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# TRADE, CAPITAL MOBILITY AND GROWTH IN THE TWO SECTOR MODEL

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*This paper analyzes an intertemporal optimization model of international trade and borrowing in which interactions between the process of capital accumulation and the pattern of international trade are more fully analyzed. We deal with the effects of trade and factor movement on the long run equilibrium values of some economic variables in a two sector model within an optimization framework. We also look into the problems related to existence and uniqueness of long run solutions.*

*In the specialization of consumption goods case, we can get the results that an increase in the relative price of investment goods or the depreciation rate reduces the steady state equilibrium level of consumption. An increase in the time preference has a lower value of consumption and net assets. In addition, an increase in the population growth rate lowers the value of assets. We have also model the comparative dynamic model. If the world price rises, the optimal path of asset is shifted down and the stationary level of consumption good could be lower.*

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## I. INTRODUCTION

The standard theory of international economics has mainly focused on the international exchange of commodities while international movements of pro-

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ductive factors have received less attention until last decade. Indeed, neoclassical trade theorists lousialer(regavel) international factor immobility as the basic cause of international trade. But in recent years, the subject of international factor movements has become a matter of importance due to a substantial increase in foreign investment and migration.

We investigate such factor movements, especially the inflow of foreign capital, and the effect of these movements on trade and economic growth. The problem of optimum trade and foreign borrowing or lending has usually been discussed in terms of static model. However, it has been recognized that a complete analysis of the benefits of foreign trade must also take into account the intertemporal pattern of allocation of all resources available to an economy, both foreign and domestic.

This paper formulates an intertemporal optimization model of international trade and borrowing in which interactions between the process of capital accumulation and the pattern of international trade are more fully analyzed.

In this paper we deal with the effects of trade and factor movement on the long run equilibrium values of some economic variables in a two sector model within an optimization framework. We also look into the problems related to existence and uniqueness of long run solutions.

We set forth to the optimization problem with trade in section II and formulate the optimal growth with trade and capital inflows in section III. Conclusions are in section IV.

## II. OPTIMAL GROWTH WITH TRADE

This section analyzes dynamic models of international trade in a small open economy and interactions between the process of capital accumulation and the patterns of trade. Ryder has considered this model in a moderately sized economy which affects its terms of trade. He extends the Uzawa - Srinivasan model by allowing the country to trade at terms dependent on the amount of trading. He introduces disembodied Harrod neutral technical change progress after the basic model and sets forth to its implication. He shows that the introduction of dynamic optimizations does not alter the nature of static allocation but merely generates a shadow price for the guidance of essentially static decisions.

The analysis of this paper is based on the assumption that the home country is too small to affect its terms of trade. Therefore the home country faces a constant terms of trade, $P^*$. In the closed economy model, the price level was endogenously determined in the domestic market.

When the economy opens to free trade at fixed world prices $P^*$, the absorption of commodity $y_i$, will be constrained by home production $q_i$, exports $x_i$, and balanced trade.

The optimization problem can be thought of as choosing time paths for the rate of change of capital accumulation ($\dot{k}$) so as to

$$\max \int_0^\infty U(y_c)\exp(-\beta t)dt \tag{1}$$

subject to

$$\begin{aligned}\dot{k} &= y_I-(n+\delta)k \\ &= q/P^*-(n+\delta)k-y_c/P^* \\ k(0) &> 0.\end{aligned} \tag{2}$$

The current value Hamiltonian is

$$H=U\{q(P^*,k)-(n+\delta)P^*k-P^*\dot{k}\}+m\dot{k} \tag{3}$$

$$\partial H/\partial \dot{k}=-P^*U'(y_c)+m=0 \tag{4}$$

$$\begin{aligned}\dot{m} &= \beta m-\partial U/\partial k=\beta m-U'\{\partial q/\partial k-(n+\delta)P^*\} \\ &= \{\beta+n+\delta-r(P^*,k)/P^*\}m\end{aligned} \tag{5}$$

We can not find the solution explicitly without specification. Nonetheless, we can qualitatively characterize the solution by sketching paths compatible with the conditions for the $m \times k$ plane or the $y_c \times k$ plane. We eliminate $m$ and consider the $y_c \times k$ plane.

