On the Theory of Credit Rationing: Further Analysis

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I. Introduction

Recently in the past, credit rationing has been examined often in the literature (see for example Jaffee and Russell [1] and Stiglitz and Weiss [6]). Common to all those analyses is the assumption that the lending institutions have imperfect information about the relevant characteristics of an individual borrower (firm). The moral hazard (incentive effects) and adverse selection effects pertinent in the credit market may result in a market failure. The price mechanism is sometimes inoperative even in the presence of an excess demand for loans.

Although the most extensive analysis is presented by Stiglitz and Weiss, they provide a complete analysis of an equilibrium with credit rationing only when the borrowing firms are risk neutral and the collateral

* Department of Economics, Korea University. The paper is a revised version of his paper appeared as "Credit Rationing in Markets with Imperfect Information: Further Analysis," Discussion Paper No. 5, The Institute of Economic Development, Korea University. He is grateful to Andrew Weiss for his comments on an earlier version of this paper and to the Asan Foundation for financial support.

requirement is fixed at an arbitrary level. They then examine the theory of collateral assuming that risk averse borrowers have an identical utility function but differ in their initial wealth. They also assume that the loan rate of interest as well as the deposit rate of interest are fixed at an arbitrary level. The theory of limited liability (or collateral) is presented assuming further that banks have no information about the initial wealth of each borrower.

The natural step to consider at this stage is the competitive choice of both collateral requirements and interest rate simultaneously. Extending our analysis, we also allow for the possibility of a set of separating loan contracts that differentiates each loan classes. We will show that in this more general framework, there can still exist an equilibrium with credit rationing. But with risk neutral firms, the theory of limited liability or collateral is not well defined. In the model with risk averse borrowers, we examine both an Walrasian equilibrium and an equilibrium with credit rationing. We discard the rather unrealistic assumption that banks have no knowledge about the initial wealth of each borrower. Instead, we assume that banks are informed about the initial wealth of loan applicants but not about the diverse risk preferences of individual borrowers. We show that in a single contract equilibrium, an equilibrium amount of collateral is less than the initial wealth of loan applicants. Finally in the concluding comments, we briefly examine the effects of a regulatory credit policy.

II. Credit Rationing Equilibrium When Borrowers Are Risk Neutral

We consider a model of a competitive loan market. The firms are borrowers in the loan market and each firm is viewed as undertaking an indivisible project denoted symbolically by θ. The firm needs to borrow one unit of loan to finance the given cost of a project. The borrowing rate of interest is denoted by r and the collateral requirement, c. We assume that for each project θ, there exists an associated probability distribution $F(R, \theta)$ of gross returns R. Following Stiglitz and Weiss, we assume $R \geqq 0$ and that the bank is not informed about the riskiness of a firm's project other than the mean return, μ. For notational convenience, we say that $\theta_1 > \theta_2$ if the project θ_1 is riskier than the project θ_2. The risk ordering employed here is in the sense of mean preserving spreads of returns (See Stiglitz and Weiss[6]).

We say that the firm is bankrupt if $R + c \leqq 1 + r$ so that the probability of bankruptcy is given by $F(1+r-c;\theta)$. Focusing our analysis to bankruptcy possibilities, we assume that $c \leqq 1+r$. Let $E_\theta[\cdot]$ denote the expectation operator taken with respect to $F(R, \theta)$. Then expected profits of each firm undertaking a project θ, $\bar{\pi}(r, c; \theta)$ can be written as

$$\bar{\pi}(r, c; \theta) = E_\theta[\mathrm{Max}(-c, R-(1+r))]. \tag{1}$$

On the other hand, the expected gross return to the representative competitive bank $\bar{\rho}(r, c; \theta)$ when it loaned to the firm is given as

$$\bar{\rho}(r, c; \theta) = E_\theta[\mathrm{Min}(R+c, 1+r)]. \tag{2}$$

It is not difficult to show that when $E_\theta[R] = \mu$,

$$\bar{\pi}(r, c; \theta) + \bar{\rho}(r, c; \theta) = \mu. \tag{3}$$

It follows that the maximum collateral requirement per unit loan should not exceed μ. Otherwise $\bar{\rho}(r, c; \theta) > \mu$ or $\bar{\pi}(r, c; \theta) < 0$ for any θ. At any given level of expected profits $\bar{\pi}$, consider an indifference curve $I_i(\bar{\pi}) = \{(r, c) \mid c \leqq \mu,\ c \leqq r+1, \text{ and } \bar{\pi}(r, c; \theta_i) = \bar{\pi}\}$. For notational convenience, let $F(R;\theta_i) = F_i(R)$. It is straightforward to prove that the marginal rate of substitution of c for r, MRS^i_{cr}, defined as

$$-\frac{dr}{dc}\bigg|_{\bar{\pi}} = \frac{\partial\bar{\pi}(r, c; \theta_i)/\partial c}{\partial\bar{\pi}(r, c; \theta_i)/\partial r} = \frac{F_i(1+r-c)}{1-F_i(1+r-c)} \tag{4}$$

is an increasing function of a probability of bankruptcy $F_i(1+r-c)$. In other words, the firm undertaking a riskier project is willing to accept a larger increase in interest rate in return for a lower collateral. Differentiating (4) at any point in $I_i(\bar{\pi})$, we have

$$\frac{d}{dc}\left(-\frac{dr}{dc}\right)_{\bar{\pi}}=\frac{-f_i(1+r-c)}{(1-F_i(1+r-c))^3}<0, \tag{5}$$

where $f_i(1+r-c)=\frac{dF_i(R)}{dR}\Big|_{R=1+r-c}$ so that an indifference curve $I_i(\bar{\pi})$ is convex to the origin. It is also easy to see that if $\bar{\pi}'<\bar{\pi}$, $I_i(\bar{\pi}')$ lies everywhere above $I_i(\bar{\pi})$.

