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WELFARE COSTS OF INFLATION IN A MONETARY ECONOMY WITH TRANSACTIONS COSTS

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This paper examines the welfare cost of inflation in an economy where agents hold money by the motivation of transaction service. I formulate and estimate the transactions cost function. Using a set of the estimated parameters, the welfare cost of inflation is measured. In addition, the model economy is used to analyze whether the cyclical properties are different from those of business cycle literature.

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I. INTRODUCTION

The question of the relationship between money and real activities is a classical topic in monetary theory. Beginning with the pioneering work of Tobin (1965), the analysis of the real effects of anticipated inflation has been not only one of the central issues but also controversial. Theoretically, there is no consistent conclusion when money is introduced into the system in different ways.[1] Even within an approach, for example transactions-cost approach, there

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[1] Tobin(1965) finds a positive relationship between money growth and real output in a monetary model where people increase the portion of capital and decrease the portion of money in the composition of their portfolios facing inflation. Sidrauski(1967) overturns Tobin's result in the money-in-the-utility model: the steady-state capital stock is independent of the inflation rate because inflation does not affect the agent's subjective discount rate, which determines the steady-state capital stock. In the cash-in-advance models studied by Stockman(1981) and Cooley and Hansen(1989), inflation will cause people to substitute away from activities that require cash, such as consumption, for activities that do not require cash, such as leisure. The main results of this approach is that higher inflation decreases capital and output, so that neither Tobin's nor Sidrauski's results apply.

is no consistent conclusion on the effects of inflation on the real activities.

The transactions-cost approach introduces the money into the system by departing from the assumption that payments must be made in cash and from the distinction between cash goods and credit goods. Saving(1971) introduces money as a transaction device through a shopping-time technology, whereas Dornbusch and Frenkel(1973) employ a pecuniary transactions cost function: In a shopping-time approach, labor input is lost as a transactions cost, while consumption or output is lost as a transactions cost in a pecuniary transactions cost model. In their model, the effect of money growth on consumption and capital stock remains ambiguous if the return on capital is a function of real balances.

In spite of these theoretical differences, Feenstra(1986) and Wang and Yip (1992) have demonstrated an equivalence between alternative approaches under some conditions. Feenstra(1986) showed a functional equivalence between the money-in-the-utility-function approach and the transactions-cost approach. He proposed a general class of transactions cost functions approximately derived from conventional models of money demand, such as Baumol(1952) and Tobin (1956), and showed an exact duality between it and a broad class of utility functions which include real balances as an argument. Wang and Yip(1992) also showed a qualitative equivalence between the money-in-the-utility-function, cash-in-advance, and transactions-cost approach. They derived the restrictions imposed by the structure of various approaches such that all approaches yield exactly identical comparative statics results. That is, higher money growth rates are associated with lower consumption, hours worked, capital stock and output under their conditions.[2]

Thus, so far, there is no consistent conclusion on the real effects of inflation and their welfare consequences, even within the transactions-cost approach. As a complement to the transactions-cost approach, this paper formulates and estimates a transactions cost function in a general equilibrium monetary model. I also measure the private loss due to transactions-costs, and further measure a welfare cost of inflation by examining the relationship between money supply growth and real variables in the model economy. Some related quantitative analyses can be found in the papers of Marshall(1992) and Bansal and Coleman(1993). They both introduce transactions costs in an endowment economy and estimate parameters of the transactions cost function. Their primary purpose is to explore the implications for the behavior of interest rates and equity returns in a monetary economy with transactions costs. Our primary interest is, however, placed on a welfare cost of inflation in a model with money holdings motivated by transactions costs.

[2] The conditions include (i) Pareto complementarity between consumption and leisure; (ii) Pareto complementarity between consumption and money; (iii) Pareto substitutability between leisure and money; and (iv) weakly dominant consumption effect of money growth compared to the real balance effect.