Differentiating eq.(4) with respect to time, we get $P^*U''\dot{y}_c=\dot{m}$. Substituting this result into eq.(5), we obtain :

$$\dot{y}=(r(P^*,k)/p^*-\beta-n-\delta)y_c/\sigma(y_c). \tag{6}$$

Eqs. (2) and (6) form a pair of differential equations in $k$ and $y_c$. In the two sector model, we usually have different solutions depending on factor intensities and the initial capital stock.

To sketch the directions of movement compatible with these two equations for the cases where consumption goods production, we first consider the $\dot{y}_c=0$ locus. Since $P^*$ is constant and $r$ is a only function of $k$, the $\dot{y}_c=0$ condition implies that $\hat{k}$ is unique in the case of specialization. If we consider the specialization case in either goods, the unique steady state solution $(\hat{y}_c, \hat{k})$ will be asymptotically approached by the system.

On the other hand, in the case of diversification, for a given $k$ value in the interval $[k_i, k_c]$ where $k_i$ is the capital labor ratio of the i sector given $P^*$ when the economy begins to specialize in the production of the i goods, the optimal long run solution depends on the initial value of the factor endowment ratio $k_0$. If $k_0 > k_i$ and $r(P^*)/P^* < (>)\ \beta + n + \delta$, the system asymptotically approaches the steady state capital-labor ratio $\hat{k}$ because of a negative (positive) $\dot{y}_C$. If $r(P^*)/P^* = \beta + n + \delta$, the solution can be obtained by $P^*$ and $k_0$. We have similar results when investment goods production is more capital intensive ($k_I > k_C$).

Figure 1 illustrates the phase diagram for per capita capital and consumption under the different assumptions when $k_C$ is greater than $k_I$. Figure 2 reflects the analysis that the economy specializes the production of the investment goods ($y_0 < k_I$) but the steady state value of per capita capital ($\hat{k}$) is in the region of specialization in the production of the consumption goods.

This result shows that convergence of the capital-labor ratio to a steady state value occurs regardless of the assumption of factor intensities in the two sectors. It differs from descriptive two sector growth models that are stable only if consumption goods are at least as capital-intensive as investment goods.

## III. OPTIMAL GROWTH WITH TRADE AND BORROWING

In this section the open economy model is extended by allowing foreign capital inflows. The home country trades with and borrows from the rest of the world. The main difference of this model from the previous one is the balance of payments relation, which now changes to $\dot{D} = -(X_I - 1/P^*) X_C + r^* D$ where the trade balance, $X_I + (1/P^*) X_C$, is added to service payment to foreign nations, $r^* D$, to obtain the current balance. We abstract from changes in international reserves and assume that the current balance is always equal to the capital balance, $\dot{D}$.

We assume that home country is confronted with a given terms of trade. The price of the investment goods in terms of the consumption goods is given exogenously. We also assume that home country faces imperfect world capital markets. The cost of foreign capital is an increasing function of indebtedness.

In brief, we may rewrite the relationships underlying the model,

$$Y_i = Q_i - X_i, \qquad i = I, C, \tag{7}$$

the absorption of commodity i, is equal to the home production plus the imports of commodity.

$$Y = Y_C + P^* Y_I, \tag{8}$$

total absorption in terms of the consumption goods.

$$Q = Q_C + P^* Q_I, \tag{9}$$

total production in terms of the consumption goods.

$$\dot{D} = -X_I - (1/P^*)X_C + r^*D, \tag{10}$$

the balance of payment relationship.

$$\dot{K} = Y_I - \delta K, \tag{11}$$

capital formation determines the net change in capital stock as absorption of the investment good minus the depreciation.

$$\dot{w} = \dot{k} - \dot{d}, \tag{12}$$

the change in net assets is equal to the change in capital minus the change in debt.

By reformulating, we obtain:

$$\begin{aligned}
\dot{w} &= \dot{k} - \dot{d} \\
&= y_I - (n+\delta)k + x_I - (r^* - n)d + 1/(P^*)x_C \\
&= q_I - (n+\delta)k - (r^* - n)(k - w) + 1/(P^*)x_C \\
&= q_I - (1/P^*)(y_C - q_C) - (r^* + \delta)k + (r^* - n)w \\
&= q/P^* - y_C/P^* - (r^* + \delta)k + (r^* - n)w
\end{aligned}$$

where $q_C = q_C(P^*, k)$, $q_I = q_I(P^*, k)$.

