Repeat Applications in College Admissions

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Introduction

- Matching markets: medical residency match, public school allocations, labor markets, college admissions.
- Many markets involve "repeat applicants" and matching is not a static game.
    - Reentering job markets for professionals.
    - Repeat taking civil service exams in Japan, Korea, US.
    - Reapplying (or transferring) colleges in China, France, Japan, Korea, Turkey, US

- Economic implications of repeat applications and their welfare consequences are not well understood, however.
- In this paper, we will
  - model repeat applications problems;
  - analyze equilibrium properties and welfare implications; and
  - draw some policy implications

  in the context of college admissions.

Repeat applications in college admissions

- Repeat applicants:
  - Korea: 23% of 3,057,983 applicants for four-year colleges were repeat applicants in 2016.
  - France: more than two years in CPGE ("prépa school") for Grandes Ecoles
  - US: 18% of 119,408 applicants in UCLA; 3% of 31,671 applicants in Duke are transfer students in 2016.
- It is costly to repeat apply.
  - Additional preparation, opportunity cost of staying behind a year
  - Korea: private tutoring institution $\$750 \sim \$2,800$ per month.
  - US: transfer students can "lose" credits when they move to the new school and typically attend for an extra year or more.

Key observations

- Sorting effect:
  - Students self-select whether to repeat apply.
  - High type students are more likely to repeat apply, and they pursue better college.
  - Repeat applications enable better matching.
- Congestion effect:
  - College admission is a situation in which individuals compete for fixed resources (i.e., “good” colleges).
  - Repeat application enlarges a pool of applicants at any given time and thereby increases competition, which causes future students to repeat apply, and so on...
  - Reapplicants do not take into account for (negative) externality of taking away seats from others, which causes repeat application to be excessive.

Related Literature

- Avery and Levin (2010), Lee (2009), Che and Koh (2016)
  - Colleges' admission strategies.
- Chade and Smith (2006), Chade, Lewis and Smith (2011)
  - Students' application decisions with application cost.
- Frisancho, Krishna, Lychagin and Yavas (2016), Krishna, Lychagin and Frisancho (forth)
  - Retaking university entrance exam/repeat applications in Turkey.
- Vigdor and Clotfelter (2003), Törnkvist and Henriksson (2004)
  - Retaking SAT and Swedish-SAT.

Model

- A unit mass of students with type (ability) $\theta \in [0,1]$, according to a distribution $G(\cdot)$.
  - Each of type-θ student draws score $s \in [0,1]$ from $F(\cdot|\theta)$
  - The density $f(\cdot|\theta)$ satisfies MLRP, i.e., for $s < s'$ and $\theta < \theta'$,

$$\frac{f(s'|\theta')}{f(s|\theta')} > \frac{f(s'|\theta)}{f(s|\theta)}.$$

- Two colleges, 1 and 2, each with capacity κ_i and quality q_i.
  - Type-θ student obtains payoff $q_i\theta$ from attending college i.
  - $q_1 > q_2 > 0$, $\kappa_1 < 1$ and κ_2 is sufficiently large.
  - If a student doesn't attend 1 or 2, he goes to the "null" college, ø, and gets zero payoff.

- Students can apply to at most one college.
  - Limiting applications is not unusual (Che and Koh, 2016).
  - E.g. Korea (at most one in each group), Japan (at most two public universities), UK (Cambridge vs. Oxford).
  - Later, we will study multiple applications.

- Timing (in each year)
  1. Students observe their types and decide which college they apply to (with no cost for initial application).
  2. Scores are (publicly) observed and colleges admit students whose scores are above some cutoffs.
  3. Students who fail to get into a (desired) college can take another year to repeat apply.
  4. When reapplying, type θ student draws another score from $F(\cdot|\theta)$ and pays reapplication cost c.

- Focus on stationary equilibrium
  - College i employs the same cutoff $\hat{s}_i$ and admits students with $s \geq \hat{s}_i$ for each year. ($\hat{s}_1 > 0 = \hat{s}_2$.)
  - The set of types (repeat) applying to each college remains the same in each year.

