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EGARCH OPTION PRICING WITH ASYMMETRIES IN THE MEAN EQUATION

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Black's option pricing model systematically misprices actual option premiums. The biases of Black's model may result from assuming non-stochastic volatility and normality. A generalized autoregressive conditional heteroskedastic(GARCH) option pricing model relaxes the assumptions of Black's model. This paper uses an asymmetric ARCH-type models. Among ARCH-type alternatives, the asymmetric EGARCH(0,1)-t model fits Chicago wheat futures prices better than several alternative ARCH-type models. A Monte Carlo study shows that Black's model underprices deep out-of-the-money put options relative to the EGARCH option pricing model when the true underlying process is an EGARCH process. Differences between Black's model and the EGARCH option pricing model increase as time to maturity increases. When used to forecast actual option premiums of Chicago wheat futures contracts, the mean squared errors of the EGARCH option pricing model with deep in-the-money put and call options and with deep out-of-the-money put options are significantly smaller than those of Black's model.

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I. INTRODUCTION

Ignoring non-normality and stochastic volatility likely leads to biased esti-

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mates of option premiums. Therefore, a correct option pricing model should model not only the stochastic volatility but also the non-normality. Since the Black-Scholes option pricing model(hereafter 'OPM') is based on constant volatility, Black-Scholes model yields systematically biased estimates of deep in-the-money and deep out-of-the-money options(Johnson and Shanno; Hull and White). These inaccuracies may be due to inappropriate assumptions about the futures price distribution.

Time-varying variance models can explain nonlinear dependence and leptokurtosis. Bollerslev(1986) suggested the generalized autoregressive conditional heteroskedasticity(GARCH) process as an effective way of modeling the dynamics of volatility. Bollerslev(1987) showed the GARCH(1,1)-t model provided a better fit to data. The GARCH(1,1)-t process fits most empirical data better than the GARCH(1,1)-normal or a mixed diffusion-jump process. Myers and Hanson also provided evidence that the GARCH OPM with a student distribution performs better than Black's OPM in predicting soybean option premiums. Engle and Mustafa estimates GARCH parameters from option premiums and finds the GARCH OPM outperforms the Black model with implied volatility. However, the GARCH process does not model skewness. An Exponential GARCH(EGARCH) model that captures skewness was suggested by Nelson. The EGARCH model and the GQARCH model consider skewness by allowing the ARCH process to be asymmetric. However, asymmetry in the dynamics of the mean returns has not been considered in past research. This study considers the asymmetry in the mean returns with the GARCH, the EGARCH, and the GQARCH models so that skewness in the mean equation can be captured, and determines the most descriptive model of daily Chicago wheat futures price distributions among several models that consider non-normality and conditional heteroskedasticity. The study also seeks to determine whether time-varying stochastic volatility and conditional non-normality can explain the biases in Black's option pricing model.

II. STATISTICAL MODELS

Many competing statistical distributions have been proposed to model the departures from normality: a symmetric stable Paretian distribution(Mandelbrot, B or and Fama), student t-distribution(Blattberg and Gonedes), a mixture of normal distributions (Kon), and a mixed diffusion-jump process(Akgiray and Booth). However, since these models assume the independence of successive asset returns, they are inconsistent with empirical data that is known to be linearly or nonlinearly dependent. Further, these models are focused on capturing leptokurtosis. Jorion found that combining a jump process with a simple ARCH process provides a significantly better fit of the distribution of weekly exchange rates than either process alone. The mixed jump-diffusion process also models skewness. However, combining the GARCH(1,1)-t process with a jump process is not always significantly better than the GARCH-t process alone(Brorsen and Yang). Thus, a GARCH(1,1)-t process is used as the benchmark model, and other alternative models are compared with it. While the GARCH model elegantly captures the volatility clustering in asset returns, it ignores the possible asymmetric response of variance to positive and negative residuals and restricts the parameters in the variance equation to be non-negative. Nelson suggested the Exponential GARCH(EGARCH) model that overcomes these objections. LeBaron reported that the EGARCH model explains skewness better in the distribution of weekly and monthly stock indices than the GARCH model. Sentana suggested the Quadratic ARCH model ensuring that estimated conditional variances are non-negative. The Generalized QARCH (GQARCH) model provides a simple way of calibrating and testing for dynamic asymmetries in the conditional variance function. Sentana found that GQARCH(1,1)-M model revealed a better fit than the GARCH(1,1)-M for U.K. monthly excess stock returns. However, either Nelson's EGARCH model or Sentana's GQARCH model considers asymmetry only in the volatility structure. Investors' reaction to the price changes is asymmetric not only in terms of volatility but also in terms of mean returns. If the asymmetric reaction of investors is effectively captured in the mean equation, the asymmetries in the volatility structure may not be significant.

The GARCH, the EGARCH, and the GQARCH, each under a student t distribution, are considered here. Each model is estimated with and without asymmetry in the mean equation. The ARCH-type process can model well-documented market anomalies such as day-of-the-week, seasonality in mean and variance, and maturity in the variance equation. In the GARCH process, the futures price changes, Yt, can be expressed as a stochastic process:

$$Y_t = f(I_{t-1}\,;\,\theta) + \varepsilon_t\,, \tag{1}$$

where $f(I_{t-1}\,;\,\theta)$ denotes a function of I_{t-1} (the information set at time $t-1$) and the parameter vector θ, and ε_t has a discrete time stochastic process

$$\varepsilon_t = \begin{cases} z_t h_t & \text{in the GARCH-normal model and} \\ (\nu-2/\nu)^{1/2} w_t h_t & \text{in the GARCH-t model,} \end{cases} \tag{2}$$

