

## SUBJECTS IN EXPERIMENTAL BESTSHOT GAMES BEHAVE LIKE CASE-BASED PLAYERS\*

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*We reexamine subjects' behavior in the sequential best-shot game experiments. In our model, unmodeled factors including rationality determine players' initial aspirations and then a modified version of case-based learning process governs subsequent adaptations. The paper shows that "satisficing" can explain the actual subjects' adaptive behavior surprisingly well. Precisely, it is shown that the proportion of aggregate periods in which plays are consistent with satisficing ranges from 86.0 to 96.25 percent depending on the experimental treatment. We also provide an intuitive explanation of why subgame perfection is observed in the bestshot game but not in the ultimatum bargaining, albeit the similarity of equilibrium predictions.*

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### I. INTRODUCTION

Experimental studies make it clear that models of game behavior demanding perfect rationality of the players seem to perform poorly with even very simple games. Consider the situation in which rational players play a finite-horizon exten-

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sive form game of perfect information. Traditional theory can generically propose a sharp prediction, called the subgame perfect equilibrium. The trouble is that experimental studies have reported drastically different behavior in some well-known games, such as the ultimatum bargaining games<sup>1</sup> and the centipede game as studied by McKelvey and Palfrey (1992). However, subgame perfection does not always fail. In the sequential best-shot public goods provision as of Harrison and Hirshleifer (1989) and Prasnikar and Roth (1992), and the market bidding game as studied by Prasnikar and Roth (1992), subjects' behavior turned out to converge to subgame perfection, despite similarity of equilibrium predictions to the ultimatum bargaining game. Why does subjects' behavior converge to the equilibrium prediction in some experimental games, but not in others?

We cast doubt on the backward induction argument.<sup>2</sup> This paper focuses on the best-shot games and investigates how well "satisficing" players do in place of rational players. The model can be described as follows. Players form their own initial aspiration levels right after being informed about the game but before playing it. This is meant to capture that, without explicitly modeling how, each subject's initial aspiration level is determined by factors such as the informational condition, the first mover advantage or disadvantage, the subject's model of his opponent's play, his experience of having faced similar situations before, his knowledge about game theory, and so forth. Once the initial aspiration levels are formed, players adapt in subsequent periods to their experience. We assume that players make decisions according to a modified version of the Gilboa and Schmeidler's case-based decision theory (CBDT henceforth).

In our framework, a player chooses a pure strategy that maximizes the objective function which is the sum of the differences between experienced payoffs and the current aspiration level. The objective function, which Gilboa and Schmeidler (1994, 1995) derived from a set of axioms, has an element of bounded rationality and habit persistence. The aspiration evolves according to a weighted average of its previous value and the best average payoff over all pure strategies.<sup>3</sup> Using Binmore's (1987) terms, in our model the 'eductive' process determines the initial aspiration level, and the 'evolutive' process governs the subsequent learning process. Traditional theory, to verify whether a particular outcome is indeed an equilibrium, often presupposes an introspection process that seems beyond the cognitive limit of

<sup>1</sup> Refer to Guth (1995) for a survey.

<sup>2</sup> Camerer *et al.* (1993) used software system to record the information subjects looked at and to draw inferences about how people think in ultimatum bargaining game. They concluded that 'any model that assumes players backward induct is a poor descriptive account of how people think. Such a model might make reasonable predictions, but models that incorporate better representations of human cognition could make even better predictions.' (p.45)

<sup>3</sup> Besides the aspiration update, we could have introduced another type of dynamics, namely expansion of the set of choices. Roughly speaking, if actions that were available resulted in unsatisfactory outcomes, the decision-maker begins searching for new available actions, which leads to an enlarged set of choices. Simon (1955; pp. 111-13) informally mentioned this adjustment dynamic, and pointed out its crucial role in guaranteeing convergence of a satisficing solution. However, in order to make the model parsimonious, we do not formalize this idea.

human subjects. On the other hand, recent developments in learning and evolution, showing how an equilibrium gets to be played, often assume agents who can do nothing but make occasional adaptations to previous history. Our model may be an approach to incorporate these two features into one framework.

This paper shows that "satisficing" can explain the subjects' observed behavior surprisingly well. We define the minimally inconsistent path(s) by the path that minimizes the number of periods in which subjects' actual play is inconsistent with the model. It is shown that the aggregate proportion of periods in which the proposer's behavior is consistent with the model ranges from 86 to 96.25 percent depending on the information conditions and the subject's role. We define the minimally imperfect initial aspiration to be the one that, among the initial aspiration levels consistent with some minimally inconsistent path, is nearest to the perfect equilibrium payoff. If there is some minimally inconsistent aspiration level equal to the perfect equilibrium payoff, we can not reject the hypothesis that the subject made his initial choice based on rational expectations. It is shown that the second movers' minimally imperfect aspiration levels are significantly close to the perfect equilibrium payoff while first movers' minimally imperfect of aspiration levels are widely dispersed.