The optimization problem is to maximize the social welfare function over the per capita consumption level. Thus, the problem the economy faces is to choose $\dot{w}$ and $k$ (neither $k$ nor $d$ are control variables since $k$ or $d$ can jump at any point in time).

$$\max \int_0^\infty U(y_C)\exp(-\beta t)dt$$

subject to

$$\begin{aligned}
&\dot{w} = q(P^*, k)/P^* - y_C/P^* - (r^* + \delta)k + (r^* - n)w. \\
&\quad k(0) = k_0,\ d(0) = d_0,\ k > 0,\ y_C > 0.
\end{aligned}$$

By using the current value of Hamiltonian,

$$H = U\{q - (r^* + \delta)P^*k + (r^* - n)P^*w - P^*\dot{w}\} + m\dot{w}$$

*where* $m$ is the current value multiplier, $\lambda\exp(\beta t)$, that is, the marginal value of net assets. The necessary conditions are:

$$\frac{\partial H}{\partial \dot{w}}=-P^{*}U'(y_c)+m=0 \tag{13}$$

$$P^{*}U'(y_c)=m$$

$$\frac{\partial H}{\partial k}=U'\left\{\frac{\partial q}{\partial k}-(r^{*}+\delta)P^{*}-r^{*\prime}P^{*}k+r^{*\prime}P^{*}w\right\} \tag{14}$$

$$=U'\left\{\frac{\partial q}{\partial k}-(r^{*}+\delta)P^{*}+P^{*}r^{*\prime}(w-k)\right\}$$

$$=U'\left\{\frac{\partial q}{\partial k}-(r^{*}+\delta)P^{*}-P^{*}r^{*\prime}d\right\}=0$$

The condition in eq.(14) can be written

$$\frac{\partial q}{\partial k}=(r^{*}+\delta)P^{*}r^{*\prime}d$$

$$\frac{r(P^{*},k)}{P^{*}}=r^{*}+r^{*}d+\delta=\Phi(d)+\delta$$

We derive per capita debt $(d)$ in terms of $k$ by totally differentiating above equation which results in $d=d(k)$ and $d'=r'(P^{*},k)/\Phi'(d)$. The sign of $d'$ depends on the sign of $r'$ and $\Phi'$. In the complete specialization case $r'<0$, so $d<0$. If we consider the case of diversification, $r'=0$, thus $d'=0$ this implies that per capita debt is constant in case of diversification in production, and decreasing in case of specialization. Therefore we must distinguish these two cases when we draw the phase diagram. Noting,

$$\dot{m}=\beta m-\frac{\partial U}{\partial w}=\beta m-U'(r^{*}+r^{*}d-n)P^{*} \tag{15}$$

$$=(\beta+n-r^{*}-r^{*\prime}d)m$$

$m$ can be replaced by $y_c$ since $U''<0$, $U'$ is a monotone thus, from eq.(13)

$$y_c=U'^{-1}(m)/P^{*}=g(m)/P^{*}.$$

The properties of $g$ are readily obtained from the properties of $U'$. Differentiating eq.(13) gives $U''dy_c=dm$, so that $U''dy_c/dm=1/U''=g'<0$. Differentiating eq.(13) with respect to time, we get $P^{*}U''\dot{y}_c=\dot{m}$, and substituting this result into eq.(15), we obtain:

$$\dot{y}_c=(\beta+n-r^{*}-r^{*\prime}d)\{(U'/U'')(1/y_c)\}y_c \tag{16}$$
$$=(r(P^{*},k)/P^{*}-\beta-n-\delta)y_c/\sigma(y_c)$$

where $\sigma(y_c)$ is the elasticity of marginal utility. For simplicity, we again assume tent $\sigma$ is a matant.

The relevant dynamics of foreign borrowing and consumption are contained in equations (12) and (16), rewritten below:

$$\dot{w}=q/P^{*}-y_{c}/P^{*}-\{r^{*}(k-w)+\delta\}k+\{r^{*}(k-w)-n\}w \qquad (12')$$

$$\dot{y}_{c}=\{r(P^{*},\ k)/p^{*}-\beta-n-\delta\}y_{c}/\sigma. \qquad (16')$$

Notice first that eqs.(12′) and (16′) do not contain any discount terms, and the steady state is defined by $\dot{w}=\dot{y}_{c}=0$.