Following the work of Bailey(1956), Fisher(1981) and Lucas(1981) analyzed welfare costs of anticipated inflation by computing the triangle under the demand for money curve. On the other hand, Cooley and Hansen(1989) estimated the welfare cost of inflation by comparing discounted streams of utilities of two economies that display different money supply paths. Following their approach, Gomme(1993) estimated the welfare cost of inflation in an endogenous growth model. Cooley and Hansen(1989), and Gomme(1993) have adopted Cash-in-Advance constraints as a way to introduce the money into the economy. But, money has only one channel to generate real effects in this setting: anticipated inflation causes people to substitute away from activities that require cash, for activities that do not require cash. Thus, anticipated inflation results in minor effects on real activities. Furthermore, this structure abstracts from the fact that banking sector provides transaction service enabling people to economize on cash balances. In the real world, banking sector uses a non-negligible portion of resources. Value added share in GNP of U.S. banking and credit sector amounts to about 2%.[3] To capture this feature, the banking sector is incorporated into the model through a transactions cost function, which provides a measure of the misallocation cost to the banking sector with inflation.

In addition to the welfare analysis of inflation, I also examine the cyclical properties of a monetary economy with transactions costs. Until now, monetary business cycle models fail to outperform the standard business cycle model. Accordingly, I analyze whether the cyclical behavior of the model economy differs from the standard real business cycle model.

In the next section, I set up a general equilibrium model which motivates money holding by transactions costs. Section 3 describes the strategy for estimating the transactions cost function and presents the estimation results. The cyclical behavior of the model economy is analyzed in the section 4. I estimate the welfare cost of inflation in various monetary regimes in the section 5. Concluding remarks are presented in the section 6.

II. THE MODEL

The economy to be studied is populated by a continuum of identical and infinitely lived households. Each household maximizes the expected value of a discounted stream of utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t), \tag{1}$$

[3] See Aiyagari, Braun and Eckstein(1995)

where E_0 denotes expectations conditional on information available at time 0 and the discount factor β is between 0 and 1, and c_t is consumption at time t and l_t is leisure in period t, and the current period utility function, $U(\cdot)$, is strictly increasing and strictly concave in both arguments and twice continuously differentiable. The household's time endowment is normalized to one so that l_t is the fraction of time the household allocates to leisure.

Consumption goods are not fully consumed, but some part of consumption goods are lost as transactions costs. I distinguish two concepts of consumption: gross consumption, which denotes the amount of output devoted to consumption, and net consumption, which is the actual amount of consumption achieved by the household. Households allocate gross consumption and investment out of their income, and use some part of gross consumption as a transaction service. Their final net consumption is net of the transactions cost. The greater is net consumption, the larger the portion of gross consumption that is lost as transactions cost. Money is introduced since money provides a transaction service. Transactions costs are reduced at a decreasing rate as money balances increase. When real balance holdings equal to $\frac{m}{p}$ consumption of c units of good requires expending an additional $\varphi\left(c, \frac{m}{p}\right)$ units of consumption as transaction costs, where c is therefore net consumption. The function $\varphi\left(c, \frac{m}{p}\right)$ is assumed to be non-negative and twice continuously differentiable and satisfy:

$$\varphi_1 \geq 0,\ \varphi_2 \leq 0,\ \varphi_{11} \geq 0,\ \varphi_{22} \geq 0 \text{ and } \varphi_{12} \leq 0, \tag{2}$$

where subscripts 1 and 2 denote differentiating with respect to consumption(c) and real money balances $\left(\frac{m}{p}\right)$, respectively.

It is assumed that the government makes a lump-sum transfer, x, to the household, each period. Households enter period t, with nominal money balances equal to m_t that is carried over from the previous period. In addition, these balances are augmented with a lump-sum transfer equal to $x_t \equiv (\mu_t - 1)M_t$,

where M_t is the per capita money supply in period t.[4]

The money stock follows a law of motion

$$M_{t+1} = \mu_{t+1} M_t \tag{3}$$

where the gross growth rate of money, μ_{t+1} evolves according to

$$\log \mu_{t+1} = \phi \log \mu_t + \xi_{t+1}, \tag{4}$$

[4] Throughout the present paper, small letters refer to the individual agent's decision variables while capital letters refer to the per-capita aggregate variables.

where ξ_t is an iid random variable with mean $\log\bar{\mu}(1-\phi)$ and variance σ_ξ^2, where $\log\bar{\mu}$ is the unconditional mean of the logarithm of the growth rate μ_t.