where z_t is i.i.d. normal with expected value of $E(z_t)=0$ and variance of Var $(z_t)=1$ and ε_t is i.i.d. student with degrees of freedom ν, $E(w_t)=0$ and Var $(w_t)=\nu/(\nu-2)$. Therefore, h_t^2 is the time varying variance of ε_t. The GARCH(p, q) model expresses h_t^2 as a linear function of past variance and past squared values of the process,

$$h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}^2 \tag{3}$$

Equation (1) is the mean equation and equation (3) is the variance equation. Since h_t^2 is the conditional variance of ε_t, it must be non-negative. The GARCH model ensures non-negativity by making h_t^2 a linear combination of positive random variables. The EGARCH model ensures that h_t^2 remains non-negative by replacing h_t^2 with $\ln(h_t^2)$ in equation (3). For example, the conditional variance of EGARCH(0, 1)-t with one lag on the conditional variance is

$$\ln(h_t^2) = \alpha_0 + \beta \ln(h_t^2) + \eta(\varepsilon_{t-1}/h_{t-1}) + \varphi(\mid \varepsilon_{t-1}/h_{t-1} - (2/\pi)^{1/2}). \tag{4}$$

Over the range $0 < \varepsilon_{t-1}/h_{t-1} < \infty$, $\ln(h_t^2)$ is linear in $\varepsilon_{t-1}/h_{t-1}$ with slope $\eta + \varphi$, and over the range $-\infty < \varepsilon_{t-1}/h_{t-1} < 0$, $\ln(h_t^2)$ is linear in $\varepsilon_{t-1}/h_{t-1}$ with slope $\eta - \varphi$. If $\eta = 0$, $\ln(h_t^2)$ responds symmetrically to $\varepsilon_{t-1}/h_{t-1}$, but if $\eta \neq 0$, $\ln(h_t^2)$ responds asymmetrically.

The variance equation of GQARCH(1, 2) is

$$h_t^2 = \alpha_0 + \alpha_1(\varepsilon_{t-1} - \psi_1)^2 + \alpha_2(\varepsilon_{t-2} - \psi_2)^2 + \beta h_{t-1}^2, \tag{5}$$

where ψ_1 and ψ_2 measure dynamic asymmetries (Sentana).

Asymmetric price transmission models have been used in research on farm-retail price transmission. Restrictions on short selling, asymmetries in information, preferences of investors, and market psychology might cause differing responses to past price rising or falling. The mean equation of the asymmetric ARCH-type model is a special case of the Threshold Autoregressive Model of Tong and Lim.

The model with asymmetry in the mean equation is obtained by segmenting lagged price changes into one set for rising changes and another set for falling changes. The logarithmic changes in returns, Yt, are segmented as,

$$YP_t = \begin{cases} Y_t, & Y_t \geq 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$YN_t = \begin{cases} Y_t, & Y_t < 0 \\ 0, & \text{otherwise,} \end{cases}$$

where Y_t is the logarithmic difference of daily returns at time t. In the asymmetric GARCH process, the mean and the variance equations to be estimated are,

$$
\begin{aligned}
Y_t &= \alpha_0 + \sum_{i=1}^{m} \delta_t \, YP_{t-i} + \sum_{i=1}^{m} w_t \, YN_{t-i} + a_1 D_{MON} + a_2 D_{TUE} \\
&\quad + a_3 D_{WED} + a_4 D_{THU} + a_5 SIN(2\pi K/252) + a_6 COS(2\pi K/252) \\
&\quad + a_7 \mathrm{SIN}(2\pi K/126) + a_8 COS(2\pi K/126) + \varepsilon_t, \\
h_t^2 &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} h_{t-j}^2 + b_1 D_{MON} + b_2 D_{TUE} \\
&\quad + b_3 D_{WED} + b_4 D_{THU} + b_5 SIN(2\pi \mathrm{K}/252) + b_6 CON(2\pi \mathrm{K}/252) \\
&\quad + b_7 SIN(2\pi \mathrm{K}/126) + b_8 COS(2\pi \mathrm{K}/126) + b_9 \mathrm{TTM},
\end{aligned} \tag{6}
$$

where δ_i and w_i represent the net effect of the i-th positive and negative changes of Yt, respectively, and m is the length of lags. The length of lags in the mean equation is identified with the Schwarz criterion.[1] D denotes dummy variables for each day of the week; $D_{MON}=1$ if Monday and 0 otherwise, $D_{TUE}=1$ if Tuesday and 0 otherwise, $D_{WED}=1$ if Wednesday and 0 otherwise, and $D_{THU}=1$ if Thursday and 0 otherwise. SIN and COS represent the sine and cosine functions, respectively, and π is approximated as 3.14. K in the sine and cosine functions is the number of trading days after January 1 of the particular year. Denominators in the sine and cosine functions are the specified cycle length in trading days, so 252 indicates a one year cycle and 126 a half year cycle. TTM is the time to maturity measured in the number of trading days prior to maturity. The variance equation for the EGARCH model is obtained by adding day-of-the-week, seasonality, and time to maturity variables into equation(4). Maximum likelihood estimates of alternative models are obtained using the statistical software package GAUSS(Aptech Systems Inc.)[2]

The asymmetry hypothesis is tested in two ways. One is that the total impact of past price increases is the same as that of past price decreases:

$$
H_0 \; : \; \sum_{i=1}^{m} \delta_i = \sum_{i=1}^{m} w_i \tag{8}
$$

[1] Schwartz's criterion is obtained by SC(m)=ln(SSEm)+Qm *ln(T)/T, where m is the length of lags SSEm is the sum of squared residuals, Qm is the number of parameters, and T is the number of observations. The value of m that minimizes SC is selected as the length of lags in the model.

