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PRODUCTIVITY-ENHANCING EFFECT OF GOVERNMENT EXPENDITURE IN A TWO-SECTOR INTERTEMPORAL OPTIMIZING MODEL

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This paper analyzes the productivity-enhancing effects associated with the government expenditure by directly applying it to the production function as one of the factors of production. Since the model incorporates a foreign-capital-dependent developing economy in an intertemporal optimizing setup, it shows an explicit dynamic relationship between external debt and capital stock accumulation. Unlike the demand-side analysis of the government expenditure, it is found that the increase in government expenditure accelerates the rate of investment, which results in more current account deficits. This implies the government expenditure affects the dynamic variables, whereas the demand-side analysis of government expenditure does not. In addition, the rise in the government expenditure may increase consumption under a certain restraint through the change in the outputs, not through the change in the aggregate demand. This is because the supply-side effect of the government expenditure dominates the demand-side effect. In general, consumption falls in a demand-side analysis of the government expenditure.

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I. INTRODUCTION

The economic problems in the developing economies are mainly concerned about the economic growth, external debt, and the trade-related disputes. Historically, many developing countries are guided and even controlled by the

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centralized economic planner (the government) to pursue their specific economic goals. This intervention has been inevitable since those economies needed foreign capital and advanced technology to activate the economies due to the lack of domestic capital formation and the inability of the private sector to access to the world capital market as well as low-level of technology.

Based upon the observations and evidences in those developing countries, several economists already pursued to give suggestions (Brock(1990)) and to explain the dramatic economic achievements theoretically and empirically [Aizenman(1989, 1990), Alesina and Tabellini(1989), Bhandari, Haque, and Turnovsky(1989)]. Among them, Brock(1990) utilized the externality of govern- ment expenditure and aggregate capital stock in a two-sector infinite time horizon model following and extending Barro(1990). His model presents two stable equilibria depending upon the degree of government intervention in the tradition of 'Big Push' growth theory. However, all previous papers do not focus on the growth aspect in a government-controlled developing economy where, in fact, external debt financed by the government window played a key role for the accumulation of capital stock. Any of the papers does not describe some possible relationship between the external debt and capital stock accumulation which was significantly influenced by the government's strategic macroeconomic policies.

In addition, none of the previous works did consider the role of government explicitly. Recently several papers [King and Rebelo(1990), Rebelo(1991), Turnovsky and Fisher(1995)] discuss the relationship between the economic growth and public policy to explain the different growth rates across the world. In this paper, we apply the government expenditure to the production side of the economy explicitly as one of the factors of production in the two-sector exportable and import-competing model to characterize the productivity-enhancing effects associated with government expenditure in developing countries with external debt. Since we assume the two goods are both tradable, the model is different from the traditional tradable-nontradable model. The model also differs from the conventional exportable-importable model due to the factor intensity assumption. In our model, exportable sector is assumed to be relatively capital intensive to reflect the government's strategic development policy. The treatment of government expenditure as one of the production factors is originated from Barro(1990), where he developed the model in the context of endogenous growth theory. The government expenditure in our model represents any activity provided by the government in order to affect the productivity of capital and labor. For domestic capital formation the economy must import foreign capital . We shall call this type of economy a foreign-capital-dependent economy. There are two important differences from the previous papers associated with the government expenditure. Firstly, the direct application of government expenditure to the production function differs from the treatment of government expenditure as one of the factors generating the positive externality because the share of

government expenditure allocated to each sector affects the productivity of capital and labor in that sector only. This implies that the share of government expenditure in one sector does not influence the productivity of the other sector. As a result, the share is also important in determining the productivity of the corresponding sector in addition to total amount of government expenditure itself.[1]

Secondly, the explicit application of government expenditure to the production side of the economy is different from the previous analysis of government expenditure in the demand side of the economy. The supply side incorporation of government expenditure reflects the fact that the government spending on the infrastructure of the economy has accelerated the economy's capital accumulation and thus the rapid economic growth by affecting the productivity of capital and labor in some developing countries. For simplicity, we assume that the government expenditure and the share are exogenously determined.

The rest of the paper is organized as follows: Section II describes the economic structure and leads to the macroeconomic equilibrium in Section III. Equilibrium dynamics and steady-state equilibrium are presented in Section IV and V, respectively. In Section VI, long-run analysis is done. Short-run dynamics is analyzed in Section VII. Section VIII concludes.

II. THE ECONOMIC STRUCTURE

The economy under consideration is inhabited by one single infinitely lived agent, who provides one unit of inelastic labor supply at a competitive wage and accumulates capital stock for rental at a competitively determined rental rate. The exogenously determined government expenditure is allocated between two sectors by the given fraction α, i.e., $0<\alpha<1$. The α fraction of government expenditure (αG) is devoted to the exportable sector and the remaining, $(1-\alpha)G$, goes to the import-competing sector. Then, we have standard neoclassical production functions in the following ways; for exportable good, $F(K_1, L_1, \alpha G)$ with the properties of $F_{K>0}, F_L>0, F_G>0$ and $F_{KK}<0$, $F_{LL}<0, F_{GG}<0$ and also the cross partials are positive, $F_{KL}>0, F_{KG}>0$, $F_{LG}>0$, assuming any two inputs are cooperative, and for import-competing good, we have $H(K_2, L_2, (1-\alpha)G)$, where $K_1+K_2=K$ and. $L_1+L_2=\bar{L}=1$.[2] The

[1] To justify this argument, we can illustrate the sector-specific government expenditure: for examples, specialized areas such as seaport or airport for the use of exportable good, power plants exclusively for exportable sector and preferred or biased government administration to the exportable sector only.