It has been recognized that satisficing can capture many salient features of experimental results. Examples include repeated prisoner's dilemma (Selten and Stoecker (1986)), bargaining problems (Mitzkewitz and Nagel (1993)), a repeated zero-sum game (Mookherjee and Sopher (1994)), and so forth. The most widely applied framework of satisficing behavior is the stochastic learning mechanism due to Bush and Mosteller (1955) and Harley (1981). In this approach, each player is assumed to adapt his strategy from one round to the next by increasing the probability assigned to the chosen action if it resulted in a payoff exceeding the aspiration level, and decreasing it otherwise. Fixing the aspiration level to be zero over time, this approach presupposes positive payoff in economic term or stimulus in psychological term. Roth and Erev (1995) have shown by simulations that the intermediate term predictions of modified Harley's models track well the observed behavior in three games: the sequential best-shot game, the market game and the ultimatum bargaining game. Roth and Erev (1995) suggested by simulations that subjects used essentially the same learning rules regardless of game structure and location and that 'the observed differences reflect different patterns of adaptations.'

We apply a modified version of CBDT on the following ground. Above all, we believe that the environment CBDT presumes is similar to the environment subjects often face in actual laboratory experiments. Gilboa and Schmeidler remark that CBDT is particularly appropriate in analyzing situations involving ignorance, which refers to the situation in which neither the states of the world nor probabilities on them are naturally defined.<sup>4</sup> In many experimental studies, subjects are

<sup>4</sup> Gilboa and Schmeidler added the notion of "ignorance" to the Knightian distinction between risk and uncertainty. To demonstrate how backward induction breaks down in some repeated strategic form games, Dow and Werlang (1994) apply concept of Schmeidler's non-additive probabilities, which is a model of uncertainty.

asked to make certain decisions, while being placed in unfamiliar situations especially in initial stages. Hence, this paper attempts to test how well CBDT works in situations that we believe are the most pertinent. Second, since CBDT is deterministic, except in tie-breaking incidents we are able to check explicitly whether actually observed paths of subjects' choices are consistent with theory. This is in sharp contrast to Roth and Erev (1995) that relies heavily on simulations, although our approach can convey the same insight and intuition. Last, CBDT is indeed informationally less demanding relative to other learning models. For instance, belief formation models, such as Fudenberg and Kreps (1992) and Canning (1992), require that players observe the empirical frequencies of opponents' realized strategies in order to form beliefs against which to choose a best response. Evolutionary dynamics, such as Kandori, Mailath and Rob (1993) and Young (1993), require that the aggregate characteristic of the population be observable. The case-based player needs to know nothing about others' choices or characteristics, nor the structure of the game at hand, nor even the set of actions available to himself in advance. All he must remember are the number of times that each strategy was chosen, if ever tried, and its average performance.<sup>5</sup> The structure of the paper is organized as follows. The next two sections build up the model and explain central notions for subsequent analysis. Section IV reports the calibration results. Section V explains why the equilibrium behavior is observed in the best-shot game, but not in the ultimatum bargaining, despite the similarities of equilibrium predictions. The final section concludes.

## II. THE MODEL

### 2.1 The Stage Game

The best-shot game is a sequential public-goods provision game. The rules are that player 1 states a quantity  $x$ , after which player 2, informed of  $x$ , states a quantity  $y$ . Players can contribute any nonnegative integer. An amount of public good equal to the maximum of  $x$  and  $y$  is provided, and the corresponding redemption value is given by  $0.025q(41-q)$ , where  $q = \max\{x, y\}$ . Each player receives the redemption value minus 0.82 times his own contribution.

With the payoffs as specified, the best-shot game has the property that if the other player makes any contribution at all, it is optimal to contribute nothing. There are two Nash equilibria to this game. One Nash equilibrium prescribes that player 1 contributes nothing and player 2 contributes 4, while the other Nash equilibrium prescribes that player 1 contributes 4 and player 2 contributes nothing re-

<sup>5</sup> Belief formation models, such as Fudenberg and Kreps (1992) and Canning (1992), require that players observe the empirical frequencies of opponents' realized strategies in order to form beliefs against which to choose a best response. Evolutionary dynamics, such as Kandori, Mailath and Rob (1993) and Young (1993), require that the aggregate characteristic of the population be observable.

gardless of player 1's play. Only the first Nash equilibrium is subgame perfect, yielding player 1 a payoff of 3.70 and player 2 a 0.42 payoff. Notice that this extreme equilibrium behavior is very similar to that of ultimatum bargaining.

