The Korean Economic Review
Volume 40, Number 2, Summer 2024, 315-348.
DOI : 10.22841/kerdoi.2024.40.2.004

Pricing Third-Party Access to Essential Facilities under Asymmetric Information*

Seok Yang** · Myeonghwan Cho***

We investigate a market wherein a dominant firm owns an essential facility and other firms use this facility by paying an access fee. Loss in social welfare occurs regardless of whether the cost of operating the essential facility is public or private information to the dominant firm. The government can improve social welfare by regulating the access fee, but it has limitations in improving social welfare when the cost of operating the essential facility is private information to the dominant firm. An appropriate tax-subsidy scheme can resolve the limitations by inducing the dominant firm to disclose its private information completely.

JEL Classification: C70, D43, L13
Keywords: Third-party access, Essential facility, Asymmetric information

I. Introduction

In some industries, firms have to use certain essential facilities, which are owned by one firm or a small number of firms, to supply goods to the market. Traditional examples of essential facilities include satellites in the telecommunication industry, transition lines in the electricity industry, and pipelines in the LNG industry. Recently, platforms and data owned by a few firms (e.g., Google, Apple, Microsoft, Amazon, etc.) in the rapidly evolving IT industry are also attracting attention as essential facilities for providing various contents and services to consumers.[1] Since

---

Received: Aug. 16, 2023. Revised: Feb. 14, 2024. Accepted: April 20, 2024.
* This work was supported by the 2022 sabbatical year research grant of the University of Seoul. We would like to thank the anonymous referees for their helpful comments.
** First Author, Graduate Student, Department of Economics, University of Seoul, 163 Seoulsiripdaero, Dongdaemun-gu, Seoul, Republic of Korea; Tel: +82-2-6490-2051, E-mail: tortoise97@uos.ac.kr
*** Corresponding Author, Professor, Department of Economics, University of Seoul, 163 Seoulsiripdaero, Dongdaemun-gu, Seoul, Republic of Korea; Tel: +82-2-6490-2067, E-mail: chomhmh@uos.ac.kr
[1] For a discussion on essential facilities in the IT industry, see Graef (2016), Stucke and Grunes (2016), and Tucker (2019). Although there is a debate on whether data and platforms play the role of

the goods in these industries are, in general, very important aspects for maintaining quality of life, it is desirable to ensure that these goods are supplied at low prices whenever possible. However, since installing essential facilities requires a huge capital, only a few large firms can operate them. This prevents sufficient competition from arising in these industries, which increases the prices of goods for consumers. Moreover, from a social point of view, it may not be desirable for every firm to individually install and operate their own essential facilities, which require significant installation costs. Therefore, governments in many countries have attempted to increase market efficiency by promoting competition in markets wherein a few firms have essential facilities and allowing new entrants to access the essential facilities of existing firms. A major policy in these efforts is the third-party access (TPA) policy, which legally ensures that firms can access essential facilities owned by other firms by paying an appropriate access fee. This policy also ensures that firms that do not own essential facilities have nondiscriminatory access to the essential facilities that they need to supply goods.

One of the main issues in TPA policies is who decides the access fees for essential facilities. If the firm that owns the essential facilities decides the access fee, then it can block other firms from entering the market by choosing a sufficiently high access fee. Even if the firm sets an access fee at a level that other firms are willing to pay to enter the market, it is not guaranteed that the level of access fee is appropriate in terms of social welfare. Meanwhile, if the government decides the access fee for an essential facility, then it may be possible to set a socially optimal level of access fee. However, for this to be possible, it may necessitate the government to have comprehensive information about industries and the market. If the cost of operating an essential facility is private information known only to the firm that owns it, then it is difficult to set a socially optimal level of access fee even if the government decides it. This study attempts to demonstrate the characteristics of the equilibrium in situations wherein the firm with an essential facility and the government set the access fee for the essential facility and compare the equilibria in these situations in terms of social welfare. This study also investigates whether it is possible for the government to set access fees at a socially optimal level by inducing the firm that owns the essential facility to disclose its private information.

In the analysis, we consider a homogeneous good market where two firms are engaged in Cournot competition. We also assume that one firm, say Firm 1, owns an essential facility but the other firm can produce goods at a cost lower than that of Firm 1. We first focus on the case in which the cost of operating the essential facility is public information. In this case, if Firm 1 decides the access fee, then it sets the access fee for the essential facility to be higher than the socially desirable level but

essential facilities in the IT industry, OECD (2015) reports that data can act as a barrier to entry, leading to a winner-take-all market.

still allows the other firm to supply goods to consumers. This is because it is beneficial for firm 1 to allow the other firm to supply goods and take some of the surplus of producing goods at a lower cost through the access fee rather than preventing other firms from entering the market by setting the access fee to be excessively high. The access fee set by Firm 1 in this case is also higher than the marginal cost of operating the essential facility. We can interpret such results that firm 1 provides the essential facility in a discriminate manner to the other firm in the sense that Firm 1 makes the other firm use the essential facility at a cost higher than Firm 1's cost to use the essential facility. The access fee set by Firm 1 is undesirable in terms of social welfare, and thus, the government may be able to improve social welfare by regulating the access fee for the essential facility. To regulate the access fee for the essential facility to maximize social welfare, the government sets the access fee at the level of the marginal cost of operating the essential facility, which can be interpreted as a nondiscriminatory provision of the essential facility.

Next, we consider the case in which the cost of operating the essential facility is private information to Firm 1. If Firm 1 determines the access fee for the essential facility, then the firm that does not own the essential facility will never enter the market regardless of the access fee set by Firm 1 for the essential facility. Under the existence of asymmetric information regarding the cost of operating the essential facility, if Firm 1 sets the access fee to be low, then the firm that does not own the essential facility believes that it cannot gain a positive profit by entering the market because Firm 1 has an advantage in the Cournot competition due to the low cost of operating the essential facility. In this case, the government can also improve social welfare by regulating the access fee for the essential facility. However, since the government does not know the cost of operating the essential facility, it cannot set the access fee at the marginal cost of operating the essential facility, which also leads to inefficiency in social welfare. Of course, the government can have Firm 1 disclose its private information before determining the access fee. However, if the government cannot verify the information that Firm 1 discloses, then Firm 1 has no incentive to reveal its private information. However, if the government can provide an appropriate tax-subsidy scheme to Firm 1 based on the information that Firm 1 reveals, then it may be possible to make Firm 1 truthfully reveal its private information. If this is possible, then the government can resolve the information asymmetry that damages social welfare and improve social welfare by regulating the access fee for the essential facility.

As TPA for essential facilities has received considerable attention in many industries, many relevant studies have been conducted. Weisman (1995) considers a market wherein firms engage in a Stackelberg competition with the leading firm having an essential facility, and shows that the restriction on the leading firm's market share can strengthen the incentive for the leading firm to discriminate against competitors with regard to the usage of its essential facility. Armstrong et al. (1996) and Armstrong and Vickers (1998) investigate markets wherein one firm owns an essential facility and sets the price of goods and another firm is a price-taker and pays an access fee to use an essential facility. Through the analysis, they find that under the constraint that the firm that owns the essential facility should not suffer loss, the socially optimal access fee for the essential facility is higher than the marginal cost of operating the essential facility. Valletti (1998) analyzes two-part pricing (fixed access fee and usage charge) for an essential facility in a situation wherein an upstream monopolist provides an essential facility to downstream firms. He finds that the upstream firm's price discrimination for the downstream firms may have a negative effect on social welfare and the government can improve social welfare by regulating the price for the essential facility. Economides (1998) considers an industry wherein the upstream monopolist has a subsidiary in the downstream market and sells monopolized input to downstream firms at a nondiscriminatory price, and finds that the monopolist will impose an additional cost to the rivals of the subsidiary until the rivals exit the market. Buccirossi (1999) compares a regime in which the government regulates the access price for essential facilities and a regime in which the firm that owns essential facilities can decide which firms can use essential facilities and at what price, and finds that these regimes are equivalent in terms of firms' entrances given that transfer of the property right of the facilities is not allowed. These studies are in line with our study in that they also examine the welfare effect of regulation on the access fee for essential facilities. However, our study differs from these studies in that we consider asymmetric information on the cost of operating an essential facility.[2]

Our study is also related to the literature that investigates the vertical integration of firms, particularly when the upstream firm produces the essential inputs. Examples include Salinger (1988), Hart et al. (1990), Ordover et al. (1990), and Chen (2001), which investigate the vertical integration of upstream firms that provide essential inputs and downstream firms that produce and supply goods in the market. These studies mainly focus on the incentives for firms to engage in vertical integration and the effects of vertical integration on market equilibrium and social welfare, while we focus on the access fee charged by the firm that owns an essential facility through vertical integration and government regulation on access fee. Arya et al. (2008a) and Moresi and Schwartz (2017) also investigate the oligopoly competition between firms that own essential facilities through vertical

[2] More broadly, our study is also related to the literature that explores markets wherein a firm has a dominant position relative to other firms. Examples include von Ungern-Sternberg (1996), Chen (2003), Erutku (2005), and Christou and Papadopoulos (2015), which investigate how the dominant firm's countervailing power affects market equilibrium. In our model, the firm with an essential facility can be interpreted as having market power since it can block the other firms from entering the market by restricting their access to an essential facility.

integration and firms that do not, and compare the equilibria when firms are engaged in Cournot competition and when they are engaged in Bertrand competition. These studies do not take into account information asymmetry and do not discuss the possibility of improving social welfare through government regulation.

Studies that consider information asymmetry in analyzing the market with essential facilities have also been conducted. For example, Lewis and Sappington (1999) analyze the welfare effect of government's regulation in a situation wherein one incumbent firm provides essential facilities and competes with its rival in the downstream market under the assumption that the marginal cost of producing goods of a firm that does not own essential facilities is the private information of the firm. They assumed that the marginal cost of operating essential facilities and the incumbent firm's marginal cost of downstream production are constant and common knowledge. Caillaud and Tirole (2004) study a situation wherein an incumbent firm owns essential facilities and competes with a potential entrant in the market. They assume that the incumbent firm has private information about market profitability and investigate the decisions of the regulator that determines the market structure through the choice of access rights and investment contributions (access fees) paid by the entrant to maximize social welfare. Gautier and Mitra (2008) also consider information asymmetry in analyzing competition between the incumbent firm that owns essential facilities and a potential entrant firm. In their model, the incumbent firm's marginal cost of producing goods is private information, and the regulator is allowed to decide the incumbent firm's output level and access fees for an essential facility. They find that allowing open access to an essential facility for the potential entrant can improve social welfare compared with the monopoly case, even if entry is not efficient. In contrast to these studies, we consider a case with information asymmetry about the cost of operating the essential facility and the government regulates the access fee for the essential facility. We also aim to find a mechanism through which information asymmetry can be resolved and social welfare can be improved.

This study is also related to the literature that explores the information sharing of firms in oligopoly markets. For example, Gal-Or (1986), Vives (1984), Li (1985), Sakai (1986), Shapiro (1986), and Ziv (1993) focus on the information sharing of firms among themselves. In particular, Ziv (1993) endogenizes the decision of firms with regard to sharing information and shows that firms will always send misleading information if they can do so. He also proposes a (costly) mechanism that induces firms to truthfully reveal their private information even in the absence of an outside verification system, which is analogous to our tax-subsidy scheme for a firm with an essential facility.

For information transmission from a firm that owns an essential facility to the government, this study is also related to Crawford and Sobel (1982), and Melumad and Shibano (1991). Crawford and Sobel (1982) consider a situation wherein an agent (sender) with private information sends non-verifiable messages to a decision maker (receiver) who takes an action based on the messages, and they construct an equilibrium wherein a privately informed agent partially, but not completely, reveals its private information. Melumad and Shibano (1991) consider a model similar to that of Crawford and Sobel (1982) and compare the equilibria when the decision maker can and cannot commit to a decision rule. Farrell and Gibbons (1989), Goltsman and Pavlov (2011), and Cho (2019) extend the model of Crawford and Sobel (1982) to cases with one sender and multiple receivers. In our model with asymmetric information, the firm with an essential facility serves as the sender and the government serves as the receiver in Crawford and Sobel (1982). However, our model differs from previous studies in that after the sender sends a message and the receiver takes an action, the sender and another agent make decisions in a strategic context.