To sketch the directions of movement compatible with these two equations, we first consider the $\dot{y}_{c}=0$. locus, namely, points satisfying

$$\{r(P^{*},\ k)/P^{*}-\beta-n-\delta\}y_{c}/\sigma=0. \qquad (17)$$

Differentiating $\dot{y}_{c}=0$ with respect to $y_c$ and $w$, we get that $dw/dy_c=$ 0.

Then the locus $\dot{y}_{c}/y_{c}=0$ is independent of $y_c$. At a point $(\hat{y}_{c},\ \hat{w})$ on the locus, eq.(17) is satisfied. At any point $(\hat{y}_{c},\ \hat{w}+t)$, where $t>0$, to the right of the locus, we have

$$\{r(P^{*}\cdot k(\hat{w}+t))/P^{*}-\beta-n-\delta\}\hat{y}_{c}/\sigma<0.$$

Since $r$ is an decreasing function $\dot{y}_{c}<0$ at such a point. In the same way, we get $\dot{y}_{c}>0$ to the left of the $\dot{y}_{c}=0$ locus.

Next, consider the points comprising the $\dot{w}=0$ locus, namely,

$$(q-y_{c})/\mathrm{P}^{*}-(n+\delta)k-(r^{*}-n)d=0. \qquad (18)$$

Totally differentiating eq.(18) with respect to $y_c$ and $w$, we have

$$-\frac{1}{P^{*}}dy_{c}+\left[\frac{1}{P^{*}}\frac{\partial q}{\partial k}\frac{\partial k}{\partial w}-(n+\delta)\frac{\partial k}{\partial w}-(r^{*}-n)\frac{\partial d}{\partial k}\frac{\partial k}{\partial w}-r^{*\prime}\frac{\partial d}{\partial k}\frac{\partial k}{\partial w}\right]dw=0$$

$$-\frac{1}{P^{*}}dy_{c}+\left[\frac{r}{P^{*}}-(n+\delta)-(\Phi(d)-n)\frac{\partial k}{\partial w}\right]\left(\frac{\partial k}{\partial w}\right)dw=0$$

The slope of the $\dot{w}=0$ locus has different values under the different assumptions, in general: $\dfrac{dw}{dy_{c}}=P^{*}(r/P^{*}-n-\delta)(1-\partial d/\partial k)(\partial k/\partial w)$.

The slope of the $\dot{w}=0$ locus depends on the sign of $r/P^{*}-(n+\delta)$ in the diversification case ($w_{I}'<w_{0}<w_{C}'$ where $w_{i}'=k_{i}'-d(k_{i}')$).

The $\dot{w}=0$ curve is downward (upward) sloping if $r/P^{*}<n+\delta(r/P^{*}>n+\delta)$. the intercept of the $\dot{w}=0$ curve is always positive in the $y_c$ x $w$ plane.

We can rule out a downward sloping curve in our discussion since it implicitly implies that our country is a capital lender. This results from a combination of assumptions. Consider the two conditions $r/P^{*}<\delta+n$ and $\beta+n=\Phi$ holding simultaneously. These conditions imply that $\beta$ must be negative which violates the initial assumption that $\beta>0$. Therefore we can concentrate our analysis on the upward sloping curve case. Figure 3 illustrates the structure of the solution paths consistent with the differential equations in the specialization case. This intersection is unique and the determination of the optimal time path of $y_c$ and $w$ also determine the time path of per capita debt, the capital labor ratio, output, and the patterns of trade. In the diversification of production case, the steady state value of $w$ is not unique as we have already analyzed.

