Endogenous Managerial Capital and Financial Frictions

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Abstract

Aggregate total factor productivity TFP differences across countries have been widely recognized as the major source of huge differences in income per capita across countries. The misallocation literature has found that distortions that inefficiently allocate resources across production units can result in a significant aggregate productivity loss even without deterioration in the underlying productivity distribution. However, with endogenous managerial capital investment decision, distortions affect underlying productivity distribution as well as reallocate resources across production units. In this paper, I examine the effect of credit constraint in a model with endogenous managerial capital investment decision. If agents can optimally invest in their managerial capital, limited access to physical capital will encourage managers to substitute away from physical capital to investment in managerial capital. The accumulation of managerial capital and the change in the underlying productivity distribution will mitigate the adverse effect of misallocation caused by the credit constraint on the economy. In fact, measured TFP could improve with a tighter credit constraint.

Keywords: Misallocation, Endogenous managerial capital, TFP, Financial frictions, Credit constraint

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1 Introduction

Aggregate TFP differences across countries have been widely recognized as the major culprit for huge differences in income per capita across countries.[1] The misallocation literature (Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)) has shown that distortions that inefficiently allocate resources across production units can result in a significant aggregate productivity loss even without deterioration in the underlying productivity distribution. A key challenge is to identify the quantitatively important sources of this misallocation. This will allow us to come up with policy advice that is effective and powerful in promoting income growth of the economies that are suffering from below par aggregate efficiency.

One of the popular sources of distortion that the misallocation literature has focused on is financial frictions. [2] In the existing literature, credit constraints distort the allocation of physical capital across heterogeneous production units holding the underlying productivity distribution fixed. This leads to dispersion of marginal productivity of physical capital across production units and thus worsens aggregate productivity. In this paper, I examine the effects of credit constraints in a model that features investment in managerial skills as in Bhattacharya et al. (2013). I find that with optimal managerial capital investment decisions, the adverse effect of credit constraint on an economy is substantially mitigated. In fact, with managerial capital investment decisions, measured TFP could improve with a tighter credit constraint.

Key to this result is that in my model financial frictions affect both the underlying productivity distribution of production units as well as the resource allocation among those production units. Over the life cycle, managers can optimally invest in managerial capital, which affects the underlying productivity distribution.[3] With the endogenous managerial

[1] This is a standard result in the development accounting literature. See, for example, Hall and Jones (1999), Caselli (2005), and Hsieh and Klenow (2009).

[2] The most relevant works are, Buera, Kaboski, and Shin (2011), Midrigan and Xu (2014), Moll (2014).

[3] In my paper, the production unit is a manager combined with some workers. Thus, hereafter, I will refer to a production unit as a manager and firm-level productivity as managerial capital. I call it managerial capital because it can be utilized only if an agent becomes a manager.

capital investment decision, if tighter credit constraint restricts the access to physical capital, managers will substitute away from physical capital to investment in managerial capital. The accumulation of managerial capital and the change in the underlying productivity distribution will dampen the adverse effect of the credit constraint on the economy.

To study quantitative and qualitative implication of credit constraints in a model with endogenous managerial capital investment decisions, I use the Lucas span of control model with optimal managerial capital investment decisions as in Bhattacharya, Guner, Ventura (2011) and impose collateral constraint in the form of Buera, Kaboski and Shin (2011). I calibrate the parameters of the model assuming that the U.S is a distortion-free, perfect credit benchmark. The calibration successfully matches the U.S firm size distribution statistics and physical capital output ratio.

In my main exercise I hold the calibrated parameters fixed, and vary a single parameter ϕ that governs the tightness of the credit constraint to examine the effect of the constraint on an economy. To isolate the role of endogenous managerial capital investment decisions in the model, I compare the results from this exercise with those from 'exogenous' set up without a managerial capital investment decision. In the exogenous set up, I assume that agents are forced to invest in their managerial capital the same amount as in the perfect credit benchmark case. Through the comparison I highlight the extent to which the model with endogenous managerial capital decision differs from the model without it and the underlying mechanism behind the difference.

My key finding is that, endogenous managerial capital decisions dampen the adverse effect of credit constraints on the economy. In fact, TFP rises with tighter credit constraints. In particular, TFP increases by 2.3 percent with a credit constraint that lowers the external finance to GDP ratio by 19 percent. Unlike TFP, output falls, but it falls less than a model without optimal managerial capital investment decisions. This is because the adverse effect of tighter credit constraints through misallocation of physical capital is offset by accumulation of managerial capital. Tighter credit constraint depresses physical capital demand for managers and lowers factor prices. The lower cost of production leads to higher profits of managers and stronger incentive to invest in managerial capital for future profits. As TFP captures both the allocative efficiency among production units and the total amount of managerial capital present in the economy, it improves with tighter constraints.

Another notable feature of my model is that tightness of the credit constraint and firm size dispersion show a non-monotonic relation. Tighter credit constraints and active accumulation of managerial capital by managers lead to a larger mass of more productive managers. However, tighter credit constraint will limit the ability of those managers to increase the size of the firm and the actual firm size could be bigger or smaller. As a result, the firm size dispersion is non-monotonic to tighter credit constraints.

In my benchmark analysis I assume that managerial capital is non-stochastic. I aso consider a case in which the skill accumulation function has a stochastic component. In this case, I find that an increase in uncertainty coming from the stochastic component discourages managers from accumulating managerial capital and can wipe away the dampening effect of endogenous managerial capital decisions mentioned above if the uncertainty is sufficiently large.

