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# MONEY GROWTH UNCERTAINTY AND REAL OUTPUT: TRIVARIATE VAR GARCH-M MODEL

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*Unlike previous literatures investigating the relationship between inflation uncertainty and real variables, we investigate the Friedman's hypothesis by observing empirically the relationship between money growth uncertainty and real output. We generated money growth uncertainty data by employing GARCH-M model and at the same time examined the empirical validity of the hypothesis in the U.S. economy. We found that the empirical results did provide support on our claim that money growth uncertainty in stead of the inflation uncertainty negatively affects real output growth. Also, allowing parameters of the money growth equation in the VAR GARCH-M model to exhibit discrete changes at the third quarter of 1979 and 1982 brings richer specification for capturing deterministic shifts in the monetary regime.*

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## I. INTRODUCTION

The attention given to the impact of inflation uncertainty on real economic variables like real output and unemployment has been growing since Friedman (1977) hypothesized a positive relationship between the mean of inflation and inflation uncertainty, and a negative relationship between inflation uncertainty and real activity. Friedman(1977) claimed that the link between inflation and inflation uncertainty occurs because political preferences involved in the deci-

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sion process of monetary policy would confuse the public to forecast the direction of future policy. The increasing uncertainty of inflation, combined with the sluggish adjustment of prices and wages, reduces the efficiency of the price system. Friedman mentioned two reasons that lead the price system to be inefficient. First, the increasing uncertainty of inflation causes long-term wage contracts written in nominal terms to be more costly to the public because it heavily discounts the future income of risk averse individuals. Second, the public cannot easily extract the relative price movement from the aggregate price movement because the uncertainty of inflation reduces the ability of aggregate price to carry information about relative price, and the cost of extracting the relative price movement from the aggregate price becomes higher. Hence, the resulting inefficiency of the price system would reduce real output and employment at least in the short run.

There have been many empirical studies on the relationship between inflation, uncertainty of inflation and real output. Several different types of approach have been applied for constructing the proxy for inflation uncertainty. The first approach takes simple sample variance from the data and constructs it as a measure of inflation uncertainty.((Okun(1971), Logue and Sweeny (1981)), This approach seems to be problematic in capturing uncertainty about future inflation because the sample variance of inflation confounds predictable and unpredictable changes in the inflation rate. For example, if the inflation rate moves in a perfectly predictable way, inflation variability would be zero, but estimated sample variance would be positive.

The second approach uses cross-sectional variance of expected inflation rates in survey data of individual inflation forecast, for example, the Livingston survey and the SRC survey.(Cukierman and Watchel(1979), Fischer(1981), Evans and Watchel(1993), Mullineaux(1980), Levi and Makin(1980), Cukierman and Watchel(1982), Hafer(1986)) However, a crucial problem with this approach is that since the measure of variability is directly taken from cross-sectional variance of expected inflation rates in the survey data, it does not necessarily capture the degree of uncertainty about future inflation rates.

The third approach employs time varying unconditional variance of inflation for the proxy of inflation uncertainty.(Katsimbris(1985), Ball and Cecchtti (1990)) This approach lacks a parametric model for the time varying variance of inflation. The moving averag for the mean inflation rate does not necessarily capture the predictable components of inflation process. Therefore, the uncertainty estimated via the simple moving average mean may be overestimated because the forecastable elements translates into the uncertainty.

The fourth measure uses time varying conditional variance as a proxy for inflation uncertainty.(Engle(1983), Jansen(1989), Evans(1991), Brunner and Hess(1993)) However, this approach may lead uncertainty about inflation to be underestimated because it can overestimate the degree to which agents can forecast inflation.

Despite many efforts with the different method, empirical results turned out to be mixed because as demonstrated by Holland(1993), the results of existing studies seem to heavily rely on the way to construct proxy for uncertainty about future inflation.

Unlike the literature investigating the inflation-inflation uncertainty relationship and inflation uncertainty-real variables relationship, the literature on the money growth uncertainty-real variables relationship is sparse.(Belongia(1984), Evans(1984), Aizenman and Marion(1993)) As an alternative, We investigate the hypothesis by observing empirical relationship between money growth uncertainty and real output. Over the period of the mid-1950s to 1980, there have been experiences of supply-side shocks such as the oil price shock and institutional rigidities and sluggish adjustments of prices and wages via increases in uncertainty. Accordingly, prices would be unlikely to respond systematically to the forces of supply and demand in the market. Inflation may not be an appropriate measure of monetary policy. Hence, investigating the monetary stock directly would provide a cleaner measure of the direct policy influence on real variables for the policy makers, because the money stock has often been employed as the instruments of monetary policy.