The set of player  $i$ 's pure strategies is  $S^i = \{0, 1, 2, \dots\}$  for  $i=1, 2$ . Let  $s^i$  and  $s$  denote typical elements of the sets  $S^i$  and  $S = S^1 \times S^2$ , respectively. Each strategy profile determines a terminal node giving a vector of payoffs, one to each player. Let  $\pi^i: S \rightarrow \mathbb{R}$  denote player  $i$ 's payoff function. Let  $\Pi_{spe}^i$  denote the set of payoffs that player  $i$  receives on the subgame perfect equilibria, then it is easy to check that  $\Pi_{spe}^1 = \{3.7\}$  and  $\Pi_{spe}^2 = \{0.42\}$  in the best-shot game.

The actual experiments of Prasnikar and Roth (1992; PR henceforth) were carried out under two information conditions, namely full and partial. The full information experiment is conducted under the standard conditions, with players informed of the monetary payoffs that would be given to their opponents. In the partial information case, players were not informed of their opponent's payoffs. PR provided experimental results that the subjects' behaviors in the best-shot game rapidly converge to the theoretical prediction and, moreover, that this observation is fairly robust to information conditions.

## 2.2 A Sketch of CBDT

Our model is inspired by Gilboa and Schmeidler (1994, 1995, 1996), in which the reader can find a full description of CBDT. Only a very sketchy outline of CBDT will suffice for the understanding of our model.

The primitives of CBDT consist of a set of problems  $P$ , a set of available acts  $A$ , and a set of possible results  $R$ . The set of cases is defined to be  $C = P \times A \times R$ . A "case" is a triple  $(p, x, r)$ , where  $p$  is the decision problem,  $x$  is the act chosen, and  $r$  is the result that was obtained in this case. At any point in time, a decision-maker is equipped with some memory  $M$ , which is the set of cases the decision-maker can remember.

The "utility" is a function  $u: R \rightarrow \mathbb{R}$ . The notion of "similarity" measures the extent to which one pair of problem and act is similar to another; that is, it is a function  $s: (P \times A)^2 \rightarrow [0, 1]$ . Faced with a problem  $p$  the decision-maker chooses an act  $x \in A$  maximizing the function<sup>6</sup>

$$U(x) = \sum_{(p, y, r) \in M} s((p, x), (p, y)) [u(r) - H].$$

<sup>6</sup> There are several versions of the CBDT functional form. The current version is a combination of the act similarity (1994) and a non-zero aspiration (1996). In the Gilboa and Schmeidler's 1995 paper, the similarity function is of the form  $s(p, q)$ , and aspiration  $H$  is identically zero.

In our model, player 1's problem at period  $t$  is to decide how much to contribute. Exactly the same problem is repeated in each period, so "problem" and "period" can be used interchangeably. Player 1's act  $x$  is the amount of contribution. Two pairs of problem-act are perfectly similar one if the contributions are identical, while perfectly dissimilar otherwise. At each period the result is the size of the opponent's contribution and the utility is the realized payoff. Player 1 evaluates an act by the sum of the differences between experienced payoffs and the current aspiration level. Formally, at each period  $t$ ,

$$s((t, x)(\tau, x')) = \begin{cases} 1, & \text{if } x = x' \\ 0, & \text{otherwise} \end{cases}, \text{ for all } \tau = 1, 2, \dots, t-1,$$

$$r \in S^2 = \{0, 1, 2, \dots\},$$

$$u(r) = 0.025q(41 - q) - 0.82x, \text{ where } q = \max\{x, r\}$$

The second-mover's problem  $p$  is identified with the outstanding contribution. We assume that a problem  $p$  at period  $t$  is perfectly similar to a problem  $q$  at period  $\tau$  if the amount of the first-mover's contribution the second-mover faces at  $t$  is identical to the amount at  $\tau$ . Otherwise, there is no similarity at all. Player 2's act is the amount of contribution. The result is the pair of contributions, and the utility are equal to the realized payoff.

### 2.3. The Dynamic Model

Imagine that a pair of randomly matched players repeatedly play the stage game at dates  $t = 1, 2, \dots, T$ , where  $T$  may be finite or infinite. Player  $i$  updates his strategy at period  $t$  according to player  $i$ 's payoff realized up to  $(t-1)$ , relative to his present aspiration level. Let  $H_t^i$  denote player  $i$ 's aspiration level at the beginning of the period  $t$ . The set of  $T$ -histories experienced by player  $i$  is a subset of  $\Omega' \equiv (I \times S \times D)^N$ , where  $I$  is the interval  $[0, 3.7]$  and  $N$  is the set of natural numbers up to  $T$ . A  $T$ -history  $\omega = ((H_t^i, s_t^i, \pi_t^i))_{1 \leq t \leq T} \in \Omega'$  will be interpreted as follows: for all  $t \geq 1$ , the aspiration level is  $H_t^i$  at the beginning of the period, a strategy  $s_t^i$  is chosen, and the strategy profile  $s_t = (s_t^1, s_t^2)$  yields a payoff  $\pi_t = (\pi_t^1(s_t), \pi_t^2(s_t))$ . The projection functions  $s_t^i: \Omega' \rightarrow S^i$  and  $H_t^i: \Omega' \rightarrow I$  have the obvious meaning.

