Where is the GE? Consumption Dynamics in DSGEs

Dan Cao, Jean-Paul L'Huillier, Donghoon Yoo*

September 2017

Abstract

This paper provides a partial equilibrium perspective on the behavior of consumption in DSGEs. It considers two benchmark dynamic general equilibrium models, one closed economy, and the other open economy. It shows that, in the calibrated versions of these models, the real interest rate is essentially fixed. One manifestation of this assumption is that, with separable preferences, the reaction of consumption to TFP shocks is flat: the random-walk permanent income hypothesis holds almost exactly, pretty much as in a *partial* equilibrium consumption-savings problem. These results explain the prominent role of aggregate demand, and how it is achieved, in modern DSGEs.

Keywords: Nominal rigidities, real rigidities, monetary policy.

JEL codes: E10, E27, E37.

*Georgetown University, Assistant Professor; Einaudi Institute for Economics and Finance (EIEF), Assistant Professor; University of Lausanne and Korea Labor Institute, Post Doctoral Fellow and Research Economist. Dan thanks EIEF for hospitality and support during the preparation of this paper. This is a revised version of an earlier draft entitled "The New Keynesian Model and the Small Open Economy RBC Model: Equivalence Results for Consumption". We thank Giulia Felice and seminar and conference participants at University of Rome "Tor Vergata", University of Lausanne, the 20th Spring Meeting of Young Economists, the 30th Annual Congress of the European Economic Association, and the 69th European Meeting of the Econometric Society for helpful suggestions.

1 Introduction

Modern DSGEs are viewed as the epitome of modern *general equilibrium* macroeconomic analysis. Complex dynamic-programming problems are solved by a number of interacting optimizing agents, leading to equilibrium relations immune to the Lucas critique. These relations balance general equilibrium forces coming from the supply-side effects of exogenous technological change and from the demand-side effects of exogenous changes in spending, together with the effects of other shocks. These forces are finely amplified (or moderated) by a host of frictions as nominal rigidities, collateral constraints, balance-sheet effects, etc. There is nowadays a menu of models that one can choose from in order to simulate the effect of shocks, and in order to study optimal policy responses. This constitutes an impressive intellectual achievement.

In this paper we will argue that the emphasis of the literature on general equilibrium (GE) may be somewhat overstated. The reason is the following, and it is the focus of the paper. In the class of one-consumption-good DSGE models, either closed or open economy, one price, mainly the real interest rate, generally features very limited general equilibrium behavior, being essentially *fixed* by the parametrization used to match the data. The class of closed economy models considered to prove this point stems from the New Keynesian model with so-called "bells and whistles" introduced by Christiano, Eichenbaum, and Evans (2005) and later used by Smets and Wouters (2007) and Justiniano, Primiceri, and Tambalotti (2010), among others (henceforth NK-BW). The class of open economy models considered stems from the one-good Small Open Economy RBC model, originally introduced by Mendoza (1991), and subsequently used by Schmitt-Grohe and Uribe (2003) and Aguiar and Gopinath (2007) (among others) (henceforth SOE-RBC).

The implication of a fixed interest rate is, of course, that consumption dynamics in these models are no different than in earlier generations of *partial equilibrium* models of permanent income consumption, as for instance, Hall (1978), Campbell (1987), Campbell and Deaton (1989), etc. We believe that this observation is worth highlighting, given the primary role consumption plays in modern DSGE analysis. First, at some level, consumption plays a key modeling and conceptual role, considered by many a central aggregate demand determinant. In fact, precursors to modern NK models *only* included consumption on the demand side (see for instance the model used by Campbell 1987 to study the behavior of savings, or the bare bones NK model, presented recently by Gali 2008.) Second, consumption plays a key empirical role in the case of U.S. business cycles, where the lion's share of GDP (above 60% during the post-war period) is composed by private consumption expenditures. This is a fact shared by other countries[1].

But why is it that quite different strands of models use parametrizations in which consumption is essentially determined in partial equilibrium? An interpretation based on our findings is that this allows both literatures to rely on *aggregate demand* channels in order to explain fluctuations in consumption. This is achieved either via direct *exogenous* shifters in consumption (Smets and Wouters 2007; Justiniano, Primiceri, and Tambalotti 2010)[2], or by shocks to the trend of productivity, which move consumption (and the current account) in anticipation of future income (Aguiar and Gopinath 2007). Without an essentially fixed real rate, this would not be possible.[3]

[1]Just to name a few, the share of private consumption to GDP is above 50% in Canada, Germany, France, Argentina, and Mexico.

[2]These shocks are called preference or risk premium shocks, among other names.

[3]In the final remarks of the paper we develop the specifics of each case, and make precise links of this interpretation to the literature.

The result that many ingredients in these models work towards limiting movements in the real rate is not easy to see because this mechanism is muddled by a number of departures from the simple dynamic consumption-savings problem under certainty equivalence. The contribution of our paper is precisely to isolate the combination of functional forms and shocks that allow to clearly visualize this feature of the models, without modifying the conceptually key building blocks of these classes of models, as for instance the presence of capital accumulation and production, nominal and real frictions as price stickiness in the NK-BW model, capital adjustment costs, or the use of an elastic debt premium in the case of the SOE-RBC model. In fact, it is a bit surprising that none of these latter ingredients matter for the result.

The isolation of the ingredients necessary for our purposes relies on previously obtained theoretical results in the case of simpler models. The random walk behavior of consumption was first established (to the best of our knowledge) in Blanchard, L'Huillier, and Lorenzoni (2013) (Online Appendix Section 6.4.2) in the case of a bare bones New Keynesian model. Similar results were established in Cao and L'Huillier (2017) for a Small Open Economy RBC model without capital and inelastic labor. These results form the basis for the numerical exploration of the full-blown models addressed in this paper.

Specifically, we proceed as follows. First, we identify a parameter region in both classes of models where the real interest rate is exactly fixed and thus consumption behaves exactly as in a random-walk permanent income model. For both models, the parameter region features no habit formation. Moreover, in the case of the NK-BW model, the parameter region is one where a1) prices are very sticky or the interest rate rule is very accommodative to inflation, and b1) the interest rule does not react to output. In the case of the SOE-RBC model, the parameter region is one where a2) the discount factor is close to one, b2) the elasticity of the interest premium is close to zero, and c2) the steady state level of debt is zero[4], with the requirement that b2) holds more strongly than a2), i.e. the elasticity tends to zero faster than the discount factor tends to one. We refer to this parametrization (for each model respectively) by "limit parametrization". Thus, in this region, consumption's reaction to technology shocks is entirely and solely determined by the long-run level of income (or, equivalently, TFP). Indeed, after a permanent impulse to TFP, consumption jumps to that level and stays there. By the same logic, consumption does not react to temporary TFP shocks, because they do not raise long-run income. So, consumption is flat and features no fluctuations around the long-run level of income.

Second, we show that *for the usually adopted calibration of both models in the literature*, the behavior of the real rate and consumption is quite close to the one obtained in the limit parametrization. The implication is that consumption is almost entirely driven by permanent shocks, as established by variance decomposition.

The intuition for why these parameter regions deliver an almost constant real interest rate is as follows. In the NK-BW model, when prices are very sticky inflation does not move, and therefore the rate determined by the nominal interest rate rule does not move either (so long as the rate does not react to output.) As a result, the real rate is constant. In the SOE-RBC model, when the elasticity of the interest premium is small, the domestic real interest rate is constant and equal to the foreign rate. Note that these findings are not entirely obvious given that there are multiple differences between the two models. To

[4]This last condition is not necessary for obtaining that consumption dynamics are flat, but they are necessary to obtain that consumption jumps to the same level than long-run productivity.

mention some of these differences, consider first goods markets. The NK-BW model features monopolistic competition and Calvo price stickiness, while the SOE-RBC model features a competitive market with flexible prices. In terms of labor markets, the NK-BW model features Calvo wage stickiness and the SOE-RBC model features, again, a competitive market. Moreover, the NK-BW model is closed economy, and the SOE-RBC is open economy. Because our limit parametrization is close to standard, our results imply that these elements of the specification (among others not described) are largely irrelevant for the conditional behavior of consumption after TFP shocks. Importantly, as shown in the body, the parameter regions identified above are relevant because they are close to standard paramterizations obtained either by calibration or estimation of both the NK-BW and SOE-RBC models in the literature.

If consumption is essentially determined in partial equilibrium, what general equilibrium effects are then present in these models? In fact, these models feature rich general equilibrium effects on other endogenous variables. In the NK-BW model, general equilibrium effects produced by changes in consumption successfully generated procyclical movements of investment, labor supply, and output. When consumption rises, firms hire more labor and invest more. This increases output. The reaction of the monetary authority to dampen these movements is quite modest. In the SOE-RBC model, general equilibrium effects generate, instead, a fall in output when consumption rises. This is due to the well known income effect. Investment, instead, rises as firms anticipate higher profits in the future. In the body, we show simulations that illustrate these effects.

The rest of the paper is organized as follows. The next Section lays down both models. Section 4 presents first the optimality conditions of the models, and then all numerical results. Section 5 contains final remarks.

2 The Models

In this Section we present the models. We start by the elements common to both models. We then specify the remaining elements of the NK-BW. Immediately after we specify the remaining elements of the SOE-RBC model.

2.1 Elements Common to Both Models: Total Factor Productivity and Preferences

Productivity a_t (in logs) is the sum of two components, permanent, x_t, and temporary z_t

$$a_t = x_t + z_t \tag{1}$$

The permanent component follows the unit root process

$$\Delta x_t = \rho_x \Delta x_{t-1} + \varepsilon_t \tag{2}$$

The temporary component follows the stationary process

$$z_t = \rho_z z_{t-1} + \eta_t \tag{3}$$

The coefficients ρ_x and ρ_z are in $[0, 1)$, and ε_t and η_t are i.i.d. normal shocks with variances σ_ε^2 and σ_η^2.