If projects are risk ordered by a mean preserving spread of distribution of returns, it is not possible to select those firms undertaking only less risky projects. Given any loan contract, the firm will undertake the project if it yields nonnegative expected profits. Let $I(\theta)=\{(r,c)\,|\,c\leqq\mu,$ $c\leqq r+1$ and $\bar{\pi}(r,c;\theta)=0\}$. Then it is easy to see that $(\mu-1,\ \mu)$ belongs to $I(\theta)$ for all θ and $I(\theta_i)$ and $I(\theta_j)$ never crosses except at $(\mu-1,\ \mu)$. In fact if $\theta_i>\theta_j$, $I(\theta_i)$ lies above $I(\theta_j)$. This follows because with limited liability, Max $(-c,\ R-1-r)$ is concave in R so that $\frac{\partial\bar{\pi}(r,c;\theta)}{\partial\theta}>0$, while $\frac{\partial\bar{\pi}(r,c;\theta)}{\partial c}<0$ and $\frac{\partial\bar{\pi}(r,c;\theta)}{\partial r}<0$. It implies that even if the bank chooses both collateral and interest rate there is no way for the bank to sort out riskier projects retaining less risky projects. When banks are

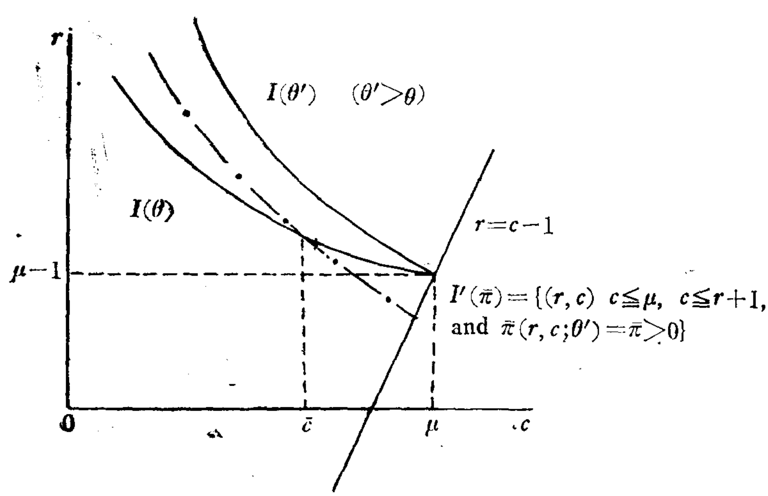

Fig. 1

allowed to vary both collateral and interest rate, they can also offer multiple loan contracts that can differentiate each loan class. But later in this section, we will show that under risk neutrality assumption, the bank optimal contract consists of a single contract. Although the marginal rate of substitution between collateral and interest rate monotonically increases as the riskiness of the firm's project increases, it is not profitable for banks to offer a set of separating contracts to risk neutral firms.[1)]

The bank will determine both r and c to maximize an average expected gross return to a unit loan defined as

$$\hat{\rho}(r,c)=\frac{\int_{\theta_0}^{\infty}\bar{\rho}(r,c;\theta)\ dG(\theta)}{1-G(\theta_0)}, \qquad (6)^{2)}$$

where $G(\theta)$ denotes a distribution of θ among the potential loan applicants and θ_0, a critical level of θ, below which individuals do not apply for loans. θ_0 is determined from $\bar{\pi}(r,c;\theta_0)=0$ so that $\theta_0=\theta_0(r,c)$. Since $\frac{\partial\bar{\pi}(r,c;\theta)}{\partial\theta}>0$, we have $\frac{\partial\theta_0}{\partial r}>0$ and $\frac{\partial\theta_0}{\partial c}>0$. For later references, let $g(\theta)=\frac{dG(\theta)}{d\theta}$. Assuming safely that each bank always offers a single contract, we introduce the following definition. Let $Ls\ (\hat{\rho})$ denote a supply of loanable funds while $L_D(r,c)$, a demand for funds. Assume $L_S'(\cdot)>0$.[3)] Since only those firms with θ greater than $\theta_0(r,c)$ apply for unit loan, it follows from the previous argument that $L_D(r,c)$ is decreasing in r and c.

Definition 1: Let (r^0,c^0) maximize $\hat{\rho}(r,c)$ as defined in (6). A loan contract (r^0,c^0) and a deposit rate $\hat{\rho}^c=\hat{\rho}(r^0,c^0)$ constitutes an equilibrium with credit rationing if $L_D(\hat{\rho}^0)>L_D(r^0,c^0)$.[4)]

1) Thus an equilibrium in the credit market with imperfect information differs with a screening equilibrium in the labor market (see Stiglitz [5] and Spence [4]).

2) We use the notation $\hat{\rho}\ (r,c)$ to denote an average expected gross return. It differs from the expected gross return from any given borrower $\bar{\rho}(r,\ c;\ \theta)$.

3) Except that Ls increases with $\hat{\rho}$, Ls is determined outside of this model. Given $\hat{\rho}$, Ls might increase if real income of individual households increases.

4) In an equilibrium with credit rationing, all the competitive banks earn normal profits and they have no incentive to change either r^0 or c^0 even in the presence of an excess demand for loans. We implicitly assumed that all the banks offer the same

