Specifications and Forecasts of Money Demand Functions

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《Abstract》

Hamburger's (1977) money demand function outperformed other specifications in post-sample simulations. This paper attempts to find the sources of his success through statistical comparison with Goldfeld (1976) models. We found that restrictions embedded in his function are strongly rejected in the estimation sample, but substantially improve forecasting accuracy. A probable cause of this seemingly conflicting result is discussed. The role of long-term rate is not clear. A comparable or better forecasting accuracy can be achieved with a moving average of short-term rate or a time trend.

1. Introduction

In his extensive examination of the specification of demand for money (M_1) Goldfeld (1976) attempted to find, without much success, "an improved specification…capable of explaining the current shortfall in money demand [Goldfeld (1976, p. 725)]." His "conventional" money demand function or its variations have failed to explain adequately the post-1973 behavior of money demand in his simulation experiments. The root-mean-square error (RMSE) of forecasts in these experiments were excessively large. Summarizing his results he concludes that some sort of shift in parameters has occured, and that this increase in instability of money demand "should tilt policy in the direction of an interest-rate policy [Goldfeld (1976, p. 728)]."

A challenge to this conclusion came from Hamburger (1977), who

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proposed a "more general money demand function." He replaced the short-term rate of interest with a long-term rate, and added the dividend-price ratio on common stocks as a proxy for the rate of return on equity. The functional form of his specification is the same as that of Goldfeld nominal adjustment model, except for an additional restriction of unitary long-run real income elasticity of demand for nominal money stock. These modifications produced a remarkable improvement in the RMSE of post-1972 simulation, though the in-sample fit was not as good as the conventional equation. This improvement has led him to conclude that Goldfeld's problem was "not so much to the instability in the public's asset preferences but rather to the restrictive specifications of the functions employed [Hamburger (1977, p. 265)]." Hamburger seems to ascribe his success to the use of the long-term rate instead of the short-term rate and the use of the rate of return on equity as an additional regressor.

Although Hamburger function may be considered "more general" in using the rate of return on equity as an additional regressor, his function imposes more restrictions on certain elasticities of money demand, Thus, the term "more general" is rather misleading. A function, which is truely more general in the sense that it includes both Goldfeld and Hamburger functions as a subset, does not perform as well as these special functions in its predictions. This suggests that the restrictions on elasticities embedded in Hamburger function may be a more important source of his success.[1]

This paper attempts to identify and analyze the sources of the success of Hamburger money demand function through statistical comparisons with Goldfeld real and nominal adjustment models.

There are two basic differences between these money demand functions: difference in the sets of interest rates used, and difference in functional forms. These differences are discussed in section 2. It is shown there

1) Hafer and Hein (1979) also suggest that Hamburger's restriction of unitary long-run income elasticity may be the source of the success of his function.

that different functional forms impose different restrictions on the income and price level elasticities of demand for nominal money stock.

These restrictions are tested in section 3. An interesting result of these tests is that the elasticity restrictions of Hamburger function are strongly rejected in the estimation sample period, while the restriction in Goldfeld nominal adjustment model with the same set of interest rates is easily accepted. Furthermore, with the same set of interest rates, Hamburger function is a nested subset of Goldfeld nominal adjustment model, and the test rejects Hamburger function in favor of Goldfeld model.

Although Hamburger's elasticity restrictions are statistically rejected in the estimation sample period, these unacceptable restrictions substantially improve forecasting accuracy. The magnitude of this improvement in terms of the change in the RMSE seems to be much larger than the improvement due to the use of the long-term rate. This is an interesting example of the conflict between conventional specification tests and the mean square error criterion. A possible cause of this conflict is discussed in section 4.