Preferences are given by

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\ln\left(C_t - hC_{t-1}\right) - \frac{1}{1+\varphi}\int_0^1 N_{jt}^{1+\varphi} dj\right)\right]$$

where C_t is consumption, the term hC_{t-1} captures internal habit formation, and N_{jt} is the supply of specialized labor of type j. As explained in the introduction, using the same separable preferences in both models will allow us to 1. establish the random walk property of consumption, and 2. compare accross models.

2.2 Remaining Specification of the NK-BW Model

The model is standard. The household budget constraint is

$$P_t C_t + P_t I_t + P_t \mathcal{C}(U_t) \bar{K}_{t-1} + B_t = R^c_{t-1} B_{t-1} + \int_0^1 W_{jt} N_{jt} dj + R^k_t K_t$$

where P_t is the price level, I_t is investment, $\mathcal{C}(U_t)$ is the cost associated with capital utilization in terms of current production, $\bar{K}_t$ is the stock of capital, B_t are holdings of one-period bonds, R^c_t is the one-period nominal interest rate, W_{jt} is the wage of specialized labor of type j, and R^k_t is the capital rental rate, and K_t are capital services rented (and used).

The Fischer equation gives the definition of the gross real rate of interest R_t:

$$R_t = R^c_t E\left[\frac{P_t}{P_{t+1}}\right]$$

The capital stock $\bar{K}_t$ is owned and rented by the representative household and the capital accumulation equation is

$$\bar{K}_t = (1-\delta)\,\bar{K}_{t-1} + \left[1 - \chi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where adjustment costs in investment are captured by

$$\chi\left(\frac{I}{I_-}\right) = \frac{\chi}{2}\left(\frac{I}{I_-} - 1\right)^2$$

The model features variable capital utilization and the capital services provided by the capital stock $\bar{K}_{t-1}$ are

$$K_t = U_t \bar{K}_{t-1}$$

where U_t represents the degree of capital utilization. The cost associated with capital utilization in terms of current production is given by

$$\mathcal{C}(U) = \frac{1}{1+\xi} U^{1+\xi}$$

Final good producers. The final good is produced using intermediate goods with the CES production function

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{1+\mu_p}} di \right]^{1+\mu_p}$$

where μ_p captures a constant elasticity of substitution across goods.

Final good producers are perfectly competitive and maximize the profits subject to the above production function, taking intermediate good prices P_{it} as given and final good price P_t such that final good producers profit maximization problem is

$$\max_{Y_{it}} P_t Y_t - \int_0^1 P_{it} Y_{it} di$$

Intermediate goods producers. There is a continuum of intermediate goods producers where each producer produces good i with following production technology

$$Y_{it} = K_{it}^{\alpha} \left(A_t N_{it} \right)^{1-\alpha}$$

where K_{it} and N_{it} are capital and labor services employed and $\alpha \in (0,1)$ represents capital's share of output. As in the SOE-RBC model below, the parameter

$A_t = e^{a_t}$.

Intermediate good prices are assumed to be Calvo-sticky. Each period intermediate firm i can freely adjust the nominal prices with probability $1-\theta$ and firms that cannot adjust prices (with probability θ) set their price according to

$$P_{it} = P_{it-1}\Pi_{t-1}^{\iota}\Pi^{1-\iota}$$

where Π is the steady state level of inflation.

Labor markets. A representative competitive firm hires the labor supplied by household j and aggregates the specialized labor supplied by the households with the following technology

$$N_t = \left[\int_0^1 N_{jt}^{\frac{1}{1+\mu_w}}\,dj\right]^{1+\mu_w}$$

where μ_w is a constant elasticity of substitution among specialized labor.

Wages are Calvo-sticky. For each type of labor j, the household can freely adjust the price W_{jt} with probability $1-\theta_w$ and those that cannot adjust prices (with probability θ_w) set their price according to

$$W_{jt} = W_{jt-1}\left(\Pi_{t-1}e^{\Delta a_{t-1}}\right)^{\iota_w}(\Pi)^{1-\iota_w}$$

where Π_{t-1} is inflation at $t-1$.

Monetary policy. Monetary policy follows the following interest rate rule

$$\frac{R_t^c}{R^c} = \frac{R_{t-1}^c}{R^c}^{\rho_r}\left(\left(\frac{\Pi_t}{\Pi}^{\gamma_\pi}\right)\left(\frac{Y_t/A_t}{Y/A}\right)^{\gamma_y}\right)^{1-\rho_r}$$

Market clearing. Market clearing in the final good market requires

$$C_t + I_t + \mathcal{C}\left(U_t\right)\bar{K}_{t-1} = Y_t$$

and market clearing in the market for labor services requires

$$\int_0^1 N_{jt}dj = N_t$$

As Christiano et al. (2005), we define output gross of capital utilization costs.

2.3 Remaining Specification of the SOE-RBC Model

The specification is standard. Here, following the literature, there is no labor heterogeneity and thus $N_{jt} = N_t$.

Maximization of household utility is constrained by

$$C_t + I_t + B_{t-1} = Y_t + Q_t B_t$$

where C_t, I_t and Y_t are the consumption, investment, and output of the country, B_t is the external debt of the country and Q_t is the price of this debt.

Output is produced with capital and labor inputs through a Cobb-Douglas production function

$$Y_t = K_{t-1}^{\alpha}(A_t N_t)^{1-\alpha}$$

where $\alpha \in (0,1)$ represents capital's share of output. The parameter $A_t = e^{a_t}$.

The resource constraint is

$$C_t + I_t + NX_t = Y_t$$

where NX_t are net exports. Following Aguiar and Gopinath (2007), the law of motion for capital is

$$K_t = (1-\delta)K_{t-1} + I_t - \frac{\nu}{2}\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}$$

Capital depreciates at the rate δ, and the (quadratic) capital adjustment cost is captured by

$$\frac{\nu}{2}\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}$$

Following Schmitt-Grohe and Uribe (2003) and Aguiar and Gopinath (2007), among others, the price of debt is sensitive to the level of debt outstanding

$$\frac{1}{Q_t} = R_t = R^* + \psi\left\{e^{\frac{B_t}{Y_t} - b} - 1\right\}$$

where R_t denotes the real interest rate, R^* denotes the exogenous world interest rate, and b represents the steady state level of the debt-to-output ratio.

3 Optimality Conditions and Steady State Conditions

Following standards steps, we derive the optimality conditions (for both models) and log-linearize. Here we present the FOC and deterministic steady state relations for both models, which allows us to comment on the assumption that the steady state level of debt in the SOE-RBC needs to be set to zero in order to more easily compare across models (on p. 21). All details of the log-linearization are given in the Appendix.

3.1 NK-BW Model: Optimality Conditions

Households. The FOC for consumption is

$$\frac{1}{C_t - hC_{t-1}} - \beta h \frac{1}{C_{t+1} - hC_t} = \Lambda_t$$

where Λ_t is the multiplier of the budget constraint. The FOC for bond holdings is

$$-\frac{\Lambda_t}{P_t} + \beta R_t^c \frac{\Lambda_{t+1}}{P_{t+1}} = 0$$

The FOC for investment

$$-\Lambda_t + \Phi_t \left[\chi \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] + \beta \Phi_{t+1} \left[1 - \frac{\chi}{2} \left(\frac{I_{t+1}}{I_t} - 1 \right)^2 + \frac{\chi}{2} \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right] = 0$$

where Φ_t is the multiplier for capital accumulation. The FOC for capital

$$\Phi_t - \beta \Lambda_{t+1} \mathcal{C}(U_{t+1}) - \beta (1 - \delta) \Phi_{t+1} \bar{K}_t = 0$$

Remain the budget constraint and the capital accumulation equation

$$\frac{B_{t-1}}{P_t}R^c_{t-1} - \frac{B_t}{P_t} + \frac{W_t}{P_t}N_t + \frac{R^k_t}{P_t}K_t - C_t - I_t - \mathcal{C}(U_t)\bar{K}_{t-1} = 0$$

$$\bar{K}_t - (1-\delta)\bar{K}_{t-1} - \left[1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_{t-1} = 0$$

The optimality condition for capacity utilization U_t is the solution to

$$\max_{U_t} \left(R^k_t/P_t\right) U_t\bar{K}_{t-1} - \mathcal{C}(U_t)\bar{K}_{t-1}$$

which yields

$$R^k_t/P_t = \mathcal{C}'(U_t) = \xi U_t^{\xi}$$

Final good producers. Final good producers maximize

$$\max_{Y_{it}} P_tY_t - \int_0^1 P_{it}Y_{it}di$$

which gives the demand for intermediate good i

$$Y_{it} = Y_t\left(\frac{P_{it}}{P_t}\right)^{-\frac{1+\mu_p}{\mu_p}}$$

Since final good producer derives zero profit,

$$P_t = \left[\int P_{it}^{\frac{1}{\mu_p}} di\right]^{\mu_p}$$

Intermediate goods producers. From the demand for intermediate good Y_{it}, μ_p is the constant markup for monopolist i. The cost minimization problem for the monopolist is

$$\min_{K_{it},N_{it}} R_t^K K_{it} + W_{it} N_{it}$$

subject to the production technology

$$K_{it}^{\alpha} \left(A_t N_{it}\right)^{1-\alpha} = Y_{it}$$

From the cost minimization problem, we obtain following FOCs:

$$R_t^k - MC_t \alpha K_{it}^{\alpha-1} \left(A_t N_{it}\right)^{1-\alpha} = 0$$

$$W_t - MC_t \left(1-\alpha\right) K_{it}^{\alpha} A_t^{1-\alpha} N_{it}^{-\alpha} = 0$$