Suppose now there exists an upper limit on the required collateral, $\bar{c}(\bar{c}<\mu)$. We now show that an equilibrium contract (r^0, c^0) is such that $r^0>\mu-1$ and $c^0=\bar{c}$. An equilibrium deposit rate of interest is then defined as $\hat{\rho}^0=\hat{\rho}\ (r^0, c^0)$. Suppose (r^0, c^0) is optimal while $c^0<\bar{c}$ and $r^0>\mu-1$. Whatever the required amount of collateral, r_0 cannot be less than $\mu-1$. Otherwise $\bar{\pi}(r_0, c_0;\theta)>0$ for all θ and the bank can increase r_0 to $\mu-1$ with no adverse selection effects. Let θ_0 denote the least risky project to be loan financed at an equilibrium. In a symmetric equilibrium that entails credit rationing, the project, θ_0 must yield only zero expected profits. Then any other firm that can undertake a riskier project $\theta'>\theta_0$ earns strictly positive expected profits by loan-financing. But at (r^0, c^0), the marginal rate of substitution of c for r is greater for θ' than for θ_0. Then if we decrease collateral and increase interest rate along $I(\theta_0)$, $\bar{\pi}(r, c;\theta')$ decreases for $\theta'>\theta_0$. Since $\bar{\pi}(r, c;\theta')+\bar{\rho}(r, c;\theta')=\mu$, it follows that the bank's return $\bar{\rho}(r, c;\theta')$ increases while $\bar{\rho}(r, c;\theta_0)$ is always fixed at μ. It follows that the bank optimal collateral requirement is always $\bar{c}$. It also follows that if there exists no upper limit on the collateral requirement, the bank will certainly offer a contract $(\mu-1, \mu)$ and will be able to earn $\hat{\rho}(\mu-1, \mu)=\mu$, the maximum average expected return. Assume now $\bar{c}<\mu$. $\hat{\rho}(r, \bar{c})$ is maximized by setting $\frac{\partial\hat{\rho}(r, \bar{c})}{\partial r}=0$. It is straightforward to see that

$$\frac{\partial\hat{\rho}(r, \bar{c})}{\partial r}=\frac{[\hat{\rho}(r, \bar{c})-\bar{\rho}(r, \bar{c}:\theta_0]g(\theta_0)\frac{\partial\theta_0}{\partial r}}{1-G(\theta_0)}+\frac{\int_{\theta_0}^{\infty}\frac{\partial\bar{\rho}(r, \bar{c};\theta)}{\partial r}dG(\theta)}{1-G(\theta_0)}. \tag{7}$$

Since $\hat{\rho}(r, \bar{c})<\bar{\rho}(r, c;\theta_0)$ and $\frac{\partial\theta_0(r, \bar{c})}{\partial r}>0$, the first term is certainly negative. If there exists $r^0=r^0(\bar{c})$ such that the first term dominates the second term for r greater than r^0, then certainly $\frac{\partial\hat{\rho}(r, \bar{c})}{\partial r}$ becomes negative. The

type of loan contract. It can be shown that in this type of an equilibrium, there does not exist a loan contract (r', c') and a deposit rate $\hat{\rho}'$ which attracts both depositors and firms. It is in fact a core of the market game (see Stiglitz and Weiss [6, pp. 219-20].

bank will not raise interest rate above $r^0(\bar{c})$ even in the presence of an excess demand for loans. The credit market can be characterized by an equilibrium with rationing. The upper limit on the required collateral, $\bar{c}$ may exceed the cost of the firm's project. Only if collateral consists of the firm's physical capital that it does not want to liquidate, our model of a competitive loan market becomes meaningful. If collateral consists of liquid assets, it certainly must be less than the amount of loan. If $c>1$, it is easy to see that every firm prefers self-financing the project to loan financing. Hence $c\leqq 1$. We have proved the following theorem.

Theorem 1: Suppose every firm has the identical amount of initial capital. Assume that projects are indivisible and yield the same mean return. Assume further that every firm is risk neutral and initial capital is less than the required amount of fund to finance the project. If the bank has no information about the riskiness of the project undertaken by an individual firm, the credit market can be characterized by an equilibrium with rationing. An equilibrium loan rate of interest is greater than the mean gross rate of return to the project but an equilibrium collateral is the firm's initial capital.

Hildegard Wette showed that increasing collateral requirement alone may result in an adverse selection effect and the theory of limited liability presented by Stiglitz and Weiss can be extended when the borrowing firms are risk neutral. Theorem 1 states that when the bank can freely choose both interest rate and collateral, an equilibrium level of collateral tends to be a theoretical upper ceiling, namely the required amount of fund to finance the project. We now search for the possibility of an equilibrium where a bank optimal strategy consists of a set of separating loan contracts, rather than a single contract. We first examine the following obvious lemma.

Lemma 1: If two firms, each undertaking the project θ_1 and θ_2 respectively where $\theta_1 < \theta_2$, face the same set of loan contracts, then the firm contemplating the riskier project θ_2 will never choose a contract requiring greater collateral.

Proof: Given the set of loan contracts, let (r_1, c_1) maximize $\bar{\pi}(r, c; \theta_1)$. For $i=1, 2$, let I_i denote an indifference curve passing through (r_1, c_1) such that at any point on I_i, each project θ_i yields the same expected profits. Then by definition, and from the property of an indifference curve, every loan contract must lie on or above I_1. Since the $\mathrm{MRS}_{c,r}(\theta)$ increases with θ, the indifference curve I_2 must cut I_1 from above, and the two curves cross only once. It implies that the set of preferred contracts by the firm undertaking θ_2 lying below I_2 and above I_1 is nonempty, which completes the proof.

Although the above lemma suggests the existence of a set of separating contracts that can differentiate each loan class, a set of separating contracts cannot be an equilibrium in the credit market unless it must be bank optimal among all feasible types of contract offers. The next theorem states that at least in a static model of a credit market with the discrete distribution of θ, each competitive bank need not search for separating contracts. However, the results obtained in this section will be modified when firms are risk averse.

Theorem 2: Assume that firms are risk neutral. For each set of bank optimal separating contracts, there exists a single contract offer which yields the greater expected return to the competitive bank.