[2] Since the exportable sector is assumed to be relatively capital intensive and import-competing sector relatively labor intensive, we have the following factor intensity rankings, $a_{KF}/a_{KH}>a_{GF}/a_{GH}>a_{LF}/a_{LH}$, as discussed by Jones and Easton(1983), where a_{ij}, denotes the input of factor i required to produce a unit of output in the jth sector. Factor

two inputs, capital and labor, are mobile between the two sectors, while the capital stock evolves over time, the labor supply is always fixed at one.

While we apply the government expenditure to the production functions of both sectors explicitly, we exclude the externality associated with the government expenditure discussed by Brock(1990). With the assumption of no externality, only the amount of government expenditure devoted to each sector affects the productivity of both capital and labor of the corresponding sector, independently of the share of government expenditure devoted to the other sector. Since the model considers a small open economy, the relative price of import-competing good (σ) in terms of exportable good, which is a numeraire, is exogenously fixed and the economy must import for investment at a given world price (p) from the rest of the world because of the shortage of domestically produced capital good. The exportable good is either consumed or exported, while the import-competing good is consumed and also used for the cost of installation of investment according to the convex and increasing function, $\phi(I)$, with the properties of $\phi'>0$, $\phi'>0$ and $\phi(0)=0$, $\phi'(0)=0$ are assumed by choice of units.

In the asset side of the economy, the private agent is not allowed to access to the world capital market due to poor credibility so that the private agent can finance the foreign capital through the government window.[3] Thus the private agent accumulates the domestic government bonds (b) which pay the fixed world interest rate (r), instead the government holds both domestic and internationally tradable bonds (z). In addition, the private agent should pay the lump-sum tax (T) to the government.

Therefore, we have the following budget constraint of private agent expressed in terms of exportable good.

$$\dot{b}=F(K_1,L_1,\alpha G)+\sigma H(K-K_1,1-L_1,(1-\alpha)G)+\bar{r}b \\ -x-\sigma y-pI-\sigma\phi(I)-T \tag{1}$$

where G in the production function denotes exogenously determined government expenditure and α is the given share of government expenditure devoted to the exportable sector, and x, y denote the consumption levels of exportable and import-competing good, respectively.

In addition, the capital stock accumulates without depreciation according to

$$\dot{K}=I \tag{2}$$

intensity reversals are assumed not to occur.

[3] In order to characterize this inaccessibility of the private agent to the world capital market, the model employs two types of bonds: domestically tradable government bond and internationally tradable external bond, which are perfect substitutes.

The private agent's problem is to choose the consumption of two goods (x, y), the allocation of capital stock and labor between the two sectors, the rate of investment (I) and domestic bond holding to maximize the intertemporal utility function.

$$Max \int_0^{\infty} U(x, y)e^{-\beta t}dt \tag{3}$$

subject to equation (1), (2) and the initial conditions; $b(0) = b_0$, $K(0) = K_0$. We assume that the instantaneous utility function is strictly concave, i.e., $U_{xx} < 0, U_{yy} < 0$ and the two goods are Edgeworth complementary, so that $U_{xy} > 0$. The rate of time preference of the private agent (β) is a constant which, in a perfect capital mobility, must be equal to the given world interest rate to ensure the steady state. In order for solving the standard intertemporal optimization problem, we set up the current value Hamiltonian.

$$\begin{aligned} H = U(x, y) + \lambda[F(K_1, L_1, aG) + \sigma H(K - K_1, 1 - L_1, (1 - a)G) \\ + \bar{r}b - x - \sigma y - pI - \sigma\phi(I) - T] + q'(I) \end{aligned} \tag{4}$$

where λ and q' are two costate variables associated with domestic bond holding and capital stock. Letting $q'/\lambda \equiv q$ be the market value of installed capital, the usual optimality conditions for the private agent are

$$U_x(x, y) = \lambda \tag{5}$$

$$U_y(x, y) = \sigma\lambda \tag{6}$$

$$q = p + \sigma\phi'(I) \tag{7}$$

Equation (5) and (6) describe the usual marginal rate of substitution conditions for consumers. Equation (7) equates the marginal cost of investment to the market price of installed capital, which is essentially a *Torbin's q* theory of investment.

In addition, the shadow value (marginal utility) of wealth and the market price of installed capital evolve according to

$$\dot{\lambda} = \lambda(\beta - \bar{r}) \tag{8}$$

$$\dot{q} = \bar{r}q - r^K(K, \sigma, a, G) \tag{9}$$

where r^K is the rental price of capital which is determined by that the marginal product of capital and labor between the two sectors must be equal, thus the

following efficiency conditions in production should be satisfied.

$$F_K(K_1, L_1, aG) = \sigma H_K(K-K_1, 1-L_1, (1-a)G) \equiv r^K(K, \sigma, a, G) \quad (10)$$

$$F_L(K_1, L_1, aG) = \sigma H_K(K-K_1, 1-L_1, (1-a)G) \equiv w(K, \sigma, a, G) \quad (11)$$

Since the exportable sector is defined to be relatively capital intensive and import-competing sector to be labor intensive, and the exportable sector uses the services of government expenditure more intensively described in the above, factor returns will depend upon the exogenous government expenditure and the fixed share as well as the capital stock and the relative price of import-competing good. Thus the rental price of capital, r^K, depends negatively upon both the capital stock and the relative price of import-competing good, while depends positively on the two exogenous variables, G and a, because of $F_{KG}>0$.

Notice that the dependence of rental price of capital on G and a makes an important channel that the government can affect the economy's capital accumulation through the induced change in market price of installed capital q which determines the rate of investment.

