \left[\sum_{i=0}^{\infty} \delta^i R_{it} Z_{t+i} + (1-\delta)(A_{t+i} - D_{t+i}) \right]. \tag{15}$$

Now from the equation (10) and (15), the current account can be obtained

$$CA_t = W_t - G_t + r_t(A_{t-1} - D_{t-1}) \tag{16a}$$
$$- (1/H_i)\left[\sum_{i=0}^{\infty} \delta^i R_{it} W_{t+i} + (1-\delta)(A_{t+i} - D_{t+i}) \right].$$

From the equation (10) and (14), the current account can be obtained

$$CA_t = W_t - G_t + [r_t(1 - 1/H_i) - (1/H_i)](A_{t-1} - D_{t-1}) \tag{16b}$$
$$- (1/H_i)[\sum_{i=0}^{\infty} \delta^i R_{it}(W_{t+i} - G_{t+i})].$$

Now suppose that the government changes the time pattern of taxes and debt issue while holding the path of government spending unchanged. In order to analyze the effects of the change in the time profile of taxes on the level of current consumption and current account, differentiating equations (14) and (16a) with respect to the total asset $(B_{t+j} = A_{t+j} - D_{t+j})$ respectively, we have

$$\frac{dZ_t}{dB_{t+j}} = (1/H_i)(1-\delta)[\delta^j / (1 + r_{t+j})],$$
$$\frac{dCA_t}{dB_{t+j}} = -(1/H_i)(1-\delta)[\delta^j / (1 + r_{t+j})]. \tag{17}$$

From equation (17), we know that an increase in government debt in period j which is followed by corresponding tax hike in the more distant period $(>j)$ raises the current value of wealth so that the current consumption will increase and current account will worsen with the rise in current consumption unless the private agents have infinite horizons. Thus they depend on time profiles of $\{B_{t+j}\}$ or $\{T_{t+j}\}$.

While if the agents have the infinite horizons $(\delta = 1)$, current consumption and current account do not depend on time profiles of $\{B_{t+j}\}$ or $\{T_{t+j}\}$ since the agents can fully internalize the government's intertemporal budget constraint. In this case Ricardian Equivalence Proposition(REP) holds as Barro (1974,1991) and Evans (1988, 1990) pointed out.

Now assume that the government budget deficit arises from an increase in government consumption, the effect will be different. Suppose that there is a tax-financed increase in the government consumption which is in the current or future period. In order to see this effects on current consumption and current account, from taking differentiation of equations (14) and (16b) with respect to B_{t+j}, respectively, we have

$$\frac{dZ_t}{dG_{t+j}} = -(1/H_i)[\delta^j/(1+r_{t+j})],$$
$$\frac{dCA_t}{dG_{t+j}} = (1/H_i)[\delta^j/(1+r_{t+j})] > 0, \qquad j>0 \tag{18}$$
$$= -1+(1/H_i)[\delta^j/(1+r_t)] < 0, \qquad j=0.$$

The interpretation of equation (18) is that if government increases its consumption, the current consumption will decrease. On the other hand, since the current consumption does not fall enough to offset fully increased current $(j=0)$ government consumption due to imperfect substitution, this will deteriorate the current account in the current period. An increase in future $(j>0)$ government consumption, however, will improve the current account in the current period.

Ⅳ. EMPIRICAL IMPLICATION

In this section, I derive the empirical implication of the model laid out in the previous section following Evans (1988) and Cambell and Shiller (1988).

4.1. Forward real interest rates are constant and equal

From the nation-wide budget constraint (9), we have

$$W_t = [Z_t + (A_t - D_t) - (1+r_t)(A_{t-1} - D_{t-1}) + G_t]. \tag{19}$$

Assuming that the forward real interest rates are constant and equal at every horizon, then

$$R_{it} = [1/(1+r)]^i = R^i,$$

where R is a parameter satisfying $0<R<1$. Thus in order to get the econometric implication, recall that an uncertain stochastic income stream. From the equation (15), we can have the stochastic version of total consumption can(see the appendix)

$$Z_t = \left[\sum_{i=0}^{\infty}(\delta\beta)^i\right]^{-1}\left[\sum_{i=0}^{\infty}\delta^i R^i E_t\{Z_{t+i} + (1-\delta)B_{t+i}\}\right]. \tag{20}$$

Now lagging equation (20) one period, multiplying both sides by (1/δR), subtracting the resulting equation from (20), the stochastic version of total consumption can be derived as

$$Z_t = [\{1 - 1/(\delta\beta)^i\}/\delta R]Z_{t-1} - [1/\{\delta R(\delta\beta)^i\}][(1-\delta)B_{t-1}] + e_{1t}, \quad (21)$$

where

$$e_{1t} = \sum_{i=0}^{\infty} \delta^i R^i [(E_t - E_{t-1})(1/(\delta\beta)^i)\{Z_{t+1} + (1-\delta)B_{t+1}\}].$$

Hence, the estimate of coefficient of B_{t-1} in equation (21) has zero probability limit if households have the infinite horizons ($\delta = 1$), and negative probability limit if they have the finite horizons ($0 < \delta < 1$).