Proof: Let the set of projects undertaken by firms be denoted as $\{\theta_i\}_{i=1}^{N}$, and the set of contracts $\tilde{C}$ offered by banks as $\tilde{C} = \{(\tilde{r}_i, \tilde{c}_i)\}_{i=1}^{N}$, where each firm financing θ_i chooses $(\tilde{r}_i, \tilde{c}_i)$. Assume $\theta_i < \theta_j$ for $i \leq j$. Then $\tilde{r}_i \leqq \tilde{r}_j$ and $\tilde{c}_j \leqq \tilde{c}_i$ whenever $i < j$. For $i \geq 2$, let $\tilde{\pi}(\tilde{r}_{i-1}, \tilde{c}_{i-1}; \theta_i) = \bar{\pi}_i$. Define an indifference curve $I_i = \{(r, c) \mid r \leqq c+1, \tilde{\pi}(r, c; \theta_i) = \bar{\pi}_i\}$ for $i = 2, \ldots, N$ and $I_1 = \{(r, c) \mid r \leqq c+1, \tilde{\pi}(r, c; \theta_1) = \tilde{\pi}(r, c, ; \theta_1)\}$. Let $P_{ij} = \{(r, c) \mid r \leqq c+1, \tilde{\pi}(r, c; \theta_i) \geqq \bar{\pi}_i$ and $\tilde{\pi}(r, c; \theta_j) \leqq \bar{\pi}_j\}$. Then P_{ij} denotes a set of (r, c) lying above I_i and below I_j (see Figure 2). Since each $(\tilde{r}_i, \tilde{c}_i)$ is the most preferred contract for θ_i, it follows immediately that $(\tilde{r}_i, \tilde{c}_i) \varepsilon P_{i-1,i}$ for $i = 2, 3, \ldots, N$. But then if $\tilde{C}$ maximize an average expected return, $\hat{\rho}(\tilde{C})$, to the bank, $(\tilde{r}_i, \tilde{c}_i)$ must lie on the curve I_i. This follows because otherwise an increase in collateral increases the bank's profits from any given borrower. Now if $(\tilde{r}_i, \tilde{c}_i) \in I_i$, $\hat{\rho}(\tilde{r}_i, \tilde{c}_i; \theta_i) \leqq \hat{\rho}(\tilde{r}_1, \tilde{c}_1, ; \theta_i)$ for $i \geqq 2$. It follows that $\hat{\rho}(\tilde{C}) \leqq \hat{\rho}(\tilde{r}_1, \tilde{c}_1)$, which completes the proof.

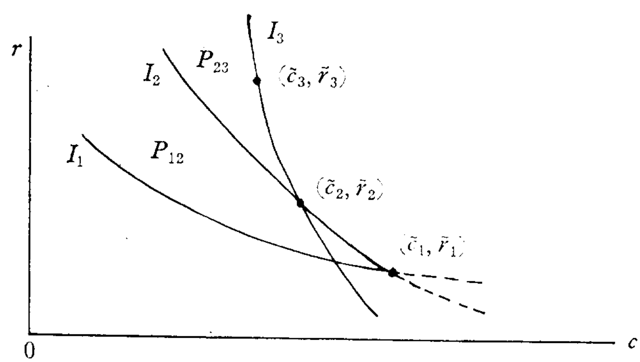

Fig. 2

III. Incentives, Rationing and an Equilibrium in the Credit Market

One way out of difficulties in modelling collateral encountered in the previous section is to assume that borrowers are risk averse. It is easy to see that an increase in collateral and a decrease in interest rate keeping expected profits constant is easily seen to represent a mean preserving spread of profits and lowers borrower's expected utility. The competitive bank may not be in a position to raise collateral at will. The adverse selection effect of an increase in collateral is more prominent when borrowers are risk averse.

In this section, we assume that borrowers are risk averse and banks have information about the initial wealth of each borrower. Although each competitive bank can classify loan applicants by their initial wealth, the bank is not informed about risk preference of each loan applicant.

Let W_0 denote the initial wealth and $V(U(W_0),\alpha)$, a borrower's Von Neuman-Morgenstern utility of wealth W_0, α being a degree of risk aversion. α denotes a risk preference and is assumed to be unobservable by the bank. Absolute risk aversion $R_A(W_0;\alpha)$ is defined as $\frac{d \log V(U(W),\alpha)}{dW}$. For notational convenience, we assume that $\frac{\partial R_A(W;\alpha)}{\partial \alpha}>0$. It implies that $\frac{\partial}{\partial \alpha}\left(\frac{V_{UU}}{V_U}\right)>0$, where $V_U=\frac{\partial V}{\partial U}$ and $V_{UU}=\frac{\partial^2 V}{\partial U^2}$. It is convenient to assume $V(U(W),\ 0)=U(W)$, $V_U>0$ and $V_{UU}<0$, while $U'(W)>0$ and $U''(W)<0$. Since the order of partial differentiation can be interchanged, $\frac{\partial}{\partial \alpha}\left(-\frac{V_{UU}}{V_U}\right)=\frac{\partial}{\partial U}\left(-\frac{V_{\alpha U}}{V_U}\right)>0$, where $V_{\alpha U}=\frac{\partial}{\partial \alpha}\left(\frac{\partial V}{\partial U}\right)$. Let R denote the gross return to the project when successful. Let $P(R)$ denote the probability of success and assume $P'(R)<0$. As in the previous section, the project is indivisible and the required amount of loan financing is normalized to unit.

We further assume that $W_0<1$ and neglect the possibility of self-finance. Each borrower chooses her own project and applies for loans to maximize expected utility. Let ρ denote a return to the depositors. Define

$$S(r,c;\alpha)=\operatorname*{Max}_{R}\{P(R)\,V(U(W_0\rho+R-1-r);\ \alpha)+(1-P(R))\,V(U((W_0-c)\rho);\alpha)\}. \tag{8}$$

Then every individual borrows from the bank if and only if

$$S(r,\ c;\alpha)\geqq V(U(W_0\rho);\alpha). \tag{9}$$

Define $\pi(R,r,c)=\mathrm{Max}\ [R-1-r,\ -c\rho]$.

Then $S(r, c; \alpha) = \underset{R}{\text{Max}}\ E[V(U(W_0\hat{\rho} + \pi(R, r, c)); \alpha)]$.

Given the interest rate r and the amount of collateral c, let R_α maximize $S(r, c; \alpha)$. Let α_0 denote the degree of a borrower's risk aversion defined as

$$S(r, c; \alpha_0) = V(U(W_0\hat{\rho}); \alpha_0). \tag{10}$$

We present the following lemma. (see the Appendix for the proof)

Lemma 2: $\left.\frac{\partial S(r, c; \alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0} < \left.\frac{\partial V(U(W_0\hat{\rho}); \alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0}$.