My paper is related to several literatures. Restuccia and Rogerson (2008) show that idiosyncratic policy distortions could lead to a substantial decrease in aggregate production. Using Chinese and Indian manufacturing firm data, Hsieh and Klenow (2009) show that reallocating resources within those countries to equalize marginal products to the same extent as in the U.S, would result in TFP gains of at least 30% to 40% in those countries. In these models, distortions do not affect the underlying productivity distributions. Bhattacharya et al. (2011) assume that the distortions do not only affect the allocation of resources across production units but also the underlying productivity distribution through investment in managerial capital. They show that if distortions are correlated with the size of production units, endogenous managerial capital investment decisions amplify the distortive effects. However, they dont examine financial frictions as source of miallocation. Literature is unclear about whether or not losses from misallocation generated by financial frictions are big. Using two-sector model, Buera, Kaboski and Shin (2011) shows that financial friction alone can bring down aggregate TFP by 36% and can account for a substantial part of TFP and income differences across countries. Midrigan and Xu (2014), using Korean plant level data, present that financial friction does not generate much losses in TFP from misallocation. They show that losses come from low levels of entry and technology adoption when there is credit constraint. Moll (2014) shows that if productivity shock is persistent, steady state TFP losses from credit constraint is small as agents save out of their credit constraints. In my paper, losses from financial frictions are further mitigated by accumulation of managerial capital.

My paper is closest to Fattal-Jaef (2015). In his paper he shows that the output gains from relaxing miallocation friction is reduced because firm entry and exit decision offsets it. The number of firms in his model is comparable to the amount of aggregate managerial capital accumulated in my model.

My paper is also related to the literature identifying importance of management practice for the productivity of a firm. Bloom and Van Reenen (2007) show that management practices display significant cross-country differences and are strongly associated with firm-level productivity. Caselli and Gennaioli (2012) show that aggregate TFP might be negatively affected by dynastic management with which less developed countries are more comfortable. They find that poor management correlated to dynastic management could account for a large part of TFP losses in those countries.

An outline of this paper is as follows. In section 2, I present a benchmark model with endogenous managerial capital investment decisions and collateral constraints. In section 3, I show steady state equilibrium of the benchmark. In section 4, I calibrate the model. In section 5, I present the main results. In section 6, I add stochastic component in the model. In section 7, I conclude.

2 Benchmark Model

In this section, I describe the benchmark model, which is taken from Bhattarcharya et al. (2013). It is life-cycle version of the Lucas span of control model. Each period, an overlapping generation of heterogenous agents are born an and live for J periods. They work for the first J_R periods, retire and live on their savings for the rest of their life. In the benchmark model, there are no financial frictions. Agents can borrow and save freely at the market interest rate. We assume that each cohort is $1+n$ bigger than the previous cohort. The population structure is stationary in the sense that the age j cohort is a fraction μ_j of whole population at any time t, with

$$\mu_{j+1} = \mu_j/(1+n) \qquad \forall j, \qquad \sum_{j=1}^{J} \mu_j = 1 \tag{1}$$

The objective of each agent is to maximize lifetime utility from consumption of the following form.

$$\sum_{j=1}^{J} \beta^{j-1} log(c_j) \tag{2}$$

When agents are born they are endowed with managerial capital z which is drawn from an exogenous log normal distribution with mean μ_z and variance σ_z^2. Until retirement, each agent is endowed with 1 unit of time which they spend inelastically as a manager or a worker. At the beginning of each period, given their capital level z and asset level a, agents decide whether to become a worker or a manager. Agents are born with zero assets. Each period on agent decides whether to become a worker or a manager, how much to save and consume and how much to invest in their managerial ability if they become a manager. Only managers can invest in managerial capital. Labor and capital markets are competitive.

A worker supplies labor inelastically throughout the whole working period and earns the market wage. A worker chooses how much to save and consume each period to maximize his utility. If an individual becomes a manager, he has to also choose how much capital or labor to employ to produce output, and how much to invest in improving managerial skills. All individuals are equally productive as workers.

2.1 Technology

Each manager has access to a span-of-control technology of production. A plant with managerial ability z will produce output with labor and capital with the following production function.

$$y = z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} \tag{3}$$

where γ is span of control parameter and α is the share of capital. Managers can enhance their future ability by investing their income into managerial capital accumulation. Managerial capital is accumulated with the function given below.

$$z' = z + g(z, x) = z + z^{\theta_1}x^{\theta_2} \tag{4}$$

where z' is next period's managerial capital level and x is investment in skill accumulation. The function g is such that current managerial capital level and investment in future managerial capital display complementarities: $g_{zx} > 0$, i. e., the higher the current level of skill, the more beneficial it is for an agent to invest in skill accumulation. Also, it is assumed that g_{xx} is negative so that there is diminishing returns to skill investment.

2.2 Decisions

I focus on a steady state equilibrium with constant factor prices R and w. Let a denote assets that pay the risk-free rate of return $r = R - \delta$, where δ is the depreciation rate for capital. In a steady state equilibrium, agents born with ability over some threshold ability level $\hat{z}$ will become managers and the rest will become workers. Agents with the same ability level will make the same decision regarding their career choice and will end up with exactly the same resource allocation along their life cycle. I next describe the optimization problems for workers and managers.

2.3 Managers

The problem of a manager of age j is given by

$$M_j(z,a) = \max_{x,a'}\{log(c) + \beta V_{j+1}(z',a')\} \tag{5}$$

subject to

$$c + x + a' = \pi(z;r,w) + (1+r)a \quad \forall 1 \leq j < J_R - 1, \tag{6}$$

$$a' \geq 0 \tag{7}$$

and

$$z' = z + g(z,x) \quad \forall j < J_R - 1,$$

with

$$V_{J+1}(z,a) \begin{cases} 0 & \text{if } a \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

where $V_j(z,a)$ is a value function at period j defined as the maximum continuation value of becoming a manager at age j and becoming a worker at age j.