On the choice between the short-term and the long-term rate of interest the conventional test again favors the short-term rate. However, Hamburger prefers the long-term rate obviously because it yields a smaller RMSE of forecasts. It is certainly disputable to use the RMSE of forecasts as the criterion of selecting regressors. If we adopt this criterion, both the short and long-term rates should be excluded from the money demand function because, as shown in section 5, a function without these rates yields a smaller RMSE of forecasts. Section 5 also shows that, with a proper adjustment for the distributed lag structure, the short-term rate can generate a RMSE of forecasts which is comparable with or better than that of Hamburger function. Section 6 summarizes the discussion.

2. Specification

A general specification of money demand function which includes as a subset Goldfeld real adjustment and nominal adjustment models and Hamburger function can be written as

$$\log(M/P)=\beta_0+\beta_1\log RCP+\beta_2\log RTD+\beta_3\log RGL+\beta_4\log DPR$$
$$+\beta_5\log Y+\beta_6\log(M_{-1}/P_{-1})+\beta_7\log(P/P_{-1})+u, \qquad (2.1)$$

where M is the nominal money stock (M_1), P the implicit GNP deflator, Y real income (GNP), RCP the commercial paper rate, RTD the rate on time deposits, RGL the yield on long-term government bonds, and DPR the divident-price ratio of common stocks. u is the first-order autoregressive error term.

Certain restrictions on (2.1) yieild Goldfeld and Hamburger functions. The restrictions we consider are short-run and long-run income elasticities (denoted by SRE_y and LRE_y, respectively) and price level elasticities (denoted by SRE_p and LRE_p, respectively) of demand for *nominal* money stock. In (2.1), $SRE_y=\beta_5$, $LRE_y=\beta_5/(1-\beta_6)$, $SRE_p=1+\beta_7$, and $LRE_p=1$. The only restriction in (2.1) is the unitary long-run price level elasticity.

Goldfeld real adjustment model imposes on (2.1) the unitary SRE_p (i.e., $\beta_7=0$):

$$\log(M/P)=\beta_0+\beta_1\log RCP+\beta_2\log RTD+\beta_3\log RGL+\beta_4\log DPR$$
$$+\beta_5\log Y+\beta_6\log(M_{-1}/P_{-1})+u. \qquad (2.2)$$

Further restrictions on (2.2) for the choice of interest rates, i.e., $\beta_3=\beta_4=0$ yield Goldfeld real adjustment equation.

Goldfeld nominal adjustment model imposes on (2.1) restriction $LRE_y=SRE_y/SRE_p$ (i.e., $\beta_6+\beta_7=0$):

$$\log(M/P)=\beta_0+\beta_1\log RCP+\beta_2\log RTD+\beta_3\log RGL+\beta_4\log DPR$$
$$+\beta_5\log Y+\beta_6\log(M_{-1}/P)+u. \qquad (2.3)$$

Further restrictions $\beta_3=\beta_4=0$ yield Goldfeld's nominal adjustment equation.

Hamburger model imposes on (2.1) restrictions $SRE_y=SRE_p$ (i.e., $\beta_5-\beta_7=1$), and $LRE_y=1$ (i.e., $\beta_5+\beta_6=1$):

$$\log(M/PY)=\beta_0+\beta_1\log RCP+\beta_2\log RTD+\beta_3\log RGL$$
$$+\beta_4\log DPR+\beta_6\log(M_{-1}/PY)+u. \qquad (2.4)$$

A further restriction $\beta_1=0$ yields Hamburger function.

It is to be noted that Hamburger model (2.4) is a nested subset of

Goldfeld nominal adjustment model (2.3). To make (2.3) more comparable with (2.4) we may rewrite (2.3) as

$$\log(M/PY)=\beta_0+\beta_1\log\mathrm{RCP}+\beta_2\log\mathrm{RTD}+\beta_3\log\mathrm{RGL}$$
$$+\beta_4\log\mathrm{DPR}+\beta_5{}^*\log Y+\beta_6\log(M_{-1}/PY)+u. \quad (2.3')$$

where $\beta_5{}^*=\beta_5+\beta_6-1$.

