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EFFECT OF EXCHANGE RATE UNCERTAINTY IN PRESENCE OF FUTURES MARKETS

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I consider a multinational firm(MNF) which produces and sells in domestic and foreign markets with monopolistic power in both markets. I assume that there exists a risk from the volatility of foreign exchange. The volatility of the exchange rates affects the firm's behavior in various ways, depending upon the situation in which the firm engages. When a currency futures market is introduced, agents have more opportunity to avoid risk from the volatility of the exchange rate since the firm can hedge against exchange rate risk on its exports in foreign exchange futures markets. This opportunity may lead the firm to produce more at home and export more, holding other things constant, because one of the major reasons for an exporting firm to produce abroad is to avoid risk from the exchange rate variability. The purpose of the study is to analyze the production and hedging decisions of the firm under exchange rate uncertainty. I examine how exchange rate uncertainty affects the firm's output and hedging decisions. The availability of foreign exchange futures markets encourages the risk averse firm to produce more in the domestic plant except when futures markets are perceived as normal backwardation. In addition, the optimal futures position is short for linear demands and unbiased futures markets, and full hedging only with futures contract cannot be attained in this nonlinear profit model.

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I. INTRODUCTION

Generally, multinational firms'(MNFs') decisions are made under uncertainty from exchange rate variability. Thus, the profits of these firms will fluctuate with a move of exchange rates, and it may seriously influence their production

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decision. To reduce this risk, the firms may move their production to foreign countries, or use some of hedging instruments, such as forward, futures, options markets. Since the Bretton Woods international monetary system, the fixed exchange rate system formalized in 1945 to provide a stable monetary framework for international trade, was broken during early 1970s, the foreign exchange rates have freely fluctuated over time, and the volatility of exchange rates has sharply increased. We witness that the major currencies have been very volatile in the last decades. Especially, the recent construction of the WTO may force countries to be more open. In a more open world, capital and technology are more likely to move across countries, and so the firms producing and/or selling in foreign countries increasingly need to manage the volatility of exchange rates. Furthermore, there may be also price or demand uncertainty for both imported and exported goods. Since many internationally traded goods do not have forward, futures, or options markets, firms cannot hedge uncertain prices in those markets. In this situation, they may want to hedge the risks in the forward, futures, or options markets through foreign exchange. Therefore, the introduction of foreign exchange futures markets provides a good opportunity to hedge exchange rate risk. For these reasons, the MNFs have increasingly used hedging instruments to protect against exchange rate risk.

There have been many contributions to the area of hedging problem of international firms under stochastic exchange rates. How exchange rate risk affects the volume of trade has been examined in various situations. The role of hedging instruments also has been analyzed in previous research. Kawai and Zilcha(1986) examined the trade behavior of a risk averse firm in the presence of exchange rate and commodity price uncertainties. They showed that the optimal forward-futures contracting is a full double-hedge, assuming that the forward foreign exchange market is unbiased and the forward foreign exchange and commodity futures markets are jointly unbiased.[1] Moreover, they investigated the role of the existence of both forward exchange and commodity futures markets in comparison to the case where only one or no market is available. They found that the existence of forward exchange and commodity futures markets increases the volume of international trade. Paroush and Wolf(1986) showed that full hedging even in the presence of basis risk under commodity price uncertainty could be attained when forward and futures markets are available. They showed that when the agent hedges in the futures markets with basis risk, her or his production decisions are risk free due to additional hedging opportunity in the forward market. Wolf(1995) examined a competitive firm's import, production, and hedging decisions under input price and exchange rate

[1] In their paper, unbiased forward market is characterized by $E[R]=R_f$, and "jointly unbiased" forward-futures markets are characterized by $E[RP]=R_fP_f$ where R is the random exchange rate, P is the price of the commodity, R_f is the forward exchange rate, and P_f is the futures price of the commodity.

uncertainties in a flexible exchange rate regime. He found that risk aversion and the variance of the exchange rate have a negative impact on imports, while their effect on hedging decision depends on the nature of the equilibrium price structure - on whether one faces a contango or backwardation - as well as on the initial magnitude of hedging. Zilcha and Eldor(1991) studied the behavior of a competitive risk-averse firm which faces uncertain exchange rate in a multiperiod framework. They showed that in the unbiased forward exchange market the firms tend to overhedge in both time periods when the exchange rates are positively correlated over time. Donoso(1995) investigated how introducing a perfectly competitive and possibly biased forward currency market affects the export and hedging decisions of a risk-averse exporting firm in a multiperiod framework under exchange rate risk. He showed that the introduction of a biased forward currency market does not always lead to an increase in the volume of exports. This result is most substantially different from the others in terms of an unbiased forward currency market.