In this paper we use ARCH-M and GARCH-M models in VAR system. The model facilitates estimation and statistical inferences about the effects of variances on mean parts. In this sense, this type of models may be appropriate to investigate the relationship between conditional variance of money growth uncertainty, inflation uncertainty, and real output growth.

The remainder of the paper is organized as follows. Section II describes data and preliminary test results. Section III presents VAR model and diagnostic tests, and section IV specifies Trivariate ARCH-M and GARCH-M model. Section V includes model selection and estimation results and section VI summarizes results of this studies.

## II. DATA AND PRELIMARY TEST RESULTS

The data used in this study consisted of seasonally adjusted quarterly observations on GDP, GDP deflater(GDPD), and M1. All of the analysis covers the period from 1959.I through 1992.Ⅳ.[1] All variables are transformed into natural logarithm. The log differenced variable multiplied by 100 is used as a percentage growth rate of that variable.

We employed the standard Dickey-Fuller test, and also the augmented Dickey-Fuller test with four lags of the variable under investigation. The test statistics are reported in Table 1. All of the variables have a unit root. Upon differencing data, growth rate of GDP and M1 clearly have no further unit root.

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[1] The source of the data is Citibase.

Hence, growth rate of GDP and M1 are integrated of order one. However, GDPD seems to be inconclusive because the augmented Dickey-Fuller test statistics for those variables do not lead to rejection of an additional unit root in the rate of change of GDPD, while the standard Dickey-Fuller test statistics indicate rejection of further unit roots.

However, it is important to note that the Dickey-Fuller critical values are calculated under the assumption that the data generating process is a random walk without drift. Schmidt(1988) provides critical values for non-zero drifting process. With non-zero drift, Schmidt critical values are lower in absolute value than those calculated by Dickey(1976) and Fuller(1976) and in fact tend toward the standard normal critical values as the values of standardized drift approaches infinity. For our GDPD series, the standardized drift[2] for GDPD is 0.63, and the 5% critical value tabulated by Schmidt for 100 sample size and a standardized drift of 0.60 is $-1.96$($-1.92$ with standardized drift of 0.70). With this Schmidt critical value, the hypothesis of a unit root in the GDPD processes is rejected at the 5% level for the augmented Dickey-Fuller test.[3]

Also, tests for time trends were conducted in the first differences of each process. The $\Phi 3$ statistic of Dickey-Fuller(1981) was used to test the joint null hypothesis of a unit root and no trend for the level of GDP, GDPD, and M1. Only the case of M1 can reject the null hypothesis. Hence, we include a time trend variable in the M1 growth equation.

The cointegration test is required to analyze the trivariate vector autoregression(TVAR) that includes GDP, GDPD, and M1. The VAR model is appropriate as long as the three variables are not cointegrated. If GDP, GDPD, and M1 level are cointegrated, a pure VAR in differences will be misspecified because the cointegration is implied by the presence of the levels of variables. Thus VARs estimated with cointegrated variables will be misspecified if the data are differenced, and will have omitted important constraints if the data are used in level.

We first selected the lag length before the cointegration tests were conducted.

For the dynamic specification, the system was initially estimated using the full data set with lag length($k$) arbitrarily set equal to 8. The unrestricted model($k=8$) then was tested against a restricted model where $k$ is reduced to 7. Then, unrestricted model($k=7$) was tested against a restricted model where $k$ is reduced to 6, and so on. The results of these tests are reported in Table 2.

We then test for cointegration by performing Johansen and Juselius(1990) version of cointegration test which provides a procedure to examine the ques-

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[2] The standardied drift was calculated by dividing the estimated intercept coefficient by the standard deviation.

[3] Since Schmidt calculated the critical values for the standard Dickey-Fuller test, applying this to the augmented test may not be completely correct. However, we follow this approach with the drift correction because the data are assumed to be generated by the random walk with drift.

tion of cointegration in a multivariate setting. The results of trace test are also reported in Table 2. The trace test allows us to evaluate the null hypothesis that there are r or fewer cointegrating vectors against a general alternative.

The reported test statistic[4] indicates that the model is specified with $k=8$ which indicates appropriate lag length. The results of the trace test on the specification with $k=8$ do not reject the null hypothesis of $r=0$, $r<=1$, and $r<=2$. Thus, these results refute the hypothesis that the variables are cointegrated.(see Table 2) Hence, the conventional VAR model can be appropriately specified in differenced series.