Note that managers can save but not borrow from the future. (a is nonnegative.) This is assumed so that one to one comparison between perfect credit benchmark case and less than perfect capital rental market easier. (In a less than perfect credit market, a cannot be negative.) In the absence of financial frictions when managers can freely rent physical capital at market rental rate R, a manager's optimal demand for inputs depend on their managerial capital only and do not depend on their savings. Managerial income for a manger with ability z is given by

$$\pi(z;r,w) \equiv \max_{n,k}\{z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} - wn - (r+\delta)k\} \tag{8}$$

Taking F.O.Cs, factor demands are given by

$$k(z;r,w) = ((1-\alpha)\gamma)^{\frac{1}{1-\gamma}} \frac{\alpha}{1-\alpha}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}} (\frac{1}{r+\delta})^{\frac{1-\gamma(1-\alpha)}{1-\gamma}} (\frac{1}{w})^{\frac{\gamma(1-\alpha)}{1-\gamma}} z \tag{9}$$

and

$$n(z;r,w) = ((1-\alpha)\gamma)^{\frac{1}{1-\gamma}} (\frac{\alpha}{1-\alpha})^{\frac{\alpha\gamma}{1-\gamma}} (\frac{1}{r+\delta})^{\frac{\alpha\gamma}{1-\gamma}} (\frac{1}{w})^{\frac{1-\alpha\gamma}{1-\gamma}} z \tag{10}$$

Substituting these into the profit function, profits are a linear function of managerial ability, z

$$\pi(z;r,w) = \Omega \quad (\frac{1}{r+\delta})^{\frac{\alpha\gamma}{1-\gamma}} (\frac{1}{w})^{\frac{\gamma(1-\alpha)}{1-\gamma}} z \tag{11}$$

Where Ω is a constant given by

$$\Omega \equiv (1-\alpha)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}} \alpha^{\frac{\gamma\alpha}{(1-\gamma)}} \gamma^{\frac{1}{1-\gamma}} (1-\gamma)$$

The solution to the dynamic programming problem is characterized by two conditions. First, the solution for next period's asset level, a', equation given below:

$$\frac{1}{c_j} \geq \beta(1+r)\frac{1}{c_j+1}$$

Second, investment is determined by the no arbitrage condition below:

$$(1+r) = \pi_z(z_j; r, w) g_x(z_j, x_j) \tag{12}$$

The left hand side is next period's gain in income from one unit of current savings. The right hand side is the gain in income to the j-period old manger from investing one unit of current consumption in managerial capital accumulation. As noted previously g_{xx} is negative. This implies that the marginal benefit of investing in skill accumulation is monotonically decreasing in the level of skill investment while the marginal cost $(1+r)$ is constant. Thus, a unique interior optimum level of x is determined from the equation above.

2.4 Workers

The problem of an age j worker is given by following

$$W_j(a) = \max_{a'} \{log(c) + \beta V_{j+1}(z, a')\} \tag{13}$$

subject to

$$c + a' = w + (1+r)a \quad \forall 1 \leq j < J_R - 1, \tag{14}$$

and

$$c + a' = (1+r)a \quad \forall j < J_R, \tag{15}$$

With

$$W_{J+1}(a) \begin{cases} 0 & \text{if } a \geq 0 \\ -\infty, & \text{otherwise} \end{cases}$$

Like managers, workers cannot borrow.

And finally,

$$V_j(z,a) = \max[W_j(a), M_j(z,a)] \tag{16}$$

2.5 Financial Friction

Now assume that renting of physical capital is limited by imperfect enforceability of contracts a la Buera, Kabokski, and Shin (2009). After production, agents can renege in which case they can keep a fraction $1-\phi$ of undepreciated capital and revenue net of labor payments, but all financial assets deposited in the bank are confiscated. Thus, ϕ is the strength of an economy's legal instiutions for enforcing contracts. Banks will rent capital only if agents will repay thus agents can borrow up to the amount that she is better to abide the contract than to defalut it. And agents gain access to the next period's financial market without any penalty. Agents choose to abide the contract if and only if

$$\max_n \{z^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn - (r+\delta)k\} + (1+r)a \geq \max_n (1-\phi)\{z^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn + (1-\delta)k\} \tag{17}$$

Which simplifies to

$$(1+r)a \geq -\phi[\max_n (z^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn) - \frac{(1-\phi+r+\phi\delta)}{\phi}k] \tag{18}$$

This inequality decides the limit of capital each agent can borrow. The RHS is minimized for some $\hat{k}(z;\phi)$ less than the unconstrained optimal demand of k. One can think of the k that maximizes the value in the bracket in the inequality above as an optimal capital demand under a higher rental rate $\frac{(1-\phi+r+\phi\delta)}{\phi} > r+\delta$. This eliminates the case that capital constraint only allows bigger amount of capital rental than is desired. Let upper bound on capital that is consistent with entrepreneurs to abide the contract is $\bar{k}(a,z;\phi)$.

Then $\bar{k}(a, z; \phi)$ is given by the max of 0 and largest root of the equation

$$(1+r)a = -\phi[\max_{n}(z^{1-\gamma}(\bar{k}^{\alpha}n^{1-\alpha})^{\gamma} - wn) - \frac{(1-\phi+r+\phi\delta)}{\phi}\bar{k}(a, z; \phi)] \tag{19}$$

The capital constraint thus reduces to $k \leq \bar{k}(a, z; \phi)$.

It is obvious that larger the amount of asset held by the entreprenuer (a), higher the current managerial capital z, and larger the ϕ fraction taken away by the contract enforcing intermediaries more the valuable the entreprenuer' collateral is and thus he can borrow more physical capital to put into the production. The proof is the same as in Buera, Kaboksi, and Shin (2009).

The problem of a manager of age j with a capital constraint is then given by

$$M_j(z, a) = \max_{x, a'}\{log(c) + \beta V_{j+1}(z', a')\} \tag{20}$$

subject to

$$\begin{aligned} c + x + a' &= \pi(a, z; r, w) + (1+r)a \quad \forall 1 \leq j < J_R - 1 \\ c + a' &= (1+r)a \quad \text{for} \quad j \geq J_R - 1 \end{aligned} \tag{21}$$

$$\bar{k}(a, z; \phi) \geq k \tag{22}$$

and

$$z' = z + g(z, x) \quad \forall j < J_R - 1,$$

with

$$V_{J+1}(z,a)\begin{cases} 0 & \text{if } a \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

Thus, capital and labor demand with financial constraints are a function of (a, z) instead of z only.

