Bargaining in Financial Over-The-Counter Markets*

Preliminary Draft

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Abstract

I study bargaining over prices between two investors in financial over-the-counter markets with asymmetric information. A buyer and a seller meet and bargain over the price of a single asset at the interim stage when the buyer is uncertain about the seller's true motive for trading—whether it is because of a liquidity need or because of a low asset value. I use the concepts of interim efficiency and neutral optimum to characterize the prices at which investors may reasonably trade with each other. I discuss the impact of bargaining powers on negotiated prices, and derive welfare implications.

JEL classification: C78, D82, G12, G14.

Keywords: Bargaining, asymmetric information, mechanism design, interim efficiency, neutral optimum, asset pricing, over-the-counter markets.

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1. Introduction

In financial over-the-counter (OTC) markets, asset prices are negotiated directly between investors. Asset owners may wish to sell either because of a high holding cost arising from illiquidity or because of a low holding benefit arising from poor asset quality (or both). Nonowners who wish to buy may want to identify an asset owner's motive for selling because, while the asset owner's liquidity state would not affect nonowners' values of the asset, the asset quality would. In such situations, if asset owners have private information about their personal values of holding the asset, whatever the sources of motives for selling might be, then they would always want to preserve some private information in an attempt to get a better price; hence prices negotiated between investors will be influenced by those incentives.

This paper applies the powerful tools of mechanism design to the study of bilateral bargaining over prices in OTC markets with asymmetric information. My general question is: What bargaining mechanisms can be designed so that the resulting bargained prices have reasonable economic properties? The goal of this paper is to develop a theory of optimal asset pricing in OTC markets where prices are negotiated at the interim stage of bargaining.

To make headway on this problem, I build a static baseline model of bargaining between two investors—a dealer (seller) and a trader (buyer). The dealer has private information about her type along two dimensions: liquidity state and asset quality, and so the trader is uncertain about why the dealer wants to sell. The two investors meet and bargain over the price of a single asset to be traded. I use the cooperative solution concepts of interim efficiency and neutral optimum to define optimal bargaining mechanisms for OTC trading. In particular, I explicitly characterize the efficient and neutral prices that are reasonably negotiated between investors under the optimal mechanisms. I illustrate the implications of the results in a simple example of the baseline model. Finally, I show how negotiated asset prices under neutral mechanisms depend on investors' bargaining powers, and discuss welfare implications.

My contributions are as follows. First, by using the mechanism design approach, I provide a theory of optimal asset pricing for financial OTC markets in the presence of information asymmetry. Second, I show that the neutral price is obtained as a unique solution that has the properties that are desirable to counterparties, i.e., fairness and efficiency. Third, I demonstrate that neutral mechanisms are computed by the tractable set of conditions, and are insightful and easy to use from a practical perspective in terms of applications to the design of bargaining protocols in OTC markets. More broadly, the results I derive within my framework suggest that there is a caution to be added to the approach that uses the Nash bargaining solution for characterizing the terms and prices, or that uses ex ante measures for evaluating welfare of the bargaining outcome, in OTC markets where uncertainty about the counterparty is often pervasive during a bargaining process.[1]

There is a growing body of work on search and bargaining models of OTC markets. My paper is most closely related to the works of Duffie, Gârleanu, and Pedersen (2005, 2007), which use the Nash bargaining solution to explicitly compute the prices that form a dynamic stationary equilibrium. They look at a richer environment than I do; richer in the dimension that a dynamic asset-pricing model with both search and bargaining frictions is considered.[2] In their model, an investor's type is characterized by whether he owns the asset or not and whether he has high or low liquidity; so the presence of asymmetric information about the investor's type is irrelevant for the analysis because trade occurs only between low-liquidity asset owner and high-liquidity nonowner. The next step for this paper would be to extend my baseline model to a dynamic or sequential bargaining setting, and to use the incomplete-information version of the Nash bargaining solution (i.e., neutral optimum) in characterizing dynamic optimal mechanisms. This extension would allow us to study the combined impact

[1]Tsoy (2017) studies the alternating-offer bilateral bargaining model with private correlated values, and suggests that the application of the Nash bargaining solution may be less compelling in environments with scarce public information, such as OTC markets.

[2]Tsoy (2016) incorporates endogenous bargaining delays, arising from precise private but coarse public information about the asset quality, into Duffie, Gârleanu, and Pedersen's (2005) dynamic model of OTC markets; and shows that search and bargaining delays have opposite effects on the set of traded assets.

of investors' preferences toward time and risk on asset prices in OTC markets.[3]

In terms of the focus of the analysis and methodological approach, this paper is related to Duffie and Wang (2017). Their work focuses on bilateral bargaining over the terms and prices of contracts in OTC network markets. They develop two solution concepts, an equilibrium refinement for a noncooperative bargaining game and a cooperative solution concept that extends the axioms of Nash bargaining to a network setting; and show that the solutions coincide in a simple three-player network bargaining problem. Also related is the work of Dworczak (2018), which combines mechanism design with information design and introduces a class of optimal cutoff mechanisms that have practical applications to the design of trading protocols and transparency in OTC markets.[4]

This draft is preliminary. In addition to the dynamic extension mentioned above, there are several other questions to be addressed. (See Section 6.) For example, if the implementation of neutral mechanisms in OTC markets protects investors' private information, then what will be the impact of information disclosure on investors' bargaining positions?

