Endogenous Information and Ramsey Policy in Lemons Markets*

Seungjun Baek†
Florida State University

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Abstract

This paper characterizes the Ramsey policy to revive financial markets in which there is a lemons problem. Policymakers recognizes that insurance policy on lemons may cause efficiency losses, possibly due to moral hazard or taxation. Agents can acquire information about the mean quality of assets traded in the market to decide whether to buy the assets. It is found that the Ramsey policy requires careful considerations of the two factors: the cost of information acquisition and the endogenous effect of an asset price on asset quality. When such an endogenous effect is high, the structure of the optimal policy is hump-shaped. When the information cost is high, the optimal policy provides greater protections to both the best and worst states, thereby increasing an upside risk and decreasing a downside risk.

The positive analysis shows that asset market volatility is higher in markets where the information acquisition cost is greater. In such markets, government direct asset purchases are inefficient as they crowd out private liquidity supply greatly, thereby causing distortion in private liquidity allocation. In contrast, those issues do not arise if the Ramsey policy is implemented.

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†Email: seungjun.baek@fsu.edu.

1 Introduction

The U.S. financial crisis of 2008-2009 led the Federal Reserve (FED) to implement a new set of unconventional policies (Reis 2009). In particular, the Fed conducted large-scale asset purchases of asset-backed securities (ABS), with the goal of relieving concern that some private sectors may not have access to the credit market. While the government large-scale asset purchases may be effective to inject liquidity into the private sector (Cúrdia and Woodford 2011; Krishnamurthy and Vissing-Jorgensen 2011), there is concern that a large amount of public liquidity may cause distortions that arise from inefficient uses of private liquidity. This paper studies an alternative policy instrument, loss insurance, which is not subject to such a problem. In doing so, this paper takes into account policymakers' concern about potential efficiency losses from policy interventions, which may arise due to moral hazard or distortions from taxation/subsidies.

To that end, I build a model that incorporates a financial market in which private securities are traded. I focus particularly on the role a financial sector plays in reallocating resources to their most productive uses (e.g., Rajan and Zingales 1998; Levine and Zervos 1998). In the model, entrepreneurs face heterogeneous investment opportunities upon which they base their borrowing and lending decisions. Being constrained from borrowing, entrepreneurs who face favorable investment opportunities (marginal sellers) seek to pledge their legacy assets in order to obtain liquidity, whereas entrepreneurs who have poor investment opportunities (marginal buyers) need to store their perishable liquidity by acquiring legacy assets from a financial market. In a hypothetically frictionless world, well-functioning financial markets are able to successfully reallocate all resources to the highest value use in all states.

I consider asymmetric information about the quality of legacy assets (Akerlof 1970) to be the main cause of financial friction, in the light of both the recent crisis (Gorton 2009; Duffie 2010) and the historical evidence (Calomiris and Gorton 1991; Mishkin 1991). Some legacy assets traded in a market are useless, and jeopardizes the existence of the financial market. The mean fraction of useless legacy assets (lemons) traded in the market is the key state variable in the model as it affects asset returns.

There are two key aspects in the model. First, the mean quality of assets traded in a market is endogenous. While the total amount of lemons existing in an economy is subject to an exogenous aggregate risk, the amount of non-lemons traded in a market is driven by an asset price. A high asset price induces marginal sellers to sell their non-lemons to initiate their investment projects, which improve the mean quality of assets traded.

Second, marginal buyers can acquire information about the mean quality of the assets traded as an endogenous response to their imperfect knowledge of the state. If they obtain more information, a signal indicating the state of the economy becomes more precise, but acquiring more accurate information is costly (Sims 2003; Woodford 2008).[1]

The positive analysis shows that there may exist multiple equilibria in the economy with imperfect information about the state, which is not the case in the economy with rational expectations. Multiplicity arises only when there is high correlation between asset price and asset quality. In such a case, which equilibrium is selected depends on the agents' forecast of others' information choices. If agents choose information collectively in ways that increase demand for assets, the quality of assets traded is improved with a high equilibrium asset price, which justifies high asset demand. Likewise, the opposite may be the case if agents forecast low asset demand. It causes deterioration in the mean quality of the assets traded, which in turn leads to insufficient private liquidity supply in the financial market.

The other positive result is that higher information acquisition costs increase the volatility of asset demand in response to arbitrary exogenous disturbances. Intuitively, higher information costs decrease the responsiveness of asset demand to the unknown state variable. At the same time, however, asset demand becomes more sensitive to arbitrary outside risks which are known to agents. As a result, the variance of asset demand conditional on the unknown state is increasing in the information acquisition cost. Accordingly, the infor-

[1] There are good reasons to believe that market participants have imperfect information on this kind of the aggregate state. At the onset of the recent crisis, for instance, financial market participants were not able to agree on prices for legacy assets, and this led to a sudden collapse of the secondary asset markets. However, since 2008, when the Fed started buying up financial assets, its profits, accrued from the spread between the interest rate paid on its reserves and the average return on its bond holdings, have skyrocketed. In 2009, the Fed posted \$59 billion, a 50 percent increase over 2008. In 2012, it sent record earnings of \$88.9 billion to the Treasury, three times greater than its typical profits.

mation acquisition cost acts as a force to increase the conditional variance of the asset price as well.

In order to determine whether there is a need for policy intervention, one needs to understand how, and under what conditions, this private choice results in inefficient decision-making at the social level. In this paper, that issue is addressed by focusing on private information acquisition and the corresponding financial decisions, which are endogenous responses to an economic environment. I define constrained efficiency by considering a fictitious planner who is not allowed to directly transfer liquidity among entrepreneurs, but who can dictate to each entrepreneur how to acquire and use information. It serves as a benchmark for the evaluation of government policies. Then I study the Ramsey policy, considering loss insurance as a main policy instrument.

On the one hand, policymakers want to maximize private liquidity supply to improve efficiency in the allocation of private liquidity. On the other hand, policymakers may be constrained if there is concern about potential effiency losses from policy interventions. For instance, providing government protections against lemons may cause moral hazard or unforeseen, undesirable outcomes which are deemed costly. This paper demonstrates that the Ramsey policy requires careful considerations of the cost of information acquisition and the endogenous effect of an asset price on asset quality. In particular, if a degree of concern about the efficiency loss is high, the structure of the Ramsey policy becomes hump-shaped. This non-linearity is prominent especially when the endogenous relationship between asset price and asset quality is strong. In other words, when fewer resources are available to policymakers, due to concern with potential efficiency losses, the welfare of an economy can be maximized by providing more protections against middle states, not the worst states.

The other consideration for the design of the Ramsey policy is the cost of information acquisition. With higher information costs, the optimal policy provides greater protections to both the best and the worst states. The optimal policy increases an upside risk by increasing subsidies to better states, while it decreases a downside risk by increasing subsidies to worse states. This policy induces agents to enter into a market and maintain a high level of private liquidity supply across states. Moreover, the effect of the insurance policy on private liquidity is greater when the information cost is higher.

In comparison to direct asset purchases which are alternative instruments for policy interventions, the government direct asset purchases are less efficient as they crowd out private liquidity supply. The government demand competes with the private demand, which acts as a force to decrease the asset return. It subsequently reduces the private demand and causes distortion in liquidity allocation. Moreover, our positive analysis about the asset market volatility indicates that large-scale direct asset purchases may exacerbate distortion in private liquidity allocation. When the information acquisition cost is large, a decreased expected asset return caused by the competing government asset demand strongly affects the private sector's information choices in ways that lead to a large reduction in the private liquidity supply.

This paper is related to an extensive literature on adverse selection initiated by Akerlof (1970). Recent applications to the financial crisis include Bolton, Santos, and Scheinkman (2011), Kurlat (2013) and Malherbe (2013). I build on the contribution of Kurlat (2013), who studies a financial market plagued with asymmetric information as an amplification mechanism by which aggregate shocks propagate. I endogenize information acquisition, and evaluate the efficacy of government interventions in the financial market.

The idea that a sudden change in information production can trigger a large consequence is similar to the one in Ordonez and Gorton (2013), who study the dynamic effects of information production but abstract from the trading motive for assets. I explicitly model such a motive, and this allows us to discuss the welfare implications of allocation of liquidity among agents. Those researchers also suppose that agents have rational expectations about the aggregate state and are allowed to be fully informed about the riskiness of each individual trading partner with some fixed costs, whereas I suppose that agents are allowed to obtain information only about the unknown aggregate state and that the cost of the information is tied to its accuracy.

This paper is closely related to the literature on government interventions in financial markets that suffer from asymmetric information. Minelli and Modica (2009) focus on optimal policies between a monopolistic bank and borrowers. Reis (2011) considers different sectors in which credit policies to be implemented, and shows that the injection of liquidity into the shadow banking system can be highly effective. Chari, Shourideh, and Zetlin-Jones

(2011) argue that asset purchases, which overcome adverse selection problems, must bring negative profits to the government. Philippon and Skreta (2012) and Tirole (2012) study cost-minimizing bank bailouts in the context of the mechanism design framework, when the government bails out first prior to opening an asset market. An important difference is that in our model, borrowing and lending decisions are endogenized with heterogeneous investment productivity, which allows us to analyze from the perspective of a social planner the impact of alternative interventions, such as asset purchases and loss insurance, on the reallocation of liquidity.[2] House and Masatlioglu (2015) favor direct asset purchases over equity injections, because equity injections may contaminate the quality of assets traded. Loss insurance studied in this paper is not subject to such a criticism and better in that it does not distort private liquidity allocation. Camargo, Kim, and Lester (2016) study the effect of government intervention on private information production. In their paper, a government uses a non-state-contingent strategy as a tool of intervention. As noted previously, the majority of this literature focuses on the theoretical analysis of government interventions on private liquidity. In this paper, I take a more quantitative approach to solve for the state-contingent Ramsey policy, when policymakers have limited resources with concern about potential efficiency losses.

2 Model

The economic environment is close to the one in Kurlat (2013). The key new feature of my model is that agents acquire information endogenously on an unknown state of the economy.

2.1 Description of the Economy

There are three dates, $t = 0, 1, 2$. There is a single perishable consumption good, an apple, and a single factor of production, a tree. The economy is populated by two groups of agents, entrepreneurs and households. Households are of measure h, and they are homogeneous.[3]

[2] In Philippon and Skreta (2012) and Tirole (2012), asset purchases (or direct lending) and debt guarantees have an equivalent impact on welfare.

[3] In the model, households exist only for a technical reason. They have no role other than ensuring that the first order condition of the information choice problem is well defined by ruling out the case in which the asset price is equal to 0.

Each of the entrepreneurs and the households is born with an endowment equal to one unit of the apple tree at date 0, and receives no further endowment in the subsequent periods. Each of them is risk neutral and consumes only at date 2; the utility from consumption is given by $E[c_2]$, where c_2 denotes consumption at date 2.

Investment: There are two types of entrepreneurs: marginal buyer and marginal seller. The population of each type is given by 1 and N respectively.[4] Each entrepreneur, indexed by j, can turn apples (consumption goods) into apple trees (capital goods), but the opposite is not feasible. Investing i_j with investment productivity A_j, each entrepreneur produces $A_j i_j$ units of the tree, which yields $A_j i_j$ units of the apple in the subsequent period. Households do not have access to such investment technologies.

Investment productivity A_j of the marginal seller is randomly distributed with distribution function $G(A)$. Investment productivity of the marginal buyer is given simply by constant $\underline{A}$.

Production Technology: Each unit of the apple tree delivers one unit of the apple at date 1.[5] After production, a fraction λ_j of each marginal seller's trees become lemons.[6] λ_j is randomly drawn from a distribution with finite mean λ. In addition, the mean λ is also a random variable which drawn from a distribution $F(\lambda)$. This is the key source of aggregate uncertainty. Only an apple tree that has not become a lemon tree produces one unit of the apple in the subsequent period. Lemon trees produce nothing useful at date 2. All trees vanish at date 2 after production. Aggregate output equals the sum of apples produced from trees, and apples cannot be stored.

No Storage Technology, Collateralized Borrowing and Asset Markets: Since agents value their consumption only at date 2, they need to transfer the apples from trees at date 1. Because there is no storage technology, each agent j needs to either transform his own apples into trees with his investment technology A_j, or exchange his own apples for trees traded in the market. The competitive market for buying and selling trees opens at date 1.

[4] The measure of the marginal buyer is normalized.

[5] Trees are perfectly divisible.

[6] For notational simplicity, it is assumed that the others' trees do not turn into lemons. It does not change any results even if we suppose that a fraction of all agents' trees turn into lemons. Buyers' information decisions do not depend at all on the ownership of lemons.

Each agent is a price taker.[7] All trees are traded at the same price, p, which is the asset price in terms of units of the apple.[8] Note that each agent has a unit of the apple at the beginning of date 1, which can be used to buy a tree.[9] Each agent chooses how many lemon and apple trees to sell in the market, denoted by d_j^L and d_j^{NL} respectively. Short sales are not allowed and new investment is not pledgeable, $d_j^L \in [0, \lambda_j]$ and $d_j^{NL} \in [0, 1 - \lambda_j]$. Each of the agents also decides the amount of assets he wishes to purchase in a market, denoted by $b_j \geq 0$, of which the quality is unknown to buyers. The agent's budget constraint is given by

$$i_j + p \cdot b_j \leq 1 + p \cdot (d_j^{NL} + d_j^L) \tag{1}$$

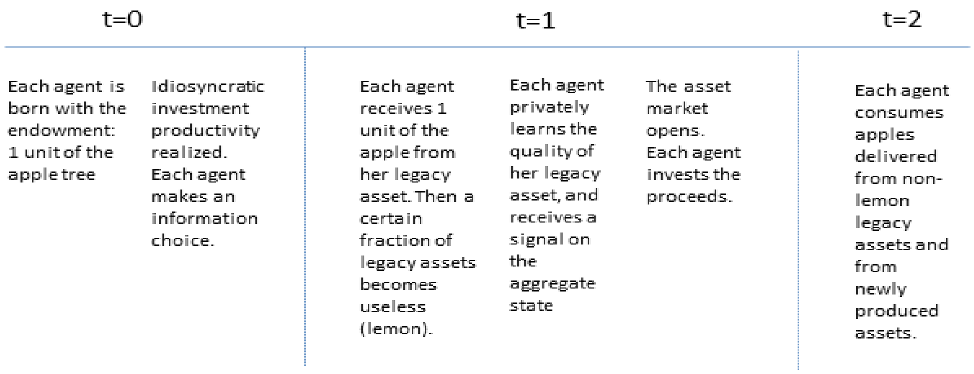
where p is an equilibrium asset price.[10] It implies that the sum of the expenditure on new investment and legacy assets bought in the market must be equal or less than the sum of the endowment and revenue from selling legacy assets, which include apple and lemon trees.

[7] I focus on a case in which a market is competitive. There might be other cases in which buyers ration credit by offering a higher price if there is any benefit to do so (Stiglitz and Weiss 1981). However, there are a few reasons why credit rationing may not occur. One possibility is that information acquisition costs for optimal credit rationing may be much higher than the strategy with a simple binary choice as in our model (See Section 3.1.3). If credit rationing incurs large information costs, an agent would rather be a simple price taker. Another possibility is that credit rationing causes borrowers to be discouraged from applying for a loan because of high application costs, which include financial, time and psychological costs (Levenson and Willard 2000; Kon and Storey 2003).

[8] There is no separating equilibrium in our framework. The intuition is that if lemon tree sellers were separated, they have to sell their assets at the price 0, which is strictly lower than the other price. Because the bad type can mimic the good type, the bad type never attempts to sell at the price 0.

[9] Another interpretation is that buyers seek to lend their money to profitable entrepreneurs, and they receive $\frac{1}{p}$ units of assets as collateral in exchange for one unit of the loan. It is equivalent to the repurchase agreement that provides a seller with the funds of p backed by a unit of collateral, together with the agreement to repurchase the asset at the price of one unit of the consumption good from a lender at a later date, although the seller can default on the agreement and it is the case whenever the collateral is lemon.

[10] In this setup, there are no insurance providers against losses from lemons. However, the presence of insurance providers does not affect the nature of our results. Appendix C considers an environment with private insurance providers.



Timeline

The fraction of non-lemon trees traded in the market is denoted by $X(\lambda)$. I focus on the parametric case in which investment productivity A satisfies the following assumption.

Assumption 1 *(i)* $\underline{A} < \frac{N}{1+h}$; *(ii)* $\max[\frac{X(\lambda)}{p}] < G^{-1}(0)$.

Assumption 1 ensures that each type of entrepreneur behaves as a marginal buyer or a marginal seller in a financial market.[11] This assumption is a convenient shortcut to maintain tractability. A detailed explanation for the role of the assumption is provided in Section 3.1.1.

2.2 Information

In period 1, each agent privately identifies lemon trees among the legacy assets he owns. However, information is asymmetric in the sense that the rest of the agents are not aware of the quality of assets that other agents attempt to sell in the market. Moreover, agents are unaware of the realization of the mean λ, which represents the total amount of lemon trees existing in the economy. The distribution function $F(\lambda)$ serves agents as a prior belief over λ.

Since each entrepreneur's portfolio can be sufficiently diversified, the aggregate state λ is the unique variable about which an agent should learn if needed. An agent can acquire more

[11] Households can be considered as entrepreneurs with 0 investment productivity. It turns out that they become buyers for sure (Section 3.1.1).

precise information about the aggregate state λ, but acquiring more accurate information is costly. The quantity of information is measured as in the information theory of Shannon (1948) and rational inattention literature starting from Sims (2003). Mathematically, collecting more accurate information on the current unknown state reduces the entropy of the agents' posterior over the state space conditional on a signal s, $f_j(\lambda|s)$, relative to the agent's prior, $f(\lambda)$. Moreover, each agent can choose the set of values of signals he will receive ex ante upon which his decision is based, while signals are random variables of the current state. Then the mutual information, which represents the average quantity of information conveyed by a set of possible signals, is given by

$$\begin{aligned} I(f_j) \equiv & \int_s \bar{f}_j(s) \int_\lambda f_j(\lambda|s) \log f_j(\lambda|s) d\lambda ds - \int_\lambda f(\lambda) \log f(\lambda) d\lambda \\ = & \int_\lambda \int_s f_j(s|\lambda) \log f_j(s|\lambda) ds dF(\lambda) - \int_{s_i} \int_\lambda f_j(s|\lambda) dF(\lambda) \ln[\int_\lambda f_j(s|\lambda) dF(\lambda)] ds \end{aligned} \tag{2}$$

where $\bar{f}_j(s)$ is the prior density function of the signal s.[12] Given $\theta > 0$, which is the cost of acquiring an additional unit of information, the cost of information acquisition is given by $\theta \cdot I(f_j)$.

Once each agent makes an information choice at date 0, he receives a signal s_j at date 1, which is independent of the signal other agents receive.[13] One interpretation is that a firm needs to hire competent analysts who can effectively learn about unknown economic fundamentals, which is costly, to be more responsive to an uncertain economic environment.

For simplicity, agents are assumed not to be allowed to use the current asset price and other variables to infer the current economic fundamental λ. While it does not affect the qualitative nature of main results, it greatly simplifies the formulation of the problem and numerical solutions. Appendix C shows that how the problem can be formulated if agents learn from the asset price.[14]

[12] Bayes' theorem was applied in the derivation of equation (2).

[13] Given $\{s_j\}$, agents may have heterogeneous posterior beliefs about λ. In reality, people seem to have different opinions on fundamentals even after they acquire information, especially when the cost of information is large. The case in which agents have heterogeneous prior beliefs is beyond the scope of this paper.

[14] By introducing noise traders in financial markets, the asset price depends on the state and unobservable random variables. For information spillovers, see Angeletos, Lorenzoni, and Pavan (2010). As far as agents cannot learn about the state perfectly from the aggregate action, multiplicity still arises (Section 3.3).

The investment productivity A_j is private information to entrepreneur j. While it is private one, it will be shown in the next section that entrepreneurs do not need to obtain any information about investment opportunities that other agents face.

2.3 Definition of Equilibrium

An equilibrium asset price depends on the economy-wide state, which is given by the aggregate amount of lemon trees existing in the economy, λ. Given the asset price, each agent wishes to make a choice which depends on the economy-wide state λ, and his own individual states A_j and λ_j. However, since λ is unknown to each agent, his decision should be based upon a particular signal s_j he receives, which may be informative on λ, and the individual states A_j and λ_j. Thus, each agent solves the following problem:

$$\max_{i_j, b_j, d_j^{NL}, d_j^L, f_j(s|\lambda)} \int_\lambda \int_s c_j(s,\lambda) f_j(s|\lambda) ds dF(\lambda) - \theta \cdot I(f_j) \tag{3}$$

subject to the budget constraint (1) and

$$c_j = 1 \cdot k_j \tag{4}$$

$$k_j = [(1 - \lambda_j - d_j^{NL}) + b_j X(\lambda)] + A_j i_j \tag{5}$$

$$0 \leq d_j^{NL} \leq 1 - \lambda_j, 0 \leq d_j^L \leq \lambda_j \tag{6}$$

$$0 \leq b_j, 0 \leq i_j \tag{7}$$

where the quantity of information $I(f_j)$ is given by (2)[15]; $X(\lambda)$, which is the function of the aggregate state λ, represents gains from a unit of an asset traded in the market (identically, proportion of apple trees (nonlemon)).

Constraint (4) implies that each agent is allowed to consume apples that are produced only at date 2 because of the non-existence of storage technology. Constraint (5) states that the total amount of assets available at date 2 to agent j is equal to the sum of nonlemon legacy assets he keeps, $1 - \lambda_j - d_j^{NL}$, and the ones bought in the market $b_j X(\lambda)$, plus the

[15] I, here, write a model in which agents choose how much information to acquire (θ is exogenous) rather than how to allocate their attention with an upper bound $\bar{I}$ on $I(f_j)$ ($\bar{I}$ is exogenous, but θ corresponds to the Lagrange multiplier for the information constraint). Our specification allows us to derive simple analytical expressions that characterize equilibrium. Note that there is a one-to-one relation between $\bar{I}$ and θ.

assets newly produced from his own investment, $A_j i_j$. Constraint (6) represents the borrowing constraint; since new investment is not pledgeable, each agent can obtain liquidity only to the extent to which he is able to sell his own legacy assets in the market. Investment and purchases of assets must be nonnegative as indicated by constraint (7).