To secure the steady state, the exogenous variable, β, must be equal to the given world interest rate, $\bar{r}$ in equation (8), which implies $\dot{\lambda}=0$ everywhere. Thus λ, the marginal utility of wealth in the form of domestically traded bond is always constant at its steady state value, i.e., $\bar{\lambda}=\lambda$(λ is a steady state value, which will be determined below).

Finally, we need to impose the transversality conditions to satisfy the private agent's intertemporal budget constraint.

$$\lim_{t\to\infty} \lambda b e^{-\beta t} = \lim_{t\to\infty} qKe^{-\bar{r}t} = 0 \quad (12)$$

Turning to the government, its budget constraint, expressed in terms of the exportable good, is given by the equation

$$\dot{a} = \bar{r} + g - T \quad (13)$$

where a is the total outstanding debt of the government, i.e., $a=b+z$ in which z is the stock of external debt held solely by the government. Two types of bonds, z and b are perfect substitutes until the model introduces the risk premium associated with the finance of foreign capital. The reasons why we introduce two types of bonds into the model; firstly, to characterize the government's behaviors of the developing countries where the private agent can only access to the foreign capital through the government window since the agent is not allowed to access to the foreign capital market due to both the

government's capital control policy and the agent's poor credibility in world capital market; secondly, to allow for the incorporation of the risk premium associated with foreign capital into the model, in which the two types of bonds are no more perfect substitutes. If b is negative, it becomes private agent's debt to the government. In this model, we presume this case. And G denotes the exogenously determined government expenditure, which can be any good. In order to finance the government expenditure and interest payment, the government can issue bonds domestically or internationally, and alternatively, adjust lump-sum taxes.

Invoking the government budget constraint together with the private agent budget constraint, the economy's external debt accumulation, expressed in terms of exportable good, is given by

$$\dot{z} = x + \sigma y + pI + \sigma\phi(I) + G + \bar{r}z - F(K_1, L_1, aG) - \sigma H(K - K_1, 1 - L_1, (1-a)G) \tag{14}$$

In fact, equation (14) also describes the current account deficit, which is the sum of trade balance deficit, $[x + \sigma y + pI + \sigma\phi(I) + G - F(K_1, L_1, aG) - \sigma H(K - K_1, 1 - L_1, (1-a)G)]$ and the debt service payment, $\bar{r}z$.

To rule out the possibility that the country can run up infinite debt or credit with the rest of the world, we shall impose the following intertemporal budget constraint.

$$\lim_{t\to\infty} ze^{-\bar{r}t} = 0 \tag{15}$$

This constraint and the transversality conditions in private agent's optimization impose the corresponding constraint on the government, that is,

$$\lim_{t\to\infty} ae^{-\bar{r}t} = 0 \tag{16}$$

Given the specified factor intensity rankings, the output of exportable sector depends positively upon the stock of capital, the exogenous government expenditure and the fixed share, and depends negatively on the relative price of import-competing good, $Y^F(K, \sigma, a, G)$. In the same fashion, the output of import-competing good depends negatively upon the stock of capital, the exogenous government expenditure and the given share, and depends positively on the relative price of import-competing good, $Y^H(K, \sigma, a, G)$.[4]

[4] Under the current factor intensity assumptions, both Rybczynski theorem and Stolper-Samuelson theorem are applied in order to determine the output responses to the changes in K, σ, a and G in a small open economy where the output prices are fixed.

Ⅲ. MACROECONOMIC EQUILIBRIUM

The macroeconomic equilibrium is obtained when the planned demand and supply functions derived from the respective optimizations, consistent with the accumulation equations, clear all markets. Setting $\lambda = \bar{\lambda}$ provides the following set of equations together with equation (12), (15) and (16) to describe the macroeconomic equilibrium.

$$U_x(x, y) = \bar{\lambda} \tag{17}$$

$$U_y(x, y) = \sigma\bar{\lambda} \tag{18}$$

$$q = p + \sigma\phi'(I) \tag{19}$$

$$F_K(K_1, L_1, aG) = \sigma H_K(K - K_1, 1 - L_1, (1-a)G) \equiv r^K(K, \sigma, a, G) \tag{10}$$

$$F_L(K_1, L_1, aG) = \sigma H_K(K - K_1, 1 - L_1, (1-a)G) \equiv w(K, \sigma, a, G) \tag{11}$$

$$\dot{K} = I \tag{20}$$

$$\dot{q} = \bar{r}q - r^K(K, \sigma, a, G) \tag{21}$$

$$\dot{z} = x + \sigma y + pI + \sigma\phi(I) + G + \bar{r}z - F(K_1, L_1, aG) - \sigma H(K - K_1, 1 - L_1, (1-a)G) \tag{22}$$

$$\dot{a} = \bar{r} + G - T \tag{23}$$

The first five equations describe the short-run static equilibrium. The first three equations, static part of the equilibrium, can be solved for x, y and I, in terms of $\bar{\lambda}, \sigma, q$ and p.