Next we define a function  $C: \Omega' \times S \times N \rightarrow 2^N$  to be the set of periods, up to a given time, in which player  $i$  played a particular strategy, according to a given history. That is,

$$C^i(\omega, s^i, t) = \{\tau < t \mid s_\tau^i(\omega) = s^i\}.$$

We also define the number of times player  $i$  played a particular strategy  $s^i$  by

$$K^i(\omega, s^i, t) = \# C(\omega, s^i, t) \in N \cup \{0\}.$$

The strategy is evaluated by the sum of the differences between experienced pay-offs and the current aspiration level. Formally, player  $i$  at period  $t$  chooses a pure strategy that maximizes the following objective function:

$$U^i(\omega, s^i, t) = \begin{cases} 0, & \text{if } K^i(\omega, s^i, t) = 0 \\ \sum_{\tau \in C^i(\omega, s^i, t)} [\pi_\tau^i(\omega) - H_t^i(\omega)], & \text{otherwise} \end{cases} \quad (1)$$

If the  $U^i$ -maximizing actions are multiple, then player  $i$  is assumed to choose among all those actions by a deterministic rule. Notice that the value  $U^i$  directly depends only on the current aspiration level  $H_t^i$ . It will be often convenient to express the  $U^i$  function in the following form:

$$U^i(\omega, s^i, t) = \begin{cases} 0, & \text{if } K^i(\omega, s^i, t) = 0 \\ K^i(\omega, s^i, t) [\bar{\Pi}^i(\omega, s^i, t) - H_t^i(\omega)], & \text{otherwise} \end{cases} \quad (2)$$

where  $\bar{\Pi}^i(\omega, s^i, t) = \frac{\sum_{\tau \in C^i(\omega, s^i, t)} \pi_\tau^i(\omega)}{K^i(\omega, s^i, t)}$  is the average payoff of the strategy  $s^i$ .

If the average performance of a particular action exceeds the current aspiration level then this action has a positive  $U^i$  value. This fact implies that the player sticks to that particular action even if other actions with a better average performance exist. Thus, the objective function captures the idea of "satisficing" as of Simon (1955). The dynamics is deterministic, given the parametric values. Markovian property does not apply, if the 'state' is defined as the current pure strategy profile and aspiration levels. In other words, path-dependence matters.

Now consider the aspiration revision rule. For a given adjustment weight  $\alpha \in [0, 1)$  and the initial aspiration level  $H_1^i \equiv H^i \in [0, \infty)$  player  $i$  updates his aspiration level in an adaptive manner, i.e. the weighted average of its previous value and the maximal average payoff, where the maximum is taken over all pure strategies that were chosen in the past. More formally,

$$H_t^i(\omega) = (1 - \alpha^i) H_{t-1}^i(\omega) + \alpha^i \max_{s^i \in S^i(t)} \{\bar{\Pi}^i(\omega, s^i, t)\}, \quad (3)$$

where  $S^i(t)$  is the set of player  $i$ 's strategies that were played before  $t$ .

### III. THE ANALYSIS

In this section we develop the theoretical framework. Experimental data at individual level provide the sequence of realized outcomes. There is a set of pure strategies which is consistent with a given outcome. As a result, corresponding to a sequence of realized outcomes there is the set of the sequences of pure strategies that is consistent with the actual data. We call it as the set of actual paths. Let  $S^i$  and  $s = (s^i)_{s, t \in T}$  be the set of actual paths for player  $i$  and its typical element, respectively.

Now we want to characterize the paths of strategies which are consistent with our model, namely the sequences of strategies that a player who behaved as if he was a modified DBDT decision-maker would have chosen. To this end, fix the initial actual choice  $s_i^0$  as given. We can then generate the subspace of  $T$ -histories that, starting from strategy  $s_i^0$  are compatible with  $U^i$ -maximization and aspiration revision rule. Focus on a particular subject playing player  $i$ 's role, so we suppress the superscript  $i$  when there is no confusion. Formally,

$$\Omega = \{\omega \in \Omega' \mid s_t(\omega) = \hat{s}_t, s_t(\omega) \in \arg \max_{s \in S} U(s, t), \forall t \geq 2, \text{ and} \\ \exists a \in [0, 1) \text{ such that } (H_t)_{1 \leq t \leq T} \text{ evolves by Eq.(3)}\}. \quad (4)$$

We say that a play of player  $i$  is *consistent with* the model if  $\hat{s} \in \{s^i(\omega) \mid \omega \in \Omega\}$ , where  $s^i(\omega)$  is the projection function.