The remainder of this paper is organized as follows. Section 2 explains the model. In Section 3, we consider the situation wherein the cost of operating an essential facility is public information and compare the equilibria when a firm that owns the essential facility decides the access fee and when the government regulates the access fee for the essential facility. Section 4 considers the situation wherein the cost of operating an essential facility is private information to the firm that owns it and compares the equilibria between when the firm that owns the essential facility decides the access fee and when the government regulates it. We also suggest a mechanism through which the firm that owns an essential facility voluntarily reveals its private information completely. Finally, we conclude with some remarks in Section 5.

II. Model

We consider a homogenous good market wherein the market demand is linearly given by

$$p = 1 - Q, \tag{1}$$

where p is the market price and Q is the market quantity. This market demand is induced by the behavior of consumers who behave as price takers. The behavior of consumers can be represented by a representative consumer, who enjoys a benefit $U(Q) = Q - (1/2)Q^2$ by consuming Q of goods. Note that the demand in (1) is obtained by maximizing the representative consumer's net benefit (or, consumer surplus)

$$CS = U(Q) - pQ = Q - \frac{1}{2}Q^2 - pQ \tag{2}$$

given p.

Two firms, Firms 1 and 2, supply goods in the market.[3] Firm 1 owns an essential facility that the firms have to use to supply goods in the market. The marginal cost for each firm $i = 1,2$ to produce goods is constant as c_i. We assume that $c_2 < c_1$. Thus, Firm 1 is less competitive in terms of production efficiency although it has a relative competitive advantage over Firm 2 in that it has an essential facility. Firm 1 has to pay a marginal cost θ to operate the essential facility. c_1 and c_2 are fixed and publicly known, but θ is randomly drawn from a continuous distribution F with a support $[0, \overline{\theta}]$. Let E_θ be the expectation of θ and V_θ be the variance of θ (i.e., $E_\theta = \mathbb{E}[\theta]$ and $V_\theta = \mathrm{var}(\theta)$). The distribution F of θ is publicly known, but the realization of θ is known only to Firm 1. To ensure interior solutions for the problems that we will consider later, it is assumed that

$$0 < c_2 < c_1 < 1 - \overline{\theta}\,. \tag{3}$$

Moreover, the difference between c_1 and c_2 is not sufficiently large to satisfy

$$\frac{c_1 - c_2}{1 - c_1 - \overline{\theta}} < 1\,. \tag{4}$$

Firm 2 has to pay Firm 1 an access fee r per unit of goods to use the essential facility of Firm 1. Let q_i be the output of firm i. Given the price p of goods determined by (1) and the access fee r, the profits Π_i of each firm i are

$$\Pi_1 = (1 - q_1 - q_2)q_1 + rq_2 - c_1 q_1 - \theta(q_1 + q_2)\,, \tag{5}$$

$$\Pi_2 = (1 - q_1 - q_2)q_2 - c_2 q_2 - rq_2\,. \tag{6}$$

Social welfare is defined as the benefit of the representative consumer from the goods minus the cost of the firms to supply the goods:

$$\begin{aligned} SW &= U(q_1 + q_2) - c_1 q_1 - c_2 q_2 - \theta(q_1 + q_2) \\ &= (q_1 + q_2) - \frac{1}{2}(q_1 + q_2)^2 - c_1 q_1 - c_2 q_2 - \theta(q_1 + q_2)\,. \end{aligned} \tag{7}$$

Social welfare depends on the output (q_1, q_2), not on the monetary transfers

[3] We can think of a more generalized model, in which $n \geq 1$ firms have to use the essential facility of Firm 1. The results in this study are preserved with such generalization of the model.

between the representative consumer and the firms and between Firms 1 and 2. Since the consumer surplus and the profits of the firms are linear in monetary transfers, any output that maximizes social welfare in (7) is (Pareto) efficient.

Before analyzing the pricing of an essential facility, we find the efficient output (q_1^*, q_2^*) that maximizes social welfare. Under the assumption of $c_1 > c_2$, the first-order necessary conditions for maximizing SW and the concavity of SW in (7) lead to

$$(q_1^*, q_2^*) = (0, 1 - c_2 - \theta), \tag{8}$$

which is the output that maximizes social welfare. Intuitively, to maximize social welfare, Firm 2 that can supply goods at a lower cost must supply goods to the entire market by using the essential facility of Firm 1. When social welfare is maximized, the market quantity is

$$Q^* = q_1^* + q_2^* = 1 - c_2 - \theta. \tag{9}$$

Plugging (q_1^*, q_2^*) in (8) into (7), we obtain the maximized social welfare as

$$SW^* = \frac{1}{2}(1 - c_2 - \theta)^2. \tag{10}$$

III. Equilibrium under Perfect Information

In this section, we investigate a benchmark situation wherein the marginal cost $\theta \in [0, \overline{\theta}]$ to operate an essential facility is fixed and publicly known. To this end, we consider two cases. In the first case, Firm 1 can choose the access fee r for the essential facility without any restriction. Note that the marginal cost for Firm 1 to use the essential facility to supply q_1 is θ and the marginal cost for Firm 2 to use the essential facility to supply q_2 is r. Since Firm 1 can choose an access fee $r \neq \theta$, we can interpret this case as Firm 1 being able to discriminate against the other firm in providing its essential facility. In the second case, the government regulates the access fee r considering social welfare. If the government regulates the access fee r to be $r = \theta$, then we can interpret it as a regulation of the nondiscriminatory provision of the essential facility.

3.1. Discriminatory Provision of Essential Facility

We first focus on the first case. The firms make decisions in two stages. In the first stage (Stage 1), Firm 1 chooses the access fee r for the essential facility, which is publicly informed. In the second stage (Stage 2), each firm i simultaneously chooses q_i. Firm 1's strategy in Stage 1 can be represented as ρ_1, which is the access fee for the essential facility. Each firm i's strategy in Stage 2 is a function $\sigma_i(r)$, where $\sigma_i(r)$ is firm i's output when Firm 1 chooses r in Stage 1. In this case, we are interested in the outcomes that can be achieved as a subgame perfect equilibrium $(\rho_1^\circ;\sigma_1^\circ,\sigma_2^\circ)$.

Utilizing the backward induction, consider Stage 2 after Firm 1 chooses r in Stage 1. From the first-order necessary conditions for maximizing firms' profits in (5) and (6), it is straightforward to see that the equilibrium strategies $(\sigma_1^\circ,\sigma_2^\circ)$ for the firms in Stage 2 are[4]

$$\sigma_1^\circ(r) = \frac{1}{3}(1-2c_1+c_2-2\theta+r)\,, \tag{11}$$

$$\sigma_2^\circ(r) = \frac{1}{3}(1+c_1-2c_2+\theta-2r)\,. \tag{12}$$

Given that the firms play $(\sigma_1^\circ,\sigma_2^\circ)$ in Stage 2, Firm 1's profit when it chooses r is

$$\begin{aligned}\Pi_1(r) &= (1-\sigma_1^\circ(r)-\sigma_2^\circ(r))\sigma_1^\circ(r)+r\sigma_2^\circ(r)-c_1\sigma_1^\circ(r)-\theta(\sigma_1^\circ(r)+\sigma_2^\circ(r)) \\ &= -\frac{1}{9}(5r^2-(5-c_1-4c_2+5\theta)r-\theta^2 \\ &\quad -(5c_1+2c_2-7)\theta+(2-2c_1+c_2)(2c_1-c_2)-1)\,.\end{aligned} \tag{13}$$

Since $\Pi_1(r)$ is concave in r, the first-order necessary condition to maximize $\Pi_1(r)$ implies that Firm 1's equilibrium strategy ρ_1° in Stage 1 is

$$\rho_1^\circ = r^\circ = \frac{1}{10}(5-c_1-4c_2+5\theta)\,. \tag{14}$$

Note that the assumptions in (3) and (4) ensure that $\rho_1^\circ > \theta$, which means that Firm 1 gains additional profits by allowing Firm 2 to use its facilities while paying access fees.

Proposition 1 summarizes the previous arguments.[5]

Proposition 1 *Suppose that Firm 1 chooses the access fee r for the essential facility.*

[4] Here, we assume that r is chosen in Stage 1 so that the equilibrium in Stage 2 is obtained as an interior solution.

[5] The proof of Proposition 1 is obvious from the previous arguments, and so omitted. In this paper, we omit the proofs that are obvious from the text and include the other proofs in the Appendix.

Then, the equilibrium is $(\rho_1^\circ;\sigma_1^\circ,\sigma_2^\circ)$*, which is defined in (11), (12), and (14).*

In the equilibrium $(\rho_1^\circ;\sigma_1^\circ,\sigma_2^\circ)$, we can derive the firms' output (q_1°,q_2°) and market quantity Q° as follows:

$$(q_1^\circ,q_2^\circ)=(\sigma_1^\circ(\rho_1^\circ),\sigma_2^\circ(\rho_1^\circ))=\left(\frac{5-7c_1+2c_2-5\theta}{10},\frac{2(c_1-c_2)}{5}\right), \tag{15}$$

$$Q^\circ=q_1^\circ+q_2^\circ=\frac{1}{10}(5-3c_1-2c_2-5\theta). \tag{16}$$

Note that (3) and (4) ensure $q_1^\circ>0$ and $q_2^\circ>0$. As the difference between c_1 and c_2 decreases, q_2^o decreases and converges to 0. This means that, only when Firm 2 produces goods more efficiently than Firm 1, Firm 1 allows Firm 2 to enter the market and takes part of the surplus from Firm 2's produced goods as its profit. The equilibrium market price p° is determined at

$$p^\circ=1-Q^\circ=\frac{1}{10}(5+3c_1+2c_2+5\theta). \tag{17}$$

In the equilibrium $(\rho_1^\circ;\sigma_1^\circ,\sigma_2^\circ)$, Firm 1 sets the access fee r° for the essential facility above the marginal cost θ of operating it, but it allows Firm 2 to supply the goods to the market. Firm 1 can exclude Firm 2 from the market by setting the access fee r to be extremey high. However, Firm 1 does not implement this in the equilibrium, because it is more beneficial for Firm 1 to allow Firm 2, which is more efficient in producing goods, to supply goods to the market and absorb some of the surplus from Firm 2's supply of goods as its own through the access fee for the essential facility than to exclude Firm 2 from the market.[6] In addition, Firm 1's cost of using the essential facility is θ, while Firm 2's cost of using the essential facility is the access fee r. Thus, $r>\theta$ can be interpreted as Firm 1 discriminating against Firm 2 in providing the essential facility.

