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ASYMMETRIC FLUCTUATIONS WITH A STOCHASTIC GROWTH

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Assuming a stochastic trend, the asymmetric structure of business cycles is examined whether downturns are less frequent and more severe than upturns. The results indicate that the old observation about asymmetric fluctuations is empirically true while for some variables, asymmetric frequency is found to be weak. We examine the possibility that an equilibrium business cycle model subject to a stochastic growth assumption can generate the observed third moments. The results are not satisfactory in matching the third moments and suggest a direction for future research of real business cycle theory.

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I. INTRODUCTION

This paper aims to discover the asymmetric structure of cyclical fluctuations under the assumption of stochastic growth, and explain it in the equilibrium business cycle model. The basic view is that downturns are brief and severe while upturns are longer and gradual, or contractions are more violent, but tend to last for a shorter period.[1] "Being more severe" means large decreases during the downturn

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[1] Probably, under some conditions, "more severe" may mean "brief" or "short", for which a formal proof requires a substantial work, but an intuitive explanation is possible. Other authors looks at different asymmetric structure:Turning point asymmetry (Keynes)=the sudden and violent substitution of a downward for an upward tendency, but no such sharp turning point when a upward is substituted for an downward tendency (Keynes), Dynarski and Sheffrin (1986)=Gradual upward slopes during expansions and steep downward slopes during recessions, deepness asymmetry (Sichel (1989))=Relatively deep troughs and low peaks, the asymmetry of duration (Keynes) and Falk

and small increases during the upturn (for a pro-cyclical variable).[2] The above statement implies that business cycles are asymmetric in two aspects of frequency (less frequent downturns than upturns) and amplitude (more severe downturns than upturns). The asymmetric fluctuations imply asymmetric distribution of the variable, which is measured by a nonzero third moment. While the notion of asymmetric fluctuations is a very old one, the topic has not been fully discussed so far, especially, in the context of equilibrium business cycle theory. Recently, economists have given more attention to the asymmetric fluctuations of economic variables over business cycles. For examples among others, see Neftci (1984), Falk (1986), Brock and Sayer (1988), Dynarski and Sheffrin (1986), Pfann (1991) and McQueen and Thorley (1993). One finding common to those studies is that there are sort of asymmetric and nonlinear movements in the business cycles.

The contribution of this paper is to discover the asymmetric structure (and its significance) and explain it in the real business cycle model. Importantly, we assume no deterministic trend, but a stochastic trend. It means that we need a model implying a stochastic growth. To begin, in an effort to provide the empirical evidence of the asymmetric fluctuations, this paper looks at the major cyclical variables to assess the above observation about the asymmetry of business cycles. The variables examined are output, consumption, investment, total hours worked, real wage and capital stock. First, in examining whether downturns are more frequent or not, the sample frequencies of upturns and downturns of variables are estimated. Secondly, in measeasuring the asymmetric frequency, transition probabilities over economic fluctuations are estimated. Thirdly, the question of whether downturns are more severe or not is examined by estimating the sample amplitudes of all variables during the expansion and contraction. The asymmetric amplitude is measured by ratio of the estimated means of the negative to the positive movements (i. e., in terms of the first moments) and by the ratio of the estimated variances of the negative to the positive movements (i.e., in terms of the second moments). They are supposed to be greater than 1. Finally, one statistics useful for measurement of asymmetric business cycles is the third moment, more precisely, the coefficient of skewness which is supposed to reflect the asymmetric frequency and amplitude together (In this paper, the third moment and skewness coefficient are used equivalently). We estimate skewness coefficient and interpret it in terms of asymmetric business cycles. After measuring the asymmetry using third moments, we address the question of whether a business cycle (RBC) model can mimic the observed third moments. For this experiment, we choose the real business cycle model with a stochastic growth. The finite sampling distribution of the population third moments is also provided by a simulation experiment.

(1986))=shorter contraction than the expansion. This paper concentrates on the asymmetric frequency and amplitude.

[2] Such a movement may be called an upward rigidity in this paper. If a variable is countercyclical, then, it means the inverse asymmetry of small decreases and large increases. In this paper, discussions are based on the pro-cyclical variables unless otherwise specified.