The agents solve for an information choice taking as given common beliefs on the asset price function, $p(\lambda)$. Since all the agents are infinitesimal, they do not internalize the general equilibrium impact of their information choice on the asset price. Taking all the considerations into account, I define equilibrium as follows.

Definition 1 *A competitive equilibrium is given by an asset price $p(\lambda)$, a market proportion of non-lemons $X(\lambda)$, an individual information choice $f_j(s|\lambda)$ and individual decision rules $\{i(s_j, A_j, \lambda_j, p), b(s_j, A_j, \lambda_j, p), d^{NL}(s_j, A_j, \lambda_j, p),$
$d^L(s_j, A_j, \lambda_j, p)\}$ that jointly satisfy the following conditions:*
(i) Each entrepreneur's information choice $f_j(s|\lambda)$ maximizes (3) given the asset price and the individual decision rules.
(ii) The individual decision rules maximize expected consumption subject to constraints (1), (4), (5), (6), and (7) given the asset price.
(iii) The asset market clears: $\int_i b(s_i, A_i, \lambda_i, p)di \leq \int_i [d^L(s_i, A_i, \lambda_i, p) + d^{NL}(s_i, A_i, \lambda_i, p)]di$.
(iv) The market proportion of non-lemons $X(\lambda)$ (equivalently, gains from buying an asset traded in the market) is consistent with the individual decision rules

$$X(\lambda) = 1 - \frac{\int_i d^{NL}(s_i, A_i, \lambda_i, p)di}{\int_i [d^L(s_i, A_i, \lambda_i, p) + d^{NL}(s_i, A_i, \lambda_i, p)]di}.$$

3 Equilibrium

To characterize the equilibrium, I will first derive the optimal decision rules and information choice of the entrepreneur. Next, I will show existence and multiplicity of solutions. Then, I will present the relation between various parameters and endogenous variables in equilibrium.

3.1 Solving for Optimal Entrepreneur Decision Rules

The problem can be solved using backward induction, starting with deriving the entrepreneurs' decision rules for a given signal. Before proceeding to the characterization of the solution, it is useful to begin with the following observations: (i) Each household becomes a buyer of assets, and (ii) Each entrepreneur always sells his entire lemon assets in the market.

The former observation (i) reflects the fact that, since households possess neither storage technology nor investment technology, buying assets in the market is the sole option they have in order to transfer their apples into the future.[16] The latter observation (ii) is a consequence of asymmetric information between sellers and buyers. Selling lemons to other agents is always profitable, as lemons are worthless and buyers have no ability to distinguish lemons from nonlemons.

3.1.1 The Period 1 Optimal Choice

With those observations, I start by solving for the period 1 optimal decision rules taking information choices as given. The solution is similar to Lemma 2 of Kurlat (2013). Given an asset price p and a signal s_j, each entrepreneur j decides how many assets they carry into the future by choosing d_j^{NL}, d_j^L, b_j and i_j. The solution to the relevant decision rules are reproduced in the following for completeness.

Lemma 1 *(i) (Seller) An entrepreneur becomes a seller of a nonlemon tree* ($d_j^{NL} + d_j^L = 1$ *and* $b_j = 0$) *if* $A_j > \frac{1}{p}$;
(ii) (Buyer) An entrepreneur becomes a buyer ($d_j^{NL} = 0$, $d_j^L = \lambda_j$ *and* $b = \frac{1}{p} + d_j^L$) *if* $A_j < \frac{1}{p} E[X(\lambda)|s_j]$;
(iii) (Keeper) An entrepreneur becomes a keeper ($d_j^{NL} = b_j = 0$ *and* $d_j^L = \lambda_j$) *if* $\frac{1}{p} E[X(\lambda)|s_j] < A_j < \frac{1}{p}$.

This result is best understood by comparing the return to investment A_j and the return from buying assets, $\frac{X}{p}$, or the opportunity cost from selling nonlemon assets, $\frac{1}{p}$. One unit of the apple, which is an endowment at date 1, can be used to produce A_j apple trees,

[16] Households play no role other than ensuring the first order condition of the information choice problem being well defined. Their presence rules out the case in which the asset price equals 0.

or to buy $\frac{1}{p}$ trees, among which only non-lemon trees produce consumption goods of $\frac{X}{p}$ at date 2. As long as the return from buying assets is greater than the return to investment, a marginal buyer supplies the entire liquidity he owns in exchange for assets traded in the market. Similarly, if the return to investment is higher than the cost from selling non-lemon assets, a marginal seller sells the whole of nonlemon assets to finance his new investment project.

The wedge $1 - X(\lambda)$, which is difference between the opportunity cost from selling non-lemon assets and the return from buying assets, emerges as $X(\lambda) < 1$. This occurs because of the asymmetric information on asset quality between buyers and sellers; each seller knows the quality of assets he sells in the market, while buyers cannot tell nonlemons apart from lemons. This implies that, from a buyer's perspective, each asset traded in the market is risky in the sense that it will be nonlemon only with uncertain probability $X(\lambda) < 1$. In contrast, from the perspective of a seller of a nonlemon, the asset he sells in the market is risk-free. As a consequence, if the entrepreneur's productivity falls between $\frac{X}{p}$ and $\frac{1}{p}$, it is neither optimal for the entrepreneur to sell nonlemons nor to buy assets. In such a case, the entrepreneur keeps his own legacy asset and invests his own liquidity utilizing his own investment productivity.

While deriving the solution to this problem seems to be quite complicated, Assumption 1 ensures tractability without affecting the qualitative results of our analysis.

$\underline{A}$ is parameterized to be sufficiently low (Assumption 1-(i)) so that marginal buyers (low productive entrepreneurs) do not become sellers; because of their low productivity, selling their non-lemon assets to finance new investment projects is never profitable. Therefore they actively seek to buy profitable assets traded in the market, as far as the return to the asset satisfies the condition in Lemma 1-(ii).

On the other side, the investment productivity of marginal sellers is high enough (Assumption 1-(ii)) so that they do not become buyers. They eager to look for liquidity to finance their promising investment projects. They are willing to sell their non-lemon assets, as far as the asset price satisfies the condition in Lemma 1-(i).

It might be more elegant to permit full continuity on investment productivity A between marginal buyers and marginal sellers. However, it would complicate the fixed point analysis without gaining meaningful intuition. For the current purpose, I opt for tractability and expositional simplicity.

3.1.2 The Market Clearing Conditions

Next, I illustrate the determination of the market clearing price function $p(\lambda)$ and the gain from an asset, $X(\lambda)$.

Marginal sellers' decisions on whether to sell or not hinge on an asset price, and thus the financial decisions of marginal buyers. If an asset price is low due to the lack of demand, more marginal sellers become keepers who do not sell their nonlemons (Lemma 1-(iii), $A < \frac{1}{p}$). In contrast, if strong asset demand leads to a high asset price, more marginal sellers become sellers, $A > \frac{1}{p}$. Therefore, a fraction of the marginal sellers who opt out of the market is given by $G\left(\frac{1}{p}\right)$.

The equilibrium proportion of non-lemons in the market, $X(\lambda)$, can be expressed as

$$X(\lambda) = 1 - \frac{\lambda N}{S^A(\lambda)}, \tag{8}$$

where the total asset supply $S^A(\lambda) = \left\{[1 - G\left(\frac{1}{p}\right)](1-\lambda) + \lambda\right\} N$. The numerator in the second term of (8) is the aggregate supply of lemons. $S^A(\lambda)$ in the denominator represents the aggregate asset supply, which is the sum of aggregate supply of lemons λN and nonlemons $[1 - G\left(\frac{1}{p}\right)](1-\lambda)N$. The quality of assets, $X(\lambda)$, is increasing in the price p, as $G(1/p)$ is decreasing in p.

Let us denote a fraction of the marginal buyers who become buyers in the state λ by $\delta(\lambda)$. A market clearing asset price p in each state λ must satisfy

$$\frac{S^L(\lambda)}{p} = S^A(\lambda), \tag{9}$$

where $S^L(\lambda) = \delta(\lambda) + h$. The numerator $S^L(\lambda)$ in the left-hand side of equation (9) is the aggregate liquidity supply (or aggregate asset demand), which is the sum of the liquidity supply by the marginal buyers and

households. The right-hand side is the aggregate asset supply.

3.1.3 The Optimal Information Choice

I next turn to the description of the information acquisition problem at date 0. Assumption 1 ensures that I will be able to focus on a case in which the marginal buyers acquire information on the underlying state, while the marginal sellers optimally choose not to obtain any information on the state; Lemma 1 indicates that the financial decisions of sellers and keepers do not depend on the unknown state λ – all relevant information that is needed to make optimal decisions for them is common knowledge. In contrast, knowing the current state λ is crucial for the marginal buyers to decide whether to buy assets or not.[17] Therefore, it suffices to illustrate the optimal information choice problem of the marginal buyers.

In the proposed framework, each marginal buyer faces the binary choice problem of whether to buy assets or not. The following lemma is useful to simplify the problem further, which is the same as Lemma 1 of Woodford (2008).

Lemma 2 *The optimal structure of the information choice involves either signals that are completely uninformative; or only two possible signals,* $\{0, 1\}$, *and a decision rule under which a marginal buyer becomes a buyer if and only if the signal 1 is received.*

This lemma states that marginal buyers choose to receive either completely uninformative signals or a binary signal, and their decision rule is deterministic upon receiving a signal. Providing that a signal indicates which action is optimal, no more accurate information on the state is needed; finer information does not expand the set of optimal strategies, and it only increases the cost of information. In addition, the decision rule is nonrandom given a signal. If the entrepreneur received the signal s that makes him indifferent between the two actions (i.e., $\underline{A} = \frac{1}{p}\int_{\lambda} X(\lambda)\, dF(\lambda|s)$), his decision would be random. In such a case, however, he can decrease information acquisition costs by choosing not to receive such signals without incurring any loss of utility.

Each entrepreneur acquires information privately and independently; none of the entrepreneurs can observe signals received by others. In addition, signals received are uncorrelated each other. This is because the objective of the entrepreneur does not directly depend on the decision of others; although others' information choices will affect the realized asset price at date 1, each entrepreneur takes beliefs about the future asset price as given. Therefore the law of large number applies; a fraction of the marginal buyers who become buyers, $\delta(\lambda)$ – equivalently, the probability of being a buyer in state λ– is equal to the conditional proba-

[17] Agents correctly understand the model and therefore, knowing $X(\lambda)$ is equivalent to knowing λ.

bility $f(s|\lambda)$ of receiving the signal 1 in state λ.[18] Then, the optimal information acquisition problem consists of choosing $\delta(\lambda)$ which maximizes the expected utility taking the asset price $p(\lambda)$ and the prior distribution $F(\lambda)$ as given.

$$\max_{\delta(\cdot)} J(\delta) - \theta I(\delta) \tag{10}$$

where

$$J(\delta) \equiv \int_\lambda [1 + \frac{X(\lambda)}{p(\lambda)}]\delta(\lambda) + (1 + \underline{A})(1 - \delta(\lambda))dF(\lambda); \tag{11}$$

$$I(\delta) = \int_\lambda \delta(\lambda) \log \delta(\lambda) + (1 - \delta(\lambda)) \log(1 - \delta(\lambda))dF(\lambda) - \bar{\delta} \log \bar{\delta} - (1 - \bar{\delta}) \log(1 - \bar{\delta}); \tag{12}$$

$$\bar{\delta} = \int_\lambda \delta(\lambda)dF(\lambda).$$

Note that $1 + \frac{X}{p}$ is the consumption of an entrepreneur who buys assets at date 2, and $1 + A_j$ is the consumption of an entrepreneur with productivity A_j who neither buys nor sells. Then the opportunity costs of being a buyer in state λ, equivalently the profit function, is given by

$$L(\lambda) \equiv \frac{X(\lambda)}{p(\lambda)} - \underline{A},$$

and the solution to (10) is equivalent to the one to the following problem:

$$\max_{\delta(\cdot)} \int_\lambda \delta(\lambda)L(\lambda)dF(\lambda) - \theta I(\delta) \tag{13}$$

where $p(\lambda)$ and $F(\lambda)$ are given as common knowledge.

This problem can be solved using a similar method as shown in Woodford (2008). However, because the shape of the loss function $L(\lambda)$ depends on the price $p(\lambda)$, further qualifications are required to have the complete set of necessary and sufficient conditions. I shall focus on a stable equilibrium.

Proposition 1 *(Sufficient and Necessary Conditions) For each λ, let $\delta^*_{\bar{\delta}}(\lambda)$ be the highest solution of (14) taking $\bar{\delta}$ as given:*

$$\frac{X(\lambda)}{p(\lambda)} - \underline{A} = \theta[\log \frac{\delta^*_{\bar{\delta}}(\lambda)}{1 - \delta^*_{\bar{\delta}}(\lambda)} - \log \frac{\bar{\delta}}{1 - \bar{\delta}}] \tag{14}$$

[18] Subscript j is omitted since the low productive entrepreneurs are homogeneous at date 0. However, their decision at date 1 may be different if a signal is drawn from a stochastic process.

where $X(\lambda)$ and $p(\lambda)$ are consistent with (8) and (9). Then the information choice $\delta^*_{\bar{\delta}^*}(\lambda)$ is a stable equilibrium if and only if

(i) $\bar{\delta}^* = J^*\left(\bar{\delta}^*\right)$ where $J^*\left(\bar{\delta}\right) \equiv \int \delta^*_{\bar{\delta}}(\lambda)dF(\lambda)$;

(ii) If $\bar{\delta}^* > 0$, then there exits $\varepsilon > 0$ such that $J^*(\bar{\delta}) > \bar{\delta}$ for all $\bar{\delta} \in (\bar{\delta}^* - \varepsilon, \bar{\delta}^*)$. If $\bar{\delta}^* < 1$, then there exits $\varepsilon > 0$ such that $J^*(\bar{\delta}) < \bar{\delta}$ for all $\bar{\delta} \in (\bar{\delta}^*, \bar{\delta}^* + \varepsilon)$.

Equation (14) implies that the marginal benefit of being a buyer must be equal to the marginal information acquisition cost which is the product of the cost of acquiring an additional unit of information, θ, and the marginal quantity of information which is needed to become a buyer given $\bar{\delta}$. By defining δ^* as the highest solution, I suppose that the equilibrium price is given by the highest solution provided $\bar{\delta}$.[19] Then for fixed $\bar{\delta}$, there exists a unique price function $p_{\bar{\delta}}(\lambda)$ that is consistent with the market clearing condition (9). This notation will be useful in the remainder of this paper.

Condition (i) in the proposition states that an equilibrium $\bar{\delta}^*$ is a fixed point, although it does not guarantee that an information choice is optimal. Condition (ii) imposes the optimality of an information choice as well as the stability of equilibrium at a fixed point. Because the stability issue arises when there are multiple fixed points, I will revisit this matter in Section 3.3.

3.2 Existence

In the next two sections, I describe the existence and multiplicity of equilibria. The next proposition shows the existence of equilibria and characterizes the optimal information choice in the special cases.

Proposition 2 *(Existence) There exists a fixed point $\bar{\delta}^*$ that satisfies the sufficient and necessary conditions for an equilibrium.*

[19] Chari, Shourideh, and Zetlin-Jones (2011) and Kurlat (2013) adopt similar assumptions to determine an equilibrium asset price. Doing so, we can eliminate obvious bilateral gains from trade that are not being exploited when they form price expectations given expected liquidity $\bar{\delta}$. (It is easy to see that the price is increasing in $\delta(\lambda; \bar{\delta})$ from (9). Sellers gain profits from a higher price. Also, buyers' gain is maximized at the highest price. To see this, notice that the return to a buyer $\frac{X}{p}$ is increasing in $\delta^*(\lambda; \bar{\delta})$ by (14). Furthermore, our multiplicity result in Section 3.3 does not rely on this setup.

Let Δ *be a set of potential equilibria, and* $E_{\theta}^{\bar{\delta}}$ *be the* θ*-adjusted expectation operator defined by* $E_{\theta}^{\bar{\delta}}[R(\lambda)] \equiv Z_{\theta}^{-1}[\int_{\lambda} Z_{\theta}(R(\lambda;\bar{\delta}))dF(\lambda)]$*, with* $Z_{\theta}(R(\lambda;\bar{\delta})) \equiv \exp(\frac{R(\lambda;\bar{\delta})}{\theta})$ *and* $R(\lambda;\bar{\delta})$ *being evaluated at* $\bar{\delta}$*. Then,*

(i) $\bar{\delta} = 0 \in \Delta$ *if and only if*

$$\underline{A} \geq E_{\theta}^{\bar{\delta}=0}[\frac{X}{p}]; \tag{15}$$

(ii) $\bar{\delta} = 1 \in \Delta$ *if and only if*

$$\underline{A} \leq E_{-\theta}^{\bar{\delta}=1}[\frac{X}{p}]; \tag{16}$$

and (iii) $\bar{\delta} \in \Delta$ *such that* $0 < \bar{\delta} < 1$ *if*

$$E_{-\theta}^{\bar{\delta}=1}[\frac{X}{p}] < \underline{A} < E_{\theta}^{\bar{\delta}=0}[\frac{X}{p}]. \tag{17}$$

There are three important variables to focus on here: fixed point $\bar{\delta}$, the asset price $p_{\bar{\delta}}$ which is evaluated at $\bar{\delta}$, and information cost adjusted expected returns, $E_{\theta}^{\bar{\delta}=0}[\frac{X}{p}]$ and $E_{-\theta}^{\bar{\delta}=1}[\frac{X}{p}]$.[20] The first part of the proposition states that the case in which the marginal buyers become keepers surely $-$ $\bar{\delta} = 0 \Leftrightarrow \delta(\lambda) = 0$ almost surely $-$ qualifies as an equilibrium if and only if the returns from being a keeper, $\underline{A}$, is equal or greater than the θ adjusted expected return from being a buyer taking $p_{\bar{\delta}=0}$ as given. The θ-adjusted expectation operator places greater weights on the states where an asset return is higher. This implies that zero liquidity supply without acquiring information qualifies as an equilibrium, only if there is no upside potential in an asset return.

The second part indicates that the case in which the entrepreneurs become buyers surely $-$ $\bar{\delta} = 1 \Leftrightarrow \delta(\lambda) = 1$ almost surely $-$ qualifies as an equilibrium if and only if $\underline{A}$ is equal or less than the $-\theta$ adjusted expected returns from buying assets taking $p_{\bar{\delta}=1}$ as given. The $-\theta$-adjusted expectation operator places greater weights on the states where an asset return is lower. This implies that the marginal buyers purchase assets for sure without acquiring information, only if there is no significant downside risk.

The third part describes a situation in which there are significant upside and downside risks. As neither buying nor keeping surely is justifiable in terms of the adjusted expected returns, there is an interior solution of $\bar{\delta}$, with which an optimal information choice of $\delta(\lambda)$ depends on λ.

[20] $p_{\bar{\delta}}$ is the market clearing price given the asset demand function $\delta_{\bar{\delta}}(\lambda)$ that satisfies equation (14).

Note that information is obtained if and only if an interior $\bar{\delta}$ is chosen, in which case the probability of the action depends on the state. In contrast, if either of the two polar cases, $\bar{\delta} = 0$ or $\bar{\delta} = 1$, is chosen as an equilibrium outcome, the action of the entrepreneur is independent of the state; in those cases, at date 0, the entrepreneurs predetermine their financial decisions at date 1; no information is obtained, and signals received, if any, are uninformative.

Let us call liquidity that is supplied by the marginal buyers by private liquidity. Then $\bar{\delta}$, which is the outcome of individual information choices, represents the measure of the expected amount of *private liquidity* available at date 1, and $\delta_{\bar{\delta}}(\lambda)$ the corresponding measure of private liquidity in the state λ. $\bar{\delta} = 0$ describes the situation in which private liquidity reaches its minimum level, as there are no agents who buy assets from other agents who wish to sell their assets in the market. Similarly, private liquidity reaches its maximum level if $\bar{\delta} = 1$, as all the marginal buyers spend their entire liquidity on purchasing assets without knowing actual λ. I shall use, interchangeably, 'private liquidity,' 'asset demand,' and 'information choice.'

The asset price $p_{\bar{\delta}}$ represents liquidity available to sellers, which reflect both private and public liquidity (possibly with government asset purchases). To gain more intuitions from the expressions, let us consider the following example.

Example 1 *Assume that the prior distribution of the gain X from an asset is normally distributed with mean μ and variance σ^2.*[21] *Then expressions (15) and (16) become*

$$\underline{A} \geq E^{\bar{\delta}=0}\left[\frac{X}{p}\right] + \frac{1}{2}\frac{\sigma_0^2(\sigma)}{\theta} \tag{18}$$

and

$$\underline{A} \leq E^{\bar{\delta}=1}\left[\frac{X}{p}\right] - \frac{1}{2}\frac{\sigma_1^2(\sigma)}{\theta} \tag{19}$$

where $E^{\bar{\delta}}\left[\frac{X}{p}\right]$ is the expected return evaluated at $\bar{\delta}$; $\sigma_0^2(\sigma)$ and $\sigma_1^2(\sigma)$ are the variance of the corresponding asset return, both of which are increasing in σ^2. Then,

(i) for sufficiently large $\frac{\sigma^2}{\theta}$, the entrepreneurs do obtain information, i.e., $0 < \bar{\delta} < 0$;

[21] For the convenience of using a normal distribution, X can be slightly generalized: $X' = \varepsilon X$, where ε can be thought of as an idiosyncratic shock.