For any α and c, define $r(\alpha, c)$ as

$$S(r(\alpha, c),\ c; \alpha) = V(U(W_0\hat{\rho}); \alpha). \tag{11}$$

Since $\frac{dS(r, c; \alpha)}{dr} < 0$, $r(\alpha, c)$ is the maximum rate of interest that will induce a borrower with risk aversion, α to apply for loans. The following theorem is now immediate.

Theorem 3: $\frac{\partial r(\alpha, c)}{\partial \alpha} < 0$ for any α and c.

Proof: Differentiating (11) with respect to α, we have

$$\frac{\partial r(\alpha, c)}{\partial \alpha} = -\frac{\frac{\partial S(r, c; \alpha)}{\partial \alpha} - \frac{V(U(W_0\hat{\rho}); \alpha)}{\partial \alpha}}{\frac{\partial S(r, c; \alpha)}{\partial r}}. \tag{12}$$

Using Lemma 2, it follows at once that $\frac{\partial r(\alpha, c)}{\partial \alpha} < 0$.

Define now an indifference curve

$$I(\alpha) = \{(r, c) \mid c \leqq r + 1,\ \ S(r, c; \alpha) = V(U(W_0\hat{\rho}); \alpha)\}. \tag{13}$$

Using the envelope theorem,

$$\begin{aligned} \frac{dS(r, c; \alpha)}{dr} &= \frac{\partial E[V(U(W_0\hat{\rho} + \pi(R_\alpha, r, c)); \alpha)]}{\partial r} \\ &\quad + \frac{\partial R}{\partial r} \cdot \frac{dE[V(U(W_0\hat{\rho} + \pi(R_\alpha, r, c)); \alpha)]}{dR} \\ &= \frac{\partial E[V(U(W_0\hat{\rho} + \pi(R_\alpha, r, c)); \alpha)]}{\partial r} < 0. \end{aligned} \tag{14}$$

In a similar way, we have

$$\frac{dS(r,c;\alpha)}{dc}=\frac{\partial E[V(U(W_0\rho+\pi(R_\alpha,r,c));\alpha)]}{\partial c}<0. \tag{15}$$

It follows that

$$-\frac{dr}{dc}\bigg|_{S(r,c;\alpha)=\text{constant}}<0 \tag{16}$$

so that an indifference curve $I(\alpha)$ is negatively sloped. Assume $V(U(O);\alpha)=0$. When $c=W_0$, and $r=\mu-1$, μ being a gross mean return to the project, the optimum project for each borrower is the sure project defined as $R=\mu$ and $P(R)=1$. Since $S(\mu-1,W_0;\alpha)=V(U(W_0\rho);\alpha)$, it follows that $(\mu-1,\ W_0)\in I(\alpha)$ for all α. The situation is well depicted in Figure 3.

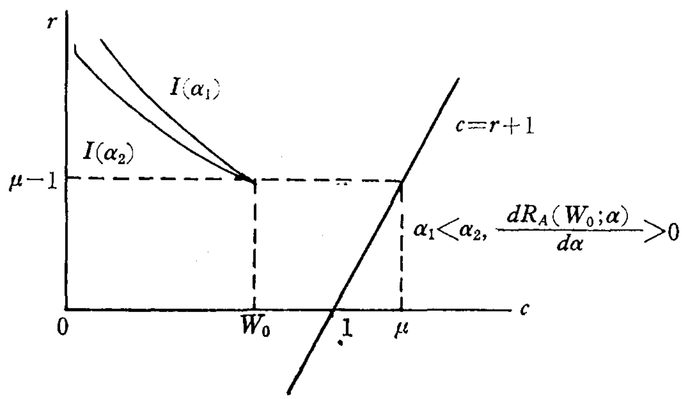

Fig. 3

Since $\frac{\partial r(\alpha,c)}{\partial\alpha}<0$ for any $c<W_0$, the indifference curve $I(\alpha_1)$ lies everywhere above $I(\alpha_2)$ whenever $\alpha_1<\alpha_2$ except at $(\mu-1,\ W_0)$. The following theorem is also immediate. We omit the proof.

Theorem 4: Assume that banks offer a single contract (r,c) such that $r>\mu-1$ and $c<W_0$. Given any contract (r,c), there exists a critical value of $\hat{\alpha}=\hat{\alpha}(r,c)$ such that only those borrowers with absolute risk aversion α less than $\hat{\alpha}(r,c)$ apply for loans. Furthermore $\frac{\partial\hat{\alpha}}{\partial r}<0$ and $\frac{\partial\hat{\alpha}}{\partial c}$ <0.

Up to this point, each borrower is assumed to take ρ, r, and c as given and to choose an optimum project R_α to maximize expected utility. Based on borrowers' behavior, the representative competitive bank chooses r, and c under imperfect information about risk preferences of borrowers. To analyze a bank optimal rate of interest and collateral requirement, we have to understand how the optimum project R_α chosen by an individual borrower depends on r and c. Let's define an optimum project R_α as a function of α, r and c so that $R_\alpha = R(\alpha, r, c)$. We now prove the following theorem on incentives. The proof is presented in the Appendix.

Theorem 5: Assume that borrowers are risk averse. The more risk averse an individual borrower is, the less risky project he undertakes. In other words, $\frac{\partial R(\alpha, r, c)}{\partial \alpha} < 0$ or $\frac{\partial P(R(\alpha, r, c))}{\partial \alpha} > 0$, where $P(R)$ denotes a probability of success. Furthermore $\frac{\partial R(\alpha, r, c)}{\partial r} > 0$ and $\frac{\partial R(\alpha, r, c)}{\partial c} < 0$.

The bank's return from any given brrower is

$$\bar{\rho}(r, c; \alpha) = P(R)(1+r) + c(1-P(R)), \tag{17}$$

where $R = R(\alpha, r, c)$.