From the equation (19), we have

$$[A_t - D_t] = -\sum_{i=0}^{\infty} R^i E_t(W_{t+i} - G_{t+i} - Z_{t+i}). \quad (22)$$

The equation (22) represents that the stock of total net foreign assets, at point in time, equals the present discounted value of the stream of future trade deficit. Now using $CA_t = \Delta(A_t - D_t)$, lagging equation (22), and subtracting the resulting equation from (22), we can derive

$$CA_t = (1/R)CA_{t-1} + \triangle(W_t - G_t - Z_t) + e_{2t}, \quad (23)$$

where $e_{2t} = -\sum_{i=0}^{\infty} R^i [(E_t - E_{t-1})\Delta(W_{t+i} - G_{t+i} - Z_{t+i})]$.

Now substituting equation (21) into (23) and arranging this, we have

$$\triangle CA_t - \triangle(W_t - G_t) = [(1/R) - 1]CA_{t-1} + [\{1 - (1/(\delta\beta)^i) - \delta R\}/\delta R]Z_{t-1} \quad (24)$$
$$+ [(1-\delta)/\{\delta R(\delta\beta)^i\}]B_{t-1} + e_{3t}$$

where

$$e_{3t} = -(1/R)\sum_{i=0}^{\infty} \delta^i R^i [(E_t - E_{t-1})(1/(\delta\beta)^i)(Z_{t+i} + (1-\delta)B_{t+i}] - e_{2t}.$$

By construction, e_{1t} is the revision of expectations about future variables ($[E_t - E_{t-1}]X_{t+1}$) as the households move from period $t-1$ to t so that these are uncorrelated with all information available in period $t-1$($i.e.$ $E_{t-1}[E_t - E_{t-1}]X_{t+1} = 0$). Thus e_{1t}, e_{2t}, and e_{3t} are uncorrelated with all information in period $t-2$.

Hence, if the instrumental variable estimates of coefficients of B_{t-1} in the equations (21) and (24) have zero probability limit, then REP holds. Otherwise, instrumental variable estimates of coefficients of B_{t-1} have negative or positive probability limit REP does not hold.

4.2. Forward real interest rates vary

When forward real interest rate vary, from equation (20) taking the logarithm of the both members of equation (20), and arranging this, we can derive as in Evans (1988)

$$\ln Z_t = \ln(1/H_i) + \ln[Z_t + (1-\delta)B_t] + \ln E_t\left[1 + \sum_{i=1}^{\infty} \delta^i \exp\left(\sum_{j=1}^{i} X_{jt}\right)\right], \quad (25)$$

where $X_{it} = \varDelta \ln[Z_{t+i} + (1-\delta)B_{t+i}] - v_{it}$, and $v_{it} = \ln(1+r_{t+i})$. (26)

Following Evans (1988) and Campbell and Shiller (1988), we can approximate equation (25) as

$$\begin{aligned}&\triangle(Z_t/Z_{t-1}) - \triangle(1+r_t)\\ &= \triangle\zeta_1 - \triangle\{(1-\zeta_2)/\zeta_2\}[(1-\delta)B_{t-1}/Z_{t-1}] + e_{4t},\end{aligned} \quad (27)$$

where $\zeta_2 = \delta\exp(E(x))$, $E(x)$ is the unconditional mean of X_{it}, and
$\zeta_1 = E(x) - \{(1-\zeta_2)/\zeta_2\}\ln[(1/H_i)/(1-\zeta_2)]$, and

$$\begin{aligned} e_{4t} = &\sum_{i=0}^{\infty} \zeta_2^i \triangle \ln(E_t - E_{t-1})\triangle\{Z_{t+i} + (1-\delta)B_{t+i}\} - \triangle(1-E_{t-1})\\ &\ln(1+r_t) - \sum_{i=1}^{\infty} \zeta_2^i \triangle(1-E_{t-1})v_{it} - \triangle\{E_{t-1}\ln(1+r_{t+i}) - v_{it}\}\\ &- \sum_{i=0}^{\infty} \zeta_2^i \triangle(E_{t-1}v_{it} - v_{i+1,t-1})\end{aligned}$$

If we assume that the term premium $E_{t-1}\{\ln(1+r_t) - v_{it-1}\}$, $v_{i+1,t-1}$ contribute negligible to the variance of e_{4t}, then e_{4t} can be taken serially uncorrelated, and uncorrelated with all information available in period $t-2$ as well. Hence when the households have the finite horizons, the instrumental variable estimate of the coefficient on (B_{t-1}/Z_{t-1}) has zero probability limit and REP holds.