In the equilibrium, the profits of Firms 1 and 2 are

$$\Pi_1^\circ=p^\circ q_1^\circ+r^\circ q_2^\circ-c_1q_1^\circ-\theta(q_1^\circ+q_2^\circ)=\frac{1}{4}(1-c_1-\theta)^2+\frac{1}{5}(c_1-c_2)^2, \tag{18}$$

$$\Pi_2^\circ=p^\circ q_2^\circ-r^\circ q_2^\circ-c_2q_2^\circ=\frac{4}{25}(c_1-c_2)^2. \tag{19}$$

Consumer surplus in the equilibrium is

[6] To see this, note that q_2° has a value of 0 in (15) when $c_1=c_2$.

$$CS^{\circ} = Q^{\circ} - \frac{1}{2}(Q^{\circ})^2 - p^{\circ}Q^{\circ} \tag{20}$$
$$= \frac{1}{8}(1-c_1-\theta)^2 + \frac{1}{50}(c_1-c_2)^2 + \frac{1}{10}(1-c_1-\theta)(c_1-c_2).$$

Social welfare in the equilibrium is

$$SW^{\circ} = (q_1^{\circ} + q_2^{\circ}) - \frac{1}{2}(q_1^{\circ} + q_2^{\circ})^2 - c_1 q_1^{\circ} - c_2 q_2^{\circ} - \theta(q_1^{\circ} + q_2^{\circ}) \tag{21}$$
$$= \frac{3}{8}(1-c_1-\theta)^2 + \frac{19}{50}(c_1-c_2)^2 + \frac{1}{10}(1-c_1-\theta)(c_1-c_2).$$

3.2. Nondiscriminatory Provision of Essential Facility

Next, we find a subgame perfect equilibrium for the case in which the government regulates the access fee r with social welfare in mind. The government and the firms make decisions through two stages. In Stage 1, the government chooses the access fee r for the essential facility, which is publicly announced. Here, we assume that the government cannot set r to be lower than θ (i.e., $r \geq \theta$ must be satisfied) to reflect that the government should not harm Firm 1 by regulating the use of the essential facility. In Stage 2, each firm i simultaneously chooses its output q_i. In this case, the government's strategy ρ_0 in Stage 1 is its choice of access fee r, and each firm i's strategy $\sigma_i(r)$ is its choice of output q_i in Stage 2 when the government chooses r in Stage 1.

To find a subgame perfect equilibrium $(\rho_0^{\bullet};\sigma_1^{\bullet},\sigma_2^{\bullet})$, we first consider Stage 2 given that the government sets the access fee r for the essential facility in Stage 1. From the same arguments in Section 3.1, we obtain the equilibrium strategies $(\sigma_1^{\bullet},\sigma_2^{\bullet})$ for the firms as

$$\sigma_1^{\bullet}(r) = \frac{1}{3}(1-2c_1+c_2-2\theta+r), \tag{22}$$

$$\sigma_2^{\bullet}(r) = \frac{1}{3}(1+c_1-2c_2+\theta-2r). \tag{23}$$

Given that the firms play $(\sigma_1^{\bullet},\sigma_2^{\bullet})$ in Stage 2, the government in Stage 1 chooses the access fee r for the essential facility to maximize social welfare

$$SW(r) = (\sigma_1^{\bullet}(r)+\sigma_2^{\bullet}(r)) - \frac{1}{2}(\sigma_1^{\bullet}(r)+\sigma_2^{\bullet}(r))^2 \tag{24}$$
$$-c_1\sigma_1^{\bullet}(r) - c_2\sigma_2^{\bullet}(r) - \theta(\sigma_1^{\bullet}(r)+\sigma_2^{\bullet}(r))$$

$$= -\frac{1}{18}(r^2 + 2(1 - 2\theta + 4c_1 - 5c_2)r - 5\theta^2 - 16c_1\theta$$
$$+2c_2\theta + 14\theta - 11c_1^2 + 14c_1c_2 + 8c_1 - 11c_2^2 + 8c_2 - 8) .$$

under the constraint of $r \geq \theta$. Since $SW(r)$ in (24) is strictly concave, the first-order necessary condition for maximizing $SW(r)$ implies that the equilibrium strategy of the government is

$$\rho_0^\bullet = r^\bullet = \theta . \tag{25}$$

Proposition 2 summarizes the previous arguments.

Proposition 2 *Suppose that the government chooses the access fee r for the essential facility under the constraint of* $r \geq \theta$ *. Then, the equilibrium is* $(\rho_0^\bullet;\sigma_1^\bullet,\sigma_2^\bullet)$*, which is defined in (22), (23), and (25).*

In the equilibrium $(\rho_0^\bullet;\sigma_1^\bullet,\sigma_2^\bullet)$ in Proposition 2, the government, which is concerned with social welfare, regulates the access fee for the essential facility at $r^\bullet = \theta$, which is lower than the access fee r° that Firm 1 chooses without regulation as in Section 3.1 (i.e., $r^\bullet < r^\circ$). We can interpret $r^\bullet = \theta$ as the firms having nondiscriminatory access to the essential facility in the sense that they pay the same cost to use the essential facility. In this equilibrium, since the access fee for the essential facility is set at the marginal cost of operating it, Firm 1 cannot receive rent from Firm 2's use of the essential facility.

In the equilibrium $(\rho_0^\bullet;\sigma_1^\bullet,\sigma_2^\bullet)$, the firms' output $(q_1^\bullet,q_2^\bullet)$ and market quantity $Q^\bullet$ are determined as follows:

$$(q_1^\bullet,q_2^\bullet) = (\sigma_1^\bullet(\rho_0^\bullet),\sigma_2^\bullet(\rho_0^\bullet)) = \left(\frac{1-2c_1+c_2-\theta}{3},\frac{1+c_1-2c_2-\theta}{3}\right) \tag{26}$$

$$Q^\bullet = q_1^\bullet + q_2^\bullet = \frac{1}{3}(2 - c_1 - c_2 - 2\theta) . \tag{27}$$

Under the assumptions (3) and (4), $q_1^\bullet > 0$ and $q_2^\bullet > 0$ are ensured. The equilibrium market price $p^\bullet$ is determined at

$$p^\bullet = 1 - Q^\bullet = \frac{1}{3}(1 + c_1 + c_2 + 2\theta) . \tag{28}$$

Comparing the firms' output $(q_1^\bullet,q_2^\bullet)$ in this section and (q_1°,q_2°) in Section 3.1, we have $q_1^\bullet < q_1^\circ$ and $q_2^\bullet > q_2^\circ$. It is intuitively clear that the government's regulation to lower the access fee r for the essential facility reduces Firm 2's cost in supplying goods to the market, which has a positive effect on Firm 2's output q_2 and a negative effect on its competitor Firm 1's output q_1. Furthermore, the increase in Firm 2's output due to the lowering of the access fee for the essential facility is greater than the decrease in Firm 1's output, resulting in an increase in the total output Q in the market and a decrease in the market price p of goods. Thus, the equilibrium market price $p^{\bullet}$ in this section is lower than p° in Section 3.1 (i.e., $p^{\bullet} < p^{\circ}$).

In the equilibrium $(\rho_0^{\bullet};\sigma_1^{\bullet},\sigma_2^{\bullet})$, the profits of Firms 1 and 2 are

$$\Pi_1^{\bullet} = p^{\bullet}q_1^{\bullet} + r^{\bullet}q_2^{\bullet} - c_1q_1^{\bullet} - \theta(q_1^{\bullet}+q_2^{\bullet}) = \frac{1}{9}(1-2c_1+c_2-\theta)^2, \tag{29}$$

$$\Pi_2^{\bullet} = p^{\bullet}q_2^{\bullet} - r^{\bullet}q_2^{\bullet} - c_2q_2^{\bullet} = \frac{1}{9}(1+c_1-2c_2-\theta)^2. \tag{30}$$

The consumer surplus in the equilibrium is

$$CS^{\bullet} = Q^{\bullet} - \frac{1}{2}(Q^{\bullet})^2 - p^{\bullet}Q^{\bullet} = \frac{1}{18}(2-2\theta-c_1-c_2)^2. \tag{31}$$

The social welfare in the equilibrium is

$$\begin{aligned} SW^{\bullet} &= CS^{\bullet} + \Pi_1^{\bullet} + \Pi_2^{\bullet} \\ &= \frac{4}{9}(1-c_1-\theta)^2 + \frac{11}{18}(c_1-c_2)^2 + \frac{4}{9}(1-c_1-\theta)(c_1-c_2). \end{aligned} \tag{32}$$

By comparing (18) to (21) and (29) to (32), we can see that Firm 1 gains higher profit when it decides the access fee for its essential facility (i.e., $\Pi_1^{\circ} > \Pi_1^{\bullet}$), while Firm 2 obtains a higher profit when the government regulates the access fee for the essential facility (i.e., $\Pi_2^{\circ} < \Pi_2^{\bullet}$). This is clear because the government's regulation lowers the access fee for the essential facility, which reduces Firm 2's cost of supplying goods and eliminates Firm 1's profit from its essential facility. In addition, since the government's regulation increases the market output and reduces the market price for goods, it is obvious that it improves consumer surplus and social welfare (i.e., $CS^{\circ} < CS^{\bullet}$ and $SW^{\circ} < SW^{\bullet}$). This is because the government encourages Firm 2, which is more efficient in producing goods, to produce more by preventing Firm 1 from benefiting from the essential facility.

IV. Equilibrium under Asymmetric Information

In this section, we investigate the situation wherein the realization of marginal cost θ to operate an essential facility is private information to Firm 1. To this end, we consider three cases. In the first case, Firm 1 chooses the access fee r for the essential facility without any regulation. In the second case, the government receives information about θ from Firm 1 and regulates the access fee r considering social welfare. For this case, we show that the government cannot obtain accurate information about θ from Firm 1, resulting in social welfare losses. In the last case, we discuss how the government can resolve information asymmetry between Firm 1 and itself by introducing a mechanism. For the situation wherein the realization of θ is Firm 1's private information, we adopt the *perfect Bayesian equilibrium* as a solution concept.[7]

4.1. Discriminatory Provision of Essential Facilities

Consider the case wherein Firm 1 chooses the access fee r for the essential facility without regulation. As in Section 3, the decisions are made in two stages. In Stage 1, Firm 1 that observes θ chooses the access fee r for the essential facility, which is publicly known. In Stage 2, each firm i simultaneously chooses its output q_i.

In this case, Firm 1's strategy in Stage 1 can be represented as a function $\rho_1(\theta)$, which is the access fee that Firm 1 chooses when observing θ. Firm 1's strategy in Stage 2 can be represented as a function $\sigma_1(\theta,r)$, which is Firm 1's choice on its output when it observes θ and chooses an access fee r for the essential facility in Stage 1. Firm 2's strategy is also a function $\sigma_2(r)$, which is Firm 2's choice of output when Firm 1 chooses an access fee r for the essential facility.[8]

To find a perfect Bayesian equilibrium $(\rho_1^\dagger,\sigma_1^\dagger,\sigma_2^\dagger)$, we assume that $\rho_1^\dagger$ is strictly increasing in θ, which will be confirmed after we construct $\rho_1^\dagger$. Let $\mathcal{R}(\rho_1^\dagger)$ be the set of access fees r that Firm 1 can choose in Stage 1 under $\rho_1^\dagger$. That is,

$$\mathcal{R}(\rho_1^\dagger)=\{r:\rho_1^\dagger(\theta)=r \text{ for some } \theta\in[0,\overline{\theta}]\}. \tag{33}$$

Since $\rho_1^\dagger(\theta)$ is strictly increasing, its inverse $\rho^{\dagger-1}(r)$ is well-defined for $r\in\mathcal{R}(\rho_1^\dagger)$.

[7] The perfect Bayesian equilibrium requires that a strategy profile satisfy two properties for some belief system: sequential rationality and consistency. Sequential rationality means that the firms choose the actions that maximize their payoffs under their beliefs. Consistency means that firms form their beliefs using Bayes' rule from the strategy whenever possible.

[8] The firms may be able to use mixed strategies (behavioral strategies) in Stages 1 and 2. However, since they will not use mixed strategies in equilibrium, we can restrict our attention to pure strategies.

Let $\mathbb{E}[\cdot\,|\,r]$ be Firm 2's expectation after observing that firm 1 chooses an access fee r in Stage 1. Under the belief consistent with $\rho_1^{\dagger}$, it should be satisfied that $\mathbb{E}[\theta\,|\,r]=\mathbb{E}[\theta\,|\,\rho_1^{\dagger}(\theta)=r]=\rho_1^{\dagger-1}(r)$ for $r\in\mathcal{R}(\rho_1^{\dagger})$.