Thanks to Theorem 4, the expected average return to the bank is written as

$$\hat{\rho}(r, c) = \frac{\int_0^{\hat{\alpha}} \bar{\rho}(r, c; \alpha)\ dG(\alpha)}{G(\hat{\alpha})}. \tag{18}$$

Let the demand for loanable funds be $L_D(r, c)$. Theorem 4 implies that $\frac{\partial L_D}{\partial r} < 0$ and $\frac{\partial L_D}{\partial c} < 0$ whenever $r > \mu - 1$ and $c < W_0$. When $r = \mu - 1$, and $c = W_0$, every borrower has to undertake the sure-project if feasible. We will mainly assume that the sure project is not technologically feasible so that $L_D(\mu - 1, W_0) = 0$. Otherwise it is trivial to see that the bank optimal rate of interest and collateral is given as $\mu - 1$ and W_0. The supply of loanable funds is assumed to be an increasing function of the competitively determined deposit rate of interest so that $L_S = L_S(\hat{\rho})$, $L_S'(\cdot) > 0$.[5] We introduce the following definition of an equilibrium in the

5) See the footnote 3.

credit market, which is essentially a variant of an equilibrium concept introduced already when borrowers are risk neutral.

Definition 2: (A) An equilibrium for the representative competitive bank is defined as

$$\underset{(r,c)}{\text{Max}} \frac{\int_0^{\alpha} \bar{\rho}(r,c;\alpha)\ dG(\alpha)}{G(\hat{\alpha})}, \text{ where } \hat{\alpha}=\hat{\alpha}(r,c),\ \frac{\partial\hat{\alpha}}{\partial r}<0 \text{ and } \frac{\partial\hat{\alpha}}{\partial c}<0. \quad (19)$$

Each bank takes $\hat{\rho}$ as given and decides on r and c to maximize $\hat{\rho}(r,c)$.

(B) An equilibrium for the banks as a whole is defined as $(r_0, c_0, \hat{\rho}_0)$. such that $\hat{\rho}_0=\hat{\rho}(r_0, c_0)$, where (r_0, c_0) is a solution to (19).

(C) An equilibrium for the banks as a whole is an Walrasian equilibrium if $L_D(r_0, c_0)=L_S(\hat{\rho}_0)$. It is an equilibrium with credit rationing if $L_D(r_0, c_0)>L_S(\hat{\rho}_0)$.

With these definitions in mind, we first examine an equilibrium for the competitive bank. Given $\hat{\rho}$, define $B(\alpha)=\{(r,c)\,|\,S(r,c;\alpha)\geqq V(U(W_0\hat{\rho});\alpha))\}$. Then whenever $(r,c)\in D(\alpha)$, an individual with risk aversion α applies for loans. Notice that Theorem 4 implies that $B(\alpha_2)\subset B(\alpha_1)$ if $\alpha_1<\alpha_2$. $B(\alpha)$ is depicted as a shaded region in Figure 4. When $r<\mu-1$ and $c<W_0$, every individual wishes to loan finance his project. Adverse selection effects of raising collateral or the loan rate of interest appear only if $r>\mu-1$. The following lemma is useful for understanding the bank optimal interest rate and collateral.

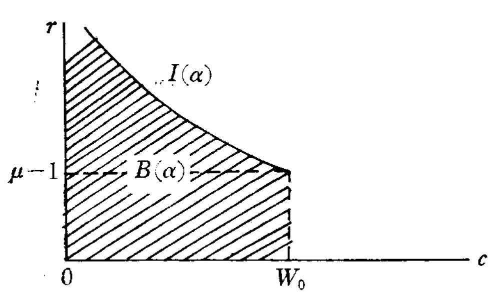

Fig. 4

Lemma 3: Assume that the sure project is not technologically feasible. Given any deposit rate $\hat{\rho}$, an equilibrium rate of interest exceeds the mean gross rate of return to the project. Furthermore the collateral requirement is less than the initial wealth of loan applicants.

Proof: Given any $\hat{\rho}$, let $(\hat{r}, \hat{c})$ solve (19). Then for any $\hat{c} \leqq W_0$, $\hat{r} \geqq \mu - 1$. Suppose $\hat{r} = \mu - 1$. Since

$$\frac{d\bar{\rho}(r, c; \alpha)}{dc} = P'(R) \text{ and } \frac{\partial R}{\partial c}(1 + \hat{r} - c) + 1 - P > 0, \tag{20}$$

competitive banks can increase collateral up to W_0 to maximize $\hat{\rho}(\hat{r}, c)$ defined in (18). Notice that $(\hat{r}, \hat{c}) \in B(\alpha)$ for all α and $\hat{c} < W_0$ so that an increase in collateral does not change the demand for loan. But if $\hat{c} = W_0$, every individual deposits his wealth since the sure project is not available. We conclude that $r > \mu - 1$. But then $\hat{c} > W_0$ since otherwise $L_D(\tilde{r}, \hat{c}) = 0$.

Suppose now there exists an equilibrium for the banks as a whole as defined in Definition (B). Lemma 3 implies that $r_0 > \mu - 1$ and $c_0 < W_0$. The following theorem is immediate from Theorem 4, 5 and Lemma 3.

Theorem 6: In either an Walrasian equilibrium or in an equilibrium with credit rationing, an equilibrium loan rate of interest exceeds the mean gross rate of return to the project while an equilibrium amount of collateral requirements is less than the initial wealth. In both types of an equilibrium, there exists a critical level of $\hat{\alpha} = \hat{\alpha}(r_0, c_0)$ such that only those borrowers with absolute risk aversion $R_A(W_0; \alpha)$ less than $R_A(W_0; \hat{\alpha})$ are able to finance their risky projects.