The government is also assumed to consume g each period.[5]

The representative household chooses consumption, investment(i_t), and nominal money holdings subject to the following budget constraint:

$$c_t+\varphi\left(c_t,\ \frac{m_t}{p_t}\right)+i_t+\frac{m_{t+1}}{p_t}\leq w_t h_t+r_t k_t-g+\frac{m_t+x_t}{p_t}, \tag{5}$$

where w_t, $h_t(\equiv 1-l_t)$ and r_t are the wage rate, labor hours and rental rate of capital, respectively. Investment is undertaken to augment the capital stock(k) owned by the household. The capital stock obeys the following law of motion:

$$k_{t+1}=(1-\delta)k_t+i_t. \tag{6}$$

The firm in the economy produces output(Y) using the constant returns to scale technology:

$$Y_t=\exp(z_t)K_t^{\alpha}H_t^{1-\alpha}, \tag{7}$$

where z_t is a productivity shock, assumed to evolve as

$$z_{t+1}=\rho z_t+\varepsilon_{t+1},\ 0\leq\rho\leq 1, \tag{8}$$

where ε_t is iid random variable with mean 0 and variance σ_ε^2

Firms maximize profits at each point in time. Since their technology is characterized by constant returns to scale, first-order conditions for profit maximization imply that

$$w(z_t,\ K_t,\ H_t)=(1-\alpha)\exp(z_t)K_t^{\alpha}H_t^{-\alpha}, \tag{9}$$

$$r(z_t,\ K_t,\ H_t)=\alpha\exp(z_t)K_t^{\alpha-1}H_t^{1-\alpha}. \tag{10}$$

To induce stationarity in variables, a transformation of variables is introduced. That is, let $\hat{m}_t=m_t/M_t$ and $\hat{p}_t=p_t/M_t$. With this transformation in variables, the problem faced by households in the model economy is a dynamic programming problem of the following form:

[5] Government consumption is incorporated into the model just to be consistent with the data set from Christiano and Eichenbaum(1992)

$$V(z, \mu, K, k, \hat{m}) = \max\{U(c, h) + \beta EV(z', \mu', K', k', \hat{m}')\} \tag{11}$$

subject to

$$c + \varphi\left(c, \frac{\hat{m}}{\hat{p}}\right) + i + \mu\frac{\hat{m}'}{\hat{p}} = w(z, K, H)h + r(z, K, H)k - g + \frac{\hat{m} + \mu - 1}{\hat{p}}, \tag{12}$$

$$z' = \rho z + \varepsilon' \tag{13}$$

$$\log \mu' = \phi \log \mu + \xi', \tag{14}$$

$$K' = (1-\delta)K + I, \tag{15}$$

$$k' = (1-\delta)k + i, \tag{16}$$

where h and H are the functions of $(z, \mu, K, k, \hat{m}, I, i, C, c, \hat{p}, \hat{m}')$. In addition, I, C, and $\hat{p}'$ are given functions of (z, μ, K).

A stationary competitive equilibrium for this economy consists of a set of decision rules for the household, $c = c(z, \mu, K, k, \hat{m})$, $i(z, \mu, K, k, \hat{m})$ and $h(z, \mu, K, k, \hat{m})$; a decision rule determining money balances, $\hat{m}' = m(z, \mu, K, k, \hat{m})$; a set of aggregate decision rules $I(z, \mu, K)$ and $C(z, \mu, K)$; a function determining the aggregate price level, $\hat{p}' = P(z, \mu, K)$; and a value function, $V(z, \mu, K, k, \hat{m})$ such that:

(i) Given the functions C, I and P, the value function, V, satisfies (11) and c, i, h and $\hat{m}'$ are the associated decision rules.