Because of constant returns to scale all intermediate good firms choose the same capital labor ratio

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}$$

and the nominal cost of producing Y_{it} is $MC_t \cdot Y_{it}$ where MC_t is the marginal cost

$$MC_t = \frac{1}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha}} \left(R_t^k\right)^{\alpha} \left(\frac{W_t}{A_t}\right)^{1-\alpha}$$

Calvo pricing implies that with probability $(1-\theta)$ firms change price and maximize

$$\mathbb{E}\left[\sum_{s=0}^{\infty} \theta^s \beta^s \frac{\Lambda_{t+s}}{P_{t+s}} \left(P_{it} \tilde{I}_{t+s} - MC_{t+s}\right) Y_{it+s}\right]$$

where $\tilde{I}_{t+s}$ is the indexing function

$$\tilde{I}_{t+s} = \Pi_{k=1}^{s} \left(\Pi_{t+l-1}^{\iota_p} \Pi^{1-\iota_p}\right)$$

The optimality condition is then

$$\mathbb{E}\left[\sum_{s=0}^{\infty}\theta^s\beta^s\frac{\Lambda_{t+s}}{P_{t+s}}\left(\frac{1}{\mu_p}\tilde{I}_{t+s}-\frac{1+\mu_p}{\mu_p}\frac{MC_{t+s}}{P_{it}}\right)Y_{it+s}\right]=0$$

Labor markets. Demand for variety j is

$$N_{jt}=N_t\left(\frac{W_{jt}}{W_t}\right)^{-\frac{1+\mu_w}{\mu_w}}$$

and the equilibrium wage is

$$W_t=\left[\int W_{jt}^{\frac{1}{\mu_w}}\right]$$

Calvo pricing for wage implies that with probability $(1-\theta_w)$ workers can adjust their wage and maximize

$$\mathbb{E}\left[\sum_{s=0}^{\infty}\theta_w^s\beta^s\frac{\Lambda_{t+s}}{P_{t+s}}\left(P_{jt}\tilde{I}_{wt+s}W_{jt}N_{jt+s}-\frac{1}{1+\phi}N_{jt+s}^{1+\phi}\right)\right]$$

The optimality condition is then

$$\mathbb{E}\left[\sum_{s=0}^{\infty}\theta_w^s\beta^s\frac{\Lambda_{t+s}}{P_{t+s}}\left(\frac{1}{\mu_w}\tilde{I}_{wt+s}-\frac{1+\mu_w}{\mu_w}\frac{N_{jt+s}^{\phi}}{W_{jt}}\right)N_{jt+s}\right]=0$$

3.2 NK-BW Model: Steady State Relations

We look for a steady state of stationary variables. This means that, because of the unit root in a_t, we need to normalize some of the variables. We denote steady state values by removing the time subindex.

From the intertemporal condition we obtain

$$R^c = \frac{\Pi}{\beta}$$

From the optimality condition for investment, we can see that

$$\Lambda A = \Phi A$$

where ΛA is the multiplier multiplied by A, which is stationary, and similarly for ΦA. In addition, the steady state value of the investment adjustment cost is

$$\chi(0) = 0$$

Also, in steady state

$$U = 1$$

so

$$\mathcal{C} = 1$$

and from the Euler condition for capital we get

$$R^k/P = \frac{1}{\beta} - (1 - \delta)$$

where R^k/P is the steady state real rental rate. From optimality of prices

$$P_i/P = (1 + \mu_p)\, MC/P$$

where MC/P are real marginal costs, and

$$P_i/P = 1$$

Thus

$$W/AP = \left[\frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}{1+\mu_p} \frac{1}{(R^k/P)^{\alpha}} \right]^{1/(1-\alpha)}$$

where W/AP are normalized real wages. From optimal factor combination, we get

$$K/AN = \frac{\alpha}{1-\alpha} \frac{W/AP}{R^k/P}$$

where K/AN is the normalized capital-to-labor ratio. From the production function

$$\frac{Y}{AN} = \left(\frac{K}{AN} \right)^{\alpha}$$

where Y/AN is the normalized output-to-labor ratio. From resource constraints, we have

$$I/AN = \delta K/AN$$

where I/AN is the normalized investment-to-labor ratio. Since

$$K/AN = \bar{K}/AN$$

then

$$C/A = Y/A - I/A - K/A$$

3.3 SOE-RBC Model: Optimality Conditions

FOCs are obtained with respect to $C_t, N_t, I_t, K_t, B_t, \Lambda_t, \Phi_t$, respectively:

$$\Lambda_t = \frac{1}{C_t}$$

$$(1-\alpha)\Lambda_t = N_t^{1+\varphi}\frac{1}{Y_t}$$

$$\Lambda_t + \Phi_t = 0$$

$$\Phi_t\left[1+\nu\left(\frac{K_t}{K_{t-1}}-1\right)\right]+\alpha\beta\mathbb{E}\left[\Lambda_{t+1}\left(\frac{Y_{t+1}}{K_t}\right)\right]-\beta\mathbb{E}\left[\Phi_{t+1}\left((1-\delta)-\frac{\nu}{2}\left(1-\left(\frac{K_{t+1}}{K_t}\right)^2\right)\right)\right]=0$$

$$Q_t\Lambda_t = \beta\mathbb{E}[\Lambda_{t+1}]$$

$$K_{t-1}^{\alpha}(A_tN_t)^{1-\alpha} - C_t - I_t + Q_tB_t - B_{t-1} = 0$$

$$K_t - (1-\delta)K_{t-1} - I_t + \frac{\nu}{2}\left(\left(\frac{K_t}{K_{t-1}}\right)^2 - 1\right)K_{t-1} = 0$$

3.4 SOE-RBC Model: Steady State Relations

As in the NK-BW model, we look for a steady state of stationary variables. We denote steady state values by removing the time subindex.

From the intertemporal condition, obtain the condition

$$R = \frac{1}{\beta}$$

The steady state ratio of capital to output can be obtained from the FOCs with respect to K_t and I_t:

$$\frac{K}{Y} = \frac{\alpha}{1/\beta-(1-\delta)}$$

From the capital accumulation equation we have

$$\frac{I}{Y} = \delta \frac{K}{Y} = \frac{\alpha\delta}{1/\beta - (1-\delta)}$$

The budget constraint gives

$$1 - \frac{C}{Y} - \frac{I}{Y} = (1-\beta)\,\frac{B}{Y}$$

The resource constraint gives

$$\frac{C}{Y} + \frac{I}{Y} + \frac{NX}{Y} = 1$$

From these steady state relations one can conclude that the steady state level of current account surplus is given by

$$\frac{NX}{Y} = (1-\beta)\,\frac{B}{Y}$$

Thus, in this model, the steady state level of normalized debt B/Y is determined exogenously. For comparability of the model with the closed economy model above, we assume $B/Y = 0$.

4 Results

We focus mainly on the behavior of the real interest rate and consumption in both the log-linearized NK-BW model and the log-linearized SOE-RBC model (the log-linearizations are presented in the Appendix.) We consider two alternative parametrizations of both models. Parametrization I is one in which the response of the real interest rate is exactly fixed and consumption is flat (limit parametrization). Parametrization II is the typically used in the literature (standard parametrization). We simulate the models using both parametrizations.

4.1 Limit Parametrization (or Parametrization I)

Parametrization I is shown in Table 1. This parametrization defines a parameter region, for both models, in which the response of the real rate is fixed and the one of consumption is flat: it jumps to the long-run level of TFP after a permanent shock, computed as the cumulated sum of the growth rates implied by the shock, i.e.

$$a_\infty = \frac{\sigma_\varepsilon}{1-\rho_x}$$

and does not move after a temporary shock (because the long-run level of TFP does not change after a temporary shock.)

We discuss the parametrization of the NK-BW model first, and then we discuss the parametrization of the SOE-RBC model. The logic follows two existing theoretical results. Blanchard, L'Huillier, and Lorenzoni (2013) proved (Online Appendix Section 6.4.2) that a baseline NK model (without capital and no bells and whistles) converges to a simple permanent income model with a fixed real interest rate, in which consumption is equal to expectations about the

Table 1: Parametrization I (limit parametrization)

	Parameter	Value
Common to Both Models		
h	Consumption habit	0
α	Capital share	0.17
φ	Inv. Frisch elasticity	3.79
ξ	Elasticity capital utilization cost	5.30
NK-BW Model		
β	Discount rate	0.9987
θ	Calvo prices	≈ 1
θ_w	Calvo wages	0.70
μ	Price markup	0.23
μ_p	Wage markup	0.15
γ_π	Interest rate rule inflation	2.09
γ_y	Interest rate rule output	1×10^{-7}
ϕ_{dy}	Interest rate rule output growth	0
ι	Price indexation	0.24
ι_w	Wage indexation	0.11
χ	Investment adjustment costs	2.85
	Policy	
ρ_r	Persistence nominal interest rate	0.82
SOE-RBC Model		
β	Discount rate	≈ 1
ψ	Elasticity of the interest rate	1×10^{-12}
B/Y	Steady state level of normalized debt	0
ν	Capital adjustment costs	2.85
Shock Processes		
	Technology	
ρ_x	Persistence permanent shock	0.20
ρ_z	Persistence temporary shock	0.20
σ_x	Standard dev. permanent shock	1.00
σ_z	Standard dev. temporary shock	1.00

Notes: Under this parametrization both models deliver flat responses of consumption. β for SOE-RBC model is set to 0.99999, and θ to 0.99999999.

long-run level of labor productivity. We do not attempt to prove an equivalent theoretical result in the case of the NK-BW model but instead numerically obtain that the result generalizes to this model as well. Following the conditions of the theorem in Blanchard et al. (2013), we set in Parametrization I the Calvo parameter very close to 1. We also set habit formation to 0 and we impose that the interest rate rule does not react to output (both restrictions follow from the baseline NK model).[5]

Regarding the SOE-RBC model, Cao and L'Huillier (2017) theoretically obtained the same result in the case of a small open economy model without capital and fixed labor supply. We do not attempt to prove an equivalent proposition in the case of both elastic labor supply and capital, but focus instead on numerical simulations. Following the conditions imposed in the statement of Proposition 1 in Cao and L'Huillier (2012), we set the discount rate β close to 1, the elasticity of the interest rate ψ close to 0, and the ratio $\beta/(1-\psi)$ close to 0. (Specifically, we set $\beta = 0.99999$ and $\psi = 1 \times 10^{-12}$.)[6]

The remaining structural parameters of the models are the estimates by Justiniano et al. (2010). (They take a period to represent a quarter.) The standard deviation of the shocks is normalized to 1, and their persistence is set to 0.20.