On the other hand, taking the first difference of equation (16), and substituting equation (27) into $\varDelta Z_t$ of the resulting equation, it can be approximated as

$$\begin{aligned}&\triangle CA_t - (1+r_t) - \triangle[(Y_t - G_t)/CA_{t-1}]\\ &= \zeta_1 \triangle(Z_{t-1}/CA_{t-1}) - \{(1-\zeta_2)/\zeta_2\}(1-\delta)\triangle\{B_{t-1}/CA_{t-1}\} + e_{5t}\end{aligned} \quad (28)$$

where $e_{5t} = (Z_{t-1} / CA_{t-1}) e_{2t}$.

and e_{5t} is also uncorrelated with all information available in period $t-2$ as before. REP implies that the instrumental variable estimate of the coefficient on (B_{t-1}/CA_{t-1}) has the zero probability limit and current account is independent of government debt. Otherwise REP does not hold.

V. EMPIRICAL RESULTS

In this section, I tested the empirical implications of the equations (21), (24), (27), and (28). If households have the infinite horizons ($\delta = 1$), then there are not the wealth effects of government debt, and the estimates of coefficients of asset terms including B_t in the equations (21), (24), (27), and (28) will have zero probability limit be zero, respectively(null hypothesis). Thus the changes in the relative amounts of tax and debt finance for given amount of government spending would have no effect on the private consumption and the current account.

If they have the finite horizons ($0 < \delta < 1$), there are the wealth effects of government debt, and those estimates of coefficients of asset terms will be not zero, respectively (alternative hypothesis).

Before I test these empirical implications, I will describe the characteristics of data used in this paper, and then present the properties of time series data to test these equations.

5.1. Data Descriptions

This section describes the data that will be used in the empirical analysis and presents econometric tests on the implications derived from previous section. I use the Korean quarterly data for the period 1976:I-1997:IV since some quarterly data are not available for pre-1976. The data on Z and G in terms of exportable good are calculated by dividing the total nominal private consumption and government consumption, respectively, in IFS or in the Monthly Statistical Bulletin(the Bank of Korea) if not available in IFS by the price of exportable good.

The current account(CA) in terms of exportable good is calculated by dividing nominal current account in the IFS by the price of exportable good. All variables except price indexes are also divided by the population and seasonally adjusted. All variables except current account and real interest rate are used in logarithm. Current account is calculated by also dividing by real GNP.

The real government debt (D) is calculated by dividing the nominal government debt in the Monthly Statistical Bulletin by the price of exportable good. The series ΔF is obtained by dividing nominal current account by the price of exportable good. Then the series B is the summation of D and F. The series of real interest rate(r) is calculated with dividing the discount rate in IFS less CPI increase rate by the price of exportable good.

5.2. Unit Root Test

To formally test for the presence of nonstationarity I perform the tests suggested by the augmented Dickey-Fuller(ADF) test. I also perform the Phillips-Perron test for unit root. Phillips(1987), and Phillips-Perron(1988) allow for weekly dependent errors with a time trend.

Table 1 reports the results of augmented Dickey-Fuller(ADF) tests and Phillips-Perron tests for stationarity of Korean private consumption(Z), government consumption(G), current account(CA), asset(B), and real interest rate(r or $\ln(1+r)$) in terms of exportable good for period 1976:I-1997:IV.[3] I choose the number of lags in those tests by Akaike's information criteria(AIC).

The economic series Z, B, G, and r(or $\ln(1+r)$) cannot reject the null hypothesis of unit root at 5% significance level, respectively, according to ADF and Phillips-Perron tests. Therefore they all have unit roots, which represents they are nonstationary in levels and they are difference-stationary.

On the other hand, the current account can reject the null hypothesis at 5% significance level. Thus the current account does not have unit roots as shown in table 1.