(ii) if $\underline{A} < E^{\bar{\delta}=1}\left[\frac{X}{p}\right]$ *or* $\underline{A} > E^{\bar{\delta}=0}\left[\frac{X}{p}\right]$, *then for sufficiently small* $\frac{\sigma^2}{\theta}$, *there exists an equilibrium in which the entrepreneurs do not obtain any information.*

The example shows the conditions under which the information is not acquired. The adjustment terms in inequalities (18) and (19) indicate that information choices depend on the uncertainty σ^2 and the information cost θ, as well as the expected return in the two polar cases. For large enough $\frac{\sigma^2}{\theta}$, inequalities (18) and (19) are violated; information is obtained, for gains from obtaining information increase as economic uncertainty rises. In such a case, the entrepreneur's financial decision will be probabilistically tied to the state λ. This implies that the uncertainty σ^2 should not be too high relative to the information cost θ in order for information not to be acquired.

3.3 Multiplicity

Multiple equilibria would emerge if there are multiple fixed points $\bar{\delta} = J^*(\bar{\delta})$ that are consistent with the equilibrium conditions.[22] For instance, one observes that in Example 1, if $E^{\bar{\delta}=0}\left[\frac{X}{p}\right] < \underline{A} < E^{\bar{\delta}=1}\left[\frac{X}{p}\right]$, then for sufficiently large θ, there are at least the two corner solutions, $\bar{\delta} = 0$ and $\bar{\delta} = 1$.

Proposition 3 *(Multiplicity) If the information cost is greater than 0,* $\theta > 0$, *there may exist multiple equilibria.*

Figure 1 describes a case in which multiple equilibria exist. In this figure, there are multiple equilibrium fixed points: two stable equilibria, $\bar{\delta}^1 = 1$ and $0 < \bar{\delta}^2 < 1$, which are indicated by arrows, and one unstable equilibrium, $0 < \bar{\delta}^3 < 1$, which is indicated by dots. One observes that only stable equilibria satisfy the condition (ii) in Proposition 1.

Note that a set of the private liquidity $\bar{\delta}$ that satisfies the conditions in Proposition 1 constitutes a set of equilibria.[23] This implies that there is a set of the equilibrium price $p_{\bar{\delta}}$

[22] Yang (2013) also shows that multiplicity may arise with the flexible information acquisition mechanism in a different context. In his context, there exists a unique equilibrium $\bar{\delta}$, but multiple $\delta(\lambda)$ given $\bar{\delta}$ are possible. In my framework, there may be multiple equilibria $\bar{\delta}$, but $\delta(\lambda)$ is unique given $\bar{\delta}$.

[23] In Woodford (2008), there must be a unique equilibrium choice of information. The reason for this difference is that the profit function here depends on $\bar{\delta}$. See also Lemma A.1 in Appendix B.

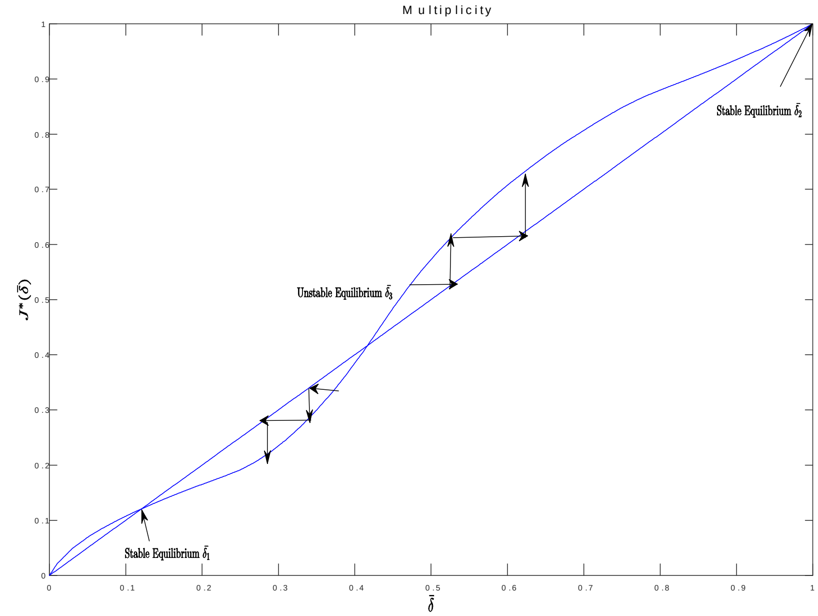


Figure 1

with being indexed by $\bar{\delta}$.[24] The asset price $p_{\bar{\delta}}$ is increasing in the private liquidity $\bar{\delta}$, which is maximized when $\delta(\lambda) = 1$. However, for any fixed $\bar{\delta}$, the equilibrium price function $p_{\bar{\delta}}$ and the individual information choice $\delta(\lambda)$ are unique.[25] Therefore we can index equilibria by the private liquidity $\bar{\delta}$.

Nevertheless, it does not imply that the agents are able to choose whatever equilibrium they wish to reach. Individual agents do not take into account the general equilibrium impact of their information choices on the asset price as they take it as given. Individual decisions on the information $\delta(\lambda)$ depend on their common beliefs about the price $p_{\bar{\delta}}$.

Alternatively, as $p_{\bar{\delta}}(\lambda)$ is the function of $\bar{\delta}$, one can interpret it as follows. When each marginal buyer makes the information choice $\delta(\lambda)$, he needs to forecast others' information choices, thereby the level of private liquidity $\bar{\delta}$ in the economy. The forecast of the private

[24] When $J^*(\bar{\delta})$ is computed to seek an equilibrium asset price function, one substitutes all those market clearing conditions into equation (14) to solve for $\delta^*_{\bar{\delta}}(\lambda)$. In so doing, one associates each $\bar{\delta}$ with a unique corresponding $p_{\bar{\delta}}(\lambda)$; the shape of the profit function depends on $p_{\bar{\delta}}(\lambda)$ and thereby $\bar{\delta}$ itself. See also Proposition 1 in Appendix B.

[25] For any fixed $\bar{\delta}$, the profit function L does not depend on $\bar{\delta}$, in which case solutions are unique as in Woodford (2008).

liquidity $\bar{\delta}$ is important when there are multiple equilibria. In such cases, there are *strategic complementarities* in the information choice across the marginal buyers, in a sense that equilibrium is driven by the agent's forcast about private liquidity $\bar{\delta}$. When a marginal buyer forecasts that private liquidity $\bar{\delta}$ is high, he increases his likelihood of liquidity supply by arranging his information choice $\delta(\lambda)$. Different forecasts of others' information choices can bring about different information choices.

One natural question is why such multiple equilibria emerge in some cases. First, multiplicity does not arise in the case of a rational expectation equilibrium, where the cost of information θ equals 0. If there are no costs associated with learning about λ, information choices are trivial. Multiplicity arises only because there may be multiple information choices consistent with equilibrium, which is more likely if the information cost θ is high.

Second, multiplicity is more likely if there is a high positive correlation between the the gain from buying an asset, $X(\lambda)$, and the private liquidity supply $\bar{\delta}$. The correlation between $X(\lambda)$ and $\bar{\delta}$ is high when an economy is densely populated with marginal sellers. In such a case, a higher asset price, which is led by greater private liquidity $\bar{\delta}$, will have a greater impact on the gain $X(\lambda)$ as more marginal sellers sell their non-lemons.

Observation 1 *(i) Multiple equilibria are more likely if the information cost θ is greater; (ii) Multiple equilibria exist only if there is a high positive correlation between the gain from buying an asset, $X(\lambda)$, and the private liquidity supply $\bar{\delta}$.*

For example, let us consider the two stable equilibria, $\bar{\delta}^1 = 1$ and $0 < \bar{\delta}^2 < 1$. In the case $\bar{\delta}^1 = 1$, the marginal buyers anticipate that the high asset price will induce marginal sellers to sell their nonlemons in the market, which leads to the improvement of the mean quality X of the assets traded. Given that the gain X from buying assets traded is high enough, it is optimal for the marginal buyers not to obtain any information to minimize the cost of information and become buyers surely. At date 1, the marginal sellers do sell in the market facing such a high asset price caused by high demand, which is consistent with the anticipation by the marginal buyers. The equilibrium $\bar{\delta}^1 = 1$ is self-fulfilling, in which case private liquidity is maximized.

Likewise, in the case that $0 < \bar{\delta}^2 < 1$, the expected asset price, equivalently asset demand, is not high enough in some states to induce some marginal sellers to sell; the marginal buyers anticipate the deterioration of the gain X from buying assets traded in some states. Compared to the case $\bar{\delta}^1 = 1$, such relatively low expected demand not only decreases the expected asset return, but also increases the dispersion of the return, which increases benefits of information acquisition as shown in Example 1. At date 1, in some states, the marginal sellers are discouraged to enter, as some marginal buyers with negative information on λ leave the market; the forecast of $\bar{\delta}^2$ turns out to be correct.

A natural question is whether a policymaker can induce a desirable equilibrium if one equilibrium is more efficient than the others, and what kinds of policy instruments are useful to achieve this goal if that is possible. I will examine this question in Appendix D.

Because of multiplicity, all endogenous variables such as $X(\lambda)$ and $\delta(\lambda)$ depend on the shape of the price function $p_{\bar{\delta}}(\lambda)$ (or $\bar{\delta}$ identically), i.e., $X_{\bar{\delta}}(\lambda)$. For better readability, such notation is omitted wherever possible.

3.4 Asset Market Volatility and Information Cost

How does the cost of information acquisition affect asset market volatility? In the model, the lemon shock λ is the unknown state which agents attempt to learn about. It is intuitively clear that asset demand $\delta(\lambda)$ becomes less sensitive to the unknown state λ as the information cost θ increases (Figure 2). When the information cost θ is low, the asset demand $\delta(\lambda)$ varies widely across the state λ, especially around the cutoff value of $\bar{\lambda} \approx 0.45$, at which the net asset return is 0. When the information cost θ is high, the asset demand $\delta(\lambda)$ becomes flatter across the state. As equation (14) indicates, agents are less willing to deviate from the mean level $\bar{\delta}$ with a higher information cost. Therefore, the volatility of asset demand $\delta(\lambda)$ with respect to the unknown state λ is decreasing in the information cost θ.

Then, how much does the asset demand $\delta(\lambda)$ respond to arbitrary exogenous shocks which affect the asset return? Example 1 indicates that if the information cost θ is large, exogenous shocks that affect the asset return can have significant consequences in information choices, thereby private liquidity supply. I explore the influence of arbitrary exogenous shocks that are known to agents and are orthogonal to the state λ (e.g., shock to investment productivity

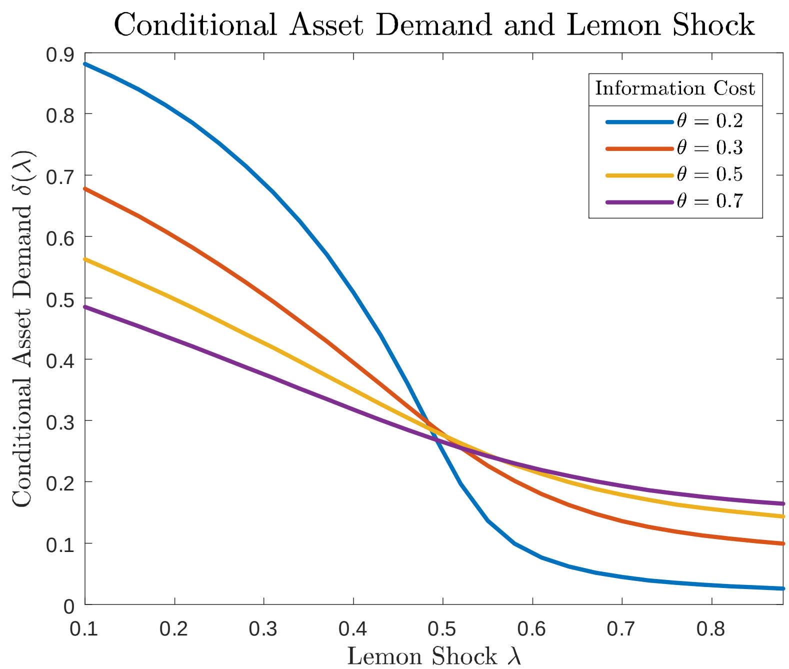


Figure 2

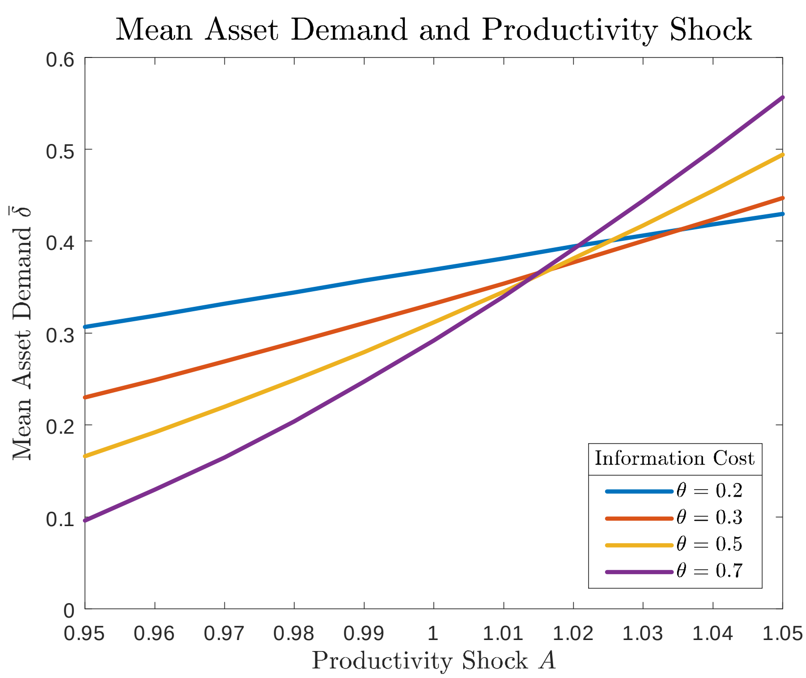


Figure 3

A, government demand shock). The following proposition shows that the sensitivity of the mean asset demand $\bar{\delta}$ with respect to arbitrary exogenous disturbances is increasing in the information cost θ.

Proposition 4 *Let ξ be an arbitrary exogenous disturance which increases the asset return, $\frac{\partial \frac{X}{p}}{\partial \xi} > 0$. The mean asset demand $\bar{\delta}$ is increasing in ξ for $0 < \bar{\delta} < 1$, $\frac{\partial \bar{\delta}}{\partial \xi} > 0$. Moreover, the responsiveness of the mean asset demand $\bar{\delta}$ is increasing in the information cost, $\frac{\partial^2 \bar{\delta}}{\partial \xi \partial \theta} > 0$.*

The proposition shows that higher information acquisition cost θ increases the sensitivity of the mean asset demand $\bar{\delta}$ to known exogenous disturbances. While the conditional demand $\delta(\lambda)$ deviates less from the mean demand $\bar{\delta}$, the mean itself fluctuates more in response to exogenous disturbances.

In order to further investigate implications on asset market volatility, I turn to numerical methods to simulate the model, due to the complications from the fixed point problem. In the numerical experiment, I use the Pareto distribution for investment productivity A, $G(A;\xi) = 1 - \left(\frac{A_{\min}}{\xi A}\right)^{\alpha}$, where ξ is an exogenous shock to the investment productivity. The Pareto distribution is useful to understand the model as its density is monotone and has one parameter α. I set α at 2, in which case multiplicity does not arise.[26]

Then I allow ξ to vary from 0.95 to 1.05, and compute a market equilibrium for each $\theta \in \{0.2, 0.3, 0.5, 0.7\}$. Figure 3 is consistent with Proposition 4. Even though higher information costs dampen the responsiveness of conditional demand $\delta(\lambda)$ to λ (Figure 2), the mean asset demand $\bar{\delta}$ becomes more susceptible to an orthogonal exogenous disturbance.

This insight motivates us to evaluate the volatility of the other key variables. Figure 4 reports the variance of the asset demand $\delta(\lambda)$ conditional on the state λ. For any given state of λ, the conditional variance of the asset demand is higher with a greater information cost. When the information cost θ is low, the conditional volatility is more concentrated around the cutoff value of $\bar{\lambda}$, at which the net profit of a buyer is 0. As the information cost θ increases, there is not only an increase in the conditional variance around the cutoff value,

[26] With higher α, the investment productivity of marginal sellers is more densely distributed toward $A_{\min}$. In such a case, multiplicity emerges as there is some range of asset prices, in which its impact on the quality X is high.

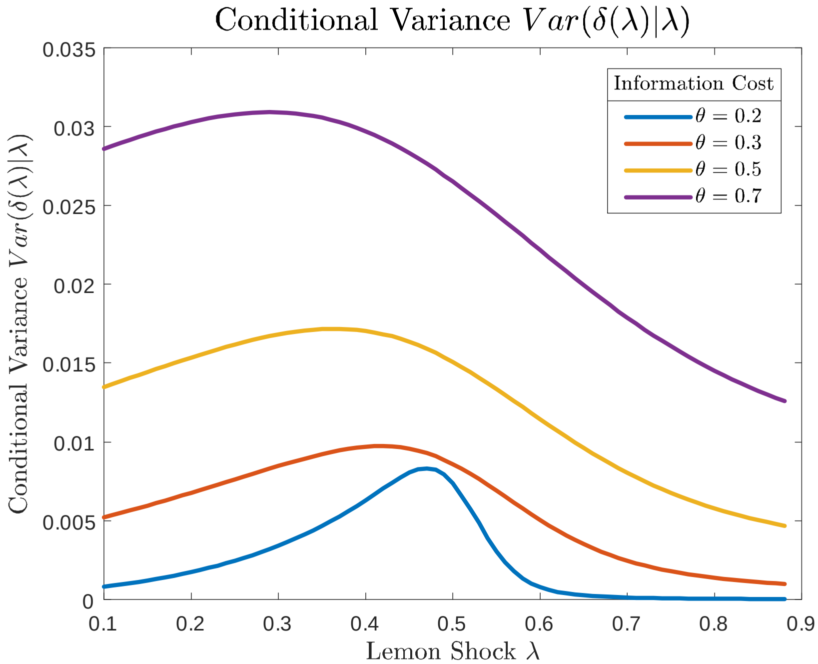


Figure 4

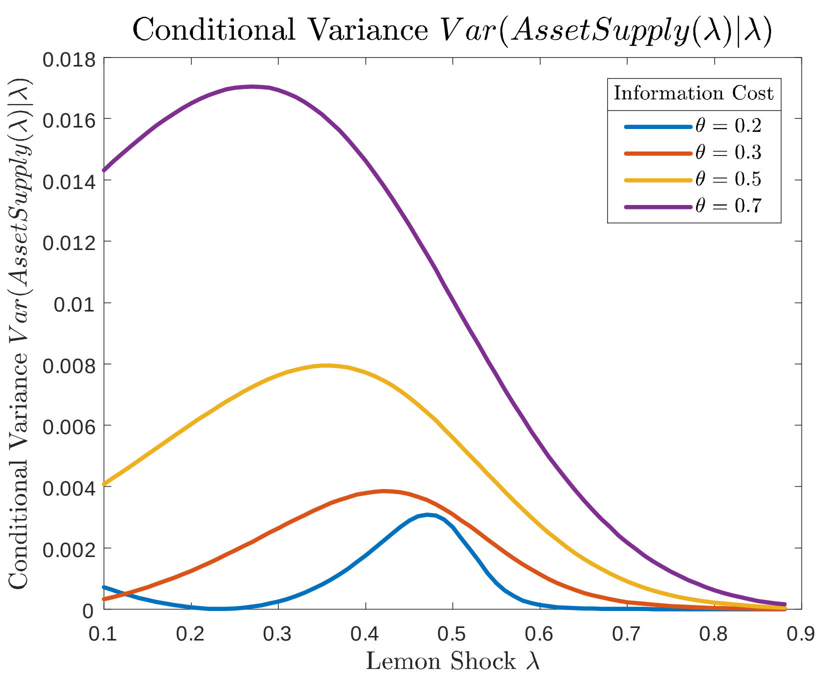


Figure 5

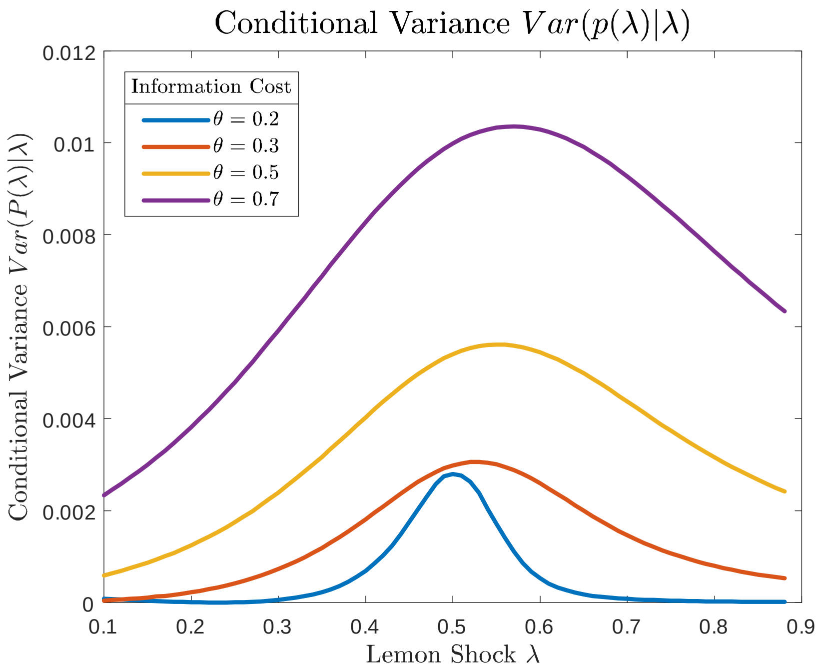


Figure 6

but also is a significant increase in the conditional variance in the states where they are far from the cutoff value.

How does asset demand volatility affect an asset price? To assess this question, we need to observe asset supply volatility as well. Figure 5 indicates that asset supply covaries highly with asset demand for a low value of λ, the state in which the volatility of the asset supply is high. For a higher value of λ, the volatility of the asset supply is low, as the composition of assets existing in an economy is dominated by lemons. For such a state, the asset supply covaries less with asset demand.