$$\bar{x} = x(\bar{\lambda}, \sigma), \quad x_{\bar{\lambda}} < 0, \; x_\sigma < 0 \tag{24}$$

$$\bar{y} = y(\bar{\lambda}, \sigma), \quad y_{\bar{\lambda}} < 0, \; y_\sigma < 0 \tag{25}$$

$$I = I(q, p, \sigma), \quad I_q > 0, I_p < 0, I_\sigma < 0 \tag{26}$$

The consumption of both goods depends upon the constant marginal utility of wealth, $\bar{\lambda}$, and the exogenously determined relative price of import-competing good, σ. Therefore, the consumption is always constant at the steady state values, i.e., $\bar{x} = \tilde{x}$, $\bar{y} = \tilde{y}$ ($\tilde{x}$, $\tilde{y}$ will be determined in the below), during the adjustment path until no further information is arrived. Thus the dynamics of consumption in this model is degenerated since the consumption does not depend

upon the dynamic variables, either K or q. The model, however, can generate the dynamics of consumption by relaxing the assumption of either inelastic labor supply or the exogenous relative price of import-competing good.[5]

The consumption of import-competing good depends negatively upon the marginal utility of wealth, $\bar{\lambda}$, with the two goods being complementary in utility, $U_{xy}>0$. This is because the increase in the marginal utility of wealth reduces the consumption of exportable good and in turn the consumption of import-competing good falls due to $U_{xy}>0$. The increase in the relative price of import-competing good, σ, reduces the consumption of import-competing good, and thus discourages consumption of exportable good as well. Equation (26) shows the investment depends positively upon the market value of installed capital, q, which is consistent with the *Tobin's q* model, and negatively depends on the two exogenous variables, p and σ as well.

Substituting equation (26) together with the efficiency conditions in production, equation (10) and (11), into equation (20) and (21), leads to a pair of dynamic equations in q and K. Note that since this pair of equations is in part determined by two exogenous policy variables, a and G, these policy variables in part determine the entire dynamic adjustment path. This is the channel that the government affects in part the economy's capital accumulation and external debt accumulation. Besides, the exogenous relative price of import-competing good, σ, and the given price of capital good, p, in part determine the dynamic adjustment path because of the dependency of the rate of investment and the rental price of capital on σ and p. The economy in this model employs the current account to finance foreign capital for domestic capital accumulation as discussed in equation (22).

Using equation (24) - (26) together with the outputs of two sectors, we can rewrite the external debt accumulation equation.

$$\begin{aligned}\dot{z} = {} & \bar{x}(\bar{\lambda},\sigma)+\sigma\bar{y}(\bar{\lambda},\sigma)+pI(q,\sigma,p)+\sigma\phi(I(q,\sigma,p))+G+\overline{rz} \\ & -Y^F(K,\sigma,G,a)-\sigma Y^H(K,\sigma,G,a)\end{aligned} \tag{27}$$

This equation will be used to describe the dynamic structure of the economy combined with equation (20) and (21).

Ⅳ. EQUILIBRIUM DYNAMICS

The dynamic structure of the economy is a simple block recursive one with the balanced budget policy of the government, i.e., $\dot{a}=0$ and $\overline{a}=b+z$. Thus $\dot{K}$ and $\dot{q}$ constitute the core dynamics and $\dot{z}$ is solved by substituting the solutions

[5] For further discussion, see Sen and Turnovsky(1989).

of K and q into $\dot{z}$. The dependence of the rental price of capital on two exogenous policy variables, G and α in part determines the entire dynamic adjustment path of the economy.

Linearizing $\dot{K}$ and $\dot{q}$ around the steady state makes two linear differential equations system.

$$\begin{pmatrix} \dot{K} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\sigma\phi''} \\ -r_K^K & r \end{pmatrix} \begin{pmatrix} K-\tilde{K} \\ q-\tilde{q} \end{pmatrix} \tag{28}$$

The system represents a saddle point with eigenvalues $\mu_1<0$ and $\mu_2>0$ because the determinant of the coefficient matrix in the above system $(r_K^K/\sigma\phi'')$ is negative and the trace of that $(\bar{r})$ is positive. Therefore, the stable solutions for K and q are given by

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu_1 t} \tag{29}$$

$$q(t) = \tilde{q} + \sigma\mu_1\phi''(K_0 - \tilde{K})e^{\mu_1 t} = q + \sigma\mu_1\phi''(K - \tilde{K}) \tag{30}$$

To determine the dynamics of the economy's current account, which also describes the economy's accumulation of external debt, we linearize equation (27) around the steady state to get

$$\dot{z} = pI_q(q - \tilde{q}) - (Y_K^F + \sigma Y_K^H)(K - \tilde{K}) + \bar{r}(z - \tilde{z}) \tag{31}$$

The coefficients in equation (31) are evaluated at steady state. Substituting the solutions of $\dot{K}$ and q into equation (31), we obtain

$$\dot{z} = \bar{r}z - \bar{r}\tilde{z} + \Omega(K_0 - \tilde{K})e^{\mu_1 t} \tag{32}$$

where $\Omega \equiv (p\mu_1 - Y_K^F - \sigma Y_K^H)$ describes two channels of influence of capital on $\dot{z}$. Assuming that the economy starts out with an initial stock of external debt $z(0) = z_0$, the solution to $\dot{z}$ is

$$z(t) = \tilde{z} + \frac{\Omega(K_0 - \tilde{K})}{\mu_1 - \bar{r}} e^{\mu_1 t} + [z_0 - \tilde{z} - \frac{\Omega}{\mu_1 - \bar{r}}(K_0 - \tilde{K})]e^{\bar{r}t} \tag{33}$$

To make the solution to be consistent with intertemporal budget constraint,

$$z_0 - \tilde{z} = \frac{\Omega}{\mu_1 - \bar{r}}(K_0 - \tilde{K}) \tag{34}$$

then, the solution becomes

$$z(t) = \tilde{z} + \frac{\Omega(K_0 - \tilde{K})}{\mu_1 - \bar{r}} e^{\mu_1 t} \tag{35}$$

Equation (35) describes the relationship between the accumulation of external debt and the accumulation of capital stock. The importance is to determine the sign of Ω. The Ω reflects the effects of the change in capital stock on the outputs of exportable and import-competing sector. Given the factor intensity rankings, the change in the output of the exportable sector in response to the change in the capital stock is positive, while that of import-competing sector is negative. While either sign is possible, we assume former effects dominate and thus, $\Omega < 0$. With $\Omega < 0$, equation (35) represents a positive relationship between the accumulation of capital stock and the accumulation of external debt stock.[6] Note that equation (34) describes the steady state relationship between the stock of external debt and capital stock, in which steady state values $(\tilde{K}, \tilde{z})$ depend upon the initial values (K_0, z_0). This implies that the temporary shock produces permanent effects.