For expositional convenience we will name the restrictions as follows:

Model Restrictions

H_1 : $\mathrm{SRE}_p=1$ $(\beta_7=0)$

H_2 : $\mathrm{LRE}_y=\mathrm{SRE}_y/\mathrm{SRE}_p$ $(\beta_6+\beta_7=0)$

H_3 : SRE=SRE and LRE=1 $(\beta_6-\beta_7=1$ and $\beta_5+\beta_6=1)$

Interest Rate Restriction

R_1 : $\beta_1=0$, R_2 : $\beta_3=0$, R_3 : $\beta_4=0$

3. Tests of Restrictions

In this section we conduct conventional tests of restrictions discussed in section 2. Equations (2.1) through (2.4) are estimated by Cochrane-Orcutt iterative method[2] for the sample period II/1955-IV/1972, and the results are presented as EQ. (3.1)-(3.4) in Table 1. The last column of this table presents the root-mean-square error of dynamic post-sample simulations for the period I/1973-IV/1977. The simulation errors are computed following the procedure used in Hafer and Hein (1979), who kindly provided all data for this study.

We can test model restrictions H_1, H_2 and H_3 from the unrestricted equation (3.1) in Table 1. Goldfeld restriction H_1 is strongly rejected and his restriction H_2 is easily accepted with t-test value of 1.256. Hamburger restrictions H_3 have an F-test value of 2.691 with the marginal significance level 14.7 percent.

We can also test restriction H_3 against H_2 by testing $\beta_5{}^*=0$ in (2.3′). The t-test value of this hypothesis is -2.020, which is significant at 5

2) Admittedly, these estimates are not consistent and the usual asymptotic theory does not apply. But, this is not the issue of the paper.

percent level. Thus, H_3 is rejected in favor of H_2.

The estimated equation (3.1) in Table 1 also shows that interest rate restrictions R_1 and R_3 are strongly rejected and restriction R_2 is accepted. The same result follows regardless of the presence of model restrictions H_1, H_2 and H_3. Thus, neither Goldfeld's nor Hamburger's restrictions of interest elasticities are acceptable. As Hamburger emphasized the rate of return on equity (DPR) carries a highly significant coefficient and hence should be included as one of the regressors. On the other hand, there is no apparent reason to prefer the long-term rate to the short-term rate as in Hamburger equation. Hamburger's reason of using the long-term rate was to avoid multicollinearity. But, the conventional test favors using the short-term rate. Notice that the coefficient of the long-term rate is extremely insignificant in all specifications (3.1)-(3.4). Exclusion of RGL from these equations does not have any significant effect on the estimates of coefficients, nor on the RMSE's of the post-sample simulations.

Although the above test result is in favor of RCP, it is interesting to investigate the effect of excluding RCP as in Hamburger equation. The regression results of (2.1)-(2.4) without RCP term are presented in Table 1 as eq. (3.5)-(3.8). There are no drastic changes in the estimates of coefficients except for that of RGL, which now becomes relatively significant. However, there is a significant change in the validity of model restrictions. Goldfeld restriction H_1 is still strongly rejected with t-test value of 3.445 and his restriction H_2 is easily accepted as before with t-test value of 1.032. But, Hamburger restriction H_3 is now strongly rejected with F-test value of 4.607 and marginal significance level of 1.4 percent. The test of restriction H_3 against H_2 now more strongly rejects H_3 in favor of H_2 with t-test value of -2.976.

All these conventional tests indicate that Goldfeld nominal adjustment model with DPR as an additional regressor is the best among the three models we consider. This is true whether we include both RCP snd RGL, or only one of them.

Table 1.