Figure 6 shows the overall picture of the volatility of the asset price. The conditional variance of the asset price, $Var\left(p\left(\lambda\right)|\lambda\right)$, is increasing in the information cost θ. The conditional variance is relatively low for a low value of λ. And it remains at a relatively higher level for the highest value of λ. Figure 7 also shows that the mean of the conditional variance $E\left[Var\left(p\left(\lambda\right)|\lambda\right)\right]$ is increasing in the cost of information θ.

The discussion so far indicates that large fluctuations in the supply of private liquidity may occur in a market where the information cost θ is high. According to Proposition 4, similar implications apply for any other exogenous disturbances. Government direct aseet

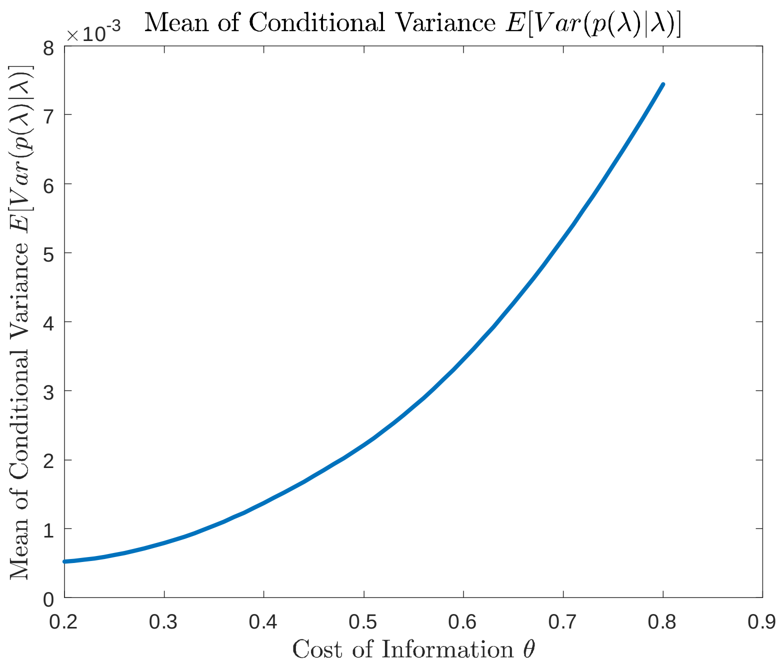


Figure 7

purchases also generate similar effects on asset market volatility. In particular, the large-scale direct asset purchases can crowd out a large amount of private liquidity. This topic will be explored in Section 4.3.3.

4 Policy Analysis

I next turn to the analysis of the normative properties of the model. I will define a notion of constrained efficiency in the context of our framework. Then I will explore the optimal policy when policymakers have a concern about potential efficiency losses.

4.1 Constrained efficiency

Although agents are heterogeneous ex post, they are identical ex ante. Let us define the welfare objective of a social planner as ex ante expected utility.

$$W = \int_{\lambda} \int_{i} c_i(\lambda) di dF(\lambda) \tag{20}$$

This welfare objective can be also regarded as a utilitarian aggregator of individual utility.[27]

[27] For a clear exposition and numerical simulations, the objective does not incorporate information costs. Those costs may be regarded as non pecuniary.

Because of the linear preferences in consumption, the efficient distribution of consumption is indeterminate. However, maximizing the welfare objective amounts to maximizing the expected aggregate output in our environment. This implies that once the social planner finds a strategy that maximizes aggregate output, she can redistribute the wealth among agents by levying lump-sum taxes.

A feasible allocation must satisfy the resource constraints.

$$\int_i i(s_i, A_i, \lambda_i) di \leq 1 + N, \tag{21}$$

$$\int_i c_i(\lambda) di \leq (1 - \lambda) N + \int_i A_i i(s_i, A_i, \lambda_i) di - T(\lambda), \tag{22}$$

after dropping some constants. Inequality (21) states that the date-1 aggregate investment must be less than the aggregate private liquidity available. Inequality (22) represents the date-2 aggregate resource constraint. The first part of the righthand side, $(1 - \lambda) N$, amounts to dividends from the legacy assets. The second part, $\int A_i i_i(s_j, A_j, \lambda_j) di$, represents dividends from the assets newly produced. The third part, $T(\lambda)$, considers potential efficiency losses from policy interventions. I do not take a strong stand on the exact source of the efficiency losses. It could arise because of problems, ranging from moral hazard to distortions from taxation.

Lemma 3 *After dropping some constants, the welfare criterion objective can be rewritten as*

$$W = \int_\lambda p(\lambda) \left[\left(\bar{A} - \underline{A} \right) - (1 - \lambda) \int_0^{1/p} \left(A - \underline{A} \right) dG(A) \right] - T(\lambda) dF(\lambda), \tag{23}$$

where $\bar{A}$ is the mean value of the investment productivity among the marginal sellers.

The welfare criterion becomes the expected value of aggregate amount of capital, which is liquidity supplied to a seller, p, times the average investment productivity (the terms inside the bracket) less the efficiency cost, $T(\lambda)$.

If the social planner can tell entrepreneurs apart by investment productivity and collect all existing liquidity available in the economy from the entrepreneurs, it is clear that the planner can improve upon a market equilibrium and achieve the first-best allocation by transfering all liquidity to the most productive entrepreneurs. However, the implementation of this policy is probably infeasible. Investment productivity as well as the type of a tree is private information in our environment. The planner may not have better knowledge than participants in a market transaction. In addition, it seems implausible that the planner can forfeit such a large scale of private property.

Therefore I consider a notion of constrained efficiency where the fictitious planner is constrained in the sense that she does not have any better knowledge than the private sector so that she is not allowed to directly transfer liquidity among entrepreneurs. However, the fictitious planner can dictate to each entrepreneur how to acquire and use information. Then private buying and selling decisions are consistent with the planner's information choice $\delta(\lambda)$, and liquidity is redistributed via the market mechanism. This notion of constrained efficiency serves to identify the best way of obtaining and using information if entrepreneurs were to internalize the impact of their information choice on the others' utility, while their decisions are solely based on their own information.

Definition 2 *(Constrained efficient allocation) An constrained efficient allocation is a collection of an information choice $\delta(\lambda)$, an asset price $p(\lambda)$, consumption $c(\lambda)$, and investment $i(s, A, \lambda)$ that maximizes (23) subject to the private sector behavior that is given by Proposition 1, (8), and (9).*

The main question is whether the fictitious planner can improve upon the market allocation by commanding a different information choice while respecting all budget, resource constraints, and letting entrepreneurs trade freely in the asset market.

Lemma 4 *(i) (Constrained efficient allocation) Suppose $T(\lambda) = 0$. The constrained efficient allocation is unique and is given by*

$$\delta(\lambda) = 1 \text{ almost surely,} \tag{24}$$

This implies that the market liquidity $p_{\bar{\delta}=1}$ is maximized, and the entire liquidity held by the low productive entrepreneurs is transferred to more productive entrepreneurs.
(ii) A market outcome is constrained inefficient unless inequality (16) is satisfied.

Efficiency is achieved only if the marginal buyers transfer their entire liquidity to more productive entrepreneurs with probability 1 in all states, i.e., $\bar{\delta} = 1$. This can be possible only if the marginal buyers do not obtain any information. Notice that inefficiency is driven by an individual information choice. For instance, even if small enough λ is realized, once acquiring information on the aggregate state λ, there are always some marginal buyers who obtain incorrect information due to the limited information production capacity. As a result, there is an insufficient amount of private liquidity in the market.

If allocation is efficient, there are no keepers in the economy: entrepreneurs either sell or buy without any information acquisition.[28] This principle is also stated in Ordonez and Gorton (2013): "Opacity can dominate transparency and the economy can enjoy a blissful ignorance."

In the following sections, I will discuss what kinds of policies are useful to improve economic welfare if policymakers have some degree of concern about potential efficiency losses, $T(\lambda) \geq 0$.

4.2 Feasible Instruments

The preceding analysis suggests that policy aimed at increasing market liquidity may achieve efficiency. In the recent financial disruption, a set of the targeted asset purchases is particularly aimed at enhancing liquidity in private securities markets (Reis 2009). Before proceeding to the evaluation of the efficacy of such government interventions, it is worth discussing which kinds of government interventions have this property.

There are a variety of polices that may drive a wedge between the asset price and the underlying private valuations. Such policies include transaction subsidies, asset purchases and loss insurance. As for transaction subsidies, it is practically difficult to implement because the government must have knowledge on the current state λ in order to determine the appropriate amount of subsidy.[29]

Instead, I will consider loss insurance as the main policy instrument. Loss insurance

[28] The constrained efficient allocation coincides with the second best allocation in which the only friction is that the planner cannot observe the types A : only individual private decision rules are binding.

[29] Also, as Kurlat (2013) noted, the same people can trade the same project several times back and forth and collect the subsidy from each transaction. In order to prevent such a subsidy collecting activity, the government needs to keep tracking of transaction records, and purchases from an original owner need to be solely subsidized, which is a difficult task.

can be understood as a policy that aims to raise the asset price indirectly. It subsidizes buyers directly against potential losses, and it subsidizes sellers indirectly because the asset price rises as demand increases. In the next section, I study the optimal loss insurance, and then compare the efficacy of the optimal loss insurance with government direct asset purchases. Unlike the loss insurance, direct asset purchases raise the asset price directly, and thus subsidizes sellers and taxes buyers indirectly.

In the implementation of such instruments, policymakers are assumed not to have better knowledge than the private sector: the quality of each asset traded as well as the current state λ at date 1 is unknown to policymakers. However, it becomes common knowledge at date 2, for agents can figure out which assets turn out to be lemon. Investment productivity is private information as well at date 1.

4.3 Ramsey Policy in Lemons Markets

In this section, I explore the efficient government interventions that can be used to remedy a market failure. Therefore, I suppose that a set of possible market equilibria does not involve the efficient allocation so as to provide scope for government interventions. In particular, more attention will be given to the case in which the market breaks down.

I study the Ramsey policy mainly in the context of loss insurance. As it will be clear in Section 4.3.3, loss insurance is a more efficient way to implement optimal allocation, because it does not crowd out private liquidity supply. Moreover, this approach sheds light on the optimal security design.

Let us consider the government's strategy of loss insurance as follows. At the beginning of period 0, the government introduces loss insurance to cover lemons that will be traded in the market.[30] Buyers of assets are guaranteed to receive $\kappa(\lambda)$ consumption goods at date 2 for each asset they bought in the previous period. Notice that the policy $\kappa(\lambda)$ is state dependent. With the loss insurance, the average payoff X is modified as

$$X(\lambda, \kappa(\lambda)) = 1 - \frac{\lambda N}{S^A(\lambda)} + \kappa(\lambda). \tag{25}$$

The loss insurance policy has a direct impact on the asset return $\frac{X}{p}$ initially, which causes

[30] The government acts before the private sector obtains information in order to affect the private sector's information choice.

higher asset demand. Higher asset demand leads to a higher asset price, which induce the marginal sellers to enter the market and sell their nonlemons. This further improves the asset quality traded in the market, which produces a virtuous circle.

The cost of the program is financed with lump sum taxes at date 2, so that it does not require policymakers to raise money at date 1. Instead, we may allow some efficiency costs $T[\kappa(\lambda)]$ which is associated with implementing the loss insurance policy, possibly due to moral hazard or taxation.

The policy $\kappa(\lambda)$ can benefit an economy through its effect on the asset price p. While it does not appear directly in the market clearing condition (9), the policy has an impact on the asset demand (equation (14)). And as can be seen the welfare criterion (23), a higher asset price increases not only the amount liquidity supplied to a seller, but also the average investment productivity by reallocating liquidity from low to high productive investors.

The Ramsey policy $\kappa(\lambda)$ maximizes the objective (23) and it must satisfy the private sector equilibrium conditions: Proposition 1 and 1 together with expression (25), and the market clearing condition (9).

4.3.1 Optimal Loss Insurance Without the Efficiency Loss

A lack of aggregate demand causes losses to both potential sellers and buyers; a low asset price implies that the sellers are left with an insufficient amount of liquidity to invest in profitable projects as well as they are likely to stay out of the market; buyers may also lose from a lack of aggregate demand if a low asset price discourages marginal sellers from selling their high quality assets. Policy is aimed at filling a lack of private demand.

I begin with a simple case in which there are no efficiency costs arising from policy interventions (e.g., moral hazard), $T(\lambda) = 0$. In this case, the maximum private liquidity, $\bar{\delta} = 1$, as in Lemma 4 is always optimal.

Proposition 5 *(Optimal Loss Insurance without the Efficiency Loss: Partial Loss Insurance) (i) The optimal loss insurance $\kappa(\lambda)$ implements the efficient allocation only if it satisfies the following equation:*

$$\underline{A} = E_{-\theta}^{\bar{\delta}=1}[\frac{X(\lambda, \kappa(\lambda))}{p(\lambda)}]. \tag{26}$$

(ii) With the credible public announcement of the asset price target $p^{\bar{\delta}=1}$ the optimal loss insurance implements the efficient allocation as the unique equilibrium.

Proposition 5-(1) is the necessary condition for the optimal loss insurance. But the insurance policy that satisfies (26) alone does not ensure the efficient allocation, for it may cause a multiplicity of equilibria to arise. To avoid an undesirable equilibrium, it is important to pin down the private sector's forecasts about the asset price, to the price that is consistent with the efficient allocation. As Appendix D shows, the credible public announcement of the asset price target $p^{\bar{\delta}=1}$ functions as a threat that pins down the private sector's expectations. As a result, this policy implements the efficient allocation as the unique equilibrium. In this regard, the public announcement is a useful dimension of policy to overcome a multiplicity of equilibria that may emerge with the partial loss insurance.

When the equation (26) is satisfied, buyers do not acquire any information. It implies that the policy should be designed in a way that reduces the variance of the asset return across states to discourage information acquisition. One example is the loss insurance policy that makes the asset return $\frac{X}{p}$ constant and equal to $\underline{A}$. One concern with this policy is that policymakers need to subsidize heavily in a bad state and raise a large amount of taxes in a good state, which may cause efficiency losses, possibly due to moral hazard or taxation. Therefore, a more interesting problem would be how to design optimal policy when there is a concern with such potential efficiency losses.

4.3.2 Optimal Loss Insurance With the Efficiency Loss

When a market breaks down ($\bar{\delta} = 0$), how should policymakers provide subsidies to resurrect the market? When there is the efficiency loss from policy interventions, $T[\kappa(\lambda)] > 0$, the optimal policy may not be to provide generous subsidies to implement the maximum private liquidity $\bar{\delta} = 1$. The optimal policy should be designed in a way that minimizes the efficiency loss while maximizing the social benefit from having a liquid financial market.

For many realistic cases, the optimal policy will invove $0 < \bar{\delta} < 1$, with some degree of concern about the efficiency loss. In such cases, buyers necessarily acquire information. Therefore equilibrium outcomes become state-dependent, unlike the outcome in the previous section. In addition to the non-linearity of the Ramsey problem, there are two variables (δ, p)

which need to be solved by fixed-point iterations. Moreover, the optimal policy function $\kappa(\lambda)$ must take into account the feedback effect of the information choice $\bar{\delta}$ on $\delta(\lambda)$. Due to those complications, I approximate the policy functions numerically to solve the Ramsey problem. The complete set of equations used for numerical appoximation is described in Appendix A.

For quantitative analysis, I focus on a case in which $\bar{\delta} = 0$ (market breakdown) if there is no policy intervention. I assume the investment productivity among marginal sellers follows the Pareto distribution, $G(A) = 1 - \left(\frac{A_{\min}}{A}\right)^{\alpha}$, where $A \in [A_{\min}, \infty)$ and $\alpha > 0$. This distribution is useful in that its density function is monotone, and it exposes the properties of the optimal policy with one parameter α; α parameterizes the marginal benefit of sustaining a high asset price through its impact on the quality X (expression (8)). With a greater value of α, marginal sellers are more densely populated toward $A_{\min}$, which increases possibilities that they opt out of the market. Therefore, from the policymakers' perspective, the marginal benefit of sustaining a high asset price is increasing in α, i.e., $\frac{d(1-G(1/p))}{dp} = \alpha \left(pA_{\min}\right)^{\alpha-1}$. Put differently, when α is high, the quality of assets traded, $X(\lambda)$, deteriorates more quickly as an asset price plummets.

As for the efficiency loss, I use the function $T = \frac{c}{2}\kappa(\lambda)^2 \delta(\lambda)$, which is quadratic in the size of subsidy $\kappa(\lambda)$. Therefore, the marginal cost of κ is given by $\frac{\partial T}{\partial \kappa} = c\kappa(\lambda)\delta(\lambda)$, where $c > 0$. c is a multiplier that indexes the degree of concern with the efficiency loss, possibly due to moral hazard or any other unexpected outcomes. Also, the size of the concern is proportional to the aggregate demand $\delta(\lambda)$.[31]

There are three parameters which are important to characterize the optimal policy: c (degree of a concern with the efficiency loss), α (effect of an asset price p on asset quality X), and θ (cost of information acquisition). I characterize the optimal policy for the different values of those parameters. The Pareto parameter α is important to characterize the optimal policy in that it governs the degree of endogeneity of the asset quality X. When α is higher, the asset quality X is driven endogenously by an asset price to a greater extent.

Also, there is a set of parameters $\{A_{\min,}\underline{A}, N, h, F(\lambda)\}$, which is of less importance in the

[31] While greater demand $\delta(\lambda)$ increases the welfare by increasing the asset price, it also increases the size of concern about the efficiency loss. Our qualitative results are not affected even if we replace $\delta(\lambda)$ with another variables associated with asset demand.

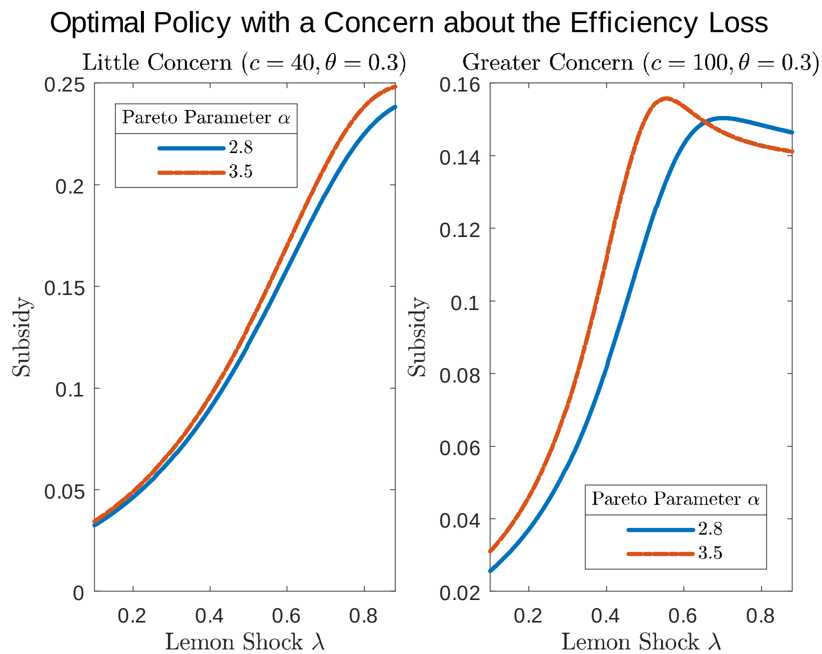


Figure 8

characterization of the optimal policy. I fix their values to satisfy Assumption 1 and $\bar{\delta} = 0$ (market breakdown) without the policy intervention. They may affect the optimal policy quantitatively, but they do not affect the qualitative nature of the policy. I pick $N = 1.5$, $\underline{A} = 0.75$, $A_{\min} = 1.25$, and $h = 0.2$, and set λ follows the uniform distribution $U[0.1, 0.9]$.

The left panel in Figure 8 shows that when a degree of concern about the efficiency loss is small ($c = 40$), there is little difference in the optimal policy functions obtained with the different Pareto parameter α.[32] The optimal subsidy is monotonically increasing across the state of λ. The right panel in the figure shows that it is no longer the case with a higher degree of concern about the efficiency losses ($c = 100$) . There are a few notable differences to note in this figure. First, the optimal policy becomes non-monotonic in λ, and the policy function has a unique critical point. In both cases, more subsidies are given in some better states than the worst state, unlike the previous case. And the value of λ in which the maximum subsidy is given is lower with the higher Pareto parameter α. Second, the amount of subsidies is decreasing beyond the critical point, but at a decreasing rate. Third, when the parameter is higher, more subsidies are given to better states, and fewer subsidies are

[32]This result is similar with a higher information cost θ.

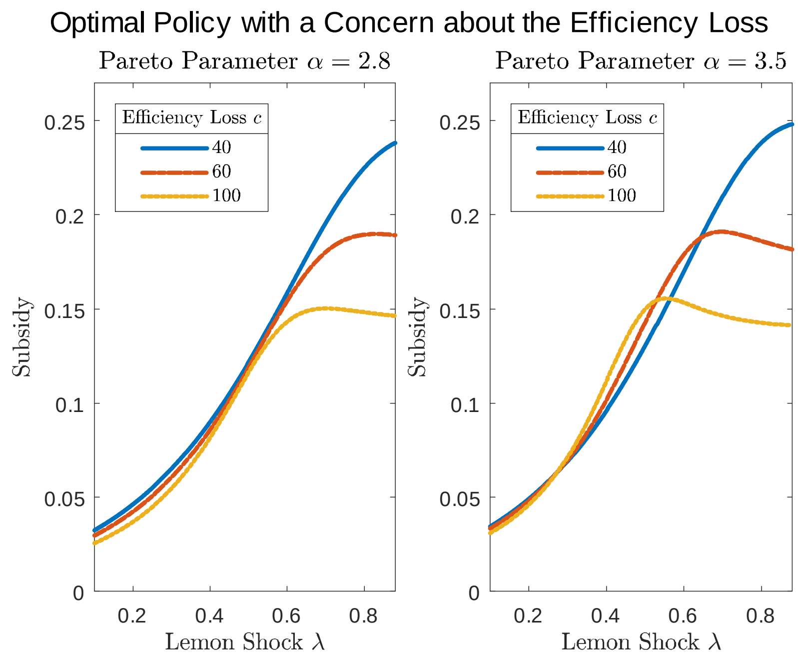


Figure 9

given to worse states.