The empirical results do show the asymmetry of more frequent upturns than downturns. They also show that downturns are more severe, i.e., there are asymmetries of large decreases and small increases in the fluctuations of all variables. And the estimated skewness are always negative, as predicted. We find, however, that the population third moments implied by the model do not match the actually observed third moment. And a simulation experiment shows that the difference between the population and actual moments may not be attributed to the finite sampling error. In sum, a real business cycle model subject to a stochastic growth is not successful in matching the third moments.

The remainder of the paper is organized as follows. Section II explains different components in the asymmetric business cycles and methodology for the empirical verification works. In section III, the sample frequencies and amplitudes of major variables are estimated and interpreted. In section IV, a business cycle model with a stochastic growth is introduced, solved and calibrated to calculate the population third moments. In section V, the calculated population third moments are provided and compared with the actually observed ones. We also report the result of simulation experiment, establishing the finite sampling distribution property of third moments. In section VI, concluding remarks close the study.

II. THE STRUCTURE OF ASYMMETRICAL FLUCTUATIONS: METHODS

As explained above, the hypothesized observation about asymmetric cycles implies two different things. These asymmetries can be written in terms of probability structure as follows. First, the observation that downturns are less frequent than upturns means

$$P[Z(t) - Z(t-1) \leq 0] \leq P[Z(t) - Z(t-1) \geq 0], \quad \text{or} \tag{1}$$

$$\pi(+/-) = P[Z(t) - Z(t-1) \geq 0]/P[Z(t) - Z(t-1) \leq 0] \geq 1. \tag{1$'$}$$

where P is a probability measure, t denotes time. $Z(t)$ is logged value of a pro-cyclical variable. Thus, an upturn (downturn) is the period when $Z(t) - Z(t-1) \geq (\leq) 0$. (Note that it is the first difference in logarithm).

Empirically, $\pi(+/-)$ is the relative frequency of good and bad times. As a more statistically rigorous method for the estimation of the sample frequency, the method developed by Neftci (1984) can be also used to examine the asymmetry of relative frequency. This method is basically a refinement of the above simple method estimating the relative frequency. He writes the asymmetry of relative frequency using the theory of finite Markov process. To begin, define another variable as

$$I(t) = 1 \text{ or } -1 \text{ if } Z(t)-Z(t-1)\geq 0 \text{ or } \leq 0. \tag{2}$$

Then, assume that the sequence $\{I(t)\}$ is a stationary stochastic process representable as a second-order Markov process. And define the transition probabilities as follows:

$$\begin{aligned} \Gamma_{11} &= P[I(t)=1 \mid I(t-1)=I(t-2)=1] \text{ and} \\ \Gamma_{00} &= P[I(t)=-1 \mid I(t-1)=I(t-2)=-1]. \end{aligned} \tag{3}$$

The asymmetric business cycle means that Γ_{00} is not equal to Γ_{11}. Specifically, the above hypothesized asymmetry of more frequent upswings can be written as $\Gamma_{00} \leq \Gamma_{11}$ (for a procyclical variable). Note that the interpretation by Neftci (1984) about the asymmetric business cycles is a little different from that chosen by this paper. Neftci (1984) does not distinguish two different asymmetries of frequency and amplitude. He uses a different word like "persistent". So, the asymmetric business cycles means that a procyclical variable tends to show persistent positive runs. As proven in table 1, however, once we distinguish two different asymmetries, the Neftci (1984)'s method is to measure the asymmetric frequency (thus, maybe the asymmetric duration, but not the asymmetric amplitude). Γ_{00} and Γ_{11} are estimated maximizing the log-likelihood function corresponding to a given realization of $\{I(t)\}$.

$$\begin{aligned} L(S_T, \Gamma_{ij}, \pi_0) = \log\pi_0 + T_{11}\log\Gamma_{11} + S_{11}\log(1-\Gamma_{11}) + T_{00}\log\Gamma_{00} + S_{00}\log(1-\Gamma_{00}) \\ + T_{10}\log\Gamma_{10} + S_{10}\log(1-\Gamma_{10}) + T_{01}\log\Gamma_{01} + S_{01}\log(1-\Gamma_{01}) \end{aligned} \tag{4}$$

where S_T is the realization of $\{I(t)\}$, $\Gamma_{10} = P[I(t)=1 \mid I(t-1)=1, I(t-2)=-1]$, $\Gamma_{01} = P[I(t)=-1 \mid I(t-1)=-1, I(t-2)=1]$, π_0 is the probability of the initial state, and $T_{11},\cdots\cdots,S_{01}$ are the numbers of occurrences associated with the corresponding transition probabilities respectively. So, $1-\Gamma_{00} = P[I(t)=1 \mid I(t-1)=I(t-2)=-1]$ and $1-\Gamma_{11} = P[I(t)=-1 \mid I(t-1)=I(t-2)=1]$. Ignoring the information in the initial condition π_0,[3] the maximum likelihood estimates of transition probabilities are as follow

$$\Gamma_{00} = T_{00}/[S_{00}+T_{00}] \text{ and } \Gamma_{11} = T_{11}/[S_{11}+T_{11}] \tag{5}$$