2. Baseline Model

I consider a simple trading problem in an OTC market between two agents (investors): a dealer and a trader. The dealer (seller, she) is initially endowed with 1 unit of a given asset. The trader (buyer, he) does not own any asset and has high liquidity. Two agents meet and bargain over the price of the asset at which they may trade with each other. The set of possible bargaining outcomes is $D = \{(q, y) | 0 \leq q \leq 1, y \in \mathbb{R}\}$, where, for each $(q, y) \in D$, q represents the probability that the asset is sold to the trader and y represents the amount of money that the trader pays to the dealer.

[3] Binmore, Rubinstein, and Wolinsky (1986) distinguish bargainers' two motives to reach an agreement—their time preferences (or impatience) and their risk preferences (or fear of breakdown).

[4] See Duffie, Dworczak, and Zhu (2018) for the transparency role of benchmarks in financial OTC markets. Also see Lee and Wang (2018) who show that OTC dealers' ability to price discriminate contributes to the prevalence of OTC trading in standardized and heavily traded assets.

The dealer can be characterized by two-dimensional types. First, the dealer who owns the asset can have either "high" or "low" liquidity. A low-liquidity dealer incurs a positive holding cost of δ when trade fails, i.e., bargaining breakdown; whereas a high-liquidity dealer has no such holding cost in case of a breakdown.[5] Second, the asset that the dealer initially owns can have either "good" or "bad" quality. The asset of good quality is associated with higher (dividend) payoff than the asset of bad quality.[6] The dealer values the asset of good quality at c_g and the asset of bad quality at $c_b < c_g$, which can be thought of as the dealer's costs of supplying the asset to the trader. The trader has values v_g and v_b for acquiring the asset of good quality and bad quality respectively, where $v_b < v_g$.[7]

At the outset, the dealer has private information about these types. The full set of dealer types is $T = \{lb, lg, hb, hg\}$, where the letters "l" and "h" indicate the dealer's liquidity-type, and "g" and "b" indicate the dealer's asset-quality-type. For simplicity, I assume that the dealer's types are independent. The trader believes that the probability of the dealer being of type $t \in T$ is p_t such that $\sum_{t \in T} p_t = 1$. I assume that all types have positive probability, so $p_t > 0$ for all $t \in T$. The trader cannot verify any claims that the dealer might make about her liquidity state or asset quality, and the two agents do not expect to make any further transactions in the future.

For notational convenience, I identify player 1 to be the dealer and player 2 to be the trader. Let u_1 and u_2 denote respectively the dealer's and the trader's utility function from $D \times T$ into $\mathbb{R}$, such that $u_i((q, y), t)$ is the utility payoff which player i would get if $(q, y) \in D$ were chosen and if t were the dealer's type. Let $d^* \equiv (q, y) = (0, 0)$ represent the outcome of bargaining breakdown, where trade fails and the dealer pays nothing. Note that the low-liquidity dealer's utility from d^* is $-\delta$, reflecting her illiquidity cost in case of

[5] Duffie, Gârleanu, and Pedersen (2005, p. 1819) mention other interpretations of the low-liquidity type, such as having high financing costs, hedging reasons to sell, a relative tax disadvantage, or a lower personal use of the asset.

[6] As in Duffie, Gârleanu, and Pedersen (2005), the OTC asset can be taken as a consol that pays the constant rate of of consumption to the asset owner.

[7] See Duffie, Dworczak, and Zhu (2018), Dworczak (2018), Lee and Wang (2018), and Tsoy (2016, 2017) for the treatment of the cost for providing the asset or the value for holding the asset.

a breakdown, whereas the high-liquidity dealer's and the trader's utilities from d^* are zero. For ease of exposition and without loss of generality, I normalize utility payoff scales so that every agent's utility from the breakdown outcome is always zero. Under this formulation, the utility functions are summarized in Table 1.

Table 1: Utility functions of baseline model

t	lb	lg	hb	hg
$u_1((q,y),t)$	$y-(c_b-\delta)q$	$y-(c_g-\delta)q$	$y-c_bq$	$y-c_gq$
$u_2((q,y),t)$	v_bq-y	v_gq-y	v_bq-y	v_gq-y

I assume that $v_b > c_b$ and $v_g > c_g$ so that gains from trade are positive for any realization of dealer types. My focus is on a class of trading problems in which not only the low-liquidity dealers but also some high-liquidity dealers have incentives to sell. A low-liquidity dealer can be anxious to sell in order to meet her liquidity needs regardless of the asset quality; and there are gains from trade in bargaining encounters with low-liquidity dealers as long as $v_b > c_b - \delta$ and $v_g > c_g - \delta$. A high-liquidity dealer can also have a reason to sell because of a low asset value, who would enter a bargain only if $v_b > c_b$. In the event that $v_b \leq c_b$ and $v_g \leq c_g$, high-liquidity dealers do not enter, and the trader can meet only low-liquidity dealers, so it simply becomes a trading problem with adverse selection. If $v_b > c_b(> c_b - \delta)$ but $v_g \leq c_g - \delta(< c_g)$, then the dealer's private information about her asset quality becomes inconsequential. Hence, essential for my analysis are the assumptions that $v_b > c_b$ and $v_g > c_g - \delta$, ensuring entry by the dealer of types lb, lg, and hb in equilibrium, but I require further that $v_g > c_g$ only for simplicity of exposition.

