TAX EVASION AND PRICING SCHEMES FOR MONOPOLIST*

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This paper analyzes some aspects of decision making of a tax-evading monopolistic firm facing a uniform pricing and two part tariffs. It is shown that an imposition of tax, in the former case. is expected to increase social welfare, while the reverse is true in the latter. On the other hand, in both tariffs, it is also shown that an imposition of tax increases the output level and does not change the fraction of over-repored cost, while if the penalty fee is imposed on the firm. it decreases the fraction of over-reported cost as well as the firm's output level.

1. INTRODUCTION

The subject of tax evasion is an area on which a number of economists' attention has recently been focused. There have been voluminous literatures that are concerned with income tax evasion. Of which the most prominent ones are written by Kolm (1973), Sandmo (1981), and Yaniv (1990), since Allingham and Sandmo (1972) demonstrated the effect of income tax evasion.

Meanwhile, a pioneering study on the profit tax evasion in the context of a monopolistic firm was undertaken by Kreutzer and Lee (1986). They explored the possibility of using a profit tax to reduce monopoly distortion.[1] They argued that monopolists can reduce their tax liability by over-reporing the costs provided that the actual unreported costs are undetectable by the authority, and that this tax evasion

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[1] It is known that a tax on profits will have no effect on the output level of a profit maximizing monopolistic firm.

due to over-reported cost induces them to increase their output level.[2]

Wang and Conant (1988) derived results that invalidate the results of Kreutzer and Lee by introducing the assumption that the cost-over-reporing monopolist may be detected and penalized. They concluded that profit taxes are neutral with respect to a monopolist's profit maximizing rate of output under an uncertainty model.

Kreutzer and Lee (1988), however, replied that the results of Wang and Conant bias the solution toward reliance on the cost overstatement factor as the dominant control variable by adopting the assumption that the probability of detected tax evasion is independent of the cost overstatement factor. They concluded that a tax on profits will be an efficient instrument to expand the output level of a monopolistic firm under the realistic assumption that the probability of the under-reporting of profits being detected is dependent on the degree to which the costs are over-reporing.

The purpose of this paper is, in particular, to explore the effect of a profit tax and a penalty rate on the output level and the fraction of over-reported cost for the profit maximizing monopolistic firm facing two-part tariffs. It is well known that the monopolistic firm prefers to use two-part tariffs rather than charging a uniform price for output (see Oi, 1971). For example, in many countries the price schedule for a local telephone or electricity company has two-part tariffs under the monopoly. The main points of differentiation between our analysis and the one with Kreutzer and Lee (1988) are as follows: First, we analyze the effect of the penalty rate as well as that of the profit tax rate on the monopolistic firm. Secondly, we consider the case of two-part tariffs adopted by many monopolistic firms adding to a uniform pricing case. Thirdly, we compare the variation of social welfare due to tax imposition on both a uniform pricing and two-part tariffs.

The remainder of this paper is organized as follows. Section 2 analyzes the behavior of the profit maximizer facing a uniform pricing. In section 3, the behavior of the profit maximizer facing two-part tariffs is examined. Section 4 provides the summary.

II. A UNIFORM PRICING AND TAX EVASION

We consider the problem of decision making of a firm facing a uniform pricing under uncertainty. A monopolistic firm can evade profit tax liability by cost overstatement (δ), which are either discovered with probability z or remain undetected with probability $(1 - z)$. While the firm's after-tax profit, assuming the tax authority is unable to detect the over-reporing cost, is Π_b, the after-tax profit, assuming the authority concerned manages to detect the evasion, is Π_a. For the detectable case,

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[2] Marrelli (1984) analyzed a monopolistic firm's decision on whether or not, and to what extent to avoid an ad valorem tax by manipulating under-reporting revenues.

an imposition of a penalty fee $(f > 1)$ is due. It is assumed that this penalty increases the tax rate (t).