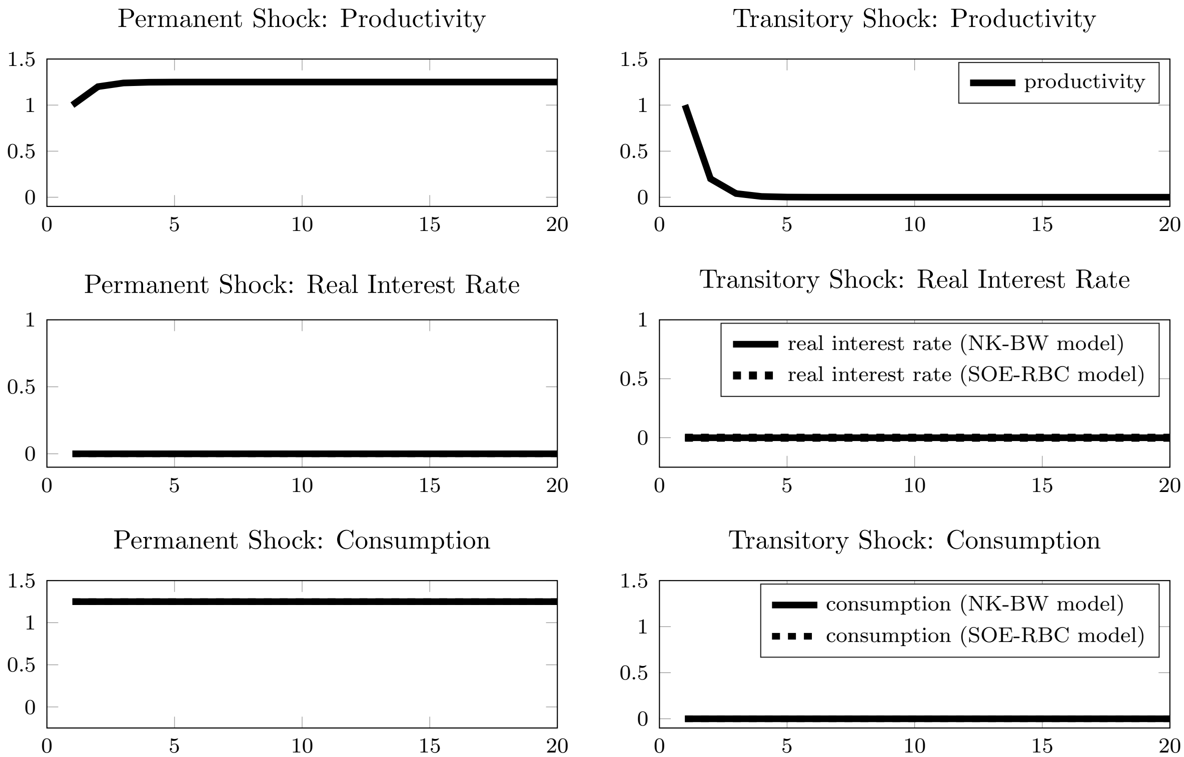
Figure 1 shows the resulting behavior of TFP, the real interest rate, and consumption in both models. We plot the IRFs of TFP and consumption following a permanent and a temporary technology shock. In both cases, the real rate does not move. The response of consumption is flat. The responses of the rate

[5]When we set the reaction of the interest rate rule to inflation to 0 (instead of the Calvo parameter close to 1) we obtain the same result.

[6]There are slight differences in the formulations of Proposition 1 between Cao and L'Huillier (2012) and Cao and L'Huillier (2017), which depend just on the normalization of endogenous variables (in the first version, C/Y is exogenous; in the second version, $\bar{b}$ is exogenous). However, the logic and extent of the proposition are essentially the same.

and consumption are indistinguishable from each other, both models delivering similar responses. In the case of a permanent shock, consumption jumps to the long-run level of TFP. In the case of a temporary shock, consumption does not move.

Figure 1: Main Impulse Responses, Parametrization I (Limit Parametrization)



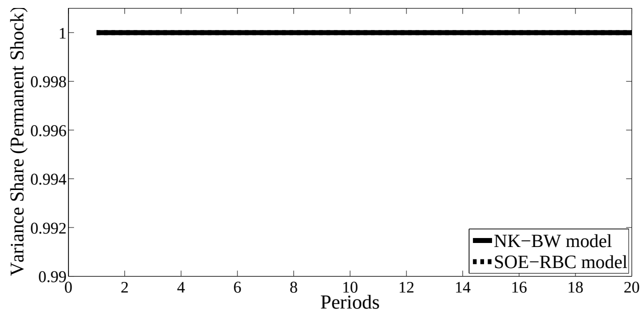
Notes: This simulation is obtained with parametrization I or 'limit parametrization'.

How both parametrizations achieve an (almost) constant real interest rate is fairly straightforward to see (and was already spelled out in the introduction). We make just a quick point about the role of separable preferences in delivering the flat response of consumption. We need separable preferences in order for the marginal utility for labor not to enter the consumption-Euler equation. For non-separable preferences, changes in the marginal utility of labor would break the random-walk property of consumption and would generate a non-flat response. This fact is also apparent after inspection of the proof in Cao and L'Huillier (2017).

What is a bit more subtle for the working of the permanent income channel is the role of the assumptions $\beta \longrightarrow 1$ and $\psi/(1-\beta) \longrightarrow 0$ in the case of the open economy model. Note that in the steady state the world interest rate R^* is $1/\beta$. So, when β is close to 1, the world interest rate is close to 1. In this case, temporary deviations of domestic demand from output lead to an accumulation of assets (or of debt). For the sake of the argument, consider a transition path in which the country has accumulated some debt. When the interest rate is 1, the domestic economy can roll-over the debt at infinity, and thus consumption can be freely set equal to the long-run level of output, determined on the steady state by the long-run level of TFP. The intuition for the requirement that $\psi/(1-\beta)$ is close to 0 comes from the fact that, in steady state, net exports finance steady state deviations of debt or assets away zero. Steady state net exports tend to infinity (in absolute value) when β goes to 1. In order to avoid the real rate from exploding when this happens, ψ needs to go to 0 even faster. This achieves the desired result. (See the proof in Cao and L'Huillier 2012.)

Another way to look into our results is to consider the variance decomposition in order to gauge which, among the permanent and the temporary shocks, accounts for a higher proportion of consumption volatility. Figure 2 shows the variance decomposition of the two models at different horizons. In both models, (almost) all consumption volatility is accounted by the permanent shock.

Figure 2: Variance Decomposition, Parametrization I (Limit Parametrization)



Notes: Percentage of forecast error explained by the permanent shock. Percentage of forecast error explained by temporary shock is just one minus the variance share depicted here in the figure. The result is obtained with parametrization I or 'limit parametrization'.

4.2 Standard Parametrization (or Parametrization II)

Parametrization II is shown in Table 2. This parametrization closely follows the literature. All structural parameters of the NK-BW model are from the benchmark estimation in Justiniano et al. (2010). Parameters specific to the SOE-RBC are set as follows. The coefficient on interest rate premium, ψ, is set to 0.0010, which is the number used in the literature (Schmitt-Grohe and Uribe 2003; Aguiar and Gopinath 2007). The steady-state level of debt-to-output ratio B/Y is set to 0.1 following Aguiar and Gopinath (2007). The standard deviation of the shocks is normalized to 1, and their persistence is set to 0.20.