[2] For the GARCH models, since the GARCH terms(α and β in the variance equation) are restricted to be non-negative, inequality restrictions are imposed by taking the exponential of the parameters, while there is no such restrictions for the EGARCH and the GQARCH models. The degrees of freedom of the student distribution is restricted to be greater than three since the likelihood function is not defined for degrees of freedom less than or equal to three. The variance of the first 15 observations of price changes is used as an initial variance. The initial algorithm is Polak-Ribiere-type Conjugate Gradient method and the initial step size is one. After a few iterations, the algorithm is switched to the Davidson-Fletcher-Powell method and the Brent step-size method is used. The final estimates are obtained with the Newton method so that the Hessian is used to estimate the information matrix. All derivatives are calculated numerically.

H_A : H_0 is not true.

The other is that the speed of adjustment to price increases and to price decreases are the same.

$$H_0 : \delta_i = w_i , \ \forall \ i \tag{9}$$

H_A : H_0 is not true.

For the hypothesis tests of equations (8) and (9), Wald-F statistics are used. The Wald test is not invariant in nonlinear models, but it is still asymptotically valid (Dagenais and Dufour). The estimated t-ratios of the parameters of the skewness terms, η in equation (4) and ψ's in equation (5) provide tests of asymmetry in the variance equations of the EGARCH model and of the GQARCH model, respectively.

III. SAMPLE DATA

The alternative statistical models are estimated with the first differences of the natural logarithms[3] of the daily futures closing prices of wheat at the Chicago Board of Trade. The first differences of the natural logarithm are rescaled by multiplying them by 100. The data are from Jan. 1982 to Sep. 1990. The data were created using Continuous Contractor from Technical Tools. Chicago wheat futures contracts are traded based on five maturities: March, May, July, September, and December. The price series used is a continuous combination of the five contracts. The rollover date is the 21st day of the month prior to delivery.[4] Log differences are taken before splicing so that no outlier is created at the rollover date. July option preminms, for example, are only predicted for April 21 to June 20, the dates the July futures contract was used to estimate the models. The daily option premiums over the simulation period are collected from the CBOT DataBank of the Chicago Board of Trade.

Using a continuous series of nearby log changes implicitly imposes

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[3] Augmented Dickey-Fuller unit-root tests are conducted for futures prices, for the first differences of the futures prices, and for the log differences of the futures prices. The test statistics are -1.80, -7.56, and -6.80, respectively. The asymptotic critical value is -2.57. Therefore, the null hypothesis that the time series has a unit root is not rejected for the futures prices, but rejected for the first differences and the log differences. Both first differences and log differences remove nonstationarity from the data, but Fama(1965) provides reasons for using log differences which are percentage changes. First, the log difference is the return, with continuous compounding, from holding the asset. Second, while the variability of simple price changes increases as the price level increases, log differences neutralize price level effects.

[4] The 21st is selected because options expire on the Friday 10 days before the first notice day. The first notice day is the last business day of the month.

restriction that the parameters are the same across contract months. One alternative would be to make a continuous series of July contracts, for example. The drawback of such an alternative approach is that distant futures contracts are often very thin. Closing prices reported will often be from a transaction earlier in the day or may be a bid or ask quote. Therefore, the models are estimated and option premiums are predicted only with near-to-maturity data.

Table 1 shows summary statistics of log differences of wheat futures prices at the Chicago Board of Trade. The departures from normality are apparent from the high kurtosis and skewness. The Kolmogorov-Smirnov test rejects normality.

Ⅳ. MODEL SELECTION AND VALIDATION

4.1. Procedure

Since the models are nested, the likelihood ratio test is used to select between the ARCH-type models, except EGARCH model, with and without asymmetries in the mean equation. Model selection among the asymmetric GARCH-t and the asymmetric EGARCH-t is conducted by selecting the model with the higher Schwarz criteria. Schwarz criterion is useful for selecting among nonnested models with different numbers of parameters, because it penalizes the model with more parameters.

If the GARCH or EGARCH models are well-specified and fit the sample data, the standardized residual generated by the models should be i.i.d. normal or student. The Ljung-Box and McLeod-Li test statistics are used to test the null hypothesis of no serial dependence in the rescaled residuals, $(\hat{\varepsilon}_t/\hat{h}_t)$ and squared rescaled residuals, $(\hat{\varepsilon}_t/\hat{h}_t)^2$ of the selected model. The Brock-Dechert-Scheinkman (BDS) test (Brock et al.) is used to test the null hypothesis that $\{Y_t\}$ is i.i.d.. The BDS statistic is based on the correlation integral. The statistic for raw data is asymptotically distributed as a standard normal random variable under the null hypothesis. However, Brock et al. have shown that the distribution of the BDS statistic is not standard normal when the data are GARCH residuals. The tables in Brock et al.(p.279) are used to obtain critical values of the test statistic. The Kolmogorov-Smirnov goodness-of-fit test is used to determine if the residuals have a t-distribution.[5]

4.2. Results

Table 2 shows the estimated log-likelihoods and the tests of no asymmetry from the models of Chicago wheat futures prices. In the asymmetric GARCH

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[5] The rescaled residuals are multiplied by $(\hat{\nu}/(\hat{\nu}-2))^{1/2}$ where ν is the estimated degrees of freedom of the model. This adjustment is needed because the variance of a t-distribution is $(\hat{\nu}/(\hat{\nu}-2))$.

[Table 1] Summary Statistics of Daily Chicago Wheat Futures Prices over January 1982 through August 1990.[a]

	Statistics
Sample Size(T)	2170
Mean (μ)	-0.0162
Standard Deviation (σ)	1.4496
Skewness[b]	0.5582*[e]
Kurtosis[c]	20.3987*
D-statistic[d]	0.0750*

[a] Units are percentages. $Y_t = [\ln(P_t) - \ln(P_{t-1})] * 100$.