The (approximate) variance and covariance matrix of the above estimates are the

[3] Falk (1986) shows that the estimates are the same without regard to the consideration of π_0. One justification for this ignorance of the initial state can be the large number of sample observations. See Neftci (1984) and its reference for this argument and the derivation of the first and second order conditions of maximizing the likelihood functions.

values of the second partial derivative of the likelihood function with respect to Γ_{00} and Γ_{11}, evaluated at the estimated values of Γ_{ij}s $(i,j=0,1)$.[4] The second asymmetry about the relative strength of "more severe downturns" may be quantified by the relative sizes of variations of each variable during upswings and downswings. The large decreases during downturns and small increases during upturns mean that

$$D^{+}[Z(t+i+1)-Z(t+i)] \leq D^{-}[Z(t+j+1)-Z(t+j)] \text{ for } i,j=1,\ 2,\ 3,\cdots. \quad (7)$$

where $D^{+}[\ ]$ is difference between $Z(t+i+1)$ and $Z(t+i)$ when $Z(t+i+1)\leq Z(t+i)$ and $D^{-}[\ ]$ is that when $Z(t+j+1)\leq Z(t+j)$. Note that $D^{+}[\ ]$ and $D^{-}[\ ]$ are random variables with different density functions. The expectation corresponding to (7) means

$$\mu(-/+)=E\{D^{-}[\ ] \mid Z(t+j+1)\leq Z(t+j)\}/E\{D^{+}[\ ] \mid Z(t+i+1)\geq Z(t+i)\}\geq 1. \quad (8)$$

The asymmetric amplitudes over the cycles can be also examined in terms of second moments. (7) may be written as (see Dynarski and Sheffrin (1986))

$$\sigma(-/+)=\sigma\{D^{-}[\ ]\}/\sigma\{D^{+}[\ ]\} \quad (9)$$

where $\sigma\{X\}$ denotes the variance of X. In the next section, two different asymmetries are examined by estimating (1)$'$, (8) and (9). Finally, the asymmetric business cycles may be summarized by the third moments, called the coefficient of skewness. Applied on a time-series of asymmetric movements explained above, the coefficients of skewness are supposed to be non-zero, specially to be negative.[5]

$$\phi\{D[Z(t)]\}\leq 0. \quad (10)$$

where ϕ denotes skewness coefficient and $D[Z(t)]=Z(t)-Z(t-1)$.

So far, several statistics have been defined in this section. One may be interested in question whether third moments, (10) can summarize all information in other statistics. While the definition of each statistics is clear and simple, it is not so to

[4] Thus, another way of testing the hypothesis is to look at the likelihood ratio test to test the restriction that $\Gamma_{00}=\Gamma_{11}$. Under the null hypothesis that, $\Gamma_{00}=\Gamma_{11}$, $2(\mid L_r\mid - \mid L_u\mid)$ may have a X^2 (1) distribution, where L denotesthe log-likelihood function, r and u denote the restricted and unrestricted cases respectively.

[5] DeLong and Summers(1986), for example, measures skewness to estimate the asymmetric business cycles. They presume that skewness coefficient measures the asymmetric structure of business cycles, which is "brief and severe" downturn.

relate them each other. So, this question is addressed empirically in next section.

III. EMPIRICAL EVIDENCES OF ASYMMETRICAL FLUCTUATIONS

This section reports the empirical results following the definitions in section 2.[6] With the data, we estimate the statistics on both the original series(=the first difference in logarithm) and its transformed series. The latter series means the transformed series of the first differenced data so that they are serially independent. The serially independent series is necessary to apply test statistics (See Dynarski and Sheffrin(1986) for this procedure). For transformation, we specify and estimate AR(I) for each series where I is chosen so that the residual is White noise. AR (2), AR(1), AR(2), AR(2), AR(1) and AR(3) are specified and estimated for output (=y), consumption(=c), investment(=i), hours(=n), real wages(=w) and capital stock(=k) respectively. Durbin-Watson for the regression of generating the serially independent series were 1.98, 2.06, 2.00, 2.02, 2.00 and 2.03 for y, c, i, n, w and k respectively.