Most papers in these studies assumed that there exists a competitive, risk-averse firm. However, some papers, such as Eldor and Zilcha(1987) and Broll and Zilcha(1992), analyzed an imperfectly competitive commodity market. Eldor and Zilcha(1987) examined a price discriminating firm, producing only in the home country and always exporting, which is a monopoly in the domestic market but a price-taker on the foreign market under exchange rate uncertainty. Since they assumed that the production decision is made under uncertainty, but the sales decision is made after the exchange rate is known, the profit function in their model is nonlinear in the exchange rate. In the presence of forward foreign exchange markets, they showed that the optimal forward hedge of the price discriminating firm is lower than that of a competitive firm. Broll and Zilcha(1992) analyzed the implications of foreign exchange futures markets in the context of a risk-averse multinational firm with monopoly power in domestic and foreign market, which produces and sells in both markets, under exchange rate uncertainty. Assuming that all decisions are made before exchange rate uncertainty is resolved, they investigated the effects of exchange rate uncertainty and the role of futures markets on the international production, sales, and direct investment. The firm hedges more (less) than its net revenue in the foreign market when the risk premium is negative(positive). Furthermore, they showed that perfect hedging with futures market can be made under the single source of risk from the exchange rate because the assumption made on the sequence of decisions leads to a linear profit function in the random exchange rate.

However, if there exists multiple sources of risk, the interaction among the sources of risk may lead to a nonlinear profit function in random variable. Another possibility for a nonlinear profit function can arise from the sequence of decisions. In contrast to their assumption that all decisions are made before exchange rate uncertainty is resolved, if some decisions are made after the resolution of uncertainty, then the profit function would be nonlinear in the random variable. It may change the effect of uncertainty on the firm's production decision. Furthermore, with nonlinear profit functions, using futures market alone as a hedging instrument will not lead to perfect hedging. Moschini and Lapan(1992) showed that perfect hedging by using futures market alone is not possible under output price risk because the sequence of decisions made in their model leads to a nonlinear profit function in random price. They assumed that the quasi-fixed input decision of a competitive firm is made under price risk, but output decision is made after output price uncertainty is resolved. With this assumption, they verified that the optimal quasi-fixed input level is larger under risk if the shadow price function of the quasi-fixed input is convex in output price. They also showed that the optimal futures hedge is a short position equal to the expected output with unbiased futures prices for the quadratic profit function.

The timing of decisions is a very important component in my model. I assume that some decisions are made under uncertainty, while other decisions are made after the resolution of uncertainty. This sequence of decisions makes my model different from Broll and Zilcha(1992). Since the assumptions in my model differ from the previous studies in various ways, the issues considered, the approach to solve them, and the results will also be different.

Thus, I study the production and hedging decision behavior of an MNF which produces and sells in domestic and foreign markets under exchange rate uncertainty, with monopolistic power in both markets. I investigate how the presence of futures markets affects the allocation of outputs between domestic and foreign markets, total output, and welfare of the risk averse firm. In addition, I examine what the optimal futures position is. The model is presented in section Ⅱ. I first investigate the effect of the presence of futures markets on outputs in section Ⅲ and Ⅳ, and then talk about the firm's welfare in section Ⅴ. Finally, I examine the optimal futures position in section Ⅵ.

II. MODEL

I consider a risk-averse multinational firm which produces and sells a homogeneous commodity in domestic and foreign markets under exchange rate uncertainty. The firm has monopolistic power in both markets. Production and hedging decisions are made when the exchange rate is not known, while the sales decisions are made after the exchange rate is known with certainty. This sequence of decisions is critical in the analysis of the issues examined in this study because it makes the profit function nonlinear in the stochastic exchange rate. The firm chooses the optimal level of sales in both markets to maximize its profit after the exchange rate uncertainty is resolved. Assuming that all outputs are sold (i.e.,), the maximization problem for the optimal sales decision can be written as

$$\max_{Y,\bar{Y}} \Pi = R(Y) + e\bar{R}(\bar{Y}) - C(q) - e\bar{C}(\bar{q}) + h(e_f - e)$$

where $Q = q + \bar{q}$, $\bar{Y} = Q - Y$, $R(Y) \equiv P(Y)Y$, and $\bar{R}(\bar{Y}) \equiv \bar{P}(\bar{Y})\bar{Y}$. $P(\cdot)$ is the inverse demand function, Y is the amount of domestic sales, e is the exchange rate measured as the home currency units per foreign currency unit, q is the amount of domestic production, h is the amount of futures foreign currency sold, e_f is the deterministic futures exchange rate, and "—" denotes the corresponding symbol for the foreign country. The cost functions in both countries, $C(\cdot)$ and $\bar{C}(\cdot)$, are assumed to be different and to have positive and nondecreasing marginal costs; $C'(\cdot)$, $\bar{C}'(\cdot) > 0$, and $C''(\cdot)$, $\bar{C}''(\cdot) \geq 0$. The revenue functions, $R(Y)$ and $\bar{R}(\bar{Y})$, are assumed to have positive and nonincreasing marginal revenues, $\bar{R}'(\bar{Y})$, $R'(Y) > 0$, and $\bar{R}''(\bar{Y})$, $R''(Y) \leq 0$. For positive values of sales in both markets, the first order condition of this problem is

$$R'(Y) - e\bar{R}'(Q - Y) = 0. \tag{1}$$

It implies that marginal revenues must be equalized between the domestic and foreign sales at the optimum. The optimal levels of sales are obtained from equation (1) as functions of Q and e,

$$Y^* = Y^*(Q, e)$$
$$\bar{Y}^* = \bar{Y}^*(Q, e).$$

Note that there is no risk in this optimal sales decision. Equation (1) provides the relationship between sales and the exchange rate. Totally differentiating equation (1) provides the following expression;

$$\frac{\partial Y}{\partial e} = R'(\bar{R}'' + e\bar{R}'')^{-1} < 0.$$

The optimal domestic sales are negatively related to the exchange rate, but the optimal foreign sales are positively related to the exchange rate.[2] With the optimal levels of sales, the production decision problem is solved under exchange rate uncertainty. The firm also chooses the optimal amount of futures contracts before the exchange rate is known. Using the von Neumann-Morgenstern utility which gives $U' > 0$ and $U'' < 0$, the production and hedging decision problems of the risk averse firm can be written as

$$\max_{q,\bar{q},h} \quad E[U(\Pi)]$$

s.t.