2.6 Occupational Choice

Each agent maximizes their lifetime utility given their ability level z and asset a. When agents are born, they supply their labor as a worker in the first period. This is assumed so that agents could accumulate physical capital for collateral. After that, agents freely choose to become a worker or a manager at the beginning of each period. Let ${z^*}_{(j,a)}$ be the ability level at which a age j agent is indifferent between being a worker and a manager if he has an asset a. This ${z^*}_{(j,a)}$ can be found by the equation below

$$M_j({z^*}_{(j,a)}, a) = W_j(a). \quad \forall a, j$$

$W_j(a)$ is a constant in a steady state equilibrium. M_j is a continous, strictly increasing function of z and a so this equation has a well defined solution $z^*_{(j,a)}$. At each period j, given their asset a, agents with managerial capital higher than $z^*_{(j,a)}$ will choose to become a manager while those under $z^*_{(j,a)}$ will become a worker.

3 Steady State Equilibrium

I focus on a steady state equilibrium in which fixed r and w are constant over time. Managerial capitals are determined endogenously after the first period since each agent optimally invests in their managerial capital level. Therefore, the upper bound for managerial capital is going to be determined endogenously. Let's call this upper bound $\bar{z}$. Then managerial capital takes values in a set $Z = [\underline{z}, \bar{z}]$ Similarly, let $A = [0, \bar{a}]$ denote the possible asset levels. Let $\psi_j(a, z)$ be the mass of age-j agents with assets a and ability level z. Given $\psi_j(a, z)$, let

$$\tilde{f}_j(z) = \int \psi_j(a, z)\,\mathrm{d}a \tag{23}$$

be the skill distribution for age j agents. In a steady state equilibrium, labor, capital and goods market must clear given the prices (r, w). The labor market equilibrium condition is given by.

$$\sum_{j=1}^{J_R-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} n(z, a; r, w)\psi_j(a, z)\,\mathrm{d}z\mathrm{d}a = F(z^*_{(j,a)}, a) \sum_{i=1}^{J_R-1} \mu_j \tag{24}$$

where μ_j is the total mass of cohort j. The left-hand side is the labor demand from the $J_R - 1$ different cohorts of managers. The right-hand side is the fraction of each cohort employed as workers. For each cohort, given a, those under ability level $z^*_{(j,a)}$ choose to become workers and there are mass of μ_j in each cohort. Labor supply comes from non-retired cohorts.

In the capital market, there are two sources of demand for savings. Managers demand capital to produce output. They also demand savings to invest in their managerial capital accumulation. Savings comes both from managers and workers of each cohort except for the oldest cohort since they have no incentive to save. Thus, the capital market equilibrium condition can be written as :

$$\begin{aligned}
&\sum_{j=1}^{J_R-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} k(z, a; r, w)\psi_j(a, z)\,\mathrm{d}z\mathrm{d}a \\
+ \sum_{j=1}^{J_R-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} x_j(z, a)\psi_j(a, z)\,\mathrm{d}z\mathrm{d}a = &\sum_{j=1}^{J-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} a^w_j(a)\psi_j(a, z)\,\mathrm{d}z\mathrm{d}a \\
&+ \sum_{j=1}^{J-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} a^m_j(a)\psi_j(a, z)\,\mathrm{d}z\mathrm{d}a
\end{aligned} \tag{25}$$

The first term of the left-hand side is physical capital demand from the working cohorts of managers. The second term is the sum of investment in managerial capital of working managers up to one period before they retire. For instance, if they retire at age 4, there are 3 investment periods. These two terms comprise the demand for savings. Each of the right-hand side terms is savings of workers and managers before they die.

The goods market equilibrium condition is that the aggregate output produced in the economy is equal to the sum of aggregate consumption plus investment in physical capital and managerial capital investments across cohorts by all managers and workers.

4 Quantitative analysis

In this section, I calibrate the parameters of the model so that the steady state equilibrium of the model matches key features of the U.S economy assuming no credit frictions. I vary the credit constraint parameter ϕ to see the effect of different levels of credit constraints on an economy if agents can optimally invest in their managerial capital over the life cycle. To assess the importance of endogenous managerial capital I also consider a model with exogenous managerial capital. In the exogenous setup, managers dont have an option to optimally invest in their managerial capital. Instead, they are forced to invest as much as they do in the perfect credit benchmark.

4.1 Calibration

Parameter values in the benchmark model are calibrated so that the steady state equilibrium of the model matches features of U.S firm size distribution and aggregate physical capital output ratio. In my calibration I assume that the U.S has a perfect credit market[4] as in Buera, Kaboski and Shin (2011). One period in the model corresponds to 10 years. Each cohort enters the model at age 20 and lives until 80. They work for 40 years and during

[4] $\phi = 1$ corresponds to perfect credit, $\phi = 0$ corresponds to no credit

working periods they supply their labor inelastically. They stay retired for the rest 20 years.

There are 9 parameters to calibrate. The share of physical capital in output is set at 0.317 as from Guner et al. (2008). Since the product of the importance of capital(α) and returns to scale(γ) responds to the share of physical capital in the model α, is determined from γ as $\alpha = 0.317/\gamma$. The depreciation rate (δ) and population growth(n) are set so that their annual rates are 0.06 and 0.011 respectively.

This leaves 6 parameters to calibrate: γ, β, θ_1, θ_2, μ_z, σ_z. I normalize the mean of the log of the skill distribution to zero and calibrate the 5 remaining parameters to match 4 moments of the U.S plant size distribution and the physical capital to output ratio: mean plant size, fraction of plants with less than 10 workers, fraction of plants with 100 or more workers, fraction of the labor force employed in plants with 100 or more employees and the physical capital to output ratio. The calibration successfully replicates the features of the U.S plant size distribution. Fraction of small establishments is large(73%) but substantial part(46%) of employment is at the large establishments. Tables 1 and 2 show the calibrated parameter values and the match to the U.S data with perfect credit. The parameter values obtained from this calibration are used for the benchmark model and the exogenous setups.