[b] Skewness is computed by $\sum_{t=1}^{T}(Y_t - \mu)^3/(n-1)\sigma^3 - 3$.

[c] Excess kurtosis is computed by $\sum_{t=1}^{T}(Y_t - \mu)^4/(n-1)\sigma^4 - 3$.

[d] The Kolmogorov-Smirnov Goodness-of-fit D-statistic. The critical value is $1.36/T^{1/2}$ at 5% significant level for T observations.

[e] Asterisks denote the null hypothesis of normality (i.e., zero skewness and zero kurtosis) are rejected at a 5% level based on the critical values by Snedecor and Cochran (1980).

model, the total impact of past rising prices is not significantly different from that of past falling prices. However, the speed of adjustment for price rising is significantly different from that of price falling, implying significant asymmetries in mean returns. Asymmetry is also present in the mean equation of the asymmetric EGARCH model and the asymmetric GQARCH model. The skewness terms in the variance equation of the asymmetric EGARCH model and the asymmetric GQARCH model are not significantly different from zero (Table 2). Thus, when the asymmetries are captured in the mean returns, no more asymmetries are found in volatility structure in the Chicago wheat futures prices.

Table 3 contains the test statistics used to select among models. Likelihood ratio statistics are used to select between nested models. Differences in Schwarz criteria are used to select between nonnested models. The asymmetric EGARCH(0,1)-t process is selected and so its estimates are used in the simulation to obtain option premiums. Table 4 reports estimates and test statistics of the asymmetric EGARCH(0,1)-t model. The effects of lagged rising prices are incorporated more quickly than those of lagged falling prices. The day of the week effect are not significant in both mean and variance equations. Significant seasonal patterns are revealed only in volatility. The Ljung-Box and McLeod-Li tests do not detect any linear or second moment autocorrelations with the standardized data, which implies the EGARCH-t process removed all the correlation in the first and second moments. The BDS statistics show that the null hypothesis of i.i.d. is rejected with the raw data (Table 4), implying that Chicago wheat futures price changes are not i.i.d.. For the rescaled residuals, however, the BDS statistics do not identify nonlinear dependence.[6] Using the Kolmogorov-Smirnov test, the null hypothesis that the EGARCH rescaled residuals follow a student distribution is not rejected at the 5% significance level.

V. IMPLICATIONS FOR OPTION PRICING

McBeth and Merville argued that the Black-Scholes formula overprices out-of-the-money options and underprices in-the-money options. However, Rubinstein argued that their conclusions did not always hold. Johnson and Shanno obtained numerical results for general cases in which the instantaneous variance obeys some stochastic processes. When volatility is a stochastic variable independent of stock price, Hull and White showed that the Black-Scholes formula overprices options that are at- or close-to-the-money and underprices options that are deep in- and deep out-of-the-money.

[**Table 2**] Estimated Log-likelihoods and Tests of Asymmetry with Alternative Models of Daily Chicago Wheat Futures Prices[a]

Models	Maximized Log-Likelihood	Test Statistics of H_0: No Asymmetries		
		Mean[b]		Variance[c]
		Total	Speed	
Asymmetric GARCH(1,1)-t	−3445.2	3.77	3.89*[d]	na[e]
GARCH(1,1)-t	−3449.1	na	na	na
Asymmetric GARCH(1,2)-t	−3442.4	3.37	4.48*	na
Asymmetric GARCH(2,2)-t	−3442.4	5.29*	6.29*	na
EGARCH(0,1)-t	−3437.3	na	na	0.68
Asymmetric EGARCH(0,1)-t	−3433.5	3.34	3.62*	0.30
Asymmetric EGARCH(1,1)t	−3433.5	3.19	3.54*	0.34
Asymmetric EGARCH(0,2)-t	−3433.1	5.17*	5.17*	0.31
GQARCH(1,2)-t	−3446.8	na	na	0.03
Asymmetric GQARCH(1,2)-t	−3444.4	3.52*	3.81*	0.12

[a] The sample has 2170 observations.

[b] Test statistics for the null hypothesis that there is no asymmetry in the mean equation.

[c] Test statistics of H_0: no asymmetry in the variance equation are the test statistics of the parameter representing skewness (η in equation (4) and ψ's in equation (5)).

[d] Asterisks denote rejection of the null hypothesis of no asymmetry at the 5% level.

[e] Not applicable.

6 Since BDS test statistics for EGARCH(0,1)-t model is not yet developed, the statistics for GARCH(1,1)-t model is used as an alternatives statistics. It allows only a rough conclusion. One way to make it clear is to show the distributions of EGARCH(0,1)-t and GARCH(1,1)-t to be asymptotically the same.

[Table 3] Test Statistics of Model Selection with Alternative Models

Hypotheses		
Null	Alternative	Statistics[a]
GARCH(1,1)−t	Asymmetric GARCH(1,1)−t[b]	7.77*[c]
Asymmetric GARCH(1,1)−t	Asymmetric GARCH(1,2)−t	5.77*
Asymmetric GARCH(1,2)−t	Asymmetric GARCH(1,3)−t	0.22
Asymmetric GARCH(1,2)−t	Asymmetric GARCH(2,2)−t	0.00
Asymmetric GARCH(1,2)−t	Asymmetric EGARCH(0,1)−t	17.66*
EGARCH(0,1)−t	Asymmetric EGARCH(0,1)−t	7.55*
Asymmetric EGARCH(0,1)−t	Asymmetric EGARCH(0,2)−t	0.91
Asymmetric EGARCH(0,1)−t	Asymmetric EGARCH(1,1)−t	0.13
GQARCH(1,2)−t	Asymmetric GQARCH(1,2)−t	8.90*
Asymmetric GARCH(1,2)−t	Asymmetric GQARCH(1,2)−t	17.79*

[a] Likelihood ratio test statistic is obtained by $2T*(L_1-L_0)$, where $T(=2170)$ is the number of observations, L_1 is the loglikelihood values under alternative hypothesis, and L_0 under null hypothesis.