- Students' payoffs for a given $\hat{s} = (\hat{s}_1, \hat{s}_2)$,
  - $u_i(\theta; \hat{s}) := q_i\theta\big(1 - F(\hat{s}_i|\theta)\big)$ is the static payoff from applying to i.
  - $u_{ij}(\theta; \hat{s}) := u_i(\theta) + F(\hat{s}_i|\theta)\big(u_j(\theta) - c\big)$ is the payoff from applying to i and reapplying to j.
  - If $j = \emptyset$, $u_{ij}(\theta; \hat{s}) \equiv u_i(\theta; \hat{s})$ is the payoff from applying to i and do not reapply.

Characterization of Equilibrium

- A stationary equilibrium consists of $\hat{s}$ and α such that

  (i) For all $\theta \in [0,1]$,

$$\alpha(\theta) = (i,j) \in \underset{k,\ell \in \{1,2\}\times\{1,2,\emptyset\}}{\arg\max} u_{k\ell}(\theta; \hat{s}).$$

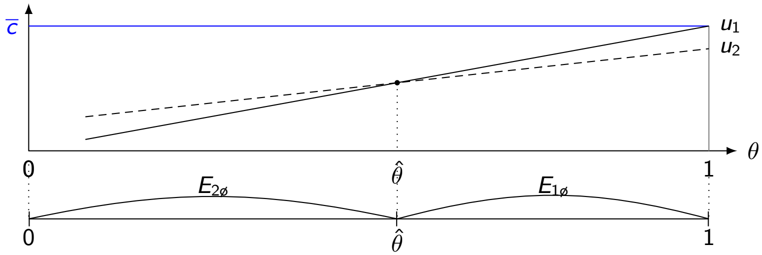
  (ii) For each school i, $m_i \leq \kappa_i$ (with equality if $\hat{s}_i > 0$).

- Mass of students enrolling in college i,

$$m_i = \int_{\alpha(\theta)=(i,j)} (1-F(\hat{s}_i|\theta))dG(\theta) + \sum_{j\in\{1,2\}} \int_{\alpha(\theta)=(j,i)} F(\hat{s}_j|\theta)(1-F(\hat{s}_i|\theta))dG(\theta)$$

(Unique) Equilibrium without reapplications: $c \geq \overline{c}$

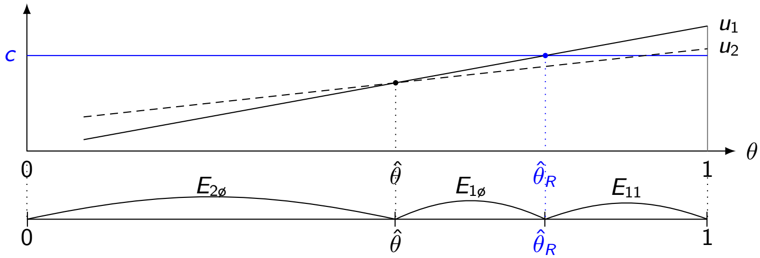
- There is $\overline{c}$ such that for $c > \overline{c}$, $u_i(\theta) < c$ for all θ, where $i = 1, 2$.



$$\alpha(\theta) = \begin{cases} (1, \varnothing) & \text{for } \theta \in [\hat{\theta}, 1] \\ (2, \varnothing) & \text{for } \theta \in [0, \hat{\theta}) \end{cases}$$

- No reapplications.
- $\hat{\theta}$ is indifferent between 1 and 2 in terms of static payoffs.
- High types take risk to enjoy higher q.