[2] From $Q = Y(Q, e) + \bar{Y}(Q, e)$, $\frac{\partial Y}{\partial e} = -\frac{\partial \bar{Y}}{\partial e}$,

$$\Pi = R(Y^*) + e\bar{R}(Q - Y^*) - C(q) - e\bar{C}(\bar{q}) + h(e_f - e),$$
$$Y^* = Y^*(Q, e), \ \bar{Y}^* = \bar{Y}^*(Q, e) \text{ and } Q = q + \bar{q}.$$

It is assumed that the distribution of the exchange rate, e, is known. For positive values of q, $\bar{q}$, and h, the first order conditions for this maximization problem can be derived as

$$\frac{\partial E[U(\Pi)]}{\partial q} = E[U' \cdot (e\bar{R}' - C')] = 0 \tag{2}$$
$$\frac{\partial E[U(\Pi)]}{\partial \bar{q}} = E[U' \cdot (e\bar{R}' - e\bar{C}')] = 0 \tag{3}$$
$$\frac{\partial E[U(\Pi)]}{\partial h} = E[U' \cdot (e_f - e)] = 0 \tag{4}$$

The optimal allocation of production in the presence of futures markets is determined by solving equations (2), (3), and (4) simultaneously, given total output, $Q = q^f + \bar{q}^f$ where q^f and $\bar{q}^f$ are the optimal domestic and foreign outputs in the presence of futures markets, respectively. Reducing equations (2), (3), and (4), I obtain

$$C'(q^f) = e_f\bar{C}'(\bar{q}^f). \tag{5}$$

This rule is the same as that in the deterministic case if the deterministic futures exchange rate is unbiased, $e_f = \bar{e}$, where e, $\bar{e} \equiv E[e]$, is used for the exchange rate in the deterministic case. Thus, the optimal allocation of outputs in the presence of unbiased futures markets must be the same as that in the deterministic case.

III. THE EFFECT ON THE ALLOCATION OF OUTPUTS

I will show how the presence of futures markets affects the composition of outputs between domestic and foreign markets, holding total output unchanged, in this section. In order to see this effect, it is necessary to compare the optimal allocation of outputs in the absence of futures markets with that in their presence. Using the assumption of positive marginal cost, the following expression can be derived from equation (5),

$$C'(q^f) \lesseqgtr \bar{e}\bar{C}'(\bar{q}^f) \text{ as } e_f \lesseqgtr \bar{e}. \tag{6}$$

Since we cannot directly compare q^f with q^*, this impact will be examined for unbiased and biased cases separately where q^* and $\bar{q}^*$ are the optimal domestic and foreign outputs in the absence of futures markets, respectively.

For the unbiased case, $e_f = \bar{e}$, equation (6) becomes

$$C'(q^f) = \bar{e}\bar{C}'(\bar{q}^f) \tag{7}$$

Given the profit function in the absence of futures markets, $\Pi = R(Y) + e\bar{R}(\bar{Y}) - C(q) - e\bar{C}(\bar{q})$, the first order conditions for the maximization problem in this uncertainty case can be given as the equations (1), (2), and (3). Then, the following equation holds for the optimal allocation of outputs of the uncertainty case without hedging instruments;

$$C'(q^*)E[U'] = \bar{C}'(\bar{q}^*)E[U'e]. \tag{8}$$

With that profit function, it is true that

$$\bar{C}'E[U'e] < \bar{e}\bar{C}'E[U'].$$[3]

Together with equation (8), this inequality provides

$$C'E[U'] < \bar{e}\bar{C}'E[U'].$$

Since the marginal utility is assumed to be positive, this can be reduced to

$$C'(q^*) < \bar{e}\bar{C}'(\bar{q}^*) \tag{9}$$

for the uncertainty case. I can now prove the following proposition.

PROPOSITION 1. *Holding the level of total production unchanged, the presence of unbiased futures markets results in higher domestic production and lower foreign production for the risk averse firm,* $q^*(Q) < q^f(Q)$ and $q^*(\bar{Q}) > q^f(\bar{Q})$.

PROOF. Let's assume $\bar{q}^* \leq \bar{q}^f$. Equations (7) and (9) together with $\bar{q}^* \leq \bar{q}^f$ implies $C'(q^*) < \bar{e}\bar{C}'(\bar{q}^*) \leq \bar{e}\bar{C}'(\bar{q}^f) = C'(q^f)$, given $\bar{C}'' > 0$. However, $C'(q^*) < C'(q^f)$ implies $q^* < q^f$, given $C'' > 0$. Under the assumption that Q is fixed, that contradicts the assumption, $\bar{q}^* \leq \bar{q}^f$. It proves $q^* < q^f$ and $\bar{q}^* > \bar{q}^f$. Q.E.D.