The dynamic path of K and q is described by a saddlepoint in the $K-q$ space. The phase diagram with the stable arm X_1X_1 and the unstable arm Y_1Y_1 is illustrated in Figure 1. Figure 2 represents the relationship between the accumulation of external debt and the accumulation of capital stock by noting that the slope of equation (35) is positive together with the stable arm X_1X_1 in the $K-q$ space. The positive relationship, denoted by S_1S_1 in part B of Figure 2, implies that the difference between domestic investment and national saving is financed by the imports of foreign capital using the current account. Thus the current account deficits mainly caused by the imports of foreign capital becomes the new addition to the capital stock.[7]

V. STEADY-STATE EQUILIBRIUM

The steady state of the economy is obtained when $\dot{K} = \dot{q} = \dot{z} = 0$ assuming the balanced budget policy of the government. With the assumption of no depreciation in the capital stock, the rate of investment at steady state is zero, $I = 0$. The rental rate of capital deplated by the fixed price of foreign capital

[6] This positive relationship appropriately reflects the characteristics of developing economies where the economy's capital stock accumulation is mainly financed by the foreign capital in the form of government-guaranteed external debt.

[7] In this model, the consumption of two goods (x, y) is constant at steady state values so that the main source of current account deficits may be explained by domestic investment rather than the interest payments.

good (r^K/p) is equal to the given world interest rate $(\bar{r})$. The market price of installed capital equals the fixed price of investment good, i.e., $\tilde{q}=p$. Then the steady state equilibrium of the economy is given by the following set of equations.

$$U_x(\tilde{x}, \tilde{y}) = \tilde{\lambda} \tag{36}$$

$$U_y(\tilde{x}, \tilde{y}) = \sigma\tilde{\lambda} \tag{37}$$

$$p\bar{r} = r^K(\tilde{K}, \sigma, a, G) \tag{38}$$

$$\bar{r}\tilde{z} = Y^F(\tilde{K}, \sigma, a, G) + \sigma Y^H(\tilde{K}, \sigma, a, G) - \tilde{x} - \sigma\tilde{y} - G \tag{39}$$

$$\tilde{z} - z_0 = \frac{\Omega}{\mu_1 - \bar{r}}(\tilde{K} - K_0) \tag{40}$$

$$\tilde{T} = \bar{r}a + G \tag{41}$$

The above equations (36)-(41) jointly determine the steady state equilibrium values of $\tilde{x}, \tilde{y}, \tilde{\lambda}, \tilde{K}, \tilde{z}$ and $\tilde{T}$. Several aspects of this steady state should be highlighted. First, the rental price of capital is a constant at $p\bar{r}$ so that the steady state value of capital stock should be higher to maintain equation (38) than the steady state value of capital stock when the rental price of capital stock does not depend upon two exogenous variables (G, a) due to $r_G^K > 0$ and $r_a^K > 0$. This directly requires the higher steady state value of external debt stock by making use of equation (40) since the coefficient, $[\Omega/(\mu_1 - \bar{r})]$, is positive. Thus the dependence of rental price of capital on G and a results in the higher steady-state values of both capital stock and external debt stock than the ones without such dependence. This is one of the main differences from the previous works, where the government expenditure does not affect the dynamic variables such as the capital stock and the external debt stock.

Consequently, the trade balance surplus, denoted by $\bar{r}\tilde{z}$, should be larger to meet the increased interest payment. This implies trade balance should improve together with the higher external debt stock. Equation (40) describes the equilibrium relationship between the change in the stock of capital and the change in the stock of external debt. And lump-sum tax is determined to ensure the steady state government budget constraint in accordance with equation (41).

Ⅵ. LONG RUN ANALYSIS

The dynamics of the system involves forward-looking behavior. Thus the short run transition is determined in part by the long run steady state. Therefore, we

will start with the long run analysis. The long run effects of unexpected permanent external shocks (increase in the world interest rate) and fiscal policies are reported in Table 1 and shall be discussed in turn.

6.1. Increase in World Interest Rate

At a higher world interest rate, the steady state value of capital stock should be lowered to maintain equation (38) due to $r_K^K < 0$. This directly requires the stock of external debt, $\tilde{z}$, should be reduced to a lower level according to equation (40) since we assumed the coefficient, $\Omega / (\mu_1 - \bar{r})$, is positive. The decline both in the stock of external debt and in the capital stock stems from the fact that the investment in the economy is made by the capital good imported from the rest of the world. Thus the increase in the world interest rate reduces the investment by increasing the cost of capital, which leads to a lower level of capital stock at a steady state, and the depressing effect on investment decreases the current account deficit, which results in the lower steady state level of external debt.