Suppose that the observed play of a particular player is not perfectly consistent with the model. We postulate that his intended play is the path minimizing the number of periods at which he behaved inconsistently. More formally,

**Definition 1** Define a *minimally inconsistent* (MIC, in short) histories that is associated with the actual path  $\hat{s}^i$  as follows:

$$\Omega^*(\hat{s}^i) \equiv \{\omega \in \Omega \mid \#\{t \mid \hat{s}_t^i \neq s_t^i(\omega)\} \leq \#\{t \mid \hat{s}_t^i \neq s_t^i(\omega')\}, \forall \omega' \in \Omega\}. \quad (5)$$

Let  $\Omega^* \equiv \bigcup_{s \in S} \Omega^*(\hat{s})$  be the set of all MIC histories. Define  $\sigma[\Omega^*(\hat{s}^i)]$  denote the set of MIC paths that is associated with the actual path  $\hat{s}^i$ , where  $\sigma(\cdot)$  is the projection function from histories to strategy choices. Clearly,  $\Sigma^* = \bigcup_{s \in S} \sigma[\Omega^*(\hat{s})]$  is the set of all MIC paths.

Corresponding to each MIC paths, there exist pairs of the initial aspiration level and aspiration revision coefficient,  $(H, a)$ , that are consistent with the given path. Let a mini-



minimally inconsistent aspiration level (MICA, in short) be the initial aspiration level which is compatible with some MIC path for some revision coefficient,  $\alpha$ .

With respect to the decision-making process, we keep in mind the following hypothetical story. Subjects form their initial aspiration levels after being informed of the game but before ever playing it. We do not attempt to model how players form their initial aspirations. We believe that un-modeled factors, such as rationality of players, the first mover (dis)advantage, whether the player has faced a similar situation before, affect the level of initial aspiration. For this reason, if there is some MICA equal to the perfect equilibrium payoff, we may not be able to reject the hypothesis that the subject made his initial choice on the basis of rational expectations. Hence, we are interested in the MICA that is nearest to the perfect equilibrium payoff. We call it to be the minimally imperfect initial aspiration level (MIPA, in short) and the MIC paths associated with MIPA to be the minimally imperfect (MIP, in short) paths.

We want to formally define these notions. Let  $\Lambda^1 = \{H^1(\omega) \in [0.4, 6.4] \mid \omega \in \Omega\}$  and  $\Lambda^2 = \{H^2(\omega) \in [0.1, 3.2] \mid \omega \in \Omega\}$  be the set of player 1's and 2's initial aspiration levels that are compatible with some MIC path. Now the notion of minimal imperfection is defined as follows.

**Definition 2** Define the *minimally imperfect initial aspiration level* (MIPA, in short) to be:

$$\lambda^i = \arg \inf_{\substack{\pi \in \Pi_{spe} \\ H \in \Lambda^i}} \|\pi^i - H^i\|, \quad (6)$$

where  $\Pi_{spe}^i$  is the set of subgame perfect equilibrium payoff to player  $i$  and the argument is taken over  $H^i$ 's. Also define a *minimally imperfect path* to be an element of the set  $\{s^i \in \Sigma^i \mid H^i(\omega) = \lambda^i\}$ , where  $H^i(\cdot)$  is the projection function mapping from histories to initial aspiration levels.

Recall that  $\Pi_{spe}^1 = \{3.7\}$  and  $\Pi_{spe}^2 = \{0.42\}$  in the best-shot game. Notice that the set of MIP paths is a subset of MIC paths.

#### IV. THE CALIBRATION RESULTS

We calibrate our model with the actual subjects' behavior in Roth *et al.* (1991) and Prasnikar and Roth (1992). Since those papers reported only the aggregate statistics, the individual level panel data on which our analysis are based were personally provided by Professor Al Roth.

For the purpose of comparisons, a sketch of experimental studies might be helpful.

The actual experiments were carried out under two information conditions, namely full and partial. The full information experiment is conducted under the standard conditions, with players informed of the monetary payoffs that would be given to their opponents. In the partial information case, players were not informed of their opponent's payoffs. Each subject participated in only one of the information conditions in ten consecutive encounters, keeping anonymity. Hence,  $T=10$  in our model specification.

One of the most striking features about the bestshot game experiments is that subjects' behavior rapidly converges to the subgame perfect equilibrium in the full information case, and even in the partial information case the data are concentrated along the two axes, that is, only infrequently do both players make positive contributions. In the full information case, player 1s always provide nothing (i.e.  $x=0$ ) after period 6. Player 2s choices are more dispersed, but the perfect equilibrium strategy ( $y=4$ ) was chosen 15 times out of 32 between period 7 to 10. In the partial information case, there is no instance in which both players provide positive contributions after period 5.

Table 1 presents the summary statistics of minimal inconsistency. The first column indicates the number of periods at which, along the MIP, subjects behaved inconsistently with the theory. The first row and the second indicate player's roles and information conditions, respectively. For example, 3 subjects of the player 1's role in the full information treatment chose strategies that our model is unable to explain.

