Is Household Heterogeneity Important for Business Cycles?*

Youngsoo Jang† Takeki Sunakawa‡ Minchul Yum§

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Abstract

Several seminal studies in the business cycle literature have found that heterogeneity at the micro level is not much relevant for aggregate dynamics at the macro level. In this paper, we present an incomplete markets environment in which household heterogeneity alters the dynamics of macroeconomic aggregates substantially. Specifically, we find that our heterogeneous-agent model, which incorporates a nonlinear government transfer schedule, accounts for the two well-documented facts on aggregate labor market fluctuations, both of which canonical real business cycle models are hard to explain: (1) weakly procyclical average labor productivity; and (2) a large cyclical volatility of aggregate hours relative to output. Our quantitative success is due to the interaction between household heterogeneity and the presence of government transfers in incomplete markets, and does not require additional exogenous shocks.

Keywords: Heterogeneity, government transfers, labor supply, business cycles

JEL codes: E32, E24, E21

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†Institute for Advanced Research, Shanghai University of Finance and Economics, China, e-mail: jangys724@gmail.com

‡Graduate School of Economics, Kobe University, Japan, e-mail: takeki.sunakawa@gmail.com

§Department of Economics, University of Mannheim, Germany, e-mail: minchul.yum@uni-mannheim.de.

1 Introduction

There has been growing interest in micro-level heterogeneity in the macroeconomics literature (see e.g., Heathcote, Storesletten, and Violante, 2009 for an extensive review). Although it is obviously crucial to incorporate household or firm heterogeneity in order to study distributional issues within a macroeconomic framework, it is less clear whether such heterogeneity at the micro level matters for aggregate dynamics at the macro level. In fact, according to seminal papers in the recent business cycle literature, it appears that incorporating micro-level heterogeneity has only limited impacts on the business cycle fluctuations of macroeconomic aggregates.[1]

This paper contributes to the literature by presenting an economic environment in which household heterogeneity considerably shapes the dynamics of aggregate economic variables. Our economic environment extends a standard incomplete-markets model with heterogeneous households by incorporating progressive government transfers. We find that our model accounts for the two long-standing anomalies on aggregate labor market fluctuations in the real business cycle literature: (i) weakly procyclical average labor productivity; and (ii) a large cyclical volatility of aggregate hours relative to output. We highlight that the key to our quantitative success is the interaction of household heterogeneity and the presence of government transfers in incomplete markets. Specifically, we require neither additional source of exogenous shocks to reduce the cyclicality of labor productivity (e.g., Benhabib, Rogerson, and Wright, 1991; Christiano and Eichenbaum, 1992; Braun, 1994; and Takahashi, 2017) nor the high curvature of utility function to obtain a large volatility of aggregate hours relative to output (see discussions in Keane and Rogerson, 2015).[2]

Our model economy is based on a standard incomplete markets model with heterogeneous households that make consumption-savings and extensive-margin labor supply decisions in the presence of both idiosyncratic productivity risk and aggregate risk (Chang and Kim, 2007; 2014).[3]

[1] Such studies include Krusell and Smith (1988), Thomas (2002), Khan and Thomas (2008) and Chang and Kim (2007; 2014) among others.

[2] Most existing quantitative theoretical explanations for acyclical labor productivity generally rely on the role of newly introduced stochastic processes. More specifically, Benhabib et al. (1991) consider home-production technology shocks; Christiano and Eichenbaum (1992) suggest government spending shocks; Braun (1994) introduces income tax shocks; and Takahashi (2017) incorporates idiosyncratic wage uncertainty shocks into a real business cycle model.

[3] This class of models in turn builds on a standard incomplete markets model without aggregate risk, pioneered by Imrohoroğlu (1988), Huggett (1993) and Aiyagari (1994).

Our model additionally incorporates progressive government transfers, captured by a nonlinear income-dependent transfer schedule. We calibrate our model economy to match salient features in the micro-level data including degree of progressiveness in the U.S. transfers from the Survey of Income and Program Participation data and the persistence of idiosyncratic wage risk from the Panel Study of Income Dynamics data.

We find that our baseline model features the aggregate labor market dynamics that differ considerably from its nested versions, abstracting from either government transfers (a model similar to Chang and Kim, 2007; 2014) or household heterogeneity (a model similar to Hansen, 1985). Specifically, we find several significant improvements in the business cycle statistics on aggregate labor market fluctuations. First, our baseline model with the nonlinear government transfer schedule generates considerably lower correlations of average labor productivity with output (0.40 vs 0.35 in the data) than the nested versions of the model (0.84 in the absence of government transfers and 0.79 in the absence of household heterogeneity). At the same time, in our baseline model, the cyclical volatility of aggregate hours relative to output is 0.94 in line with 0.91 in the data. This finding is particularly notable since household heterogeneity seems to make it more challenging to generate a large fluctuation of aggregate hours relative to output, according to the findings in the recent business cycle literature with household heterogeneity.[4]

To understand the mechanism underlying our quantitative success, we conduct impulse response exercises. We find that, in our baseline model following a negative aggregate productivity shock, a sharp fall in aggregate hours takes more time to return to its steady state level whereas a fall in average labor productivity is mitigated. This is in sharp contrast to the nested versions of the baseline model abstracting from either transfers or household heterogeneity, both of which show that aggregate hours recover more quickly (and even overshoot) whereas average labor productivity closely follows the inverse hump-shape of smooth consumption responses. In addition, we compute the impulse responses of aggregate hours by individual labor productivity to better understand the aggregate dynamics using the heterogeneous-agent models. We find that, in our baseline model, labor supply responses are generally stronger at the low and medium productivity levels.

[4] For example, Chang and Kim (2014) reports that the volatility of aggregate hours relative to output is 0.58 in their model with indivisible labor.

Furthermore, and more importantly, the employment rate among households with relatively low productivity levels falls in our baseline model whereas it stays nearly constant in the model without government transfers. This finding is consistent with the role of social insurance in dampening the precautionary motive of labor supply among the poor household, which in turn would make their labor supply more elastic (Yum, 2018). Since the labor supply of households with relatively higher productivity is highly inelastic in both specifications, this implies that our baseline model generates asymmetric labor supply responses across individual productivity whereas the model without transfers does not. This clearly shows that it is the interplay of heterogeneous labor supply responses and the presence of government transfers that is crucial for our quantitative success.

Our main result suggests that household heterogeneity at the micro level is important for the dynamics of macroeconomic variables. This result is broadly in line with recent papers such as Krueger, Mitman and Perri (2016) and Ahn, Kaplan, Moll, Winberry and Wolf (2017), both of which find that heterogeneity at the micro level is indeed relevant for the impact of aggregate shocks on macroeconomic variables.[5] Although the distribution of wealth plays an important role in all the studies, it is important to note that Krueger et al. (2016) and Ahn et al. (2017) focus on the consumption-savings channel whereas our paper focuses on the labor supply channel as a key mechanism through which micro-level heterogeneity matters for the business cycle fluctuations of macroeconomic aggregates such as labor productivity.

Our underlying mechanism regarding the labor supply channel in this paper builds upon Yum (2018) who finds that providing social insurance to the poor households who lack savings for self-insurance in the incomplete markets environment reduces their precautionary motives of labor supply, and makes their labor supply more elastic. Our results herein suggest that the presence of government transfers in this class of incomplete markets environments not only matters for the long run employment effects of labor taxes, as studied in Yum (2018), but also has important implications for the volatility of hours over the business cycle and the dynamics of other related macroeconomic aggregates.

Finally, we note that our quantitative business cycle model builds upon Chang and Kim (2007;

[5] See also Kim (2017) among others.