EQ.	DEP. VAR.	Regressors[d] Const.	RCP	RTD	RGL	DPR	Y	M-1/P-1	P/P-1	ρ	SEE	RMSE[c]
3.1	M/P	−0.686 (0.169)	−0.014 (0.004)	−0.035 (0.011)	0.001 (0.011)	−0.016 (0.007)	0.141 (0.035)	0.771 (0.078)	−0.589 (0.155)	0.529	0.0036	20.997 (20.896)
3.2	M/P	−0.773 (0.175)	−0.013 (0.004)	−0.039 (0.011)	−0.005 (0.010)	−0.014 (0.007)	0.164 (0.036)	0.695 (0.077)		0.417	0.0039	23.297 (23.389)
3.3[a]	M/P	−0.742 (0.166)	−0.014 (0.004)	−0.038 (0.011)	0.002 (0.011)	−0.016 (0.008)	0.152 (0.035)	0.749 (0.078)	−0.749 (0.078)	0.564	0.0036	20.740 (20.714)
3.4[b]	M/P	−0.507 (0.119)	−0.016 (0.004)	−0.024 (0.008)	0.004 (0.011)	−0.016 (0.007)	0.097 (0.022)	0.903 (0.022)	−0.903 (0.022)	0.502	0.0037	16.934 (16.404)
3.5	M/P	−0.729 (0.183)		−0.035 (0.012)	−0.018 (0.011)	−0.024 (0.008)	0.159 (0.038)	0.698 (0.084)	−0.544 (0.158)	0.630	0.0038	15.519
3.6	M/P	−0.852 (0.190)		−0.043 (0.012)	−0.023 (0.011)	−0.021 (0.008)	0.187 (0.039)	0.617 (0.083)		0.562	0.0041	18.676
3.7[a]	M/P	−0.771 (0.180)		−0.037 (0.012)	−0.018 (0.011)	−0.025 (0.008)	0.169 (0.037)	0.671 (0.083)	−0.671 (0.083)	0.668	0.0038	15.424
3.8[b]	M/P	−0.441 (0.136)		−0.019 (0.009)	−0.019 (0.010)	−0.024 (0.008)	0.089 (0.025)	0.911 (0.025)	−0.911 (0.025)	0.549	0.0040	5.531

a. Goldfeld's constraint $LRE_y = SRE_y / SRE_p$ is imposed.
b. Hamburger's constraints $SRE_y = SRE_p$ and $LRE_y = 1$ are imposed.
c. The RMSE in the parantheses are the RMSE of each equation without RGL term.
d. Estimated standard errors of coefficients are in parantheses.

However, if we use prediction accuracy measured by the RMSE as the criterion to evaluate the models, Hamburger model restriction H_3 yields the best result regardless of the choice between RCP and RGL. The performance of Hamburger equation, eq. (3.8), is truely outstanding. This has led him to claim that his specification is better than other models, and that the long-term leads to a more stable money demand function than the short-term rate.

Hamburger's claim is obviously in conflict with the conventional test results. This conflict raises two questions about the success of Hamburger equation (3.8). Even if we agree with his choice of interest rate RGL to avoid possible multicollinearity, we have seen that his model restriction H_3 is strongly rejected, suggesting that eq. (3.5) or (3.7) should be used. However, unacceptable restriction H_3 reduces the RMSE by almost 10 points. This is rather a counter-intuitive result. Intuitively, we may expect a smaller RMSE with a valid restriction.

Secondly, if we have to choose between RCP and RGL to avoid multicollinearity, why is RGL much more successful than RCP in prediction? Judging from unrestricted equations exclusion of RCP reduces the RMSE by about 5 points [from eq. (3.1) to eq. (3.5)], while exclusion of RGL from eq. (3.1) does not have much effects.

In the following sections we suggest some explanations to the causes of these unexpected results. But, it does not by any means imply that our suggestions are the only or true causes. It is possible that the success of Hamburger's equation comes from some other sources.