Figure 3 presents the IRFs of productivity, the real interest rate, and consumption (in both models) under parametrization II. The response of the real rate is no longer nil as under parametrization I. However, the response on impact is quite small and reverts to zero quickly. To get a sense of how small this impact response is, we simulated a standard RBC model subject to the same shocks. Figure 4 shows the responses of the real interest rate in this model, and compares it to the responses of the NK-BW model (the responses in the SOE-RBC model are even smaller.) This figure clearly illustrates the main point

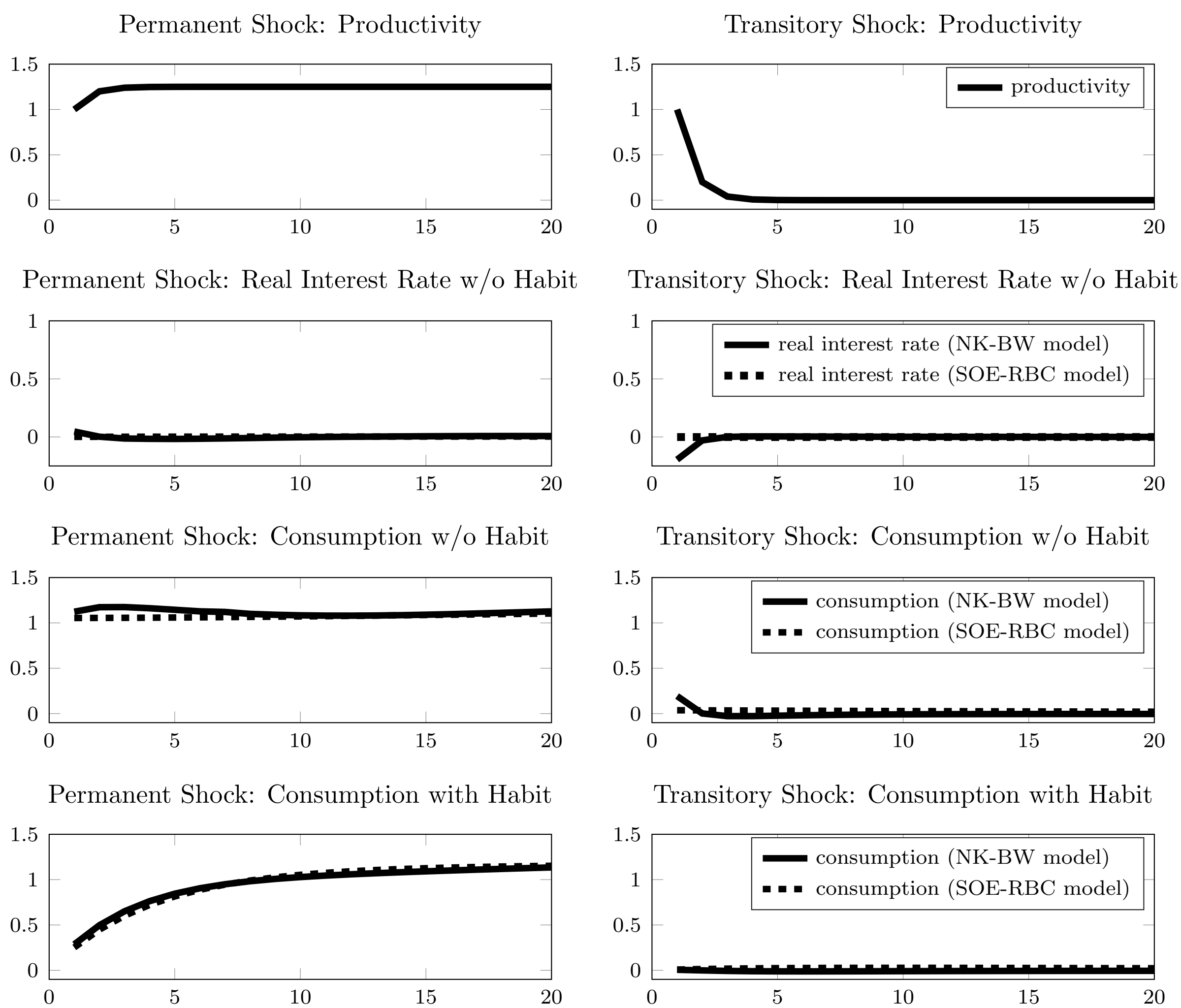
Table 2: Parametrization II (Standard Parametrization)

	Parameter	Value
Common to Both Models		
β	Discount rate	0.9987
h	Consumption habit	0
α	Capital share	0.17
φ	Inv. Frisch elasticity	3.79
ξ	Elasticity capital utilization cost	5.30
NK-BW Model		
θ	Calvo prices	0.84
θ_w	Calvo wages	0.70
μ	Price markup	0.23
μ_p	Wage markup	0.15
γ_π	Interest rate rule inflation	2.09
γ_y	Interest rate rule output	0.07
ϕ_{dy}	Interest rate rule output growth	0.24
ι	Price indexation	0.24
ι_w	Wage indexation	0.11
χ	Investment adjustment costs	2.85
	Policy	
ρ_r	Persistence nominal interest rate	0.82
SOE-RBC Model		
ψ	Elasticity of the interest rate	0.0010
B/Y	Steady state level of normalized debt	0.1
ν	Capital adjustment costs	2.85
Shock Processes		
	Technology	
ρ_x	Persistence permanent shock	0.20
ρ_z	Persistence temporary shock	0.20
σ_x	Standard dev. permanent shock	1.00
σ_z	Standard dev. temporary shock	1.00

Notes: Parametrization based on Justiniano, Primiceri, and Tambalotti (2010), Schmitt-Grohe and Uribe (2003), and Aguiar and Gopinath (2007).

of the paper, which is that the NK-BW has achieved a very muted real rate response by adding frictions to the RBC model. Indeed, most of the bells and whistles are real rigidities that, coupled with some nominal rigidity, constitute a powerful force that limit the general equilibrium propagation of technology shocks into the real rate of interest. This way, the NK-BW model essentially approaches a *partial equilibrium* model.

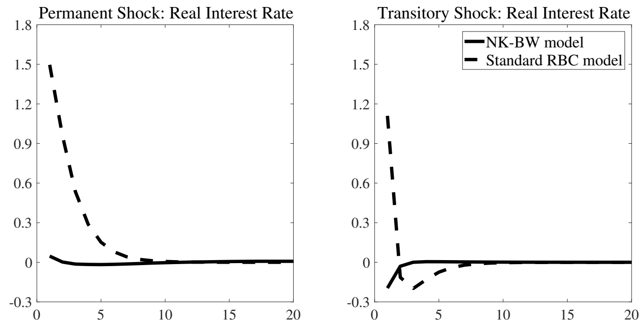
Figure 3: Main Impulse Responses, Parametrization II (Standard Parametrization)



Notes: This simulation is obtained with parametrization II, based on Justiniano, Primiceri, and Tambalotti (2010), Schmitt-Grohe and Uribe (2003), and Aguiar and Gopinath (2007).

Notice that also the SOE-RBC features very limited propagation to the

Figure 4: Comparison With Real Interest Rate in Standard, Closed Economy, RBC



Notes: For the standard RBC model, we set $\beta = 0.9987$, $\varphi = 3.79$, and $h = 0$. Productivity persistence for both permanent and transitory components (ρ_x and ρ_z) are set to 0.2. The simulation for the NK-BW model is obtained with parametrization II.

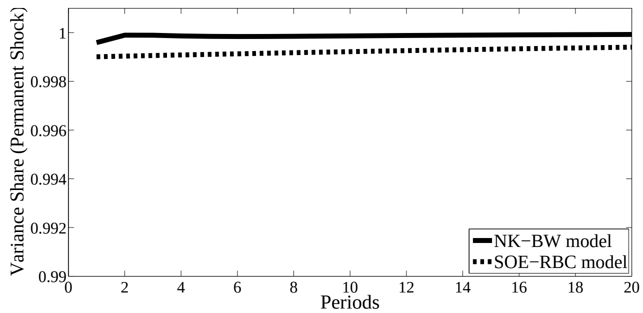
real rate. We note, importantly, that this is *not* because this is a small open economy model, but mainly because the value of ψ used in the literature is so small. One could set ψ to a higher value and this would still be a small open economy, because the world interest rate R^* is exogenous.[7]

Going back to Figure 3, we observe that the responses of consumption in the case of no-habit formation are close to flat in both models. In the case of the NK-BW model, nominal and real rigidities allow consumption to be disconnected from actual productivity by almost shutting down the general equilibrium effect on the real interest rate. Thus, it is not crucial for our purposes that the Calvo parameter θ tends to one: So long as it is fairly high, real rigidities will help get an important effect of permanent shocks on consumption. In the case of the SOE-RBC model, $\psi > 0$ achieves stationarity of the debt holdings as discussed in the literature. However, our analysis shows that setting it to a small value also disconnects consumption from the rest of the model, pretty much as in a NK economy with sticky prices.[8]

[7] See Schmitt-Grohe and Uribe (2003), pp. 172–4, for a related discussion on how to complete financial market for a small open economy.

[8] See Garcia-Cicco, Pancrazi, and Uribe (2010) for a related feature of the model with small ψ in terms

Figure 5: Variance Decomposition



Notes: Percentage of forecast error explained by permanent shock. Percentage of forecast error explained by temporary shock is just one minus the variance share depicted here in the figure. The result is obtained with parametrization II, based on Justiniano, Primiceri, and Tambalotti (2010), Schmitt-Grohe and Uribe (2003), and Aguiar and Gopinath (2007).