## III. VAR MODEL AND DIAGNOSTIC TESTS

The lag structure of the VAR model was specified with the aid of Schwarz Criterion(SC) based on Bayesian treatment. We investigated lag lengths up to 12 quarters in our VAR models. The specification for each equation with minimum SC is reflected in Table 3. SC indicates a specification for the VAR model that the growth rate of GDP depends on a lagged value of itself, and a lag of the inflation rate; the rate of GDPD depends on a lag of the growth rate of GDP, and three lags of the growth rate of GDPD; the M1 growth equation depends on a lag of the GDPD rate, and a lag of the M1 growth. (see Table 3)

Beside these specified VAR models, we attempted to specify models by adding intercept and slope dummies on money growth equations because there have been monetary regime changes in the third quarter of 1979 and in the third quarter of 1982.[5] This change is manifest in the response of money growth to lags of output growth, inflation, and money growth in the period after the change in operating procedure. Thus, we allowed parameters of money growth equation in the VAR model to exhibit discrete changes at these time periods by employing intercept and slope dummies.[6](see Table 3)

The Breusch-Godfrey test was conducted to test the lack of serial correlation for each equation. The hypothesis of no serial correlation on each equation could not be rejected versus alternative hypothesis of first through fourth order serial correlation at 5%.(see Table 4)

The existence of ARCH disturbances on each equation was investigated by using Engle's ARCH test. The tests indicate third through sixth order of autoregressive conditional heteroskedasticity(ARCH) are found in the GDPD

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[4] Likelihood ratio tests statistics are used for the test statistics.

[5] The Fed announced a change in operating procedure to a nonborrowed reserve instrument from the previously employed federal funds rate instrument.

[6] The intercept dummies take the value of one from the third quarter of 1979 and 1982 to the fourth quarter of 1992. The slope dummies take their own value from the third quarter of 1979 and 1982 to the fourth quarter of 1992.

equation. Also first through third order ARCH are found in the M1 growth equation.(see Table 4)

In the VAR model with dummies, the hypothesis of no serial correlation on each equation could not be rejected versus alternative hypothesis of first through fourth order serial correlation at 5%.(see Table 5) Also, the ARCH test results indicate that significant ARCH exists in third through sixth order in the inflation equation.(see Table 5)

## Ⅳ. SPECIFYING TRIVARIATE ARCH-M AND GARCH-M MODEL

We proceed to specify and estimate ARCH-M and GARCH-M models based on the results of the VAR specification. Since our primary interest is the effect of the monetary and the inflation uncertainty on real output growth and on inflation, we allow GDP growth and GDPD equation to be a function of conditional variances of M1 growth and GDPD equation that are specified through ARCH or GARCH specification. As a further check of the specification, we also allow conditional variances of M1 growth and GDPD to enter the GDP growth equation. Further, conditional variance of the GDP growth equation is included in each equation for the same specification reason. The trivariate ARCH-M model for real output growth, inflation, and money growth is as follows:

$$\Delta Y_t = c_y + a_{11}\,\Delta Y_{t-1} + b_{11}\,\Delta P_{t-1} + e_1 h_{yt}^{1/2} + f_1 h_{pt}^{1/2} + g_1 h_{mt}^{1/2} + \varepsilon_{y,t} \quad (1)$$

$$\Delta P_t = c_p + b_{21}\,\Delta P_{t-1} + b_{22}\,\Delta P_{t-2} + b_{23}\,\Delta P_{t-3} + e_2 h_{yt}^{1/2} + f_2 h_{pt}^{1/2} + g_2 h_{mt}^{1/2} + \varepsilon_{p,t} \quad (2)$$

$$\Delta M_t = c_m + \delta T + b_{31}\,\Delta P_{t-1} + c_{31}\,\Delta M_{t-1} + e_3 h_{yt}^{1/2} + f_3 h_{pt}^{1/2} + g_3 h_{mt}^{1/2} + \varepsilon_{m,t} \quad (3)$$

where, $\varepsilon_{it} / \psi_{i,t-1} \xrightarrow{N} (0,\ h_{it}),\ i = y,\ p,\ m,$

$$h_{yt} = \alpha_1^2 + \alpha_{11}^2 \left[\sum_{i=1}^{4} (5-i)\varepsilon_{y,t-i}^2 / 10\right] \quad (4)$$

$$h_{pt} = \alpha_2^2 + \alpha_{21}^2 \left[\sum_{i=1}^{4} (5-i)\varepsilon_{p,t-i}^2 / 10\right] \quad (5)$$

$$h_{mt} = \alpha_3^2 + \alpha_{31}^2 \left[\sum_{i=1}^{4} (5-i)\varepsilon_{m,t-i}^2 / 10\right] \quad (6)$$

and

$$h_t = \begin{bmatrix} h_{yt} & h_{yp} & h_{ym} \\ h_{py} & h_{pt} & h_{pm} \\ h_{my} & h_{mp} & h_{mt} \end{bmatrix}$$