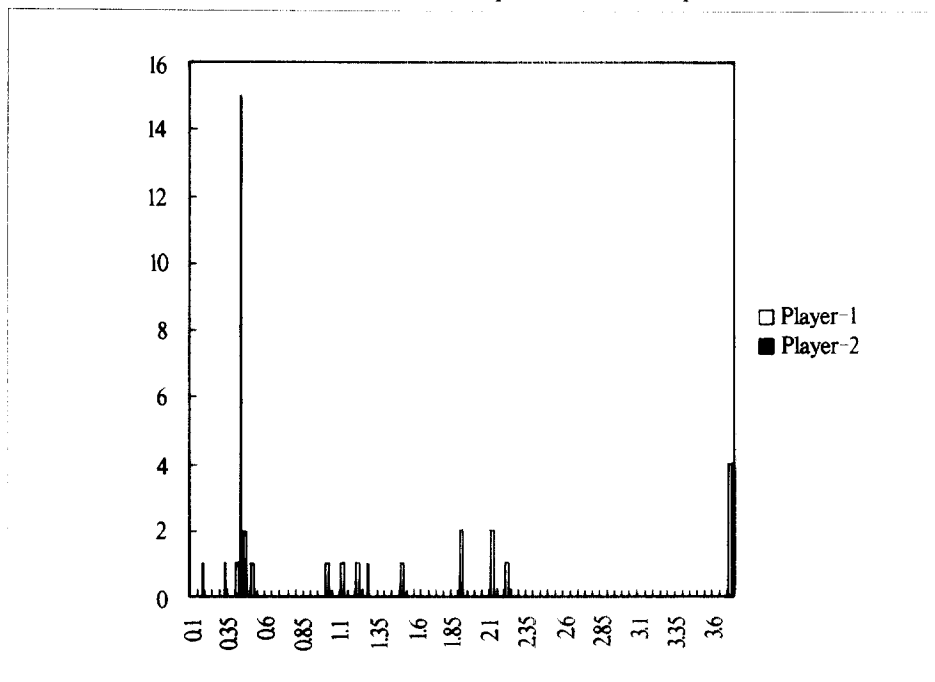
It is apparent from Table 1 that the results of minimal inconsistency are not significantly affected by information conditions. Our model can explain the first mover's plays surprisingly well, as implied by the fact that plays are consistent for only 93.0 to 96.25 percent of all periods depending on information conditions. On the other hand, case-based theory can explain the second mover's plays a little worse, which is 86 to 90 percent depending on information conditions. In terms of strategic uncertainty, the situation player 2 faces is much more straightforward than that player 1 faces. Thus, it is not entirely convincing why player 2 fails to choose a conditional optimizing strategy. Naturally, our model of bounded rationality does worse with player 2 than player 1.

Figure 1 summarizes the frequency distribution of MIPAs. We pool the data across informational conditions. Looking at the first-movers' behavior, we see that

Table 1. Results of Minimal Inconsistency

	Player 1		Player 2	
	Full	Partial	Full	Partial
None	5	4	4	3
1	3	4	1	2
2		2	2	2
$\geq 3$			1	3
Proportion of consistent periods(%)	96.25	93.0	90.0	86.0