Table 1: Calibrated parameter values

Parameter	Value
Population Growth Rate (yearly) (n)	0.011
Depreciation rate (yearly) (δ)	0.06
Importance of Capital (α)	0.417
Returns to Scale (γ)	0.7601
STD of log-managerial Ability (σ_z)	2.2731
Discount Factor (yearly) (β)	0.94
Skill accumulation technology (θ_1)	0.9102
Skill accumulation technology (θ_2)	0.5172

Table 2: Fit of the benchmark model and data with parameter values in table 1

Statistic	Data	Model
Average Firm Size	17.9	17.9
Physical Capital Output ratio	0.23	0.23
Fraction of small (0-9 workers) establishments	0.73	0.74
Fraction of large (100+ workers) establishments	0.026	0.022
Employment Share of Large Establishments	0.46	0.46

5 Results

Having calibrated the model to match the firm size distribution of the U.S and the physical capital output ratio, I now use the calibrated parameter values and vary the parameter ϕ, that governs the strictness of credit constraints. First, I will look at the steady state equilibrium statistics at different values of ϕ and address the effects of credit constraints on an economy when agents can optimally invest in their managerial capital over the life cycle. Results are presented in Table 3. TFP is measured as follwing: $TFP = Y/(K^{\alpha}L^{(1-\alpha)})^{\gamma}$.[5]

Table 3: Financial friction with managerial capital investment decisions: Denoted as percentage of $\phi = 1$ value

Statistic	$\phi = 1$	$\phi = 0.5$	$\phi = 0.4$	$\phi = 0.3$
TFP	100	102.5	102.6	102.7
Y	100	99.3	0.954	962
K/Y	100	0.91	0.83	85
X/Y	100	127	1.36	144
K+X	100	104	87	90
H	100	114.8	117.7	121.5
Mean Firm Size	100	84	73	65
Mean Profit	100	115	118	118
Manager fraction	100	117	134	149
Mean ability	100	98	88	88
MPK variance(level)	0	0.087	0.243	0.350
EF/Y	100	90	79	77

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
MPK	Marginal productivity of physical capital
EF/Y	External finance to GDP ratio

In a model without managerial capital investment decision, tighter credit constraint misallocate resources across production units, lowering aggregate output, aggregate physical capital, and aggregate measured TFP. However, in my model, aggregate TFP increases with a tighter credit constraint. Quantitatively, the TFP measure increases by 2.7% with a credit constraint that reduces external finance to GDP ratio by 23%. With the endogenous

[5]I tried with different TFP measures. The directoin of change is robust to many of those measures. For detail, see section 5.2

managerial capital investment decisions, tighter credit constraints worsen the allocation of physical capital across production units but improve underlying productivity distribution by encouraging managers to substitute away from physical capital to investment in managerial capital.[6] While total amount of physcial capital falls with a tighter credit constraint, total amount of managerical capital increases.[7] The traditional TFP measure improves because the accumulation of managerial capital offsets the adverse effects of credit constraint that is caused by misallocation. Quantitatively, the total amount of investment in managerial capital increases by more than 38 percent while total amount of physical capital decreases by 13.8 percent with the credit constraint that lowers external finance to GDP ratio by 23 percent.[8]

In the model, there are two distinct incentives for a manager to invest in his managerial capital. First, having more managerial capital in the next period will allow him to borrow more physical capital in the next period since collateral is a function of productivity. Second, with the same amount of physical capital, he can have higher profits with higher current productivty and managerial capital investment improves one's current productivity. I call the first incentive the collateral incentive and the latter the profit incentive. The collateral incentive is stronger the higher ϕ is because ϕ governs the fraction of a manager's profit that can be redeemed for collateral. If $\phi = 0$, the collateral incentive is gone and only the productivity incentive is present. Therefore, all else equal, tighter credit constraint will

[6]Managers are able to channel their resources toward managerial capital with tighter constraint because credit constraint does not restrict investment in managerial capital while it limits the borrowing of physical capital directly. Managerial capital investment is also restricted indirectly through inter-temporal borrowing constraint since borrowing is not allowed in my model. Managers cannot borrow to invest in their managerial capital. However, this indirect restriction is not positively correlated with the strictness of the physical credit constraints.

[7]Although total amount of managerial capital increases, average managerial capital falls as less productive managers enter the market with tighter credit constraitnts. average is taken using mass of firms at each managerial capital level as weight. However, large firms with more than 100 workers produce more than 45% of total production. Using the mass of production of firms as weight, average managerial capital rises and then decreases with tighter credit constraints.

[8]However, the investment in managerial capital is much smaller than the amount of physical capital in level. Thus sum of physical capital and managerial capital investment decreases (by 9.9 percent) with tighter credit constraints. Managers are substituting away from physical capital to managerial capital but by less than one-to-one.

discourage managers from investing in managerial capital. However, credit constraint will depress factor prices as it restricts total demand for inputs at a given underlying productivity distribution. Lower costs of production lead to higher profits for managers holding other state variables and credit constraint parameter constant. Therefore, in a general equilibrium in which factor prices adjust to clear capital markets, the profit incentive grows stronger with tighter credit constraints.[9] Thus, depending on which incentive dominates, managerial capital investment of a manager of a particular characteristic (age, asset, current managerial capital), could either increase or decrease with tighter credit constraints. In my model's calibration, profit incentive dominates and managers engage in more active investment in managerial capital as credit tightens.

Another important feature of my model is that credit constraint and dispersion of firm size distribution has a non-monotonic relation. Credit constraints limit the size of firms and induce entrance of less productive agents into entrepreneurship. As a result, there is a larger mass of smaller firms with tighter credit constraints. At the same time, lower factor costs encourages managers to invest more in their managerial capital and there is a larger mass of very productive managers in the economy. The dispersion in the underlying productivity distribution has the potential to give rise to a larger dispersion in firm size with tighter credit constraints. However, severe credit constraints induce productive managers to optimally choose to reduce the amount of workers they hire in spite of their higher managerial capital and therefore the dispersion in actual firm size could also shrink with tighter credit constraints. In my model, as credit tightens, the mass of large firms (with more than 100 employees) increases first and then decreases.