Let Γ denote the bargaining problem of interest, as described above. This class of problems is motivated by the intuition that the trader (buyer) may be uncertain about *why the dealer wants to sell.* If both agents have a known value of the asset, then the dealer's private information only about her liquidity state will not play a role because only the dealer with low liquidity would want to sell. As noted by Duffie, Gârleanu, and Pedersen (2005), it would be common knowledge that a gain from trade arises only between a low-liquidity dealer and a high-liquidity trader. While there is publicly observed information about asset classes (e.g., credit ratings, benchmarks, past quotes), the asset quality reflects various factors that affect asset payoffs but that are not captured by public information. So if the dealer has private information about the asset's value for the trader or about the cost of providing the asset, then whether she has high or low liquidity would now matter. The dealer may wish to sell because of a liquidity need, or because of a low asset payoff. With asymmetric information on both asset quality *and* liquidity state, the dealer then have incentives to hide her true intention for selling the asset in order to execute trade and receive a better price.[8]

Some basic features of the model described here are borrowed from various models of Duffie, Dworczak, and Zhu (2018), Duffie, Gârleanu, and Pedersen (2005, 2007), and Tsoy (2016, 2017). I do not consider the effects of search frictions, agents' impatience for liquidity, or information disclosure about agents' costs for supplying the asset. In my model, the incentive to trade is provided by the dealer's attitudes toward risk of breakdown, which reflects the dealer's reason for selling, rather than due to the time preferences or impatience of the agents. This paper thus focuses on (1) how asset prices are influenced by asymmetric information on the dealer's preferences toward the outcome of breakdown (or "outside option"), defined by the dealer's private types on asset quality and liquidity state; (2) how asset prices are set depending on agents' bargaining powers; and (3) the associated welfare implications of bargaining over the price.

3. Bargaining Mechanism

In this section, I characterize "reasonable" transaction prices negotiated directly between two agents. The transaction price is determined through a bargaining process between agents, where the agents' utility payoffs from a success or breakdown of the bargaining process depend on the dealer's type. If I build the bargaining process explicitly into a

[8]For example, both *hb*-type and *lb*-type dealers may pretend to be *lg*-type in order to sell.

non-cooperative bargaining game, then the equilibrium correspondence of this game can be characterized. However, the results of this analysis might depend very strongly on the precise form of the game and might be driven by its protocols of bargaining encounter. Hence, I abstract away from procedural details, and instead delineate the properties of the bargaining outcomes in terms of prices using the cooperative solution concepts under the mechanism design approach. The cooperative solution concepts without a precisely defined structure for the actual bargaining game can ably make predictions of reasonable transaction prices.

My goal is to identify a set of optimal mechanisms that give predictions of prices for which agents should reasonably bargain. In particular, I use two criteria to refine reasonable bargaining solutions of prices: interim incentive efficiency and neutral optimum. First, interim incentive efficiency is clearly a minimal requirement and the appropriate concept of efficiency for problems with incomplete information. But the set of efficient prices may be quite large; and when the feasible term or price of trade that is best for an agent depends on what her type is, the problem of information leakage may arise during the bargaining process. So a reasonably negotiated price should be a non-revealing kind of solution by itself, at the same time reflecting equitable compromise between the different goals of different possible types of each agent as well as between conflicting incentives of two agents. Hence, to get a more useful theory, I investigate the concept of neutral optimum, which captures the idea of inscrutable and equitable intertype compromise. Before proceeding with the analysis, I begin by formally defining a bargaining mechanism and an implementable price for the simple trading problem given in Section 2.

3.1. Implementable Prices

By the revelation principle, I can set up the trading problem (bargaining game) as a direct-revelation mechanism, without loss of generality. That is, the agents in my problem do not have to agree on a specific price; instead they may agree some bargaining mechanism for determining the allocations and payments. After the dealer is informed of her type, each agent chooses whether to participate in the mechanism or not. If both agents agree to participate, the dealer confidentially sends a report $t \in T$ to the mechanism, which then determines whether the asset is traded and how much the trader must pay.

A pair $(Q(\cdot), Y(\cdot))$ represents a *bargaining mechanism* for determining the bargaining outcome as a function of the dealer's type, where $Q(t)$ is the probability that the asset is transferred from the dealer to the trader (i.e., the probability of trade occurring) and $Y(t)$ is the expected transfer payment from the trader to the dealer if the dealer's reported type is t. This mechanism must satisfy $0 \leq Q(t) \leq 1$ for all $t \in T$. I call $Q(\cdot)$ an allocation rule and $Y(\cdot)$ a payment rule. For each $t \in T$, $P(t) \equiv Y(t)/Q(t)$ represents a transaction price per unit of the asset when the dealer's type is t, under the terms of the mechanism (Q, Y).