(ii) Given the pricing function, P, individual decision rules are consistent with aggregate outcomes:

$C(z, \mu, K) = c(z, \mu, K, K, 1)$, $I(z, \mu, K) = i(z, \mu, K, K, 1)$ and $1 = m(z, \mu, K, K, 1)$

Assuming that the solution is an interior, we get the following first-order-conditions:

$$\frac{U_1(c_t, l_t)F_2(k_t, h_t)}{1+\varphi_1\left(c_t, \frac{m_t}{p_t}\right)} = U_2(c_t, l_t), \tag{17}$$

$$\frac{U_1(c_t, l_t)}{1+\varphi_1\left(c_t, \frac{m_t}{p_t}\right)} = \beta E_t \frac{U_1(c_{t+1}, l_{t+1})}{1+\varphi_1\left(c_{t+1}, \frac{m_{t+1}}{p_{t+1}}\right)}[F_1(k_{t+1}, h_{t+1}) + (1-\delta)], \tag{18}$$

$$\frac{U_1(c_t, l_t)}{1+\varphi_1\left(c_t, \frac{m_t}{p_t}\right)} = \beta E_t \frac{U_1(c_{t+1}, l_{t+1})}{1+\varphi_1\left(c_{t+1}, \frac{m_{t+1}}{p_{t+1}}\right)} \frac{1-\varphi_2\left(c_{t+1}, \frac{m_{t+1}}{p_{t+1}}\right)}{(p_{t+1}/p_t)}, \tag{19}$$

where E_t is the mathematical expectation operator taking into account the information available to the household at the beginning of period t.

Eq.(17) shows how introduction of money distorts the decisions on consumption and labor input. Without the transactions cost, marginal utility of consumption obtained from marginal unit of labor input is equated to the marginal disutility of working. However, transaction costs introduce a wedge of inefficiency in Eq.(17) since some part of consumption is lost due to a transactions cost.

Transactions costs also alter the accumulation of physical capital that is governed by Eq.(18). The marginal utility of one unit of the good consumed in the current period is represented by the left-hand-side of Eq.(18). The right-hand-side of Eq.(18) represents the expected utility of one unit of the good that is invested and then consumed in the next period. Both sides reflect the loss of consumption due to transactions cost when the good is consumed.

Eq.(19) illustrates how money balances are intertemporarily allocated. Eq.(19) equates the expected utility costs and benefits of reducing current-period consumption by one unit and allocating that unit to money holdings and then to consumption in the next period. The left-hand-side of Eq.(19) represents utility loss of reducing current-period consumption by one unit, while right-hand-side of Eq.(19) represents discounted next period marginal utility of money balances carried over from this period. Notice that money carried over to the next period gives more utility in the next period by reducing transactions cost, while the value of money balances are eroded by inflation(p_{t+1}/p_t).

III. ESTIMATION

Quarterly data are used, which cover from 1959:1 to 1984:1. The money supply is measured by M1(CITIBASE FM1). The set of the other data comes from Christiano and Eichenbaum(1992).[6] The measured private consumption is assumed to equal consumption net of transaction costs(c) plus the costs of transaction services used in purchasing these consumption goods(φ). Thus the net consumption is not measurable, but GMM estimation procedure makes it possible to calculate the value of net consumption at each step of iteration given the parameters which are assigned by initial guess or readjusted from previous trial.

[6] Private consumption is measured as the sum of private sector expenditures on non-durable goods plus the imputed service flow from the stock of durable good. Capital stock is measured as the sum of consumer durables, producer structures and equipment, and government and private residential capital plus government non-residential capital. Gross investment series are the flow data that conceptually match the capital stock data. Government consumption is measured by real government purchases of goods minus real government investment. Gross output is measured as the sum of private consumption, gross investment, government consumption plus inventory investment.

For the specification of the utility function, indivisible labor is assumed following Hansen(1985).

$$U(c, l) = \log c - \mathrm{B}h. \tag{20}$$

And the transactions cost function is specified as

$$\varphi\left(c, \frac{m}{p}\right) = \frac{\varphi}{r} c^{\gamma}\left(\frac{m}{p}\right)^{1-\gamma} \tag{21}$$

where $\phi > 0$ and $\gamma > 1$ such that Eq. (2) holds.