This proposition shows that a risk averse firm produces more in the domestic plant and less in the foreign plant when there exist unbiased futures markets,

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[3] Assume that foreign net revenue is positive for all $e > 0$, $\frac{\partial \Pi}{\partial e} = \bar{R}(\bar{Y}) - \bar{C}(\bar{q}) > 0$. Using $\frac{\partial \Pi}{\partial e} > 0$ and the assumption of $U'' < 0$, I obtain $\frac{\partial U'}{\partial e} = U'' \frac{\partial \Pi}{\partial e} < 0$, and thus $cov(U', e) < 0$. Then I get $E[U'e] < (E[U'])(E[e])$ from $cov(U', e) = E[U'e] - (E[U'])(E[e]) < 0$. Therefore, $\bar{C}'E[U'e] < \bar{e}\bar{C}'E[U']$ where $\bar{C}' > 0$.

given total output unchanged. It implies that the firm moves some of its production from the foreign plant to the domestic plant because it can avoid the exchange rate risk by using the futures markets. When the firm engages in international trade, it faces risk from the variability of the exchange rate which makes its profit unstable. In order to avoid the risk and to make the profit more stable, the risk averse firm may want to use futures markets, instead of moving some of its production to foreign plant.

On the other hand, for the biased case, the allocation of outputs in the absence of futures markets is changed in a different way. In order to compare q^f with q^*, I will show that a higher deterministic futures exchange rate leads to higher domestic production in the presence of futures markets, $\frac{dq^f}{de_f}>0$. The comparison depends on how the futures exchange rate is biased, normal backwardation ($e_f<e$) or contango ($e_f>e$). Equation (5) tells us that when e_f increases, $C'(q^f)$ must increase to satisfy the equation, and thus q^f increases with the assumption of positive and increasing marginal costs. This can be mathematically shown with equation (5), $C'(q^f)-e_f\overline{C}'(Q-q^f)=0$. Given total output constant, total differentiation of equation (5) provides

$$C''dq^f+e_f\overline{C}''dq^f-\overline{C}'de_f=0 \quad \text{or} \quad \frac{dq^f}{de_f}=\frac{\overline{C}'}{C''+e_f\overline{C}''}>0.$$

This implies that the deterministic futures exchange rate and domestic production in the presence of futures markets move together. This result can now proceed to explain the impact of the presence of futures markets on outputs for the biased case. When the futures currency markets are contango, $e_f>\overline{e}$, q^f increases because of a larger q^f than that in the unbiased case ($e_f=\overline{e}$), holding total output constant. That is, for the case of contango, the firm produces more domestically in the presence of futures markets, $q^*<q^f$ and $\overline{q}^*>\overline{q}^f$. However, when the futures currency markets are normal backwardation, $e_f<\overline{e}$, q^f decreases because of a smaller q^f than that in the unbiased case. Since this effect offsets the initial increase in domestic production due to the presence of unbiased futures markets, the impact of the presence of futures markets is indefinite, depending on how much e_f decreases with a smaller e_f.

After all, we can conclude that the optimal level of domestic production of the firm in the presence of futures markets will be higher than it is in the absence of futures markets except for the case of normal backwardation.

IV. THE EFFECT ON TOTAL OUTPUT

I have showed that exchange rate uncertainty differently affects the composition of outputs for the risk averse firm when currency futures markets exist. In this section, I study the effect of the presence of futures markets on total output. When a firm use futures markets to avoid risk from uncertainty, it is highly concerned to see how much risk can be reduced. Is it possible to completely eliminate the risk? I will examine it by comparing total output in the presence of futures markets with total output under certainty. A risk averse firm produces more or less under uncertainty in the presence of futures markets, depending on the curvature of the marginal revenue functions. That is, the introduction of futures markets may alter the effect of uncertainty on total output.

PROPOSITION 2. *Given unbiased futures markets, the effect of the presence of futures markets on total output depends on the curvature of the domestic marginal revenue curve in the exchange rate, i.e.,* $\bar{Q}^c \lesseqgtr Q^f$ *as* $\frac{\partial^2 R'}{\partial e^2} \gtreqless 0$. *If the marginal revenue curves are non-convex,* R''', $R'''\leq 0$, *then* $\bar{Q}^f < Q^c$. *However, if the marginal revenue curves are strictly convex,* R''', $R'''>0$, *then the effect on total output is ambiguous. Specifically,*

(*a*) if the demand functions are linear, then $Q^f < Q^c$.

(*b*) *if the demand functions are constant elasticity, then* $Q^f > Q^c$.

From equations (1), (2), and (3), I derive the equation

$$E[U' \cdot (R'(Y) - C'(q))] = 0.$$

After plugging the optimal solutions for sales and the domestic output in the presence of futures markets back into the equation, this becomes

$$\frac{\partial E[U(\Pi)]}{\partial Q} = E[U' \cdot (R'(Y^*(Q, e)) - C'(q^f(Q, e_f)))] = 0 \qquad .(10)$$