Table 1 (θ=0.3)

Pareto Parameter (α)	2.8			3.5		
efficiency loss (c)	40	60	100	40	60	100
Mean Liquidity	0.8371	0.6812	0.5156	0.8367	0.6004	0.4004
Mean Subsidy	0.1273	0.1156	0.1005	0.1354	0.1261	0.1118
Welfare	0.7419	0.6342	0.5224	0.7124	0.5849	0.4610
Welfare with No Loss	1.0604	0.9178	0.7590	1.0721	0.8634	0.6705

Then what causes a difference in the optimal policy? Figure 9 demonstrates the optimal policy for a different degree of concern about the efficiency loss, and Table 1 shows key statistics for each case.[33] With a higher degree of such a concern, the mean subsidy is lower (Table 1), while relatively more subsidies are given in the middle states of λ (Figure 9). It tells us that when the amount of subsidies given to the private sector is limited by concern

[33] The welfare with no loss is computed without considering the efficiency loss $T(\lambda)$ from the welfare criterion.

Optimal versus Suboptimal Policy ($\alpha = 3.5, \theta = 0.3, c = 100$)

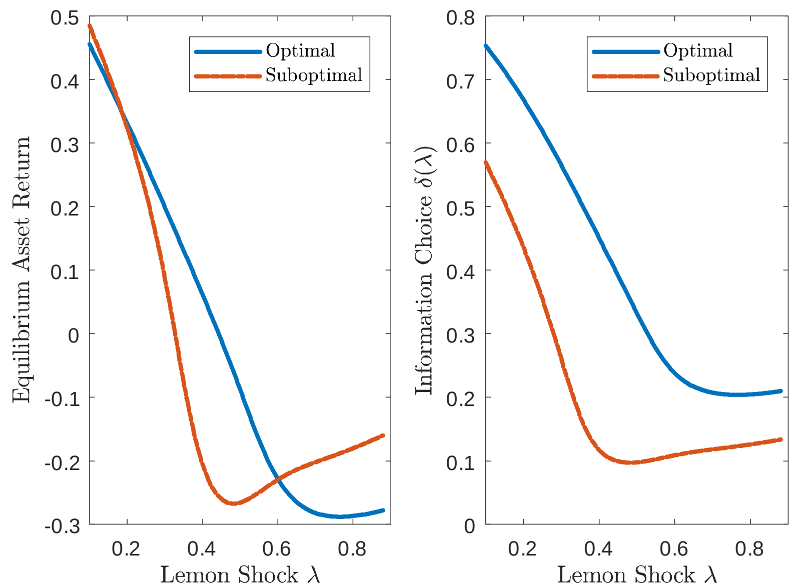


Figure 10

about the efficiency loss, private liquidity supply can be maximized with policy that allocates more resources toward the middle states of λ.

To understand this result, note that there are two considerations that should be taken into account for the design of the optimal policy. First, more subsidies should be provided in states in which their influence on private liquidity supply is maximized. To this end, policymakers need to assess an endogenous impact of the asset price on the asset quality X. The optimal policy dictates that more subsidies are provided to support the asset price in states where the asset quality X starts to deteriorate quickly. In the case in which the Pareto parameter α is higher, the asset quality X deteriorates more rapidly with a decrease in the asset price, as more marginal sellers with non-lemons leave the market.

Table 2: α=3.5, θ=0.3,c=100

	Optimal	Suboptimal
Mean Liquidity	0.4004	0.2080
Mean Subsidy	0.1118	0.1005
Welfare	0.4610	0.3937

Figure 10 shows this point with the comparisons of the asset return and the information

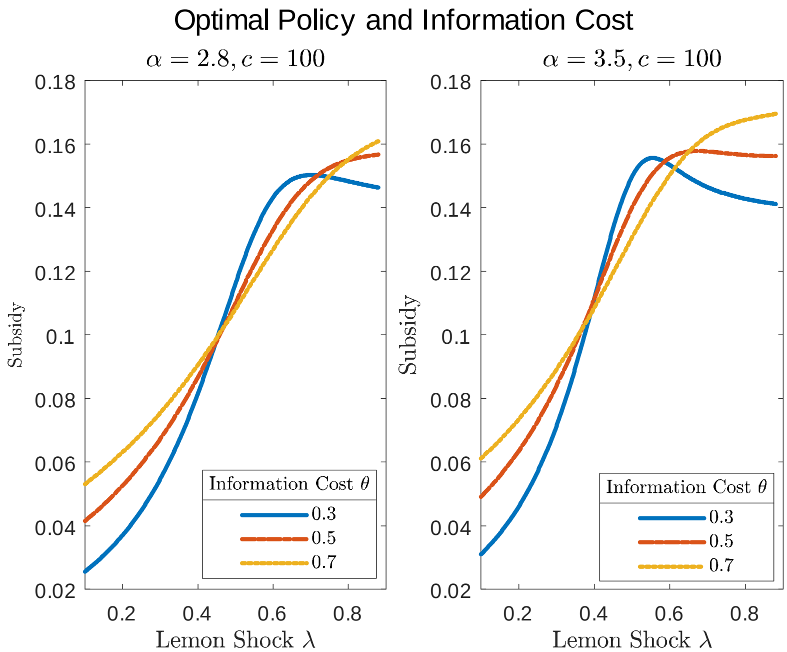


Figure 11

choice between the optimal and the suboptimal policy. To compute a market equilibrium when $\alpha = 3.5$ with the suboptimal policy, I plugged in the policy computed with $\alpha = 2.8$ as shown in Figure 9. With the suboptimal policy, the equilibrium asset return declines rapidly, and thus there is less incentive to buy assets. It leads to lower private liquidity supply $\bar{\delta}$ ($\bar{\delta}_{optimal} = 0.4004$ versus $\bar{\delta}_{suboptimal} = 0.2080$). The suboptimal case will occur when policymakers fail to take into account the endogenous effect of the asset price on the asset quality X or adopt a monotone policy instead of a non-monotone policy as demonstrated.

Table 3

Pareto Parameter (α)	2.8			3.5		
Information Cost (θ)	0.3	0.5	0.7	0.3	0.5	0.7
Mean Liquidity	0.5156	0.5741	0.6506	0.4004	0.4393	0.5493
Mean Subsidy	0.1005	0.1051	0.1075	0.1118	0.1200	0.1233
Welfare	0.5224	0.5070	0.5107	0.4610	0.4238	0.4133
Welfare with No Loss	0.7590	0.8210	0.8956	0.6705	0.7186	0.8280

The other consideration for the design of the optimal policy is the cost of information acquisition, θ. Figure 11 reveals the relationship between optimal policy and information acquisition costs. There are two observations to note. First, the optimal policy with higher information costs dictates that more subsidies are provided in better states in which λ is small. In order to revive the market from $\bar{\delta} = 0$, inequality (15) should not be satisfied. The $\theta-$adjusted expectation operator in the inequality puts greater weights to the better states, but those weights are decreasing in θ. It implies that $\bar{\delta} = 0$ occurs only if there is no significant upside risk, so that the opportunity cost of not participating in a market is small. To break inequality (15), some subsidies should be given even in better states. It induces buyers to enter into a market and acquire information. Moreover, in order to attract marginal buyers into a market in which the information acquisition cost is high, greater subsidies should be allocated toward better states.

Second, when the information cost is higher, the optimal policy dictates that more subsidies are allocated to worse states as well. While greater subsidies in better states induce marginal buyers to enter into a market, adequate subsidies in worse states reduce a downside risk. As inequality (16) implies, when a downside risk is smaller, the private liquidity supply is greater especially when the information cost θ is higher. Therefore, relatively more subsidies are given in states in which λ is closer to either of the corners. By increasing an upside risk and decreasing a downside risk, the optimal policy maximizes private liquidity supply with relatively small subsidies.

As can be seen in Table 3, the mean private liquidity is greater with a higher information cost. This is because a higher information cost raises the effect of subsidies on the mean liquidity $\bar{\delta}$, if the insurance policy is properly designed; the mean liquidity becomes more sensitive to the amount of subsidies given.[34][35]

[34]This result is also associated with the result in Section 3.4. Subsidies increase the conditional demand $\delta(\lambda)$, which again raises the mean demand $\bar{\delta}$. With a greater information cost, the feedback effect of $\bar{\delta}$ on $\delta(\lambda)$ is stronger (equation 14), and thus the effect of subsidies is amplified.

[35]Despite the mean liquidity is higher with a higher information cost, the overall welfare is slightly lower. It is because greater subsidies are given to states where asset demand is large, and thus the concern with the efficiency loss is higher.

Optimal versus Suboptimal Policy ($\alpha = 2.8, \theta = 0.7, c = 100$)

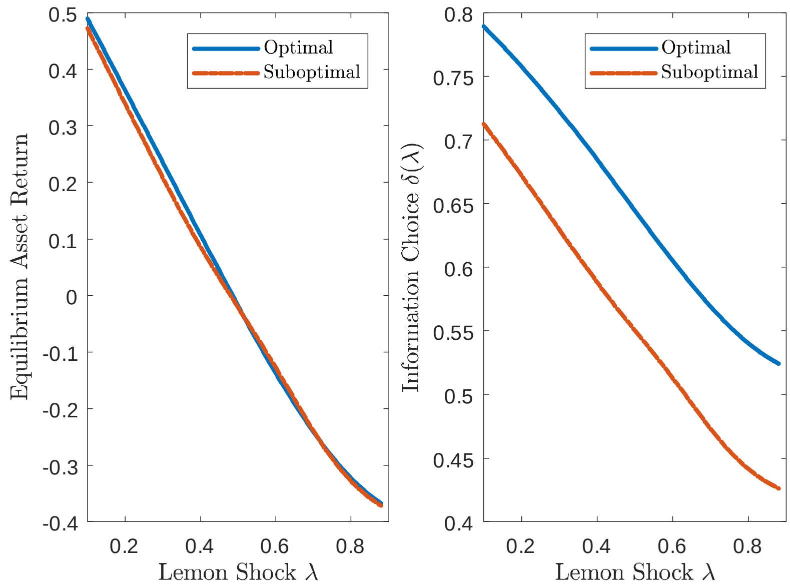


Figure 12

Optimal versus Suboptimal Policy ($\alpha = 3.5, \theta = 0.7, c = 100$)

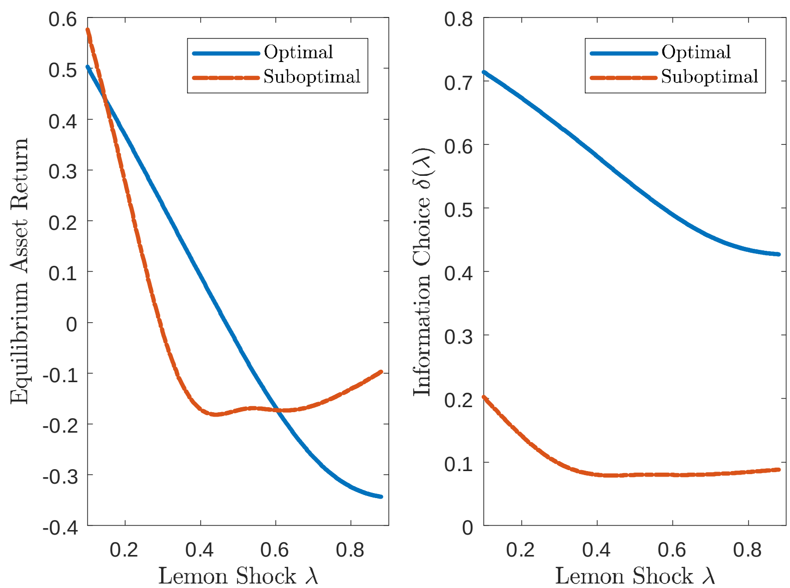


Figure 13

Table 4

Pareto Parameter (α)	2.8		3.5	
	Optimal	Suboptimal	Optimal	Suboptimal
Mean Liquidity	0.6506	0.5580	0.5493	0.1002
Mean Subsidy	0.1075	0.1051	0.1233	0.1118
Welfare	0.5107	0.5031	0.4133	0.2978

Figure 12-13 and Table 4 contrast the optimal policy with the suboptimal policy. To compute a market equilibrium when $\theta = 0.7$ with the suboptimal policy, I plugged in the policy computed with $\theta = 0.3$ as shown in Figure 11. As can be seen in the tables, the difference between the optimal and suboptimal policy is greater when the endogenous relationship between the asset quality and the asset price is higher ($\alpha = 3.5$).[36] In both cases, the high information cost allows little variation in asset demand $\delta(\lambda)$, compared to the previous one (Figure 10). Accordingly, the asset price exhibits little variation. Therefore, rather than supporing the asset price in the middle states of λ, allocating more subsidies to the corners of states is optimal to maximize private liquidity supply.

4.3.3 Direct Asset Purchases

In this section, we will consider direct asset purchases as alternative policy instruments, and compare them to the optimal policy discussed in the previous section. Let us consider the government's strategy of direct asset purchases as follows.[37] At the beginning of date 0, the government promises to purchase D dollars of assets regardless of the state λ at date 1.[38] After the announcement, the private sector makes an information choice. At date 1, D dollars are financed by a government deficit to fulfill its promise. The shadow cost of public

[36]Even though the optimal policy provides greater protection in the worst state, the equilibrium asset return is lower (the left panel in Figure 13). It is because a relatively high asset price is maintained even in the worst state with high demand (the right panel). It lowers the equilibrium asset return.

[37]This policy is equivalent for the government to post price p^G at which the government is willing to buy assets from the private sector, i.e., direct lending. Suppose the government posts price p^G for a legacy asset. Agents sell their assets to the government only if $p^G > p$ where p is a market price of the asset. Note that the government needs liquidity to implement this policy: in the model, there is a one-to-one mapping between the policy of direct lending and the policy of asset purchases.

[38]To obtain public funds which are needed to implement such policies, the government can pledge economic resources that will be available only in the future in order to borrow from outside capital markets. Unlike

funds is given by r, and therefore $T(\lambda) = (1 + r)D$. The market clearing price p^D with the government asset purchases satisfies the following condition:

$$\frac{S^L(\lambda) + D}{p^D} = S^A(\lambda).$$

At date 2, the government deficit D is repaid with lump sum taxes and revenues generated by the purchased assets.

The next proposition investigates an efficieny loss when some liquidity is supplied by a government.

Proposition 6 *(Direct Asset Purchases) Suppose $0 \leq \bar{\delta} < 1$ is the unique market equilibrium before intervention. Then,*
(i) If $\frac{\partial(1-G(1/p))}{\partial p} < c$ for $p \geq p_{\bar{\delta}=1}$, direct asset purchases cannot implement the constrained efficient allocation.
(ii) (Welfare loss) The efficiency loss relative to the optimal policy is given by

$$\Xi(D) = \int \underbrace{\left[\delta^*(\lambda) - \delta^D(\lambda)\right](A^*(\lambda) - \underline{A})}_{\text{liquidity misallocation}} - \underbrace{\left[\delta^D(\lambda) + h\right]\left(A^D(\lambda) - A^*(\lambda)\right)}_{\text{adjustment term for the productivity difference}}$$
$$+ \underbrace{\left[(1+r) - A^D\right] D(\lambda) - T(\kappa(\lambda))}_{\text{difference in the cost of the interventions}} dF(\lambda),$$

where $\delta^*(\lambda)$ is private asset demand with the optimal policy, $\delta^D(\lambda)$ is the one with direct asset purchases, and A^* is the average investment productivity with the optimal policy, and A^D is the one with direct asset purchases.

As the government demand competes with the private demand, the government asset purchases act as a direct force to raise an equilibrium asset price p^D. A higher asset price has two opposing effects on the asset return $\frac{X}{p}$. On the one hand, a higher asset price lowers the asset return $\frac{X}{p}$ by increasing the cost of asset purchases. On the other hand, a higher asset price may increase the asset return, for it may improve the quality of assets X as more nonlemons are traded.

the private sector, the government can exercise its taxation power to pay its debt back; the government is able to borrow without such severe collateral requirements, which are imposed on the private sector.

To some extent, government direct purchases may increase private liquidity $\bar{\delta}$ due to the endogenous effect of the asset price on its quality. However, the government cannot achieve the constrained efficient allocation with this type of policy. The condition in Proposition 6-(i) holds when the total amount of liquidity endowed with marginal buyers is large enough. In such a case, their liquidity endowment has potential to drive an asset price to the point at which a high enough fraction of marginal sellers enter into a market.

To see this point, note that the marginal buyers do not acquire information and become buyers surely $\bar{\delta} = 1$ only if the $-\theta$ adjusted expected return is sufficiently high, $\underline{A} \leq E^{\bar{\delta}=1}_{-\theta}[\frac{X}{p}]$ (Proposition 2). But when an asset price is high beyond $p_{\bar{\delta}=1}$, it has a limited impact on the quality X, while it decreases the asset return. Therefore, if $\bar{\delta} = 1$ is not an initial market equilibrium, the $-\theta$ adjusted expected return cannot be any higher with the government purchases, $\underline{A} > E^{\bar{\delta}=1}_{-\theta}[\frac{X}{p}] > E^{\bar{\delta}=1}_{-\theta}[\frac{X}{p^D}]$; private demand cannot coexist with competing government demand. As competing government asset demand decreases the asset return, private demand is crowded out.

The second part of the proposition provides the expression for the welfare loss relative to the optimal policy discussed in the previous section. The welfare loss due to liquidity misallocation depends on the crowding out effect $\delta^* - \delta^D$ and the productivity dispersion $A^* - \underline{A}$. The liquidity misallocation represents the utility loss resulting from the marginal buyers who keep their liquidity, wasting private liquidity in less productive projects. Regarding the social costs of both kinds of the policy interventions, I will discuss about the shadow cost of public funds, as well as about the productivity dispersion in Section 4.4.

Moreover, the analysis in Section 3.4 indicates that the crowding out effect is large in a market where the information acquisition cost is high. In such a case, the private sector's information choice is highly sensitive to a change in the asset return; a relatively small change in the asset return caused by the direct asset purchases can induce a large reduction in private liquidity supply in financial markets. For instance, in asset-backed securities market where the information cost θ is large, large-scale asset purchases may cause a large welfare loss due to a greater crowding out effect.

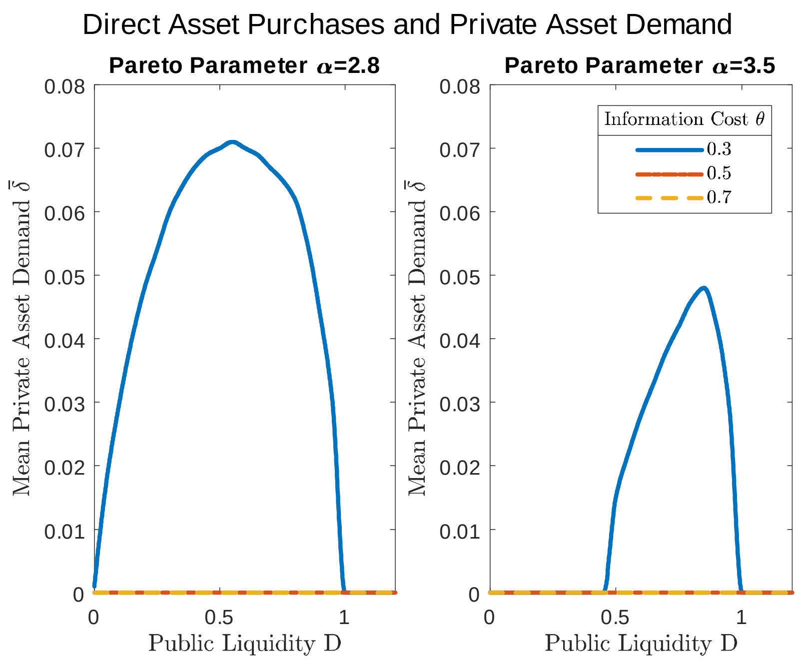


Figure 14

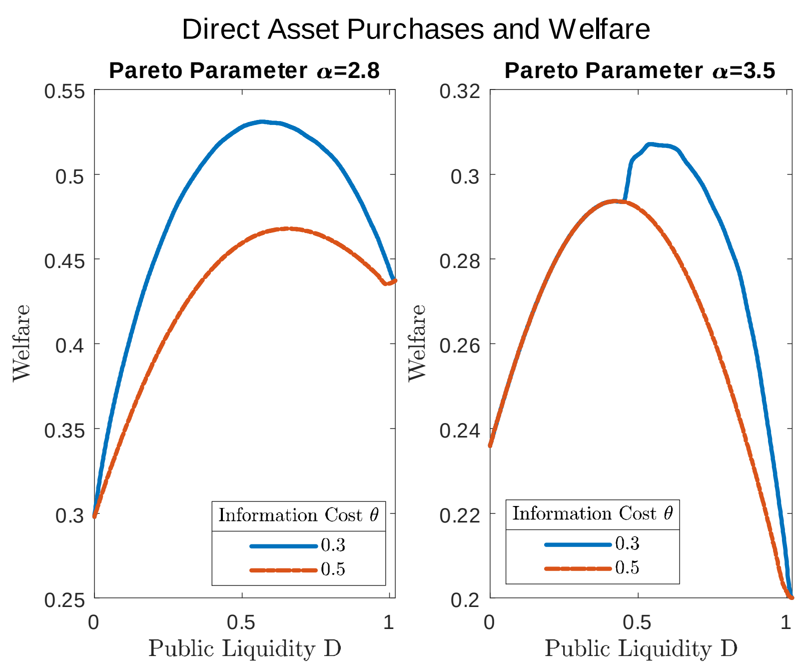


Figure 15

Table 5

Pareto Parameter (α)	2.8		3.5	
Information Cost (θ)	0.3	0.5	0.3	0.5
Max Mean Liquidity	0.0710	0	0.0490	0
Max Welfare with No Loss	0.5310	0.4722	0.3071	0.2937

Figure 14 shows numerical results obtained with the same parameter values used in the previous section. As it shows, the direct asset purchases may revive private liquidity supply to small extent in certain cases. However, the maximum private liquidity supply is less than 0.1 in all the cases. In addition, private liquidity supply becomes positive only when the information cost θ is small. In the cases in which the information cost is higher, private liquidity supply is 0 all the time regardless of the size of public liquidity supply, and this policy fails to revive the private market.