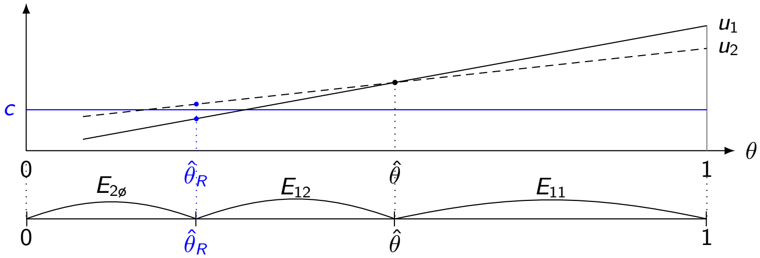
Equilibrium with reapplications-Case 1: $c \in [\hat{c}, \overline{c})$



$$\alpha(\theta) = \begin{cases} (1,1) & \text{for } \theta \in [\hat{\theta}_R, 1] \\ (1,\emptyset) & \text{for } \theta \in [\hat{\theta}, \hat{\theta}_R) \\ (2,\emptyset) & \text{for } \theta \in [0, \hat{\theta}) \end{cases}$$

- $\hat{\theta}_R$ is indifferent between reapplying and not.
- $u_{11}(\theta) = u_1(\theta) + F(\hat{s}_1|\theta)(u_1(\theta) - c) > u_1(\theta)$ for $\theta > \hat{\theta}_R$.
- High types have incentive to reapply to 1.

Equilibrium with reapplications-Case 2: $c \in [0, \hat{c})$



$$\alpha(\theta) = \begin{cases} (1,1) & \text{for } \theta \in [\hat{\theta}, 1] \\ (1,2) & \text{for } \theta \in [\hat{\theta}_R, \hat{\theta}) \\ (2,\emptyset) & \text{for } \theta \in [0, \hat{\theta}_R) \end{cases}$$

- Except for low types ($E_{2\emptyset}$), students have incentive to reapply.
- Middle types (E_{12}) consider it worthwhile to take a chance on 1, since c is low and there is a safe option in the next period.
- High types (E_{11}) keep pursuing college 1.

Existence of Equilibrium

- Characterization so far based on a fixed $\hat{s} = (\hat{s}_1, \hat{s}_2)$.
- Associate the equilibrium with a fixed point in the cutoff score under a map Φ.
  - Fix any cutoff scores $\hat{s} = (\hat{s}_1, \hat{s}_2)$.
  - Pin down $\alpha(\theta)$ as constructed above.
  - This in turn determines the mass of applicants to each college at any given score profile $s = (s_1, s_2)$.

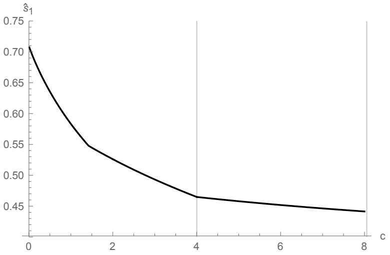
$$
\begin{aligned}
m_i(s; \hat{s}) = & \int_{\{\theta | \alpha(\theta; \hat{s}) = (i,j)\}} (1 - F(s_i|\theta)) dG(\theta) \\
& + \sum_{j \in \{1,2\}} \int_{\{\theta | \alpha(\theta; \hat{s}) = (j,i)\}} F(s_j|\theta)(1 - F(s_i|\theta)) dG(\theta)
\end{aligned}
$$

  - Equating them to capacities yields new cutoff scores $\tilde{s} = (\tilde{s}_1, \tilde{s}_2)$.

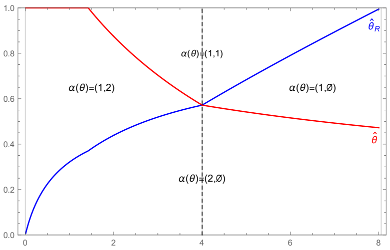
- The map Φ from $\hat{s}$ to $\tilde{s}$ admits a fixed point by Brouwer.
  - Show that $\hat{s}$ lies within a compact set
  - Show that Φ is continuous.
- Each step of the proof requires subtle care.
  - Need to rule out the possibility that mass of students are indifferent between any two application strategies.