Then the optimal level of total output in the presence of futures markets, Q^f, solves this equation. In order to compare Q^f with Q^c, I evaluate equation (10) at Q^c where Q^c is the optimal level of total output under certainty. Assuming that the deterministic futures exchange rate is unbiased, $e_f = \bar{e}$, the evaluation at Q^c can be expressed as

$$\left.\frac{\partial E[U(\Pi)]}{\partial Q}\right|_{Q^c} = E[U' \cdot (R'(Y(Q^c, e)) - R'(Y(Q^c, \bar{e})))],$$

using the first order condition of the production problem in the deterministic case, $R'(Y(Q^c, \bar{e})) = C'(q(Q^c, \bar{e}))$. Then we can conclude that

$$Q^c \lesseqgtr Q^f \text{ as } \left.\frac{\partial E[U(\Pi)]}{\partial Q}\right|_{Q^c} \gtreqless 0 = \left.\frac{\partial E[U(\Pi)]}{\partial Q}\right|_{Q^f},$$

assuming that the second order condition of total output decision problem is satisfied. Now define a function H as $H(e) \equiv R'(Y(Q^c, e))$. Using Taylor series expansion, the function $H(e)$ can be expanded around the mean of the exchange rate as

$$H(e) = H(\bar{e}) + H_e(\bar{e}) \cdot (e - \bar{e}) + \frac{1}{2} H_{ee}(\tilde{e})(e - \bar{e})^2 \text{ where } \tilde{e} \in [\bar{e}, e].$$

Then, the evaluation at Q^c can be written as

$$\frac{\partial E[U(\Pi)]}{\partial Q}\bigg|_{Q^c} = \frac{1}{2} E[U' H_{ee}(\tilde{e}) \cdot (e - \bar{e})^2].$$ [4]

Therefore,

$$\frac{\partial E[U(\Pi)]}{\partial Q}\bigg|_{Q^c} \gtreqless 0 \text{ as } H_{ee}(\tilde{e}) \gtreqless 0, \text{ and thus}$$
$$Q^c \lesseqgtr Q^f \text{ as } H_{ee} \gtreqless 0.$$

It proves

$$Q^c \lesseqgtr Q^f \text{ as } \frac{\partial^2 R'}{\partial e^2} \gtreqless 0, \text{ given } H_{ee}(e) = \frac{\partial^2 R'(Y(Q^c, e))}{\partial e^2}.$$

Thus, the sign of the second derivative of the domestic marginal revenue with respect to the exchange rate must be determined to compare Q^c with Q^f; i.e., $Q^f \gtreqless Q^c$ as $\frac{\partial^2 R'}{\partial e^2} \gtreqless 0$. By taking the first derivative of the domestic marginal revenue function, $R'(Y(Q^c, e))$, with respect to the exchange rate, we can obtain

$$\frac{\partial R'}{\partial e} = \frac{\partial R'}{\partial Y}\frac{\partial Y}{\partial e} = R''\bar{R}' \cdot (R'' + e\bar{R}'')^{-1}$$

where

$$\frac{dY}{de} = R' \cdot (R'' + e\bar{R}'')^{-1} < 0.$$

Then the second derivative of the domestic marginal revenue function with respect to the exchange rate is obtained as

[4] $\frac{\partial E[U(\Pi)]}{\partial Q}\bigg|_{Q^c} = E[U' \cdot (H(e) - H(\bar{e}))]$
$= E[U' \cdot (H(\bar{e}) + H_e(\bar{e}) \cdot (e - \bar{e}) + \frac{1}{2} H_{ee}(\tilde{e}) \cdot (e - \bar{e})^2 - H(\bar{e}))]$
$= E[U' \cdot (e - \bar{e})]H_e(\bar{e}) + \frac{1}{2} E[U' H_{ee}(\tilde{e}) \cdot (e - \bar{e})^2]$
$= E[U' \cdot (e - e_f)]H_e(\bar{e}) + \frac{1}{2} E[U' H_{ee}(\tilde{e}) \cdot (e - \bar{e})^2] = \frac{1}{2} E[U' H_{ee}(\tilde{e}) \cdot (e - \bar{e})^2]$

$$\frac{\partial^2 R'}{\partial e^2} = R''' \cdot \left(\frac{\partial Y}{\partial e}\right)^2 + R'' \frac{\partial^2 Y}{\partial e^2}$$
$$= (R'' + e\bar{R}'')^{-2} \frac{\partial Y}{\partial e} [e\bar{R}' \cdot (\bar{R}''R''' + R''\bar{R}''') - 2R''\bar{R}'' \cdot (R'' + e\bar{R}'')].$$

where $\dfrac{\partial^2 Y}{\partial e^2} = -(R'' + e\bar{R}'')^{-1} \dfrac{\partial Y}{\partial e}\left(\bar{R}'' + R'' + (R''' - e\bar{R}''')\dfrac{\partial Y}{\partial e}\right).$

The sign of this expression depends on the curvature of marginal revenue functions, R''' and $\bar{R}'''$. If the marginal revenue curves are non-convex, the second derivative of the domestic marginal revenue with respect to the exchange rate will be negative, and thus the optimal level of total output will be less under uncertainty in the presence of futures markets than under certainty. For example, if the demand functions are linear, then the marginal revenue functions are also linear, $R''' = \bar{R}''' = 0$, and the second derivative becomes negative. Therefore, the firm produces less under uncertainty in the presence of futures markets than under certainty. On the other hand, if the marginal revenue curves are strictly convex, then the second derivative can be positive, negative, or equal to zero, and so the effect on total output is ambiguous. However, if the demand functions are constantly elastic, then the second derivative eventually turns out to be positive, $\frac{\partial^2 R'}{\partial e^2} > 0$, even though the marginal revenue functions are strictly convex. Therefore, with constant elasticity demands, the optimal level of total output is greater in the presence of futures markets under uncertainty than it is under certainty.