[b] The statistic reported is the difference in Schwarz criteria which is obtained by $2T*(L1-L_0)-(K_1-K_0)*\ln(T)$, where K_1 and K_0 are the number of parameters under alternative and null hypothesis, respectively.

[c] Asterisks denote rejection of the null hypothesis in favor of the alternative hypothesis at the 5% significance level.

Black's model specifies the option premium as a function of five underlying parameters: the current futures price (Ft), the exercise price of the option (X), the time to maturity of the option (T-t), the risk-free rate of interest (r), and the variance of futures prices (σ^2).

The Black premium (B) is

$$B=\begin{cases} e^{-r(T-t)}[X*N\ (-d_2)\ -F_t*N(-d_1)] & \text{for put,} \\ e^{-r(T-t)}[F_t*N(-d_1)-X*N\ (-d_2)\ ] & \text{for call,} \end{cases} \tag{10}$$

where $d_1=[\ln(F_t/X)+(\sigma^2/2)(T-t)]/\sigma(T-t)^{1/2}$, $d_2=[\ln(F_t/X)-(\sigma^2/2)(T-t)]/\sigma(T-t)^{1/2}$, and $N(\cdot)$ is the normal cumulative density function.

The EGARCH OPM can not be solved in a closed form, since the stochastic volatility adds risk which can not be diversified into a riskless hedge portfolio (Johnson and Shanno; Hull and White; Myers and Hanson). Thus, the EGARCH OPM approximates the premium by providing expected option prices at maturity with Monte Carlo integration. The EGARCH OPM is an European option premium. Only European options are considered since estimates of American option premiums cannot be obtained with Monte Carlo methods (Hull).

[Table 4] Parameter Estimates and Test Statistics of the Asymmetric EGARCH (0,1)-t Process with Chicago Wheat Futures Log Price Changes

	Estimated Coefficients	(t-ratio)		Estimated Coefficients	(t-ratio)
Mean			Variance		
Intercept	−0.129 *[a]	(−0.152)	Intercept	−0.0706	(−0.76)
Lag 1 positive	0.036	(1.011)	β	0.9795 *	(120.82)[b]
Lag 1 negative	0.044	(1.257)	η	0.1074 *	(4.91)[b]
Lag 2 positive	0.021	(0.610)	ψ	0.0043	(0.30)[b]
Lag 2 negative	−0.128 *	(−3.839)	D_{Mon}	0.058	(0.399)
D_{MON}	0.016	(0.216)	D_{Tue}	0.171	(1.19)
D_{TUE}	0.035	(0.492)	D_{Wed}	−0.025	(−0.17)
D_{WED}	0.159 *	(2.294)	D_{Thu}	−0.163	(−1.022)
D_{THU}	0.016	(0.224)	SIN252	0.049	(0.389)
SIN252	−0.017	(−0.52)	COS252	−0.012	(−0.251)
COS252	0.055	(1.78)	SIN126	0.081	(1.123)
SIN126	−0.017	(−0.53)	COS126	0.106 *	(2.144)
COS126	−0.047	(−1.47)	Maturity	−0.060	(−0.405)
Degrees of Freedom					
ν	4.61[c]				
Wald F statistics					
Day of Week in Mean	1.80				
Seasonality in Mean	1.38				
Day of Week in Variance	1.41				
Seasonality in Variance	2.33 *				
Ljung-Box and Mcleod-Lid					
$\varepsilon^{\tau/2} h_t(12)$[d]	9.10				
$\varepsilon_t/h_t(12)$[d]	3.04				
BDS tests ($\varepsilon=\sigma$)[e]					
Raw Data					
Dimension = 3	7.43 *				
Dimension = 6	9.08 *				
Dimension = 9	11.63 *				
Rescaled Data					
Dimension = 3	0.72				
Dimension = 6	0.13				
Dimension = 9	0.02				
Goodness-of-fit[f]					
D	0.025				

[a] Asterisks denote the rejection of the null hypothesis at the 5% significance level. t-values are in parentheses.

[b] Since the GARCH terms are restricted to be positive, the null hypothesis lies on the boundary of the parameter space. Under the assumptions of Moran, the t-statistic is distributed as a mixture of a degenerate distribution and a half t-distribution. Hypothesis tests can still be conducted in the usual fashion with t-tests. However, the t ratio of α is computed as $t=e^{\hat{a}}/(e^{\hat{a}} S^2_{e^a} e^{a}\ e^{\hat{a}})^{1/2}$, where $S^a_{e^a}$ is the standard error of $e^{\hat{a}}$, because the inequality constraint was imposed on the parameter α using an exponential transformation.

[c] The residuals are assumed to have a student-t distribution. The degrees of freedom of this student t distribution is restricted to be greater than three.

[d] Both null hypotheses that ε_t/h_t are not autocorrelated and that $\varepsilon_t^2\ h_t^2$ are not autocorrelated are tested with twelve degrees of freedom. Test statistics are distributed asymptotically as $\chi^2(12)$ under the null hypothesis.

[e] H_0 : The standardized residuals are i.i.d. The hypothesis test is based on Table F.4 in Brock et al. (p.279).

[f] Kolmogorov-Smirnov D statistic. The critical value is $D_c = 1.36/T^{1/2} = 0.029$ where T is the sample size.