Figure 15 and Table 5 compute the welfare with direct asset purchases assuming that the cost of public funds r equals 0. They indicate that the welfare with direct asset purchases is greater only if there is a large concern with the efficiency loss associated with loss insurance, while the cost of public funds r is very small.[39]

4.4 Discussion

The main role of a financial market in our model is the reallocation of resources among agents. A well-functioning financial market effectively reallocates resources to more productive entrepreneurs, which leads to greater aggregate output and growth. If a financial market breaks down, and if the misallocation from such a market failure causes a large welfare loss, government interventions are justified to the extent to which such interventions are effective in improving upon market equilibrium. The previous analysis indicates that both the direct asset purchases and the loss insurance are useful instruments to some extent in the case of the market failure. I have shown that the Ramsey policy with loss insurance may be better than direct asset purchases in regards to efficiency.

[39] The welfare is decreasing beyond a certain amount of public liquidity. With a higher asset price, more low productive entrepreneurs become sellers, which puts downward pressure both on the asset price and the average productivity.

Direct asset purchases lead to distortion in private liquidity allocation, causing inefficient uses of private liquidity. By crowding out private liquidity supply, direct asset purchases cause welfare losses. One natural question is then whether the magnitude of the welfare loss from the direct asset purchases is large enough, and therefore whether policymakers should adopt the optimal strategy to intervene in the ABS markets. The loss insurance here should be interpreted more broadly. It may cover prepayment risk or liquidity risk as well as credit risk. For instance, if financial institutions hesitate to buy ABS due to liquidity risk, policymakers may offer them liquidity backstop proportional to the size of the ABS holdings. Also, policymakers may buy Collateralized Mortage Obligation tranches that involve more prepayment risk so that private sectors can buy tranches that involve less prepayment risk at lower prices.

Our model predicts that inefficiency caused by the direct asset purchases can be particularly large in the ABS markets. If the cost of information acquisition is large in those markets, the private sector's information choice is highly sensitive to a change in an asset price; a relatively small increase in an asset price initiated by the direct asset purchases can induce a large reduction in private liquidity provision in the ABS markets. This is the pitfall of the direct asset purchases, because the scale of the direct asset purchases that is required to fill a lack of demand in the ABS markets can be significantly larger than policymakers' estimate due to such crowding out effects.

The extent of the resulting inefficiency depends further on the investment productivity of less productive entrepreneurs $\underline{A}$ and the shadow cost of public funds r. I will provide further reasons why the magnitude of such inefficiency is potentially large.

Regarding the former, financial market disruptions, as seen in the financial crisis of 2007-2008, are usually followed by a prolonged period of recession (Reinhart and Rogoff 2009). Arguably, $\underline{A}$ is lower in such a recession compared to that in normal periods. Any policies that discourage resource transfer from lower to higher productive entrepreneurs necessarily lead to inefficient uses of valuable resources, and such losses from the misallocation of resources are particularly large in recession.

Regarding the latter, there are at least three reasons to believe that the shadow cost of public funds r is significantly large. First, the direct asset purchases cause wealth redistribution. As more private assets are held by the government, the asset returns are reaped by the government, not by private agents who are in need of valuable storage technology. This is especially a problem when a good state is realized. If such assets were not in the hand of the government, those resources delivered from the assets would be better used by private agents. Even if the government tries to redistribute those resources to the private sector, it is likely to encounter considerable political controversy as it raises a question about the way in which those transfers ought to be distributed.

Second, it may be difficult to unload large assets from the Fed's balance sheet. It is hard to reach a consensus when to start to use "Exit strategies," for there is a lot of uncertainty about the consequences of such strategies. Among those assets in the Fed's balance sheet, reducing its longer-term mortgage backed securities holdings are particularly difficult, because it is unclear when is optimal to sell those securities to unload; the Fed is subject to unexpected capital losses (Hall and Reis 2013). Moreover, after Chairman Bernanke mentioned in May 2013 that the Fed would begin to cut back on its stimulus program once the economy had improved, speculation over the Fed's future policy became another unnecessary source of financial market instability.

The loss insurance has, in contrast, virtue in this regard. It is easy for the private sector to observe the current default rate. Once it goes back to the one in normal periods, it may not be difficult to reach a consensus between the government and the private sector as to when to undo such policy accommodation. Furthermore, since the government does not hold any assets in its hand, it is unlikely to incur any capital losses to the government when it tries to retreat from such unconventional policy regime.

Third, as the Fed is exposed to many financial institutions by holding private securities, the Fed's independence can be jeopardized (Reis 2013). Many financial institutions that are in financial relations with the Fed may lobby to manipulate the Fed's action in an attempt to obtain private rents that are socially costly, which was not a problem when the Fed dealt with small numbers of heavily regulated financial institutions. In anticipation of such activities, there may be considerable political pressure to control the Fed's action, in which case the ability of the Fed to accomplish its dual mandate will be questioned.

To summarize, it is simple to state the principle of the efficient intervention that intends to inject liquidity in the private sector: to make private agents trade with each other. Such principle prevents policymakers neither from providing liquidity to financial institutions that are in urgent need nor from using the size and the composition of the central-bank's balance sheet combined with forward guidance as an instrument of monetary policy to combat deflation at the zero lower bound (Eggertsson and Woodford 2003). The central-bank is still able to extend liquidity via a set of emergency lending facilities, such as the Term Auction Facility (TAF), the Term Structure Lending Facility (TSLF) and the Primary Dealer Credit Facility. As the terms of those loans are at most three months, by letting them expire, the central bank's effort to inject liquidity is unlikely to have a seriously negative impact on efficiency (Reis 2013). It is rather the central-bank's direct purchases of longer-term private securities that should be reassessed. In addition, the central-bank can rely on forward guidance to fight deflation at the zero lower bound by buying more government bonds than would be required to set the interest rate to zero for an extended period of time, while letting private securities trade among private agents in a private financial market. Such policy not only can satiate the private sector with enough liquidity, but also let the private agents who seek profitable investment opportunities reap the benefits of valuable financial assets.

5 Concluding Remarks

This paper studies the Ramsey policy in a model of the financial market with asymmetric information. Private agents can choose the accuracy of information about the state of the world. The positive analysis shows that asset market volatility is higher in markets where the cost of information acquisition is greater.

In the normative analysis, policymakers have a concern with insurance policy on lemons, as it may cause potential efficiency losses. It is found that the Ramsey policy requires careful considerations of the information acquisition cost and the endogenous effect of the asset price on its quality. When the asset price has a greater impact on the quality of assets traded, the optimal policy is hump-shaped. With a greater information cost, the optimal policy increases an upside risk with more subsidies in better states, and decreases a downside risk with more subsidies in worse states.

While this paper assumes potential efficiency losses from policy interventions, it does not identify a specific source of the efficiency losses. Different policies may incur different kinds of efficiency losses, which is one direction of future research.

Instead of providing general insurance policy, a government may limit the provision of insurance to specific kinds of securities. How to, and where to, provide such insurance is another direction for future research.

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Appendix A: The Complete Set of Equations for the Ramsey Policy and the Numerical Solution Method

The Complete set of equations for the Ramsey policy

The social planner chooses $\left\{\kappa(\lambda), \delta(\lambda), \bar{\delta}, p(\lambda)\right\}$ to maximize expression (23), subject to

$$\delta(\lambda) = \frac{\bar{\delta}}{\bar{\delta} + (1-\bar{\delta}) \exp\left(-\theta^{-1}\left[\frac{X(\lambda,p)+\kappa(\lambda)}{p(\lambda)} - \underline{A}\right]\right)}, \tag{27}$$

$$\bar{\delta} = \int \delta(\lambda)\, dF(\lambda), \tag{28}$$

$$X(\lambda, p) = 1 - \frac{\lambda}{\lambda + (1-\lambda)\left[1 - G\left(\frac{1}{p(\lambda)}\right)\right]}, \tag{29}$$

$$p(\lambda) = \frac{\delta(\lambda) + h}{\left\{\lambda + (1-\lambda)\left[1 - G\left(\frac{1}{p(\lambda)}\right)\right]\right\} N}. \tag{30}$$

Note that equation (30) defines $p = p(\delta(\lambda)|\lambda)$ implicitly. Plugging it and equation (29) into equation (27), we can define δ implicitly, $\delta = \delta\left(\kappa(\lambda), \bar{\delta}|\lambda\right)$. Then the welfare criterion can be rewritten as

$$\begin{aligned} \max_{\kappa(\lambda),\bar{\delta}} W = \int_{\lambda} & p\left(\kappa(\lambda), \bar{\delta}|\lambda\right) N \left[\left(\bar{A} - \underline{A}\right) - (1-\lambda)\int_0^{1/p} (A - \underline{A})\, dG(A)\right] \\ & -T[\kappa(\lambda)] + \eta\left(\delta\left(\kappa(\lambda), \bar{\delta}|\lambda\right) - \bar{\delta}\right) dF(\lambda), \end{aligned} \tag{31}$$

where η is the lagrangian multiplier associated with equation (28).

The first order conditions are given by

$$\begin{aligned} & \frac{\partial p\left(\kappa(\lambda), \bar{\delta}|\lambda\right)}{\partial \kappa(\lambda)} \left[\left(\bar{A} - \underline{A}\right) - (1-\lambda)\int_0^{1/p\left(\kappa(\lambda),\bar{\delta}|\lambda\right)} (A - \underline{A})\, dG(A)\right] \\ & -p\left(\kappa(\lambda), \bar{\delta}|\lambda\right) N (1-\lambda) \frac{\partial}{\partial \kappa(\lambda)} \int_0^{1/p\left(\kappa(\lambda),\bar{\delta}|\lambda\right)} (A - \underline{A})\, dG(A) + \eta \frac{\partial \delta\left(\kappa(\lambda), \bar{\delta}|\lambda\right)}{\partial \kappa(\lambda)} = \frac{\partial T}{\partial \kappa(\lambda)}, \end{aligned} \tag{32}$$

and

$$\int_{\lambda} \frac{\partial p\left(\kappa(\lambda), \bar{\delta}|\lambda\right)}{\partial \bar{\delta}} \left[\left(\bar{A} - \underline{A}\right) - (1-\lambda)\int_0^{1/p\left(\kappa(\lambda),\bar{\delta}|\lambda\right)} (A - \underline{A})\, dG(A)\right]$$

$$-p\left(\kappa\left(\lambda\right),\bar{\delta}|\lambda\right)N\left(1-\lambda\right)\frac{\partial}{\partial\bar{\delta}}\int_{0}^{1/p\left(\kappa(\lambda),\bar{\delta}|\lambda\right)}\left(A-\underline{A}\right)dG\left(A\right)dF\left(\lambda\right) \tag{33}$$
$$=\frac{\partial T}{\partial\delta\left(\lambda\right)}+\eta\left(1-\int_{\lambda}\frac{\partial\delta\left(\kappa\left(\lambda\right),\bar{\delta}|\lambda\right)}{\partial\bar{\delta}}dF\left(\lambda\right)\right).$$

Note that $\frac{\partial p}{\partial\kappa}=\frac{\partial p}{\partial\delta}\frac{\partial\delta}{\partial\kappa}$, and $\frac{\partial p}{\partial\bar{\delta}}=\frac{\partial p}{\partial\delta}\frac{\partial\delta}{\partial\bar{\delta}}$. $\frac{\partial p}{\partial\delta}$ can be obtained by applying the implicit function theorem to the expression (30), $\frac{\partial p(\lambda)}{\partial\delta(\lambda)}=-\frac{\frac{\partial g}{\partial\delta}}{\frac{\partial g}{\partial p}}$, where $g\left(p,\delta\right)=-p+\frac{\delta+h}{[1-G\left(\frac{1}{p}\right)](1-\lambda)+\lambda}=0$, and

$$\frac{\partial g}{\partial p}=-1-\left[\frac{1}{\left\{[1-G\left(\frac{1}{p}\right)](1-\lambda)+\lambda\right\}N}\right]^{2}\left(\delta+h\right)\left(G'\left(\frac{1}{p}\right)\left(1-\lambda\right)\frac{1}{p^{2}}\right), \tag{34}$$

$$\frac{\partial g}{\partial\delta}=\frac{1}{\left\{[1-G\left(\frac{1}{p}\right)](1-\lambda)+\lambda\right\}N}. \tag{35}$$

Likewise, $\frac{\partial\delta(\lambda)}{\partial\kappa(\lambda)}=-\frac{\frac{f}{\kappa}}{\frac{f}{\delta}}$, where $f=-\delta\left(\lambda\right)+\frac{\bar{\delta}}{\bar{\delta}+(1-\bar{\delta})\exp\left(-\theta^{-1}L(\lambda)\right)}$, and

$$\frac{\partial f}{\partial\kappa}=\frac{\bar{\delta}(1-\bar{\delta})\exp\left(-\theta^{-1}L(\lambda)\right)\frac{\theta^{-1}}{p(\lambda)}}{\left[\bar{\delta}+(1-\bar{\delta})\exp\left(-\theta^{-1}L(\lambda)\right)\right]^{2}}, \tag{36}$$

$$\frac{\partial f}{\partial\delta}=\frac{\bar{\delta}(1-\bar{\delta})\exp\left(-\theta^{-1}L(\lambda)\right)\theta^{-1}\frac{1}{p(\lambda)}\left(\frac{\partial X(\lambda)}{\partial p(\lambda)}\frac{\partial p(\lambda)}{\partial\delta(\lambda)}-\frac{X+\kappa}{p(\lambda)}\frac{\partial p(\lambda)}{\partial\delta(\lambda)}\right)}{\left[\bar{\delta}+(1-\bar{\delta})\exp\left(-\theta^{-1}L(\lambda)\right)\right]^{2}}-1, \tag{37}$$

$$\frac{\partial X\left(\lambda\right)}{\partial p\left(\lambda\right)}=\left[\frac{1}{\left\{[1-G\left(\frac{1}{p}\right)](1-\lambda)+\lambda\right\}}\right]^{2}\lambda\left(G'\left(\frac{1}{p}\right)\left(1-\lambda\right)\frac{1}{p^{2}}\right)N, \tag{38}$$

$$L\left(\lambda\right)=\frac{X\left(\lambda\right)+\kappa\left(\lambda\right)}{p\left(\lambda\right)}-\underline{A}=\frac{[1-G\left(\frac{1}{p}\right)](1-\lambda)N}{\delta+h}+\frac{\kappa\left(\lambda\right)}{p\left(\lambda\right)}-\underline{A}. \tag{39}$$

Also, $\frac{\partial\delta(\lambda)}{\partial\bar{\delta}}=-\frac{\frac{f}{\bar{\delta}}}{\frac{f}{\delta}}$, where $\frac{f}{\bar{\delta}}=\frac{\left[\bar{\delta}+(1-\bar{\delta})\exp\left(-\theta^{-1}L(\lambda)\right)\right]-\bar{\delta}\left[1-\exp\left(-\theta^{-1}L(\lambda)\right)\right]}{\left[\bar{\delta}+(1-\bar{\delta})\exp\left(-\theta^{-1}L(\lambda)\right)\right]^{2}}$.

Description of the solution algorithm

The problem is to find policy functions $\left\{\kappa\left(\lambda\right),\eta,\delta\left(\lambda\right),\bar{\delta},p\left(\lambda\right)\right\}$ to satisfy the equations (27), (28), (29), (30), (32), (33), and the other expressions (34), (35), (36), (37), (38), and (39) which are needed to compute $\frac{\partial p}{\partial\kappa}$, $\frac{\partial p}{\partial\bar{\delta}}$ and $\frac{\partial\delta}{\partial\bar{\delta}}$ as well. We solve for the solutions using policy function iteration.

(a) Start with initial guess for policy functions $\left\{\kappa\left(\lambda\right),\eta,\delta\left(\lambda\right),\bar{\delta},p\left(\lambda\right)\right\}$.

(b) For every λ, find $\kappa\left(\lambda\right)$ that solves equation (32). This becomes new value of $\kappa\left(\lambda\right)$.

- To ensure that solution $\kappa(\lambda)$ is a global maximum, solve the equations using multiple initial points. Then pick a solution that maximizes the objective (31).

(c) For every λ, find η that solves equation (33). This becomes new value of $\kappa(\lambda)$.

(d) For every λ, find $\left\{\delta(\lambda), \bar{\delta}, p(\lambda)\right\}$ that solves equation (27), (28), (29), and (30),. This becomes new value of $\left\{\delta(\lambda), \bar{\delta}, p(\lambda)\right\}$.

(e) Check convergence criterion. If satisfied, end. If not, go to (b).

I applied this method to find the optimal solution and verified that there is a unique equilibrium under the solution.

Appendix B: proofs

For simplicity of notation, the subscript j is dropped if there is no confusion.

Proof of Lemma 1. The objective of entrepreneur j given a signal s_j at date 1 is

$$\int_\lambda c(s_j, \lambda) dF(\lambda|s_j). \tag{40}$$

As the budget constraint (1) holds with equality, the objective can be written as follows by substituting (1), (4), and (5) into this objective (40).

$$U(b_j, d_j^{NL}, d_j^L) = \int_\lambda [(1-\lambda_j - d_j^{NL}) + b(X(\lambda))] + A_j[1 + p(d_j^{NL} + d_j^L - b_j)] dF(\lambda|s), \tag{41}$$

where $X(\lambda) = 1 - \lambda^M(\lambda)$, and $\lambda^M(\lambda)$ is the market fraction of lemon.

Income is increasing in d_j^L, but d_j^L does not contribute to capital accumulation. This implies that agents sell all their lemons, $d_j^L = \lambda_j$. Notice that the marginal return to sell an additional unit of non-lemon assets, $\frac{\partial U}{\partial d_j^{NL}}$, is equals to $A_j p - 1$, and the marginal return to buy an additional unit of assets in the market, $\frac{\partial U}{\partial b_j}$, is given by $\int_\lambda X(\lambda) dF(\lambda|s) - A_j p$. Therefore,

$$\text{seller: } d_j^{NL} = 1 - \lambda_j \text{ and } b_j = 0 \text{ if } A_j > \frac{1}{p},$$

$$\text{buyer: } d_j^{NL} = 0 \text{ and } b_j = \frac{1}{p} + d_j^L \text{ if } A_j < \frac{1}{p}\int_\lambda X(\lambda) dF(\lambda|s),$$

$$\text{keeper: } d_j^{NL} = b_j = 0 \text{ if } \frac{1}{p}\int_\lambda X(\lambda) dF(\lambda|s) < A_j < \frac{1}{p}.$$

Entrepreneurs who are in the boundary are indifferent between those choices.

$$0 \le d_j^{NL} \le 1-\lambda_j \text{ and } b_j = 0 \text{ if } A_j = \frac{1}{p},$$

$$d_j^{NL} = 0 \text{ and } 0 \le b_j \le \frac{1}{p} + d_j^L \text{ if } A_j = \frac{1}{p}\int_\lambda X(\lambda)\, dF(\lambda|s).$$

■

Suppose $p(\lambda)$ is given. Let $\delta(\lambda;\bar{\delta})$ be a solution to equation (14) given $p(\lambda)$ and $\bar{\delta}$, and $J(\delta) \equiv \int_\lambda \delta(\lambda;\bar{\delta})dF(\lambda)$.

Lemma A.1. (Woodford 2008) Suppose the information cost $\theta > 0$ is given, and $F(L(\lambda) \neq 0) > 0$ where F is a probability measure associated with the prior distribution of λ. Then [1] there is a unique equilibrium; and [2] there are three kinds of possible solutions: (i) $\delta(\lambda) = 0$ almost surely if and only if

$$\int \exp\{\frac{L(\lambda)}{\theta}\}dF(\lambda) \le 1, \qquad \int \exp\{-\frac{L(\lambda)}{\theta}\}dF(\lambda) > 1,$$

which implies $J(\bar{\delta}) < \bar{\delta}$ for all $0 < \bar{\delta} < 1$; (ii) $0 < \delta(\lambda) < 1$ almost surely if and only if

$$\int \exp\{\frac{L(\lambda)}{\theta}\}dF(\lambda) > 1, \qquad \int \exp\{-\frac{L(\lambda)}{\theta}\}dF(\lambda) > 1$$

; (iii) $\delta(\lambda) = 1$ almost surely if and only if

$$\int \exp\{\frac{L(\lambda)}{\theta}\}dF(\lambda) > 1, \qquad \int \exp\{-\frac{L(\lambda)}{\theta}\}dF(\lambda) \le 1.$$

Proof) See Lemma 2 in Woodford (2008).