2014) yet differs from theirs in two major ways. First, as highlighted above, we bring the institutional feature of progressive government transfers, as observed in the micro-level data, into the model.[6] Second, we deal with selection problems in labor supply at the extensive margin and potential temporal aggregation bias (quarterly model vs. micro-level annual data) using the model simulation directly, whereas Chang and Kim (2007; 2014) deal with these issues outside the model using an econometric technique. Specifically, we use the simulated data where selection is endogenously taken care of within the model and then perform temporal aggregation using the simulated quarterly data to obtain the simulated annual data. Our calibration strategy leads to a fairly high persistence estimate of idiosyncratic shocks (in line with estimates in Heathcote, Storesletten and Violante, 2010 among others), which crucially interacts with the presence of government transfers in improving the performance of the incomplete-markets business cycle model.

The paper is organized as follows. Section 2 describes the model environment, defines equilibrium, and discusses the numerical solution methods. In Section 3, we describe how parameters are calibrated and show the steady-state properties of the model economy. Section 4 presents the main results from the quantitative analysis. Section 5 concludes.

2 Model

In this section, we describe the model environment of the business cycle models studied in this paper.

2.1 Baseline model

The baseline model economy extends Chang and Kim (2007, 2014) by incorporating labor taxes and nonlinear government transfers.

Households:

[6] Chang, Kim, and Schorfheide (2013) consider a version of the model in Chang and Kim (2007) with flat lump-sum transfers. However, given the different focus of their paper, they report limited number of standard business cycle statistics, which are rather the main focus of our paper.

The model economy is populated by a continuum of infinitely-lived households. It is convenient to describe the infinitely-lived household's decision problem recursively. At the beginning of each period, households are distinguished by their asset holdings a and productivity x_i. We assume that x_i takes a finite number of values N_x and follows a Markov chain with transition probabilities π^x_{ij} from the state i to the state j. In addition to the individual state variables, a and x_i, there are aggregate state variables including the distribution of households $\mu(a, x_i)$ over a and x_i and aggregate total factor productivity shocks z_k. We also assume that z_k takes a finite number of values N_z following a Markov chain with transition probabilities π^z_{kl} from the state k to the state l. We assume that the Markov processes for individual productivity and aggregate productivity capture the following continuous AR(1) processes in logs.

$$\log x' = \rho_x \log x + \varepsilon'_x \tag{1}$$

$$\log z' = \rho_z \log z + \varepsilon'_z \tag{2}$$

where $\varepsilon_x \sim N(0, \sigma_x^2)$ and $\varepsilon_z \sim N(0, \sigma_z^2)$. Finally, we assume competitive markets; in other words, households take as given the wage rate per efficiency unit of labor $w(\mu, z_k)$ and the real interest rate $r(\mu, z_k)$, both of which depend on the aggregate state variables. Households take as given government policies.

The dynamic decision problem of households can be written as the following functional equation:

$$V(a, x_i, \mu, z_k) = \max\left\{V^E(a, x_i, \mu, z_k), V^N(a, x_i, \mu, z_k)\right\}$$

where

$$V^E(a, x_i, \mu, z_k) = \max_{a' > \underline{a},} \left\{ \log c - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi^x_{ij} \sum_{l=1}^{N_z} \pi^z_{kl} V(a', x'_j, \mu', z'_l) \right\} \tag{3}$$

$$\text{subject to} \quad c + a' \leq (1-\tau) w(\mu, z_k) x_i \bar{n} + (1 + r(\mu, z_k)) a + T(m)$$

$$\mu' = \Gamma(\mu, z_k).$$

and

$$V^N(a, x_i, \mu, z_k) = \max_{a' > \underline{a},} \left\{ \log c + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \mu', z_l') \right\} \tag{4}$$

$$\text{subject to} \quad c + a' \leq (1 + r(\mu, z_k))a + T(m) \tag{5}$$

$$\mu' = \Gamma(\mu, z_k). \tag{6}$$

Households maximize utility by choosing optimal consumption c, asset holdings in the next period a', and labor supply n.[7] The labor supply decision is assumed to be discrete $n \in \{0, \bar{n}\}$. The total disutility of work is captured by $B\bar{n} > 0$. Households understand that the expected future value, discounted by a discount factor β, is affected by stochastic processes for individual productivity x' and aggregate productivity z' as well as the whole distribution μ'. The evolution of μ is governed by the law of motion in (6). The budget constraint states that the sum of current consumption c and asset demands for the next period a' should be less than or equal to the sum of net-of-tax earnings $(1-\tau)w(\mu, z_k)x_i n$, current asset holdings and capital income $(1 + r(\mu, z_k))a$, and government transfers $T(m)$, which may depend on the sum of labor and capital income:

$$m = w(\mu, z_k)x_i n + r(\mu, z_k)a.$$

Household face a borrowing limit $\underline{a} \leq 0$.

Government:

There is a government that taxes labor earnings at a fixed rate of τ. The government uses the collected tax revenue to finance transfers $T(m)$ to households. As in Yum (2018), the size of government transfers is assumed to be determined by the following simple functional form

$$T(m) = \omega_s(1 + m)^{-\omega_p} \tag{7}$$

where ω_s, ω_p and m is the sum of labor and capital income. This parametric assumption adds two

[7] A variable with a prime denotes its value in the next period.

parameters. First, $\omega_s \geq 0$ is a scale parameter in the sense that it determines the size of transfers for the zero-income households ($T(0) = \omega_s$). Second, $\omega_p \geq 0$ captures the degree of progressivity since a higher ω_p implies a more negative slope of transfers over income. The other extreme case with $\omega_p = 0$ would imply that the transfer schedule is independent of income, which is a common assumption in the literature.

Firm:

Aggregate output Y is produced by a representative firm. The firm maximizes its profit

$$\max_{K,L} \{z_k F(K,L) - (r(\mu, z_k) + \delta)K - w(\mu, z_k)L\} \tag{8}$$

where $F(K,L)$ captures a standard neoclassical production technology in which K denotes aggregate capital, L denotes aggregate efficiency units of labor inputs, and δ is the capital depreciation rate. As is standard in the literature, we assume that the aggregate production function follows a Cobb-Douglas function with constant returns to scale:

$$F(K,L) = K^{\alpha} L^{1-\alpha}. \tag{9}$$

The first-order conditions for K and L give

$$r(\mu, z_k) = z_k F_1(K,L) - \delta, \tag{10}$$

$$w(\mu, z_k) = z_k F_2(K,L). \tag{11}$$

Equilibrium:

A recursive competitive equilibrium is a collection of factor prices $r(\mu, z_k), w(\mu, z_k)$, the household's decision rules $g_a(a, x_i, \mu, z_k)$, $g_n(a, x_i, \mu, z_k)$, government policy variables $\tau, G, T(\cdot)$, a value function $V(a, x_i, \mu, z_k)$, a measure of households $\mu(a, x_i)$ over the state space, the aggregate capital and labor $K(\mu, z_k), L(\mu, z_k)$, and the aggregate law of motion $\Gamma(\mu, z_k)$ such that

1. Given factor prices $r(\mu, z_k), w(\mu, z_k)$ and government policy $\tau, G, T(\cdot)$, the value function $V(a, x_i, \mu, z_k)$ solves the household's decision problems defined above, and the associated household decision rules are

$$a'^{*} = g_a(a, x_i, \mu, z_k) \tag{12}$$

$$n^{*} = g_n(a, x_i, \mu, z_k). \tag{13}$$

2. Given factor prices $r(\mu, z_k), w(\mu, z_k)$, the firm optimally chooses $K(\mu, z_k)$ and $L(\mu, z_k)$ following (10) and (11).