3.1. Empirical Results: Asymmetric Frequency

The estimation results about the asymmetric frequency are in Table 1. Overall, the results in Table 1 show the asymmetry of more frequent upturns and less frequent downturns for all variables. For all variables,

$$\pi(+/-)\geq 1 \text{ and } \pi^T(+/-)\geq 1.^{7)} \text{ and (except real wages) } \Gamma_{11} \geq \Gamma_{00}.$$

In the above discussion, two methods for the estimation of the asymmetric frequency have been introduced. Note that two methods are basically the same. They make the same results empirically, as seen in Table 1. One can see that (except real wage) $\pi(+/-)\geq 1$ if $\Gamma_{11} \geq \Gamma_{00}$.

3.2. Empirical Results: Asymmetric Amplitude

The ratios of the negative to the positive movement measured in terms of the first and second moments are in Tables 2. The results are consistent with the inequalities of (8) and (9). They show that all variables experience the asymmetric am-

[6] The data examined are U.S.A. data which are quarterly and seasonally adjusted for the sample period of 1953-1984, which are collected from Citibase data set. Thee observational period is chosen for a convenience of data collection. For output, gross national product (GNP82; code number in Citibase tape) is used and personal consumption expenditure (GC82) are consumption data. Hours are total employee hours worked in non-agricultural establishment (LPMHU). The total private investment (GPI82) is used for the investment, from which the capital stock is measured. Hourly compensation for all employees on nonagricultural payrolls (LPCNAG) is chosen for the wage.

[7] Hereafter, supscript T denotes the transformed series.

plitude of large decreases and small increases, or upward rigidity: $\mu(-/+)$ and $F_\sigma(-/+)\geq 1$. Thus, the asymmetry hypothesis about amplitude is consistent with data. Among the variables, the asymmetry is stronger in investment, consumption, total hours worked and employment than other variables like real wages and output. The test statistics of the ratios of second moments is F_σ $(-/+)$ which is the ratio of negative movement to positive movement of transformed series.[8] With F_σ $(-/+)$, we reject the null at 5 percent level for all variables (except for real wages) and reject it at 1 percent level for consumption, investment, hours and capital.

3.3. Empirical Results: Third Moments

Table 3 reports the skewness measurement. They show the negative value for all variables, meaning the left skewness of the variable: ϕ and $\phi_r' \leq 0$. Empirically, the interpretation of skewness in terms of two asymmetries seems clear because of consistent relationship among skewness, asymmetric amplitude and asymmetric frequency. One can see that

$$\phi \leq 0 \text{ with } \mu(-/+)\geq 1,\ \sigma(-/+)\geq 1 \text{ and } \pi(+/-)\geq 1^{9} \tag{11}$$

To examine the significance of estimated third moments, we apply three test procedures. First one is to compare the estimated third moments to the null distribution of third moments. The latter is constructed using the samples from the normal population. This procedure has been well developed in statistics literature. (See Sach (1982), D'Agostino and Tietjen (1973), Dynarski and Sheffrin (1986).) According to the null distribution, the 10 percentage point is 0.27 and 5 percentage point is 0.35. The estimated third moments (from transformed series) of all variables are greater than 0.27 (except for real wages). They are greater than 0.35 for consumption, investment, hours and capital stock. Certainly, we reject the null of symmetric distribution based on this procedure. Now consider t and t. They are the same as the usual t statistics, applied on the third moments.[10] The difference between t and t is that the former assumes the normal population and the latter does not assume it. Based on them, we can reject the null of symmetric distribution at 10 percent level (with the critical value of 1.28 from standard normal distribution) except for real wages. The null is also rejected at 5 percent (1.65) for consumption, investment, hours and capital stock.

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[8] There is no good test statistics for the ratio of the first moments because of the well known problem in testing the equality of means with unknown and unequal variances (This is known as the Behrens isher problem).

[9] Based on this observation, we can focus on the third moment characteristics of economic fluctuations in the simulation experiment of RBC model in sections 4 and 5.