Two sets of T-t random numbers are generated: one from a t-distribution with $\hat{v}$ degrees of freedom and another from a standard normal distribution. Time is measured in number of trading days. Then, the futures prices F_t are simulated for T-t periods using estimates from the selected model to get the futures price at maturity. Denoting this price at maturity $\{F_T\}_i$, the simulated option prices are

$$E=\begin{cases} e^{-r(T-t)}(1/n)\sum_{i=1}^{n} max[X-\{F_T\}_i,\ 0] \text{ for call,} \\ e^{-r(T-t)}(1/n)\sum_{i=1}^{n} max[\{F_T\}_i-X,\ 0] \text{ for call,} \end{cases} \tag{11}$$

where n = 10,000 is the number of replications.

One efficient way to improve the accuracy of this calculation is to use the control variate technique (Hammersley and Handscomb; Boyle). This technique requires solving a problem which is similar to the one under consideration, but has an analytical solution. The solution of the simpler problem is used to increase the accuracy of the solution to the more complex problem. Essentially, a term is added to the EGARCH option price, E in equation (11). The term has expected value zero, but is negatively correlated with the error in E. The control variate is B - B_{MC}, where B is the analytical solution to Black's model in equation (10) and B_{MC} is the Monte Carlo solution to Black's model. Two random number streams are used for B_{MC} and E. The stream for B_{MC} is generated from a standard normal distribution. The stream for E is generated as a t-distribution with the degrees of freedom from the estimated model, using the standard normal random number stream for B_{MC}. The two random number streams are generated using the same seed so that the random errors in E and B_{MC} are positively correlated. The EGARCH option price calculated with the control variate technique (E^*) is

$$E^*=E+(B-B_{MC}). \tag{12}$$

VI. MONTE CARLO STUDY

A Monte Carlo study is conducted to determine the differences between Black's OPM and the EGARCH OPM. Differences between Black's OPM and the EGARCH OPM may be caused by not considering the observed conditional heteroskedasticity and non-normality in Black's OPM. The extent of the difference can be measured by the absolute difference $(B-E^*)$ or by the percentage difference $\zeta=(B-E^*)/E^*$. The differences for short-lived option premiums are not the same as those for long-lived option premiums. The differences are also different by how much the option is in the money or out of the money. Most commodity futures option exercise prices are within 10% of the actual price. Futures-exercise price ratios (F_t/X) of 0.90, 0.95, 1.0, 1.05, and 1.10 are considered. In the Monte Carlo study, exercise price is \$1.00. Differences are measured from six months through two weeks prior to maturity.

The average unconditional variance of the EGARCH process $e^{\frac{a_0 - \eta\sqrt{2/\pi}}{1-\beta}}\left[1+\frac{(1-\beta)(\eta^2+\varphi^2)(4/\pi)-\beta\eta^2+2\alpha_0\eta\sqrt{2/\pi}}{2(1-\beta)^2(1+\beta)}\right]$ is used as an initial volitility to generate conditional variances for E, and as the constant volatility for Black's analytical solution B and Black's Monte Carlo sulution B_{MC}. The seasonality and the day of the week effects are not included in the Monte Carlo study. Thus the difference in option premiums with the Black and GARCH models in the Monte Carlo study reflect only the different distributional assumptions and non-constant volatilities. Maturities up to six months are considered to illustrate the model's predictions. But, of course, the model is only valid in predicting actual option premiums which are within two or three months of maturity. The asymptotic t-statistics for the simulated differences are provided. These could be used to determine whether the reported pricing biases are significantly different from zero. They are the ratios of the simulated differences to the standard deviations of the differences.

6.1. Findings of Monte Carlo Study

In this section, the differences in option pricing between Black OPM and the EGARCH OPM are calculated. Table 5 presents absolute and percentage differences between put option premiums with Black's OPM and the EGARCH OPM. Black's OPM yields significantly lower premiums than the EGARCH OPM for deep in- and deep out-of-the-money put options. Absolute differences increase as time to maturity increases in deep in- and deep out-of-the-money put options (Table 5, Panel A). Percentage differences for deep in-the-money options also increase as time to maturity increases, but those for deep-out-of-the money options decrease as time to maturity increases (Table 5, Panel B). As time to maturity decreases, the time-value of deep out -of-the-money options decreases very quickly and eventually becomes zero. Therefore, deep out-of-the-money options close to maturity show extremely high percentage differences. At-the-money option premiums are higher with Black OPM than with the EGARCH OPM, but not significantly. Percentage differences for at-the-money put options decrease as the time to maturity increases.[7]

The simulation results confirm Hull and White's findings that the Black-

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[7] When European options are considered, put-call parity must hold. The results of the Monte Carlo study with call options are similar and, therefore, are not reported.

[Table 5] Differences between Black's Option Pricing and EGARCH Option Pricing for Put Options When the True Process Is EGARCH