A profit tax requires that the enterprise pay the authority a specified proportion of the difference between the firm's total revenue and total cost. If the tax rate is flat, the firm's profit after tax payment when it over-reports its costs and is not detectable by the authority is given by

$$\Pi_1 = p(q)\,q - c(q) - t[p(q)\,q - (1 - \delta)c(q)],$$

where $p(q)$ is the inverse demand function for the monopoly output q and $c(q)$ is the monopolist's cost function. On the other hand, the firm's after-tax profit when its over-reporing cost is discovered by the authority is:

$$\Pi_2 = \Pi_1 - ft[\delta c(q)].$$

It is assumed that the probability of the under-reporting of profits being detected is a function of the cost over-reporing factor. The firm's problem is to select q and δ so as to maximize its expected profits;[3] which can be written as

$$MAX\ E = (1 - z(\delta))\Pi_1 + z(\delta)\Pi_2.$$

The first order necessary conditions for the expected profit maximization with respect to q and δ are

$$F_q = (1 - t)(MR - MC) + t\delta(1 - zf)MC = 0 \qquad (1)$$

and

$$E_\delta = tc[1 - f(z'\delta + z)] = 0, \qquad (2)$$

where *MC* and *MC* are marginal revenue and marginal cost, respectively, and where $(')$ is the first derivative and $('')$, the second derivative.

The second order sufficient conditions are assumed to be satisfied provided the firm's inverse demand and cost functions are linear:

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[3] Wang and Conant (1988) assumed that the preference function of a monopolist is given by a Von Neuman-Morgenstern utility function. Kreutzer and Lee (1988), however, introduced expected profit function instead of that, because they consider that expected profit maximization is superior to expected utility maximization in terms of realism. We also adopt the latter assumption.

$$E_{qq} = 2(1 - t)p' < 0$$

$$E_{\delta\delta} = - ft(z''\delta + 2z')c < 0,$$

and

$$D = \begin{vmatrix} E_{qq} & E_{q\delta} \\ E_{\delta q} & E_{\delta\delta} \end{vmatrix} > 0,$$

where the notation is that $E_{qq} = \partial^2 E / \partial q^2$ and similarly for $E_{q\delta}$, E_{qq} and E_{qq}, and where it is assumed that $z'(\delta) > 0$ and $z''(\delta) > 0$. On the other hand, we can get the following second-order derivatives for $E_{q\delta}$ and $E_{\delta q}$

$$E_{q\delta} = E_{\delta q} = t[1 - f(z'\delta + z)]MC = 0.$$

Since equations (1) and (2) do have continuous partial derivatives with respect to all the endogenous and exogenous variables, and since the relevant Jacobian deter-minant is always nonzero, we can take q and δ to be implicit functions of (f, t) at and around any point that satisfies equations (1) and (2). If we now hold all the ex-ogenous variables and parameters fixed except for t, then the following equation system will realize:

$$\begin{bmatrix} E_{qq} & E_{q\delta} \\ E_{\delta q} & E_{\delta\delta} \end{bmatrix} \begin{bmatrix} \partial q^*/\partial t \\ \partial \delta^*/\partial t \end{bmatrix} = - \begin{bmatrix} E_{qt} \\ E_{\delta t} \end{bmatrix},$$

from which two comparative-static derivatives with respect to t can be calculated. They will be come out to be

$$\frac{\partial q^*}{\partial t} = \frac{1}{D} \begin{vmatrix} -E_{qt} & E_{q\delta} \\ -E_{\delta t} & E_{\delta\delta} \end{vmatrix} > 0$$

and

$$\frac{\partial \delta^{*}}{\partial t} = \frac{1}{D} \begin{vmatrix} E_q & -E_{qt} \\ E_{\delta q} & -E_{\delta t} \end{vmatrix} = 0,$$

where $E_{qt} = -(MR - MC) + (1 - fz)^{2} MC/fz' > 0$ and $E_{\delta t} = [1 - f(z'\delta + z)]c = 0$.