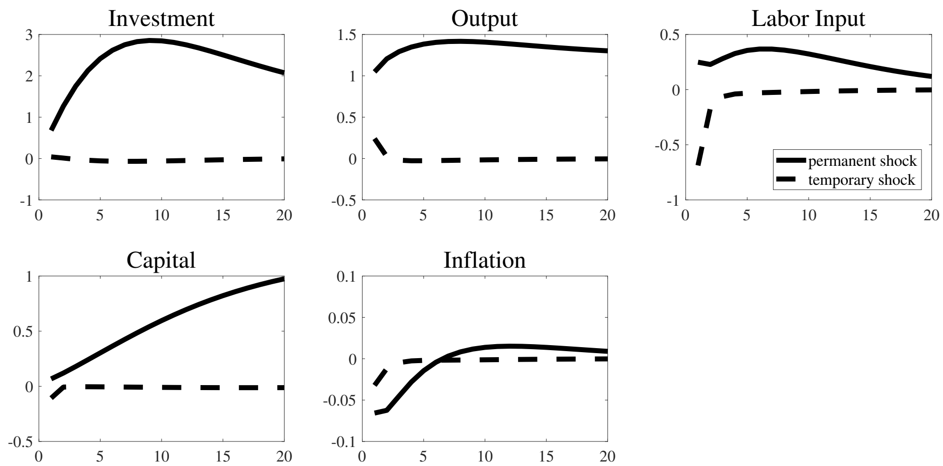
In the presence of habits (last row of Figure 3), the response of consumption to a permanent shock is gradual, but notice that both models deliver similar responses. Even though in the presence of habit formation the responses of both models are no longer flat under parametrization II, most of the action in the reaction of consumption is still dominated by permanent shocks. Indeed, under habit formation, the impact reaction of consumption after a temporary shock is also muted. To see this, it is useful to focus on the variance decomposition of consumption into both shocks, shown in Figure 5. The variance decomposition shows that most of the impact effect on consumption is due to permanent shocks ($> 99.5\%$), and a fortiori this remains true at longer horizons (because the effect of permanent shocks can only grow over time.)

4.3 Where is the GE?

The previous results clearly established non-existent or very limited general equilibrium effects in the case of the real interest rate and consumption in the both NK model with bells and whistles, and in the RBC open economy model.

of the autocorrelation function of net exports.

Figure 6: Impulse Responses of other Variables: NK-BW Model, Parametrization II (Standard Parametrization)



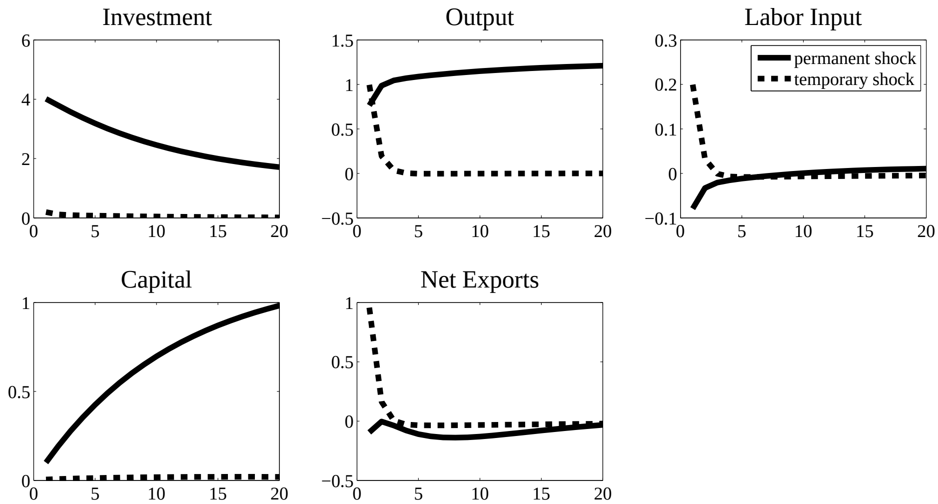
Notes: This simulation is obtained with parametrization II, based on Justiniano, Primiceri, and Tambalotti (2010), Schmitt-Grohe and Uribe (2003), and Aguiar and Gopinath (2007).

But then, which variables present general equilibrium effects? Here, we will simulate other variables to explore this question.

Figure 6 plots the IRFs of the investment, output, the labor input, capital, and inflation in the NK-BW model. Different from the response of consumption, in the case of a permanent shock, the response of investment and output is gradual. The simulation does *not* include habits, so what is happening is generated by GE. Capital needs to rise, but this happens gradually. Thus, output increases gradually. The response of labor is slightly hump shaped. The responses comove. Inflation does not move much due to the high amount of nominal and real rigidities. Notice also the limited reaction of all of these variables in the case of a temporary shock. This is a demand determined economy, and since consumption does not move, all the rest moves little as well.

Figure 7 plots the IRFs of investment, output, the labor input, capital and net exports in the SOE-RBC model. In the case of a permanent shock, invest-

Figure 7: Impulse Responses of other Variables: SOE-RBC Model, Parametrization II (Standard Parametrization)



Notes: This simulation is obtained with parametrization II, based on Justiniano, Primiceri, and Tambalotti (2010), Schmitt-Grohe and Uribe (2003), and Aguiar and Gopinath (2007).

ment and output comove positively. Capital accumulates, to reach a new steady state in the long run. Net exports fall slightly. The labor input falls as well, due to the income effect. In this case, the economy can finance an investment boom from abroad, so the fall in labor does not limit it. Notice also how, in the case of a temporary shock, the benefits of the temporary productivity increases are exported to the rest of the world.

The simple conclusion from these simulations is then that, all among investment, output, labor input, capital, inflation (NK-BW) and net exports (SOE-RBC) feature rich GE effects.

5 Final Remarks

We comment on the role played by the near partial equilibrium behavior of consumption in each of these two literatures. Why is it that both feature models with an almost fixed behavior of the real rate (conditional on technology shocks)? As argued in the introduction, this feature is an enabler of aggregate demand as an explanation of consumption fluctuations.

First, in the closed-economy literature built around the NK model with bells and whistles introduced by Christiano et al. (2005), the almost fixed real rate is perhaps an unintended consequence of building and calibrating a model in which monetary policy shocks deliver large and persistent responses of the economy, successfully fitting the standard definition of a business cycle in which several variables comove. These ingredients and parametrization also help to obtain suitable effects of demand shocks. Indeed, given a shock that rises consumption away from steady state, a muted response of the real rate (either because nominal and real rigidities imply a muted response of inflation, or because the monetary authority is accommodative), allows for a strong deviation of consumption. This is a point mentioned, for instance, by Blanchard et al. (2013), pp. 3064–5, in the case of demand shifts modeled by noise shocks.[9]

Regarding the SOE-RBC model, a literature following the seminal work by Aguiar and Gopinath (2007) has embarked in explaining the volatile current account behavior of emerging market. This has been achieved using trend shocks, which by the permanent income assumption move consumption by the anticipation of changes in future income. But because these shocks mostly change future, and not contemporaneous TFP, one can correctly think about them as

[9] A related paper here is by Kocherlakota (2012), who studied the implications of a fixed real interest rate for labor markets.

contemporaneous demand shocks.[10] But for this, it is indeed essential use a low elasticity interest rate (low ψ), as pointed out by Garcia-Cicco, Pancrazi, and Uribe 2010, and other papers following. We hope our paper ultimately provides a useful perspective on these channels based on a partial equilibrium reasoning.

[10]These shocks are in fact close cousins of the ones called news by Beaudry and Portier (2007), or noise by Lorenzoni (2009).

A Log-linear Approximations

A.1 NK-BW Model

First we define log-deviations for variables in the approximation. Here we also normalize variables in the model to ensure their stationarity. Specifically, we define

$$c_t \equiv \log(C_t/A_t) - \log(C/A))$$

We define in a similar way y_t, k_t, $\bar{k}_t$, and i_t. N_t and U_t are already stationary, therefore we define

$$n_t \equiv \log(N_t) - \log(N)$$

$$u_t \equiv \log(U_t) - \log(U)$$

For nominal variables, we also need to consider non-stationarity in the price level and therefore define

$$w_t \equiv \log\left(\left(W_t/P_t\right)/A_t\right) - \log\left(\left(W/P\right)/A\right)$$

$$r_t^k \equiv \log(R_t^k/P_t) - \log\left(R^k/P\right)$$

$$r_t \equiv \log(R_t)$$

$$\pi_t \equiv \log(P_t/P_{t-1}) - \Pi$$

Finally, for Lagrange multipliers, we define

$$\lambda_t \equiv \log(\Lambda_t A_t) - \log(\Lambda A)$$

$$\phi_t \equiv \log(\Phi_t A_t/P_t) - \log(\Phi A/P)$$

Households. The marginal utility of consumption is

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h \frac{1}{C_{t+1} - hC_t}$$

multiplying both sides by A_t

$$\Lambda_t A_t = \frac{1}{C_t/A_t - h\left(C_{t-1}/A_{t-1}\right)\left(A_{t-1}/_t\right)} - \beta h \frac{1}{\left(C_{t+1}/A_{t+1}\right)\left(C_{t+1}/A_t\right) - h\left(C_t/A_t\right)}$$

which with log-linearization yields

$$\lambda_t = \frac{h\beta}{(1-h\beta)(1-h)}\mathbb{E}_t c_{t+1} - \frac{1+h^2\beta}{(1-h\beta)(1-h)}c_t + \frac{h}{(1-h\beta)(1-h)}c_{t-1} + \\ + \frac{h\beta}{(1-h\beta)(1-h)}\mathbb{E}_t \Delta a_{t+1} - \frac{h}{(1-h\beta)(1-h)}\Delta a_t \tag{4}$$

The Euler equation is

$$\Lambda_t = \beta R_t^c \mathbb{E}_t \left[\Lambda_{t+1} \frac{P_t}{P_{t+1}}\right]$$

Multiplying both sides of this Euler equation by A_t gives

$$\Lambda_t A_t = \beta R_t^c \mathbb{E}_t \left[\Lambda_{t+1} A_{t+1} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}}\right]$$

and approximating yields

$$\lambda_t = r_t^c + \mathbb{E}_t\left[\lambda_{t+1} - \Delta a_{t+1} - \pi_{t+1}\right] \tag{5}$$