Consider Stage 2 wherein Firm 1 that uses $\rho_1^{\dagger}$ chooses $r\in\mathcal{R}(\rho_1^{\dagger})$. Given that the other firm j plays $\sigma_j^{\dagger}$ in Stage 2, Firms 1 and 2 choose $q_1=\sigma_1^{\dagger}(r;\theta)$ and $q_2=\sigma_2^{\dagger}(r)$, respectively, which maximize their (expected) profits:

$$\Pi_1=(1-q_1-\sigma_2^{\dagger}(r))q_1+r\sigma_2^{\dagger}(r)-c_1q_1-\theta(q_1+\sigma_2^{\dagger}(r)),\tag{34}$$

$$\mathbb{E}[\Pi_2\,|\,r]=\mathbb{E}[(1-\sigma_1^{\dagger}(\theta,r)-q_2)q_2-rq_2-c_2q_2\,|\,r].\tag{35}$$

The first-order necessary conditions for maximizing Π_1 in (34) and $\mathbb{E}[\Pi_2\,|\,r]$ in (35) imply that for $\theta\in[0,\overline{\theta}]$ and $r\in\mathcal{R}(\rho_1^{\dagger})$,

$$\sigma_1^{\dagger}(r;\theta)=\frac{1}{2}(1-\sigma_2^{\dagger}(r)-c_1-\theta),\tag{36}$$

$$\sigma_2^{\dagger}(r)=\frac{1}{2}(1-\mathbb{E}[\sigma_1^{\dagger}(\theta,r)\,|\,r]-r-c_2).\tag{37}$$

Since $\mathbb{E}[\theta\,|\,r]=\rho_1^{\dagger-1}(r)$ for $r\in\mathcal{R}(\rho_1^{\dagger})$, (36) imply $\mathbb{E}[\sigma_1^{\dagger}(\theta,r)\,|\,r]=(1/2)(1-\sigma_2^{\dagger}(r)-c_1-\rho_1^{\dagger-1}(r))$. Plugging this into (37) and rearranging the equations, we obtain the equilibrium strategies of the firms in Stage 2 as

$$\sigma_1^{\dagger}(r;\theta)=\frac{1}{6}(2-4c_1+2c_2-3\theta+2r-\rho_1^{\dagger-1}(r)),\tag{38}$$

$$\sigma_2^{\dagger}(r)=\frac{1}{3}(1+c_1-2c_2-2r+\rho_1^{\dagger-1}(r)).\tag{39}$$

Next, we consider Stage 1 wherein Firm 1 that observes θ has to choose the access fee r for the essential facility. Given that the firms play $(\sigma_1^{\dagger},\sigma_2^{\dagger})$ in (38) and (39) in Stage 2, Firm 1's profit by choosing r is

$$\begin{aligned}\Pi_1(r)&=(1-\sigma_1^{\dagger}(r;\theta)-\sigma_2^{\dagger}(r))\sigma_1^{\dagger}(r;\theta)+r\sigma_2^{\dagger}(r)\\&\quad-c_1\sigma_1^{\dagger}(r;\theta)-\theta(\sigma_1^{\dagger}(r;\theta)+\sigma_2^{\dagger}(r))\\&=\frac{1}{36}(2+2c_1+2c_2+3\theta+2r-\rho_1^{\dagger-1}(r))(2-4c_1+2c_2-3\theta+2r-\rho_1^{\dagger-1}(r))\\&\quad+\frac{1}{3}r(1+c_1-2c_2-2r+\rho_1^{\dagger-1}(r))-\frac{1}{6}c_1(2-4c_1+2c_2-3\theta+2r-\rho_1^{\dagger-1}(r))\\&\quad-\frac{1}{6}\theta(4-2c_1-2c_2-3\theta-2r+\rho_1^{\dagger-1}(r)).\end{aligned}\tag{40}$$

Since $\rho_1^{\dagger-1}(r)=\theta$ and $\rho_1^{\dagger}(\theta)=r$ for $r\in\mathcal{R}(\rho_1^{\dagger})$, we can see that the first-order necessary condition to maximize $\Pi_1(r)$ is equivalent to

$$2(5-c_1-4c_2+5\theta-10\rho_1^{\dagger}(\theta))\frac{d\rho_1^{\dagger}(\theta)}{d\theta}-(2-4c_1+2c_2+2\theta-4\rho_1^{\dagger}(\theta))=0\,. \tag{41}$$

Solving this differential equation with respect to $\rho_1^{\dagger}(\theta)$, we obtain[9]

$$\rho_1^{\dagger}(\theta)=r^{\dagger}=\frac{1}{2}(1+c_1-2c_2+\theta)\,. \tag{42}$$

Note that $\rho_1^{\dagger}$ in (42) is strictly increasing in θ and its inverse is $\rho_1^{\dagger-1}(r)=-1-c_1+2c_2+2r$. Plugging $\rho_1^{\dagger-1}(r)$ into (40) and rearranging the equation, we obtain $\Pi_1(r)=(1/4)(1-c_1-\theta)^2$, which does not depend on r . Under the belief that Firm 1 plays $\rho_1^{\dagger}(\theta)$ in (42), $\Pi_1(r)$ does not depend on the access fee r that Firm 1 chooses. This means that $\rho_1^{\dagger}(\theta)$ in (42) is Firm 1's equilibrium strategy in Stage 1. Plugging $\rho_1^{\dagger-1}(r)$ into (38) and (39), we obtain the firms' equilibrium strategies in Stage 1 as

$$\sigma_1^{\dagger}(r;\theta)=\frac{1}{2}(1-c_1-\theta)\,, \tag{43}$$

$$\sigma_2^{\dagger}(r)=0\,. \tag{44}$$

Proposition 3 summarizes the previous arguments.

Proposition 3 *Suppose that Firm 1 chooses the access fee r for an essential facility. Then, the equilibrium is $(\rho_1^{\dagger};\sigma_1^{\dagger},\sigma_2^{\dagger})$, which is defined in (42), (43), and (44).*

In the equilibrium $(\rho_1^{\dagger},\sigma_1^{\dagger},\sigma_2^{\dagger})$ in Proposition 3, each firm i 's ex-post output $(q_1^{\dagger},q_2^{\dagger})$ and market quantity $Q^{\dagger}$ are

$$(q_1^{\dagger},q_2^{\dagger})=(\sigma_1^{\dagger}(\theta,\rho_1^{\dagger}(\theta)),\sigma_2^{\dagger}(\rho_1^{\dagger}(\theta)))=\left(\frac{1-c_1-\theta}{2},0\right), \tag{45}$$

[9] We note that $\rho_1^{\dagger}(\theta)$ in (42) is not a unique solution for (41). Indeed, any $\rho_1^{\dagger}(\theta)$ satisfying

$$6(c_1-c_2)\ln\left(\rho_1^{\dagger}(\theta)-\frac{1}{2}\theta-\frac{1}{2}c_1+c_2-\frac{1}{2}\right)+10\rho_1^{\dagger}(\theta)=2\theta-10c_1+5c_2+C$$

for a constant $C\in\mathbb{R}$ is a solution for the differential equation in (41). Unfortunately, we cannot find the closed form of $\rho_1^{\dagger}(\theta)$ that satisfies the above equation, which makes it difficult to find an equilibrium strategy for Firm 1 in Stage 1. Thus, we focus on the equilibrium wherein Firm 1 plays $\rho_1^{\dagger}(\theta)$ in (42) in Stage 1 in this section.

$$Q^{\dagger} = q_1^{\dagger} + q_2^{\dagger} = \frac{1}{2}(1 - c_1 - \theta). \tag{46}$$

The assumptions (3) and (4) ensure that $q_1^{\dagger} > 0$. The ex-post equilibrium market price $p^{\dagger}$ is determined by

$$p^{\dagger} = 1 - Q^{\dagger} = \frac{1}{2}(1 + c_1 + \theta). \tag{47}$$

In this equilibrium, whatever Firm 1 chooses as the access fee r for the essential facility, Firm 2 never enters the market under the belief that it cannot gain a positive profit by entering the market. That is, Firm 1 cannot improve its profit by allowing Firm 2 to enter the market and absorbing some of the surplus from the entry of Firm 2 as its own profit through the access fee for the essential facility. Indeed, when the marginal cost θ of operating the essential facility is private information to Firm 1, the access fee r chosen by Firm 1 serves as a signal for θ to Firm 2. Thus, if r is set low, then Firm 2 believes that θ is low and that it will suffer a loss from entering the market because Firm 1 produces more. Meanwhile, if r is set high, Firm 2 believes that θ is high and that Firm 1 produces less but that Firm 2 will nevertheless suffer a loss from entering the market because of the high cost of using the essential facility.

Comparing the access fees $r^{\dagger}$ in (42) and r° in (14) in Section 3.1, we can see that, for any $\theta \in [0, \overline{\theta}]$, $r^{\dagger} > r^{\circ}$. This means that Firm 1 sets a higher access fee r for its essential facility when the realization of θ is its private information than when it is publicly known. This intuitively makes sense as Firm 1 has to set a high access fee for the essential facility to exclude Firm 2 from the market. Comparing the equilibrium market prices $p^{\dagger}$ in (47) and p° in (17) in Section 3.1, we have that, for any $\theta \in [0, \overline{\theta}]$, $p^{\dagger} > p^{\circ}$. This is clear because by excluding Firm 2 from the market, Firm 1 can set a monopolistic market price, which is higher than the equilibrium price when θ is publicly known.

In the equilibrium, the profits of Firms 1 and 2 are

$$\Pi_1^{\dagger} = p^{\dagger} q_1^{\dagger} + r^{\dagger} q_2^{\dagger} - c_1 q_1^{\dagger} - \theta(q_1^{\dagger} + q_2^{\dagger}) = \frac{1}{4}(1 - c_1 - \theta)^2, \tag{48}$$

$$\Pi_2^{\dagger} = p^{\dagger} q_2^{\dagger} - r^{\dagger} q_2^{\dagger} - c_2 q_2^{\dagger} = 0. \tag{49}$$

The consumer surplus in the equilibrium is

$$CS^{\dagger} = Q^{\dagger} - \frac{1}{2}(Q^{\dagger})^2 - p^{\dagger} Q^{\dagger} = \frac{1}{8}(1 - c_1 - \theta)^2. \tag{50}$$

The social welfare in the equilibrium is

$$SW^{\dagger} = CS^{\dagger} + \Pi_1^{\dagger} + \Pi_2^{\dagger} = \frac{3}{8}(1 - c_1 - \theta)^2. \tag{51}$$

By comparing (48) to (51) and (18) to (21) in Section 3.1, we obtain Proposition 4.

Proposition 4 *Suppose that Firm 1 chooses the access fee r without regulation. Let Π_i°, CS°, and SW° be Firm i's profit, consumer surplus, and social welfare, respectively, in the equilibrium when the marginal cost of operating the essential facility is public information. Let $\Pi_i^{\dagger}$, $CS^{\dagger}$, and $SW^{\dagger}$ be Firm i's profit, consumer surplus, and social welfare, respectively, in the equilibrium when the marginal cost of operating the essential facility is private information to Firm 1. Then, (i) $\Pi_1^{\circ} > \Pi_1^{\dagger}$, (ii) $\Pi_2^{\circ} > \Pi_2^{\dagger}$, (iii) $CS^{\circ} > CS^{\dagger}$, and (iv) $SW^{\circ} > SW^{\dagger}$.*

Proposition 4 is a direct consequence of comparing (18) to (21) and (48) to (51). In Proposition 4, (i) and (ii) state that the profits of the firms under asymmetric information are lower than those under perfect information. Firm 2 is more efficient in producing goods than Firm 1, so allowing Firm 2 to supply goods in the market may improve social welfare. However, Firm 2 exits the market under asymmetric information, possibly reducing social residual in the market and removing Firm 1's chance to take the social residual from Firm 2's efficiency in producing goods through the access fee for the essential facility. Meanwhile, (iii) states that the consumers under asymmetric information obtain lower consumer surplus, which is clear from the fact that the price of goods is higher under asymmetric information than under perfect information. Since the firms and the consumers acquire lower welfare under asymmetric information, it is also clear that social welfare is lower under asymmetric information than under perfect information. The decline in social welfare under asymmetric information is due to Firm 1 monopolizing the supply of goods by preventing Firm 2 from entering the market.