Note that substituting $\delta^{*} = (f^{-1} - z)/z'$, obtained from (2), into (1) yields

$$MR - MC = -[t(1 - fz)^{2}/(1 - t)fz']MC < 0.$$

Intuitively, from the above equations, we can observe that the output level will be increased, but the fraction of over-reporing cost will be unchanged with the increase in the tax rate. Hence, we might conclude that it is possible for us to employ the profit taxation as an effective policy instrument to expand the output level, though it has a neutral effect on the fraction of over-reporing cost.

On the other hand, supposing all the exogenous variables and parameters are fixed except for f, we get the following equation system:

$$\begin{bmatrix} E_{qq} & E_{q\delta} \\ E_{\delta q} & E_{\delta \delta} \end{bmatrix} \begin{bmatrix} \partial q^{*}/\partial f \\ \partial \delta^{*}/\partial f \end{bmatrix} = - \begin{bmatrix} E_{qf} \\ E_{\delta f} \end{bmatrix},$$

from the above equation, we get the following equations:

$$\frac{\partial q^{*}}{\partial f} = \frac{1}{D} \begin{vmatrix} -E_{qf} & E_{q\delta} \\ -E_{\delta f} & E_{\delta \delta} \end{vmatrix} < 0$$

and

$$\frac{\partial \delta^{*}}{\partial f} = \frac{1}{D} \begin{vmatrix} E_{qq} & -E_{qf} \\ E_{\delta q} & -E_{\delta f} \end{vmatrix} < 0,$$

where $E_{qf} = -tz\delta MC < 0$ and $E_{\delta f} = -t(z'\delta + z)c < 0$.

Froms the preceding equations, we can see that the output level and the fraction of over-reporing cost are decreased with the increase in the rate of penalty fee.

III. TWO-PART TARIFFS AND TAX EVASION

We shall, at this juncture, consider the monopolistic firm faces two-part tariffs wherein the firm charges consumers a constant usage charge, p, per unit purchased and a fixed charge, l, per period for the right to purchase at price p. For simplicity, it is assumed that both p and l are the same for all consumers and that the firm can extract consumer's surplus completely as a fixed charge. This discriminating two-part tariff is equivalent to Pigou's perfect first-degree price discrimination structure, which maximizes monopoly profits by absorbing all consumers' surpluses. We assume that consumer's surplus, $u(q)$, is continuous, has continuous first and second order partial derivatives, and is a strictly quasi-concave function with respect to q. By Roy's identity, the demand function, q, is derived by $q = -\partial l/\partial p$.

Applying the assumptions adopted in the case of the uniform pricing into the case of two-part tariffs, the expected profits of the monopolist will be given by

$$V = (1 - z(\delta))[l(p(q)) + p(q)q - c(q) - t\{l(p(q)) + p(q)q - (1 + \delta)c(q)\}]$$
$$+ z(\delta)[l(p(q)) + p(q)q - c(q) - t\{l(p(q)) + p(q)q - (1 + \delta)c(q)\} - ft\delta c(q)].$$

The optimal output level and the fraction of over-reporing cost, which maximize the above equation, necessarily satisfy the following conditions:

$$V_q = (1 - t)(p - MC) + t\delta(1 - zf)MC = 0 \qquad (3)$$

and

$$V_\delta = tc[1 - f(z'\delta + z)] = 0. \qquad (4)$$

We obtain $\delta^* = (f^{-1} - z)/z'$ from (4). Substituting the value of δ^* into (3), and rearranging yields

$$p - MC = -\left(\frac{t}{1 - t}\right)\frac{(1 - zf)}{fz}MC.$$

$p - MC$ will be negative, since $0 < t < 1$ and $z' > 0$. This implies that the after-tax profit maximizing output, q^*, will be larger than the output which would maximize

profits in the absence of a tax.