[Table 1] Tests of a Unit Root

variable	ADF	Mackinnon Critical Value(5%)	Phillips-Perron	Mackinnon Critical Value(5%)
Z	-1.2092	-2.8963	-0.4455	-3.4652
lnZ	-1.5756	-3.4666	-0.0278	-3.4629
B	-1.7899	-3.4632	-1.3192	-3.4652
CA	-3.1863*	-2.8963	-9.4481*	-3.4652
G	-2.3576	-2.8963	-2.5003	-3.4652
lnG	-2.1225	-3.4639	-2.5390	-3.4614
r	-1.5276	-3.4652	-1.5056	-3.4652
lnr	-2.0235	-3.4862	-1.3356	-3.4614
ln(1+r)	-2.3808	-3.4652	-	-

Note : The null hypothesis is nonstationarity, i.e., each variable has a unit root. The Mackinnon critical values for unit root tests are obtained with the time trend and intercept, respectively, at 5 % significance level.
The number of lags are chosen to eliminate the serial correlation in the error terms and also are considered by minimum AIC. The statistic in the parenthesis represents the nominal economic variables.
* signifies the rejection of null hypothesis at 5 % significance level.

[3] Where all variables are the logarithms of real variables except current account. Since current account is often negative, I divide current account by national income and the price of exportable good.

5.3. Cointegration Tests

In the previous section, I found no evidence that the time series such as Z, B, and G do not have unit roots at the 5% significance level. That is, they are nonstationary in their levels. Cointegration means that nonstationary time series variables tend to move together such that a linear combination of them is stationary and thus it is interpreted as representing a long-run equilibrium. It is pioneered by Granger(1983), and Engle and Granger(1987). Thereafter, testing for the existence of cointegration among economic variables has been an increasingly popular approach to study the economic interrelations.

The estimation method can be different according to whether they are cointegrated or not. Since the Korean economic variables seem to have the unit roots, unless cointegration obtains, OLS may yield inconsistent estimates and face Granger and Newbold's spurious regression problem(see Granger and Newbold (1974)). If they are cointegrated, however, since OLS yields the superconsistent estimates, we can avoid the spurious regression problem and apply OLS method to estimate the implications of conventional model.

Engle-Granger cointegration test is based on the fact that of any economic vector $X(t)$ is cointegrated, then the residuals from that regression are stationary.

Following Engle and Granger(1987), and Phillips and Ouliaris(1990), I attempt to test the null hypothesis of no cointegration by testing the null that there is a unit root in the residual from each regression equation against the alternative that the root is less than unity. It does obtain by applying the DF or ADF test to the residuals. If the null of a unit root is rejected, then the null of no cointegration is also rejected. If there is a unit root, then they are not cointegrated.

Thus I attempt to estimate whether there is a long-run stationary equilibrium in the equations (21), (24), (27), and (28). I apply OLS estimation to the cointegrating regression equations (21), (24), (27), and (28), as shown in table 2. Table 2 shows the Engle and Granger's residual based tests for cointegration by Sims, Stock, and Watson's (1990) generalized cointegration method.

We can reject the null hypothesis of no cointegration at 5% significance level in equations (21), (24), (27) and (28), respectively. This cointegration test results are found to be robust to the numbers of lags. Thus since there exists a generalized cointegration among economic variables in equations (21), (24), (27), and (28), respectively, OLS yields superconsistent estimates.

I also perform Johansen and Juselius (1990) cointegration test. Johansen (1988), and Johansen and Juselius (1990) take a general maximum likelihood approach to test the cointegration as well as the number of independent cointegrating vectors. They suggested the following the vector autoregressive model:

$$X(t) = \Pi(1)X(t-1) + \Pi(2)X(t-2) + \cdots + \Pi(k)X(t-k) + \varepsilon(t), \tag{29}$$

[Table 2] Residual-based Generalized Cointegration Test

	ADF Test Statistic		
Cointegrating equation	lag=1	lag=2	lag=3
(21)	-10.387	-10.547	-10.326
log (21)	-11.288	-8.245	-9.803
(24)	-15.530	-9.339	-20.286
(27)	-6.221	-6.595	-4.672
(28)	-6.997	-5.941	-5.642

Note: The critical values at 5 % significance level are -4.11(-3.93) for number of observation 50(100)(see Engle and Yoo (1987)). The null hypothesis is that there is no cointegration in each equation.

which can be reparameterized as follows

$$\varDelta X(t) = \Xi(1)\varDelta X(t-1) + \Xi(2)\varDelta(2)\varDelta X(t-2) + \cdots + \Xi X(t-k) + \varepsilon(t) \quad (30)$$

where $X(t)$ is n integrated time series and

$$\Xi(i) = -[I - \Xi(1) - \Xi(2) - \cdots - \Xi(i)],\ i = 1,\ 2,\ \cdots,\ k-1$$
$$\Pi = -[I - \Pi(1) - \Pi(2) - \cdots - \Pi(k)].$$

If the rank of impact matrix Π is $m < n$, the Π can be expressed as

$$\Pi = S \cdot Q'$$

for suitable $(n \times m)$ matrices S and Q. Thus the linear combinations given by the rows of $Q'X(t)$ are stationary and $X(t)$ is cointegrated with cointegrating vectors, the rows of Q'. In this case the impact matrix Π can be interpreted as an error-correction model of Engle and Granger(1987).