3. Markets clear

$$K(\mu, z_k) = \sum_{i=1}^{N_x} \int_a a d\mu \tag{14}$$

$$L(\mu, z_k) = \sum_{i=1}^{N_x} \int_a x_i g_n(a, x_i, \mu, z_k) d\mu. \tag{15}$$

4. Government balances its budget

$$G + \sum_{i=1}^{N_x} \int_a T(m) d\mu = \tau w L(\mu, z_k).$$

5. The law of motion for the measure of households over the state space $\mu' = \Gamma(\mu, z_k)$ is consistent with individual decision rules and the stochastic processes governing x_i and z_k.

Solution method:

Our model cannot be solved analytically, is thus solved numerically. Several key features of our model make the numerical solution method nontrivial. First, a key decision in our model economy is a discrete employment choice. Therefore, our solution method is based on the nonlinear method (i.e., the value function iteration) applied to the recursive representation of the problem described above. Second, the aggregate law of motion involves an infinite-dimensional object: the distribution μ. Therefore, we solve the model by approximating it by the mean of the distribution following Krusell and Smith (1998). In addition, since market-clearing is nontrivial in our model with endogenous labor, our solution method incorporates a step to find market clearing prices in each period when simulating the model.

We describe the solution method briefly.[8] Following Krusell and Smith (1998), we assume that households use a smaller object that approximates the distribution when they forecast the future state variables to make current decisions. More precisely, we approximate $\mu(a, x_i)$ by its mean of the asset distribution $K = \int_a \sum_{i=1}^{N_x} a d\mu$. Also, the next period's aggregate capital K', real wage rate w and real interest rate r are assumed to be functions of (K, z) instead of (μ, z). We impose the parametric assumptions on the aggregate law of motion $K' = \Gamma(K, z)$ and $w = w(K, z)$ following log-linear equations:

$$\log \hat{K}' = a_0 + a_1 \log K + a_2 \log z \tag{16}$$

$$\log \hat{w} = b_0 + b_1 \log K + b_2 \log z. \tag{17}$$

Based on these forecasting rules, households obtain the forecasted $\hat{r}$ through the first-order conditions of firm's profit maximization.

Given the above forecasting rules, the model is solved in the two steps. First, we solve for the individual policy functions given the forecasting rules using the value function iterations (*the inner loop*). Then, we update the forecasting rules by simulating the economy using the individual policy functions (*the outer loop*). As noted above, it is important to note that, since our model environment with endogenous labor supply involves non-trivial factor market clearing unlike the benchmark Krusell-Smith (1998) setting wherein factor prices depend only on aggregate capital and the aggregate shock, we make sure that factor prices implied by the converged approximate forecasting rules clear the markets (Chang and Kim, 2014; Takahashi, 2014).[9] The consistency

[8] See Appendix for more details.

[9] We have also checked the results when market-clearing is ignored. We find that although R^2 and Den-Haan statistics look reasonably good, the key business cycle statistics are considerably different from those obtained with market-clearing prices. In particular, we find that the difference is even larger than those reported in Takahashi (2014) in our model with more persistent idiosyncratic shocks.

between the law of motion and individual decision rules is obtained as we repeat this procedure until the coefficients in the forecasting rules converge. We provide more details in Appendix.

2.2 Alternative model specifications

In addition to the baseline model (Model (a) hereafter), we consider two alternative specifications to illustrate the importance of the interplay of government transfers and household heterogeneity. Relative to Model (a), Model (b) shuts down government transfers by setting $\omega_s = 0$. In other words, Model (b) is a nested specification of the baseline model (called Model (a) henceforth), corresponding to a standard incomplete-markets real business cycle model with household heterogeneity and endogenous labor supply at the extensive margin (Chang and Kim, 2007; 2014). Therefore, Model (b) shares the same economic environment as the baseline model.

Next, Model (c) shuts down household heterogeneity while maintaining the fiscal environment including taxes and transfers. Given the indivisible labor assumption in the baseline model, our representative agent version of the model is the business cycle model studied in Hansen (1985) augmented with tax and transfers. We consider the decentralized competitive equilibrium given the fiscal policy. We briefly explain Model (c) below.

Representative-agent model environment:

At the beginning of each period, the stand-in household has the current period's asset k. The aggregate state variables are the aggregate capital K and the aggregate productivity z_k. The aggregate productivity follows the same stochastic process as in the baseline model. Taking the wage rate $w(K, z_k)$ and the real interest rate $r(K, z_k)$, as well as the aggregate law of motion $\Gamma(K, z_k)$ as given, the dynamic decision problem of the representative household can be written as the following functional equation:

$$V(k, K, z_k) = \max_{\substack{k' \geq 0, \\ n \in [0,1]}} \left\{ \log c - Bn + \beta \sum_{l=1}^{N_z} \pi_{kl}^z V(k', K', z_l') \right\}$$

subject to

$$
\begin{aligned}
c + k' &\leq (1-\tau)w(K, z_k)n + (1 + r(K, z_k))k + \omega_s \\
K' &= \Gamma(K, z_k)
\end{aligned}
$$

The household maximize utility by choosing optimal consumption c, the next period's capital k' and labor supply n. Our stand-in household has a linear disutility of work B due to the aggregation in Rogerson (1988). The budget constraint states that the sum of consumption c and the next period's capital k' should be less than or equal to the sum of net-of-tax labor income $(1-\tau)w(K, z_k)n$, current capital k, capital income $r(K, z_k)k$ and government transfers ω_s.

As in the baseline model, government collects taxes on labor earnings τwn to finance transfers ω_s and government expenditure G. We assume the amount of government transfers is constant, implying that the ratio of transfers to output is countercyclical, as in the data. We maintain the firm side as in the baseline model. The resulting first-order conditions for K and L are the same as those in (10) and (11).

Equilibrium:

A recursive competitive equilibrium is a collection of factor prices $r(K, z_k)$, $w(K, z_k)$, the household's decision rules $g_k(k, K, z_k)$, $g_n(k, K, z_k)$, government policy variables τ, G, ω_s, the household's value function $V(k, K, z_k)$, the aggregate capital and labor K, L and the aggregate law of motion $\Gamma(K, z_k)$ such that

1. Given factor prices $r(K, z_k)$, $w(K, z_k)$ and government policy τ, G, ω_s, the value function $V(k, K, z)$ solves the household's decision problem, and the associated decision rules are

$$
\begin{aligned}
k'^* &= g_k(k, K, z_k) \\
n^* &= g_n(k, K, z_k).
\end{aligned}
$$

2. Given factor prices $r(K, z_k)$, $w(K, z_k)$, the firm optimally chooses K and L following (10) and (11).

3. Market clear

$$K = k,$$

$$L = n.$$

4. Government balances its budget

$$G + \omega_s = \tau w(\mu, z_k)L.$$

5. The law of motion is consistent with individual decision rules and the stochastic process governing z_k:

$$K' = \Gamma(K, z_k) = g_k(K, K, z_k).$$

3 Setting model parameters

The model is calibrated to U.S. data. A period in the model is a quarter, as is standard in the business cycle literature. There are two sets of parameters. The first set of parameters is calibrated externally in line with the business cycle literature. These parameter values are commonly set in the two specifications. The second set of parameters is calibrated to match the relevant target statistics. Therefore, the calibrated parameter values differ in the two specifications.