Proof of Proposition 1 (Sufficient and Necessary Conditions). Define $\phi(\delta,\bar{\delta}) \equiv \delta\log(\frac{\delta}{\bar{\delta}}) + (1-\delta)\log(\frac{1-\delta}{1-\bar{\delta}})$. Then

$$\min_{\bar{\delta}^*}\int_\lambda \phi(\delta(\lambda),\bar{\delta}^*)dF(\lambda) = \int_\lambda \phi(\delta(\lambda),\bar{\delta})dF(\lambda) = I(\delta)$$

where $\bar{\delta} = \int_\lambda \delta(\lambda)dF(\lambda)$. Therefore,

$$\max_{\delta(\cdot)}\int_\lambda [\delta(\lambda)L(\lambda) - \theta\phi(\delta(\lambda),\bar{\delta})]dF(\lambda) = \max_{\delta(\cdot)}\max_{\bar{\delta}^*}\int_\lambda [\delta(\lambda)L(\lambda) - \theta\phi(\delta(\lambda),\bar{\delta}^*)]dF(\lambda).$$

Moreover, it can be easily shown that

$$\max_{\delta(\cdot)}\max_{\bar{\delta}^*}\int_\lambda [\delta(\lambda)L(\lambda) - \theta\phi(\delta(\lambda),\bar{\delta}^*)]dF(\lambda) = \max_{\bar{\delta}^*}\max_{\delta(\cdot)}\int_\lambda [\delta(\lambda)L(\lambda) - \theta\phi(\delta(\lambda),\bar{\delta}^*)]dF(\lambda).$$

This implies that we can solve an inner problem separately for each λ given $\bar{\delta}$, and then maximize the object over $\bar{\delta}$.

Taking $p(\lambda)$ as given, private agents maximizes their utility. Provided $\bar{\delta}$ ($\bar{\delta}$ can be interpreted as the expected liquidity), the first order necessary condition requires that

$$L(\lambda; p(\lambda)) - \theta\phi_1(\delta(\lambda), \bar{\delta}) = 0, \tag{42}$$

which implies that

$$\frac{X\left(\lambda; p\left(\lambda\right)\right)}{p(\lambda)} - \underline{A} = \theta[\log\frac{\delta}{1-\delta} - \log\frac{\bar{\delta}}{1-\bar{\delta}}] \tag{43}$$

where $X\left(\lambda; p\left(\lambda\right)\right)$ is given by equation (8). Given $\bar{\delta}$ and $p\left(\lambda\right)$, there is a unique solution to equation (43) for each λ, $\delta^*\left(\lambda; \bar{\delta}, p\left(\lambda\right)\right)$, and an equilibrium choice of $\delta^*(\lambda; \bar{\delta}, p\left(\lambda\right))$ must satisfy $\bar{\delta}^* = \int_\lambda \delta^*(\lambda; \bar{\delta}^*, p\left(\lambda\right))dF(\lambda)$. However, $\bar{\delta}^*$ which satisfies the above conditions does not necessarily correspond to a local maximum. To see this point, substitute the solution $\delta^*(\lambda; \bar{\delta}, p\left(\lambda\right))$ into the objective (13) and differentiate with respect to $\bar{\delta}$. We obtain

$$\begin{aligned} U'(\bar{\delta}) &\equiv \int_\lambda \{\delta_{\bar{\delta}}(\lambda)[L(\lambda; p(\lambda)) - \theta\phi_1(\delta^*(\lambda; \bar{\delta}, p\left(\lambda\right)), J(\bar{\delta}))] - \theta\phi_2(\delta^*(\lambda; \bar{\delta}, p\left(\lambda\right)), J(\bar{\delta}))J'(\bar{\delta})\}dF(\lambda) \\ &= \int_\lambda \delta_{\bar{\delta}}(\lambda)[\phi_1(\delta^*(\lambda; \bar{\delta}, p\left(\lambda\right)), \bar{\delta}) - \phi_1(\delta^*(\lambda; \bar{\delta}, p\left(\lambda\right)), J(\bar{\delta}))]dF(\lambda) \\ &= \int_\lambda \delta_{\bar{\delta}}(\lambda)[\log\frac{J(\bar{\delta})}{1-J(\bar{\delta})} - \log\frac{\bar{\delta}}{1-\bar{\delta}}]dF(\lambda). \end{aligned} \tag{44}$$

for any $0 < \bar{\delta} < 1$, where $\delta_{\bar{\delta}}(\lambda) \equiv \frac{\partial\delta^*(\lambda; \bar{\delta}, p(\lambda))}{\partial\bar{\delta}} > 0$. The first equality holds by (42) and from the fact that $\int_\lambda \phi_2(\delta^*(\lambda; \bar{\delta}, p\left(\lambda\right)), J(\bar{\delta}))dF(\lambda) = 0$.

Note that $U'(\bar{\delta}) > 0$ if and only if $J(\bar{\delta}) > \bar{\delta}$. Therefore, a local maximum $\bar{\delta}^*$ requires (i) if $\bar{\delta}^* > 0$, then there exits $\varepsilon > 0$ such that $J(\bar{\delta}) > \bar{\delta}$ for all $\bar{\delta} \in (\bar{\delta}^* - \varepsilon, \bar{\delta}^*)$; and (ii) if $\bar{\delta}^* < 1$, then there exits $\varepsilon > 0$ such that $J(\bar{\delta}) < \bar{\delta}$ for all $\bar{\delta} \in (\bar{\delta}^*, \bar{\delta}^* + \varepsilon)$. Moreover, by Lemma A.1, there exists a unique solution given the price function $p(\lambda)$ to equation (43).

I next turn to the determination of an equilibrium asset price function $p(\lambda)$, which must satisfy equation (9). The reason why the results from Lemma A.1 does not apply here is that $L\left(\cdot\right)$ is the function of $p\left(\lambda\right)$, which is the function of $\delta(\lambda; \bar{\delta})$. Therefore, $L\left(\cdot\right)$ itself depends on $\bar{\delta}$ while it is not the case in Lemma A.1.

Let $\delta^*(\lambda; \bar{\delta})$ denote the solution that satisfies (43), (9) and (**??**) given $\delta = \bar{\delta}$. In order to eliminate obvious bilateral gains from trade, I suppose that $\delta^*(\lambda; \bar{\delta})$ is given by the highest one if there are multiple solutions. At the highest $\delta^*(\lambda; \bar{\delta})$, both the asset price and the return $\frac{X}{p}$ are the highest, which benefits both sellers and buyers. To see this, note that $p(\lambda)$

is increasing in $\delta^*(\lambda;\bar{\delta})$, and $G(p)$ is non-decreasing in $p(\lambda)$, which implies that the highest solution $\delta^*(\lambda;\bar{\delta})$ corresponds to the highest asset price. Also, from the first order condition, the return $\frac{X}{p}$ is increasing in $\delta^*(\lambda;\bar{\delta})$.

Define $J^*(\bar{\delta}) \equiv \int_\lambda \delta^*(\lambda;\bar{\delta})dF(\lambda)$. Suppose $\bar{\delta}^* = J^*(\bar{\delta}^*)$. If $\bar{\delta}^*$ is a stable equilibrium, it still requires that (i) if $\bar{\delta}^* > 0$, then there exits $\varepsilon > 0$ such that $J^*(\bar{\delta}) > \bar{\delta}$ for all $\bar{\delta} \in (\bar{\delta}^* - \varepsilon, \bar{\delta}^*)$; and (ii) if $\bar{\delta}^* < 1$, then there exits $\varepsilon > 0$ such that $J^*(\bar{\delta}) < \bar{\delta}$ for all $\bar{\delta} \in (\bar{\delta}^*, \bar{\delta}^* + \varepsilon)$. Otherwise, a small disturbance to agents' beliefs over the asset price (which is equivalent to a disturbance to $\bar{\delta}^*$) causes further divergence from the original point, and their beliefs will converge to a new stable equilibrium.

Note that an equilibrium $\bar{\delta}^*$ that satisfies the sufficient and necessary conditions needs not to be unique here. Multiplicity can be possible. If there are multiple $\bar{\delta}^*$, each of which corresponds to the unique asset price function which is denoted by $p(\lambda;\bar{\delta}^*)$, and thus there are multiple asset price functions that are consistent with equilibrium.■

Lemma A.2. Fix $0 \leq \bar{\delta} \leq 1$. Let $p(\lambda)$ be consistent with the equilibrium conditions given $\bar{\delta}$. (i) p is increasing in δ; (ii) $\delta(\lambda;\bar{\delta})$ is decreasing in λ; (iii) p is decreasing in λ; (iv) $\delta(\lambda;\bar{\delta})$ is increasing in $\bar{\delta}$.

Proof of Lemma A.2. Choose any λ such that $p(\lambda) > \frac{1}{A_M}$.

Plugging (9) into (43), we obtain

$$\frac{(1-\lambda)(1-G(1/p))N}{\delta+h} - \underline{A} = \theta[\log\frac{\delta}{1-\delta} - \log\frac{\bar{\delta}}{1-\bar{\delta}}]. \tag{45}$$

(i) The asset price function is given by

$$p = \frac{\delta+h}{[(1-G(1/p))(1-\lambda)+\lambda]N}. \tag{46}$$

While the left-hand side of equation (46) is increasing in p, the right-hand side is decreasing in p. Since the right-hand side is increasing in δ, p is increasing in δ.

(ii) The left-hand side of equation (45) is bounded. However, for any given $\bar{\delta}$, the right-hand side of equation (45) approaches $-\infty$ as $\delta \to 0$, and ∞ as $\delta \to 1$. This implies

that there exists the highest δ^* such that the net marginal benefit of δ is decreasing in δ in the neighborhood of δ^*. With the highest δ^*, both the marginal benefit of δ and the asset price are maximized, eliminating any bilateral gains unexploited. Since the left-hand side of equation (45) is decreasing in λ, $\delta(\lambda;\bar{\delta})$ is decreasing in λ.
(iii) By (i) and (ii), p is decreasing in λ.
(iv) The same logic as in (ii) applies. An increase in $\bar{\delta}$ shifts down the right-hand side of equation (45). This implies that δ^* is increasing in $\bar{\delta}$. This completes the proof. ■

Lemmm A.3. For each λ, let $\delta^*(\lambda;\bar{\delta})$ denote a solution to (14) given $\bar{\delta}$ as in Proposition 1, and define

$$J^*(\bar{\delta}) \equiv \int_\lambda \delta^*(\lambda;\bar{\delta})dF(\lambda).$$

and

(i) If

$$\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) \leq 1,$$

then $J^*(\bar{\delta}) < \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 0, in which case $\bar{\delta}^* = 0$ is an equilibrium. If

$$\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) > 1,$$

then $J^*(\bar{\delta}) > \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 0, in which case $\bar{\delta}^* = 0$ cannot be an equilibrium.

(ii) If

$$\int \exp[-\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{h} - \underline{A})]dF(\lambda) \leq 1,$$

then $J(\bar{\delta}) > \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 1, in which case $\bar{\delta}^* = 1$ is an equilibrium. If

$$\int \exp[-\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{h} - \underline{A})]dF(\lambda) > 1,$$

then $J^*(\bar{\delta}) < \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 1, in which case $\bar{\delta}^* = 1$ cannot be an equilibrium.

Proof of Lemma A.3. This lemma is similar to Lemma A.1, but the important difference here is that the loss function itself now depends on $\bar{\delta}$. To see this, let $\delta^*(\lambda;\bar{\delta})$ denotes a solution which is consistent with the equilibrium conditions given $\bar{\delta}$. Substituting them into the loss function, we get

$$L^*(\lambda,\bar{\delta}) \equiv L^*(\lambda,\delta^*(\lambda;\bar{\delta})) = \frac{(1-\lambda)\left(1-G\left(\frac{1}{p^*_{\bar{\delta}}}\right)\right)N}{\delta^*(\lambda;\bar{\delta})+h} - \underline{A},$$

where $p^*_{\bar{\delta}}$ solves equation (46) given $\bar{\delta}$.

Since p is increasing in δ, $p^*_{\bar{\delta}}$ is maximized when $\bar{\delta}=1$, in which case $\delta(\lambda)=1$ almost surely. And $p^*_{\bar{\delta}}$ is minimized when $\bar{\delta}=0$, in which case $\delta(\lambda)=0$ almost surely. We denote each case by $p^*_{\bar{\delta}=1}$ and $p^*_{\bar{\delta}=1}$ respectively.

The first order condition (45) implies that

$$\delta^*(\lambda;\bar{\delta}) = \frac{\bar{\delta}}{\bar{\delta}+(1-\bar{\delta})\exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))}. \tag{47}$$

Define

$$\begin{aligned}
A &\equiv \bar{\delta}+(1-\bar{\delta})\exp(-\theta^{-1}L^*(\lambda,\bar{\delta})) \\
B &\equiv 1-\exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))+(1-\bar{\delta})\exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))\frac{\partial L^*(\lambda,\delta^*(\lambda;\bar{\delta}))}{\partial\delta^*}\frac{\partial\delta^*}{\partial\bar{\delta}}(-\theta^{-1}) \\
A' &= 1-\exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))+(1-\bar{\delta})\exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))\frac{\partial L^*(\lambda,\delta^*(\lambda;\bar{\delta}))}{\partial\delta^*}\frac{\partial\delta^*}{\partial\bar{\delta}}(-\theta^{-1}).
\end{aligned}$$

(i) Differentiating $J^*(\bar{\delta})$,

$$\frac{\partial J^*(\bar{\delta})}{\partial\bar{\delta}} = \int \frac{A-\bar{\delta}A'}{A^2}dF(\lambda)$$

and thus

$$\begin{aligned}
\frac{\partial J^*(\bar{\delta})}{\partial\bar{\delta}}|_{\bar{\delta}=0} &= \int \exp(\theta^{-1}L^*(\lambda,\bar{\delta}))dF(\lambda) \\
&= \int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda).
\end{aligned} \tag{48}$$

The second equality comes from the fact that $\delta(\lambda)=\bar{\delta}$ almost surely if $\bar{\delta}=0$ or $\bar{\delta}=1$. Therefore, $\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) > 1$ if and only if $J^*(\bar{\delta}) > \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 0.

Claim : $\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) \leq 1$ if and only if $J^*(\bar{\delta}) < \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 0.

Proof of the claim : Note that if $\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) \leq 1$, $\frac{\partial J^*(\bar{\delta})}{\partial \bar{\delta}}|_{\bar{\delta}=0} \leq 1$. Hence, it suffices to show that $\frac{\partial^2 J}{\partial \bar{\delta}^2}|_{\bar{\delta}=0} > 0$.

Differentiating $J^*(\bar{\delta})$ twice, we obtain

$$\frac{\partial^2 J^*(\bar{\delta})}{\partial \bar{\delta}^2} = \int \frac{-\bar{\delta}A''A^2 - (A - \bar{\delta}A')2AA'}{A^4} dF(\lambda).$$

Observe

$$\begin{aligned}
\frac{\partial^2 J}{\partial \bar{\delta}^2}|_{\bar{\delta}=0} &= -2\int \frac{A'}{A^2} dF(\lambda) \\
&= -2\int \frac{[1-\exp(-\theta^{-1}L^*(\lambda,\bar{\delta})) + \exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))\frac{\partial L^*(\lambda,\delta^*(\lambda;\bar{\delta}))}{\partial \delta^*}\frac{\partial \delta^*}{\partial \bar{\delta}}(-\theta^{-1})]}{\exp(-\mu^{-1}L^*(\theta,\bar{q}))^2} dF(\lambda) \\
&= -2\int [\exp(2\theta^{-1}L^*(\lambda,\bar{\delta})) - \exp(\theta^{-1}L^*(\lambda,\bar{\delta})) \\
&\quad +(-\theta^{-1})\exp(\theta^{-1}L^*(\lambda,\bar{\delta}))\frac{\partial L^*(\lambda,\delta^*(\lambda;\bar{\delta}))}{\partial \delta^*}\frac{\partial \delta^*}{\partial \bar{\delta}}]dF(\lambda).
\end{aligned}$$

Since $g(x) = x^2$, by Jensen's inequality,

$$\begin{aligned}
\int \exp(2\theta^{-1}L^*(\lambda,\bar{\delta}))dF(\lambda) &\leq \{\int \exp(\theta^{-1}L^*(\lambda,\bar{\delta}))dF(\lambda)\}^2 \\
&= \{\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda)\}^2
\end{aligned}$$

Note that the equality holds only if $L^*(\lambda,\bar{\delta})$ is constant almost surely.

If $\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) \leq 1$,

$$\int \exp(2\theta^{-1}L^*(\lambda,\bar{\delta}))dF(\lambda) \leq \int \exp(\theta^{-1}L^*(\lambda,\bar{\delta}))dF(\lambda) \leq 1.$$

The equality holds only if $L^*(\lambda,\bar{\delta}) = 0$ almost surely, which contradicts the assumption that $F(L^*(\lambda,\bar{\delta}) \neq 0) > 0$. Therefore, the inequality must be strict.

Also, $\frac{\partial L^*(\lambda,\delta^*(\lambda;\bar{\delta}))}{\partial \delta^*} > 0$ in thie region. Combining with Lemma A.3,

$$\frac{\partial L^*(\lambda,\delta^*(\lambda;\bar{\delta}))}{\partial \delta^*}\frac{\partial \delta^*}{\partial \bar{\delta}} \geq 0.$$

Therefore,

$$\frac{\partial^2 J}{\partial \bar{\delta}^2}|_{\bar{\delta}=0} > 0.$$

End of claim.

(ii) Similar to case (i),

$$\frac{\partial J^*(\bar{\delta})}{\partial \bar{\delta}}|_{\bar{\delta}=1} = \int \exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))dF(\lambda) = \int \exp[-\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{1+h} - \underline{A})]dF(\lambda)$$

and

$$\begin{aligned}\frac{\partial^2 J}{\partial \bar{\delta}^2}|_{\bar{\delta}=1} &= -\int A'' + (1-A')2A'dF(\lambda)\\ &= -2\int \exp(-\theta^{-1}L^*(\lambda,\bar{\delta}))\frac{\partial L^*(\lambda,\delta^*(\lambda;\bar{\delta}))}{\partial \delta^*}\frac{\partial \delta^*}{\partial \bar{\delta}}\\ &\quad + \exp(-\theta^{-1}L^*(\lambda,\bar{\delta})) - \exp(-2\theta^{-1}L^*(\lambda,\bar{\delta}))dF(\lambda).\end{aligned}$$

Therefore, if $\int \exp[-\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{1+h} - \underline{A})]dF(\lambda) > 1,\ J^*(\bar{\delta}) < \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 1.

If $\int \exp[-\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{1+h} - \underline{A})]dF(\lambda) \leq 1$, then $\frac{\partial^2 J}{\partial \bar{\delta}^2}|_{\bar{\delta}=1} < 0$ by the same reasoning as the above, and thus $J(\bar{\delta}) > \bar{\delta}$ for $\bar{\delta}$ sufficiently close to 1.

By Lemma A.1, $\bar{\delta}^* = 0$ can be an equilibrium only if $\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) \leq 1$. Likewise, $\bar{\delta}^* = 1$ can be an equilibrium only if $\int \exp[-\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{1+h} - \underline{A})]dF(\lambda) \leq 1$. ■

Proof of Proposition 2 **(Existence).** Define

$\Delta \equiv \{0 \leq \bar{\delta} \leq 1 | \bar{\delta} = J\left(\bar{\delta}\right),$ and $\bar{\delta}$ is consistent with the all the conditions in Proposition 1$\}$.

I will show that Δ is non-empty, i.e., there exists a fixed point $\bar{\delta}$ which satisfies the sufficient and necessary conditions for an equilibrium.

Lemma A.5 implies that $\bar{\delta} = 0 \in \Delta$ if and only if

$$\int \exp[\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h} - \underline{A})]dF(\lambda) \leq 1.$$

If $\bar{\delta} = 0$ is chosen as an equilibrium, $\delta(\lambda; 0) = 0$ for all λ, in which case an agent does not obtain any information and he becomes a buyer with probability 0.

Likewise, $\bar{\delta} = 1 \in \Delta$ if and only if

$$\int \exp[-\theta^{-1}(\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{1+h} - \underline{A})]dF(\lambda) \leq 1.$$

If $\bar{\delta} = 1$ is chosen as an equilibrium, $\delta(\lambda; 0) = 1$ for all λ, in which case an agent does not obtain any information, but he becomes a buyer with probability 1.

Otherwise, because $J^*\left(\bar{\delta}\right)$ is increasing in $\bar{\delta}$ (Lemma A.3), Lemma A.5 implies that there exists an equilibrium $0 < \bar{\delta} < 1$, in which case an agent obtains information, and his probability of becoming an buyer depends on the state λ.

Also, notice that

$$E_\theta[\frac{X(\lambda; p^*_{\bar{\delta}=0}(\lambda))}{p^*_{\bar{\delta}=0}(\lambda)}] = Z^{-1}\int \exp[\theta^{-1}\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=0}\right)\right)N}{h}]$$

and

$$E_{-\theta}[\frac{X(\lambda; p^*_{\bar{\delta}=1}(\lambda))}{p^*_{\bar{\delta}=1}(\lambda)}] = Z^{-1}\int \exp[-\theta^{-1}\frac{(1-\lambda)\left(1-G\left(1/p^*_{\bar{\delta}=1}\right)\right)N}{1+h}]dF(\lambda).$$

■

Proof of Proposition 3. (i) Under the rational expectation in which $\theta = 0$, multiplicity does not emerge. Note that $\frac{X(\lambda; p_{\bar{\delta}}(\lambda))}{p_{\bar{\delta}}(\lambda)} - \underline{A}$ is decreasing in λ (Lemma A.2). Choose any $0 < \bar{\delta} < 1$. As $\theta \longrightarrow 0$,

$$\delta^*(\lambda; \bar{\delta}) \longrightarrow 1 \text{ for } \lambda^{\min} \leq \lambda < \lambda_1 \text{ such that } \frac{(1-\lambda)(1-G(1/p_{\bar{\delta}}))N}{1+h} - \underline{A} > 0$$
$$\delta^*(\lambda; \bar{\delta}) \longrightarrow 0 \text{ for } \lambda_2 < \lambda \leq \lambda^{\max} \text{ such that } \frac{(1-\lambda)(1-G(1/p_{\bar{\delta}}))N}{h} - \underline{A} < 0$$

For $\lambda_1 \leq \lambda \leq \lambda_2$, $0 \leq \delta^*(\lambda; \bar{\delta}) \leq 1$, and $\delta^*(\lambda; \bar{\delta})$ solves

$$\frac{(1-\lambda)(1-G(1/p_{\bar{\delta}}))N}{\delta^*(\lambda; \bar{\delta}) + h} - \underline{A} = 0$$

and equation (46).