[10] See Kendall and Stuart (1963) and D'Agostiano and Tietjen (1973).

IV. THE REAL BUSINESS CYCLES WITH A STOCHASTIC GROWTH

Here, we address the question of whether real business cycle model can mimic the above summarized asymmetry. Since we are looking at the asymmetry of the first difference in logarithm, we choose the real business cycle model with a stochastic growth. According to King et al. (1988), the neoclassical business cycle model with a stochastic growth implies that the first difference is relevant in examining the business cycles. In this section, the model is introduced with the equilibrium features.

4.1. Economic Environment

It is assumed that the household has a following utility function defined over two goods of consumption and leisure.

$$U(C,\ L) = log(C(t)) + c_1 v(L(t)) \tag{12}$$

where $C(t)$ is consumption and $L(t)$ is leisure. The utility function is increasing and concave with respect to $C(t)$ and $L(t)$.[11] The production technology is Cobb-Douglass

$$Y = F(N,\ X,\ K) = K(t)^{(1-\alpha)}\{N(t)X(t)\}^{\alpha} \tag{13}$$

where $N(t)$ is the labor input at time t, $K(t)$ is the capital stock available at time t which is predetermined at $t-1$. And $0 \leq \alpha \leq 1$. Note that there is no temporary change in productivity. $X(t)$ makes the permanent changes in the production technology. $X(t)$ is not deterministic, but stochastic, the growth of which is governed by

$$log(r_x) = logX(t) - logX(t-1) - e(t) \tag{14}$$

where e(t) is i.i.d. and r_x is the gross growth rate of $X(t)$. Both inputs of labor and physical goods have following constraints and transition path. They are written as following. In (17), $I(t)$ is gross investment accumulating the capital stock and is the constant rate of capital depreciation.

[11] See the empirical section for the more concrete specification of $v(L(t))$. We specify the elasticity of leisure with respect to real wage in section 5. Here, the general representation in (12) is enough.

$$L(t)+N(t)\leq 1,\ L(t)\geq 0,\ N(t)\geq 0 \tag{15}$$

$$C(t)+I(t)\leq Y(t),\ C(t)\geq 0,\ I(t)\geq 0 \tag{16}$$

$$K(t+1)=(1-\delta_k)K(t)+I(t),\ K(t)\geq 0 \tag{17}$$

Equilibrium Paths

The equilibrium of this model economy can be characterized by a dynamic programing problem, the solution of which is the equilibrium paths of endogenous variables. KPR (1988) shows that the approximate solutions are following.

$$z(t)=\pi_{zk}k(t),\ z=y,\ c,\ i,\ N \text{ and } w. \tag{18}$$

$$k(t)=\pi_{kk}k(t-1)-e(t) \tag{19}$$

where lower case denotes the percentage deviations from the stationary values (except N): e.g., $y(t)=log[(Y(t)/X(t))/y]$ where y is stationary value. The coefficients π_{zk} and π_{kk} are the functions of preference, technology and budget constraint parameters. Returning to the original variables with a stochastic growth, they look like following.

$$logZ(t)=logX(t)+log(Z)+z(t),\ \text{for } Z=Y,\ C,\ I,\ W \text{ and } K. \tag{20}$$

where $log(Z)$ is stationary value. With (14) and (20), it is easy to show that removing a deterministic trend does not make a variable stationary.

4.2. Measurement of Third Moments of the Equilibrium Paths

With the stochastic growth specification, one way of rendering the variables stationary is taking the first difference in logarithm.

$$DlogZ(t)=log(r_x)+(1-\pi_{zk})e(t)+\ \pi_{zk}(\pi_{kk}-1)k(t-1), \tag{21}$$
$$Z=Y,\ C,\ I \text{ and } W.$$

$$DlogK(t)=log(r_x)+(\pi_{kk}-1)k(t-1). \tag{22}$$

$$DN(t)=-\pi_{Nk}e(t)+\pi_{Nk}(\pi_{kk}-1)k(t-1). \tag{23}$$

To calculate third moments, we write each of them as a Moving Average (MA) model. (In the below, B is a backshift operator and constant term is ignored).