V. FIRM'S WELFARE

Assuming that the futures exchange rate is unbiased, the risk averse firm hedging in futures markets is better off under uncertainty than it is under certainty, regardless of the optimal level of total output. In other words, its expected utility of the profit with futures markets (Π^f) under exchange rate risk, $E[U(\Pi^f(Y^f(Q^f, e), h^*))]$, is greater than its utility of the profit with certainty (Π^c), $U(\Pi^c(Y^c(Q^c, e), e))$, where h^* is the optimal number of futures contracts sold, and Q^f and Q^c are the optimal levels of total output in the presence of futures markets and under certainty, respectively. The indirect profit functions are described as

$$\Pi^f(Y(Q^f, e), e, h^*) = R(Y(Q^f, e)) + e\bar{R}(Q^f - Y(Q^f, e))$$
$$- C(q^f) - e\bar{C}(Q^f - q^f) + (e_f - e)h^*$$
$$\Pi^c(Y(Q^c, \bar{e}), \bar{e}) = R(Y(Q^c, \bar{e})) + e\bar{R}(Q^c - Y(Q^c, \bar{e}))$$
$$- C(q^c) - \bar{e}\bar{C}(Q^c - q^c).$$

PROPOSITION 3. *Assuming unbiased futures markets* ($e_f = \bar{e}$), *the risk averse firm hedging in futures markets benefits from uncertainty, even though perfect hedging is not feasible.*

PROOF. It can be verified by showing that $E[U(\Pi^f(Y(Q^f, e), e, h^*))]$ is greater than $U(\Pi^c(Y(Q^c, \bar{e}), \bar{e}))$. Suppose the risk averse firm chooses $Q = Q^c$ and $h = \hat{h}$ where $\hat{h} = \bar{R}(Q^c - Y(Q^c, \bar{e})) - \bar{C}(Q^c - q^c)$, which represent output and net foreign revenue for the deterministic case. Then, realized profits in the presence of exchange rate risk are

$$\Pi^f(Y(Q^c, e), e, \hat{h}) = R(Y(Q^c, e)) + e\bar{R}(Q^c - Y(Q^c, e)) \\ - C(q^c) - e\bar{C}(Q^c - q^c) + (e_f - e)(\bar{R}(Q^c - Y(Q^c, \bar{e})) - \bar{C}(Q^c - q^c)).$$

Hence, $\Pi^f(Y(Q^c, e), e, \hat{h}) - \Pi^c(Y(Q^c, \bar{e}), \bar{e})$

$$= [R(Y(Q^c, e)) + e\bar{R}(Q^c - Y(Q^c, e))] - [R(Y(Q^c, \bar{e})) \\ + e\bar{R}(Q^c - Y(Q^c, \bar{e}))] + (e_f - \bar{e})(\bar{R}(Q^c - Y(Q^c, \bar{e})) - \bar{C}(Q^c - q^c)).$$

Assuming $e_f = \bar{e}$, it implies $\Pi^f(Y(Q^c, e), e, \hat{h}) > \Pi^c(Y(Q^c, \bar{e}), \bar{e})$ for all $e \neq \bar{e}$ since $[R(Y(Q^c, e)) + e\bar{R}(Q^c - Y(Q^c, e))] > [R(Y(Q^c, \bar{e})) + e\bar{R}(Q^c - Y(Q^c, \bar{e}))]$ and $Y(Q^c, e) \neq Y(Q^c, \bar{e})$ for all $e \neq \bar{e}$ by the definition of maximization problem, $[R(Y(Q^c, e)) + e\bar{R}(Q^c - Y(Q^c, e))] = \max_Y [\bar{R}(Y) + e\bar{R}(Q^c - Y)]$.
$U(\Pi^f(Y(Q^c, e), e, \hat{h})) > U(\Pi^c(Y(Q^c, \bar{e}), \bar{e}))$ and thus
$E[U(\Pi^f(Y(Q^c, e), e, \hat{h}))] > U(\Pi^c(Y(Q^c, \bar{e}), \bar{e}))$. In addition, it is also true that $E[U(\Pi^f(Y(Q^f, e), e, h^*))] \geq E(U(\Pi^f(Y(Q^c, e), e, \hat{h}))]$ by the definition of maximization problem, $E[U(\Pi^f(Y(Q^f, e), e, h^*))] = \max_{q, Q, h} E[U(\pi(Y(Q, e), e, h))]$ for all e,
where $\Pi(Y(Q, e), e, h) = R(Y(Q, e)) + e\bar{R}(Q - Y(Q, e)) - C(q) - e\bar{C}(Q - q) + (e_f - e)h.$
This proves that $E[U(\Pi^f(Y(Q^f, e), e, h^*))] > U(\Pi^c(Y(Q^c, \bar{e}), \bar{e}))$ for all $e \neq \bar{e}$.
Q.E.D.

Therefore, the risk averse firm is always better off with risk in the presence of unbiased futures markets, regardless of the optimal levels of total output and futures contracts sold.

VI. OPTIMAL FUTURES POSITION

I turn to the issue of what the optimal futures position is when other hedging instruments are not available. Again, it is not possible to perfectly hedge against risk with a single hedging instrument when the profit function is nonlinear in the random variable.[5] As mentioned earlier, the nonlinearity of the profit function may come from multiple sources of risk and/or the sequence of decisions. Since the assumption made on the sequence of decisions in this model makes the profit function nonlinear in the exchange rate, full hedging only with futures contracts is not attained here.