According to Engle and Granger's representation theorem, there will be r cointegrating vectors among n vector time series. Since $X(t)$ is cointegrated with the cointegrating vectors, the rows of Q', the linear combinations given by the rows of $Q'X(t)$ are stationary. Hence Johansen's cointegration test corresponds to the test of rank of impact matrix Π.

If the rank of impact matrix Π is $m=0$, then the impact matrix Π is null matrix and equation (15) corresponds to a typical differenced VAR. If the rank of impact matrix Π has a full rank, then $X(t)$ is stationary.

Johansen (1988) developed the likelihood ratio test for the rank of Π. Letting N denote the number of time periods available in the data, the likelihood ratio test statistic for the rank of Π, which is called trace test statistic in Johansen and Juselius (1990), is computed as

$$\text{Trace Test Statistic:} \quad -2 \ln \zeta = -N\Sigma_{i=r+1}^{n} \ln (1-\lambda_i) \tag{31}$$

where $\lambda_{r+1}, \cdots, \lambda_n$ are the $n-r$ smallest squared canonical correlations between the residuals of the regression of $X(t-k)$ and $\Delta X(t)$ on $\Delta X(t-1)$, $\Delta X(t-2)$, $\cdots \Delta X(t-p+1)$, where p is the number of lag in VAR chosen by AIC. The null hypothesis is that there are m or less(at most m) cointegrating vectors.

Table 3 reports the results of Johansen's trace tests for the null hypothesis $m=0$, and $m \leq 1$ or 2, respectively. The four lags are used in the VAR and maximum likelihood estimates by AIC. As shown in table 3, we are able to reject the null $m=0$, but are able to accept $m \leq 1$ or 2, respectively, at 5% significance level in the economic variables used in estimating equations (24) and (27).

It also reports that we cannot reject the null hypothesis of no cointegration of the economic variables included in equation (28) at 5% significance level. We are able to reject the null $m=0$ and $m \leq 1$. Thus there exist 2 cointrgration vectors in equation (28). This cointegration test results reassure that there exist cointegrations among economic variables in the equation (24), (27), and (28).

[Table 3] Johansen Cointegration Test

equation	Eigen Value	Likelihood Ratio	Mackinnon's 5% critical value	Mackinnon's 1% critical value	number of cointegration vectors
(24)	0.3039	39.90	29.68	35.65	m=0
	0.1313	11.64	15.41	20.64	m≤1
	0.0085	0.66	3.76	6.65	r≤2
(27)	0.3952	40.37	29.68	35.65	m=0
	0.1150	11.20	15.41	20.64	m≤1
	0.0685	3.12	3.76	6.65	m≤2
(28)	0.3112	52.56	15.41	20.64	m=0
	0.2665	23.86	3.76	6.65	m≤1

5.4. GMM estimation Test Results

Table 4 reports the estimation results from total consumption equation (21), and current account equation (24) under the constant forward real interest rate and table 5 from equations (27) and (28) under the varying forward real interest rate assumption to the Korean data described in the previous section for the sample period 1976:I - 1997:IV, respectively.

Each regression was fitted using following instrumental variables: the constant term, Z_{t-2}, G_{t-2}, and B_{t-2} for equation (21); the constant term, and (B_{t-2}/Z_{t-2}) for equation (27); the constant term, CA_{t-2}, Z_{t-2}, and B_{t-2} for equation (24) and the constant term, Z_{t-2}/CA_{t-2}, and B_{t-2}/CA_{t-2} for equation (28).

Since the error term e_{it} is serially correlated with e_{it-1}, I obtained the consistent estimate of standard error by Hansen's (1982) generalized method of moments(GMM) to calculate t-value of asset term in each equation. The Hansen-Hodrick test using Hansen's generalized method of moment (GMM) does not impose the assumption of no serial correlation and conditional homoscedasticity.

The GMM estimation is computed by performing following procedure. For given sample size N, the method of moments estimator of $\Phi(\beta)$ is

$$\Phi(\beta)=\frac{1}{N}\sum_{t=1}^{N} f(Z_t;\beta),$$

where Z is a vector of data and instruments, β is a vector of parameter. If the model is correctly specified, each component of $\Phi(\beta)$ should be small when evaluated at the true value of the parameter vector, β. Then GMM estimator of β is the minimizer of the following quadratic form:

$$J(\beta)=\Phi(\beta)' W\Phi(\beta),$$

where W is a symmetric, positive definite weighting matrix which may depend on sample information. Suppose that $f(Z)$ is m-th order serially correlated. Then the optimal estimator which minimizes the asymptotic covariance matrix of β is obtained by choosing the inverse matrix of $W(W^{-1})$ to be a consistent estimator of H_0, where