We begin with the first set of parameters that is calibrated externally. Most of these parameters are commonly used in the real business cycle literature. The capital share, α, is chosen to be consistent with the capital share of 0.36. The quarterly depreciate rate, δ, is 2.5 percent. In our model with a binary labor supply choice, the level of hours worked, $\bar{n}$, can be arbitrarily set since it simply determines the scale of the calibrated disutility parameter B. We set it to 1/3, implying that working individuals spend a third of their time endowment on working. The labor income tax, τ_l, is set to 27.9 percent as in Yum (2018) who follows Mendoza et al. (1994) and Trabandt and Uhlig (2011). We do not allow borrowing (i.e., $\underline{a} = 0$) in our baseline model.[10] Finally, we

[10] We also considered a model with a moderate amount borrowing limit, but the main results were affected very

Table 1: Parameter values chosen internally using simulation

	Model (a) Baseline	Model (b) No transfers	Model (c) No Heterog.	Description
$B =$	.697	1.07	1.08	Disutility of work
$\beta =$	.988	.984	.990	Subject discount factor (quarterly)
$\rho_x =$	.983	.972	-	Persistence of individual productivity x
$\sigma_x =$	.087	.141	-	S.D. of idiosyncratic shocks to x
$\omega_s =$	.197	-	.223	Transfer scale
$\omega_p =$	1.72	-	-	Transfer progressiveness

Notes: Model (a) is the baseline specification: a heterogeneous-agent incomplete markets model with government transfers. Model (b) shuts down government transfers but keeps household heterogeneity.

should note that the goal of this paper is not to investigate the relative importance of different sources of aggregate fluctuations. Instead, our focus is on the transmission mechanism of aggregate shocks while taking stochastic process for aggregate productivity shocks exogenous. Hence, our baseline model employs standard values for the aggregate productivity shocks (i.e., $\rho_z = 0.95$ and $\sigma_z = 0.007$) (Cooley and Prescott, 1995), which are also used by recent related papers such as Chang and Kim (2007, 2014) and Takahashi (2017).[11]

The second set of parameters that are jointly calibrated for each specification of the model. As shown in Table 1, there are six parameters for the baseline specification, and four parameters for Model (b) which shuts down government transfers with the restriction of $\omega_s = 0$. The parameter values are calibrated to match the same number of target statistics summarized in Table 2. The first parameter is B, which captures the disutility of work. The target moment is the employment rate of 65.3 percent in our SIPP samples. The next parameter β captures the discount factor of households. As is standard in the literature, it is targeted to match the quarterly interest rate of 1 percent.

The next two parameters, ρ_x and σ_x, govern the individual labor productivity, which is closely

little.

[11] Estimating the aggregate risk within the model with household heterogeneity is an important yet difficult task partly due to the computational burden. This important task is out of scope of this paper.

Table 2: Target statistics in the data and in the model

Target	Data	Model (a) Baseline	(b) No transfers	(c) No heterog.
Employment rate	.653	.654	.653	.653
Real interest rate	.010	.010	.010	.010
Persistence of annual worker wages	.950	.950	.951	-
S.D. of log annual worker earnings	.713	.716	.709	-
Ratio of aggregate transfers to output	.092	.092	-	.092
Mean transfers by 1st income quintile relative to unconditional mean	1.95	1.95	-	-

Notes: See Table 1 for the description of the model specifications.

linked to individual wages if households choose to work. Note that there are two issues we would like to highlight when it comes to these parameters. First, there is a discrepancy in the data frequency. Specifically, the model period is a quarter while the wage data that are frequently used for estimating wage or earnings processes in the literature are at the annual frequency. This may lead to a non-straightforward temporal aggregation bias since labor supply is endogenous in the model. Moreover, and relatedly, in the data, there is a selection issue. That is, we only observe wages if households choose to work. To deal with both issues, we first estimate the persistence of annual wage using the PSID following a standard method in the literature (e.g., Heathcote et al. 2010), as described in detail in Appendix. The estimation result shows that the persistence of wages at the annual frequency is 0.950, which is in line with the estimates in the literature. Then, we calibrate the model so that the persistence of the annual wages, which are constructed by simulating the model as in the data, is also 0.950.[12] Next, the standard deviation of innovations to the AR(1) process σ_x in (1) is calibrated to match the overall dispersion of annual earnings, which are also constructed by simulating the model. Note that our calibration approach strives to construct the statistics from the simulated data as in the actual data.

[12] More precisely, we simulate the model and construct annual wages by dividing annual earnings by annual hours worked through explicit temporal aggregation. Note that, although labor supply is a binary choice at the quarterly frequency, there are richer variations in the *annual* hours worked driven by the number of quarters worked. Erosa, Fuster and Kambourov (2016) highlight a similar point in a stationary environment in the absence of business cycles.

Table 3: Characteristics of wealth distribution

Unit: %	Wealth quintile				
	1st	2nd	3rd	4th	5th
Share of wealth					
U.S. Data (SIPP)	-.017	.015	.077	.189	.742
Model (a): Baseline	.002	.020	.086	.216	.676
Model (b): $\omega_s = 0$	.001	.027	.103	.240	.629
Employment rate					
U.S. Data (SIPP)	.636	.708	.659	.639	.624
Model (a): Baseline	.542	.758	.731	.655	.587
Model (b): $\omega_s = 0$	.960	.724	.618	.532	.433

Notes: See Table 1 for the description of the model specifications.

The last two parameters, ω_s and ω_p, govern the nonlinear government transfer schedule in Model (a). As described above, ω_s captures the scale of transfers. The target statistic for ω_s is chosen to be the aggregate transfer to output ratio 0.092, which is obtained by taking the average over 1956Q1-2011Q4 in the U.S. The other parameter ω_p captures the progressiveness. To measure the degree of progressiveness in the U.S. transfer programs, we use the SIPP data to construct a broad measure of government transfers (see details in Appendix). Next, we compute the ratio of the average transfers received by the first income quintile to the unconditional mean transfers. This ratio of 1.95 is used to calibrate ω_p. Table 2 shows that both specifications of the model do a good job of matching all the target statistics.

Before we present the main business cycle results, we present some distributional aspects of the model economy in steady state. Table 3 summarizes the share of wealth and employment rates by wealth quintile. It is important to note that these are non-targeted moments.

Overall, both model specifications do a good job of accounting for the share of wealth by wealth quintile. A closer look reveal that our baseline model that incorporates nonlinear government transfers does a noticeably better job of accounting for the wealth concentration at the top of the wealth distribution. Specifically, the relative shares of the fourth and fifth quintiles are noticeably closer to the data (18.9% and 21.6%, respectively) in the baseline model (21.6% and 67.6%, respectively)

Table 4: Transfers by income quintiles: models vs data

Unit: %	Income quintile				
	1st	2nd	3rd	4th	5th
U.S. Data	1.95	1.69	0.80	0.36	0.19
Model (a): Baseline	1.95	1.31	0.79	0.58	0.36

Notes: Reported values are the mean transfers in each income quintile relative to the unconditional mean transfers. U.S. data are based on the Survey of Income and Program Participation.

compared to Model (b) (24.0% and 62.9%, respectively). Note that, in the presence of government transfers, households' incentive to save declines (Hubbard, Skinner and Zeldes, 1995). This force is especially stronger for the low-income households since our nonlinear government transfer schedule implies that transfers decline with income. Therefore, this force tends to raise the relative share of wealth by the richer households in the baseline model.

When we look at the employment rate at each wealth quintile, it is clearer that the baseline model does a better job of accounting for the employment-wealth relationship. In particular, the relatively low employment rate of the first quintile (63.6%) compared to that of the second quintile (70.8%) and the weakly inverse-U shape of the employment rates across wealth quintiles in the data are well captured in the baseline model. On the other hand, Model (b) predicts that employment falls sharply with wealth, consistent with the findings in Chang and Kim (2007). The sharp difference in these non-targeted moments between Model (a) and Model (b) is due to the presence of government transfer, which substantially reduces the strong precautionary motive of labor supply among the poor households in this class of the incomplete markets framework (Yum, 2018).