Equation (1) describes the mean GDP growth as a function of a lag of GDP growth, a lag of GDPD, the square root of the conditional variances of GDP growth, GDPD, and M1 respectively. Equation (2) indicates that the mean GDPD is a function of three lags of GDPD, the square root of the conditional variances of GDP growth, GDPD, and M1 respectively. Equation (3) implies that the mean M1 growth is a function of a trend, a lag of GDPD, a lag of M1 growth, the square root of the conditional variances of GDP growth, GDPD, and M1 growth respectively. The equations (4), (5), and (6) are the ARCH(4) equations for the conditional variance of GDP growth, GDPD, and M1 growth respectively.[7] The non-negativity conditions are imposed on coefficients of each ARCH equation by squaring parameters to ensure positive variance. The above model allows the explicit incorporation of the square root of the conditional variances of monetary growth and inflation in the real output growth and inflation equations in order to directly test the effect of monetary and inflation uncertainty on real output growth and inflation.

The coefficient $e_1$ indicates the degree to which money growth uncertainty affects real output. The coefficient $f_1$ shows the degree to which inflation uncertainty affects real output. The coefficient $e_2$ displays the degree to which money growth uncertainty affects inflation. The coefficient $f_2$ shows the degree to which inflation uncertainty affects inflation. If there is a negative relationship between money growth uncertainty and real output growth, $e_1$ is expected to be a negative and statistically significant in equation (1). Also, if there is a negative relationship between inflation uncertainty and real output growth, $f_1$ is expected to be a negative and statistically significant in equation (1). In contrast, if inflation is positively related to money growth uncertainty, $e_2$ is expected to be a positive and statistically significant in equation (2). Also, if inflation is positively linked to inflation uncertainty, $f_2$ is expected to be negative and statistically significant in equation (2).

In our case, the variance process which is specified with a weighted average of four lagged squared realizations of the errors affects the conditional variance.[8] Thus the ARCH specification of the fourth order linear declining lag structure explicitly captures phenomenon like the observation that large disturbances tend to occur in clusters.

In the GARCH-M specification, the error variance process is specified with a lagged value of the squared realization of the error and a lagged value of

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[7] The ARCH specifications can be somewhat a restricted version suggested by Bolleslev due to the diagonal restriction on conditional variance equations.

[8] This type of specification is somewhat restrictive because it allows just one free parameter to be estimates on the four lagged squared residuals and imposes a linearly declining lag structure. However, it seems unlikely that numerical optimization of the more general model suggested by Bolleslev would be successful. Another specification that allowed each of the four lagged squared residuals to affect the conditional variance with separate coefficients was attempted. However, this specification was unsuccessful due to the nonpositive definite variance-covariance matrix.

the conditional variance. Thus this model allows the effect of the lagged conditional variance to persist over time. The postulated model consists of the same mean parts of the ARCH-M model and the different error variance process from that of the ARCH specification such as,

$$h_{yt} = \alpha_1^2 + \alpha_{11}^2 \varepsilon_{y,t-1}^2 + \alpha_{12}^2 h_{y,t-1}, \qquad (4)'$$

$$h_{pt} = \alpha_2^2 + \alpha_{21}^2 \varepsilon_{p,t-1}^2 + \alpha_{22}^2 h_{p,t-1}, \qquad (5)'$$

$$h_{mt} = \alpha_3^2 + \alpha_{31}^2 \varepsilon_{m,t-1}^2 + \alpha_{32}^2 h_{m,t-1}, \qquad (6)'$$