	Time to maturity (months)					
	0.5	1	1.5	3	4.5	6
Panel A: Absolute differences[a]						
Deep In-the-money	−0.000015	−0.00015	−0.00056	−0.00085	−0.00093	−0.001
(F_t/X=0.90)	(−0.18)	(−1.25)	(−3.19)	(−4.36)	(−3.99)	(−3.86)
In-the-money	−0.00011	−0.00019	−0.0004	−0.00089	−0.0011	−0.0019
(F_t/X=0.95)	(−1.39)	(−1.69)	(−3.03)	(4.91)	(−5.25)	(−7.98)
At-the-money	0.00037	0.00019	0.00009	−0.0009	−0.0013	−0.0015
(F_t/X=1.0)	(5.74)	(2.16)	(0.83)	(−5.50)	(−6.75)	(−6.92)
Out-of-the-money	−0.00018	−0.000098	−0.00015	−0.0007	−0.0011	−0.0014
(F_t/X=1.05)	(−5.32)	(−1.81)	(−1.98)	(−5.91)	(−7.33)	(−7.75)
Deep Out-of-money	−0.000064	−0.00027	−0.00026	−0.00079	−0.0011	−0.0014
(F_t/X=1.10)	(−5.63)	(−8.62)	(−5.72)	(−8.75)	(−9.32)	(−8.97)
Panel B: Percentage differences (%)[b]						
Deep In-the-money	−0.00	−0.15	−0.56	−0.83	−0.93	−0.939
(F_t/X=0.90)	(−12.07)	(−6.57)	(−14.39)	(−8.49)	(−3.99)	(−1.84)
In-theamoney	−0.22	−0.36	−0.73	−1.42	−1.63	−2.58
(F_t/X=0.95)	(−1.47)	(−7.51)	(−3.56)	(−1.74)	(−1.47)	(−0.47)
At-the-money	2.83	1.04	0.40	−2.57	−3.13	−3.15
(F_t/X=1.0)	(1.05)	(0.71)	(1.14)	(−1.01)	(−0.54)	(−0.19)
Out-of-the-money	−0.14	−2.48	−2.22	−4.90	−5.29	−5.10
(F_t/X=1.05)	(−6.06)	(0.70)	(−0.58)	(−2.32)	(−0.22)	(−1.62)
Deep Out-of-money	−72.95	−39.02	−16.86	−13.22	−10.60	−8.90
(F_t/X=1.10)	(−10.32)	(−14.92)	(−7.61)	(−6.50)	(−3.77)	(−0.91)

[a] Black's option price minus EGARCH option price when exercise price is set equal to $1.00. The absolute differences are measured in ₵/bushel.

[b] [(Black's price - EGARCH price)/EGARCH price]*100.

Note: Asymptotic t-statistics are in parentheses.

Scholes model underprices in- and out-of-the-money options when stochastic volatility is present. Their argument that the Black-Scholes model overprices close-to-the-money options is also confirmed. The absolute differences are small but the absolute differences between the two OPM's are larger for at-the-money options than for in- or out-of-the-money options.

The asymptotic t-statistics in parentheses provide some evidence that the reported option pricing errors are not due to sampling errors. In particular, differences between Black's option price and the EGARCH option price for deep out-of-the-money options are always significantly different from zero.

VII. PREDICTING PREMIA

To compare the performance of the EGARCH OPM and Black's OPM, option prices are estimated for 1991 Chicago wheat futures for two month periods prior to maturity for each March, May, July, September, and December contract. Both Black's OPM and EGARCH OPM provide predictions conditional on F_t, F_{t-1}, ..., F_{t-19}. Thus, they predict today's premium based on today's information. The annualized risk-free interest rate is assumed constant during the simulation period at r = 5.5%.[8] In this out-of-sample simulation[9],

[**Table 6**] Ranges of Futures Prices During the Out of Sample Simulation Period and Strike Prices of In-, At-, and Out-of-the-Money Options for Each Contract[a]

Maturity	March	May	July	September	December
Panel A: Put Option					
Price Ranges	(2.46,2.62)	(2.62,2.93)	(2.80,3.01)	(2.59,3.01)	(3.25,3.71)
Out-of Money	2.40	2.50	2.70	2.50	3.20
At-the-Money	2.60	2.80	2.90	2.80	3.50
In-the-Money	2.80	2.90	3.10	3.10	3.70
Panel B: Call Option					
Price Ranges	(2.46,2.62)	(2.62,2.93)	(2.80,3.01)	(2.59,3.01)	(3.25,3.71)
Out-of-Money	2.40	2.50	2.60	2.40	3.10
At-the-Money	2.60	2.80	2.90	2.80	3.50
In-the-Money	2.80	3.00	3.10	3.20	3.70

[a] Units are in $/bushel.

[8] During the simulation period, the range of the rate of return on treasury bills is (0.052, 0.058).

[9] The statistical model of CBOT wheat futures price changes is estimated using data from Jan. 1982 through Sept. 1990. The Black and EGARCH option premiums are simulated for the trading of Dec. 1990 through Nov. 1991, so the simulation is out of sample.

[Table 7] Out of Sample Forecasting Performance of Black and EGARCH Option Pricing for 1991 Chicago Wheat Options

	Black			EGARCH		
Monevness[a]	Out	At	In	Out	At	In
panel A: Put Option						
Mean Error[b]	−0.24	−0.36	−0.49	−0.22	−0.41	−0.48
Root Mean Squared Error[b]	0.96	1.34	1.33	0.76	1.17	1.31
panel B: Call Option						
Mean Error[b]	−0.31	0.09	0.44	−0.24	0.02	0.54
Root Mean Squared Error[b]	0.88	1.71	1.61	0.78	1.57	1.65

[a] The precise exercise prices for the in-, at-, and out-of-the-money options are given in Table 6.

[b] Mean errors and root mean squared errors are in cents per bushel.

twenty-day historical volatility[10], which is seasonally adjusted[11], is used as the starting value to generate unconditional variances in the EGARCH integration process E, and as volatility measure in Black's analytical solution B and in Black's Monte Carlo solution B_{MC}. The model is only used to predict nearby option premiums with up to about two months maturities. That is, for example, the model is used to predict about July option premiums only over the same time period that July futures contract data was used in estimating the EGARCH model.

Results are given for both put and call options. The mean errors and root mean squared errors (RMSE) of Black's OPM and EGARCH OPM are computed. Further, the Ashley-Granger-Schmalensee test is used to test if the

[10] For the purpose of forecasting option premiums using Black's OPM, the implied volatility is a better choice of variance in Black's model than historical volatility because the implied volatility implicitly contains information about all market conditions including non-normality. However, since the objective is to test distributional assumptions between Black's OPM and the EGARCH OPM, it is not appropriate to use implied volatility because then Black's OPM and the EGARCH OPM would be conditioned on different information sets. It is theoretically possible, although not yet practical(because of computer costs), to calculate an implied volatility for the EGARCH model. The EGARCH implied volatility would not be the same as the implied volatility with Black model.