4. Effects of Restrictions

When parameters are stable over periods, we may expect an improvement in forecasting accuracy if the restrictions imposed in estimation of parameters are valid, because the restricted estimates are more efficient. But, what if we impose unacceptable or invalid restrictions in estimation of parameters? To investigate the effect on forecasting accuracy of imposing restrictions in parameter estimation, we will set up the problems formally as follows. Suppose we have two subsamples for a linear regression model

$$y_1 = X_1\beta + u_1,$$

$$y_2 = X_2\beta + u_2,$$

where X_1 and X_2 are matrices of observations on exogenous variables with dimensions $T_1 \times K$ and $T_2 \times K$, respectively. We assume that the error terms u_1 and u_2 have a common variance σ^2 and zero covariances.[3] We wish to predict y_2 based on an estimate of β from the first subsample. We consider a set of linear restrictions $R\beta = r$, where R is a $q \times K$ matrix of known constants with a full row rank, and r is a q-dimensional vector of known constants. Let $\tilde{\beta}$ and $\hat{\beta}$ be, respectively, the constrained and unconstrained estimates of β from the first subsample. The corresponding mean square errors of forecasts are denoted by $\mathrm{MSE}(\tilde{\beta})$ and $\mathrm{MSE}(\hat{\beta})$. Our purpose is to compare these mean square errors under various conditions. We first consider

Proposition 1. A sufficient condition for $E[\mathrm{MSE}(\tilde{\beta})] \leq E[\mathrm{MSE}(\hat{\beta})]$ is that $F \leq 1/q$, where $F = (r - R\beta)' \ [\sigma^2 R(X_1'X_1)^{-1}R']^{-1}(r - R\beta)/q$.

Proof. See Appendix 1.

It is clear that, if restrictrons are strictly valid ($R\beta = r$), then the MSE based on the constrained estimates is expected to be smaller than the MSE based on the unconstrained estimates. If restrictions are not strictly valid the value of F in Proposition 1 must be smaller than $1/q$. Notice that, if unknown parameters β and σ^2 are replaced with their unconstained estimates, F is simply the F-test statistic for testing restrictions $R\beta = r$. Thus, we may expect a smaller MSE from the constrained estimates if

3) The money demand functions include a lagged dependent variable as a regressor and the error terms are autocorrelated. Therefore, the model we consider in this section is not exactly same as the situation of money demand function. The following discussions may be extended for the case of a dynamic equation, but the conditions we derive in the propositions below are hopelessly complicate in that case and do not offer much intuitive guidance. The following discussion and propositions, therefore, should be taken as a suggestive guide rather than a solid theory for the case of dynamic equation.

the value of F-test statistic for $R\beta=r$ is small. On the other hand, if restrictions $R\beta=r$ are strongly rejected with a large F-test value, we would not normally expect the same result.

Notice that the condition in Proposition 1 is only a sufficient condition. Even if the condition is violated it is possible to have a smaller MSE(β), depending on the sample. The proposition does not imply that the rejected Hamburger restriction H_3 in (3.5) will not reduce the MSE. It simply says that we do not normally expect it to happen on the average.

Is there a situation in which we might expect a smaller MSE by using unacceptable restrictions? Our intuition suggests that if restrictions are valid in the forecasting period, restricted parameter estimates may produce more accurate forecasts, because these restricted estimates will be closer to the true parameter values in the forecasting period. So, consider again

$$y_1=X_1\beta_1+u_1,$$
$$y_2=X_2\beta_2+u_2,$$

where β_1 is now not necessarily equal to β_2. Suppose that $R\beta_2=r$, but $R\beta_1\neq r$. In this situation we have

Proposition 2. A sufficient condition for $E[\text{MSE}(\tilde{\beta}_1)]\leq E[\text{MSE}(\hat{\beta}_1)]$ when $R\beta_1\neq r$ and $R\beta_2=r$ is that $\beta_2=\beta_1+(X_1'X_1)^{-1}R'_1[R(X_1'X_1)^{-1}R']^{-1}R\eta$ for some real vector η.

Proof. See Appendix 2.

It should be emphasized again that the condition in Proposition 2 is only a sufficient condition. As the proof in Appendix 2 shows the constrained MSE can be smaller than the unconstrained one even if the condition is not satisfied.