The trade balance, denoted by $\bar{r}\tilde{z}$, however, responds ambiguously because the stock of external debt falls to the permanent increase in world interest rate. Then, the net effects on the trade balance depends upon relative responsiveness of external debt. For example, if the investment is not so sensitive to the interest rate (plausible in some developing countries), the interest payment rises to the increase in world interest rate because the imports of capital good does change little the current account deficits. In this case, the trade balance surplus at a new steady state must increase to meet the additional debt-service obligation.

The lower level of capital stock at a new steady state reduces the output of exportable sector, while increases that of import-competing sector because of $Y_K^F > 0$ and $Y_K^H < 0$. Thus the total output in the economy shows uncertain response to the permanent shock. This implies that the marginal utility of wealth, λ, is not clear as well. This also leads to the unclear response of consumption of both goods. The lump-sum taxes, however, rise to finance additional interest payment for the outstanding debt in accordance with equation (41). In sum, the rise in the world interest rate depresses the investment and thus the accumulation of both capital stock and external debt stock slows down in a foreign-capital-dependent economy. This outcome is consistent with that of previous works (Bhandari, Haque and Turnovsky(1989)).

6.2. Increase in the Government Expenditure (G)

Unlike the demand-side analysis of the government expenditure, the alternative analysis of government expenditure makes it possible to affect the dynamic variables such as capital stock and external debt stock, still maintaining the

assumptions of a small open economy and the inelastic labor supply.[8] Thus, the increase in the exogenous government expenditure must raise the steady state value of capital stock to satisfy equation (38) due to $r_G^K > 0$. This may be explained in the following way: The increase in the government expenditure enhances the productivity of both capital and labor so that the market price of installed capital will jump through the induced change in the rental price of capital and thus lead to the acceleration of investment. The higher level of capital stock in return requires the higher level of external debt stock according to equation (40). The reasoning is that the imports of foreign capital good must increase to meet the increased investment and hence produce more current account deficits during the adjustment path.

The outputs are affected by both the change in the government expenditure directly and the change in the capital stock indirectly. Based upon the specified factor intensity rankings, the change in the output of exportable sector is assumed to exceed that of import-competing sector in response to the increase in government expenditure, i.e., $(Y_G^F + \sigma Y_G^H) > 0$. Then the gross national wealth would increase, but the disposable income is not clear due to the accompanying increase in lump-sum taxes until we impose another condition. Suppose that the government expenditure is made with the range that marginal product of government expenditure is greater than the marginal cost, $(Y_G^F + \sigma Y_G^H - 1 > 0)$, the disposable income will rise and then the marginal utility of wealth will decrease. With this assumption, the consumption of exportable good will rise and so the consumption of import-competing good because of $U_{xy} > 0$. This outcome is contrast to the effects of government expenditure where the consumption, in general, falls due to the increased lump-sum taxes. The trade balance surplus in terms of interest payment ($\overline{rz}$) should increase to meet the increased interest payments. And the lump-sum tax should rise to finance the additional government expenditure.

It is useful to summarize the effects of the government expenditure in relation to the demand-side analysis of the government expenditure. The supply-side analysis of the government expenditure captures the productivity-enhancing effect and thus accelerates the rate of investment, which also results in more current account deficits in a foreign-capital-dependent developing economy. That the government expenditure affects the dynamic variables in a small open economy with inelastic labor supply is worthwhile to be emphasized, because the demand-side analysis usually does not affect the dynamic variables in the economy. In addition, the supply-side application of the government expenditure may increase

[8] In Turnovsky and Sen(1991), they analyze the effects of increase in the government expenditure in an intertemporal optimizing model to highlight the accumulation of capital stock during the dynamic adjustment. In their model, however, they rely on the endogenous labor supply and the endogenously determined relative price of foreign good assuming a semi-small open economy. The model still considers the demand aspect of government expenditure, though.

consumption under a certain restraint through the change in the outputs, not through the change in the aggregate demand. This is because the supply-side effect of the government expenditure dominates the demand-side effect.

In addition, the change in the share of the government expenditure allocated to the exportable sector basically results in the same effects on the economy like in the case of government expenditure. The outcomes are identical to those of government expenditure except no change in lump-sum tax. Together with the effects of the increase in the share on the economy, several points deserve to be mentioned policy implications.

Firstly, the share as well as the total amount of government expenditure matters in determining the rate of investment and current account deficits, which ultimately determine the accumulation of capital stock and the accumulation of external debt stock. This implies that the choice of the share is also important to the macroeconomic performances even at the given level of government expenditure. Secondly, the share also affects the consumption levels of both goods through the change in the outputs, not through the change in disposable income. This change in consumption is attributed to the change in national wealth by the induced change in capital stock and the sectoral transfer of government expenditure. Thirdly, in this context, the share may be used as an additional policy instrument by the government in developing countries depending upon the relevant factor intensities. What this means is that the government can choose a different value of the share without changing the given level of government expenditure as the relevant factor intensities adjust to the accumulation of capital stock and the new technology transfer. Lastly, it is also possible that either G or α may be utilized to stabilize the economy facing the external shocks, for example, the increase in world interest rate as accommodating policies. The reason is that, unlike the demand-side analysis which in general affects the aggregate demand only, the supply-side application of government expenditure affects both aggregate demand and aggregate supply.