Comparative Statics

- Equilibrium cutoffs:



(a) $\hat{s}_1$



(b) $\hat{\theta}_R$ and $\hat{\theta}$

Parameters: $q_1 = 10,\ q_2 = 7,\ \kappa_1 = 0.4,\ F(s|\theta) = s^{\theta+1},\ G(\theta) = \theta$

- As it becomes more costly to repeat apply ($c \uparrow$),
  - Less students reapply ($\hat{\theta}_R \uparrow$) $\Rightarrow$ make college 1 less competitive ($\hat{s}_1 \downarrow$) $\Rightarrow$ lowers the lowest applicant type ($\hat{\theta} \uparrow$).

Welfare Analysis

- Social welfare

$$SW := Q - C = (q_1 v_1 + q_2 v_2) - c\, m_R$$

- $Q := q_1 v_1 + q_2 v_2$ captures "matching quality," where $q_i v_i$ is the value generated from matching students with college i.
  - For instance, if $c \in [\hat{c}, \overline{c})$,

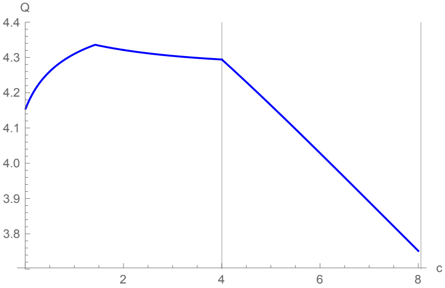
$$v_1 = \int_{\hat{\theta}}^{1} \theta\big(1 - F(\hat{s}_1|\theta)\big)\, dG(\theta) + \int_{\hat{\theta}_R}^{1} \theta F(\hat{s}_1|\theta)\big(1 - F(\hat{s}_1|\theta)\big)\, dG(\theta)$$

- $C := c\, m_R$ is the total cost of repeat applications, where m_R is the mass of reapplicants, where

$$m_R := \int_{\hat{\theta}_R}^{1} F(\hat{s}_1|\theta)\, dG(\theta)$$

  - m_R is decreasing in c ($\hat{\theta}_R$ is increasing and $\hat{s}_1$ is decreasing in c).

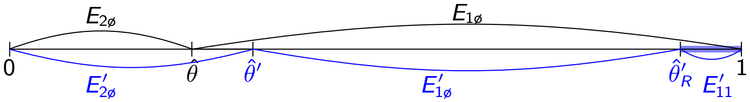
Sorting effect



- Q is maximized when an interior fraction of students reapplies.
- For sorting, it is not good for everybody to repeat apply or for nobody to repeat apply.

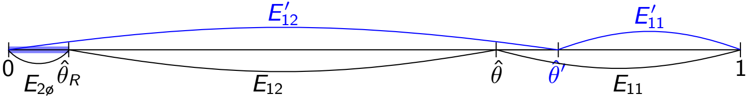
- Reduce c to $c' < c$.
  - Marginal types switch to repeat apply ($\hat{\theta}_R > \hat{\theta}'_R$) and college 1 becomes more selective ($\hat{s}_1 < \hat{s}'_1$).
  - Gain: types $[\hat{\theta}'_R, \hat{\theta}_R]$ replace low types.
  - Loss: due to fierce competition, even higher types become less likely to be admitted.

- When $c = \overline{c} \downarrow c'$



  - $[\hat{\theta}'_R, 1]$ replace lower types.
  - No higher types negatively affected by increased score.
  - Q is increasing as $c = \overline{c} \downarrow c'$.

- When $c \downarrow c' = 0$,



  - No lower types replaced by $[0, \hat{\theta}_R]$.
  - All above types are negatively affected by $\hat{s}'_1 > \hat{s}_1$.
  - Q is decreasing as $c \downarrow c' = 0$.