Now we can analyze the comparative static analysis of the steady state in case of specialization. An increase in the relative price of investment goods in terms of consumption goods $P^{*}$ lowers the $\dot{w}=0$ curve and leaves the $\dot{y}_c=0$ curve unaffected. The new intersection involves lower values of $\hat{y}_c$, and no change in $\hat{w}$. We have same results when the depreciation rate $\delta$ is changed. Thus, an increase in the relative price or the depreciation rate reduces equilibrium consumption. An increase in the time preference $\beta$ shifts the $\dot{y}_c=0$ locus leftward, so the new intersection has a lower value of $y_c$ and $w$. An increase in the population growth rate lowers the value of $\hat{w}$. While the equilibrium net assets fall, the influence on absorption of the equilibrium level of consumption is ambiguous. If $r^{*}>n$ a lower value of $y_c$ unambiguously results. These comparative static results can also be found by differentiating the system of eqs.(7) and (8). Comparative dynamic analysis is also possible. To determine the effect of an increase in the price level on the optimal path, let $P_2^{*}>P_1^{*}$. The impact on the steady state was discussed above. We derive the optimal path corresponding to $P_1^{*}$ by showing that the paths can not cross. Suppose $w_1$ is the $w$ coordinate of the intersection point. From figure 3 the slope of the optimal path associated with $P_2^{*}$ must be smaller than that associated with $P_1^{*}$ at $(y_1, w_1)$. The slope of the path is $dy_c/dw=\dot{y}_c/\dot{w}=\{(\beta+n-r^{*}-r^{*\prime}d)y_c/\sigma\}/\{q-y_c/P^{*}-(r^{*}+\delta)k+(r^{*}-n)w\}$. At the intersection, $y_1$ and $w_1$ will be the same for both paths. Therefore the required inequality holds if $P_2^{*}<P_1^{*}$, which is a contradiction and the optimal paths corresponding to $P_1^{*}$ and $P_2^{*}$ cannot cross. If the world price rises, the optimal path of $w$, and hence that of $y_c$, is shifted down and the stationary level of $w$ can not be changed. We can also get similar analyses in shifts of other parameters.

## Ⅳ. CONCLUSION

We have analyzed the two sectors model of economic growth. Especially we deal with the effects of trade on the long run equilibrium values of some economic variables within an optimization framework. We also looked into existence and uniqueness of long run solutions.

As already known, we have had results that the two sector growth model under the closed economy offers unique optimal path for per capita capital and the shadow price. And the shadow price might involve an initial phase of specialization in the production of the consumption goods if the initial capital stock is sufficiently large.

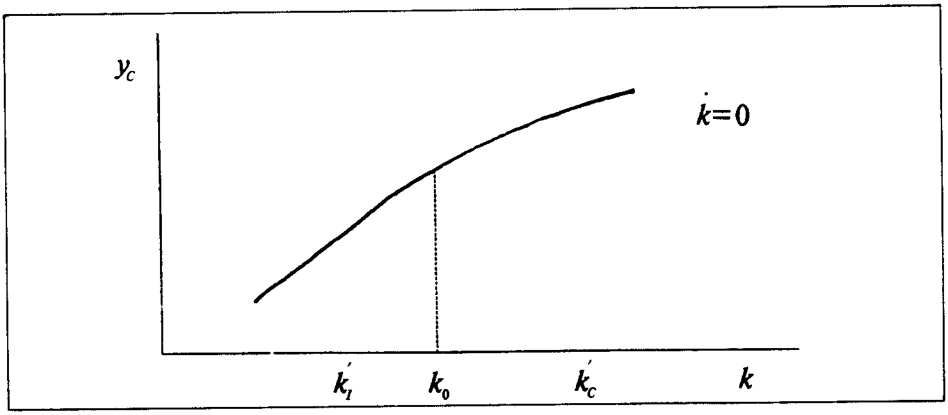
In the open economy, we have the results that convergence of the capital labor ratio to the steady state value occurs regardless of the assumption of factor intensities. It differs from the closed economy model which is stable only if consumption goods are at least as capital-intensive as investment goods.

We extended the model invohoing trade and foreign capital inflows. Then we have the results that the effect of goods is in terms of trade and foreign investment on the long run values of economic variables. In the specialization of consumption goods case, we can get the results that an increase in the relative price of investment goods in terms of consumption goods or the depreciation rate reduces the steady state equilibrium level of consumption. An increase in the time preference has a lower value of consumption and net assets. In addition, an increase in the population growth rate lowers the value of assets.