Having got this result, one can make a comparison with the case of firm facing a uniform pricing. It is clear from the above expositions that, under the absence of a tax, the firm facing two-part tariffs will maximize profits at $p = MC$, while profits will be maximized at $MR = MC$ for the firm facing a uniform pricing. The former will produce more goods than the latter does. Let us consider a case whereby a profit tax is imposed on each firm. Given this condition, if an attempt is made to compare the behavior of the firm facing two-part tariffs with the one facing a uniform pricing, the former will expand the output level from q_2^* towards the right hand side as shown in Figure 1 where the fixed charge is increased at the expense of the usage charge, while the latter will expand the output level from q_1^* towards the right hand side where the usage charge is decreased. On the other hand, an increase of output due to tax imposition will, in the two-part tariffs, decrease social welfare, meanwhile the reverse is true in the case of a uniform pricing.[4]

[**Figure 1**] Uniform pricing, two-part tariffs, and social welfare

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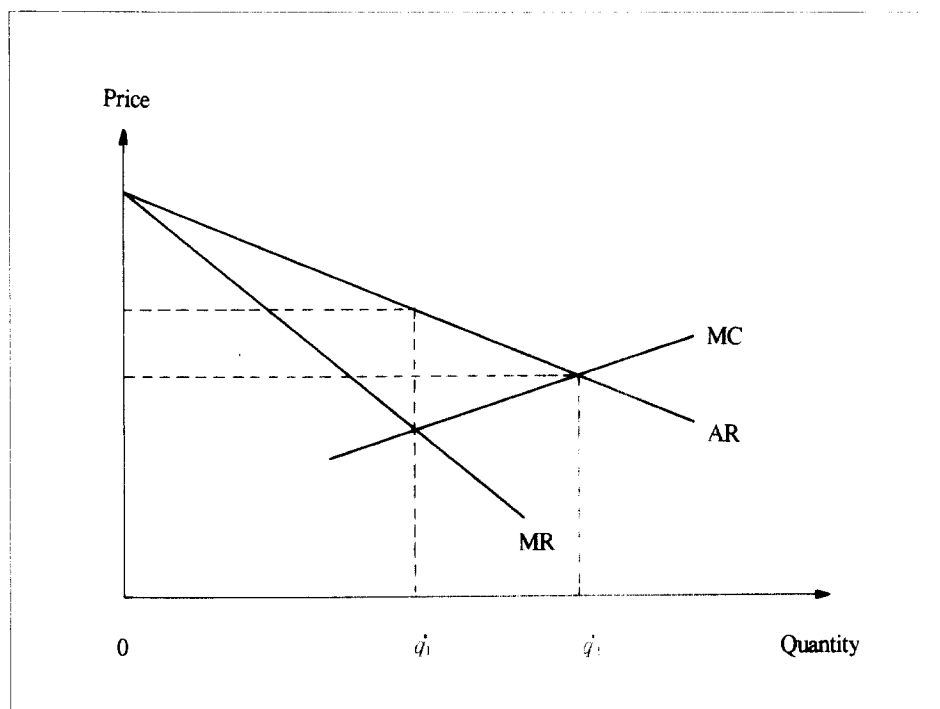
[4] Social welfare is defined as the sum of consumer surplus plus profits. To justify using consumer's surplus as a partial measure of social welfare, we assume that a population of consumers each of whom has a quasi-linear utility function in the good produced by the firm and the numeraire commodity. To simplify the welfare analysis, we assume that all consumers are identical in tastes and endowments.

Given the assumption that sufficient conditions are satisfied, the necessary conditions that define $\dot{q}$ and δ^* as functions of t and f. Utilizing the comparative-static analysis, we get

$$\frac{\partial q^*}{\partial t} > 0, \quad \frac{\partial \delta^*}{\partial t} = 0, \quad \frac{\partial q^*}{\partial f} < 0, \text{ and } \frac{\partial \delta^*}{\partial f} < 0.$$

We find that an increase in the tax rate will increase the output level, but have no effect on the fraction of over-reporing cost, by the same token an increase in the penalty rate lessens the output level and the fraction of over-reporing cost. These results are similar to the case of a uniform pricing. See the Appendix for the proof.