[Figure 1] Distribution of Minimally Imperfect Initial Aspiration Levels

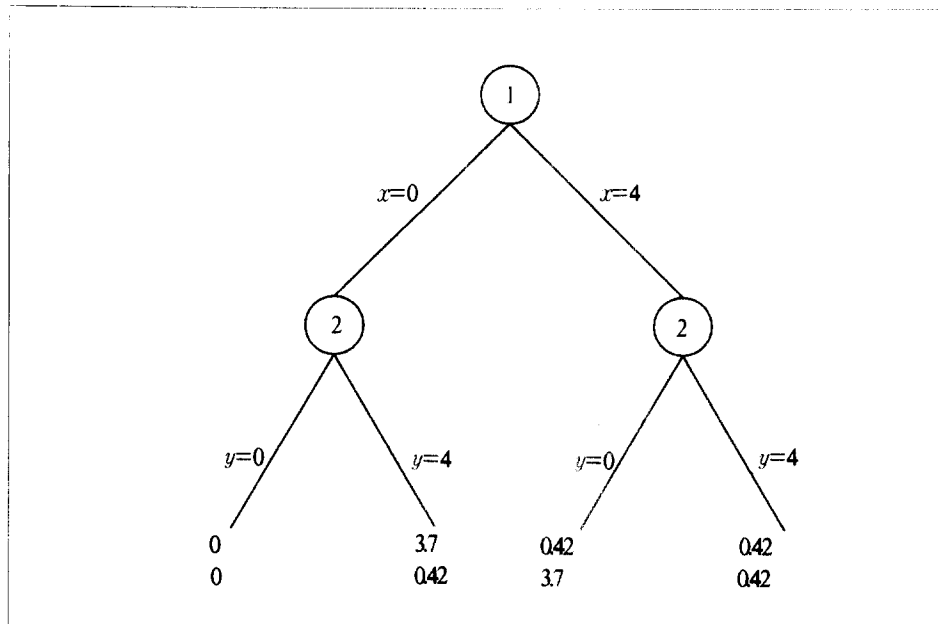


the distribution of MIPAs is widely spread between 0.4 and 3.7. In particular, only 4 subjects out of 18 have their MIPAs equal to the equilibrium payoff of 3.7. To the contrary, looking at the second-movers' behavior, 14 out of 18 subjects have their MIPAs equal to the equilibrium payoff, namely 0.42. Three out of the remaining four values (0.14, 0.29, 0.4 and 1.24) are located quite close to the equilibrium payoff. This implies that we cannot reject the hypothetical situation as follows: most subjects of the player 2's role introspect the stage game in the rational way, form their initial aspiration levels exactly equal to the perfect equilibrium payoff, and adapt strategies to experience in an adaptive way.

Figure 2 depicts the simplified best-shot game, in which each player has binary choices, i.e. either 0 or 4. This simplification can be partly justified by actual experimental observations. Although subjects are allowed to choose any nonnegative integer, a quantity level zero or 4 appeared most frequently.<sup>7</sup> Conditional on player 1's high contribution ( $x=4$ ), player 2 will choose  $y=0$  contribution more often than  $y=4$  since  $y=0$  yields the maximum payoff 3.70 and  $y=4$  yields a very low payoff 0.42. Recall that the MIPAs of player 2s are concentrated around 0.42, so  $y=0$  yields player 2 a satisfactory outcome while the other choice yields a disastrous outcome. Hence,  $y=0$  is very much reinforcing while  $y=4$  is almost discouraging. Regardless of the opponent's subsequent choice, player 1 would receive 0.42 by choosing  $x=4$ .

<sup>7</sup> Refer to PR Figure II, p.877, for full information and Figure III, p.878, for partial information case.

[Figure 2] A Simplified Best-shot Game

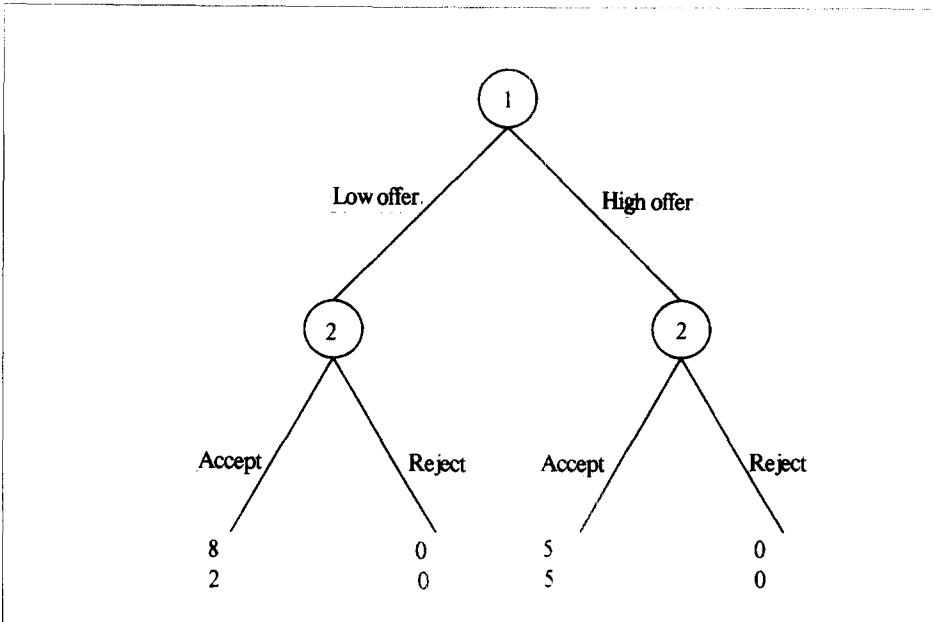


On the other hand, conditional on player 1's low contribution ( $x=0$ ), both  $y=0$  or  $y=4$  yield player 2 extremely low payoffs, namely 0 and 0.42, respectively. Both actions are initially discouraging, but player 2 tends to choose  $y=4$  more often than  $y=0$  as aspiration adapts. The low contribution yields player 1 either the maximum payoff of 3.70 or the minimum payoff 0, depending on player 2's choices. The average performance of  $x=0$  would lie somewhere in the middle of 0.42 and 3.7. Recall that the player 1's MIPAs are widely dispersed between 0.4 and 3.7. Consequently, from the viewpoint of player 1's,  $x=0$  is reinforcing while  $x=4$  is discouraging. As time passes, therefore, player 1 would choose the low contribution unless his initial aspiration level was too low, whereas player 2 would choose  $y=4$  unless her aspiration adapts too slowly. Recall that the strategy profile of  $x=0$  and  $y=4$  is the unique subgame perfect equilibrium.