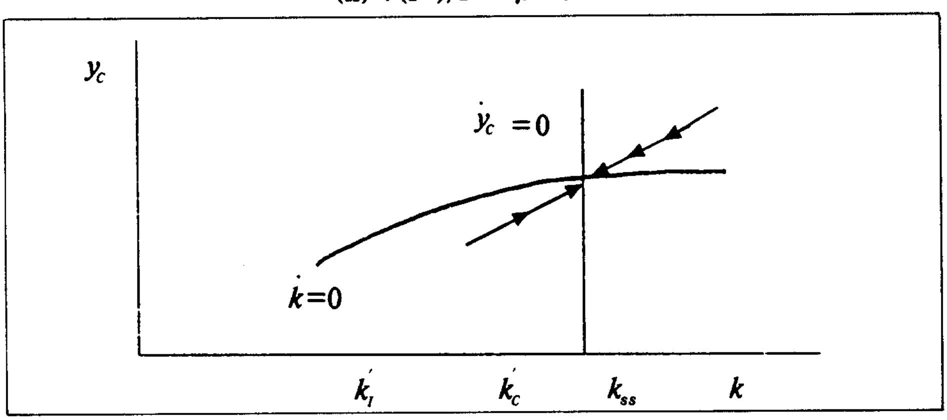
We have also analyzed the comparative dynamic analysis. If the world price rises, the optimal path of assets is shifted down and the stationary level of consumption goods could be lower.