The expected utilities for each type of the dealer if mechanism (Q, Y) is implemented are

$$\begin{aligned} U_1(Q, Y|lb) &= Y(lb) - (c_b - \delta)Q(lb), \\ U_1(Q, Y|lg) &= Y(lg) - (c_g - \delta)Q(lg), \\ U_1(Q, Y|hb) &= Y(hb) - c_b Q(hb), \\ U_1(Q, Y|hg) &= Y(hg) - c_g Q(hg), \end{aligned} \tag{1}$$

and the expected utility for the trader in mechanism (Q, Y) is

$$\begin{aligned} U_2(Q, Y) = &p_{lb}(v_b Q(lb) - Y(lb)) + p_{lg}(v_g Q(lg) - Y(lg)) \\ &+ p_{hb}(v_b Q(hb) - Y(hb)) + p_{hg}(v_g Q(hg) - Y(hg)). \end{aligned} \tag{2}$$

In this trading problem, the dealer can lie about her type and either agent can refuse to trade. Let $U_1^*(Q, Y, s|t)$ denote the expected utility for the dealer in mechanism (Q, Y) if her type were t but she pretended that her type were s in implementing the mechanism. A mechanism (Q, Y) is *incentive compatible* if and only if it satisfies the following (interim) informational incentive constraints:

$$
\begin{aligned}
U_1(Q,Y|lb) &\geq U_1^*(Q,Y,s|lb) = Y(s) - (c_b - \delta)Q(s),\ \forall s \in \{lg, hb, hg\} &\quad (3)\\
U_1(Q,Y|lg) &\geq U_1^*(Q,Y,s|lg) = Y(s) - (c_g - \delta)Q(s),\ \forall s \in \{lb, hb, hg\} &\quad (4)\\
U_1(Q,Y|hb) &\geq U_1^*(Q,Y,s|hb) = Y(s) - c_b Q(s),\ \forall s \in \{lb, lg, hg\} &\quad (5)\\
U_1(Q,Y|hg) &\geq U_1^*(Q,Y,s|hg) = Y(s) - c_g Q(s),\ \forall s \in \{lb, lg, hb\} &\quad (6)
\end{aligned}
$$

That is, no type of the dealer would be tempted to lie about her type in implementing (Q, Y). A mechanism (Q, Y) is *individually rational* if and only if it satisfies the following (interim) participational incentive constraints:

$$
U_1(Q,Y|t) \geq u_1(d^*, t) = 0,\ \forall t \in T, \quad (7)
$$

$$
U_2(Q,Y) \geq \textstyle\sum_{t\in T} p_t u_2(d^*, t) = 0. \quad (8)
$$

That is, because each agent could guarantee himself a payoff of zero by refusing to trade given the normalization of utility payoff scales, the agents would be willing to participate in mechanism (Q, Y) that is at least as good for them as the breakdown outcome.

A mechanism (Q, Y) is *feasible* if and only if it it is both incentive compatible and individually rational. Again by the revelation principle, there is no loss of generality in focusing on incentive compatible and individually rational mechanisms. I say that a payment rule Y is *implementable* if (Q, Y) is feasible, and I call a quadruplet $\mathbf{P} \equiv \big(P(t)\big)_{t\in T}$ an implementable transaction price under a feasible mechanism (Q, Y).

3.2. Efficient Prices

The agents bargain over the price at the *interim* stage, in which the dealer has private information about her type but the trader does not know the dealer's information. Hence, a bargaining solution correspondence should satisfy the interim efficiency criterion. This property is a reasonable requirement to impose on a bargaining mechanism.

In my setting, a feasible mechanism (Q, Y) is defined to be *interim incentive efficient* (IIE) if and only if there is no other feasible mechanism (Q', Y') such that

$$U_1(Q', Y'|t) \geq U_1(Q, Y|t) \;\; \forall t \in T, \quad U_2(Q', Y') \geq U_2(Q, Y), \tag{9}$$

and at least one inequality is strict. Equivalently, a mechanism (Q, Y) is IIE if and only if there exist some positive numbers (utility weights) $\lambda_1(t)$ for each type t of the dealer and λ_2 for the trader, such that (Q, Y) is an optimal solution to the optimization problem:

$$\max_{(Q,Y)} \Big[\sum_{t \in T} \lambda_1(t) U_1(Q, Y|t) + \lambda_2 U_2(Q, Y) \Big] \tag{10}$$

subject to the constraints (3)–(8).

Let each $\alpha(s|t)$ denote the Lagrange multiplier for each of the dealer's incentive compatibility constraints (3)–(6). Then the Lagrangean function can be written as:

$$\sum_{t \in T} \lambda_1(t) U_1(Q, Y|t) + \lambda_2 U_2(Q, Y) + \sum_{t \in T} \sum_{s \in T} \alpha(s|t) \Big(U_1(Q, Y|t) - U_1^*(Q, Y, s|t) \Big) \tag{11}$$

Let

$$\begin{aligned} v_1((q, y), t, \lambda, \alpha) =& \Big(\big(\lambda_1(t) + \sum_{s \in T} \alpha(s|t) \big) u_1((q, y), t) - \sum_{s \in T} \alpha(t|s) u_1((q, y), s) \Big) / p_t, \\ v_2((q, y), t, \lambda, \alpha) =& \lambda_2 u_2((q, y), t). \end{aligned} \tag{12}$$