Beside these specified trivariate ARCH-M and GARCH-M models, as we mentioned in the above, we attempted to specify models by adding intercept and slope dummies on money growth equations to account for the two monetary regime changes in the third quarter of 1979 and in the third quarter of 1982. Thus the money growth equation of the ARCH-M and GARCH-M model is modified by including intercept and slope dummies. The modified version of the money growth equation(3) is as follows,

$$\Delta M_t = c_m + \delta T + b_{31} \Delta P_{t-1} + c_{31} \Delta M_{t-1} + d_1 DUM_{y1} + d_2 DUM_{y2} + d_3 DUM_{p1}$$
$$+ d_4 DUM_{p2} + d_5 DUM_{m1} + d_6 DUM_{m2} + e_3 h_{yt}^{1/2} + f_3 h_{pt}^{12} + g_3 h_{mt}^{\frac{1}{2}} + \varepsilon_{m,t}' \qquad (3)'$$

where

$DUM_{i1}$ = if time t is after the third quarter of 1979
= 0 otherwise,

and

$DUM_{i2}$ = 1 if time t is after the third quarter of 1982
= 0 otherwise.

Thus, we allowed parameters of the money growth equation in the ARCH-M and GARCH-M model to exhibit discrete changes at these time periods-the third quarter of 1979 and 1982 by employing intercept and slope dummies.

## V. MODEL SELECTION AND ESTIMATION RESULTS

After estimating these ARCH-M and GARCH-M models without or with dummies through maximum likelihood estimation, we selected the most appropriate model which has the significantly increased value of maximum likelihood, as the likelihood ratio tests were conducted for a restricted version of the model against a unrestricted version of the model.(e.g. the VAR model versus the unrestricted version of ARCH-M model) The test results are reported in Table 6.

We first conducted a likelihood ratio test on the VAR model against the VAR model with dummies(VARD). The likelihood ratio indicates that setting all coefficients of dummies in VARD to zero is not a valid restriction. Next, we tested if the homoskedastic VARD is a valid restriction to the ARCH-M and GARCH-M model. The null hypothesis of the homoskedatstic specification -VARD- cannot be rejected at any reasonable significance level in favor of the alternative hypothesis of the heteroskedastic specification- ARCH-M with dummies. However, the null hypothesis of the homoskedatstic specification- VARD is rejected at 1 percent level in favor of the alternative hypothesis of the heteroskedastic specification- GARCH-M with dummies. Also, the likelihood ratio shows that the ARCH-M with dummies is better specified than the ARCH-M model.(see Table 6) The conditional variance equations from GDP and GDPD equation in the GARCH-M with dummies exhibit significant a weighted average of four lagged squared realizations of the errors and the one lagged conditional variance. In spite of the significant lagged conditional variance, the conditional variance process for M1 growth exhibits insignificant and almost zero coefficient of the weighted average of four lagged squared realizations of the errors. This phenomenon implies that the conditional variance process is almost homoskedastic because it does not have much variation. Nevertheless, the conditional variances for GDP, GDPD, and M1 tend to persist over time.

The GARCH-M with dummies model turns out to be an appropriate model in order to investigate the negative relationship between real output, money growth uncertainty, and inflation uncertainty. At same time, this model shows the evidence of the relationship between inflation, monetary and inflation uncertainty. In the GDP equation, the money growth uncertainty has a significant negative impact on real output, while the inflation uncertainty has an insignificant negative coefficient. Also, the conditional variance from the output GARCH specification positively affects real output. This evidence supports our claim that monetary aggregate is a cleaner measure of monetary policy influence on real variables. In contrast, the estimated GDPD equation shows that the inflation uncertainty negatively affects inflation. In addition, the estimated coefficient of money growth uncertainty is a negative number even though it is insignificant.(see Table 7)

## Ⅵ. CONCLUSION

First, according to the Engle's ARCH test results for single equation, there exists ARCH effects in GDP equations. However, the M1 growth equation without dummies does have ARCH, while M1 growth equation with dummies hoo no ARCH.

We found that the money growth uncertainty, measured as a conditional variance from the M1 growth equation, negatively affects real output growth in the trivariate GARCH-M model of GDP, GDPD, and M1. Hence, the empirical evidence from observing GDP, GDPD, and M1 is to support our claim that monetary aggregate might be cleaner measure of monetary policy influence on real economic movement. The estimation results from the model show that money growth uncertainty is negatively linked to inflation. These results are somewhat unexpected and suprising. This phenomenon does not seem to be easily explained by macroeconomic theory if ARCH or GARCH specification is valid in defining uncertainty term. Also, money growth uncertainty tends to persist over time.

Hence, the inflation process might not be a robust measure of monetary policy influence on real economic activity due to its sluggishness in adjusting demand and supply shock. Financial variables- federal fund rates, commercial paper rates, and treasure bill rates- would be testable alternatives to inflation process.