From the efficiency conditions from the previous section with these specifications, the orthogonality conditions are defined as[7]

$$E\left[(1-\alpha)\frac{y_t}{h_t c_t} - \mathrm{B}\left(1 + \phi\left(\frac{c_t}{m_t/p_t}\right)^{\gamma-1}\right)\right] = 0, \tag{22}$$

$$E\left[(1-\beta)\frac{(y_t/h_t)}{(y_{t+1}/h_{t+1})}\left(\alpha\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right)\right] \otimes Z_t = 0, \tag{23}$$

$$E\left[(1-\beta)\frac{(y_t/h_t)}{(y_{t+1}/h_{t+1})(p_{t+1}/p_t)}\left(1 - \phi\left(\frac{1}{\gamma} - 1\right)\left(\frac{c_{t+1}}{m_{t+1}/P_{t+1}}\right)^{\gamma}\right)\right] \otimes Z_t = 0, \tag{24}$$

$$E\left[\delta - 1 + \frac{k_{t+1}}{k_t} - \frac{i_t}{k^t}\right] = 0, \tag{25}$$

where $Z_t = \{1,\ y_t/k_t,\ (y_{t-1}/h_{t-1})/(y_t/h_t),\ c_t/(m_t/p_t),\ p_t/p_{t-1}\}$.
Eqn. (22)-(24)[8] come from the efficiency conditions, Eqns. (17)-(19) by applying the law of iterated expectations. Eqn. (25) is the law of motion for the capital stock. It is assumed that the instrumental variables, Z_t, are applied only for the Eqns. (23) and (24).

Note that these disturbance terms are not serially correlated because all variables are included in the information set available at time t+1. Since the discount factor, β is set to 1.03^(−1/4), the set of parameters to be estimated is $\Psi = \{\alpha, \mathrm{B}, \phi, \gamma, \delta\}$, where α is capital share, B is the coefficient on the leisure in the utility function, ϕ and $\gamma$are the coefficients on the transactions cost function, and δ is the depreciation rate, respectively.

By stacking Eqns. (22)-(25), we obtain a system of twelve moment conditions, which can be summarized as a vector equation.

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[7] Formally, Eq.(22) and Eq.(25) are not the moment conditions because these equations hold even without the expections operator if the model is true.

[8] Eq.(24) is obtained by substituting Eq.(17) into Eq.(19).

$$E[f(X_t,\ \Psi_0)]=0, \tag{26}$$

where Ψ_0 is the true value of Ψ and X_t is the vector of random variables,
which are assumed to be stationary.

Under some regularity conditions discussed in Hansen(1982), Ψ_0 can be consistently estimated by choosing the value of Ψ, say Ψ_T that minimizes a quadratic form of the sample means

$$J_T=\left\{\frac{1}{T}\sum_{t=1}^{T}f(X_t,\ \Psi)\right\}' W_T\left\{\frac{1}{T}\sum_{t=1}^{T}f(X_t,\ \Psi)\right\} \tag{27}$$

with respect to Ψ; where W_T is a positive definite matrix that depends on sample information.

Hansen(1982) showed that the optimal weighting matrix, W_T, in the sense of the smallest asymptotic covariance matrix for GMM estimators, is the inverse of the long-run covariance matrix of the disturbance terms. With this optimal weighting matrix, $\sqrt{T}(\Psi_T-\Psi_0)$ has a normal distribution with mean zero and the covariance matrix

$$Cov(\Psi_T)=\frac{1}{T}(\Gamma'_T W_T \Gamma_T)^{-1}, \tag{28}$$

where $\Gamma_T=\frac{1}{T}\sum_{t=1}^{T}(\partial f(X_t,\ \Psi_T)/\partial\Psi)$.

Since five parameters are to be estimated, there are seven overidentifying restrictions, which are testable by Hansen's J-test. Hansen(1982) showed that T times the minimized value of the objective function, $TJ_T(\Psi_T)$, has a chi-square distribution with 7 degrees of freedom.

Table 1 provides the result of estimation. The minimal value of the GMM criterion function, which is distributed as x^2(xi-squiare) with 7 degrees of freedom, has a value of 6.666, with p-value of 46.4 percent. This implies that the model's overidentifying restrictions are not rejected at the 10% significance level.