[9] If I set the price level equal to that of perfect credit benchmark and tightens the credit market, I can erase the profit incentive and factor out how credit constraint discourages investment in managerial capital through weaker collateral incentives. Under this partial equilibrium, managerial capital to output ratio falls by more than 20 percent while physical capital output ratio falls by more than 34 percent. Hence, the general equilibrium has substantially different implication from partial equilibrium regarding investment in managerial capital. Managerial capital investment increases in a general equilibrium while it falls in a partial equilibrium.

5.1 Endogenous vs. Exogenous

Having seen the features of the benchmark model, I will compare the benchmark model with a model without optimal managerial capital investment decisions. Through the comparsion, I try to quantify the importance of endogenous managerial capital investment decision in the model. Credit constraints quantitatively and qualitatively have different implications with and without endogenous managerial capital investment decisions. In the endogenous investment economy, as credit tightens, managers can substitute away from physical capital to managerial capital. Tighter credit constraints reduce the demand for physical capital and depress factor prices for both. And lower costs of production lead to larger average profits of managers.

In the endogenous case, managers use the profits to invest in their managerial capital increasing the total amount of investment in managerial capital, and also total amount of managerial capital present in the economy. Thus, the economy can maintain higher average managerial capital in spite of entry of less productive marginal managers with tighter constraint. Unlike managers in endogenous economy in which managers can increase their profits/consumption through investing in managerial capital and use external finance to borrow physical capital, managers in exogenous economy are more likely to save and use less external finance. This is because managerial capital of mangers in exogenous economy is fixed over time, and therefore the only way to increase managers' profits/consumption in the future is by saving more.

Table 4: Effects of Credit Constraints; Endogenous vs. Exogenous:
Denoted as a percentage of benchmark case without credit constraint: $\phi = 1$

Statistic	$Endo : \phi = 1$	$Endo : \phi = 0.455$	$Fixed : \phi = 1$	$Fixed : \phi = 0.4$
TFP	100	102.5	100	99.7
Y	100	95.1	100	93.3
K/Y	100	83	100	87
X/Y	100	1.30	100	1.11
X	100	123	100	103
H	100	114.6	100	104.2
Average Firm Size	100	78	100	70
Manager fraction	100	127	100	140
Average Managerial Capital	100	90	100	75
EF/Y	100	80.6	100	80.8

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
EF/Y	External finance to GDP ratio

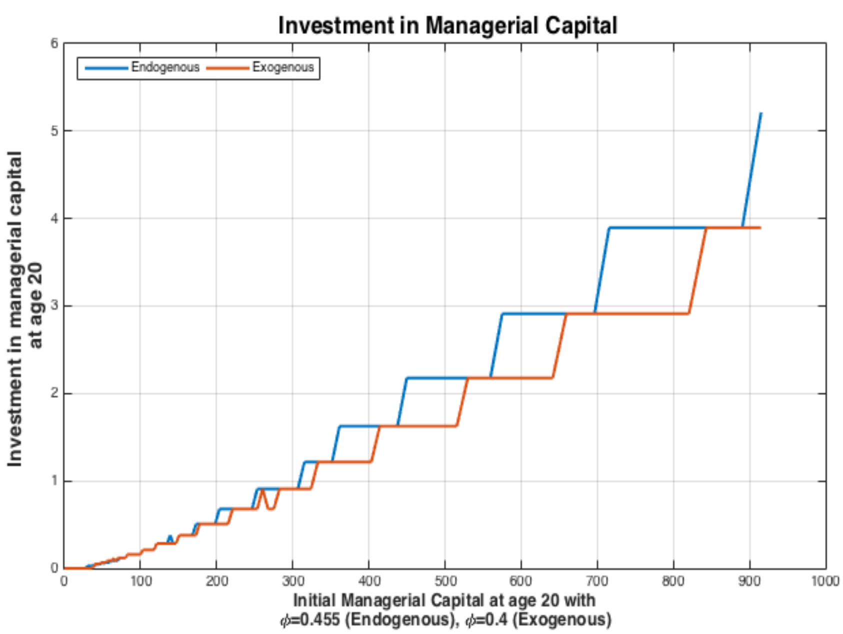
In the exogenous setup, agents invest in managerial capital but they are forced to invest the same amount as in the distortion-free benchmark model. Through the comparison I doument some notable quantitative and qualitative difference between the two setups. Specifically I will set credit constraint parameter for each case so that the external finance to GDP ratio drops by the same proportion with respect to that of the perfect credit case for each. Then I compare the the proportional change with respect to the distortion-free benchmark case of each for aggregate statistics.[10]

The result shows that accumulation of additional managerial capital induced by credit constraint in the endogenous case dampens the adverse effect of credit constraint on the

[10] In this particular exercise, I matched the drop to 19% for each

economy. Output falls less in endogenous case (4.9% vs. 6.7%), whereas physical capital output ratio falls more (16.5% vs.13%) as managers substitute away from physical capital to managerial capital. Average managerial capital falls much less for the endogenous case (9.5% vs. 25%) for two reasons. First, total investment in managerial capital increases substantially more (23% vs. 3.8%) for endogenous case. Second, fraction of managers is much lower (127% vs. 140%) for endogenous case and thus marginal manager is more productive.

With additional accumulation of managerial capital TFP increases by 2.3 % in the endogenous case despite distortion created by the tighter credit constraint while TFP decreases slightly (0.7 percent) in the exogenous case. Endogenous managerial capital investment decisions also contribute to larger dispersion in managerial income by inducing more productive agents to invest more in their managerial capital and enhance their productivity more rapidly than the less productive managers over the life cycle. Large dispersion in managerial capital leads to large dispersion in managerial income and also larger gap between income of workers and income of average managers. So the adverse effects of credit constraint on income inequality is larger with endogenous managerial investment decisions.