Proposition 4 implies that if Firm 1 can reveal its private information truthfully, then Firm 1 has the incentive to do so, improving its profits, consumer surplus, and social welfare. However, as long as Firm 1 is able to choose the access fee for its essential facility, its truthful revelation of its private information does not eliminate the inefficiency caused by the discriminatory provision of the essential facility.

4.2. Nondiscriminatory Provision of Essential Facilities

Next, we consider the case wherein the government regulates the access fee r for the essential facility. The government cannot observe θ. Hence, it may ask Firm 1 to (partially) reveal its information on θ, but it cannot verify whether Firm 1's revelation is true. We model this case by considering a three-stage decision procedure. In Stage 1, Firm 1 sends a message $m \in [0,\overline{\theta}]$. The message m sent by Firm 1 is publicly known but is not verifiable in the sense that message m may not convey exactly how θ is realized. In Stage 2, the government chooses the access fee r for the essential facility, which is publicly known. Here, we also assume that the government cannot set the access fee r to be lower than the expectation of the marginal cost θ to operate the essential facility, indicating that the government should not harm Firm 1 through regulation. In Stage 3, Firms 1 and 2 choose their output (q_1,q_2) simultaneously.

We allow Firm 1 to send a random message m in Stage 1. Thus, Firm 1's strategy in Stage 1 can be represented as a function $\mu_1:[0,\overline{\theta}]\to\Delta[0,\overline{\theta}]$, where $\Delta[0,\overline{\theta}]$ is the set of distributions on $[0,\overline{\theta}]$. $\mu_1(\theta)$ represents a strategy of Firm 1 that sends a random message following a distribution $\mu_1(\theta)$ on $[0,\overline{\theta}]$ after observing θ. The government's strategy in Stage 2 is represented as a function $\rho_0:[0,\overline{\theta}]\to\mathbb{R}_+$, where $\rho_0(m)$ is the access fee r that the government chooses when observing a message m from Firm 1. Firm 1's strategy in Stage 3 is a function $\sigma_1:[0,\overline{\theta}]\times\mathbb{R}_+\times[0,\overline{\theta}]\to\mathbb{R}_+$, where $\sigma_1(m,r;\theta)$ is the output that Firm 1 observing θ chooses when it sends a message m and the government chooses an access fee r for the essential facility. Firm 2's strategy in Stage 3 is a function $\sigma_2:[0,\overline{\theta}]\times\mathbb{R}_+\to\mathbb{R}_+$, where $\sigma_2(m,r)$ is the output that Firm 2 chooses when Firm 1 sends a message m and the government chooses an access fee r for the essential facility. For this case, we also find a perfect Bayesian equilibrium $(\mu_1^{\ddagger},\rho_0^{\ddagger},\sigma_1^{\ddagger},\sigma_2^{\ddagger})$.

Let $\mathcal{M}(\mu_1^{\ddagger})$ be the set of messages that can be sent under $\mu_1^{\ddagger}$ and $\mathcal{E}(\mu_1^{\ddagger})$ be the set of expectations of θ that can be formed under $\mu_1^{\ddagger}$. That is,

$$\mathcal{M}(\mu_1^{\ddagger})=\{m: m\in \operatorname{supp}(\mu_1^{\ddagger}(\theta)) \text{ for some } \theta\in[0,\overline{\theta}]\}, \text{ and} \tag{52}$$

$$\mathcal{E}(\mu_1^{\ddagger})=\{\varepsilon:\varepsilon=\mathbb{E}[\theta\,|\,\mu_1^{\ddagger}(\theta)=\mathrm{m}] \text{ for some } m\in\mathcal{M}(\mu_1^{\ddagger})\}. \tag{53}$$

Utilizing backward induction, consider Stage 3 after Firm 1 sends a message $m\in\mathcal{M}(\mu_1^{\ddagger})$ in Stage 1 and the government chooses $r=\rho_0^{\ddagger}(m)$ in Stage 2.[10] Under the belief consistent with $\mu_1^{\ddagger}$, the expectation of θ conditioning on message $m\in\mathcal{M}(\mu_1^{\ddagger})$ sent in Stage 1 is $\mathbb{E}[\theta\,|\,m]=\mathbb{E}[\theta\,|\,\mu_1^{\ddagger}(\theta)=m]$. For notational simplicity, let $\varepsilon_m=\mathbb{E}[\theta\,|\,\mu_1^{\ddagger}(\theta)=m]$ for a message $m\in\mathcal{M}(\mu_1^{\ddagger})$. Then, arguments similar to those in Section 4.1 yield the firms' equilibrium strategy in Stage 2 as

[10] Here, we consider a history that can appear along the path of the equilibrium. Because the perfect Bayesian equilibrium does not require consistency for the information sets out of the equilibrium path, it is easy to construct equilibrium strategies for the information sets out of the equilibrium path. We focus on comparing the equilibrium outcomes in various situations. Thus, we do not provide the equilibrium strategies for the information sets out of the equilibrium path.

$$\sigma_1^{\ddagger}(m,r;\theta)=\frac{1}{6}(2-4c_1+2c_2-3\theta-\varepsilon_m+2r), \tag{54}$$

$$\sigma_2^{\ddagger}(m,r)=\frac{1}{3}(1+c_1-2c_2+\epsilon_m-2r). \tag{55}$$

Next, consider Stage 2, wherein the government is supposed to decide r after observing a message $m\in\mathcal{M}(\mu_1^{\ddagger})$ sent by Firm 1 in Stage 1. Given that the firms play $(\sigma_1^{\ddagger},\sigma_2^{\ddagger})$ in (54) and (55) in Stage 3, the expected social welfare when the government chooses r is

$$\begin{aligned}\mathbb{E}[SW\,|\,m]=\;&\mathbb{E}[(\sigma_1^{\ddagger}(m,r;\theta)+\sigma_2^{\ddagger}(m,r))-\frac{1}{2}(\sigma_1^{\ddagger}(m,r;\theta)+n\sigma_2^{\ddagger}(m,r))^2\\ &-c_1\sigma_1^{\ddagger}(m,r;\theta)-c_2\sigma_2^{\ddagger}(m,r)-\theta(\sigma_1^{\ddagger}(m,r;\theta)+\sigma_2^{\ddagger}(m,r))\,|\,m]\\ =\;&\mathbb{E}[\frac{1}{6}(4-2c_1-2c_2-3\theta+\varepsilon_m-2r)-\frac{1}{72}(4-2c_1-2c_2-3\theta-2r+\varepsilon_m)^2\\ &-\frac{1}{6}c_1(2-4c_1+2c_2-3\theta-\varepsilon_m+2r)-\frac{1}{3}c_2(1+c_1-2c_2+\varepsilon_m-2r)\\ &-\frac{1}{6}\theta(4-2c_1-2c_2-3\theta-2r+\varepsilon_m)\,|\,m].\end{aligned} \tag{56}$$

Since $\mathbb{E}[SW\,|\,m]$ is concave in r, the first-order necessary condition for maximizing $\mathbb{E}[SW\,|\,m]$ subject to $r\geq\varepsilon_m$ induces that the government's equilibrium strategy in Stage 2 is

$$\rho_0^{\ddagger}(m)=\varepsilon_m. \tag{57}$$

Lastly, consider Stage 1, wherein Firm 1 that observes θ is supposed to decide message m. Given that the government and the firms play $(\rho_0^{\ddagger},\sigma_1^{\ddagger},\sigma_2^{\ddagger})$ in Stages 2 and 3, Firm 1's profit when observing θ and sending a message $m\in\mathcal{M}(\mu_1^{\ddagger})$ is

$$\begin{aligned}\Pi_1(m;\theta)=\;&(1-\sigma_1^{\ddagger}(m,\rho_0^{\ddagger}(m),\theta)-\sigma_2^{\ddagger}(m,\rho_0^{\ddagger}(m)))\sigma_1^{\ddagger}(m,\rho_0^{\ddagger}(m),\theta)\\ &+\rho_0^{\ddagger}(m)\sigma_2^{\ddagger}(m,\rho_0^{\ddagger}(m))-c_1\sigma_1^{\ddagger}(m,\rho_0^{\ddagger}(m),\theta)\\ &-\theta(\sigma_1^{\ddagger}(m,\rho_0^{\ddagger}(m),\theta)+\sigma_2^{\ddagger}(m,\rho_0^{\ddagger}(m)))\\ =\;&\frac{1}{36}(2+2c_1+2c_2+3\theta+\varepsilon_m)(2-4c_1+2c_2-3\theta+\varepsilon_m)\\ &+\frac{1}{3}\varepsilon_m(1+c_1-2c_2-\varepsilon_m)-\frac{1}{6}c_1(2-4c_1+2c_2-3\theta+\varepsilon_m)\\ &-\frac{1}{6}\theta(4-2c_1-2c_2-\varepsilon_m-3\theta).\end{aligned} \tag{58}$$

For $\mu_1^\ddagger$ to be Firm 1's equilibrium strategy in Stage 1, Firm 1 has to send a message $m \in \mathcal{M}(\mu_1^\ddagger)$, such that $\varepsilon_m = \mathbb{E}[\theta \mid \mu_1^\ddagger(\theta) = m]$ maximizes $\Pi_1(m;\theta)$ in (58) on $\mathcal{E}(\mu_1^\ddagger)$. This observation is used to prove Lemma 1.

Lemma 1 *For any equilibrium* $(\mu_1^\ddagger, \rho_0^\ddagger, \sigma_1^\ddagger, \sigma_2^\ddagger)$, $\mathcal{E}(\mu_1^\ddagger) = \{E_\theta\}$ *holds.*

Lemma 1 means that, in any equilibrium, Firm 1 does not reveal its information on θ through its messages in Stage 1. An example of such strategy is $\mu_1^\ddagger$, such that for any $\theta \in [0, \overline{\theta}]$, Firm 1 sends a random message $\mu^\ddagger(\theta)$ that is uniformly distributed on $[0, \overline{\theta}]$. Another example is strategy $\mu_1^\ddagger$, such that $\mu_1^\ddagger(\theta) = 0$ for any $\theta \in [0, \overline{\theta}]$. Under such uninformative strategies $\mu_1^\ddagger$, messages $m \in \mathcal{M}(\mu_1^\ddagger)$ do not reveal any information about θ, so the government always forms an expectation of θ as E_θ regardless of the messages.

Due to the previous arguments and Lemma 1, we can characterize the equilibrium in Proposition 5.

Proposition 5 *Suppose that the government chooses the access fee* r *for the essential facility under the constraint that* r *is not less than its expectation on* θ. *Then, there is an equilibrium* $(\mu_1^\ddagger, \rho_0^\ddagger, \sigma_1^\ddagger, \sigma_2^\ddagger)$ *that satisfies (i) for any* $m \in \mathcal{M}(\mu_1^\ddagger)$, $\mathbb{E}[\theta \mid \mu_1^\ddagger(\theta) = m] = E_\theta$, *(ii)* $\rho_0^\ddagger(m) = E_\theta$ *for all* $m \in \mathcal{M}(\mu_1^\ddagger)$, *and (iii)* $\sigma_1^\ddagger$ *and* $\sigma_2^\ddagger$ *are as in (54) and (55) with* $\varepsilon_m = E_\theta$. *In addition, every equilibrium* $(\mu_1^\ddagger, \rho_0^\ddagger, \sigma_1^\ddagger, \sigma_2^\ddagger)$ *satisfies (i) through (iii).*

In Proposition 5, (i) means that Firm 1 does not reveal any of its private information through the messages, which is a direct consequence of Lemma 1. (ii) follows from (57) and Lemma 1. This means that the government always chooses $r = E_\theta$ as the access fee for the essential facility in any equilibrium. We can interpret it as Firm 1 nondiscriminantly providing the essential facility to Firm 2 in the sense that the marginal costs of the firms to use the essential facility are ex-ante equal.[11] Finally, (iii) describes how the firms choose their output in equilibrium.