The parameter estimates for α(capital share), B(coefficient on labor in the utility function), δ(depreciation rate) are estimated to be 0.338, 5.032 and 0. 021, respectively. All of these are plausible values, in keeping with those used in many calibration exercises. The parameters in the transactions cost technology are estimated somewhat imprecisely at 0.032 and 1.362 for ϕ and γ, respectively. But these point estimates satisfy the conditions of the transactions cost function, i.e., Eq.(2), implying that 1.2 percent of output is lost in transactions service.[9]

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[9] Bansal and Coleman(1993) shows 1.4% loss of income in the endowment economy.

Ⅳ. CYCLICAL PROPERTIES

This section studies the cyclical properties of the model economy. How does the introduction of money into the standard neoclassical economy motivated by transaction services change the cyclical behavior of the economic variables? The answer to this question is important for both understanding the contribution of the model to the general equilibrium business cycle literature and providing some confidence in the welfare results.

Table. 2 presents statistics summarizing the cyclical properties of the model economy under various monetary regimes as well as those of actual U.S. time-series. To facilitate comparison with other real business cycle exercises, both the U.S. and model data series are logged and detrended using the Hodrick-Prescott filter.

Parameter values for the artificial economies, are based on the GMM estimation from the previous section. The autoregressive coefficient of technology shock process is set at 0.97. The standard deviation of the innovation to the technology shock, σ_ε is set equal to 0.0094 so that the standard deviation of the simulated output series is close to that of the actual output series.[10] The autoregressive coefficient of money growth, Ψ, and the variance of its innovations, σ_ζ^2, are obtained by estimating a first-order autoregressive process for money growth. The resulting values are 0.466 and 0.0077, respectively.[11]

Given a set of parameter values, simulated time-series are computed using linear-quadratic approximation method.[12]

I simulate the economy 500 times and report the averages of the statistics over these simulations as well as the sample deviations of these statistics, which is given in parentheses. The second panel of Table. 2 shows the percent standard deviations and correlations from the nonmonetary model economy, and the third and fourth panel present the standard deviations and correlations from the model economy where the money supply grows at an erratic rate.

The third panel, which experiments the model economy with the postwar average money growth rate, reproduces major characteristics of macroeconomic time series. The volatility of consumption is less than that of output while investment is volatile than output. Investment is highly correlated with output as observed in the data. Consumption is also highly correlated with output, but its correlation with output is bigger than in the data. As found in the real business cycle literature, productivity is found to be procyclical, which is contractive to the data.

[10] Hansen(1985) has also used this method.

[11] In the estimation, average quarterly value of M1 is used as Cooley and Hansen(1989).

[12] For the details of the algorithm, see Hansen and Prescott(1993).

By comparing the second and third panel, the features of the business cycle are unaffected by the introduction of money into the model economy. The standard deviations and correlations are almost the same for the nonmonetary and monetary economies. However, comparing the third and fourth panel, we find that increases in the average growth rate of money increase the volatility of price level and negative correlations with output in the absolute level, even though the cyclical properties of the real variables are little affected.[13]

V. WELFARE RESULTS

Table.3 presents the welfare costs for various annual inflation rates, along with the associated steady-state values for output, consumption, investment, capital stock and hours worked. The welfare measure to be used is based on the increase in consumption that is necessary to make individuals indifferent between Pareto optimal allocation and each monetary regime. The Pareto optimal allocation for the present economy is equivalent to the equilibrium allocation for a similar economy without transactions costs. It turns out that for the model studied in this paper, the transactions costs do not appear if money balances are large enough relative to consumption that the marginal benefit of money holding in reducing transactions cost is zero($\varphi_2\left(c, \frac{m}{p}\right)=0$), and the marginal cost of consumption in increasing transactions cost is also zero($\varphi_1\left(c, \frac{m}{p}\right)=0$) or the gross rate of money growth is equal to the discount factor, β under the Pareto optimal policy.[14]

To obtain a measure of the welfare loss associated with various monetary regimes other than the Pareto optimal allocation, I solve for $\triangle c^a$ in the equation

$$U(c^*, l^*)=U(c^a+\triangle c^a, l^a), \tag{29}$$

where asterisk superscripts denote the steady state associated with an Pareto optimal policy and superscript *a*'s denote an competitive equilibrium of each alternative monetary regime. Thus $U(c^*, l^*)$ represents the level of utility attained under the Pareto optimal allocation.