The optimality condition for capacity utilization is

$$R_t^k / P_t = U_t^{\xi}$$

In logs,

$$r_t^k = \xi u_t \tag{6}$$

The provision of the capital services is

$$K_t = U_t \bar{K}_{t-1}$$

dividing both sides by A_t

$$\frac{K_t}{A_t} = \frac{U_t}{A_t} \frac{\bar{K}_{t-1}}{A_{t-1}} \frac{A_{t-1}}{A_t}$$

Approximating it leads to

$$k_t = u_t + \bar{k}_{t-1} - \Delta a_t \tag{7}$$

The capital accumulation equation is

$$\bar{K}_t = (1-\delta)\,\bar{K}_{t-1} + \left[1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] I_t$$

dividing both sides by A_t

$$\frac{\bar{K}_t}{A_t} = (1-\delta)\,\frac{\bar{K}_{t-1}}{A_{t-1}}\left(\frac{A_{t-1}}{A_t}\right) + \left[1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] \frac{I_t}{A_t}$$

Approximating this leads to

$$\frac{\bar{K}}{A}\left(1 + \bar{k}_t\right) = (1-\delta)\,\frac{\bar{K}}{A}\left(1 + \bar{k}_{t+1} - \Delta a_t\right) + \left[1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}}\right)^2\right] \frac{I}{A}\left(1 + i_t\right)$$

which summarizes to

$$\frac{\bar{K}}{A}\left(1+\bar{k}_t\right) = (1-\delta)\frac{\bar{K}}{A}\left(1+\bar{k}_{t+1}-\Delta a_t\right) + \frac{I}{A}(1+i_t)$$

Then, the log-linear approximation is

$$\bar{k}_t = (1-\delta)\left(\bar{k}_{t+1}-\Delta a_t\right) + \delta i_t \tag{8}$$

The optimality condition for capital is

$$\Phi_t - \beta\Lambda_{t+1}\mathcal{C}(U_{t+1}) - \beta(1-\delta)\Phi_{t+1}\bar{K}_t$$

which is log-linearly approximated to

$$\phi_t = (1-\delta)\beta\mathbb{E}_t\left[\phi_{t+1}-\Delta a_{t+1}\right] + \left[1-(1-\delta)\beta\right]\mathbb{E}_t\left[\lambda_{t+1}-\Delta a_{t+1}+r^k_{t+1}\right] \tag{9}$$

Similarly, for the optimality condition for investment

$$-\Lambda_t+\Phi_t\left[\chi\left(\frac{I_t}{I_{t-1}}-1\right)\right]+\beta\Phi_{t+1}\left[1-\frac{\chi}{2}\left(\frac{I_{t+1}}{I_t}-1\right)^2+\frac{\chi}{2}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t}-1\right)\right]=0$$

The log-linear approximation obtained is

$$\lambda_t = \phi_t - \chi\left(i_t - i_{t-1} + \Delta a_t\right) + \beta\chi\mathbb{E}_t\left[i_{t+1}-i_t+\Delta a_{t+1}\right] \tag{10}$$

From the Fisher equation, we also obtain

$$r_t = r^c_t - E_t[\pi_{t+1}] \tag{11}$$

Final good producers. Total output

$$Y_t = K_t^{\alpha} \left(A_t N_t\right)^{1-\alpha}$$

dividing both sides by A_t

$$\frac{Y_t}{A_t} = \frac{K_t^{\alpha}}{A_t^{\alpha}} N_t^{1-\alpha}$$

Taking logs,

$$y_t = \alpha k_t + (1-\alpha)\, n_t \tag{12}$$

Intermediate goods producers. The optimal factor proportions between capital and labor is

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}$$

rearranging and dividing both sides by A_t,

$$(1-\alpha) \frac{K_t}{A_t} R_t^k = \alpha \frac{W_t}{A_t} N_t$$

Log-linearizing it leads to

$$k_t - n_t = w_t - r_t^k \tag{13}$$

The marginal cost equation is

$$MC_t = \frac{1}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha}} \left(R_t^k\right)^{\alpha} \left(\frac{W_t}{A_t}\right)^{1-\alpha}$$

dividing both sides by P_t

$$\frac{MC_t}{P_t} = \frac{1}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha}} \left(\frac{R_t^k}{P_t}\right)^{\alpha} \left(\frac{W_t}{P_t A_t}\right)^{1-\alpha}$$

Log-linearizing, it leads to

$$mc_t = \alpha r_t^k + (1-\alpha)\, w_t \tag{14}$$

Finally, from the optimality conditions for price setters, we have

$$\pi_t = \frac{\iota}{1+\iota\beta}\pi_{t-1} + \frac{\beta}{1+\iota\beta}\mathbb{E}_t\pi_{t+1} + \kappa mc_t \tag{15}$$

where $\kappa = (1-\theta\beta)(1-\theta)/\theta(1+\iota\beta)$.

Labor market. Similar to the optimality conditions for price setters, aggregating individual optimality conditions for the wage setter lead to

$$\begin{aligned} w_t =& \frac{1}{1+\beta}w_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t w_{t+1} + \frac{\iota_w}{1+\beta}\pi_{t-1} - \frac{1+\iota_w\beta}{1+\beta}\pi_t + \frac{\beta}{1+\beta}\mathbb{E}_t\pi_{t+1} \\ &+ \frac{\iota_w}{1+\beta}\Delta a_{t-1} - \frac{1+\iota_w\beta}{1+\beta}\Delta a_t + \frac{\beta}{1+\beta}\mathbb{E}_t\Delta a_{t+1} - \kappa_w mc_t^w \end{aligned} \tag{16}$$

where

$$k_w = \frac{(1-\theta_w\beta)(1-\theta_w)}{[\theta(1+\beta)(1+\varphi(1+1/\mu_w))]}$$

and

$$mc_t^w = w_t - \varphi n_t + \lambda_t \tag{17}$$

Monetary policy. The interest rate rule is

$$\frac{R_t^c}{R} = \left(\frac{R_{t-1}^c}{R}\right)^{\rho_r}\left[\left(\frac{\Pi_t}{\Pi}^{\gamma_\pi}\right)\left(\frac{Y_t/A_t}{Y/A}\right)^{\gamma_y}\right]^{1-\rho_r}\left(\frac{Y_t/A_t}{Y_{t-1}/A_{t-1}}\right)^{\phi_{dy}}$$

we obtain

$$r_t^c = \rho_r r_{t-1}^c + (1-\rho_r)(\gamma_\pi\pi_t + \gamma_y y_t) + \phi_{dy}(y_t - y_{t-1}) \tag{18}$$

Market clearing Market clearing in goods market implies

$$\mathcal{C}\left(1\right) = R^k/P$$

and

$$C_t + I_t + \mathcal{C}\left(U_t\right)\bar{K}_{t-1} = Y_t$$

Dividing both sides by A_t and approximating

$$\frac{C}{A}c_t + \frac{I}{A}i_t + \frac{R^k K}{PA}u_t = \frac{Y}{A}y_t$$

which leads to

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{R^k K}{PY}u_t \tag{19}$$

A.1.1 Summary

- There are 19 variables in the model. The endogenous variables are

$$\lambda_t, \phi_t, y_t, c_t, i_t, \bar{k}_t, k_t, n_t, u_t, r_t^c, r_t, r_t^k, w_t, \pi_t, mc_t, mc_t^w$$

  and the exogenous variables are

$$a_t, x_t, z_t$$

- Equations (1)-(3) and (4)-(19) constitute the log-linearized model.

A.2 SOE-RBC Model

In order to log-linearize the model, we define the following log-deviations:

$$c_t \equiv \log(C_t/A_t) - \log(C/A))$$

$$y_t \equiv \log(Y_t/A_t) - \log(Y/A))$$

$$k_t \equiv \log(K_t/A_t) - \log(K/A))$$

$$i_t \equiv \log(I_t/A_t) - \log(I/A))$$

$$n_t \equiv \log(N_t) - \log(N)$$

$$\lambda_t \equiv \log(\Lambda_t A_t) - \log(\Lambda A)$$

$$\phi_t \equiv \log(\Phi_t A_t) - \log(\Phi A)$$

the log of the interest rate

$$r_t \equiv \log(R_t)$$

and the following absolute deviations:

$$b_t \equiv \frac{B_t}{Y_t} - \frac{B}{Y}$$

$$nx_t \equiv \frac{NX_t}{Y_t} - \frac{NX}{Y}$$

In addition to the ten endogenous variables defined above, we also have additional exogenous variables x_t, z_t, a_t summarizing the productivity process. Log-linearization of the equilibrium conditions proceeds as follows. The marginal utility of consumption is

$$\Lambda_t = \frac{1}{C_t}$$

multiplying both sides by A_t

$$\lambda_t = \frac{A_t}{C_t}$$

where $\lambda_t = \Lambda_t A_t$. In logs the condition is

$$\lambda_t = -c_t \tag{20}$$

The production function is

$$Y_t = K_{t-1}^{\alpha} \left(A_t N_t\right)^{1-\alpha}$$

dividing both sides A_t

$$\frac{Y_t}{A_t} = \left(\frac{K_{t-1}}{A_{t-1}}\right)^{\alpha} \left(\frac{A_{t-1}}{A_t}\right)^{\alpha} N_t^{1-\alpha}$$

The log-linearized production function is

$$y_t = \alpha k_t + (1-\alpha) n_t - \alpha \Delta a_t \tag{21}$$

The resource constraint is

$$C_t + I_t + NX_t = Y_t$$

multiplying both sides by Y_t

$$\frac{C_t}{A_t}\frac{A_t}{Y_t} + \frac{I_t}{A_t}\frac{A_t}{Y_t} + \frac{NX_t}{Y_t} = 1$$