[**Figure 1**] Phase Diagram in a Two Sector Model with Trade

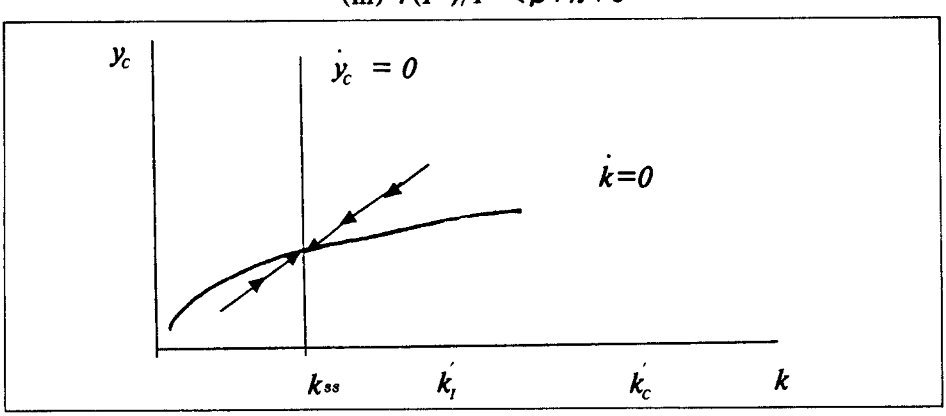
(i) $r(P^*)/P^* = \beta + n + \delta$




(ii) $r(P^*)/P^* > \beta + n + \delta$

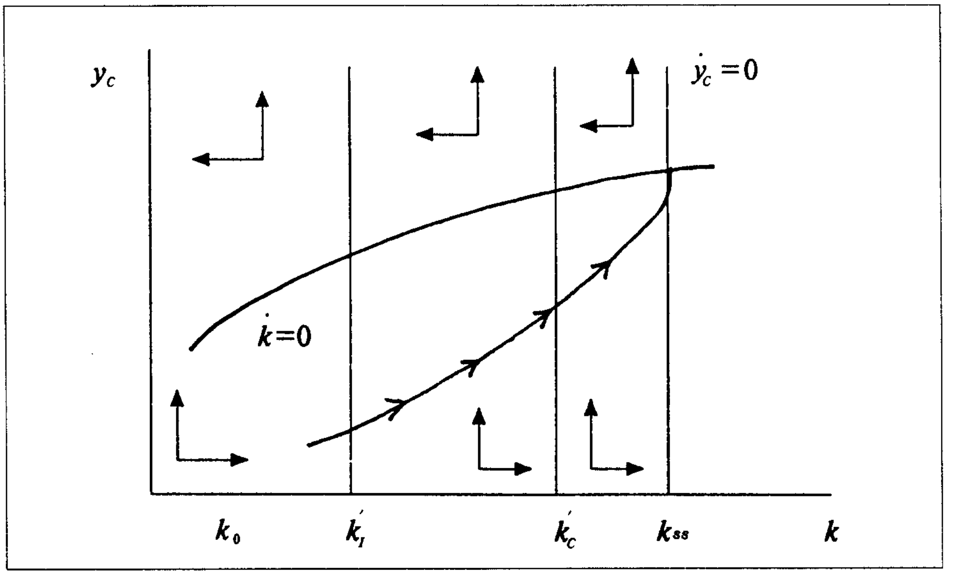



(iii) $r(P^*)/P^* < \beta + n + \delta$

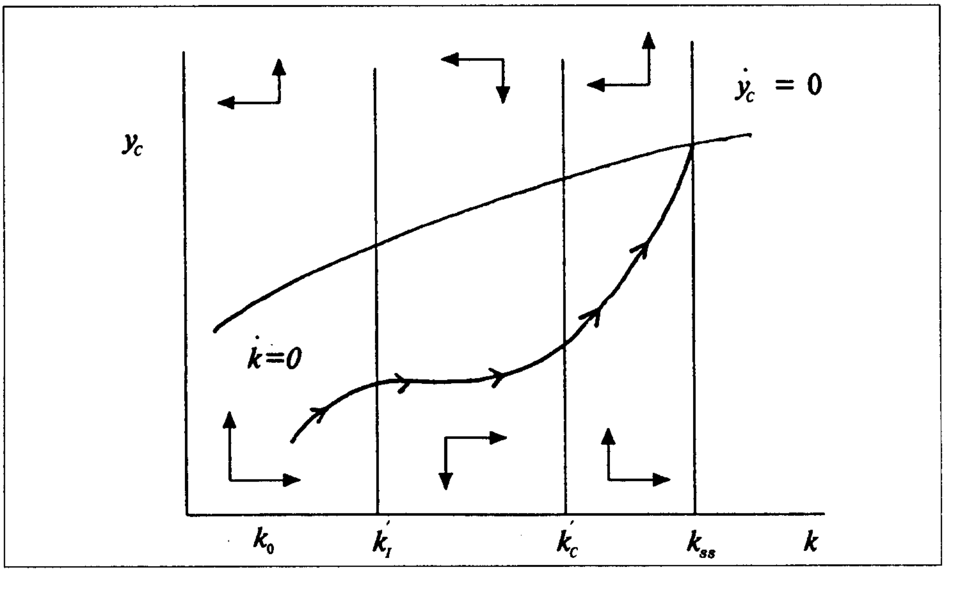


[**Figure 2**] Phase Diagram in a Two Sector Model with Trade When $k_0 < k_I' < k_{ss}$ Case

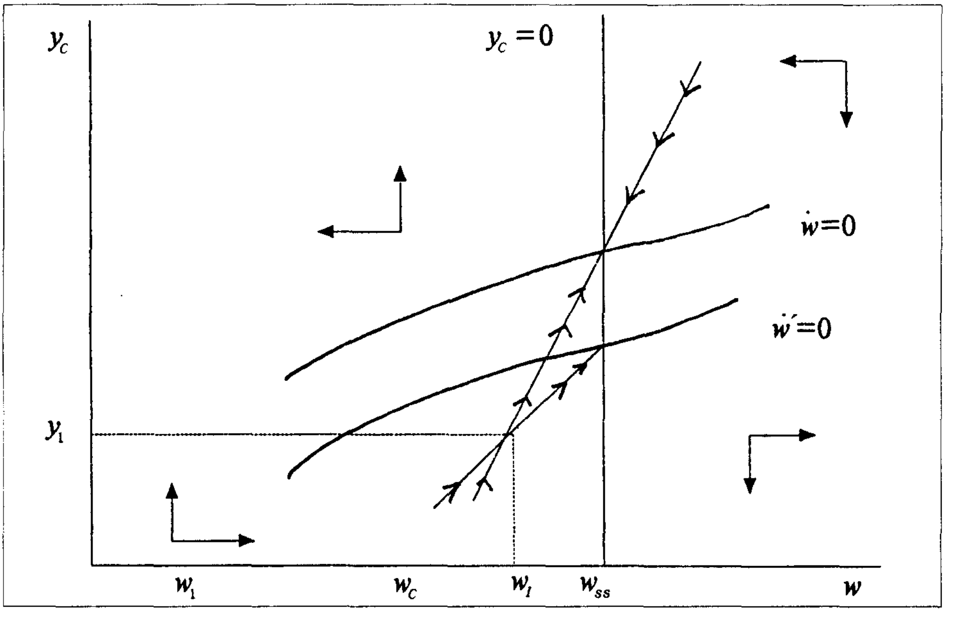
(i) $r(P^*)/P^* > \beta+n+\delta$




(ii) $r(P^*)/P^* < \beta+n+\delta$



[**Figure 3**] Comparative Dynamics and Phase Diagram in the Specialized Consumption Goods Case of a Two Sector Model with Trade and Borrowing



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