The results show that agents substitute leisure for consumption in the face of higher inflation and therefore the output decreases. That is, higher inflation are associated with lower consumption, hours worked, capital stock and output as the cash-in-advance model of Cooley and Hansen(1989). As found in Feenstra

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[13] Cooley and Hansen(1989) and Gomme(1993) show that the volitility of the money supply affects the standard deviations of consumption as well as the standard deviations of price level and their correlations with output when money is introduced via the cash-in-advance constraint.

[14] The equilibrium in the limiting economy where real money balances are infinite may be ill-defined, so we confine attention to economy without transactions cost when referring Pareto Optimum.

(1986) and Wang and Yip(1992), a transactions-cost approach adopted in the model reveals a qualitative equivalence with the cash-in-advance approach.

The welfare cost of moderate(10%) inflation is 1.733 percent of the output. This magnitude is quite a bit larger than that found in the literature. Fischer (1981) and Lucas(1981) found the welfare cost of 10% inflation to be 0.30% and 0.45% of GNP, respectively, by computing the triangle under the demand for money curve. In their analyses, inflation is shown to generate a loss since it causes an increase in transaction costs due to agents economizing on holdings of real cash balances. On the other hand, by comparing the steady-state utility between an optimal policy regime and 10% inflation regime, Cooley and Hansen(1989) report 0.39% of GNP as a welfare loss. Gomme(1993) found even a smaller welfare cost of 10% money growth rate equal to 0.03% of GNP in an endogenous growth model. In both the economies above, welfare costs are due to agents substituting leisure for consumption in the face of inflation and causing output to be lower. The magnitude of welfare cost of 10% inflation rate I find in the present paper is even bigger than 1.07% of GNP in the welfare cost found by Imrohoroglu(1992), where agents hold money in order to smooth consumption in the face of income variability under no insurance.

Fig. 1 plots the welfare costs of inflation along with the transactions costs in various monetary regimes. The transactions costs sacrificed with inflation are proportional to the welfare cost of inflation.[15] At moderate inflation rate(10% annual rate), the transactions cost amounts to 1.48 percent of output, compared to 1.73 percent of output, the total welfare cost of inflation. The difference between the total welfare cost of inflation and the transactions cost is 0.25 percent. Thus the inflation tax component of welfare cost(the difference mentioned above) is a little less than the welfare cost of inflation tax(0.39%) found in Cooley and Hansen(1989), which abstracts from the banking sector.

At high inflation, the welfare cost of inflation remains bounded, even though the welfare cost is quite large at the moderate inflation. Fig. 2 compares the welfare costs of the model economy with that of Cooley-Hansen (1989). In the CIA model of Cooley-Hansen, the welfare cost of inflation is linear in the inflation rate, implying unbounded at high inflation. But, the model economy shows the concave welfare cost in the inflation rate. Thus, at high inflation rate, welfare cost is smaller in the model economy than in the CIA model.

[15] The ratio of transactions cost to welfare cost of inflation averages 85.2%, with the standard deviation of 0.27%. The ratio slightly decreases with a higher inflation: at the zero annual inflation rate, the ratio is 86% and decreases to 85% at the 100% annual inflation rate.

VI. CONCLUSIONS

In this paper simple monetary general equilibrium models are studied to assess the welfare costs of inflation in economies where money is used to reduce the transactions costs. The transactions cost function is estimated using the post-war U.S. time-series. With these GMM estimates, I analyze the cyclical behavior of several economies that differ only in their money supply path. It is shown that the monetary model economies constructed here make little difference from the standard neoclassical model, except the increased volatility of price level and its increased negative correlation with output. The monetary model economy generates quite a large welfare cost of inflation equal to 1.7 percent of the output. But, the welfare cost of inflation remains bounded at high inflation.