$$H_0=\sum_{k=-m}^{m} E[f(Z_{-n+k};\beta_0)f(Z_{n+k};\beta_0)'].$$

To guarantee the positive definiteness of W in finite samples, Newey and West (1987) propose the following estimator:

$$W^{-1}=\Omega_0+\sum_{j=1}^{m}\omega_{j,m}(\Omega_j+\Omega_j')$$

where $\omega_{j,m}=\frac{m+1-j}{m+1}$ and $\omega_j=\frac{1}{N}\sum_{t=1}^{N-j}[f(Z_{n+k};\beta_0)f(Z_{n+k};\beta_0)']$ for $j=0,\ 1,...,$ m.

Thus $plim(W^{-1})\rightarrow H_0$ as $t\rightarrow\infty$, and the statistical inferences are asymptotically given as follows:

$$plim(\beta)\rightarrow\beta_0,\quad \sqrt{N}(\beta-\beta_0)\rightarrow N[0,(D'H_0^{-1}D)^{-1}],\ \text{and}\ NJ(\beta_N)\rightarrow\chi^2(r-q),$$

where $D=E\left[\frac{\partial f(Z;\beta_0)}{\partial\beta'}\right]'$. r is the number of orthogonality conditions used in estimation and q is the number of parameters. To test the overidentifying restrictions of the model, we use the chi-square test. The quantity $NJ(\beta_N)$ is distributed χ^2 with degree of freedom $(r-q)$. As shown in Newey and West (1987), $m=O(N^{1/4})$, and under certain regularity conditions, $m=O(N^{1/2})$ will suffice for the consistency of the inverse matrix of $W(W^{-1})$.

In table 4 and 5 the instrumental variable estimates of asset terms are not statistically and significantly different from zero at 5% significance level. Each regressor in table 4 and table 5 is stationary in its difference and the instrumental variable estimates are consistent. The statistics $(NJ(\beta_N))$ for testing overidentifying restriction are low that none of them reject the model at 5% significance level.

[Table 4] GMM estimation of the constant forward real interest rate model

regression equation	term	coefficient	t-value (prob>|t-value|)	$NJ(\beta_N)$(p-value)
(21)	Z_{t-2}	0.9858	21.773(0.0001)	15.02(0.28)
	B_{t-2}	0.2421	1.456(0.1494)	
log(21)	$\log Z_{t-2}$	0.9325	20.700(0.0001)	15.93(0.34)
	$\log B_{t-2}$	0.0612	1.442(0.1533)	
(24)	CA_{t-2}	-0.0887	-1.922(0.0583)	17.21(0.43)
	Z_{t-2}	-0.0013	-0.382(0.7044)	
	B_{t-2}	0.0022	0.169(0.8666)	

Note: Each regression equation includes the constant term.

a. Instrumental variables in Z_{t-2}, and B_{t-2} are used in equation (21).

b. Instrumental variables in CA_{t-2}, Z_{t-2}, and B_{t-2} are used in equation (24).

c. The t-values in equations (21) and (24) are calculated by GMM.

d. $NJ(\beta_N)$ is distributed as χ^2 used for testing the overidentifying restrictions.

[Table 5] GMM estimation of the varying forward real interest rate model

regression equation	term	coefficient	t-value (prob>|t-value|)	$NJ(\beta_N)$(p-value)
(27)	$\triangle(B_{t-2}/Z_{t-2})$	-0.0355	-1.759(0.0825)	13.04(0.27)
(28)	$\triangle(Z_{t-2}/CA_{t-2})$	0.0352	0.166(0.8684)	14.71(0.31)
	$\triangle(B_{t-2}/CA_{t-2})$	-0.1339	-0.198(0.8436)	

Note: Each regression equation includes the constant term. Since there are many negative values in current account, the approximate $\Delta CA_t/CA_{t-1}$ is used in equation (28).

a. Instrumental variables in B_{t-2}/Z_{t-2} are used in equation (27).

b. Instrumental variables in Z_{t-2}/CA_{t-2}, and B_{t-2}/CA_{t-2} are used in equation (28).

c. The t-values in equations (27) and (28) are calculated by GMM.

d. $NJ(\beta_N)$ is distributed as χ^2 used for testing the overidentifying restrictions.

The table 4 and 5 show that the real consumption and real current account of Korea in period 1976:I - 1997:IV seem to be independent of Korean government budget deficit and debt. This implies that the conventional Keynesian macroeconomic analysis is not consistent with Korean economic experience.