In the equilibrium $(\mu_1^\ddagger, \rho_0^\ddagger, \sigma_1^\ddagger, \sigma_2^\ddagger)$ in Proposition 5, each firm i's ex-post output $(q_1^\ddagger, q_2^\ddagger)$ and the market quantity $Q^\ddagger$ are

$$
\begin{aligned}
(q_1^\ddagger, q_2^\ddagger) &= (\sigma_1^\ddagger(\mu_1^\ddagger(\theta), \rho_0^\ddagger(\mu_1^\ddagger(\theta)), \theta), \sigma_2^\ddagger(\mu_1^\ddagger(\theta), \rho_0^\ddagger(\mu_1^\ddagger(\theta)))) \\
&= \left(\frac{2 - 4c_1 + 2c_2 - 3\theta + E_\theta}{6}, \frac{1 + c_1 - 2c_2 - E_\theta}{3} \right),
\end{aligned}
\tag{59}
$$

[11] In the equilibrium, the firms pay the same ex-ante costs to use the essential facility but may pay different ex-post costs to use the essential facility. Firm 1's ex-post marginal cost to use the essential facility is a realization of θ, and Firm 2's ex-post marginal cost to use the essential facility is $r = E_\theta$.

$$Q^{\ddagger} = q_1^{\ddagger} + q_2^{\ddagger} = \frac{1}{6}(4 - 2c_1 - 2c_2 - 3\theta - E_\theta)\,. \tag{60}$$

In addition, the ex-post price $p^{\ddagger}$ of the goods in the equilibrium is

$$p^{\ddagger} = 1 - Q^{\ddagger} = \frac{1}{6}(2 + 2c_1 + 2c_2 + 3\theta + E_\theta)\,, \tag{61}$$

and the ex-post access fee for the essential facility in the equilibrium is $r^{\ddagger} = E_\theta$.

Given that the marginal cost θ of operating the essential facility is Firm 1's private information, we can compare the ex-ante access fees for the essential facility in the equilibrium when Firm 1 decides it and when Firm 1 sends a message about θ and the government decides the access fee. By comparing the expectations of (42) and (57), we can see that the ex-ante access fee for the essential facility is lower when the government regulates it than when Firm 1 decides it (i.e., $\mathbb{E}[r^{\ddagger}] < \mathbb{E}[r^{\dagger}]$). By comparing the expectations of (47) and (61), we can also see that the ex-ante market price is lower when the government regulates the access fee for the essential facility than when Firm 1 decides it (i.e., $\mathbb{E}[p^{\ddagger}] < \mathbb{E}[p^{\dagger}]$). That is, the government's regulation of the access fee can increase the market quantity of goods. A reasoning similar to that in the situation wherein θ is publicly known can be applied to these observations.

Moreover, given that the government regulates the access fee for the essential facility, we can compare the ex-ante access fees for the essential facility and the ex-ante market prices in the equilibrium when θ is public information and when θ is private information to Firm 1. By comparing the expectations of (25) and (57), we obtain $\mathbb{E}[r^{\ddagger}] = E_\theta = \mathbb{E}[r^{\bullet}]$. When the government is not informed of the realization of θ, its regulation of the access fee for the essential facility fixes the access fee at E_θ regardless of the realization of θ, and it does not affect the ex-ante access fee for the essential facility. In addition, by comparing the expectations of (28) and (61), we can see that the ex-ante market price is the same regardless of the existence of information asymmetry (i.e., $\mathbb{E}[p^{\ddagger}] = \mathbb{E}[p^{\bullet}]$). However, the ex-post market price appears differently depending on the realization of θ, and it is less volatile when the marginal cost of operating the essential facility is private information to Firm 1.

In the equilibrium, the ex-post profits of Firms 1 and 2 are

$$\begin{aligned} \Pi_1^{\ddagger} = {} & p^{\ddagger} q_1^{\ddagger} + r^{\ddagger} q_2^{\ddagger} - c_1 q_1^{\ddagger} - \theta(q_1^{\ddagger} + q_2^{\ddagger}) \\ = {} & \frac{1}{4}(1 - c_1 - \theta)^2 + \frac{1}{9}(c_1 - c_2)^2 - \frac{1}{36}(5 - 5c_1 + 6\theta - 11E_\theta)(1 - c_1 - E_\theta) \\ & - \frac{1}{9}(2 - 2c_1 + 3\theta - 5E_\theta)(c_1 - c_2)\,, \end{aligned} \tag{62}$$

$$\begin{aligned}\Pi_2^{\ddagger} &= p^{\ddagger}q_2^{\ddagger} - r^{\ddagger}q_2^{\ddagger} - c_2 q_2^{\ddagger} \\ &= \frac{4}{9}(c_1 - c_2)^2 + \frac{1}{18}(2 - 2c_1 + 3\theta - 5E_\theta)(1 - c_1 - E_\theta) \\ &\quad + \frac{1}{9}(4 - 4c_1 + 3\theta - 7E_\theta)(c_1 - c_2)\,.\end{aligned} \tag{63}$$

The ex-post consumer surplus in the equilibrium is

$$\begin{aligned}CS^{\ddagger} &= Q^{\ddagger} - \frac{1}{2}(Q^{\ddagger})^2 - p^{\ddagger}Q^{\ddagger} = \frac{1}{72}(4 - 2c_1 - 2c_2 - 3\theta - E_\theta)^2 \\ &= \frac{1}{18}(c_1 - c_2)^2 + \frac{1}{8}(1 - c_1 - \theta)^2 + \frac{1}{72}(7 - 7c_1 - 6\theta - E_\theta)(1 - c_1 - E_\theta) \\ &\quad + \frac{1}{18}(4 - 4c_1 - 3\theta - E_\theta)(c_1 - c_2)\,.\end{aligned} \tag{64}$$

The social welfare in the equilibrium is

$$\begin{aligned}SW^{\ddagger} &= CS^{\ddagger} + \Pi_1^{\ddagger} + \Pi_2^{\ddagger} \\ &= \frac{11}{18}(c_1 - c_2)^2 + \frac{3}{8}(1 - c_1 - \theta)^2 + \frac{1}{18}(4 - 4c_1 - 3\theta - E_\theta)(c_1 - c_2) \\ &\quad + \frac{1}{72}(1 - c_1 - E_\theta)(5 - 5c_1 - 6\theta + E_\theta + 16(c_1 - c_2))\,.\end{aligned} \tag{65}$$

Proposition 6 compares the firms' profits, consumer surplus, and social welfare between the situations when Firm 1 chooses the access fee without any restriction and when the government regulates the access fee, given that the realization of θ is private information to Firm i.

Proposition 6 *Let* $\Pi_i^{\dagger}$, $CS^{\dagger}$, *and* $SW^{\dagger}$ *be firm* i*'s profit, consumer surplus, and social welfare, respectively, in the equilibrium when Firm 1 chooses the access fee without any restriction. Let* $\Pi_i^{\ddagger}$, $CS^{\ddagger}$, *and* $SW^{\ddagger}$ *be firm* i*'s profit, consumer surplus, and social welfare, respectively, in the equilibrium when the government regulates the access fee. Then, (i)* $\mathbb{E}[\Pi_1^{\dagger}] > \mathbb{E}[\Pi_1^{\ddagger}]$, *(ii)* $\mathbb{E}[\Pi_2^{\dagger}] < \mathbb{E}[\Pi_2^{\ddagger}]$, *(iii)* $\mathbb{E}[CS^{\dagger}] < \mathbb{E}[CS^{\ddagger}]$, *and (iv)* $\mathbb{E}[SW^{\dagger}] < \mathbb{E}[SW^{\ddagger}]$.

Proposition 6 states that the government's regulation on the access fee reduces Firm 1's ex-ante profit but improves Firm 2's profit and consumer surplus. The government regulation of the access fee also improves social welfare. The intuition behind Proposition 6 is similar to that in Section 3.2. That is, the government's regulation yields a negative effect on Firm 1's profit and a positive effect on Firm 2's profit by lowering the access fee for the essential facility. It also yields positive effects on consumer surplus and social welfare by inducing a lower price and a larger output of goods in the market.[12]

Proposition 7 compares the firms' profits, consumer surplus, and social welfare when the realization of θ is public information and when the realization of θ is private information to Firm 1, given that the government regulates the access fee for the essential facility.

Proposition 7 *Suppose that the government regulates the access fee* r. *Let* $\Pi_i^{\bullet}$, $CS^{\bullet}$, *and* $SW^{\bullet}$ *be firm* i*'s ex-post profit, ex-post consumer surplus, and ex-post social welfare, respectively, in the equilibrium when the marginal cost of operating the essential facility is public information. Let* $\Pi_i^{\ddagger}$, $CS^{\ddagger}$, *and* $SW^{\ddagger}$ *be firm* i*'s ex-post profit, ex-post consumer surplus, and ex-post social welfare, respectively, in the equilibrium when the marginal cost of operating the essential facility is private information to Firm 1. Then, (i)* $\mathbb{E}[\Pi_1^{\bullet}] < \mathbb{E}[\Pi_1^{\ddagger}]$, *(ii)* $\mathbb{E}[\Pi_2^{\bullet}] > \mathbb{E}[\Pi_2^{\ddagger}]$, *(iii)* $\mathbb{E}[CS^{\bullet}] > \mathbb{E}[CS^{\ddagger}]$, *and (iv)* $\mathbb{E}[SW^{\bullet}] > \mathbb{E}[SW^{\ddagger}]$.

Proposition 7 states that Firm 1's profit is smaller, while social welfare, consumer surplus, and Firm 2's profits are greater under perfect information than under asymmetric information, given that the government regulates the access fee for the essential facility. From (26), (27), (59), and (60), we see that each firm i's output q_i and the market output Q in the equilibrium have the same expectations in both cases (i.e., $\mathbb{E}[q_i^{\ddagger}] = \mathbb{E}[q_i^{\bullet}]$ for $i = 1,2$ and $\mathbb{E}[Q^{\ddagger}] = \mathbb{E}[Q^{\bullet}]$). However, the volatilities of q_1, q_2, and Q, which depend on θ, are different in both cases. Under asymmetric information, a change in θ affects Firm 1's output q_1 by changing its cost to supply goods, but it does not affect Firm 2's output q_2 because Firm 2 does not observe θ. Under perfect information, a change in θ can affect the output of Firms 1 and 2 through the access fee r for the essential facility. Since the effects of the change in θ on q_1 through Firm 1's cost and the access fee are in opposite directions under perfect information, Firm 1's output q_1 is less volatile under perfect information.[13] By contrast, Firm 2's output in the equilibrium is more volatile under perfect information, because the access fee r is fixed and Firm 2 cannot choose its output depending on θ under asymmetric information. Since each firm i's profit in equilibrium is convex in its output, the expectation of firm

[12] Note that the comparisons in Proposition 6 are based on ex-ante values. For ex-post values, the comparisons in Proposition 6 can be reversed depending on the realization of θ. For example, if E_θ is sufficiently high and the realization of θ is sufficiently low, then the government sets the access fee $r = E_\theta$ to be higher than θ. This may result in Firm 1's ex-post profit $\Pi_1^{\dagger}$ without regulation being lower than its ex-post profit $\Pi_1^{\ddagger}$ under the government's regulation.

[13] An increase in Firm 1's cost to supply goods has a negative effect on q_1, and an increase in the access fee for the essential facility has a positive effect on q_1 by inducing Firm 2 to produce fewer goods.

i 's profit is higher when its output is more volatile in the equilibrium. This explains (i) and (ii). Moreover, since Firm 2's output does not depend on θ under asymmetric information, the market output, which is the sum of the firms' outputs, is also more volatile under perfect information. Then, the expectation of consumer surplus, which appears convex in the market output in equilibrium, is greater under perfect information than under asymmetric information. This condition explains (iii). Meanwhile, (iv) in Proposition 7 means that the shift of the information structure from perfect information to asymmetric information has a greater effect on Firm 2's profit and consumer surplus than on Firm 1's profit.