Lastly, recall that our calibration strategy only targets the relative amount of transfers among the first income quintile, as shown in Table 2. The joint relationship between income and transfers is a complicated equilibrium object, which is shaped not only by the parametric assumption on the nonlinear transfer schedule (7) but also by the endogenous income distribution of households. To check whether the degree of progressivity implied by the baseline model is reasonable or not, Table 4 shows the mean transfers by all income quintiles, relative to the unconditional mean transfers.

Despite the simple functional form in (7), we can see that the model does a good job of accounting for the mean transfers relative to the unconditional mean transfers across the other income quintiles as well.

4 Quantitative results

In this section, we report the main business cycle results and inspect the mechanism underlying the main results.

4.1 Business cycle statistics

We first compare business cycle statistics of key macroeconomic variables from simulations of the two model specifications with those from the data. We filter all the series using the Hodrick-Prescott filter with a smoothing parameter of 1600. The U.S. data statistics are computed using the aggregate data from 1955Q1 to 2011Q4 (see Appendix for more details).

Table 5 summarizes the cyclical volatility of the U.S. quarterly aggregate data along with that of simulated data in the baseline model and Model (b). Recall that Model (b) is a nested version of Model (a) by shutting down government transfers ($\omega_s = 0$). We report the absolute volatility of the key aggregate variables: Y is output, C is consumption, I is investment, L is aggregate efficiency unit of labor, H is aggregate hours, and Y/H is average labor productivity. The absolute volatility is measured by the percentage standard deviation, as is standard in the literature. The relative volatility is reported in parentheses, and is computed as the absolute volatility of each variable divided by that of output.

Table 5 reveals that the baseline model incorporating progressive government transfers is able to generate a great deal of volatility in output with respect to the same stochastic process for aggregate productivity z. Specifically, the baseline model has a standard deviation of output equal to 1.76 percent, which is even larger than 1.56 percent in the data. This is in sharp contrast to the model (b), which generates considerably less volatility (the standard deviation of 1.48 percent).[13]

[13] It should be noted that our stochastic process for aggregate productivity is exogenously set, based on Cooley and Prescott (1995). It would be more ideal to estimate the productivity process within the model, yet it is out of the

Table 5: Volatilities of aggregate variables

		Model		
		(a)	(b)	(c)
	U.S. data	Baseline	No transfers	No heterog.
σ_Y	1.56	1.76	1.48	1.87
σ_C	0.89	0.33	0.40	0.38
	(0.57)	(0.19)	(0.27)	(0.20)
σ_I	4.42	4.43	4.03	5.32
	(2.82)	(2.52)	(2.73)	(2.84)
σ_L	-	1.34	0.92	-
	-	(0.76)	(0.62)	-
σ_H	1.50	1.65	1.01	1.59
	(0.96)	(0.94)	(0.68)	(0.85)
$\sigma_{Y/H}$	0.85	0.33	0.64	0.38
	(0.55)	(0.19)	(0.43)	(0.20)

Notes: See Table 1 for the description of the model specifications. In short, Model (b) is nested version of Model (a) by shutting down government transfers. Model (c) is the representative-agent model. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Volatility is measured by the percentage standard deviation of each variable. Numbers in parentheses are the ratio of volatility of each variable to that of output. The U.S. statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

To understand why our baseline model features much stronger amplification of aggregate shocks, compared to Model (b), it is useful to look at the volatility of other macroeconomic variables. In our baseline model, what stands out most is the volatility of aggregate hours (the standard deviation of 1.65 percent), which is nearly 60 percent as larger as that in Model (b) (1.01 percent). The large volatility of hours also transmits a large standard deviation of efficiency unit of labor (1.35 percent) in the baseline model, compared to 0.94 percent in Model (b).

An important finding in Table 5 is that the relative volatility of hours is 0.94 in the baseline model, compared to 0.68 in Model (b). There are two points worth noting regarding this finding. First, it shows that the model is able to generate the volatility of hours that is as large as that of output. It is fairly well known that a canonical real business cycle model has difficulties in generating a large relative volatility of hours without resorting a curvature of the utility function that imposes a Frisch elasticity inconsistent with micro evidence (see e.g., discussions in Keane and Rogerson, 2015). Moreover, according to the results in Chang and Kim (2007; 2014), the large relative volatility of hours obtained through indivisible labor (Rogerson, 1988) in Hansen (1985) may not be robust in an incomplete markets economy since their relative volatility of hours is less than 0.6 (Chang and Kim, 2014; Takahashi, 2014). Our result suggests that, once the model incorporates the government transfers providing social insurance, an indivisible labor economy is able to generate the relative volatility of hours that is in line with the data even in an incomplete market environment.

Second, the fact that an increase in output volatility is less than an increase in the volatility of hours in the baseline model compared to Model (b) implies that changes in total hours are not being perfectly transmitted to changes in output. Relatedly, if we look at the standard deviation of L, the baseline model has a standard deviation of 1.35 percent, which is approximately 40 percent as large as that of 0.94 percent in Model (b). Since the efficiency unit of labor L, which enters into the aggregate production function, is essentially a weighted average of individual worker productivity, the fact that the change in L is quite smaller than the change in total hours suggests that the worker composition (or aggregate labor productivity) changes more in Model (a) relative to Model

scope in this paper.

Table 6: Cyclicality of aggregate variables

		Model		
		(a)	(b)	(c)
	U.S. data	Baseline	No transfers	No heterog.
$Cor(Y,C)$	0.82	0.74	0.83	0.79
$Cor(Y,I)$	0.91	0.99	0.99	0.99
$Cor(Y,L)$	-	0.98	0.96	-
$Cor(Y,H)$	0.85	0.98	0.94	0.99
$Cor(Y,Y/H)$	0.35	0.40	0.84	0.79
$Cor(H,Y/H)$	-0.21	0.22	0.60	0.69

Notes: See Table 1 for the description of the model specifications. In short, Model (b) is nested version of Model (a) by shutting down government transfers. Model (c) is the representative-agent model. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Cyclicality is measured by the correlation of each variable with output. The statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

(b). We investigate this in more detail in the next subsection with impulse responses of employment across individual productivity.

Despite the considerable improvement in the quantitative performance of the baseline model, as described above, and similar performance in terms of investment being two to three times as volatile as output, we also note that the performance of the model slightly worsens along some dimensions. First, consumption becomes less volatile in the presence of government transfers. This result should not be surprising given the nature of progressive transfers, effectively providing insurance against aggregate shocks. Second, average labor productivity becomes less volatile in the presence of government transfers. This is somewhat less clear, and we attempt to understand this result in detail when we investigate the impulse responses in the next subsection.

We now move on to the cyclicality of key macroeconomic variables. The first five rows of Table 6 show the correlations of output with other aggregate variables considered in Table 5.[14] The last row shows the correlation between total hours and labor productivity. As is well known in the literature (e.g., King and Rebelo, 1999), most macroeconomic variables such as consumption, investment, and total hours tend to be highly procyclical in the U.S. except for average labor

[14] Again, all aggregate variables are detrended using the Hodrick-Prescott filter, as above.