VI. CONCLUDING REMARKS

This paper has investigated the responses of consumption and current account in a small open economy, Korea, to the fiscal policies through a discontinuous time horizon model. In the two-sector intertemporal optimizing model, the fiscal policy effects on consumption and current account can be different according to whether we have the finite or infinite time horizons. Current consumption and current account do depend on whether the agents have finite time horizons or not since they may or may not internalize the government's intertemporal budget constraint.

The empirical results suggest the following main findings in Korea. Firstly, private consumption and current account are not significantly affected by government deficit and debt in contrast to conventional Keyensian macroeconomic model. Secondly, GMM estimation suggests that the fiscal policy effects on the private consumption and current account seem not to be much effective as the finite horizon model predicted in Korea. Thirdly, the current consumption and current account do not depend on time profiles of government debt or tax since the agents may internalize the government's intertemporal budget constraint. Thus Ricardian equivalence proposition with the infinite horizons seems to be more persuasive to explaining the recent fluctuation of consumption and current account in Korea.

Even if the economic agent has the finite life, he can have the infinite horizon if there can be intergenerational transfers between parents and their sons due to the bequestive motive. In that case Ricardian equivalence proposition with the infinite horizons would be relevant. In Korea, the bequestive motive is extremely strong due to the Confucian social system. In addition, we can think of several other intergenerational transfers due to the educating children and supporting children and the parents in Korea. These social factors in Korea can explain some of reasons to derive the empirical results in this paper.

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APPENDIX

1. Derivation of the equation(9)

Combining the equation (6) and (8) yields

$$Z_t = W_t - G_t + (1+r_t)(A_{t-1} - D_{t-1}) - (A_t - D_t). \tag{A1}$$

Rearranging equation (A1) yields

$$A_{t-1} - D_{t-1} = \frac{1}{(1+r_t)}[Z_t - W_t + G_t + (A_t - D_t)] \tag{A2}$$

$$= \frac{1}{(1+r_t)}[Z_t - W_t + G_t + \frac{1}{(1+r_{t+1})}[(Z_{t+1} - W_{t+1} + G_{t+1} + (A_{t+1} - D_{t+1})]].$$

Now we easily have

$$\begin{aligned} A_t - D_t &= \frac{1}{(1+r_{t+1})}[Z_{t+1} - W_{t+1} + G_{t+1} + \frac{1}{(1+r_{t+2})} \\ &\quad [(Z_{t+2} - W_{t+2} + G_{t+2} + (A_{t+2} - D_{t+2})]] \\ &= \sum_{i=1}^{\infty} \frac{1}{\Pi(1+r_{t+i})}[Z_{t+i} - W_{t+i} + G_{t+i}] \\ &= \sum_{i=1}^{\infty} R_{i_t}\,[Z_{t+i} - W_{t+i} + G_{t+i}]. \end{aligned} \tag{A3}$$

Hence substituting equation (A3) into equation (A1), we can obtain

$$\sum_{i=0}^{\infty} R_{it} Z_{t+i} = \sum_{i=0}^{\infty} R_{it}[W_{t+i} - G_{t+i}] + (1+r_t)(A_{t-1} - D_{t-1}) \tag{A4}$$

Now we introduce the expectation operator the equation (9):

$$\sum_{i=0}^{\infty} \delta^i R_{it} Z_{t+i} = \sum_{i=0}^{\infty} \delta^i R_{it}(W_{t+i} - G_{t+i}) + (1+r_t)(A_{t-1} - D_{t-1}). \tag{9}$$

2. Derivation of equation (14)

From the Lagrange function method, we have

$$p_t = \frac{U_2}{U_1} = \frac{(1-\alpha)C_X}{\alpha C_M}, \text{ and } \frac{U_{2,t}}{U_{2,t+1}} = \beta(1+r_{t+1})\frac{p_t}{p_{t+1}}.$$

From $C = C_X^{\alpha} C_M^{1-\alpha}$, we can derive $\frac{U_{2,t}}{U_{2,t+1}} = \left[\frac{C_t}{C_{t+1}}\right]^{-\theta}\left[\frac{p_t}{p_{t+1}}\right]^{\alpha}$, thus we can have the equation (4),

$$\left[\frac{C_{t+1}}{C_t}\right]^{\theta} = \beta(1+r_{t+1})\left(\frac{P_{t+1}}{P_t}\right)^{\alpha-1}. \tag{4}$$