Allowing parameters of the money growth equation in the ARCH-M and GARCH-M model to exhibit discrete changes at the time period -the third quarter of 1979 and the third quarter of 1982 brings richer specification to this study because the model selection test results consistently favor the model with dummies over the model without dummies. Since this dummies approach captures deterministic shifts in parameters of money equation, the stochastic regime switching model introduced by Hamilton might be an alternative to attack these issues.

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[**Table 1**] Trend and Unit Root Test Results.

Sample: I/1959-IV/1992

| Variable | DF test t-ratio | ADF test t-ratio | DF Trend test $\Phi_3$-statistics |
|---|---|---|---|
| ln(GDP) | −2.32 | −1.56 | |
| ln(GDPD) | 2.31 | −0.82 | |
| ln(M1) | 4.69 | 1.91 | |
| dln(GDP) | −8.50*** | −4.56*** | 2.47 |
| dln(GDPD) | −3.94*** | −2.08 | 2.57 |
| dln(M1) | −5.71*** | −2.95** | 7.25** |

Approximate critical values:

| Significance | Critical value for DF | Critical value for Trend DF |
|---|---|---|
| 10% | −2.58 | 5.47 |
| 5% | −2.89 | 6.49 |
| 1% | −3.17 | 8.73 |

[**Table 2**] Cointegration Results(GDP, GDPD, M1).

a. Lag specification results

| Unrestricted | Restricted | Test statistics |
|---|---|---|
| K=8 | K=7 | 21.19* |
| K=7 | K=6 | 16.71 |
| K=6 | K=5 | 22.61* |
| K=5 | K=4 | 8.48 |
| K=4 | K=3 | 16.88 |
| K=3 | K=2 | 20.18* |
| K=2 | K=1 | 89.10* |

Notes: The test statistic is distributed as a chisquare(9) with a 5 percent critical value. An asterisk denotes rejection of the restriction at the 5% level.

b. Test results for cointegration

| | | | Critical values | | |
|---|---|---|---|---|---|
| Lag length | Trace test | | 1%*** | 5%** | 10%* |
| K=8 | r=0 | 24.15 | 37.30 | 31.26 | 28.44 |
| | r<=1 | 10.13 | 21.96 | 17.80 | 15.58 |
| | r<=2 | 4.57 | 11.58 | 8.08 | 6.69 |

Notes: The critical values are taken from Johansen and Juselius(1990).

[Table 3] Trivariate VAR Model

a. VAR without Dummies

$$\Delta Y_t = 0.93 + 0.27\Delta Y_{t-1}^{***} - 0.34\Delta P_{t-1}^{***}$$
(4.76)(3.20) (−2.68)

$$\Delta P_t = 0.13 + 0.41\Delta P_{t-1}^{***} + 0.24\Delta P_{t-2}^{***} + 0.24\Delta P_{t-3}^{***}$$
(1.65) (4.73) (2.64) (2.78)

$$\Delta M_t = 0.61 + 0.01T^{**} - 0.10\Delta P_{t-1} + 0.50\Delta M_{t-1}^{***}$$
(2.88) (2.52) (−0.87) (6.42)

Value of Log-likelihood = −348.59

b. VAR with Dummies

$$\Delta Y_t = 0.91 + 0.28\Delta Y_{t-1}^{***} - 0.33\Delta P_{t-1}^{***}$$
(4.72) (3.39) (−2.65)

$$\Delta P_t = 0.13 + 0.41\Delta P_{t-1}^{***} + 0.25\Delta P_{t-2}^{***} + 0.24\Delta P_{t-3}^{***}$$
(1.63) (4.70) (2.69) (2.78)

$$\Delta M_t = 0.55 + 0.01T^{**} - 0.09\Delta P_{t-1} + 0.47\Delta M_{t-1}^{***} - 0.33DUM_{y1} + 1.43DUM_{y2}$$
(2.45) (1.42) (−0.48) (3.20) ( −0.31) (1.29)

$$+ 0.73DUM_{p1} - 2.36DUM_{p2} - 0.88DUM_{m1}^{***} + 0.97DUM_{m2}^{***}$$
(1.63) (−3.57) (−3.99) (−3.57)

Value of Log-likelihood = −327.01

Note: The asterisks indicate significance at 10%, 5%, and 1% level respectively.(The critical values are 1.64, 1.96, and 2.58)