Then the function (11) can be simplified to

$$\sum_{t \in T} p_t \int_{(q,y) \in D} \Big[\sum_{i \in \{1,2\}} v_i((q, y), t, \lambda, \alpha) \Big] d\mu(q, y|t), \tag{13}$$

where $\mu : T \to \Delta(D)$ satisfies, for every $t \in T$, $Q(t) = \int_{(q,y) \in D} q d\mu(q, y|t)$ and $Y(t) =$

$\int_{(q,y)\in D} y d\mu(q,y|t)$, so that

$$\int_{(q,y)\in D} \Big[\sum_{i\in\{1,2\}} v_i((q,y),t,\lambda,\alpha) \Big] d\mu(q,y|t) = \sum_{i\in\{1,2\}} v_i((Q(t),Y(t)),t,\lambda,\alpha).$$

Such random mechanism μ would be essentially equivalent to the deterministic mechanism $(Q,Y) : T \to D$. I use the notation of μ only for the purpose of obtaining the simplified expression for the Lagrangean function; Because D is convex and utility is linear, I can restrict attention to deterministic mechanisms without loss of generality, so it suffices to consider only (Q,Y) when characterizing optimal mechanisms.[9]

With the above setup, the duality theorem of linear programming implies the following result.

Result 1 (Characterization of IIE Mechanisms). *A feasible mechanism (Q,Y) is IIE if and only if there exist vectors $\lambda = \big((\lambda_1(t))_{t\in T}, \lambda_2\big)$ and $\alpha = \big(\alpha(s|t)\big)_{t\in T, s\in T}$ such that*

$$\lambda_1(t) > 0, \quad \lambda_2 > 0, \quad \alpha(s|t) \geq 0, \quad \forall s \in T, \ \forall t \in T, \tag{14}$$

$$\alpha(s|t)\big(U_1(Q,Y|t) - U_1^*(Q,Y,s|t)\big) = 0, \quad \forall s \in T, \ \forall t \in T, \tag{15}$$

and

$$\sum_{i\in\{1,2\}} v_i((Q(t),Y(t)),t,\lambda,\alpha) = \max_{(q,y)\in D} \sum_{i\in\{1,2\}} v_i((q,y),t,\lambda,\alpha), \quad \forall t \in T. \tag{16}$$

Definition 1 (Efficient prices)**.** $\mathbf{P} = \big(P(t)\big)_{t\in T}$ is an *efficient price* for the agents in a bargaining problem Γ if a mechanism (Q,Y) is IIE.

Example 1. I select parameters for a numerical illustration of the reasonable bargained prices for a financial OTC market with asymmetric information about the dealer's type. Table 2 contains the exogenous parameters for the baseline model.

[9] The agent's expected utility under the random mechanism μ that is equivalent to (Q,Y) could be expressed in terms of (Q,Y). See Myerson (1991, p. 272).

Table 2: Parameters for baseline model

c_b	c_g	δ	v_b	v_g	p_{lb}	p_{hb}	p_{lg}	p_{hg}
20	40	10	30	50	0.4	0.3	0.2	0.1

In this example, the asset is always worth more to the trader than to the dealer, but no feasible mechanism can be ex post efficient, i.e., $Q(t) = 1$ for all $t \in T$.[10] The dealer's informational incentive constraints imply $Q(lb) \geq Q(hb) \geq Q(lg) \geq Q(hg)$. Further, if (Q, Y) is any feasible mechanism for this example, then

$$\begin{aligned} &0.1U_1(Q,Y|lb) + 0.2U_1(Q,Y|hb) + 0.5U_1(Q,Y|lg) + 0.2U_1(Q,Y|hg) + U_2(Q,Y) \\ &\quad = 8Q(lb) + 0.3\big[\,(Y(hb) - 10Q(hb)) - (Y(lb) - 10Q(lb))\,\big] \\ &\qquad\quad + 0.4\big[\,(Y(lg) - 20Q(lg)) - (Y(hb) - 20Q(hb))\,\big] \\ &\qquad\quad + 0.1\big[\,(Y(hg) - 30Q(hg)) - (Y(lg) - 30Q(lg))\,\big] \leq 8, \end{aligned}$$

where the inequality follows from the dealer's informational incentive constraints and $Q(lb) \leq 1$. Thus, a feasible mechanism (Q, Y) is IIE if

$$0.1U_1(Q,Y|lb) + 0.2U_1(Q,Y|hb) + 0.5U_1(Q,Y|lg) + 0.2U_1(Q,Y|hg) + U_2(Q,Y) = 8. \quad (17)$$

Table 3 contains one specific example of an IIE mechanism and the associated type-contingent efficient price per unit.

Table 3: An example of IIE mechanism and efficient asset price per unit

t	lb	hb	lg	hg
$Q(t)$	1	2/3	1/6	1/9
$Y(t)$	20	50/3	20/3	5
$P(t)$	20	25	40	45

[10] Because there is a positive probability that trade breakdowns under any feasible mechanism, we may allow for the dealer and trader to keep bargaining until the asset is traded. However, if there is some cost of waiting and investors use a common discount factor to measure such costs, then the set of expected utility allocations that are achievable by feasible mechanisms cannot be enlarged by allowing delay of trade. See Myerson (1991, p. 271).