$$DlogZ(t) = (1-\pi_{zk})e(t) - \pi_{zk}(\pi_{kk}-1)\{1/(1-\pi_{kk}B)\}e(t-1) \qquad (21)'$$

$$DlogK(t) = (1-\pi_{kk})\{1/(1-\}\{1/(1-\pi_{kk}B)\}e(t-1) \qquad (22)'$$

$$DN(t) = -\pi_{Nk}e(t) + \pi_{Nk}(1-\pi_{kk})\{1/(1-\pi_{kk}B)e(t-1). \qquad (23)'$$

The third moments of these variables can be calculated easily as,

$$\Phi^T_{Dz} = [(1-\pi_{zk})^3 - \{\pi_{zk}^3(\pi_{kk}-1)^3/(1-\pi_{kk}^3)\}]E[e(t)^3]. \qquad (24)$$

$$\Phi^T_{Dk} = [-(\pi_{kk}-1)^3/(1-\pi_{kk}^3)\}]E[e(t)^3]. \qquad (25)$$

$$\Phi^T_{DN} = [\pi_{zk}^3\{\pi_{kk}-1)^3/(1-\pi_{kk}^3)-1\}]E[e(t)^3]. \qquad (26)$$

The skewness coefficient is calculated by normalizing (24), (25) and (26) using the standard deviation of each variable. Thus, the skewness coefficient of each variable is defined as

$$\phi_i = \Phi_i^T/(\sigma_i^3) \qquad i = Dy,\ Dc,\ Di,\ CN,\ DW \text{ and } Dk. \qquad (27)$$

V. CALIBRATION, THE POPULATION MOMENT AND ITS FINITE SAMPLE DISTRIBUTION

In this section, we calibrate the model, and measure the population third moments which are compared to the sample third moments. And a simulation experiment examines the finite sample characteristics of the population moments.

5.1 Model Calibration

Before calculating the population third moments, we calibrate the model. For the preferences, the elasticity of marginal utility of leisure with respect to leisure, the steady state fraction of hours worked (N) and discount rate are specified. For the technology and budget structures, the capital depreciation rate, the growth rate of labor-augmenting technical progress and the labor's share in the output are specified.[12] After we get through the base line specification of model (is called Long-

[12] See King et al (1988) for parameter values used to calibrate the model.

Plosser with a realistic depreciation in KPR (1988)), to compare the implications of different model specifications, the model is calibrated in other different specifications depending on different labor supply elasticities. That is, we look at the difference in the simulation result of small and large values of labor supply elasticity, which are 0.4 and infinity. They are denoted ϕ^{p}_{l} and ϕ^{p}_{h} respectively in Table 4.

5.2. The Population Moment and Its Finite Sample Distribution

The calculated population skewness are reported in Table 4. For a comparison, it also includes the skewness coefficient estimated from the actual data and the model simulated data (see below). Note that to make the empirical work consistent with the model, the skewness are estimated from per capita series. (For this purpose, each aggregate series has been divided by the population 16 years and older (p16)). However, empirical results are identical in both cases of aggregate and per capita series, which is explained from the definition of first difference in logarithm of per capita series (See Tables 3 and 4).

In matching the observed third moments, apparently, it seems that they are not so successful in explaining the actual observation. The actual asymmetry is stronger in consumption, investment and hours than output, capital stock and real wages. All variables, however, have almost the same population skewness, while consumption and capital stock have a little weaker asymmetry than other variables. Comparing the result in Choi (1993), however, it seems that the model with a stochastic growth is better than the one with a deterministic trend. Relative to the case of deterministic trend, here, the population third moments are closer to those of the actually observed third moments, and specially, the smaller asymmetry of input is required to generate the observed asymmetry of the output.

Next we establish finite sampling distribution of the population moments through a simple experiment.[13] For this simulation, the specification of Long-Plosser with a realistic capital depreciation has been used. The experiment results are in table 4. The moments are computed with 100 simulations, each of which is consisted of 128 periods. The standard errors of skewness for all variables are small. Certainly, for example, the 95 percent confidence intervals constructed for those sampling distributions of third moments would not be large enough. So, it seems not correct to argue that the difference between the population and the actually observed skewness may be ascribed to the finite sampling distribution.