[Table 4] Diagnostic Tests

a. Test for serial correlation

| | GDP | GDPD | M1 | Critical Value | | |
|---|---|---|---|---|---|---|
| Order | Test stat | Test stat | Test stat | 1%*** | 5%** | 10%* |
| 1 | 3.07* | 0.41 | 0.62 | 6.63 | 3.84 | 2.71 |
| 2 | 3.13 | 1.89 | 1.03 | 9.21 | 5.99 | 4.61 |
| 3 | 3.84 | 3.25 | 4.44 | 11.34 | 7.82 | 6.25 |
| 4 | 3.88 | 4.99 | 8.49* | 13.28 | 9.49 | 7.78 |

b. Test for ARCH single equation tests

| | GDP | GDPD | M1 | GDP & GDPD | GDP & M1 | GDPD & M1 |
|---|---|---|---|---|---|---|
| Order of ARCH | Test stat | Test stat | Test stat | Test stat | Test stat | Test stat |
| 1 | 0.06 | 0.05 | 5.51** | 2.12 | 5.70** | 0.00 |
| 2 | 0.83 | 0.93 | 5.58* | 2.12 | 5.89* | 1.81 |
| 3 | 1.12 | 10.81** | 6.38* | 3.26 | 7.20* | 3.16 |
| 4 | 1.36 | 10.83** | 6.77 | 3.45 | 7.57 | 5.12 |
| 5 | 1.86 | 11.07** | 8.31 | 3.63 | 7.78 | 5.15 |
| 6 | 2.33 | 10.92* | 9.54 | 3.63 | 8.13 | 6.16 |
| 7 | 3.33 | 10.94 | 10.47 | 3.70 | 9.25 | 6.50 |
| 8 | 10.92 | 11.23 | 12.14 | 3.67 | 10.11 | 6.83 |

Critical values

| Order | 1%*** | 5%** | 10%* |
|---|---|---|---|
| 1 | 6.63 | 3.84 | 2.71 |
| 2 | 9.21 | 5.99 | 4.61 |
| 3 | 11.34 | 7.82 | 6.25 |
| 4 | 13.28 | 9.49 | 7.78 |
| 5 | 15.09 | 11.07 | 9.24 |
| 6 | 16.81 | 12.59 | 10.64 |
| 7 | 18.48 | 14.07 | 12.02 |
| 8 | 20.09 | 15.51 | 13.36 |

[Table 5] Diagnostic Tests on VAR with Dummies.

a. Test for serial correlation

| | GDP | GDPD | M1 | Critical Value | | |
|---|---|---|---|---|---|---|
| Order | Test stat | Test stat | Test stat | 1%*** | 5%** | 10%* |
| 1 | 3.07* | 0.41 | 0.01 | 6.63 | 3.84 | 2.71 |
| 2 | 3.13 | 1.89 | 0.74 | 9.21 | 5.99 | 4.61 |
| 3 | 3.84 | 3.25 | 0.83 | 11.34 | 7.82 | 6.25 |
| 4 | 3.88 | 4.99 | 8.15* | 13.28 | 9.49 | 7.78 |

b. Test for ARCH single equation tests

| | GDP | GDPD | M1 | GDP & GDPD | GDP & M1 | GDPD & M1 |
|---|---|---|---|---|---|---|
| Order of ARCH | Test stat | Test stat | Test stat | Test stat | Test stat | Test stat |
| 1 | 0.06 | 0.05 | 0.21 | 2.12 | 0.23 | 0.64 |
| 2 | 0.83 | 0.93 | 1.85 | 2.12 | 6.14** | 1.11 |
| 3 | 1.12 | 10.81** | 2.02 | 3.26 | 7.23* | 3.20 |
| 4 | 1.36 | 10.83** | 2.10 | 3.45 | 7.64 | 6.81 |
| 5 | 1.86 | 11.07** | 2.14 | 3.63 | 7.65 | 6.68 |
| 6 | 2.33 | 10.92* | 2.09 | 3.63 | 9.71 | 6.79 |
| 7 | 3.33 | 10.94 | 2.08 | 3.70 | 10.64 | 7.32 |
| 8 | 10.92 | 11.23 | 4.26 | 3.67 | 11.36 | 9.79 |

Critical values

| Order | 1%*** | 5%** | 10%* |
|---|---|---|---|
| 1 | 6.63 | 3.84 | 2.71 |
| 2 | 9.21 | 5.99 | 4.61 |
| 3 | 11.34 | 7.82 | 6.25 |
| 4 | 13.28 | 9.49 | 7.78 |
| 5 | 15.09 | 11.07 | 9.24 |
| 6 | 16.81 | 12.59 | 10.64 |
| 7 | 18.48 | 14.07 | 12.02 |
| 8 | 20.09 | 15.51 | 13.36 |