This mechanism, which I call (Q^1, Y^1), stipulates that the price of the asset be the average of the value to the trader and the normalized value to the dealer (taking into account her illiquidity cost) in each case of dealer's type; that is, $P(t) = \big(v_k + (c_k - \delta \mathbf{1}_{\{t \in \{lb, lg\}\}})\big)/2$ for each $t \in \{lb, hb, lg, hg\}$, where $k \in \{b, g\}$ identifies the second letter in each t and $\mathbf{1}$ is the indicator of low-liquidity type.

It can easily be shown that (Q^1, Y^1) is a feasible mechanism, satisfying all of the incentive constraints (3)–(8), where the binding incentive constraints are

$$\begin{aligned} U_1(Q, Y|lb) &= U_1^*(Q, Y, hb|lb), \\ U_1(Q, Y|hb) &= U_1^*(Q, Y, lg|hb), \\ U_1(Q, Y|lg) &= U_1^*(Q, Y, hg|lg). \end{aligned} \tag{18}$$

That is, the dealer with type lb is indifferent between honestly reporting her type and lying that her type is hb; the dealer with type hb is indifferent between honestly reporting her type and lying that her type is lg; and the dealer with type lg is indifferent between honestly reporting her type and lying that her type is hg.

The type-contingent expected utilities for mechanism (Q^1, Y^1) are

$$U_1(Q^1, Y^1|lb) = 10,\ U_1(Q^1, Y^1|hb) = 10/3,\ U_1(Q^1, Y^1|lg) = 5/3,\ U_1(Q^1, Y^1|hg) = 5/9,$$
$$U_2(Q^1, Y^1) = 97/18.$$

Note that these utility payoffs satisfy (17), so mechanism (Q^1, Y^1) is IIE. It can also be verified that the conditions in Result 1 for (Q^1, Y^1) are satisfied by the parameters

$$\begin{aligned} &\lambda_1(lb) = 0.1,\ \lambda_1(hb) = 0.2,\ \lambda_1(lg) = 0.5,\ \lambda_1(hg) = 0.2,\ \lambda_2 = 1, \\ &\alpha(hb|lb) = 0.3,\ \alpha(lg|hb) = 0.4,\ \alpha(hg|lg) = 0.1, \end{aligned}$$

and all other $\alpha(s|t)$ equal to zero. Note that in this example, there are many other IIE mechanisms and efficient prices at which investors can reasonably trade with each other. ■

3.3. Neutral Prices

The goal of this paper is to develop a formal definition of the reasonable price that should be negotiated between investors in financial OTC markets with asymmetric information.[11] However, the concept of interim efficiency may identify too large a set of bargained prices; and there is no reason a priori to expect any particular one among the many efficient prices will be agreed upon. Then an important theoretical question is whether I can determine a smaller set of solutions so as to get stronger predictions of bargained prices.

I argue that the cooperative solution concept of *neutral optimum* is a natural and theoretically appealing way to refine the solution set for any problem of bargaining over prices of the form Γ. In the context of conflict settings in which two privately-informed parties choose a mediator, Kim (2017) elaborates the analytical power of the neutral optimum as a solution concept for bargaining problems with incomplete information. The application of such concept is also pertinent to this paper's setting and suitable for giving a stronger refinement on the set of IIE mechanisms. In fact, the concept of neutral optimum selects a unique mechanism among all IIE mechanisms and hence a unique price $\mathbf{P}$.

Consider a mechanism-selection game in which agents can agree on some mechanism instead of a specific price. If the feasible mechanism that is best for each agent depends on what her type is, then an agent cannot select the one that is best for her, no matter what her type might be, unless the other agent believes that all possible types would have "inscrutably" selected the same mechanism without sharing any information during the bargaining process (see Myerson (1983) for the *inscrutability principle*). If an agent selects a mechanism in a scrutable way, then it might be giving information to the other side about her type that could be unfavorable to her bargaining position. So such mechanism should be excluded from the set of non-revealing bargaining solutions. In contrast, some other mechanisms shall be excluded because an agent might actually want to reveal information about her type

[11] For this preliminary draft, I focus on a static model.

instead of letting those mechanisms be implemented. Thus, due to the conflicting incentives of different possible types, as well as of two agents, each agent must make some sort of compromise between what she really wants and what she might have wanted if her type had been different.

The idea of this inscrutable intertype compromise is captured by the concept of neutral optimum (Myerson, 1983, 1984). The neutral optimum is an axiomatic bargaining solution; I omit detailed expositions of the axioms. In short, a neutral optimum is an equitable and efficient mechanism in terms of some virtual utilities for the two agents. Here I present the following characterization theorem, which is suitably modified from Myerson (1984) to fit into the class of problems in this paper, and stated without proof.