Finally. we look at the population third moments for alternative specifications of

[13] In generating the finite samples for this purpose, one problem is the lack of the control on the third moments of the driving force. Generally, there is no random number generating function which can control the third moment of generated series. The choice of first and second moments determines the third moments. Due to this problem, the sample means of the third moments estimated from the generated sample can not be made equal to the population third moments. What we can do is, therefore, to generate the finite sample and presume that the sample means of the third moments has shifted from the population moment by a constant.

labor supply elasticity. The results are reported in table 4. They show that the three different specifications make no significant difference in the asymmetries of variables, for a given asymmetry of the input.[14] So, alternative specification does not improve the model in matching the third moments.

VI. CONCLUSION

This paper examined the asymmetry of business cycles without assuming any deterministic trend. It attempts to verify the "stylized fact" about the asymmetry of business cycles by identifying two different components which are asymmetric frequency and amplitude. The results are consistent with the "fact" for the asymmetric fluctuations. And based on the estimated third moments, one interesting question addressed is whether any full aggregate business cycle model can generate the observed asymmetry of business cycles, or whether any special condition is required for any model to generate the asymmetry. It is found that the RBC model with a stochastic growth is not so successful in generating the observed asymmetry in the context of third moment. An implication delivered by this paper is that one should also look at third moment as well as second moments.[15]

Another future work worthwhile may be to assess the loss caused by the ignorance of the asymmetry in the practical matters like the forecasting, policy making and model building etc.. It may be important because most above cited studies find the evidence of asymmetry, but some of them conclude that the asymmetry is not significant enough to be considered in the practical matters. In that case, the interesting question is how the insignificant asymmetric structure of the business cycles can have a significant loss and inefficiency when it is ignored.

[14] This is also true for the second moment. That is, the four specifications have the same fluctuation in the growth rate of variables. This may be a restriction on this business cycle model.

[15] The discussion in this paper is based on the presumption that the main determinant of asymmetric cycles is the asymmetric input to the system. Another possibility is the case that the system is nonlinear so that the symmetric input is transformed into asymmetric output variables. This possibility is very interesting, but can not be answered easily since it needs a non-linear business cycle model which is not available currently. Choi (1995) suggests the convex structure of the economy as a possibility for nonlinear system.

APPENDIX A.

This appendix explains the derivation of the third moments in (21), (22) and (23). For a convenience, the expressions (21)′, (22)′ and (23)′ are written here.

$$DlogZ(t)=(1-\pi_{zk})e(t)-\pi_{zk}(\pi_{kk}-1)\{1/(1-\pi_{kk}B)\}e(t-1) \quad (21)'$$

$$DlogK(t)=(1-\pi_{kk})\{1/(1-\pi_{kk}B)\}e(t-1) \quad (22)'$$

$$DN(t)=-\pi_{Nk}e(t)+\pi_{Nk}(1-\pi_{kk})\{1/(1-\pi_{kk}B)\}e(t-1) \quad (23)'$$

They can be rewritten as following.

$$DlogZ(t)=(1-\pi_{zk})\,e(t)-\pi_{zk}(\pi_{kk}-1)\{1/(1-\pi_{kk}B)\}\,e(t-1)=(1-\pi_{zk})\,e(t)\pi_{zk}(\pi_{kk}-1)$$
$$\{1+\pi_{kk}B+\pi_{kk}^{2}B+\cdots\}\,e(t-1) \quad (21)'$$

$$DlogK(t)=(1-\pi_{kk})\{1/(1-\pi_{kk}B)\}\,e(t-1)=(1-\pi_{kk})$$
$$\{1+\pi_{kk}B+\pi_{kk}^{2}B+\cdots\}\,e(t-1)\} \quad (22)'$$

$$DN(t)=-\pi_{Nk}\,e(t)+\pi_{Nk}(1-\pi_{kk})\{1/(1-\pi_{kk}B)\}\,e(t-1)=$$
$$-\pi_{Nk}\,e(t)+\pi_{Nk}(1-\pi_{kk})\{1+\pi_{kk}B+\pi_{kk}^{2}B+\cdots\}\,e(t-1) \quad (23)'$$