[Table 1] The Long Run Effects of Permanent Increase

Variable	World Interest Rate	G	α
$\hat{\tilde{x}}$	?	+	+
$\hat{\tilde{y}}$	?	+	+
$\hat{\lambda}$	?	−	−
$\hat{K}$	−	+	+
$\hat{\tilde{z}}$	−	+	+
$\hat{T}$	+	1	0
TB	?	+	+

Ⅶ. SHORT RUN DYNAMICS

The dynamics of K and q are described by a saddle point in $K-q$ space in section Ⅳ. The stable arm X_1X_1 in Figure 1 is given by

$$q(t)=\tilde{q}+\sigma\mu_1\phi''(K-\bar{K}) \tag{42}$$

and is negatively sloped: the unstable arm, which is drawn by Y_1Y_1 in Figure 1, is described by

$$q(t)=\tilde{q}+\sigma\mu_2\phi''(K-\bar{K}) \tag{43}$$

and is positively sloped. The effects of an unanticipated external shock and fiscal expansion will be discussed. The dynamic responses are represented in Figure 2.

7.1. Increase in World Interest Rate ($\bar{r}$)

The economy lies initially in the stable arm X_1X_1 in Figure 2A and starts out with the initial capital stock, K_0. If the economy is hit by the unexpected permanent increase in world interest rate and no further change is anticipated, the market price of installed capital, q jumps down instantaneously, which is given by

$$\frac{dq(0)}{d\bar{r}}=-\sigma\mu_1\phi''\frac{d\bar{K}}{d\bar{r}}<0 \tag{44}$$

since the long run value of capital stock falls in response to the unexpected shock.

Part A of Figure 2 describes the adjustments in K and q, while part B describes the evolution of external debt, z, with the capital stock accumulation. Without unanticipated shock, the economy will be in steady state at point E_1 with $\tilde{q}=p$ and $\bar{K}_1$. With the arrival of the unexpected shock, the economy's new steady state will be at point E_1' with lower level of capital stock, $\bar{K}_1'$, and unchanged market price of installed capital, $\tilde{q}=p$. In the short run, q jumps from P to R on the new stable locus $X_1'X_1'$ immediately following the shock. The jump in q has an immediate contractionary effect on investment and capital accumulation slows down. The responses of consumption of two goods are identical to those of long run analysis since consumption of two goods is

determined by $\bar{\lambda}$ and σ, and is independent of K.

Upon reaching point R on a stable arm $X_1'X_1'$, q begins to start falling again, while K starts to rise; $d\dot{q}(0)<0$, $d\dot{K}(0)>0$. The rate of investment, however, should be reduced to get the lower level of capital stock at a new steady state. The consumptions of two goods are constant over time at the levels of steady state.[9] This is because we characterize a sufficiently small open developing economy where the terms of trade (real exchange rate) is not affected by the exports of domestically produced good.

Part B of Figure 2 illustrates the relationship between z and K, which combining equation (29) and (35) is given by

$$z-\tilde{z}=\frac{\Omega}{\mu_1-\bar{r}}(K-\tilde{K}) \tag{45}$$

This is a positively sloped locus, denoted by S_1S_1 in Figure 2B. Since neither K nor z are jump variables, this line remains fixed over time. Without having the unexpected permanent shock, the economy will end up with the steady state, M_1 with $\tilde{K}_1$ and $\tilde{z}_1$. As the shock hits the economy, the economy will be in a new steady state denoted by M_1' with lower levels of both capital stock accompanied by lower level of external debt stock.

Due to the reduced rate of investment caused by the jump down of the market price of installed capital, the imports of foreign capital good decrease and in turn the use of import-competing good for the installation cost of investment falls, too. The combination of these two effects improves the current account deficits during the adjustment path and leads to a lower level of external debt stock at a new steady state, which is denoted by $\tilde{z}_1$ in part B of Figure 2.

Since we assume the economy should import capital good from the rest of the world for domestic capital formation, the economy may experience current account deficits as long as capital accumulation occurs. In the early stage of dynamic adjustment, the current account deficit is mainly attributed to the trade balance deficit by the massive inflow of foreign capital good for domestic investment. As the economy approaches to the steady state, however, the interest payment for the outstanding external debt may become the main part of current account deficit rather than the trade balance deficit (possibly surplus) since the investment falls over time as the market price of installed capital drops continuously as well toward the steady state value.

[9] Sen and Turnovsky(1989) discusses the endogenous labor supply and endogenously determined relative price of import-competing good in an optimizing model to analyze the dynamic responses to the tariffs. In their model, the short run response of consumption depends upon the indirect effects through the induced jump in λ and q as well as the direct effect to the change in tariff.

7.2. Increase in Government Expenditure (G)

If the economy is hit by the unexpected permanent increase in government expenditure as the fiscal expansion and no further change is anticipated, the market price of installed capital, q, jumps up immediately, which is given by

$$\frac{dq(0)}{dG} = -\sigma\mu_1\phi'' \frac{d\bar{K}}{dG} > 0 \tag{46}$$

since the long run value of capital stock rises in response to the unexpected policy shock. On impact, q jumps up from P to Q on the stable arm locus $X_1''X_1''$. The jump in q has an immediate stimulating effect on investment and thus the capital accumulation speeds up. Then the imports of both capital good and import-competing good must increase to meet the increased demand and installation cost of investment. Thus the current account deficits will increase in the short run, which will result in the higher level of external debt at a new steady state, denoted by $\tilde{z}_1''$ in part B of Figure 2.

Ⅷ. CONCLUSION

The model in this paper incorporates a foreign-capital-dependent developing economy and thus shows the explicit relationship between external debt and capital stock accumulation which has been controversal, but has not yet been explained theoretically. Especially, the direct application of government expenditure to the production side of the economy as one of the factors of production using the two-sector exportable and importable-competing intertemporal optimizing model captures the productivity-enhancing effects associated with the government expenditure in such developing economies.