Result 2 (Characterization of Neutral Mechanisms). *A mechanism (Q, Y) is neutral if and only if, for each positive number ε, there exist vectors $\lambda = \big((\lambda_1(t))_{t\in T}, \lambda_2\big)$, $\alpha = \big(\alpha(s|t)\big)_{t\in T, s\in T}$, and $\omega = \big((\omega_1(t))_{t\in T}, \omega_2\big)$ (which may depend on ε) such that*

$$\lambda_1(t) > 0, \ \ \lambda_2 > 0, \ \ \alpha(s|t) \geq 0, \ \forall s \in T, \ \ \forall t \in T, \tag{19}$$

$$\begin{aligned} &\Big(\big(\lambda_1(t) + \sum_{s\in T} \alpha(s|t)\big)\omega_1(t) - \sum_{s\in T} \alpha(t|s)\omega_1(s)\Big)/p_t \\ &\qquad = \max_{(q,y)\in D} \sum_{i\in\{1,2\}} v_i((q, y), t, \lambda, \alpha)/2, \ \ \forall t \in T, \end{aligned} \tag{20}$$

$$\lambda_2\omega_2 = \sum_{t\in T} p_t \max_{(q,y)\in D} \sum_{i\in\{1,2\}} v_i((q, y), t, \lambda, \alpha)/2, \tag{21}$$

$$U_1(Q, Y|t) \geq \omega_1(t) - \varepsilon, \ \ \forall t \in T, \quad U_2(Q, Y) \geq \omega_2 - \varepsilon \tag{22}$$

and (λ, α) satisfies the IIE conditions in Result 1.

I briefly explain the above conditions for computing neutral mechanisms. Consider a fictitious game in which the dealer's types are verifiable and virtual-utility payoffs are transferable among the agents, where a virtual-utility payoff $v_i((\cdot,\cdot),\cdot,\lambda,\alpha)$ is defined by taking into account the shadow price of the incentive constraints. In this virtual problem, a mechanism is efficient and equitable if it maximizes the sum of the agents' transferable virtual-utility payoffs and allocates the total transferable payoff equally among the agents, in every state of dealer's types. Conditions (20) and (21) define these criteria of efficiency and equity, and condition (22) requires that a mechanism gives real expected utilities that are at least as large as the virtually equitable utility allocations $\big((\omega_1(t))_{t\in T}, \omega_2\big)$. The conditions suggest that the agents implicitly make interpersonal-equity comparisons in terms of virtual utilities.

Definition 2 (Neutral prices)**.** $\mathbf{P} = \big(P(t)\big)_{t\in T}$ is a *neutral price* for the agents in a bargaining problem Γ if a mechanism (Q, Y) is neutral.

Example 2 (Example 1 continued)**.** Table 4 shows a unique neutral mechanism and the associated neutral price per unit for the example with parameters given in Table 2.

Table 4: Neutral mechanism and neutral asset price per unit

t	lb	hb	lg	hg
$Q(t)$	1	1/2	1/6	1/6
$Y(t)$	20	15	25/3	25/3
$P(t)$	20	30	50	50

This mechanism, which I call (Q^2, Y^2), gives

$$U_1(Q^2, Y^2|lb) = 10,\ U_1(Q^2, Y^2|hb) = 5,\ U_1(Q^2, Y^2|lg) = 10/3,\ U_1(Q^2, Y^2|hg) = 5/3,$$
$$U_2(Q^2, Y^2) = 4.$$

and satisfies equation (17), so it is IIE. In addition to the three binding incentive constraints in (18), (Q^2, Y^2) has two additional binding incentive constraints:

$$U_1(Q, Y|hb) = U_1^*(Q, Y, hg|hb),\ \ U_1(Q, Y|hg) = U_1^*(Q, Y, lg|hg).$$

The conditions in Result 2 can be verified for all ε by the same λ and α that I used in

Example 1.[12] With these parameters, conditions (20) and (21) have the unique solution:

$$\omega_1(lb) = 10,\ \omega_1(hb) = 5,\ \omega_1(lg) = 10/3,\ \omega_1(hg) = 5/3,\ \omega_2 = 4,$$

which satisfy condition (22) for every positive ε. ■

3.4. Discussions

I discuss the implications of the results by providing several interesting comparisons between mechanisms (Q^1, Y^1) and (Q^2, Y^2).

First, comparing the interim expected utilities, we have $U_2(Q^1, Y^1) > U_2(Q^2, Y^2)$ but $U_1(Q^1, Y^1|t) < U_1(Q^2, Y^2|t)$ for all $t \in \{hb, lg, hg\}$, while $U_1(Q^1, Y^1|lb) = U_1(Q^2, Y^2|lb)$. Hence, neither of the mechanisms (Q^1, Y^1) and (Q^2, Y^2) is interim Pareto superior to the other; in fact, both mechanisms are IIE and the associated prices are efficient for investors.

Second, in the example, a dealer of type lb has a virtual value of 10 for a unit of the asset, which is the same as her actual (normalized) value of $c_b - \delta = 10$; whereas a dealer of type hb has a virtual value of 30 instead of an actual value of $c_b = 20$, and dealers of types lg and hg both have a virtual value of 50, instead of actual values of $c_g - \delta = 30$ and $c_g = 40$ respectively. Under the IIE mechanism (Q^1, Y^1), the prices at which investors trade are exactly halfway between the actual values of the asset to the dealer and the trader. Hence, (Q^1, Y^1) can be considered both equitable and efficient in real utility terms. However it is not equitable in virtual utility terms, because dealers of types hb, lg, and hg are selling the asset for prices that are less than their virtual values under the terms of mechanism (Q^1, Y^1). In contrast, the neutral mechanism (Q^2, Y^2) stipulates that the price of the asset be the same as in (Q^1, Y^1) when the dealer's type is lb; whereas it stipulates that the asset be traded at the transaction price per unit that is exactly the virtual value of the asset to

[12] I can use $\alpha(hg|hb) = \alpha(lg|hg) = 0$ although the associated incentive constraints are binding. The complementary slackness condition does not dictate that if the Lagrange multiplier is zero, then the constraint is not binding; although this would be a geometrically special case.

the dealer when the dealer's type is in $\{hb, lg, hg\}$.