[Table 6] Model Selection Test Results.

| Unrestricted | Restricted | Test Stat | Restrictions | 1% | 5% | 10% |
|---|---|---|---|---|---|---|
| VARD | VAR | 43.16*** | 6 | 16.81 | 12.59 | 10.64 |
| ARCH | VAR | 36.76*** | 12 | 26.22 | 21.03 | 18.55 |
| ARCHD | VAR | 53.54*** | 17 | 33.41 | 27.59 | 24.77 |
| GARCHD | VAR | 80.30*** | 21 | 38.93 | 32.67 | 29.62 |
| ARCHD | VARD | 10.38 | 11 | 24.72 | 19.68 | 17.28 |
| GARCHD | VARD | 37.14*** | 15 | 30.58 | 25.00 | 22.31 |
| ARCHD | ARCH | 16.78* | 10 | 23.21 | 18.31 | 15.99 |
| ARCH | ARCH1 | 3.72 | 5 | 15.09 | 11.07 | 9.24 |
| ARCHD | ARCHD1 | 6.33 | 10 | 23.21 | 18.31 | 15.99 |

Note: The test statistic is distributed as $x^2$(number of restrictions).
VAR=Vector Autoregression Model,
ARCH=Unrestricted ARCH-M Model,
ARCH1=Restricted ARCH-M Model,
GARCH=Unrestricted GARCH-M Model,
GARCH1=Restricted GARCH-M Model,
VARD=Vector Autoregression Model with dummies,
ARCHD=Unrestricted ARCH-M Model with dummies,
ARCHD1=Restricted ARCH-M Model with dummies,
GARCHD=Unrestricted GARCH-M Model with dummies,
GARCHD1=Restricted GARCH-M Model with dummies.

[Table 7] Estimation Results on Trivariate VAR GARCH-M[1] with Dummies

$$\Delta Y_t = 1.34 + 0.20\Delta Y_{t-1}^{**} - 0.54\Delta P_{t-1}^{***} - 1.36h_{yt}^{1/2**} - 0.88h_{pt}^{1/2} - 1.40h_{mt}^{1/2***}$$
$$(4.72)\quad(2.14)\quad(-3.51)\quad(-2.47)\quad(-1.22)\quad(-2.73)$$

$$\Delta P_t = 0.28 + 0.40\Delta P_{t-1}^{***} + 0.22\Delta P_{t-2}^{**} + 0.27P_{t-3}^{***} + 0.25h_{yt}^{1/2} - 0.57h_{pt}^{1/2**} - 0.26h_{mt}^{1/2}$$
$$(2.35)\quad(4.47)\quad(2.36)\quad(3.17)\quad(1.30)\quad(-2.06)\quad(-1.52)$$

$$\Delta M_t = -0.02 + 0.01T - 0.33\Delta P_{t-1}^{**} + 0.43\Delta M_{t-1}^{***} - 1.37DUM_{y1} + 2.49DUM_{y2}^{*}$$
$$(-0.08)\quad(0.97)\quad(-2.01)\quad(3.63)\quad(-1.18)\quad(1.85)$$
$$+ 1.15DUM_{p1}^{**} - 2.55DUM_{p2}^{***} - 0.77DUM_{m1}^{***} + 0.85DUM_{m2}^{***} + 0.70h_{yt}^{1/2**}$$
$$(2.29)\quad(-3.23)\quad(-3.55)\quad(3.89)\quad(1.29)$$
$$- 1.03h_{pt}^{1/2} - 0.26h_{mt}^{1/2}$$
$$(-1.54)\quad(-0.18)$$

$$h_{yt} = 0.01 + 0.12ey_{t-1}^{2***} + 0.88h_{yt-1}^{***}$$
$$(0.53)\quad(5.21)\quad(59.87)$$

$$h_{pt} = 0.00 + 0.42ep_{t-1}^{2***} + 0.54h_{pt-1}^{***}$$
$$(0.99)\quad(5.11)\quad(11.99)$$

$$h_{mt} = 0.01 + 0.00em_{t-1}^{2} + 0.98h_{pmt-1}^{***}$$
$$(6.68)\quad(0.00)\quad(181.35)$$

Value of Log−likelihood = −308.44

Note: The asterisks indicate significance at 10%, 5%, and 1% level respectively. (The critical values are 1.64, 1.96 and 2.58)