I assume that futures markets are unbiased ($e_f = \bar{e}$), and $e = \bar{e} + \varepsilon$ where ε has a zero mean with a symmetric distribution ($f(\varepsilon) = f(-\varepsilon)$). In addition, foreign net revenue is assumed to be positive so that the first derivative of indirect profit function with respect to the exchange rate is positive, $\frac{\partial \Pi}{\partial e} = \bar{R} - \bar{C} > 0$. Demand curves are assumed to be linear, implying that slopes of marginal revenue functions are constant, $R''' = \bar{R}''' = 0$. Then, the following proposition can be derived.

PROPOSITION 4. *The optimal futures position depends upon the shape of demand curves. With linear demands, the optimal futures position is short and less than the foreign net revenue of the deterministic case.*

Defining

$$G(Q, e) \equiv \max_{Y} [R(Y) + e\bar{R}(Q - Y)],$$

the profit function can be expressed as

$$\Pi = G(Q, e) - C(q) - e\bar{C}(\bar{q}) + h(e_f - e).$$

The indirect revenue function is obtained as $G(Q, e) = R(Y^*(Q, e)) + e\bar{R}(Q - Y^*(Q, e))$ after solving the sales decision problem. Differentiating it with respect to the exchange rate, one obtains;

$$G_e(Q, e) = \bar{R}(Q - Y^*(Q, e)),\ G_{ee}(Q, e) = -(\bar{R}')^2(R'' + e\bar{R}'')^{-1}, \text{ and}$$
$$G_{eee}(Q, e) = 3(\bar{R}')^2\bar{R}'' \cdot (R'' + e\bar{R}'')^{-2} + (\bar{R}')^2(\bar{R}'' + e\bar{R}'')^{-2}(R''' - e\bar{R}''')\frac{\partial Y}{\partial e}.$$

Using Taylor series expansion, the indirect revenue function can be expanded around the mean of the exchange rate as

[5] See Moschini and Lapan (1992, 1995).

$$G(Q,\ e)=G(Q,\ \bar{e})+G_e(Q,\ \bar{e})\varepsilon+\frac{1}{2}G_{ee}(Q,\ \bar{e})\varepsilon^2+\frac{1}{6}G_{eee}(Q,\ \tilde{e}(\varepsilon))\varepsilon^3$$
where $\tilde{e}\in[\bar{e},\ e]$.

I use this expansion to describe the profit function as

$$\Pi(\varepsilon)=A+(B-h)\varepsilon+\theta(\varepsilon)$$
where $A\equiv G(Q,\ \bar{e})-C(q)-\bar{e}\bar{C}(Q-q)$,
$B\equiv G_e(Q,\ \bar{e})-\bar{C}(Q-q)=\bar{R}(Q-Y(Q,\ \bar{e}))-\bar{C}$, and
$\theta(\varepsilon)\equiv\frac{1}{2}G_{ee}(Q,\ \bar{e})\varepsilon^2+\frac{1}{6}G_{eee}(Q,\ \tilde{e}(\varepsilon))\varepsilon^3$.

The first order condition of the hedging decision problem can be rewritten as

$$\frac{\partial EU(\Pi(\varepsilon))}{\partial h}=-E[U'\varepsilon]=\int_{\varepsilon>0}[U'(\Pi(-\varepsilon))-U'(\Pi(\varepsilon))]\varepsilon f(\varepsilon)\,d\varepsilon=0,$$ [6]
where $\Pi(+\varepsilon)=A+(B-h)\varepsilon+\theta(+\varepsilon)$ and $\Pi(-\varepsilon)=A-(B-h)\varepsilon+\theta(-\varepsilon)$.

Evaluating this first order condition at $h=B$ yields;

$$\left.\frac{\partial EU(\Pi(\varepsilon))}{\partial h}\right|_{h=B}=\int_{\varepsilon>0}[U'(\Pi(-\varepsilon))-U'(\Pi(\varepsilon))]\varepsilon f(\varepsilon)\,d\varepsilon$$
where $\Pi(+\varepsilon)=A+\theta(+\varepsilon)$ and $\Pi(-\varepsilon)=A+\theta(-\varepsilon)$.

COROLLARY. The optimal futures position can be described as follows:
$h^*\gtreqless B$ as $G_{eee}\gtreqless 0$, assuming that the second order condition of the production and hedging decision problem holds.

PROOF. Given $U''<0$, $[U'(\Pi(-\varepsilon))-U'(\Pi(\varepsilon))]\gtreqless 0$
as $[\Pi(-\varepsilon)-\Pi(+\varepsilon)]\lesseqgtr 0$.
Since $\Pi(-\varepsilon)-\Pi(+\varepsilon)=\theta(-\varepsilon)-\theta(+\varepsilon)=\frac{1}{6}[G_{eee}(Q,\ \tilde{e}(+\varepsilon))$
$+G_{eee}(Q,\ \tilde{e}(-\varepsilon))]\varepsilon^3$ for $\varepsilon>0$, $[\Pi(-\varepsilon)-\Pi(+\varepsilon)]\lesseqgtr 0$ as $G_{eee}\gtreqless 0$.
Thus, $\left.\frac{\partial EU(\Pi(\varepsilon))}{\partial h}\right|_{h=B}\gtreqless 0$ as $G_{eee}\gtreqless 0$.
Given the second order condition, it proves $h^*\gtreqless B$ as $G_{eee}\gtreqless 0$. Q.E.D.