Proposition 7 implies that the accurate disclosure of Firm 1's private information improves consumer surplus and social welfare under the government's regulation of the access fee for the essential facility. However, since such disclosure of Firm 1's private information reduces its profit in equilibrium, Firm 1 does not have an incentive to reveal its private information truthfully. This raises the question of whether it is possible for the government to induce Firm 1 to truthfully reveal its private information, which will be answered in the next section.

4.3. Truth-telling Mechanism

In this section, we introduce a mechanism under which Firm 1 truthfully reveals its private information on θ when the government regulates the access fee r for the essential facility. One such mechanism will be a tax-subsidy scheme wherein Firm 1 has to pay a tax or receive a subsidy depending on its message on θ. Such tax-subsidy scheme can be represented as $\phi(m)$, which indicates the amount of monetary transfer from Firm 1 to the government depending on Firm 1's message m. An appropriate tax-subsidy scheme $\phi(m)$ may induce Firm 1 to reveal its private information on θ truthfully through messages.

The decision procedure in this case is the same as that in Section 4.2, except that $\phi(m)$ is determined before Stage 1 begins. Given a tax-subsidy scheme $\phi(m)$, Firm 1's payoff U_1 is its profit subtracted by $\phi(m)$. That is, $U_1 = \Pi_1 - \phi(m)$, where Π_1 is Firm 1's profit in (5). We denote a perfect Bayesian equilibrium in this case as $(\mu_1^T, \rho_0^T, \sigma_1^T, \sigma_2^T)$. Our goal in this section is to find $\phi(m)$ under which μ_1^T is the truth-telling strategy (i.e., $\mu_1^T(\theta) = \theta$ for all θ).

To find an equilibrium $(\mu_1^T, \rho_0^T, \sigma_1^T, \sigma_2^T)$, consider Stage 3, where the firms are supposed to choose their output (q_1, q_2) given that Firm 1 sends a message $m \in \mathcal{M}(\mu_1^T)$ in Stage 1 and the government chooses r as the access fee for the essential facility in Stage 2. Arguments similar to those in Sections 4.1 and 4.2 yield that the firms' equilibrium strategies (σ_1^T, σ_2^T) in Stage 3 as

$$\sigma_1^T(m, r; \theta) = \frac{1}{6}(2 - 4c_1 + 2c_2 - 3\theta - \varepsilon_m + 2r), \tag{66}$$

$$\sigma_2^T(m,r) = \frac{1}{3}(1 + c_1 - 2c_2 + \varepsilon_m - 2r), \tag{67}$$

where ε_m is the expectation of θ given that a message m from Firm 1 is observed (i.e., $\varepsilon_m = \mathbb{E}[\theta \mid \mu_1^T(\theta) = m]$ for $m \in \mathcal{M}(\mu_1^T)$).

Given the firms' strategies (σ_1^T, σ_2^T) in Stage 3, consider the government's choice of access fee r in Stage 2. Since the government cares about social welfare, as see in Section 4.2, the government chooses the access fee $r = \varepsilon_m$ for the essential facility when it observes a message m from Firm 1 in Stage 1. That is, $\rho_0^T(m) = \varepsilon_m$. Under the belief that the truth-telling strategy μ_1^T is played in Stage 1, it is satisfied that $\mathcal{M}(\mu_1^T) = [0, \bar{\theta}]$ and $\varepsilon_m = m$ for any $m \in [0, \bar{\theta}]$. Thus, the government's equilibrium strategy in Stage 2 is

$$\rho_0^T(m) = m. \tag{68}$$

Next, consider Stage 1, wherein Firm 1 that observes θ is supposed to send a message m. Given that the firms and the government play $(\rho_0^T, \sigma_1^T, \sigma_2^T)$ with the belief that Firm 1 adopts the truth-telling strategy μ_1^T in Stage 1, the payoff of Firm 1 that observes θ and sends message m is

$$\begin{aligned} U_1(m;\theta) &= \Pi_1 - \phi(m) \\ &= \frac{1}{36}(2 - 4c_1 + 2c_2 - 3\theta + m)^2 + \frac{1}{3}(m - \theta)(1 + c_1 - 2c_2 - m) - \phi(m), \end{aligned} \tag{69}$$

where Π_1 is Firm 1's profit in (5) with $q_1 = \sigma_1^T(m,m;\theta)$ and $q_2 = \sigma_2^T(m,m)$. For the truth-telling strategy μ_1^T to constitute an equilibrium, the first-order necessary condition for maximizing $U_1(m;\theta)$ has to be satisfied at $m = \theta$. That is,

$$\frac{d\phi(m)}{dm} = \frac{1}{9}(4 + c_1 - 5c_2 - 4m). \tag{70}$$

Integrating both sides of (70), we obtain that, for some $\tau \in \mathbb{R}$,

$$\phi(m) = \frac{1}{9}((4 + c_1 - 5c_2)m - 2m^2) + \tau. \tag{71}$$

Plugging $\phi(m)$ in (71) into (69), we can see that $U_1(m;\theta)$ in (69) is concave in m (i.e., $d^2U_1(m;\theta)/dm^2 < 0$). Therefore, if the tax-subsidy scheme $\phi(m)$ is given as in (71), the truth-telling strategy μ_1^T is optimal for Firm 1 in Stage 1 given that ρ_0^T in (68) and (σ_1^T, σ_2^T) in (66) and (67) with $\varepsilon_m = m$ are played in Stages 2 and 3.

Proposition 8 summarizes these observations.

Proposition 8 *Suppose that the government regulates the access fee* r *for the essential facility. Let the tax-subsidy scheme* $\phi(m)$ *be given as in (71). Then,* $(\mu_1^T, \rho_0^T, \sigma_1^T, \sigma_2^T)$, *where* μ_1^T *is the truth-telling strategy,* (σ_1^T, σ_2^T) *are in (66) and (67), and* ρ_0^T *is in (68), is an equilibrium.*

Due to the assumption in (3), for any $m \in [0, \overline{\theta}]$,

$$\frac{d\phi(m)}{dm} = \frac{1}{9}(4 + c_1 - 5c_2 - 4m) > 0, \tag{72}$$

which means that the monetary transfer $\phi(m)$ from Firm 1 to the government should be increased to make Firm 1 reveal its information on θ truthfully. In the absence of a mechanism such as (71), Firm 1 has an incentive to provide false information that θ is realized higher than it actually is. This is because Firm 1 can obtain more payoff by inducing the government to set a high access fee. Hence, to induce Firm 1 to truthfully disclose its private information, the mechanism must induce Firm 1 to inform that θ is realized lower than originally intended to be announced. Thus, for the mechanism to provide such an incentive to Firm 1, the higher θ Firm 1 claims to be realized, the higher the tax levied on Firm 1 should be. (72) coincides with this intuition.

In (71), τ represents a monetary transfer from Firm 1 to the government regardless of the message that Firm 1 sends. That is, $\tau > 0$ can be interpreted as a lump-sum tax and $\tau < 0$ can be interpreted as a lump-sum subsidy. If τ is a lump-sum tax, then $\phi(m) > 0$ holds for all $m \in [0, \overline{\theta}]$. Meanwhile, if τ is a lump-sum subsidy and it is sufficiently large in absolute value, then we can make $\phi(m) < 0$ for all $m \in [0, \overline{\theta}]$. Thus, the truth-telling mechanism $\phi(m)$ can be implemented as a tax or subsidy.[14]

The expected amount of transfers from Firm 1 to the government through the mechanism $\phi(m)$ can be represented as

$$\mathbb{E}[\phi(\theta)] = \frac{1}{9}((4 + c_1 - 5c_2)E_\theta - 2\mathbb{E}[\theta^2]) + \tau \tag{73}$$

[14] We note that our model differs from typical models of mechanism design through monetary transfers, in that the government does not directly consider monetary transfers in its objective function. If we consider a government that maximizes monetary transfers from the firms and social welfare, then we can show that under the appropriate participation constraint for Firm 1, Firm 1 transfers less to the government when θ is Firm 1's private information and a truth-telling mechanism is adopted than when θ is public information. That is, Firm 1 can enjoy an information rent from its private information on θ.

$$=\frac{1}{9}((4+c_1-5c_2)E_\theta-2E_\theta^2-2V_\theta)+\tau\ ,$$

which decreases in V_θ. This implies that given a lump-sum transfer τ, the higher the uncertainty of θ, the lower the tax or the larger the subsidy needed to induce Firm 1 to truthfully reveal its private information.

The outcome of the truth-telling equilibrium $(\mu_1^T,\rho_0^T,\sigma_1^T,\sigma_2^T)$ under the truth-telling mechanism ϕ is exactly the same as that of the equilibrium $(\rho_0^\bullet,\sigma_1^\bullet,\sigma_2^\bullet)$ in Section 3.2. The firms' profits, consumer surplus, and social welfare are also the same as those in Section 3.2. As stated in Proposition 7, the government can improve consumer surplus or social welfare by introducing the truth-telling mechanism ϕ as in (71). Note that the monetary transfer $\phi(\theta)$ between Firm 1 and the government only affects Firm 1's payoff and does not affect social welfare.

V. Conclusion

We study markets in which firms are engaged in an oligopoly competition and one of these firms owns an essential facility. In such markets, since the firm that owns the essential facility can monopolize the market by preventing other firms from using it, ensuring TPA rights for essential facilities is crucial for maintaining market competition. To ensure TPA rights, it is necessary for the government to appropriately regulate the access fee for the essential facility. If no information asymmetry exists between the government and the firms, then the government can set the access fee for the essential facility at a level that is desirable in terms of social welfare. However, in reality, it is difficult for the government to fully know the private information of firms, so the government may experience difficulty in setting an access fee for an essential facility that is socially optimal.

For the analysis, we consider the cases that are classified in accordance with the existence of the government's regulation and the existence of information asymmetry on the cost of operating the essential facility. We assume that the firm that does not own the essential facility can produce goods at a cost lower than the firm that owns the essential facility. If the firm that owns the essential facility decides the access fee for the essential facility, then the access fee is set above the marginal cost of operating the essential facility, which means that the essential facility is provided discriminately to the other firm. Moreover, under perfect information, the firm that does not own the essential facility supplies small amounts of goods to consumers, while it leaves the market under asymmetric information. This difference arises from the fact that the access fee for the essential facility serves as a signal for the cost of operating the essential facility under asymmetric

information. Meanwhile, if the government can regulate the access fee, then it can improve social welfare by setting the access fee at the marginal cost of operating the essential facility. That is, the essential facility is nondiscriminately provided to the other firm. In this case, if the government accurately knows the cost of operating the essential facility, then it can set the access fee at the marginal cost of operating the essential facility, but otherwise, it has to set the access fee at the expected marginal cost of operating the essential facility, which results in the loss of social welfare. We finally show that for the latter case, if the government implements an appropriate tax-subsidy scheme for the firm that owns the essential facility, then it can induce the firm to truthfully disclose its private information on the cost of operating the essential facility, and therefore, improve social welfare.