Solving equation (4) for $t+i$, $t+i-1$, $\cdots$, $t+1$, yields

$$\left[\left(\frac{C_{t+i}}{C_{t+i-1}}\right)^{\theta}\left(\frac{C_{t+i-1}}{C_{t+i-2}}\right)^{\theta}\cdots\left(\frac{C_{t+1}}{C_t}\right)^{\theta}\right]^{i} = \beta^{i}\Pi(1+r_{t+i})\left(\frac{P_{t+i}}{P_t}\right)^{\alpha-1}, \tag{A5}$$

and

$$\left[\left(\frac{C_{t+i}}{C_t}\right)^{\theta}\right]^{i} = \beta^{i}\,\Pi(1+r_{t+i})\left(\frac{P_{t+i}}{P_t}\right)^{\alpha-1}. \tag{A6}$$

Thus we obtain

$$C_{t+i} = C_t\left(\frac{\beta^{i}}{R_{it}}\right)^{1/\theta}\left(\frac{P_{t+i}}{P_t}\right)^{(\alpha-1)/\theta}. \tag{A7}$$

Now rearranging the equation (13), we have

$$Z_t = \left(\frac{P_t}{1-\alpha}\right)\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} C_t P_t^{-\alpha}, \tag{A8}$$

and

$$Z_{t+i} = \left(\frac{P_{t+i}}{1-\alpha}\right)^{1-\alpha} C_{t+i}\,\alpha^{-\alpha}. \tag{A9}$$

Hence from equations (A1), (A7), (A9), and(9), we have

$$\sum_{i=0}^{\infty}\delta^{i}R_{it}\left(\frac{P_{t+i}}{1-\alpha}\right)^{1-\alpha}\alpha^{-\alpha}C_t\left(\frac{\beta^{i}}{R_{it}}\right)^{1/\theta}\left(\frac{P_{t+i}}{P_t}\right)^{(\alpha-1)/\theta}$$
$$= \sum_{i=0}^{\infty}\delta^{i}R_{it}\,(W_{t+i}-G_{t+i})+(1+r_t)(A_{t-1}-D_{t-1}) = Q$$

From equation (A8), we derive

$$Z_t = \left[\left(\frac{P_{t+1}}{P_t}\right)^{(1-\alpha)(1-1/\theta)}\sum_{i=0}^{\infty}(R_{it})^{(1-1/\theta)}\beta^{i/\theta}\right]^{-1} = Q. \tag{A10}$$

Now if we assume $P_t = P_{t+1}$, and $R_{it}\beta^{-i} = 1$, then we obtain equation (14):

$$Z_t=\left[\sum_{i=0}^{\infty}\delta^i\beta^i(R_{it}\beta^{-i})^{(1-1/\theta)}\right]^{-1}Q$$
$$=\left(\sum_{i=0}^{\infty}\delta^i\beta^i\right)^{-1}\left[\sum_{i=0}^{\infty}\delta^i R_{it}(W_{t+i}-G_{t+i})+(1+r_t)(A_{t-1}-D_{t-1})\right]. \qquad (14)$$

3. Derivation of equations (15) and (20)

Rearranging equation (A1) yields

$$W_t=Z_t+\Delta(A_t-D_t)-r_t(A_{t-1}-D_{t-1})+G_t. \qquad \text{(A11)}$$

Substituting this equation into equation (14), we have the equation (15), and then introducing the expectation operator in the equation (15), we can obtain the following equation (20):

$$Z_t=\left(\sum_{i=0}^{\infty}\delta^i\beta^i\right)^{-1}\left[\sum_{i=0}^{\infty}\delta^i R_{i_t}E_t(Z_{t+i}+\Delta(A_{t+i}-G_{t+i})-r_{t+i}(A_{t+i-1}-D_{t+i-1})\right]$$
$$=\left(\sum_{i=0}^{\infty}\delta^i\beta^i\right)^{-1}\left[\sum_{i=0}^{\infty}\delta^i R_{i_t}E_t Z_{t+i}+\sum_{i=1}^{\infty}\delta^i R_{i_t}E_t[\Delta(A_{t+i}-D_{t+i})\right.$$
$$\left.-r_{t+i}(A_{t+i-1}-D_{t+i-1})]+(A_t-D_t)\right]$$
$$=\left(\sum_{i=0}^{\infty}\delta^i\beta^i\right)^{-1}\left[\sum_{i=0}^{\infty}\delta^i R_{i_t}E_t Z_{t+i}+\sum_{i=1}^{\infty}\delta^i R_{i_t}E_t[(A_{t+i}-D_{t+i})\right.$$
$$\left.-(1+r_{t+i})(A_{t+i-1}-D_{t+i-1})]+(A_t-D_t)\right]$$
$$=\left(\sum_{i=0}^{\infty}\delta^i\beta^i\right)^{-1}\left[\sum_{i=0}^{\infty}\delta^i R^i E_t\,[Z_{t+i}+(1-\delta)(A_{t+i}-D_{t+i})]\right] \qquad \text{(A12)}$$