It is rather difficult to comprehend the condition intuitively. The condition implies $R(\beta_2-\beta_1)=R\eta$. Let us consider a special case. Suppose that the condition is satisfied for $\eta=\beta_2-\beta_1$. Then, the condition becomes

$$\beta_2=\beta_1+(X_1'X_1)^{-1}R'[R(X_1'X_1)^{-1}R']^{-1}(r-R\beta_1)=E(\tilde{\beta}_1).$$

Therefore, MSE$(\tilde{\beta}_1)$ is expected to be smaller than MSB$(\hat{\beta}_1)$ if constrained estimate $\tilde{\beta}_1$ is an unbisased estimate of β_2 although it is a biased estimate of β_1. This confirms our intuition.

Propositions 1 and 2 show us that invalid parameter restrictions in estimation tend to generate less accurate forecasts if parameters are stable, but they may generate more accurate forecasts if restrictions are valid in the forecasting period. As noted earlier Hamburger restriction H_3 is rejected in (3.5), but improves forecasting accuracy significantly. Proposition 2 offers an explanation of a possible cause of this success. To check the relevance of Proposition 2 to the case of Hamburger restricition H_3 we tested[4)] the following hypotheses:

	Null Hypothesis	Alternative Hypothesis	F-test Value	Marginal Significance
(a)	$R\beta_1 \neq r,\ R\beta_2 = r$	$R\beta_1 \neq r,\ R\beta_2 \neq r$	0.308	0.74
(b)	$R\beta_1 = r,\ R\beta_2 = r$	$R\beta_1 \neq r,\ R\beta_2 \neq r$	2.240	0.07
(c)	$R\beta_1 = r,\ R\beta_2 = r$	$R\beta_1 \neq r,\ R\beta_2 = r$	4.254	0.02

These tests are not independent of each other and therefore do not provide independent information. But, the basic hypothesis posited in Proposition 2 appears to be reasonably acceptable.

It is shown earlier that the condition in Proposition 2 is satisfied if $E(\tilde{\beta}_1) = \beta_2$, which implies that $E(\hat{\beta}_1) \neq \beta_2$ when $R\beta_1 \neq r$. Intuitively, the condition will be satisfied if $\tilde{\beta}_1$ is better estimate of the true value of β_2 than $\hat{\beta}_1$ is. This may be checked out by comparing $(\tilde{\beta}_1 - \hat{\beta}_2)'(\tilde{\beta}_1 - \hat{\beta}_2)$ with $(\hat{\beta}_1 - \hat{\beta}_2)'(\hat{\beta}_1 - \hat{\beta}_2)$, where $\hat{\beta}_2$ is the unconstrained estimate[5)] of β_2 in the simulation period and is used as an estimate of true value of β_2. The value of the former is 0.092 and the value of the latter 0.557. Clearly, the constrained estimate $\tilde{\beta}_1$ is a better estimate of β_2 in this sense.

These results support the presumption that the situation posited in Proposition 2 is a possible source of the success of Hamburger equation in forecasting. It is interesting to notice that the usual Chow-test of parameter stability in restricted equation (3.8) will be biased toward acceptance of stability. The formal test of parameter stability between

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4) The following tests are based on eq. (3.5) in Table 1, and are conditional on the value of ρ equal to 0.630.

5) The estimate of β_2 in the simulation period uses the autocorrelation coefficient 0.630.

two sample periods shows the F-test value 1.42 ineequation (3.5) and 0.878 in equation (3.8).

5. Short-Term vs Long-Term Rate of Interest

Economic theory suggests that both short-and long-term interest rates will have some effects on the demand for money. Hamburger cited possible multicollinearity between RCP and RGL as a reason of excluding one of these rates from the equation.[6] He chose the long-term rate because it yields a smaller RMSE of forecasts. As shown in Table 1 the RMSE of forecasts of his model is 5.531 with RGL, and 16.404 with RCP.