If a managers managerial capital was fixed over time, a managers will try to save out of the credit constraint because his profit is constant over time if he didn't save. He needs to save to obtain higher profits in the future and to smooth his consumptions. However with growing managerial capital over the life cycle and growing profits, managers find it not optimal to save. In particular, more productive the manager is, less attractive option savings become compared to investment in managerial capital. Complementarity between current managerial capital and investment allows the more productive managers to obtain bigger increase in their productivity with the same amount of managerial capital investment. As a result, their profits/managerial capital increase much rapidly than lower productive managers over the life cycle. In addition, consumption smoothing motives make more productive agents less eager to save than to consume. Therefore, marginal productivity of physical capital of productive managers remains high throughout his working periods. They rather choose to increase his managerial capital and stay constrained than to save. In fact, they dont save at all except for the last working period in which they have to save to consume during the retired periods. On the other hand, less productive managers save and do get out of credit constraints at the end of their working periods and their marginal productivity of physical capital decreases over time.

To sum up, despite the higher measured TFP with the moderate level of credit constraints in endogenous case the adverse impact of credits on aggregate economy in terms of real production and consumption is still substantial. In my model, in the economy with a credit constraint that lowers the external finance to GDP ratio by 18.6 percent has a measured TFP that is 2.3 percent higher than that of the distortion-free, first best benchmark economy. Whereas in the exogenous case, measured TFP falls by 1.8 percent relative to perfect credit TFP.

An economy with credit constraints could have larger amount of managerial capital and have higher measured TFP but will still produce less than the distortion free economy. Therefore, when investment in managerial capital decision is endogenous and thus is affected by distortions in the economy, traditional TFP measure is incapable of capturing true inefficiency of the market because it doesnt take into account of allocation of resources between (non-tangible) managerial capital and physical capital. Implication is that an economy with lower measured TFP could be producing more efficiently with better allocation of resources across tangible and non-tangible capital such as managerial capital. Large TFP could be the result of suboptimal choice of agents in the economy to invest more in non-tangible (managerial) capital as they are restricted from investing in tangible (physical) capital.

5.2 Different TFP measures

If the non-monotonicity of TFP in the model is created by how TFP measure is defined, particularly because it includes resources used for managerial capital investment as part of its output while those resources cannot be consumed, then, this is a mere problem of definition of TFP measure that is leading to this non-monotone pattern of aggregate productivity with tighter credit constraints. To verify that this is not the case, I tested several different TFP measures: [11] such as the following to exclude resources used for managerial capital from the total production.

$$TFP = (Y - X)/(K^{\alpha}L^{(1-\alpha)})^{\gamma}Z^{1-\gamma}$$

Where X = Total amount of investment in managerial capital. With above new definition of TFP the result still retained non-monotone TFP pattern as credit tightens. It shows that even after taking account of the fact that investment in managerial capital is not consumed, TFP improves with credit constraints if investment in managerial capital is endogenous. Therefore dampening effect of managerial capital accumulation of credit constraints is robust to more conservative TFP measures.

[11] The only measure of TFP that showed monotonically decreasing level with tighter credit constraint was the measure that fully takes in to account of total managerial capital held by managers:

$$TFP = Y/(K^{\alpha}L^{(1-\alpha)})^{\gamma}H^{1-\gamma})$$

Where H = Total amount of managerial capital held by managers. However, traditional TFP measures do not capture managerial capital or intangible capital as thoroughly as this measure.

6 Stochastic Managerial Capital

In this section I add a stochasticity element to the model by assuming managerial capital accumulation process has a random part. I refer to this setup as the 'stochastic case' and the benchmark setup without stochastic component as 'non-stochastic case'. The results are shown in Table 7. I recalibrate parameters to target the same statistics that I used for calibrating the benchmark non-stochastic case. I will compare the effect of credit constraints in the stochastic case with the benchmark that does not have a stochastic compenent and address how the result is different if there is uncertainty in skill accumulation. Specifically, managerial capital accumulation is assumed to follow the formula below.

$$z' = z + g(z, x) = z + z^{\theta_z} x^{\theta_x} \epsilon \qquad \epsilon \sim lnN(-0.5\sigma_\epsilon^2, \sigma_\epsilon^2)$$

Due to the stochastic component in the managerial capital accumulation function, the next period's managerial capital will not be perfectly correlated with current period's investment in managerial capital. The uncertainty will discourage managers from investing in managerial capital. If managers are accumulating managerial capital less actively, its' dampening effect on credit constraint will diminish. In this section, I set each of the credit constraint parameter so that external finance to GDP ratio in both cases falls by 20 percent.

6.1 Calibration with stochastic managerial capital accumulation

Parameters are recalibrated to match the same target as non-stochastic case. I set the variance of the log of stochastic component σ_ϵ^2 equals to 1 and set the mean of the log of ϵ so that expected value of ϵ equals zero.[12]

[12]Since the level of variance of shock that I chose is arbitrary it is important to know the right way to decide the level of uncertainties in a model. I have tried different σ_es ranging from 1/3 to 2. Depending on the level of uncertainty agents faces, the extent to which credit constraint distorts and lowers TFP and how investment in managerial capital dampens those effects is different.

Table 5: Calibrated parameter values: Stochasitc

Parameter	Value
Population Growth Rate (yearly) (n)	0.011
Depreciation rate (yearly) (δ)	0.06
Importance of Capital (α)	0.419
Returns to Scale (γ)	0.7558
STD of Log-Managerial Capital (σ_z)	2.1132
Discount Factor (yearly)(β)	0.95
Skill accumulation technology (θ_1)	0.7715
Skill accumulation technology (θ_2)	0.5666
STD of shock (σ_e)	1

Table 6: Fit of the benchmark model and data with parameter values in table 1: Stochastic

Statistic	Data	Model
Average Firm Size	17.9	17.9
Physicl Capital Output ratio	0.23	0.25
Fraction of small (0-9 workers) establishments	0.73	0.75
Fraction of large (100+ workers) establishments	0.026	0.02
Employment Share of Large Establishments	0.46	0.46

6.2 Stochastic Case vs. Non-Stochastic Case

If there is uncertainty in managerial capital accumulation, as credit tightens, managers will not invest in managerial capital as actively as in the benchmark case that does not have a stochastic component. With increased uncertainty in mangerail capial accumulation, credit constraint has more adverse effects through misallocation on overall productivity as production units cannot perfectly prepare for future shocks. In addition to that, less accumulation of managerial capital induced by uncertainty will result in less increase in measured TFP compared to non-stochastic case.