[11] The historical volatility is seasonally adjusted by multiplying it by the seasonality index which is calculated as $\sum_{t=t_0}^{\tau} S_t/(\tau-t_0+1)/(\sum_{t=t_0-19}^{t_0} S_t/20)$, where t_0 is current date, τ is the day of maturity, and S_t is unconditional variance of day t. The unconditional variance is the expected value of eq. (6) calculated using estimated parameters.

mean squared error of Black's OPM is equal to that of the EGARCH OPM. Statistical tests were based on White's heteroskedastic-consistent covariance matrix because of likely heteroskedasticity due to differences in maturity and on Newey and West's autocorrelation-consistent covariance matrix because the Ashley-Granger-Schmalensee test assumes independence.

The ranges of futures prices during the simulation period, and exercise prices

[**Table 8**] Ashley-Granger-Schmalensee Test of the Out of Sample Forecast Performance of Black and EGARCH Option Pricing for 1991 Chicago Wheat Options

	β_1^a	β_2^a	F statistics [b]	Conclusion	Model Favored
Panel A: Put Option					
Out-of-Money [c]	0.1275*	0.1164*	87.87*	reject H_0	EGARCH
	(12.95)	(8.34)			
At-the-Money [d]	0.0485*	−0.0833*	na	inconclusive	None
	(2.78)	(−6.10)			
In-the-Money [c]	0.0028	0.011	1.58	not reject H_0	None
	(0.21)	(1.74)			
Panel B: Call Option					
Out-of-Money [c]	0.0667*	0.0539*	30.40*	reject H_0	EGARCH
	(5.56)	(6.75)			
At-the-Money [c]	0.1814*	0.0968	16.02*	reject H_0	EGARCH
	(5.51)	(0.80)			
In-the-Money [d]	0.0021	0.0154	1.75	not reject H_0	None
	(0.06)	(1.13)			

[a] The Ashley-Granger-Schmalensee test is based on the following regression results:

$$\Delta_t = \beta_1 + \beta_2(\Sigma_t - \overline{\Sigma}_t) + \mu_t \,.$$

Here, $$\Delta_t = \begin{cases} e_t^B - e_t^G & \text{when } \bar{e}_t^B > \bar{e}_t^G \\ e_t^G - e_t^B & \text{when } \bar{e}_t^G > \bar{e}_t^B, \end{cases}$$

where e_t^B is Black's option price minus actual option price and e_t^G is EGARCH option price minus actual price.

$\Sigma_t = e_t^B + e_t^G$, assuming $\bar{e}_t^B > 0$ and $\bar{e}_t^G > 0$. $\overline{\Sigma}_t$, $\bar{e}_t^B$, and $\bar{e}_t^G$ are means of Σ_t, e_t^B e_t^G, respectively.

[b] If both β_1 and β_2 are negative, then F test with one half of regular significance level can be used. However, if one of β_1 and β_2 is negative, then t test of the other coefficient should be used. If one of β_1 and β_2 is significantly different from zero, then the test is inconclusive.

[c] H_0: Mean squared error of Black's OPM is equal to that of EGARCH OPM,
H_1: Mean squared error of Black's OPM is greater than that of EGARCH OPM.

[d] H_0: Mean squared error of EGARCH OPM is equal to that of Black's OPM,
H_1: Mean squared error of EGARCH OPM is greater than that of Black's OPM.

* Asterisks denote significance at 5% level. Numbers in parentheses are t statistics.

for in-, at-, and out-of-the-money options are in Table 6. Table 7 shows the out-of-sample results over Dec. 1990 to Nov. 1991. A total of 1,214 out-of-sample observations is provided. Except for at-the-money put and for in-the-money call options, the mean errors of the EGARCH OPM are smaller than those of Black's OPM. For the Chicago wheat futures prices, the root mean squared errors (RMSE) of EGARCH OPM are smaller than those of Black's OPM in all cases except for in-the-money call option. However, the differences are not large. The Ashley-Granger-Schmalensee (AGS) test (Table 8) finds the mean squared errors of Black's OPM are significantly larger than those of EGARCH OPM for out-of-the-money put and call options, and at-the-money call option. However, there is no winner for the in- and at-the-money put, and in-the-money call options. Therefore, the EGARCH OPM performs better than Black's OPM for out-of-the-money put and call options and at-the-money call option in this out-of-sample simulation.

VIII. CONCLUSIONS

This paper introduces a model that considers the asymmetries in the mean equation of the various ARCH-type models, and determines the most likely distribution of Chicago wheat futures price changes among alternative autoregressive conditional heteroskedasticity models. The asymmetric EGARCH (0, 1)-t with one lag on the conditional variance, which considers asymmetries both in the mean and variance equations, was selected as the most likely model of Chicago wheat futures price changes. This implies that considering asymmetries not only mean returns but also volatility structure is important for the fatures prices.

A Monte Carlo study using the estimated asymmetric EGARCH(0, 1)-t parameters shows that Black's model values deep out-of-the-money put options less than the EGARCH option pricing model does. However, Black's option pricing model values at-the-money put option premiums higher than the EGARCH option pricing model does. In the Monte Carlo study, differences between Black's model and the EGARCH option model increase as time to maturity increases, which confirms Hull and White's findings. The EGARCH option pricing model predicts actual option premiums of Chicago wheat more accurately than Black's model for deep out-of-the-money put and call options and at-the-money call options.

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