Table 7: Persistence of aggregate variables

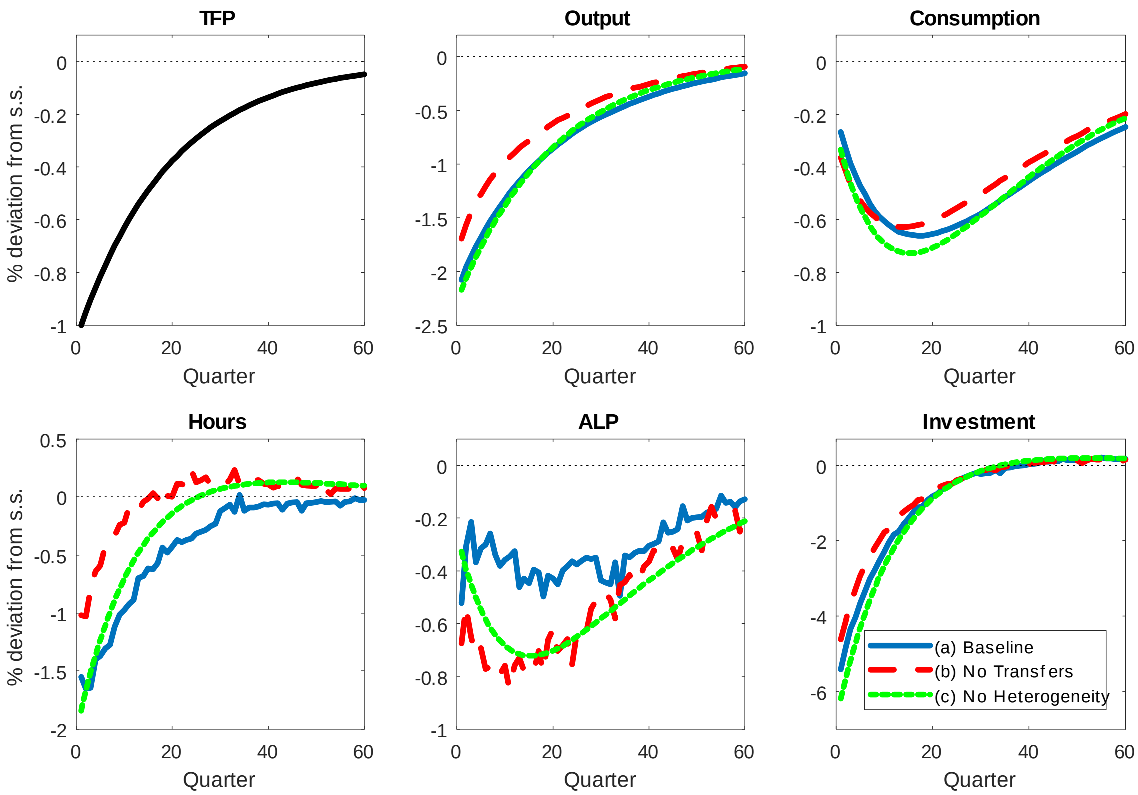
		Model		
	U.S. data	(a) Baseline	(b) No transfers	(c) No heterogeneity
$\rho(Y)$	0.85	0.70	0.69	0.68
$\rho(C)$	0.84	0.87	0.83	0.83
$\rho(I)$	0.89	0.69	0.67	0.67
$\rho(L)$	-	0.68	0.64	
$\rho(H)$	0.84	0.73	0.70	0.67
$\rho(Y/H)$	0.52	0.55	0.57	0.83

Notes: See Table 1 for the description of the model specifications. In short, Model (b) is nested version of Model (a) by shutting down government transfers. Model (c) is the representative-agent model. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Cyclicality is measured by the correlation of each variable with output. The statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

productivity (or output per total hours) which is much more moderately procyclical. A related observation is that the correlation between hours and average labor productivity is acyclical or even moderately negative. Since canonical real business cycle model generates a strong procyclicality of the macroeconomic aggregates including productivity, the literature suggests various possibilities to lower the correlation of labor productivity with output and hours (Benhabib, Rogerson, and Wright, 1991; Christiano and Eichenbaum, 1992; Braun, 1994; and Takahashi, 2017).

The performance of the model reported in Table 6 shows that both the baseline model and Model (b) generally share the strengths of the real business cycle models explaining the procyclical behavior of consumption, investment, and hours. A striking finding is that the cyclicality of labor productivity is much smaller (0.22) in our baseline model, compared to Model (b) (0.60). Importantly, in contrast to the existing literature which relies on additional exogenous shocks, the key to our result is the interaction between household heterogeneity and the presence of government transfers, both of which generates heterogenous labor supply behavior across households. In the next subsection, we investigate the mechanism underlying our quantitative success.

Figure 1: Impulse responses following a negative TFP shock



Notes: TFP denotes the total factor productivity (or aggregate productivity). ALP refers to the average labor productivity or output divided by aggregate hours.

4.2 Inspecting the mechanism

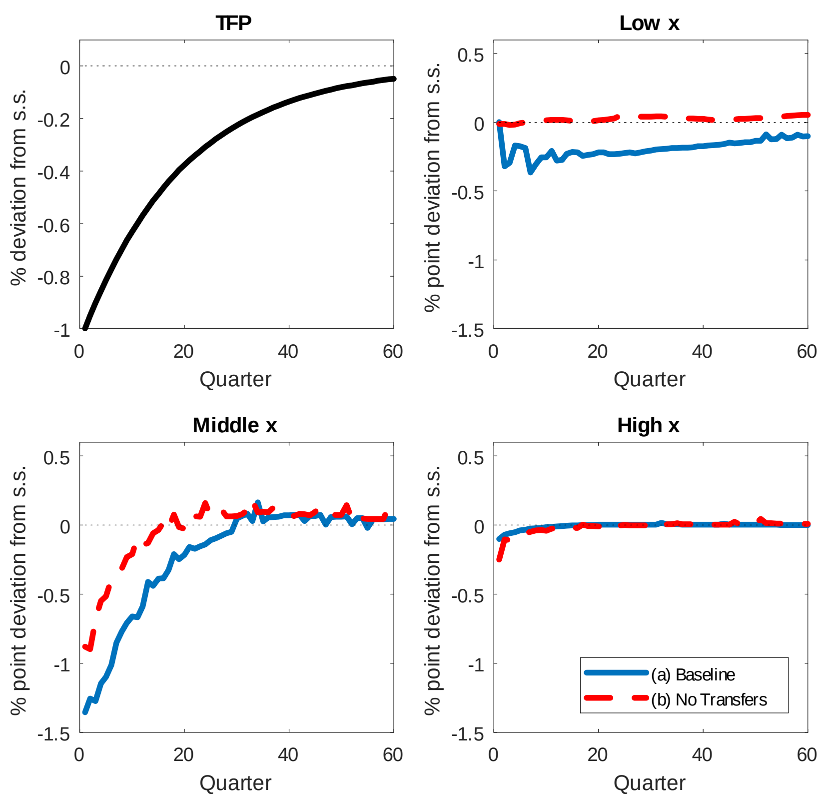
Figure 1 shows the impulse response of the key aggregate variables such as output, consumption, total hours, average labor productivity, and investment. There is a noticeable difference in the IRFs of the output between the baseline model and Model (b). Specifically, our baseline model generates a considerably large fall in output (close to -2%) immediately whereas Model (b) generates a smaller fall (-1%) in output following a -1% fall in the aggregate productivity (z). This is consistent with the larger amplification observed in Table 5. The impulse response of total hours clearly confirm that the main source of a larger fall is the labor supply channel. That is, the magnitude of a decline in aggregate hours in the baseline model is almost twice as large as that in Model (b). Again, this is consistent with the findings in Table 5 that the baseline model is able to generate a large volatility of aggregate hours. Another notable difference is that aggregate hours are more persistent, taking more time to recover its steady state level in our baseline model whereas they recover earlier and even overshoot in the nested version of the model.