Table 7: Effects of financial friction with/without stochastic component: Values are denoted as a percentage of $\phi = 1$

Statistic	$BM : \phi = 1$	$BM\phi = 0.45$	$Sto : \phi = 1$	$Sto : \phi = 0.44$
TFP	100	102.3	100	100.7
Y	100	95	100	91
K/Y	100	83	100	88
X	100	123	100	117
H	100	105.5	100	114.7
Mean Firm Size	100	77	100	86
Manager fraction	100	128	100	115
Mean ability	100	114	100	91
MPK variance(level)	0	0.1842	0	0.1322
EF/Y	100	79.9	100	79.9

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
MPK	Marginal productivity of physical capital
EF/Y	External finance to GDP ratio
BM	Benchmark, without stochastic component
Sto	With stochastic compoenent

Total output drops more (8.8% vs. 5.2%) in the stochastic case. Managers are reluctant to invest in managerial capital with uncertainty and as a result increase in investment in managerial capital in stochastic case (17 percent increase) is not as large as in non stochastic case (23 percent). With less investment in mangerial capital, traditional measured TFP increases less than 1 percent in the stochastic case while it increases by 2.3 percent in the non-stochastic case as credit tightens. On the other hand, physical capital output ratio drops less in the stochastic case because managerial capital and physical capital are subsitutes. Overall total amount of capital (both managerial capital and physical capital) present in the economy with credit constraint is lower for stochastic case than non stochastic case (78 percent vs.82 percent) with credit constraint. The larger drop in capital with tighter credit constraint leads to a larger drop in output for stochastic case.

Variance of marginal productivity of physical capital is larger for the non-stochastic case because less productive managers are more likely to save for the future. If less productive managers become more productive than they expected because of a shock, they can use those savings to dampen credit constraint and the marginal productivity of physical for those managers would be lower than without a shock. On the other hand, If more productive managers become less productive, he has not saved and he will be constrained. As a result, the marginal productivity of physical capital for those managers would be higher with the shock. Given that the marginal productivity of physical capital is higher for more productive managers if managers hold the same amount of assets, the shock will reduce the dispersion of marginal productivity across managers. Thus, in stochastic case, the variance of marginal productivity is much lower than the non-stochastic case. Physical capital output ratio falls less in the stochastic case (12% vs. 17%) as managers are not substituting away from physical capital as much as in the non stochastic case.

Uncertainty in managerial capital accumulation leads to less increase in the number of managers as credit tightens. Therefore number of managers increases less in stochastic case but because these managers are not investing in managerial capital as much as those in the non-stochastic case, the average managerial capital falls (by 8.6 percent) with credit constraint in the stochastic case while it increases (by 14.3 percent) in the non-stochastic case.

To conclude, the dampening effect of endogenous managerial capital investment on credit constraint itself is dampend with a stochastic shock in managerial capital accumulation.

6.3 Stochastic: Endogenous Case vs. Exogenous Case

In this section, I am going to compare the two setups. Both have the same stochastic component in managerial capital accumulation function but one has the optimal managerial capital investment decisions and the other has the forced investment in managerial capital at the level same as in the endogenous, no credit constraint case. By comparing these two setups, I find that even if opportunity to optimally invest in managerial capital is present in the economy, if managers are not actively investing in their managerial capital because of frictions such as uncertainty, the offsetting effect on credit constraint weakens and could become non-existent. The following result (Table 8) typically shows that having optimal investment decision in managerial capital (endogenous case) and fixed investment setup (exogenous case) do not show any pronounced difference in aggregate statistics if the variance of shock in managerial accumulation process is large enough.[13] This shows that the level of unceratinty in managerial capital investment returns could affect the optimal choice behavior of agents. If the uncertainty becomes very large, regardless of availablity of optimal investment decisions result is not so different from without optimal decisions. Therefore, for mitigating effect of endogenous managerial capital investment to distortion caused by financial frictions to be meaningful, there should be limited amount of unceratainty in the economy.

[13]The variance of shock used is 1 ($\sigma_e = 1$).

Table 8: Effects of financial friction with optimal managerial capital investment decisions vs. fixed investment in managerial capital: Both with a stochastic component with variance 1. Denoted as a percentage of $\phi = 1$

Statistic	$Endo : \phi = 1$	$Endo : \phi = 0.35$	$Fixed : \phi = 1$	$Fixed : \phi = 0.3$
TFP	100	99.9	100	99.7
Y	100	86	100	87
K/Y	100	73	100	74
X	100	117	100	112
H	100	104	100	104
Mean Firm Size	100	77	100	86
Manager fraction	100	122	100	125
Mean ability	100	85	100	83
MPK variance(level)	0	0.3295	0	0.3568
EF/Y	100	0.72	100	0.72

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
MPK	Marginal productivity of physical capital
EF/Y	External finance to GDP ratio
Endo	Endogenous investment in managerial capital
Fidxed	Fixed investment in managerail capital

7 Conclusion

In a model in which the underlying productivity distribution is also affected by the credit constraints, measured TFP do not necessarily monotonically decrease with tighter credit constraints. Limited access to physical credit will encourage agents to actively invest in managerial capital and change the underlying productivity distribution so as to improve aggregate measured TFP. The improvement in underlying productivity distribtuion will alleviate the adverse effects of credit constraint to the economy and thus lead to a less radical fall in output and average firm size. However, the dispersion in the firm size and firm profits could be larger compared to the model without optimal managerial capital investment decisions. Also, large uncertainty in managerial capital accumulation process could wipe out the mitigating effect of optimal managerial capital investment decisions on credit constraints. It would be an interesting excercise to measure how large uncertainty is in an economy and to what extent it suppresses the channel found in this paper so as to pass through the adverse effect credit constraints exert on the economy.

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