It does not seem to be appropriate to choose an interest rate based on the comparison of these RMSE's, because it implicitly assumes that the patterns of distributed lag effects of the short-term and long-term interest rates are the same. There is no theoretical reason to expect the same shape of their distributed lag effects. To allow a different distributed lag pattern of the effect of RCP we may include lagged RCP terms in the equation. However, to maintain the same number of regressors we estimated Hamburger equation, eq. (3.8) in Table 1, with the 6-quarter moving average (RCP6) of RCP, and obtained

$$\log(M/P) = \underset{(0.131)}{-0.323} \underset{(0.006)}{-0.003}\log RCP6 \underset{(0.008)}{-0.017}\log RTD \underset{(0.009)}{-0.024}\log DPR$$

$$\underset{(0.023)}{+0.064}\log Y \underset{(0.023)}{+0.936}\log(M_{-1}/P_{-1}) \underset{(0.023)}{-0.936}\log(P/P_{-1}),$$

$$\rho = 0.516, \qquad SEE = 0.0041, \qquad RMSE = 6.640$$

The forecasting accuracy of this equation is very much comparable with that of Hamburger function. If we add to the above equation the time trend term to capture gradual changes in structure and preference of the financial market the RMSE of forecasts becomes even smaller than that of Hamburger function:

$$\log(M/P) = \underset{(0.287)}{-0.512} \underset{(0.006)}{-0.002}\log RCP6 \underset{(0.009)}{-0.018}\log RTD \underset{(0.010)}{-0.025}\log DPR$$

6) The sample correlation between RGL and RCP during the estimation period is only 0.07. Thus, Hamburger's reason of excluding RCP is not very convincing.

$$+0.098\log Y + 0.902\log(M_{-1}/P_{-1}) - 0.902\log(P/P_{-1}) - 0.0003\text{TIME},$$
$$\quad(0.051) \qquad\qquad (0.051) \qquad\qquad\qquad (0.051) \qquad\qquad\qquad (0.0003)$$

$$\rho=0.540, \qquad \text{SEE}=0.0041, \qquad \text{RMSE}=4.595$$

These examples demonstrate that forecasting accuracy is not unique only to the long-term rate. In fact, if forecasting accuracy is the criterion to select variables, we may even claim that neither RCP nor RGL are determinants of real money demand. Whem these interest rates are replaced by a time trend term the RMSE of forecasts becomes even smaller:

$$\log(M/P) = -0.509 - 0.018\log\text{RTD} - 0.027\log\text{DPR} + 0.098\log Y$$
$$\qquad\qquad (0.285) \; (0.009) \qquad\qquad (0.008) \qquad\qquad (0.051)$$

$$+0.902\log(M_{-1}/P_{-1}) - 0.902\log(P/P_{-1}) - 0.0003\text{TIME},$$
$$(0.051) \qquad\qquad\qquad (0.051) \qquad\qquad\qquad (0.0003)$$

$$\rho=0.546, \qquad \text{SEE}=0.0041, \qquad \text{RMSE}=3.336$$

When the trend TIME is eliminated from this equation the RMSE increases slightly to 4.449, but coefficients do not change much.

6. Conclusion

It was a remarkable achievement for Hamburger to find a specification of money demand function which outperformed any other specifications in forecasting recent movements of money demand. He attributed the source of his success to the use of the yield on equities and long-term rate of interest.

The yield on equities appears to play a significant role in his success. For example, without the DPR term, the RMSE of eq. (3.1) in Table 1 increases to 27.349 and that of eq. (3.5) increases to 22.915.

The role of the long-term rate is, however, not clear. The conventional test favors the short-term rate RCP. Hamburger's reason of excluding one of these rates, i.e., multicollinearity, is not convincing. He chose the long-term rate RGL apparently because it leads to a smaller RMSE of forecasts. If forecasting accuracy is the criterion of choosing variables, however, the 6-quarter moving average of RCP yields a RMSE quite comparable with that of RGL, and a smaller RMSE if the time trend is also added. Furthermore, a function without these rates yields a even better RMSE of forecasts. This indicates that forecasting accuracy is not unique to the long-term rate.