Next, an important difference appears in the impulse response of average labor productivity. More precisely, in the nested versions of the model, the response of the labor productivity follows the consumption very closely. On the other hand, the baseline model implies that average labor productivity falls much less following the fall in aggregate productivity, and then it takes several quarters to fall eventually to the lowest level. Clearly, this contrasting response of average labor productivity with respect to a fall in aggregate productivity shows why the baseline model is able to predict a much weaker positive correlation between average labor productivity and output (or hours).

Next, to better understand why labor productivity increases sharply with respect to fall in aggregate productivity, it is useful to look at the impulse responses at more disaggregated level. Figure 2 plots the impulse responses of hours worked by individual productivity. We categorize households into three groups such that the measure of each group is relatively evenly distributed: (i) low productivity including x_1 to x_8; (ii) middle productivity of x_9; and (iii) high productivity including x_{10} to x_{17}.

Figure 2 shows that the middle productivity group is most responsive to the change in aggregate

Figure 2: Impulse responses of hours by individual productivity following a negative TFP shock



Notes: We group households by their individual productivity level (x). Low x refers to the first to eighth productivity levels. Middle x refers to the ninth productivity levels. High x refers to the tenth to seventeenth levels.

productivity in both specifications. More interestingly, in our baseline model, the labor supply of low productivity group is also quite responsive to the fall in aggregate productivity whereas in Model (b), the labor supply of these households changes very little. This key difference at the low productivity group explains why average labor productivity increases sharply only in our baseline model since the composition of workers changes by disproportionately larger drop in the low productivity workers with respect to a fall in aggregate productivity.[15] The reason why the labor supply of low productivity workers is not elastic to aggregate productivity (or aggregate wage) changes is the absence of social insurance, which raises their precautionary motive for labor supply (Yum, 2018). This exercise clearly shows that, in our economic environment, heterogeneity plays a key role in shaping the aggregate dynamics.

5 Conclusion

In this paper, we have presented an economic environment wherein micro-level heterogeneity shapes the dynamics of macroeconomic aggregates substantially. Our model framework extends a standard incomplete-markets model with household heterogeneity by incorporating progressive government transfers. In the presence of a simple government transfer schedule calibrated to match the degree of transfer progressiveness in the micro level data, we find that our model generates (1) a substantially weaker correlation of average labor productivity with output and total hours; and (2) a strong amplification mechanism through the labor supply channel, both of which are much more in line with what we observe in the data, compared to the models that abstract from either heterogeneity or government transfers. In particular, the impulse responses in our baseline model show that the fall in output and aggregate hours can be reconciled with a sharp rise in labor productivity with respect to a fall in aggregate productivity, as observed in the recent recessions in the U.S.

Our finding that micro-level household heterogeneity matters for the dynamics of macroeconomic variables over the business cycle has some important implications for the business cycle studies. For instance, our analysis suggests that an increase either in the scale or in the progressiv-

[15] Note that in both specifications of the model, the change in the labor supply of the high productivity group is much smaller than the other groups.

ity of transfers would reduce the cyclicality of labor productivity. This suggests that our mechanism can provide a potential explanation for the recent drop in the cyclicality of labor productivity (e.g., see Fernald and Wang, 2016 for a detailed review on the declining cyclicality of labor productivity).

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Appendix

A Aggregate data

The business cycle statistics are based on the aggregate time-series data covering from 1955Q1 to 2011Q4. For output, we use "Real Gross Domestic Product (millions of chained 2009 dollars)" in Table 1.1.6 of the Bureau of Economic Analysis (BEA). For consumption, we use "Personal Consumption Expenditure" after subtracting durable goods in Table 2.3.5 of the BEA. Investment is constructed as the sum of durable goods in Table 2.3.5 and private fixed investment in Table 5.3.5. The real values of consumption and investment are calculated using the price index for Gross Domestic Product in Table 1.1.4. Data on total hours worked are obtained from Cociuba et al. (2012). We modified all of the raw time series into those per capita by dividing the raw data by quarterly population in Cociuba et al.(2012).

B Micro data

For the statistics obtained at the micro level, we use data from the Survey of Income and Program Participation (SIPP). This data set is representative of the non-institutionalized U.S. population. The survey period is in a monthly basis. The SIPP covers a wide range of information on income, wealth, and participation in various transfer programs. We choose the samples from the first wave to the ninth wave of the SIPP in 2001, covering from 2001 to 2003.

The period of variables used in our analysis is quarterly given a structural feature in the SIPP. The data set is composed of a main module and several topical modules. While the main module contains monthly information on income and transfers, variables for medical expenses and wealth are quarterly reported in the topical modules.[16] To do this, we adjust the time frequency of variables in the main module to a quarterly basis so that information from both modules is consistent in terms of the time frequency.

We construct variables at household level. Data sets in the SIPP contain not only household

[16] Specifically, the 3rd, 6th, 9th topical modules contain the information.

variables but also individual variables. To generate a household variable from its corresponding individual variable, we take the following steps. First, we identify households with sample unit identifier (SSUID) and household address id in sample unit (SHHADID). Second, we add up the values of a variable for all members in a household. The government transfers that is used to infer the degree of progressivity is based on a broad range of transfer programs including Supplemental Security Income (SSI), Temporary Assistant for Needy Family (TANF), Supplemental Nutrition Assistance Program (SNAP), Supplemental Nutrition Program for Women, Infants, and Children (WIC), and Medicaid. We do not include Social Security and Medicare since these programs are targeted at old populations only. We construct a variable of *household income* broadly since the payment of transfer programs depends on total income in the U.S. Therefore, it consists of labor income, income from financial investments, and property income. We exclude households in which the age of head is less than or equal to 20. We convert all of their nominal values to the values in 2001 US dollar using the CPI-U.

C Estimation of the wage persistence

We estimate the persistence of wage in the United States using data from the Panel Study of Income Dynamics (PSID). We choose samples for the period of 1969-2010. Our measure of labor productivity is defined as a worker's relative hourly wage to other individuals. This labor productivity is measured by a worker's earnings divided by hours worked. To avoid the oversampling of low income household heads, we exclude households from the Survey of Economic Opportunity. We consider household heads whose age is between 23 and 65. We drop the samples whose wage is below a half of the minimum wage. The nominal values are converted into the value of US dollar in 2000 with the CPI-U.

We run the ordinary least square regression of the log of the productivity (hourly wages) on a dummy for male, a cubic polynomial in potential experience (age minus years of education minus five), a time dummy, and a time dummy interacted with a college education dummy. We take its residual, $x_{i,j}$, as an idiosyncratic productivity that contains a wide range of individual abilities in the labor market. This stochastic process is composed of the summation of a persistent, $\eta_{i,j}$ and a transitory process, $\nu_{i,j}$:

$$
\begin{aligned}
x_{i,j} &= \eta_{i,j} + \nu_{i,j}, \nu_{i,j} \sim N(0, \sigma_\nu) \\
\eta'_{i,j} &= \rho_\eta \eta_{i,j-1} + \epsilon'_{i,j}, \epsilon'_{i,j} \sim N(0, \sigma_\epsilon)
\end{aligned}
\tag{18}
$$

We use a Minimum Distance Estimator to estimate the parameters of the process. The mechanism is to find parameters that minimizing the distance between empirical and theoretical moments. We take the covariance matrix of the residual $x_{i,j}$ as our moments. Let's denote θ as a vector of $(\rho_\eta, \sigma_v, \sigma_\epsilon)$. Let $m_{j,j+n}(\theta)$ be the covariance of the labor productivity between age j and $j+n$ individuals. $\hat{m}_{j,j+n}$ is defined as the empirical counterpart of $m_{j,j+n}(\theta)$. Then,

$$
E\left[\hat{m}_{j,j+n} - m_{j,j+n}(\theta)\right] = 0 \tag{19}
$$

where

$$
\hat{m}_{j,j+n} = \frac{1}{N_{j,j+n}} \sum_{i=1}^{N_{j,j+n}} x_{i,j} \cdot x_{i,j+n}
$$