IV. CONCLUDING REMARKS

This paper has examined how the output and the fraction of over-reporing cost for the monopoly facing two-part tariffs under uncertainty are influenced by tax and penalty rates levied by the tax authority. We have shown that the tax imposition will encourage the monopolist to increase the output level, but will not change the fraction of over-reporing cost. It has also been shown that raising the penalty rate motivates the monopolist to decrease the fraction of over-reported cost as well as the output level. We have driven the same results for a monopolist facing a uniform pricing under uncertainty except for the effect of tax on social welfare and pricing: the tax imposition decreases the usage charge and increases social welfare in a uniform pricing, but decreases the usage charge, increases the fixed charge, and decreases social welfare in two-part tariffs.

APPENDIX

The first order conditions for V are satisfied in view of (3) and (4). The second order conditions, which are assumed to be satisfied, are denoted by

$$V_{qq} = (1-t)p' < 0$$

$$V_{\delta\delta} = -ft(z''\delta + 2z')c < 0$$

$$D' = \begin{vmatrix} V_{qq} & V_{q\delta} \\ V_{\delta q} & V_{\delta\delta} \end{vmatrix} > 0$$

where $V_{q\delta} = V_{\delta q} = t[1 - f(z'\delta + z)]MC = 0.$

Since second order conditions enable us to find the derivatives of implicit function, we can take q and δ to be implicit functions of (s, t) at and around any point that satisfies equations (3) and (4). If we hold all the exogenous variables and parameters fixed except for t, we get the following equation system:

$$\begin{bmatrix} V_{qq} & V_{q\delta} \\ V_{\delta q} & V_{\delta\delta} \end{bmatrix} \begin{bmatrix} \partial q^*/\partial t \\ \partial \delta^*/\partial t \end{bmatrix} = - \begin{bmatrix} V_{qt} \\ V_{\delta t} \end{bmatrix},$$

By the comparative-static analysis, the above equation system will be come out to be

$$\frac{\partial q^*}{\partial t} = \frac{1}{D'} \begin{vmatrix} -V_{qt} & V_{q\delta} \\ -V_{\delta t} & V_{\delta\delta} \end{vmatrix} > 0$$

and

$$\frac{\partial \delta^*}{\partial t} = \frac{1}{D'} \begin{vmatrix} V_{qq} & -V_{qt} \\ V_{\delta q} & -V_{\delta t} \end{vmatrix} = 0,$$

where $V_{qt} = -(p - MC) + (1 - f\bar{z})^2 MC/f\bar{z} > 0$ and $V_{\delta t} = [1 - f(\bar{z}\delta + z)]c = 0$.
Note that substituting $\delta^* = (f - z)/\bar{z}$ obtained from (4) into (3) yields

$$ p - MC = -\left(\frac{t}{1-t}\right)\frac{(1 - zf)^2}{f\bar{z}} MC $$

On the other hand, supposing all the exogenous variables and parameters are
fixed except for f, we get the following equation system:

$$ \begin{bmatrix} V_{qq} & V_{q\delta} \\ V_{\delta q} & V_{\delta\delta} \end{bmatrix} \begin{bmatrix} \partial q^*/\partial f \\ \partial \delta^*/\partial f \end{bmatrix} = - \begin{bmatrix} V_{qf} \\ V_{\delta f} \end{bmatrix}, $$

from the above system, we get the following equations:

$$ \frac{\partial q^*}{\partial f} = \frac{1}{D'} \begin{vmatrix} -V_{qf} & V_{q\delta} \\ -V_{\delta f} & V_{\delta\delta} \end{vmatrix} < 0 $$

and

$$ \frac{\partial \delta^*}{\partial f} = \frac{1}{D'} \begin{vmatrix} V_{qq} & -V_{qf} \\ V_{\delta q} & -V_{\delta f} \end{vmatrix} < 0, $$

where $V_{qf} = --t z\delta MC < 0$ and $V_{\delta f} = -t(\bar{z}\delta + z)c < 0$.

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