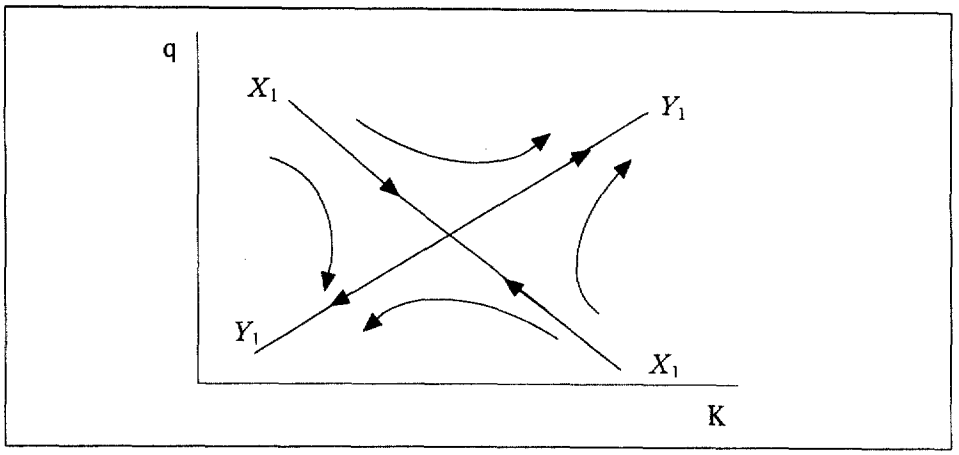
Unlike the demand-side analysis of the government expenditure, the supply-side analysis affects the rate of investment through the change in the market value of installed capital, which also results in the change in current account deficits. The fact that the government expenditure affects the dynamic variables in a small open economy with inelastic labor supply is worthwhile to be emphasized, because the demand-side analysis usually does not affect the dynamic variables in the economy. In addition, the supply-side application of the government expenditure may increase consumption under a certain restraint through the change in the outputs, not through the change in the aggregate demand. This is because the supply-side effect of the government expenditure dominates the demand-side effect.

The model, however, has some limitations. Inelastic labor supply should be relaxed to allow for unemployment and endogenous relative price of import-

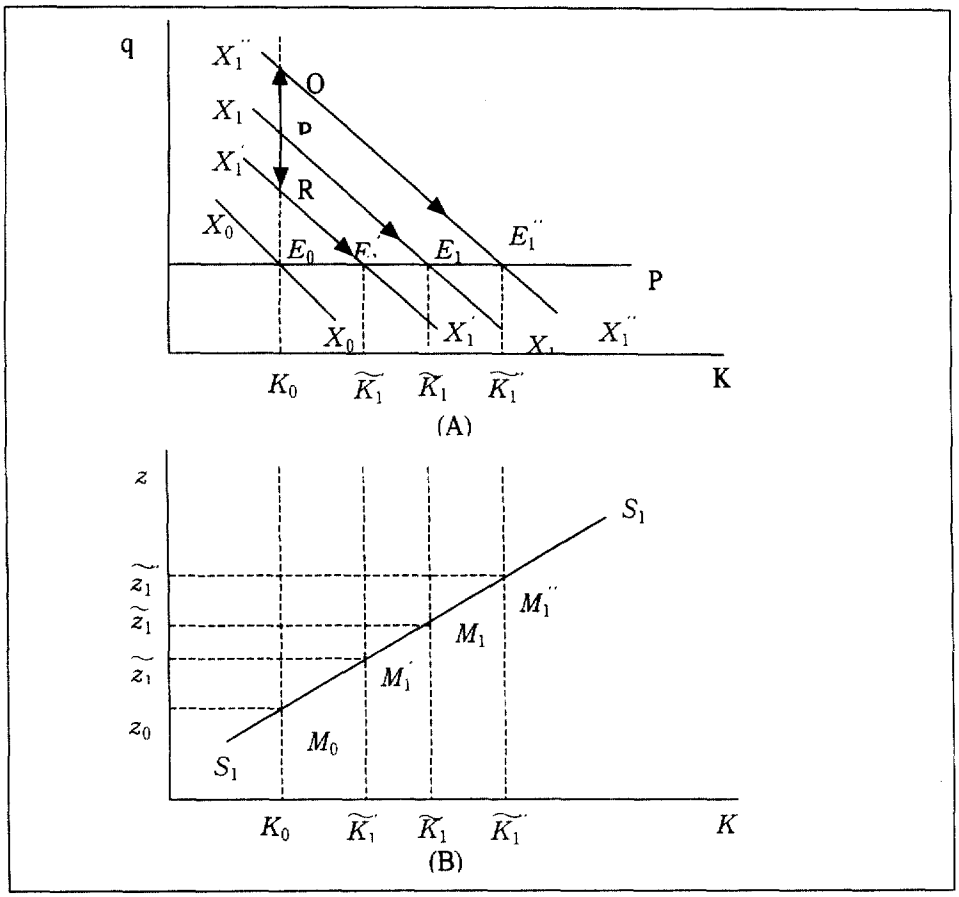
competing good rather than the fixed price could be applied by assuming the semi-small open economy. Besides, in order to make the model more useful, the government expenditure and its share can be endogenously determined by treating the government as a welfare maximizer. In this case, the mutual interactions between the private agent and the government can affect the dynamic variables and thus the adjustment paths of the economy.

Despite the above shortcomings, the current study shows the explicit dynamic relationship between external debt and capital stock accumulation, and captures the productivity-enhancing effects associated with the government expenditure in a foreign-capital-dependent developing economy using an intertemporal optimizing model. Thus, the model can partly explains the macroeconomic performances in such developing countries where the role of government has been quite significant in the process of economic developments.

[Figure 1]



[Figure 2]



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