The moments can be represented by as an upper triangle matrix:

$$
\bar{m}(\theta) = \begin{bmatrix}
m_{0,0}(\theta) & m_{0,1}(\theta) & \cdots & \cdots & m_{0,J-1}(\theta) & m_{0,J}(\theta) \\
0 & m_{1,1}(\theta) & \cdots & \cdots & m_{1,J-1}(\theta) & m_{1,J}(\theta) \\
0 & 0 & m_{2,2}(\theta) & \cdots & m_{2,J-1}(\theta) & m_{2,J}(\theta) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & m_{J-1,J-1}(\theta) & m_{J-1,J}(\theta) \\
0 & 0 & 0 & \cdots & 0 & m_{J,J}(\theta)
\end{bmatrix}
$$

We denote a vector of $\bar{M}(\theta)$ by vectorizing $\bar{m}(\theta)$ with length $(J+1)(J+2)/2$. To estimate parameters θ, we solve

$$
\min_\theta \left[\hat{\bar{M}} - \bar{M}(\theta)\right]' W \left[\hat{\bar{M}} - \bar{M}(\theta)\right]
$$

where the weighting matrix W is set to be an identity matrix.[17]

D More on numerical solution methods

The model with aggregate risk is solved in the following two steps. First, we solve for the individual policy functions given the forecasting rules (*the inner loop*). Then, we update the forecasting rules by simulating the economy using the individual policy functions (*the outer loop*).

In the inner loop, we solve for $V(a, x_i, K, z_k) = \max\left\{V^E(a, x_i, K, z_k), V^N(a, x_i, K, z_k)\right\}$. These value functions are stored on the non-evenly spaced grid for a and the evenly spaced grid for K with the number of grid points $n_a = 100$ and $n_K = 7$. Unlike Chang and Kim (2007; 2014) and Takahashi (2014), we discretize stochastic processes x_i and z_k by using Rouwenhorst (1995) method with $n_x = 17$ and $n_z = 7$. We find that the approximation of the continuous processeses for our highly persistent shocks is substantially better with the Rouwenhorst method.[18] To obtain $V(a, x_i, K, z_k)$ at each grid point, we solve the following problems

$$V^E(a, x_i, K, z_k) = \max_{a' > \underline{a},} \left\{ \log c - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \hat{K}', z_l') \right\} \tag{20}$$

$$\text{subject to} \quad c + a' \leq (1-\tau)\hat{w}(K, z_k)x_i z_k \bar{n} + (1 + \hat{r}(K, z_k))a + T(\hat{w}(K, z_k)x_i z_k \bar{n} + \hat{r}(K, z_k)a)$$

and

$$V^N(a, x_i, K, z_k) = \max_{a' > \underline{a},} \left\{ \log c + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \hat{K}', z_l') \right\} \tag{21}$$

$$\text{subject to} \quad c + a' \leq (1 + \hat{r}(K, z_k))a + T(\hat{r}(K, z_k)a)$$

As $(a', \hat{K}')$ are not on the grid points, we use the bivariate cubic splines to approximate the expected future value.

Then, in the outer loop, we simulate the model economy. It is important to find the equilibrium

[17] Using the identity matrix has been common in the literature since Altonji and Segal (1996) show that the optimal weighting matrix generate severe small sample biases.

[18] Specifically, we use the simulated data from Rouwenhorst and Tauchen's methods and estimate the persistence and the standard deviation of error terms in the AR(1) processes for both aggregate productivity shocks and idiosyncratic shocks (available upon request). See also Kopecky and Suen (2010).

factor prices and associated total employment in each period of the simulation since aggregate dynamics can be considerably misleading in this class of models with the discrete choice of endogenous labor supply (Takahashi, 2014). Our market-clearing procedure closely follows Takahashi (2014). Specifically, in each simulation period, we first use the forecasting rule (17) to obtain the guess for w used by households, depending on the current aggregate state variables (K, z_k). The guess for r is obtained using the relationship $r = z_k^{\frac{1}{\alpha}} \alpha \left(\frac{w}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta$, which is implied by (10) and (11). The measure of households $\mu(a, x_i)$ is stored in a finer (non-evenly spaced) grid on a with the number of grid points equal to 4000 (Rios-Rull, 1999). K is constructed based on the measure of households following $K = \int_a \sum_{i=1}^{N_x} a\mu(da, x_i)$. Then, the forecasting rule (16) implies $\hat{K}'$ in each period. Since $\hat{K}'$ is not on the grid points of K, we solve (20) and (21) given the expected value function in the next period using interpolation. We use the piecewise linear interpolation over K' and the cubic spline interpolation over a'. Having V^E and V^N at hand, we check whether the household indexed by (a, x_i) works or not. If $V^E > V^N$, $n = \bar{n}$ and a' is obtained by (20); otherwise, $n = 0$ and a' is obtained by (21). Then we have the individual household decision rules $a' = g_a(a, x_i)$ and $n = g_n(a, x_i)$. The aggregate labor supply is obtained by $L^s = \int_a \sum_{i=1}^{N_x} x_i g_n(a, x_i) d\mu$. We compare this with L^d implied by the forecasting rule (17) and the first-order condition to see if the labor market is cleared. If L^s is different from L^d, we use bisection to find the equilibrium $\tilde{w}$. After finding the market-clearing prices, we update the measure μ in the next period by using $a' = g_a(a, x_i)$ and the stochastic process for x_i. We simulate 3,500 periods, the first 500 periods of which are discarded when computing statistics.

Finally, the coefficients $(a_0, a_1, a_2, b_0, b_1, b_2)$ in the forecasting rules

$$\log K' = a_0 + a_1 \log K + a_2 \log z + \varepsilon_K, \tag{22}$$

$$\log w = b_0 + b_1 \log K + a_2 \log z + \varepsilon_w, \tag{23}$$

where ε_K and ε_w are error terms, are updated by ordinary least squares with the simulated sequence of $\{K', w, K\}$ sorted by the index for the aggregate shocks i. Our parametric assumption on the forecasting rules are the same as those in Chang and Kim (2007; 2014) and Takahashi (2014; 2017).

We repeat the whole procedure of the inner and outer loops until the coefficients in the forecasting rules converge.

As is clear in the forecasting rules (22) and (23), households predict prices and the future distributions of capital only with the mean capital stock. Therefore, it is important to check whether the equilibrium forecast rules are precise or not. We summarize results for the accuracy of the forecasting rules for the future mean capital stock K' in Table $A1$ and for the wage w in Table $A2$. It is clear that all R^2 are very high in both specifications of the model. We also present accuracy statistics suggested by Den Haan (2010). Since our dependent variables are in logs, we multiply the statistics by 100 to interpret them as percentage errors. We note that the mean errors are sufficiently small (far less than 0.1 percent) and the maximum errors are also reasonably small around 0.5 percent for the baseline model and 0.4 percent for Model (b).

Table A1: Estimates and accuracy of forecasting rules

Model	Dependent					Den-Haan (2010)	
specification	variable	Const.	$\log K$	$\log z$	R^2	Max (%)	Mean (%)
(a) Baseline	$\log K'$	0.1187	0.9502	0.1169	.99999	0.5376	0.0853
	$\log w$	-0.3157	0.4948	0.5034	.99911	0.5488	0.0691
(b) $\omega_s = 0$	$\log K'$	0.1281	0.9471	0.1056	.99999	0.4493	0.0715
	$\log w$	-0.3884	0.5185	0.6796	.99932	0.3805	0.0615