The most interesting result is perhaps the role of restrictions on the price level and income elasticities of demand for nominal money stock. These restrictions are strongly rejected in the estimation period, and yet substantially increase forecasting accuracy. A probable cause of this seemingly conflicting result is suggested in this paper.

Appendix 1

The constrained estimator $\tilde{\beta}$ is given by

$$\tilde{\beta}=\hat{\beta}+(X_1'X_1)^{-1}R'[R(X_1'X_1)^{-1}R']^{-1}(r-R\hat{\beta}), \qquad \text{(A. 1)}$$

where $\hat{\beta}=(X_1'X_1)^{-1}X_1'y_1$ is the unconstrained estimator of β. Let $Q=R(X_1'X_1)^{-1}R'$, $\tilde{y}_2=X_2\tilde{\beta}$, and $\hat{y}_2=X_2\hat{\beta}$. Then,

$$\begin{aligned} d &\equiv (\tilde{y}_2-y_2)'(\tilde{y}_2-y_2)-(\hat{y}_2-y_2)'(\hat{y}_2-y_2) \\ &=2(\hat{y}_2-y_2)'X_2(X_1'X_1)^{-1}R'Q^{-1}(r-R\hat{\beta})+(r \\ &\quad -R\hat{\beta})'Q^{-1}R(X_1'X_1)^{-1}X_2'X_2(X_1'X_1)^{-1}R'Q^{-1}(r-R\hat{\beta}). \end{aligned} \qquad \text{(A. 2)}$$

Using $\hat{\beta}=\beta+(X_1'X_1)^{-1}X_1'u_1$, $E(u_1u_1')=\sigma^2I$, and $E(u_1u_2')=0$, we can write, after some rearrangement,

$$E(d)=-tr(APA'), \qquad \text{(A. 3)}$$

where $A=X_2(X_1'X_1)^{-1}R'Q^{-1}$ and $P=\sigma^2Q-(r-R\beta)(r-R\beta)'$.

It is known [see Judge and Bock (1978), p. 29] that P is positive semi-definite if and only if $(r-R\beta)'(\sigma^2Q)^{-1}(r-R\beta)\leq 1$. Dividing both sides by q, and noting that the trace of a positive semi-definite matrix is nonnegative, we obtain the desired result.

Appendix 2

After adding subscript 1 to $\hat{\beta}$ in (A. 2) a straightforward algebra will give

$$E(d)=2tr[X_2(X_1'X_1)^{-1}R'Q^{-1}(r-R\beta_1)(\beta_1-\beta_2)'X_2']$$
$$-\sigma^2tr[AQA']+tr[A(r-R\beta_1)(r-R\beta_1)'A \qquad (A.4)$$

If $\beta_1=\beta_2$, (A.4) will be reduced to (A.3). Now, using $r=R\beta_2$ and rearranging terms in (A.4), we obtain

$$E(d)=(\beta_2-\beta_1)'W(\beta_2-\beta_1)-\sigma^2tr[AQA'], \qquad (A.5)$$

where $W=Z'X_2'X_2Z-X_2'X_2Z-Z'X_2'X_2$, and $Z=(X_1'X_1)^{-1}R'Q^{-1}R$. Notice that Z is a nonsymmetric idempotent matrix of rank q. Hence, $Z'WZ=-Z'X_2'X_2Z$. Since $Z'X_2'X_2Z$ is positive semi-definite, $Z'WZ$ is negative semi-definite. Therefore, if $\beta_2-\beta_1=Z\eta$ for some real vector η, $(\beta_2-\beta_1)'W(\beta_2-\beta_1)\leq 0$, and $E(d)\leq 0$ because $tr[AQA']$ is nonnegative. It is to be noted that $E(d)$ can be negative even if the sufficient condition $\beta_2-\beta_1=Z\eta$ is violated if $(\beta_2-\beta_1)'W(\beta_2-\beta_1)$ is less than $\sigma^2tr[AQA']$.

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