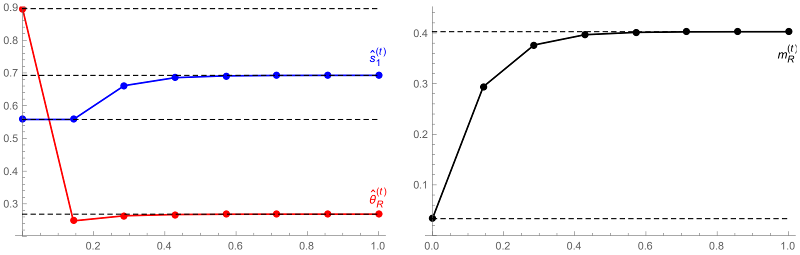
- Hence, Q is maximized in an interior value of c.

Congestion effect

- Ignoring the sorting effect, college admissions is like a "zero sum game" (when you win college 1, somebody else loses).
- Excessive reapplications
  - Private benefit from repeat application exceeds social benefit
    $\Rightarrow$ Excessive reapplication at the individual level.
- Positive feedback
  - More people in one cohort repeat apply $\Rightarrow$ college 1 becomes more selective, $\hat{s}_1 \uparrow \Rightarrow$ induces even more people in the next cohort to repeat apply $\Rightarrow \ldots$
  - Negative externalities are amplified by the chain reaction.

Congestion effect: positive feedback

- Consider two costs $c > c'$, and reduce c to c' permanently.
  - E.g. online tutoring, less weights on high school GPA, and so on..
  - $(\hat{s}_1^*, \hat{\theta}^*, \hat{\theta}_R^*)$ and $(\hat{s}_1^{*'}, \hat{\theta}^{*'}, \hat{\theta}_R^{*'})$: steady state cutoffs at c and c'.
- Dynamics:



(a) $\hat{s}_1^{(t)}, \hat{\theta}_R^{(t)}$ (b) $m_R^{(t)}$

Parameters: $q_1 = 10,\ q_2 = 2,\ \kappa_1 = 0.6,\ c = 6 > c' = 1$

- $c \downarrow c'$ at $t = 0$

  $\Rightarrow$ more students repeat apply ($\hat{\theta}_R^{(0)} < \hat{\theta}_R^*$ and $m_R^{(0)} > m_R^*$)

  $\Rightarrow$ college 1 becomes more selective ($\hat{s}_1^{(1)} \uparrow$)

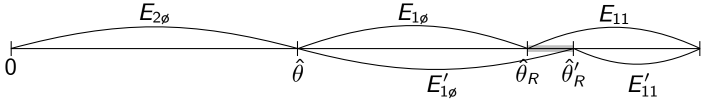
  $\Rightarrow$ even more students are rejected and forced to reapply ($m_R^{(1)} > m_R^{(0)}$), although repeat application becomes less attractive ($\hat{\theta}_R^{(1)} > \hat{\theta}_R^{(0)}$)

  $\Rightarrow \cdots$

  $\Rightarrow s_1^{(t)} \uparrow \hat{s}_1^{*\prime}, \hat{\theta}_R^{(t)} \uparrow \hat{\theta}_R^{*\prime}, m_R^{(t)} \uparrow m_R^{*\prime}$ as $t \to \infty$.

Policy Implication: Imposing tax

- Tax on repeat application: $c + \tau$.
- At $\tau = 0$, raising tax raises $\hat{\theta}_R$ and lowers $\hat{s}_1$.
- If the tax rate is slight,
  - private welfare loss is second order (because marginal types are making optimal decisions)
  - but the benefit from reducing negative externalities is first order.
- E.g., for $c \in [\hat{c}, \overline{c})$



Reducing quality gap

- Excessive reapplication comes from that students want to enroll in 1. Reducing quality gap, $q_1 - q_2$, mitigates such desires.
- Suppose q_i changes by Δq_i, $i = 1, 2$, such that

$$\Delta q_1 \leq 0 < \Delta q_2 \quad \text{and} \quad \Delta q_2 \geq -\frac{v_1}{v_2}\Delta q_1.$$