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[Table 1] Estimates of Parameters

Parameter	Point Estimate	Standard Error
α	0.3380	0.0065
B	5.0323	0.2285
ϕ	0.0320	0.0545
γ	1.3619	0.6612
δ	0.0211	0.0003
$J_T=$	6.6660	p-value=0.4645($d.f.$=7)

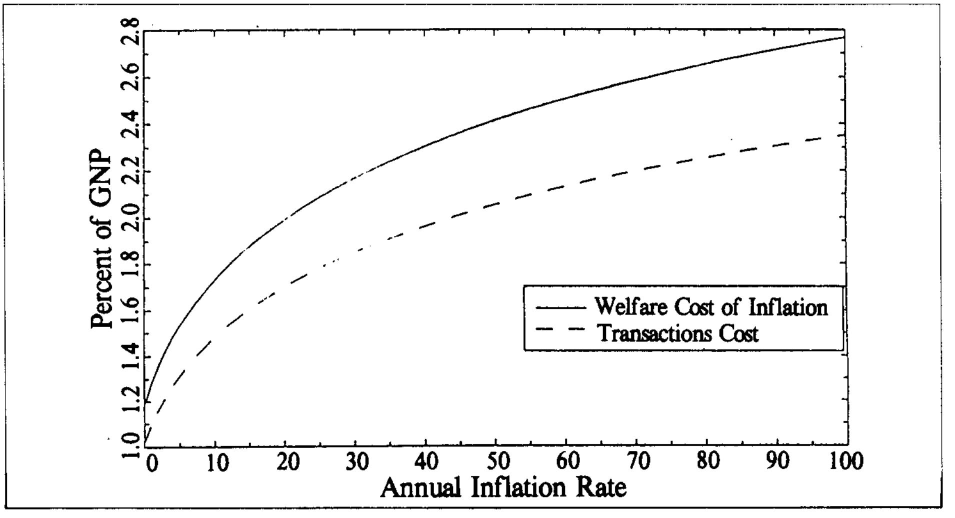
[Table 2] Selected Moments for U.S. and Artificial Economies

Series	Quarterly U.S. Data		Nonmonetary Economy	
	Standard Deviation	Correlation with Output	Standard Deviation	Correlation with Output
Output	2.04	1.00	2.06(0.28)	1.00(0.00)
Consumption	0.88	0.79	0.80(0.12)	0.94(0.01)
Investment	4.44	0.94	6.53(0.88)	0.99(0.00)
Capital Stock	0.36	0.11	0.47(0.11)	0.02(0.07)
Hours	1.67	0.75	1.33(0.18)	0.98(0.01)
Price Level(CPI)	1.65	−0.73	−	−
Productivity	1.37	0.58	0.80(0.12)	0.94(0.01)
	Economy with Autoregressive Growth Rate($\bar{\mu}$ = 1.0105)		Economy with Autoregressive Growth Rate($\bar{\mu}$ = 1.0105)	
Series	Standard Deviation	Correlation with Output	Standard Deviation	Correlation with Output
Output	2.04(0.28)	1.00(0.00)	2.03(0.27)	1.00(0.00)
Consumption	0.80(0.12)	0.94(0.01)	0.79(0.12)	0.94(0.01)
Investment	6.49(0.88)	0.99(0.00)	6.44(0.84)	0.99(0.00)
Capital Stock	0.47(0.11)	0.02(0.07)	0.46(0.11)	0.02(0.07)
Hours	1.31(0.18)	0.98(0.01)	1.31(0.17)	0.98(0.01)
Price Level(CPI)	1.41(0.12)	−0.11(0.13)	1.84(0.14)	−0.31(0.13)
Productivity	0.80(0.12)	0.94(0.01)	0.79(0.12)	0.94(0.01)

[Table 3] Welfare Results of Alternative Money Grouth Rates

	Annual Inflation Rate				
	Optimal	0%	10%	100%	400%
Output	0.817	0.811	0.808	0.803	0.799
Consumption	0.465	0.451	0.445	0.433	0.425
Investment	0.204	0.203	0.202	0.201	0.200
Capital	9.701	9.623	9.589	9.523	9.477
Hours	0.231	0.229	0.228	0.227	0.226
Transactions Cost(%)	0.0	1.023	1.484	2.350	2.958
Welfare Cost(%): ($\triangle c/c$)	0.0	2.137	3.146	5.128	6.589
($\triangle c/y$)	0.0	1.189	1.733	2.768	3.506

[**Figure 1**] Welfare Cost of Inflation



[**Figure 2**] Welfare Cost of Inflation