Since the sign of G_{eee} depends on the second derivative of the marginal

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[6] $$\frac{\partial EU(\Pi(\varepsilon))}{\partial h}=-E[U'\varepsilon]=\int_{\varepsilon<0}[-U'(\Pi(\varepsilon))]\varepsilon f(\varepsilon)d\varepsilon+\int_{\varepsilon>0}[-U'(\Pi(\varepsilon))]\varepsilon f(\varepsilon)d\varepsilon$$
$$=\int_{-\infty}^{0}[-U'(\Pi(-\varepsilon))](-\varepsilon)f(-\varepsilon)d(-\varepsilon)+\int_{\varepsilon>0}[-U'(\Pi(\varepsilon))]\varepsilon f(\varepsilon)d\varepsilon$$
$$=\int_{0}^{\infty}[U'(\Pi(-\varepsilon))]\varepsilon f(\varepsilon)d\varepsilon-\int_{\varepsilon>0}[U'(\Pi(\varepsilon))]\varepsilon f(\varepsilon)d\varepsilon$$
$$=\int_{\varepsilon>0}[U'(\Pi(-\varepsilon))-U'(\Pi(\varepsilon))]\varepsilon f(\varepsilon)\,d\varepsilon=0$$

revenue functions, the optimal futures position cannot generally be determined without knowing the shape of demand curves. For the case of linear demands, $R''' = \bar{R}''' = 0$, we obtain

$$G_{eee}(Q, e) = 3(\bar{R}')^2 \bar{R}''(R'' + e\bar{R}'')^{-2} < 0,$$

and hence the optimal amount of futures foreign currency sold must be less than the foreign net revenue of the deterministic case, $h^* < B$. In order to show that the optimal futures position is short, I evaluate the first order condition of the hedging decision problem at $h = 0$. The profit function at $h = 0$ becomes

$$\Pi = R(Y^*(Q, e)) + e\bar{R}(Q - Y^*(Q, e)) - C(q) - \bar{e}\bar{C}(q).$$

Given the assumption of the positive foreign net revenue, the evaluation at $h = 0$ becomes positive,

$$\frac{\partial EU(\Pi(\varepsilon))}{\partial h}\bigg|_{h=0} = E[U'(\Pi)(e_f - e)] = E[U'(\Pi)(\bar{e} - e)] > 0.$$ [7]

Together with the second order conditions, this implies that the optimal amount of futures foreign currency sold is greater than zero, $h^* > 0$. Therefore, the optimal futures position is

$$0 < h^* < \bar{R}(Q - Y)(Q, \bar{e})) - \bar{C}$$

with linear demands. Since the level of perfect hedging, $\bar{R}(Q - Y)(Q, e)) - \bar{C}$, is a function of the random exchange rate, complete hedging cannot be attained via futures contracts alone. It is because the assumption I made on the sequence of decisions makes the indirect profit function nonlinear in the random exchange rate, which does not allow perfect hedging with single hedging instrument.[8]

VII. CONCLUSION

I have analyzed the production and hedging behavior of a MNF under exchange rate uncertainty. The volatility of the exchange rate affects the firm's behavior in various ways, depending upon the situation in which the firm engages. The timing of decisions and the firm's attitude toward risk can be crucial components to determine the direction of this effect. When a currency

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[7] From $U'' < 0$ and $\frac{\partial \Pi}{\partial e} = \bar{R} - \bar{C} > 0$ at $h = 0$, $\frac{\partial U'}{\partial e} = U''\frac{\partial \pi}{\partial e} < 0$ at $h = 0$, and thus $cov(U', e) < 0$. Since $cov(U', e) = E[U'e] - (E[U'])(E[e]) = E[U'e] - E[U'\bar{e}]$ $= E[U' \cdot (e - \bar{e})] < 0$, $E[U' \cdot (\bar{e} - e)] > 0$ at $h = 0$.

[8] See Moschini and Lapan (1992)

futures market is introduced, agents have more opportunity to avoid risk from the volatility of the exchange rate since the firm can hedge against exchange rate risk on its exports in the foreign exchange futures market. This opportunity may lead the firm to produce more at home and export more, holding other things constant, because one of the major reasons for an exporting firm to produce abroad is to avoid risk from the exchange rate variability.

In this circumstance, I have investigated the effect of the presence of futures markets on the allocation of outputs, and total output. I also examined how the risk averse firm's welfare is affected by the presence of futures markets. The availability of futures markets encourages the risk averse firm to produce more at home except in the case of normal backwardation. Even though futures markets are available to reduce risk, the risk averse firm may produce more or less, depending upon the curvature of the marginal revenues. The firm produces less under uncertainty than under certainty with linear demands, but more with constant elasticity demands. Regardless of the optimal level of total output, however, the risk averse firm benefits from risk in the presence of futures markets. In addition, the optimal futures position depends upon the shape of marginal revenue curves. For linear demands and unbiased futures markets, the optimal futures position is short, and full hedging only with futures contract is not attained because the indirect profit function is nonlinear in the random exchange rate.

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