This theorem provides insights into the theory of limited liability. The required collateral in an equilibrium is less than the initial wealth in the presence of adverse selection effects of an increase in collateral. This can be well illustrated in the following equations. Given $r = r^0$,

$$\frac{\partial \hat{\rho}(r^0, c)}{\partial c} = \frac{\int_0^{\alpha} \frac{\partial \bar{\rho}(r^0, c; \alpha)}{\partial c} dG(\alpha) + \frac{\partial \hat{\alpha}}{\partial c} g(\alpha) [\bar{\rho}(r^0, c; \hat{\alpha}) - \hat{\rho}(r^0, c)]}{G(\hat{\alpha})}.$$

From (20), the first term is certainly positive while the second term is negative since $\frac{\partial\hat{\alpha}}{\partial c}<0$ and $\rho(r^0,c;\hat{\alpha})>\hat{\rho}(r^0,c)$. The latter inequality follows from the fact that $\frac{d\bar{\rho}(r,c;\alpha)}{d\alpha}=P'(R)\left(\frac{dR}{d\alpha}\right)(1+r-c)>0.$

We now finally examine the existence of a set of bank optimal multiple contracts such that when offered, each individual borrower who undertakes a less risky project is induced to use a self-selection strategy by differentiating himself with other borrowers undertaking riskier projects. As in risk neutrality case, the existence of a set of separating contracts depends on the following proposition. When we denote an interest function $\tilde{r}(c)$ as representing a set of contract offers, it must be clear that $\tilde{r}(c)$ is decreasing in c.

Proposition 1: Given the same set of contract offers, the less risk averse borrower will never choose a greater amount of collateral requirement than the more risk averse borrower.

Proof: The proof of the above proposition is essentially the same as the proof of Lemma 1. We only prove that at any (r,c), the marginal rate of substitution of collateral for interest rate $MRS_{c,r}(\alpha)$ decreases as the degree of risk aversion α increases. From the definition of $S(r,c;\alpha)$, $MRS_{c,r}(\alpha)$ is defined as

$$-\frac{dr}{dc}\bigg|_{S(r,c;\alpha)=\text{constant}}=\frac{(1-p)U_2'\hat{p}}{pU_1'}, \tag{21}$$

where $U_1=U(W_0\hat{p}+R-1-r)$ and $U_2=U((W_0-c)\hat{p})$.

Noticing that $R_\alpha=R(\alpha;r,c)$ and $\frac{\partial R}{\partial\alpha}<0$, we have

$$\frac{d}{d\alpha}\left(-\frac{dr}{dc}\bigg|_{S(r,c;\alpha)=\text{constant}}\right)$$
$$=\frac{-\frac{\partial R}{\partial\alpha}[pp'U_1'U_2'\hat{p}+(1-p)p'U_1U_2\hat{p}+p(1-p)U_1''U_2']}{(pU_1')^2}<0, \tag{22}$$

which completes the proof.

The existence of a bank optimal interest rate function $\tilde{r}(c)$ requires stringent assumptions on the risk preference of borrowers. This is because the isoprofit curve of the bank is not well defined. Consider a set of (r, c) which yields the same return to the bank from any given borrower, α. Then

$$-\frac{dr}{dc}\bigg|_{\bar{\rho}(r,c;\alpha)=\text{constant}}=\frac{(1-p)+p'\frac{\partial R}{\partial c}(1+r-c)}{p+p'\frac{\partial R}{\partial r}(1+r-c)}.$$

The sign of numerator is certainly positive, but the sign of denominator is indeterminate. Although we will not go into details of the existence question, we show that in the risk averse case, banks may use a self-selection strategy rather than a single contract. It is in contrast to the results obtained when borrowers are risk neutral. Let $(r^0.c^0)$ denote a bank optimal contract when banks are constrained to choose only a single contract. For illustrations, suppose only two types of borrowers, α^0 and α' obtain loans. In figure 5, we draw indifference curves, I_{α_0} and $I_{\alpha'}$ of two types of borrowers passing through (r^0, c^0). Suppose $\alpha'<\alpha^0$. Then $I_{\alpha'}$ cuts I_α from above and two curves never cross again.

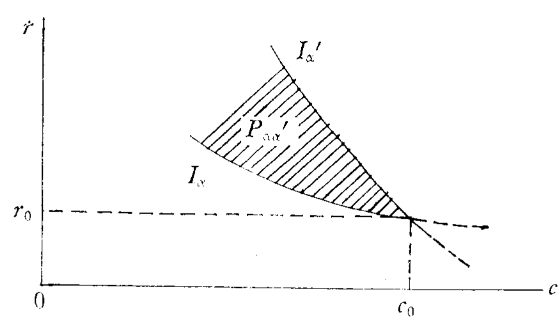

Fig. 5

Let $P_{\alpha\alpha'}=\{(r,c)\,|\,r\leq c+1,\ S(r,c;\alpha')\geq s(r_0,c_0;\alpha')\ \text{and}\ S(r,c;\alpha)\leq S(r_0,c_0;\alpha)\}$. Suppose there exists a loan contract (r', c') in $P_{\alpha\alpha'}$ such that $\bar{\rho}(r', c'; \alpha')>\bar{\rho}(r_0, c_0;\alpha')$. Then if banks offer a contract lying in the shaded region, type α' borrowers will prefer (r', c') to (r_0, c_0) while type α borrowers choose (r_0, c_0). The banks' profits certainly increase when they switch from a single contract to a self-selection strategy. Notice that without the incentive effects of raising collateral or interest rate on the choice of technology, there does not exist a contract in $P_{\alpha\alpha'}$ which yields greater return to banks from type α' borrowers. Existence of a self-selection strategy depends in an important way on incentive effects. But as we stated earlier, the isoprofit curve of the bank from any given borrower is theoretically indeterminate unless we impose more stringent assumptions on the preferences and a set of feasible technologies. But the above example helps us to understand why competitive banks often offer a set separating contracts which can be interpreted as a self-selection strategy.