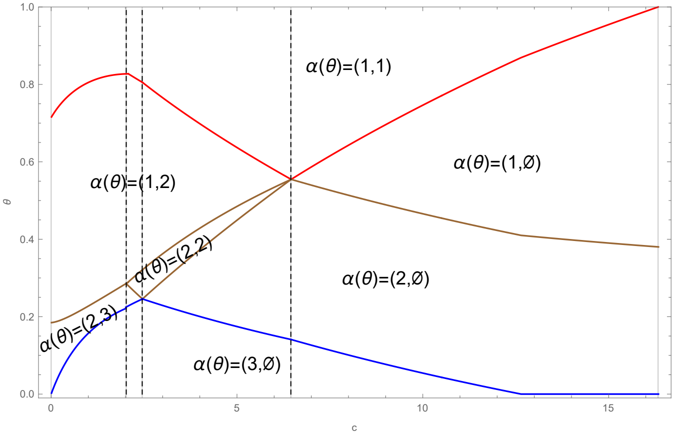
- This makes college 1 less attractive, alleviating the congestion problem. Hence, SW increases.
- NB. $\Delta q_1 < 0 = \Delta q_2$ doesn't work since this lowers matching quality, while $\Delta q_1 = 0 < \Delta q_2$ works well.

More than Two Colleges

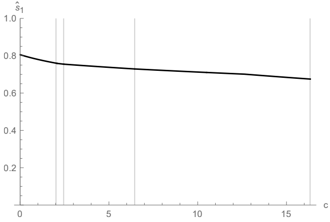
- Extend the baseline model
  - There are $n \geq 2$ colleges, each with capacity κ_i and $q_i > q_{i+1}$.
  - Soring effect and congestion effect still prevail.
- Congestion effect
  - Imposing a slight tax τ increases social welfare.
- Sorting effect
  - There exists $\overline{c}$ such that for $c \geq \overline{c}$, no student reapplies.
  - For c sufficiently close to $\overline{c}$, Q is decreasing in c.

3 colleges: $q_1 = 30 > q_2 = 15 > q_3 = 10$, $\kappa_1 = \kappa_2 = 0.3$.

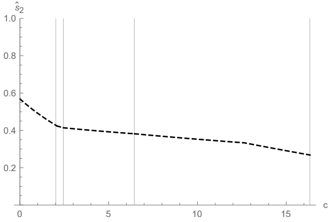
- Optimal application decisions



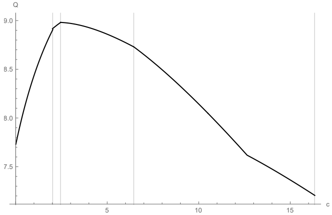
- Cutoff scores, matching quality and social welfare



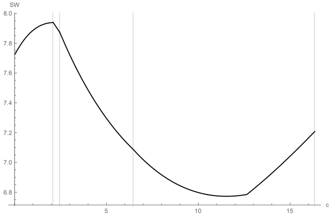
(a) $\hat{s}_1$



(b) $\hat{s}_2$



(c) Q



(d) SW

Multiple Applications / Transfers

- Students can apply to both colleges in the baseline model.
  - Students are always admitted by college 2 because κ_2 is large.
  - Reapplication is interpreted as transferring from college 2 to 1.
- Each type θ reapplies if and only if

$$u_M(\theta) := q_1\theta\big(1 - F(\check{s}_1|\theta)\big) + q_2\theta F(\check{s}_1|\theta) - c > q_2\theta$$

  - $u_M(\theta)$ is the expected payoff from reapplication.
  - $u_2(\theta) = q_2\theta$ is the current payoff.
- Welfare and policy implications
  - Both sorting effect and congestion effect.
  - The same policy implications as before.

Conclusion

- First theoretical work that analyzes repeat applications in the matching literature.
- Provide a tractable framework for analyzing
  - How students make (re)application decisions.
  - Welfare consequences of repeat applications.
  - Some policy implications.
- Further works
  - Empirical evidence.
  - Learning through repeat applications.
  - Designing admissions standards.