Then the third moment of each series is defined as follows

$$E[DlogZ(t)^{3}]=[(1-\pi_{zk})^{3}-\pi_{zk}^{3}(\pi_{kk}-1)^{3}\{1+\pi_{kk}^{3}+\pi_{kk}^{6}+\cdots\}]E[e(t)^{3}]$$
$$=[(1-\pi_{zk})^{3}-\pi_{zk}^{3}(\pi_{kk}-1)^{3}\{1/(1-\pi_{kk}^{3})\}E[\,e(t)^{3}]$$

$$E[DlogK(t)^{3}]=[(1-\pi_{kk})^{3}\{1+\pi_{kk}^{3}+\pi_{kk}^{6}+\cdots\}]E[\,e(t)^{3}]$$
$$=(1-\pi_{kk})^{3}\{1/(1-\pi_{kk}^{3})\}E[\,e(t)^{3}]$$

$$E[DN(t)^{3}]=-\pi_{Nk}^{3}+\pi_{Nk}^{3}(1-\pi_{kk})^{3}\{1+\pi_{kk}^{3}+\pi_{kk}^{3}+\cdots\}E[\,e(t)^{3}]$$
$$=-\pi_{Nk}^{3}+\pi_{Nk}^{3}(1-\pi_{kk})^{3}\{1/(1-\pi_{kk}^{3}\}]E[\,e(t)^{3}]$$
$$=\pi_{Nk}^{3}[(1-\pi_{kk})^{3}\{1/(1-\pi_{kk}^{3})\}-1]E[\,e(t)^{3}]$$

APPENDIX B.

Table 1. The Asymmetry of Upturns and Downturns: Frequency

Variable	$\pi(+/-)$	$\pi^T(+/-)$	Γ_{11}	Γ^T_{11}
y	1.23	1.02	0.70(0.08)	0.61(0.06)
c	1.05	1.10	0.69(0.08)	0.62(0.07)
i	1.08	1.12	0.63(0.08)	0.63(0.07)
n	1.35	1.16	0.81(0.07)	0.72(0.05)
w	1.08	1.07	0.50(0.08)	0.59(0.08)
k	1.22	1.77	0.84(0.05)	0.82(0.04)

Note. $\pi(+/-)$=the ratio of sample relative frequency of positive to negative movements. Γ_{11} and Γ^T_{11} are transition probabilities, T means transformed series.

Table 2. The Asymmetry of Upturns and Downturns: First and Second Moments

Variable	$\mu(-/+)$	$\mu^T(-/+)$	$\sigma(-/+)$	F(p-value)
y	1.23	1.02	1.87	1.63(0.02)
c	1.05	1.10	2.13	1.89(0.00)
i	1.08	1.11	1.87	2.15(0.00)
n	1.35	1.15	2.59	1.63(0.00)
w	1.08	1.06	1.23	1.40(0.08)
k	1.22	1.25	1.86	2.81(0.00)

$\mu(-/+)$ and $\sigma(-/+)$=the ratio of sample relative means and variances of negative to positive movements respectively, T means transformed series.

Table 3. The Asymmetry of Upturns and Downturns. Third Moment

	ϕ_i	$\phi_i^T(t_1, t_2)$
y	−0.49	−0.30 (1.35, 1.37)
c	−0.76	−0.64 (2.92, 1.69)
i	−0.68	−0.79 (3.62, 1.41)
n	−0.69	−0.39 (1.77, 2.02)
w	−0.18	−0.26 (1.19, 1.07)
k	−0.48	−1.11 (5.06, 1.68)

Note: ϕ_i=the third moments actually observed from data. T denotes transformed (serially independent) series. t_1 and t_2 are t-statistics.

Table 4. The Population Moments and Its Finite Sampling Distribution

	ϕ^0	ϕ^p_B	ϕ^s_B	ϕ^p_L	ϕ^p_H
y	− 0.49	− 0.49	− 0.90(0.03)	− 0.49	− 0.49
c	− 0.76	− 0.45	− 0.85(0.03)	− 0.45	− 0.45
i	− 0.68	− 0.49	− 0.90(0.03)	− 0.49	− 0.49
n	− 0.69	− 0.47	− 0.87(0.02)	− 0.48	− 0.47
w	− 0.18	− 0.47	− 0.87(0.02)	− 0.48	− 0.45
k	− 0.48	− 0.10	− 0.12(0.03)	− 0.10	− 0.10
e		− 0.49	− 0.90(0.03)	− 0.49	− 0.49

ϕ^0=the third moments actually observed, ϕ^p_B=the population third moments of the base model, ϕ^s_B=third moments estimated in the finite sample simulated of the base model. Standard errors are in ( ). ϕ^p_L=the population third moments when the model is calibrated with low value of labor supply elasticity. ϕ^p_H=the population third moments when the model is with high value of labor supply elasticity.

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