III. Conluding Comments

Whether the credit market is characterized by an Walrasian equilibrium or by a rationing equilibrium, it is interesting to note that the theory of limited liability holds if an equilibrium loan rate of interest exceeds the mean gross rate of return to the project. We also showed that this conclusion depends upon the assumption on borrowers' risk preferences. When the borrowing firms are risk neutral and if banks choose both collateral and interest rate, banks require the firm's initial capital as collateral.

With imperfect information, the market fails in efficient allocation of resources. In any type of an equilibrium analyzed in the paper, only riskier projects are financed by the bank and the safer projects are systematically screened out of the credit market (we assumed all types of projects have the same mean return). What is more interesting in a market with imperfect information is a question of the existence of a self-selection strategy consisting of a set of separating contracts. We showed that it can be observed in the credit market only if borrowers are risk averse and incentive effects play a major role.

Finally, in a model of a single contract equilibrium a regulatory credit policy may be effective in increasing the loan rate of interest to equate the supply and demand for loanable funds. But the bank's profits will then decrease and a competitively determined deposit rate of interest decreases. The credit supply also decreases and the less risky projects financed previously are now screened out. Suppose now the loan rate of interest is regulated at the level below an equilibrium rate. The loan demand increases, while again the bank's profits decrease. The excess demand for loans increases further. Since an adverse selection effect is lessened by regulatory fiat, those borrowers who were screened out previously may be in a position to apply for loans. But there is no guarantee that they will be loaned.

Appendix

1. Proof of Lemma 2

At $\alpha=\alpha_0$, loan financing the project is a mean utility preserving spread in wealth (see (10)). Let $u=V(U(W_0\hat{p}+\pi(R_\alpha,r,c));\alpha_0)$. Let $v=V(W_0\hat{p}+\pi(R_\alpha,r,c))$. Then $u=V^{-1}(v,\alpha)$ ($V^{-1}(\cdot)$ denotes an inverse function), and $\frac{\partial V(u;\alpha)}{\partial\alpha}=V_\alpha(V^{-1}(v,\alpha);\alpha)$.

Since $\frac{\partial V_\alpha(V^{-1}(v,\alpha);\alpha)}{\partial v}=\frac{V_{\alpha U}}{V_U}$, $\frac{\partial^2 V_\alpha(V^{-1}(v,\alpha);\alpha)}{\partial v^2}=\frac{\partial}{\partial v}\left(\frac{V_{\alpha U}}{V_U}\right)$

$=\frac{1}{V_U}\frac{\partial}{\partial U}\left(\frac{V_U}{V_U}\right)<0$. It shows that $V_\alpha(V^{-1}(v,\alpha);a)$ is concave in v.

It follows that

$$E[V_\alpha(V^{-1}(v,\alpha);\alpha)]<V_\alpha(V^{-1}(E^v,\alpha);\alpha)=V_\alpha(V^{-1}(V(U(W_0\hat{p});\alpha),\alpha);\alpha)$$

$=V_\alpha(U(W_0\hat{p});\alpha)$, which completes the proof.

2. Proof of Theorem 5

The first order condition for maximizing expected utility is

$$PV_uU_1'+P'(V(U_1;\alpha)-V(U_2;\alpha))=0, \tag{A1}$$

where $U_1=U(W_0\hat{\rho}+R-1-r)$ and $U_2=U(W_0-c)\hat{\rho})$. Let $B\equiv PV_U\cdot U_1'+P'(V(U_1;\alpha)-V(U_2;\alpha))$. Assuming that the second order condition is fulfilled, we have $\frac{\partial B}{\partial R}<0$. Differentiating (A1) with respect to R and α, we have

$$\operatorname{sign}\left(\frac{\partial R}{\partial \alpha}\right)=\operatorname{sign}[PV_{\alpha U}U_1'+(V_\alpha(U_1;\alpha)-V_\alpha(U_2;\alpha))P']. \tag{A2}$$

Using (A1) again and noticing that $PV_UU_1'>0$, it follows that

$$\operatorname{sign}\left(\frac{\partial R}{\partial \alpha}\right)=\operatorname{sign}\left(\frac{V_{\alpha U_1}}{V_{U_1}}-\frac{V_\alpha(U_1;\alpha)-V_\alpha(U_2;\alpha)}{V(U_1;\alpha)-V(U_2;\alpha)}\right). \tag{A2$'$}$$

We first notice that $\lim_{U_1\to U_2}\frac{V_\alpha(U_1;\alpha)-V_\alpha(U_2;\alpha)}{V(U_1;\alpha)-V(U_2;\alpha)}=\frac{V_{\alpha U_2}}{V_{U_2}}$, (A3)

which implies that if $U_1=U_2$, $\frac{\partial R}{\partial \alpha}=0$. If the bracked term in the right hand side of equation (A2$'$) decreases as U_1 increases from U_2, the theorem is proved. But (A3) implies that

$$\frac{\partial}{\partial U_1}\left[\frac{V_{\alpha U_1}}{V_{U_1}}-\frac{V_\alpha(U_1;\alpha)-V_\alpha(U_2;\alpha)}{V(U_1;\alpha)-V(U_2;\alpha)}\right]\Bigg|_{U_1=U_2}=\frac{\partial\left(\frac{V_{\alpha U_1}}{V_{U_1}}\right)}{\partial U_1}. \tag{A4}$$

From the assumption that $\frac{\partial R_A(W_0;\alpha)}{\partial \alpha}>0$, it is obvious that $\frac{\partial\left(\frac{V_{\alpha U_1}}{V_{U_1}}\right)}{\partial U_1}=-\frac{\partial}{\partial \alpha}\left(-\frac{V_{U_1U_1}}{V_{U_1}}\right)>0$. The first part of the proof is now completed. The second half of the theorem follows from

$$\frac{\partial R(\alpha,r,c)}{\partial c}=-\frac{P'V_UU_2'}{B}<0. \tag{A5}$$

$$\text{and}\quad \frac{\partial R(\alpha,r,c)}{\partial r}=\frac{PV_{U_1}U_1'^2+PV_{U_1}U_1''+P'V_{U_1}U_1'}{B}>0. \tag{A6}$$

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