Appendix: Proofs

Proof of Lemma 1. For m' and $m'' \in \mathcal{M}(\mu_1^{\ddagger})$, let $\varepsilon_m' = \mathbb{E}[\theta \mid \mu_1^{\ddagger}(\theta) = m']$ and $\varepsilon_m'' = \mathbb{E}[\theta \mid \mu_1^{\ddagger}(\theta) = m'']$. Suppose that $\varepsilon_m' \neq \varepsilon_m''$. Without loss of generality, let $\varepsilon_m' < \varepsilon_m''$. Since ε_m' is an expectation of θ with $m' \in supp(\mu_1^{\ddagger}(\theta))$, we can take $\theta \geq \varepsilon_m'$ such that $m' \in supp(\mu_1^{\ddagger}(\theta))$. From (58), we see that

$$\Pi_1(m';\theta) - \Pi_1(m'';\theta) = \frac{1}{36}(\varepsilon_m'' - \varepsilon_m')(11(\varepsilon_m'' + \varepsilon_m') - 4(4 + c_1 - 5c_2) - 6\theta). \tag{74}$$

Since $m' \in supp(\mu_1^{\ddagger}(\theta))$, it has to be satisfied that $\Pi_i(m';\theta) - \Pi_i(m'';\theta) \geq 0$, or equivalently

$$6\theta \leq 11\varepsilon_m' + 11\varepsilon_m'' - 4(4 + c_1 - 5c_2). \tag{75}$$

Then, since $\varepsilon_m' \leq \theta$ and $\varepsilon_m' < \varepsilon_m'' \leq \overline{\theta}$, we have a contradiction that

$$\begin{aligned} 6\theta + 5\overline{\theta} &\leq 11\varepsilon_m' + 11\varepsilon_m'' - 4(4 + c_1 - 5c_2) + 5\overline{\theta} \\ &\leq 11\varepsilon_m' + 16\overline{\theta} - (16 + 4c_1 - 20c_2) \\ &= 11\varepsilon_m' - 16(1 - c_1 - \overline{\theta}) - 20(c_1 - c_2) \leq 11\varepsilon_m'. \end{aligned} \tag{76}$$

Thus, for any m' and $m'' \in \mathcal{M}(\mu_1^{\ddagger})$, $\mathbb{E}[\theta \mid \mu_1^{\ddagger}(\theta) = m'] = \mathbb{E}[\theta \mid \mu_1^{\ddagger}(\theta) = m'']$ must hold. This means that $\mathcal{E}(\mu_1^{\ddagger})$ is a singleton. Let G be the distribution of messages m that is generated by F and $\mu_1^{\ddagger}$. Then, the fact of $\int \mathbb{E}[\theta \mid \mu_1^{\ddagger}(\theta) = m]dG(m) = \mathbb{E}[\mathbb{E}[\theta \mid \mu_1^{\ddagger}(\theta) = m]] = \mathbb{E}[\theta] = E_\theta$ establishes the result. ■

Proof of Proposition 5. It is easy to see that Firm 1's strategy $\mu^{\ddagger}$ of sending a random message m that is uniformly distributed on $[0, \overline{\theta}]$ constitutes an equilibrium with $\rho_0^{\ddagger}$ in (57) and $(\sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ in (54) and (55). Such strategy $\mu^{\ddagger}$ of Firm 1 satisfies (i) and $\rho_0^{\ddagger}$ in (57) satisfies (ii). Hence, we establish the first assertion. In addition, since Lemma 1 implies that every equilibrium $(\mu_1^{\ddagger}, \rho_0^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ satisfies (i), the arguments to obtain (54), (55), and (57) prove the second assertion. ■

Proof of Proposition 6. Comparing the expectations of (48) to (51) and the expectations of (62) to (65), we obtain

$$\mathbb{E}[\Pi_1^{\dagger}] - \mathbb{E}[\Pi_1^{\ddagger}] = \frac{1}{36}(5 - 5E_\theta - 7c_1 + 2c_2)(1 - E_\theta + c_1 - 2c_2) > 0, \tag{77}$$

$$\mathbb{E}[\Pi_2^\dagger] - \mathbb{E}[\Pi_2^\ddagger] = -\frac{1}{9}(1 - E_\theta + c_1 - 2c_2)^2 < 0\,, \tag{78}$$

$$\mathbb{E}[CS^\dagger] - \mathbb{E}[CS^\ddagger] = -\frac{1}{72}(7 - 7E_\theta - 5c_1 - 2c_2)(1 - E_\theta + c_1 - 2c_2) < 0\,, \tag{79}$$

$$\mathbb{E}[SW^\dagger] - \mathbb{E}[SW^\ddagger] = -\frac{1}{72}(5 - 5E_\theta + 17c_1 - 22c_2)(1 - E_\theta + c_1 - 2c_2) < 0\,, \tag{80}$$

where the inequalities are ensured by (3). ■

Proof of Proposition 7. Comparing the expectations of (29) to (32) and the expectations of (62) to (65), we obtain

$$\mathbb{E}[\Pi_1^\bullet] - \mathbb{E}[\Pi_1^\ddagger] = -\frac{5}{36}(\mathbb{E}[\theta^2] - E_\theta^2) = -\frac{5}{36}V_\theta < 0\,, \tag{81}$$

$$\mathbb{E}[\Pi_2^\bullet] - \mathbb{E}[\Pi_2^\ddagger] = \frac{1}{9}(\mathbb{E}[\theta^2] - E_\theta^2) = \frac{1}{9}V_\theta > 0\,, \tag{82}$$

$$\mathbb{E}[CS^\bullet] - \mathbb{E}[CS^\ddagger] = \frac{7}{72}(\mathbb{E}[\theta^2] - E_\theta^2) = \frac{7}{72}V_\theta > 0\,, \tag{83}$$

$$\mathbb{E}[SW^\bullet] - \mathbb{E}[SW^\ddagger] = \frac{5}{72}(\mathbb{E}[\theta^2] - E_\theta^2) = \frac{5}{72}V_\theta > 0\,, \tag{84}$$

where V_θ is the variance of θ. ■

References

Armstrong, M., C. Doyle, and J. Vickers (1996), "The Access Pricing Problem: A Synthesis," *Journal of Industrial Economics*, 44(2), 131–150.

Armstrong, M., and J. Vickers (1998), "The Access Pricing Problem with Deregulation: A Note," *Journal of Industrial Economics*, 46(1), 115–121.

Arya, A., B. Mittendorf, and D. E. Sappington (2008), "Outsourcing, Vertical Integration, and Price vs. Quantity Competition," *International Journal of Industrial Organization*, 26(1), 1–16.

Buccirossi, P. (1999), "Access to an Essential Facility: Efficient Component Pricing Rule or Unrestricted Private Property Rights?" *Journal of Regulatory Economics*, 16(3), 287–296.

Caillaud, B., and J. Tirole (2004), "Essential Facility Financing and Market Structure," *Journal of Public Economics*, 88(3), 667–694.

Chen, Y. (2001), "On Vertical Mergers and Their Competitive Effects," *RAND Journal of Economics*, 32(4), 667–685.

Chen, Z. (2003), "Dominant Retailers and the Countervailing-power Hypothesis," *RAND Journal of Economics*, 34(4), 612–625.

Cho, M. (2019), "Investor's Information Sharingi with Firms in Oligopoly," *Korean Economic Review*, 35(2), 439–469.

Christou, C., and K. G. Papadopoulos (2015), "The Countervailing Power Hypothesis In The Dominant Firm-competitive Fringe Model," *Economics Letters*, 126, 110–113.

Crawford, V. P., and J. Sobel (1982), "Strategic Information Transmission," *Econometrica*, 50(6), 1431–1451.

Economides, N. (1998), "The Incentive for Non-price Discrimination by an Input Monopolist," *International Journal of Industrial Organization*, 16(3), 271–284.

Erutku, C. (2005), "Buying Power and Strategic Interactions," *Canadian Journal of Economics*, 38(4), 1160–1172.

Farrell, J., and R. Gibbons (1989), "Cheap Talk with Two Audiences," *American Economic Review*, 79(5), 1214–1223.

Gal-Or, E. (1986), "Information Transmission-cournot and Bertrand Equilibria," *Review of Economic Studies*, 53(1), 85–92.

Gautier, A., and M. Mitra (2008), "Regulation of an Open Access Essential Facility," *Economica*, 75(300), 662–682.

Goltsman, M., and G. Pavlov (2011), "How to Talk to Multiple Audiences," *Games and Economic Behavior*, 72(1), 100–122.

Graef, I. (2016), *Data as Essential Facility: Competition and Innovation on Online Platforms*, Wolters Kluwer.

Hart, O., J. Tirole, D. W. Carlton, and O. E. Williamson (1990), "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity. Microeconomics*, 1990, 205–286.

Lewis, T. R., and D. E. Sappington (1999), "Access Pricing with Unregulated Downstream Competition," *Information Economics and Policy*, 11(1), 73–100.

Li, L. (1985), "Cournot Oligopoly with Information Sharing," *RAND Journal of Economics*, 16(4), 521–536.

Melumad, N. D., and T. Shibano (1991), "Communication in Settings with No Transfers," *RAND Journal of Economics*, 22(2), 173–198.

Moresi, S., and M. Schwartz (2017), "Strategic Incentives when Supplying to Rivals with an Application to Vertical Firm Structure," *Journal of Industrial Organization*, 51, 137–161.

OECD (2015), *Data-Driven Innovation: Big Data for Growth and Well-Being*, OECD Publishing.

Ordover, J. A., G. Saloner, and S. C. Salop (1990), "Equilibrium Vertical Foreclosure," *American Economic Review*, 80(1), 127–142.

Sakai, Y. (1986), "Cournot and Bertrand Equilibria under Imperfect Information," *Journal of Economics*, 46(3), 213–232.

Salinger, M. A. (1988), "Vertical Mergers and Market Foreclosure," *Quarterly Journal of Economics*, 103(2), 345–356.

Shapiro, C. (1986), "Exchange of Cost Information in Oligopoly," *Review of Economic Studies*, 53(3), 433–446.

Stucke, M., and A. Grunes (2016), *Big Data and Competition Policy*, Oxford University Press.

Tucker, C. (2019), "Digital Data, Platforms and the Usual [antitrust] Suspects: Network Effects, Switching Costs, Essential Facility," *Review of Industrial Organization*, 54(4), 683–694.

Valletti, T. (1998), "Two-part Access Pricing and Imperfect Competition," *Information Economics and Policy*, 10(3), 305–323.

Vives, X. (1984), "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal of Economic Theory*, 34(1), 71–94.

von Ungern-Sternberg, T. (1996), "Countervailing Power Revisited," *International Journal of Industrial Organization*, 14(4), 507–519.

Weisman, D. L. (1995), "Regulation and the Vertically Integrated Firm: The Case of RBOC Entry into Interlata Long Distance," *Journal of Regulatory Economics*, 8(3), 249–266.

Ziv, A. (1993), "Information Sharing in Oligopoly: The Truth-telling Problem," *RAND Journal of Economics*, 24(3), 455–465.

비대칭적 정보 아래 필수설비에 대한 제3자 접속가격 설정 방식에 대한 연구

양 석* · 조 명 환**

초 록 본 연구는 재화의 공급을 위한 필수설비를 소유한 기업과 그렇지 않은 기업의 과점경쟁 상황을 분석한다. 필수설비를 소유하지 않은 기업은 필수설비를 소유한 기업에게 접속료를 지불하고 필수설비를 이용해 재화를 공급할 수 있다. 필수설비 운영비용이 이를 소유한 기업의 사적정보인 상황과 모두에게 알려진 공적정보인 상황 모두 필수설비 소유기업은 경쟁기업에 대해 필수설비 접속료를 차별적으로 설정하고, 이로인해 사회후생의 손실이 발생한다. 이에 정부는 필수설비 소유기업이 필수설비를 비차별적으로 제공하도록 접속료를 규제하여 사회후생을 증가시킬 수 있지만, 필수설비 운영비용이 필수설비 소유기업의 사적정보인 상황에서는 사회후생을 증가시키는데에 한계가 존재한다. 그러나 정부가 필수설비 소유기업에게 적절한 세금(보조금)을 부과한다면 필수설비 소유기업으로 하여금 자신의 사적정보를 완전히 공개하도록 유도할 수 있다.

핵심 주제어: 제3자 접속, 필수설비, 비대칭적 정보
경제학문헌목록 주제분류: C70, D43, L13

투고 일자: 2023. 8. 16. 심사 및 수정 일자: 2024. 2. 14. 게재 확정 일자: 2024. 4. 20.
* 제1저자. 서울시립대학교 경제학부 석사과정, e-mail: tortoise97@uos.ac.kr
** 교신저자. 서울시립대학교 경제학부 교수, e-mail: chomhmh@uos.ac.kr