Third, and related to the second point, mechanism (Q^1, Y^1) gives positive expected gains from trade to the trader for any realization of dealer's types. However, under mechanism (Q^2, Y^2) the trader expects no gains from trade if $t \in \{hb, lg, hg\}$ and can only gain from trading with a dealer of type lb. Lastly, the ex ante probability of trade occurring, computed by $\sum_t p_t Q(t)$, is lower in (Q^2, Y^2) than in (Q^1, Y^1).

Beyond what these comparisons entail for the problem of bargaining over prices between investors, the concept of the neutral optimum gives a unique prediction of the price that should reasonably arise as an outcome of the interim bargaining process. In the simple example, the price specified under mechanism (Q^1, Y^1) may also be considered reasonable but only reasonable in the sense of interim efficiency. When prices are negotiated directly between investors in financial OTC markets and when a dealer can conceal information about her true reason for selling the asset from a potential trading partner, the two investors should reasonably bargain for the price that respects not only interim efficiency but also inscrutable and fair intertype compromise. This is because the dealer acts strategically in the bargaining process due to the possibility of information leakages, implicitly contemplating a compromise among the interim preferences of different possible types.

When the investors inscrutably agree on the neutral price, they are not going to do as well as under all other efficient prices in terms of the ex ante probability of trade occurring. That is, the neutral mechanism and the associated neutral price are not effective in maximizing the chance of trade. But such ex ante feature of the solution is irrelevant for bargaining situations with incomplete information. When bargaining takes place at the interim stage, interim incentive efficiency is the appropriate welfare criterion to evaluate the bargaining outcome, and the neutral price is indeed interim incentive efficient. The illustration in this paper suggests caution in using the Nash bargaining solution to compute the prices or in using ex ante measures to evaluate the bargaining outcome when prices might have emerged as a result of endogenous interim bargaining.

4. Bargaining Power and Prices

In this section, I discuss how negotiated asset prices under neutral mechanisms depend on agents' bargaining powers.

A vector of neutral prices in the preceding analysis is the bargaining solution that emerges when the dealer and the trader have equal negotiating ability. In my setting, this solution coincides with the bargaining solution of neutral prices that emerges when the dealer has all of the bargaining power in the context of the informed principal mechanism-selection problem (Myerson, 1983). Alternatively, the dealer with full bargaining power can be thought of as the dealer making a take-it-or-leave-it price offer to the trader.

The informed principal problem can be modeled as the following noncooperative bargaining game. After the dealer learns her own type, she selects and announces a bargaining mechanism; then the trader makes some inferences about the dealer's type, based on the announcement; and finally the mechanism is implemented, with each agent using some participation strategy that is rational given his or her information. The participation strategy profile is such that each agent chooses whether to participate in the mechanism and if both agents participate, the dealer reports her type to the mechanism, which then determines whether the asset is traded and how much the trader should pay. In this game, there is a sequential equilibrium that supports the neutral price.

On the other hand, we may assign all bargaining power to the trader making an offer. For now, I have not formally proved a result, but I conjecture that this will not change the conclusion. The rest of this section as well as subsequent sections are in progress.

5. Welfare Analysis

- Neutral mechanisms, hence associated prices, are interim Pareto optimal among all feasible mechanisms and implementable prices.

- Task: I compute the price offer that minimize the ex ante probability of trade breakdown and the price offer that maximize the ex ante efficient gains of trade. Conjectures: These two offers coincide. I compare them to neutral prices, in terms of welfare.

- Task: I discuss in detail why neutral prices are reasonable and desirable. The dealer would always want to preserve some private information (under certain conditions on the distribution of the value of the outside option) so that the dealer-optimal mechanism would not disclose any information, in order to obscure her true reason for selling and get a better deal. So the neutral price might seem "undesirable" from the ex ante point of view; however, it is interim Pareto optimal, which is the correct welfare criterion that should be used in games with incomplete information.

6. Further Tasks, Extensions

- An extension to a dynamic model: Multiple dealers and traders, randomly matched in repeated encounters. A dynamic process of dealers' types. Bargaining encounter every period, bargaining for neutral prices. Add search frictions.

- Publication of benchmarks, information disclosure problem, and policy implications: If the implementation of neutral mechanisms in OTC markets protects investors' private information, then what will be the impact of public information disclosure on investors' bargaining positions?

- Discussion on optimal trading platforms: Dealers participate in a mechanism (trading platform) which allocates a single asset. The platform is characterized as a seller of the asset.

- Discussion on a resale mechanism in the OTC market: A tractable reduced-form framework capturing a situation in which a dealer acts as an intermediary whose value for the object depends only on the resale opportunity.

7. Conclusion

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