Since in steady state

$$\frac{C}{A}\frac{A}{Y} + \frac{I}{A}\frac{A}{Y} + \frac{NX}{Y} = 1$$

approximating,

$$\frac{C}{A}\frac{A}{Y}(1+c_t-y_t)+\frac{I}{A}\frac{A}{Y}(1+i_t-y_t)+nx_t+\frac{NX}{Y}=1$$

which yields

$$\frac{C}{Y}(c_t-y_t)+\frac{I}{Y}(i_t-y_t)+nx_t=0 \tag{22}$$

The price of debt equation is

$$R_t=R^*+\psi\left(e^{\frac{B_t}{Y_t}-b}-1\right)$$

We have steady state relation

$$R=R^*$$

so approximating the equation yields

$$r_t=\psi b_t \tag{23}$$

The optimality condition for labor is

$$(1-\alpha)\Lambda_t=N_t^{1+\varphi}\frac{1}{Y_t}$$

multiplying both sides by A_t

$$(1-\alpha)\lambda_t=N_t^{1+\varphi}\frac{A_t}{Y_t}$$

In logs

$$\lambda_t=(1+\varphi)n_t-y_t \tag{24}$$

The budget constraint

$$C_t + I_t + B_{t-1} = Y_t + Q_t B_t$$

dividing both sides by Y_t,

$$\frac{C_t}{A_t}\frac{A_t}{Y_t} + \frac{I_t}{A_t}\frac{A_t}{Y_t} + \frac{B_{t-1}}{Y_{t-1}}\frac{A_t}{Y_t}\frac{Y_{t-1}}{A_{t-1}}\frac{A_{t-1}}{A_t} = 1 + Q_t\frac{B_t}{Y_t}$$

which leads in steady state

$$\frac{C}{A}\frac{A}{Y} + \frac{I}{A}\frac{A}{Y} + \frac{B}{Y} = 1 + Q\frac{B}{Y}$$

Approximating

$$\frac{C}{A}\frac{A}{Y}(1+c_t-y_t)+\frac{I}{A}\frac{A}{Y}(1+i_t-y_t)+(b_{t-1}+b)\left(1-\Delta y_t-\Delta a_t\right) = 1+\frac{1}{R}\left(1-r_t\right)\left(b_t+b\right)$$

Since $1/R = \beta$, we have that

$$\frac{C}{Y}(c_t - y_t) + \frac{I}{Y}(i_t - y_t) = \beta b_t - b_{t-1} + b(\Delta a_t + y_t - y_{t-1} - \beta r_t) \qquad (25)$$

The optimality condition for debt is

$$Q_t \Lambda_t = \beta \mathbb{E}_t\left[\Lambda_{t+1}\right]$$

multiplying both sides by A_t,

$$Q_t \lambda_t = \beta \mathbb{E}_t\left[\lambda_{t+1}\left(\frac{A_t}{A_{t+1}}\right)\right]$$

which in steady state

$$1 = \beta R$$

Then, approximating

$$\lambda_t = E_t \left[\lambda_{t+1} - \Delta a_{t+1} \right] + r_t \tag{26}$$

The optimality condition for investment is

$$\Lambda_t = -\Phi_t$$

multiplying both sides by A_t

$$\lambda_t = -\phi_t \tag{27}$$

The capital accumulation equation is

$$K_t - (1-\delta)K_{t-1} - I_t + \frac{\nu}{2}\left(\left(\frac{K_t}{K_{t-1}} \right)^2 - 1 \right) K_{t-1} = 0$$

multiplying both sides by A_t,

$$\frac{K_t}{A_t} - (1-\delta)\frac{K_{t-1}}{A_{t-1}}\frac{A_{t-1}}{A_t} - \frac{I_t}{A_t} + \frac{\nu}{2}\left(\left(\frac{K_t}{A_t}\frac{A_{t-1}}{K_{t-1}}\frac{A_t}{A_{t-1}} \right)^2 - 1 \right) \frac{K_{t-1}}{A_{t-1}}\frac{A_{t-1}}{A_t} = 0$$

which in steady state

$$\delta \frac{K}{A} - \frac{I}{A} = 0$$

Then, approximating

$$(1+k_t) - (1-\delta)\left(1 + k_{t-1} - \Delta a_t\right) - \frac{I}{K}\left(1 + i_t\right) + \frac{\nu}{2}\left(1 + k_{t-1} - \Delta a_t\right)\left(\left(1 + k_t - k_{t-1} + \Delta a_t\right)^2 - 1 \right) = 0$$

which yields

$$k_t = (1-\delta)\left[k_{t-1} - \Delta a_t \right] + \delta i_t \tag{28}$$

Finally, the optimality condition for capital is

$$\Phi_t\left[1+\nu\left(\frac{K_t}{K_{t-1}}-1\right)\right]+\alpha\beta\mathbb{E}_t\left[\Lambda_{t+1}\left(\frac{Y_{t+1}}{K_t}\right)\right]-\beta\mathbb{E}_t\left[\Phi_{t+1}\left((1-\delta)-\frac{\nu}{2}\left(1-\left(\frac{K_{t+1}}{K_t}\right)^2\right)\right)\right]=0$$

multiplying both sides by A_t

$$\begin{aligned}\phi_t\left[1+\nu\left(\frac{K_t}{K_{t-1}}-1\right)\right]+\alpha\beta\mathbb{E}_t\left[\lambda_{t+1}\frac{A_t}{A_{t+1}}\left(\frac{Y_{t+1}}{K_t}\right)\right]-\\-\beta\mathbb{E}_t\left[\phi_{t+1}\frac{A_t}{A_{t+1}}\left((1-\delta)-\frac{\nu}{2}\left(1-\left(\frac{K_{t+1}}{K_t}\right)^2\right)\right)\right]=0\end{aligned}$$

which in steady state is

$$\phi+\alpha\beta\mathbb{E}_t\left[\lambda\frac{Y}{K}\right]-\beta\mathbb{E}_t\left[\phi\left(1-\delta\right)\right]=0$$

Approximating, we end up with

$$\begin{aligned}\phi\left(1+\phi_t\right)\left[1+\nu\left(1+k_t\right)\left(1-k_{t-1}\right)\left(1+\Delta a_t\right)-\nu\right]+\\+\alpha\beta\lambda\left(1+\lambda_{t+1}\right)\left(1-\Delta a_{t+1}\right)\left(1+y_{t+1}\right)\left(1-k_t\right)\left(1+\Delta a_{t+1}\right)\frac{Y}{K}+\\+\beta\phi\left(1+\phi_{t+1}\right)\left(1-\Delta a_{t+1}\right)\left[-\left(1-\delta\right)\right)-\nu\left(k_{t+1}-k_t+\Delta a_{t+1}\right)\right]=0\end{aligned}$$

which yields after collecting terms

$$\begin{aligned}-(1+\phi_t+\nu k_t-\nu k_{t-1}+\nu\Delta a_t)+\alpha\beta\frac{Y}{K}(1-k_t+\mathbb{E}_t\left[y_{t+1}+\lambda_{t+1}\right])+\\+\beta(1-\delta)(1+\mathbb{E}_t\left[\phi_{t+1}-\Delta a_{t+1}\right])-\beta(\nu k_t-\mathbb{E}_t\left[\nu k_{t+1}+\nu\Delta a_{t+1}\right])=0\end{aligned}\tag{29}$$

A.2.1 Summary

- There are 13 variables in the model. The endogenous variables are

$$\lambda_t, \phi_t, y_t, c_t, i_t, k_t, n_t, nx_t, b_t, r_t$$

  and the exogenous variables are

$$a_t, x_t, z_t$$

- Equations (1)-(3) and (20)-(29) constitute the log-linearized model.

References

Aguiar, M. and G. Gopinath (2007). Emerging market business cycles: The cycle is the trend. *Journal of Political Economy 115*(1), 69–102.

Beaudry, P. and F. Portier (2007). When can changes in expectations cause business cycle fluctuations in neo-classical settings? *Journal of Economic Theory 135*(1), 458–477.

Blanchard, O. J., J.-P. L'Huillier, and G. Lorenzoni (2013). News, noise, and fluctuations: An empirical exploration. *American Economic Review 103*(7), 3045–3070.

Campbell, J. and A. Deaton (1989). Why is consumption so smooth? *The Review of Economic Studies 56*(3), 357–373.

Campbell, J. Y. (1987). Does saving anticipate declining labor income? An alternative test of the permanent income hypothesis. *Econometrica 55*(6), 1249–1273.

Cao, D. and J.-P. L'Huillier (2012). Technological revolutions and debt hangovers: Is there a link? *EIEF Working Paper 16/12*.

Cao, D. and J.-P. L'Huillier (2017). Technological revolutions and the three great slumps: A medium-run analysis. *Mimeo*.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Gali, J. (2008). *Monetary policy, inflation and the business cycle: An introduction to the New Keynesian framework*. Princeton: Princeton University Press.

Garcia-Cicco, J., R. Pancrazi, and M. Uribe (2010). Real business cycles in emerging countries? *American Economic Review 100*(5), 2510–2531.

Hall, R. E. (1978). Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence. *Journal of Political Economy 86*(6), 971–987.

Justiniano, A., G. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132–145.

Kocherlakota, N. R. (2012). Incomplete labor markets. *Mimeo*.

Lorenzoni, G. (2009). A theory of demand shocks. *American Economic Review 99*(5), 2050–84.

Mendoza, E. G. (1991). Real business cycles in a small open economy. *American Economic Review 81*(4), 797–818.

Schmitt-Grohe, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Smets, F. and R. Wouters (2007). Shocks and frictions in US business cycles: A bayesian DSGE approach. *American Economic Review 97*(3), 586–606.