Now we claim that the observed difference between the ultimatum bargaining and the best-shot game stems from the payoff structures. The rule of the one-round ultimatum bargaining game<sup>8</sup> is as follows. Player 1 proposes to divide 10 US dollars, of the form  $(x, 10-x)$ . Proposals take discrete values using the unit of a quarter dollar. Player 2 then either accepts or rejects this offer. If accepted, the money is divided as proposed; if rejected, each player receives zero. There are two subgame perfect equilibria, depending on whether responders accept the null offer. These equilibria prescribe that player 1 demands almost all money, and that player 2 accepts.

<sup>8</sup> Roth (1995) provided an excellent survey of bargaining experiments.

[Figure 3] A Simplified Ultimatum Bargaining



Contrary to these extreme equilibrium predictions, experimental studies showed that subjects' behavior quickly converged to a fair division (around 3.8 to 4.5 in the last few periods). Since the best-shot game and the ultimatum bargaining have very similar equilibrium structure, one might wonder what makes subjects' behavior converge to subgame perfection in the former game but not in the latter.

My companion paper studies how well a modified CBDT approach explains subjects' behavior in the ultimatum bargaining game. A reader who is interested in the results of minimal inconsistency and minimal imperfection with ultimatum bargaining may refer to the companion paper. This paper summarizes a rough idea. Figure 3 depicts the extensive-form of a simplified ultimatum bargaining game. Player 2's acceptance and rejection of the low offer, respectively, yield payoffs of 2 and 0. Since these payoffs are both unsatisfactory to player 2 if subjects' initial aspiration levels are higher than 2, subjects of the player 2's role sometimes accept and other times reject the low offer. This again implies that player 1 would obtain a payoff around 4. On the other hand, accepting a high offer yields 5 to player 2 while rejecting it yields 0, so acceptance is reinforcing if player 2's initial aspiration levels are not too large. This means that player 1s would obtain a payoff close to 5. Unless player 1s aspiration levels are too high, they tend to choose a high offer more often than a low offer. As time passes, behavior will converge to a fair division.

Gale, Binmore and Samuelson (1995) is worth mentioning. They observe that the basin of attraction of the subgame imperfect Nash equilibrium,  $(x=4, y=0)$ , relative to the unperturbed replicator dynamics is much smaller in the best-shot game than in the ultimatum game. Moreover, if perturbations are introduced, the perturbed

dynamics cannot point into the basin of attraction of the imperfect Nash equilibrium. They argue that no local attractor can therefore be found close to this Nash equilibrium and the perfect equilibrium is necessarily selected. Like our approach, the payoff structure of the games plays a central role in explaining why perfection is observed in the best-shot game but not in the ultimatum bargaining.

## V. CONCLUDING REMARKS

Our model is deterministic. Many seminar participants and referees suggest that the right direction is to allow tie-breaking randomizations and to estimate the maximum-likelihood parameters. Although I agree this MLE method makes another sense, I intentionally did not follow their advice. The reason is that, from the traditional theorist's viewpoint, the decision-maker in the modified CBDT may be silly enough. Decision-makers maximize an objective function that incorporates aspiration and cumulative payoffs, but not the average realized payoff. Aspiration evolves in an adaptive manner but not in a forward-looking way. If we allow players to randomize or tremble, the parsimony of the model might be extremely problematic.

One obvious shortcoming of this paper directly stems from lack of data. The existing experiments were conducted for nine to ten periods, on which our analysis of consistency are based. This is clearly not sufficient for a study of dynamic processes. The high performance of our model might be even due to the fact that it fits only ten-period time series data.

A number of questions are raised about works that are currently being undertaken, and problems for future research. There are other competing theories available. A model of bounded rationality is the Bush-Mosteller (1955) type stochastic learning approach. There are also attempts to reconcile experimental data with more traditional game-theoretic predictions. Camerer and Weigelt (1988) and McKelvey and Palfrey (1992) use the home-made priors approach, but such a method seems to difficult to generalize to other games. Fudenberg and Levine (forthcoming) argue that, using the self-confirming equilibrium notion to have irrational payoffs to generate the observed path is small. They report that the average loss of a player is \$0.03 to \$0.64 in a game involving stakes between \$2 and \$30. From a methodological point of view, while Fudenberg and Levine's analysis are based on the aggregate probability distribution over outcomes, we consider the complete history of each individual player's choices. It will be interesting to compare which approach fares best in applicable experimental studies provide very short periods data. An agenda for future research is to conduct laboratory experiments for a much longer periods and to test various theories by using new experimental data.

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