Since $J^*(\bar{\delta})$ is positive constant for $0 < \bar{\delta} < 1$, a fixed point $\bar{\delta} = J^*(\bar{\delta})$ is a unique equilibrium, and it coincides with a rational expectation equilibrium.

(ii) When $\theta > 0$, there may be multiple equilibria. See Example 1.■

Lemmm A.4. The price of legacy assets p, the fraction $1-G(1/p)$ of marginal sellers who sell their nonlemons in the market, and private liquidity (equivalently, the probability $\delta(\lambda)$ of becoming a buyer) are non-increasing in the fraction λ of lemons.

Proof of Lemma A.4. See Lemma A.2. As p is decreasing in λ, $1-G(1/p)$ is decreasing in λ. ■

Proof of Proposition 4

(i) Suppose $\frac{\partial L}{\partial \xi}>0$, where $L=\frac{X}{p}-\underline{A}$. Then from the first order condition for the information choice, $\frac{\partial \delta(\lambda)}{\partial \xi}>0$. Since $\frac{\partial \bar{\delta}}{\partial \delta(\lambda)}>0$ and $\frac{\partial \delta(\lambda)}{\partial \bar{\delta}}>0$, it follows $\frac{\partial \bar{\delta}}{\partial \xi}>0$.

(ii) Suppose that the asset price function $p(\lambda)$ is given. For $0<\bar{\delta}<1$, we have $0=\int \frac{1}{\bar{\delta}+(1-\bar{\delta})\exp(-\theta^{-1}L(\lambda))}dF(\lambda)-1$, integrating equation 47. Then, applying the implicit function theorem,

$$\begin{aligned}
\frac{\partial \bar{\delta}}{\partial \xi} &= -\frac{\frac{\partial f}{\partial \xi}}{\frac{\partial f}{\partial \bar{\delta}}} = -\int \frac{(1-\bar{\delta})\exp(-\theta^{-1}L(\lambda))\left(-\theta^{-1}\frac{\partial L}{\partial \xi}\right)}{1-\exp(-\theta^{-1}L(\lambda))}dF(\lambda) \\
&= \int \frac{(1-\bar{\delta})\exp(-\theta^{-1}L(\lambda))\left(\theta^{-1}\frac{\partial L}{\partial \xi}\right)}{1-\exp(-\theta^{-1}L(\lambda))}dF(\lambda) \\
&= \int \frac{(1-\bar{\delta})\left(\theta^{-1}\frac{\partial L}{\partial \xi}\right)}{\exp(\theta^{-1}L(\lambda))-1}dF(\lambda)
\end{aligned}$$

Since

$$\begin{aligned}
\frac{\partial^2 \bar{\delta}}{\partial \xi \partial \theta} &=. \int -\frac{(1-\bar{\delta})\frac{\partial L}{\partial \xi}}{\left[\exp(\theta^{-1}L(\lambda))-1\right]^2}\left[\left(\exp(\theta^{-1}L(\lambda))-1\right)-\theta^{-1}\exp(\theta^{-1}L(\lambda))L(\lambda)\right]dF(\lambda) \\
&= \int -\frac{(1-\bar{\delta})\frac{\partial L}{\partial \xi}}{\left[\exp(\theta^{-1}L(\lambda))-1\right]^2}\left[\exp(\frac{L(\lambda)}{\theta})\left(1-\frac{L(\lambda)}{\theta}\right)-1\right]dF(\lambda).
\end{aligned}$$

Notice that $\left[\exp(\frac{L(\lambda)}{\theta})\left(1-\frac{L(\lambda)}{\theta}\right)-1\right]\leq 0$. Therefore it follows $\frac{\partial^2 \bar{\delta}}{\partial \xi \partial \theta}>0$.■

Proof of Lemma 3. Substituting (22) into (20),

$$W=\int_{\lambda} 1+h+(1-\lambda)N+\int_j\int_s A_i i(s,A_j,\lambda_j)dsdjF(\lambda). \tag{49}$$

The aggregate investment output is given by

$$\begin{aligned}
\int_j A_j i(s_j,A_j,\lambda_j)dj &= (1-\delta)\underline{A}+N(1+p\lambda)\int_0^{1/p}AdG(A)+N(1+p)\int_{1/p}^{\infty}AdG(A) \quad (50)\\
&= (1-\delta)\underline{A}+N\left[(1+p)\bar{A}-p(1-\lambda)\int_0^{1/p}AdG(A)\right],
\end{aligned}$$

where $\bar{A} = \int_0^\infty AdG(A)$.

Note that the aggregate liquidity is given by

$$\delta + h = Np[1 - (1-\lambda)G(1/p)]. \tag{51}$$

Substituting it into the expression (50), we obtain

$$\int_j A_j i(s_j, A_j, \lambda_j)dj = (1+h)\underline{A} + N\bar{A} + pN[(\bar{A} - \underline{A}) - (1-\lambda)\int_0^{1/p}(A - \underline{A})\, dG(A)]. \tag{52}$$

Therefore, the lemma follows.■

Proof of Lemma 4.

Note that the welfare (52) is increasing in p. Since p is increasing in δ, the welfare is increasing in δ, and thus the welfare is maximized when $\delta(\lambda) = 1$ for all λ. When $\delta(\lambda) = 1$ for all λ, the asset price is maximized as well as private liquidity supply $\bar{\delta}$, in which case $p^*_{\bar{\delta}=1}$ solves

$$p^*_{\bar{\delta}=1} = \frac{1+h}{\left\{[1 - G\left(1/p^*_{\bar{\delta}=1}\right)](1-\lambda) + \lambda\right\} N}.$$

Also, note that Individual investment is given by

$$i^*(s, A_j, \lambda_j) = \begin{cases} 0 & \text{for all } A_j = \underline{A}, \\ 1 + p^*_{\bar{\delta}=1}(\lambda) & \text{for all } A_j \geq \frac{1}{p^*_{\bar{\delta}=1}} \\ 1 + \lambda p^*_{\bar{\delta}=1}(\lambda) & \text{for all } A_j < \frac{1}{p^*_{\bar{\delta}=1}} \end{cases}.$$

Aggregate investment and aggregate expected consumption are given by

$$I^*(\lambda) = N\left[\left(1 + p^*_{\bar{\delta}=1}\right)\bar{A} - p^*_{\bar{\delta}=1}(1-\lambda)\int_0^{1/p^*_{\bar{\delta}=1}} AdG(A)\right] \tag{53}$$

$$W = \int_\lambda 1 + h + (1-\lambda)N + I^*(\lambda)\, dF(\lambda). \tag{54}$$

(ii) It is corollary to Proposition 2. ■

Proof of Proposition 5. Suppose the government guarantees that a buyer will receive $\kappa(\lambda)$ consumption goods at date 2 for each financial transaction. The government raises lump-sum taxes from all agents at date 2 and transfers those to buyers as subsidies. Thus, it does not change the budget and resource constraints.

Notice that $E_{-\theta}^{\bar{\delta}=1}[\frac{X+\kappa(\lambda)}{p}]$ is increasing in $\kappa(\lambda)$. From inequality (16), the lower bound of $E_{-\theta}^{\bar{\delta}=1}[\frac{X+\kappa(\lambda)}{p}]$ that implements the optimal plan is given by $\underline{A}$. Therefore, the cost-minimizing insurance must satisfy $\underline{A} = E_{-\theta}^{\bar{\delta}=1}[\frac{X+\kappa(\lambda)}{p}]$. With the minimum cost intervention, $\bar{\delta} = 1$ is consistent with the equilibrium conditions. However, it does not guarantee the uniqueness of the equilibrium. If the impact of a price on the quality of assets, X, is large enough, then a multiplicity of equilibria may arise. For instance, it may be possible that

$$E_{\theta}^{\bar{\delta}=0} < \underline{A} < E_{-\theta}^{\bar{\delta}=1},$$

as Example 1 shows. ■

Proof of Proposition 6. (i) If $\frac{\partial X(\lambda,p)}{\partial p} < c$ for $p > p^{\bar{\delta}=1}$, then for $p' > p^{\bar{\delta}=1}$,

$$\int \exp[-\theta^{-1}(\frac{X(\lambda, p')}{p'} - \underline{A})]dF(\lambda) > \int \exp[-\theta^{-1}(\frac{X(\lambda, p^{\bar{\delta}=1})}{p^{\bar{\delta}=1}} - \underline{A})]dF(\lambda).$$

It implies that if $\bar{\delta} = 1$ cannot be an equilibrium with $D = 0$ (See Lemma A.3 (ii)), then $\bar{\delta} = 1$ cannot be an equilibrium with any $D > 0$. Therefore, no policy in this class can implement $\bar{\delta} = 1$ as an equilibrium.

(ii) The difference in the welfare between the optimal policy and direct asset purchases can be written as

$$
\begin{aligned}
L(\lambda) = &-(\delta^*(\lambda) - \delta^D(\lambda))\underline{A} + p^*(\lambda)\, N \left\{ \bar{A} - (1-\lambda) \int_0^{1/p^*(\lambda)} AdG(A) \right\} \\
&-p^D(\lambda)\, N \left\{ \bar{A} - (1-\lambda) \int_0^{1/p^D(\lambda)} AdG(A) \right\} - \{T(\kappa(\lambda)) - (1+r)D\} \\
= &(\delta^*(\lambda) - \delta^D(\lambda))(-\underline{A}) \\
&+ (\delta^*(\lambda) + h)\, \bar{A}^* - \left(\delta^D(\lambda) + h + D(\lambda)\right) \bar{A}^D - \{T(\kappa(\lambda)) - (1+r)D\} \\
= &\delta^*(\lambda)\left(\bar{A}^* - \underline{A}\right) - \delta^D(\lambda)\left(\bar{A}^D - \underline{A}\right) - h\left(\bar{A}^D - \bar{A}^*\right) \\
&- \left[\bar{A}^D - (1+r)\right] D(\lambda) - T(\kappa(\lambda)) \\
= &\left[\delta^*(\lambda) - \delta^D(\lambda)\right]\left(\bar{A}^* - \underline{A}\right) - \left[\delta^D(\lambda) + h\right]\left(\bar{A}^D - \bar{A}^*\right) \\
&+ \left[(1+r) - \bar{A}^D\right] D(\lambda) - T(\kappa(\lambda)),
\end{aligned}
$$

where $\bar{A}^* = \left\{ \bar{A} - (1-\lambda) \int_0^{1/p^*(\lambda)} AdG(A) \right\} N/S^A(\lambda, p^*)$ and $\bar{A}^D = \left\{ \bar{A} - (1-\lambda) \int_0^{1/p^D(\lambda)} AdG(A) \right\} N/S^A\left(\lambda, p^D\right)$, $\delta^*(\lambda)$, and $p^*(\lambda)$ are the information choice and the asset price under the optimal policy, $\delta^D(\lambda)$ and $p^D(\lambda)$ under direct asset purchases.■

Appendix C

The model with noisy price observations.

Let us introduce unobservable random supply ε which is independent of the state λ into the market clearing condition.

$$\frac{S^L(\lambda)}{p} = S^A(\lambda) + \varepsilon.$$

Then the asset price p is the multivariate function of λ and ε, $p(\lambda, \varepsilon)$; the asset price is a noisy signal on λ.

Suppose that entrepreneurs can postulate the current state from the asset price. Once they make an information choice, they observe the asset price and receive a private signal.

Since $f(\lambda, \varepsilon|s, p) = \frac{f(\lambda,\varepsilon,s|p)}{f(s|p)} = f(s|\lambda, \varepsilon, p)\frac{f(\lambda,\varepsilon|p)}{f(s|p)}$, it follows that

$$\begin{aligned}
I(f|p) &= H(f|p) - H(f|s,p) = -E[\log f(\lambda,\varepsilon|p)] + E[\log f(\lambda,\varepsilon|s,p)] = E[\log \frac{f(\lambda,\varepsilon|s,p)}{f(\lambda,\varepsilon|p)}] \\
&= E[\log \frac{f(s|\lambda,\varepsilon,p)}{f(s|p)}] \\
&= -E[\log f(s|p)] + E[\log f(s|\lambda,\varepsilon,p)] \\
&= \int_{\lambda,\varepsilon,p} \int_s f(s|\lambda,\varepsilon,p) \ln f(s|\lambda,\varepsilon,p) ds dF(\lambda,\varepsilon,p) \\
&\quad - \int_p \int_s [\int_{\lambda,\varepsilon} f(s|\lambda,\varepsilon,p) dF(\lambda,\varepsilon|p)] \ln[\int_{\lambda,\varepsilon} f(s|\lambda,\varepsilon,p) dF(\lambda,\varepsilon|p)] ds dF(p) \\
&= \int_{\lambda,\varepsilon,p} f(1|\lambda,\varepsilon,p) \log f(1|\lambda,\varepsilon,p) + f(0|\lambda,\varepsilon,p) \log f(0|\lambda,\varepsilon,p) dF(\lambda,\varepsilon,p) \\
&\quad - \int_p [f(1|p) \log f(1|p) + f(0|p) \log f(0|p)] dF(p)
\end{aligned}$$

where $F(\lambda, \varepsilon, p)$ is the prior over $(\lambda, \varepsilon, p)$. Then,

$$I(\delta) = \int_\lambda \delta(\lambda,\varepsilon,p) \log \delta(\lambda,\varepsilon,p) + (1-\delta(\lambda,\varepsilon,p)) \log(1-\delta(\lambda,\varepsilon,p)) dF(\lambda) - \bar{\delta}(p) \log \bar{\delta}(p) - (1-\bar{\delta}(p)) \log(1-\bar{\delta}(p))$$

where $\bar{\delta}(p) = \int_{\lambda,\varepsilon} \delta(\lambda, \varepsilon, p) dF(\lambda, \varepsilon|p)$.

Therefore, an equilibrium $\delta(\lambda, \varepsilon, p)$ solves the following.

$$\max_{\delta(\cdot)} \int_{\lambda,\varepsilon,p} \delta(\lambda,\varepsilon,p) L(\lambda,\varepsilon,p) dF(\lambda,\varepsilon,p) - \theta I(\delta).$$

Notice that the agents take the price $p(\lambda, \varepsilon)$ as given. For each $p(\lambda, \varepsilon) = \bar{p}$, there is an equilibrium fixed point $\bar{\delta}(\bar{p})$. An equilibrium is a set of $\bar{\delta}(p)$ for all possible $p = p(\lambda, \varepsilon)$ that satisfies $\bar{\delta} = \int_p \bar{\delta}(p)dF(p)$. In the same way, we can add an additional random variable if agents are allowed to infer the current state from an additional aggregate variable.■

Implementation of the Optimal Policy with a Private Insurance Firm

Another interesting question is whether an insurance policy provided by a private insurance firm can lead to the constrained efficient allocation $\bar{\delta} = 1$. A private insurance firm is arguably more efficient in providing loss insurance to the private sector than the government, and therefore it is important to establish under what conditions a private insurance firm can implement the efficient allocation on behalf of the government.

Suppose a private insurance firm provides buyers of assets with the protection $\kappa^P(\lambda)$ against lemons in the state λ at the premium τ. The private insurance firm participates only if its profits are greater than 0:

$$\tau - \int_\lambda \kappa^P(\lambda)dH(\lambda) \geq 0 \tag{55}$$

where the distribution $H(\lambda)$ represents the prior belief of the private insurance firm over possible states λ, which can be different from the entrepreneurs' prior, $F(\lambda)$. This condition represents the participation constraint of the private insurance firm.

This insurance policy changes the return from buying assets into

$$\frac{X(\lambda) + \kappa^P(\lambda)}{p(\lambda) + \tau}.$$

The efficient allocation is achieved only if there exists the protection $\kappa^P(\lambda)$ and the premium τ that satisfy equation (55) and

$$\underline{A} \leq E_{-\theta}^{\bar{\delta}=1}[\frac{X(\lambda) + \kappa^P(\lambda)}{p^*(\lambda) + \tau}]$$

where the $-\theta$ adjusted expectation is taken with respect to the entrepreneur's prior $F(\lambda)$.

The insurance firm needs to provide a strong protection in the state in which λ is expected to be large, but at the same time, the premium τ must remain sufficiently low. In general, the existence of this kind of the policy depends on the prior distributions of both parties over λ, $F(\lambda)$ and $H(\lambda)$. Roughly speaking, this kind of insurance is likely to exist if there is sufficient heterogeneity in beliefs over future states between the insurance firm and the potential buyers. If both the insurance firm and the potential buyers are pessimistic about future states, however, there is a comparable rise in the insurance premium τ as compensation for the insurance protection in those states. Subsequently, the insurance policy will not have much impact on the return to assets, in which case it is difficult to alter an information choice by the private sector. Hence, an insurance policy that implements the socially optimal allocation is unlikely to exist in such a case.

There are two ways for the government to circumvent this difficulty. The first one is to provide additional insurance protection $\kappa^G(\lambda)$ in such disastrous states on behalf of the insurance firm. Then minimum cost intervention $\kappa_*^G(\lambda)$ satisfies

$$\tau - \int \kappa^P(\lambda) dH(\lambda) = 0,$$

$$\underline{A} = E_{-\theta}^{\bar{\delta}=1}[\frac{X(\lambda) + \kappa^P(\lambda) + \kappa_*^G(\lambda)}{p^*(\lambda) + \tau}].$$

This policy will be less costly especially when the private sector is overly pessimistic about the economy, but the objective probabilities of such disastrous states are small.

The second one is to provide a direct subsidy s^τ on the premium τ, which reduces the premium effectively to $\tau - s^\tau$. Notice that raising the contingent tax $\tau^G(\lambda)$ on the return to assets, which imposes some taxes in good states, is helpful to partly cover the intervention cost of s^τ, especially when the pessimism among the private agents is ungrounded. Then the minimum cost intervention $\{s_*^\tau, \tau_*^G(\lambda)\}$ that implements the efficient allocation satisfies

$$\tau - \int \kappa^P(\lambda) dH(\lambda) = 0,$$

$$\underline{A} = E_{-\theta}^{\bar{\delta}=1}[\frac{X(\lambda) + \kappa^P(\lambda) - \tau_*^G(\lambda)}{p^*(\lambda) + \tau - s_*^\tau}].$$

If beliefs between the insurance firm and entrepreneurs are more pessimistically aligned, then the larger subsidy s_*^τ is needed, while the larger contingent tax $\tau^G(\lambda)$ can be imposed on good states.

In addition, a transaction tax can be imposed to sellers of assets to obtain public funds to implement the policy, if needed. The insurance policy increases demand and pushes the price up, which increases seller's profits. Thus the transaction tax, as far as it is not too large, is still effective in attracting more productive entrepreneurs to sell their nonlemons to invest in their profitable projects.

Appendix D

Can Policymakers Choose a Desirable Equilibrium?

The previous analysis shows that there may exist multiple equilibria with imperfect information on the state λ. One question is that whether the policymaker can induce a desirable equilibrium among multiple equilibria if one of them is more efficient than the others, and what kinds of policy instruments can be used to accomplish this goal. This section explores whether a simple public announcement by the policymaker can lead to a desirable equilibrium among multiple equilibria.

More specifically, let us consider the following strategy as a potential way to achieve this goal. At the beginning of date 0, the policymaker uses the model to solve for all possible equilibria given the private sector's prior beliefs over states. Suppose $\bar{\delta}$ is the most desirable equilibrium among such equilibria. The policymaker computes the asset price $p_{\bar{\delta}}$ and announces that she will intervene in the asset market to implement the asset price target $p_{\bar{\delta}^*}$, as far as an market equilibrium asset price is not equal to the asset price target. To fulfil her promise, she purchases or sells assets in the market at date 1, if needed, to manipulate the asset price.

The effectiveness of the public announcement depends on its ability to alter the private sector's expectation of the asset price. Such a public announcement is most effective in the case that the implementation of the policy does not require any knowledge on λ.

In this regard, it is useful to consider two cases separately to address this question: case (i) a set of multiple equilibria involves the efficient equilibrium, $\bar{\delta} = 1$, and case (ii) a set of multiple equilibria does not involve the efficient allocation. I will show that in the case (ii), the public announcement is less likely to be effective than the case (i).

Case (i): Note that the aggregate private liquidity supply (aggregate asset demand) that is consistent with $\bar{\delta} = 1$ is *independent* of the state variable λ. This implies that the policymaker is not required to have knowledge on λ at date 1 to implement the target asset price $p_{\bar{\delta}=1}$. With the demand externalities with multiplicity, One can state policy in terms of the target aggregate asset demand alternatively, as the key source of inefficiency in the model is the aggregate demand externalities. If the private sector is convinced that the policymaker will fulfil her promise, this policy changes the private sector's beliefs about the future asset price at date 0; the private sector takes $p_{\bar{\delta}=1}$ as given when it makes an information choice.[40] Since this expectation of the asset price is self-fulfilling, the policymaker actually does not need to intervene in the market at date 1; the commitment to the price target $p_{\bar{\delta}=1}$ improves the social welfare, while it does not incur any costs for the policymaker. Therefore, the public announcement is effective to achieve its goal in this case.

Case (ii): Note that if $0 < \bar{\delta} < 1$, the aggregate private liquidity supply $\delta(\lambda)$ depends on λ, as private information is acquired. As a consequence, unless the policymaker has perfect knowledge on the current state, the policymaker may not precisely achieve the announced target $p_{\bar{\delta}}$ at date 1. Even though the policymaker announces $p_{\bar{\delta}}$ as the target asset price, it would not be credible to the private sector as far as the policymaker has imperfect knowledge on λ; the private sector's expectation of the asset price is unlikely to coincide precisely with the announced target price.

Nevertheless, the policymaker may change the private sector's expectation of the asset price in an attempt to induce a better equilibrium. For instance, suppose the policymaker receives a noisy signal s^G on the current state λ, and intervenes with her own estimate on λ, $E[\lambda|s^G]$ where s^G is drawn from the conditional distribution $F(s^G|\lambda)$. Denote the asset price target by $p^T(\lambda)$. Then, the asset price with the intervention in the state λ given s^G is $p(\lambda, s^G) \equiv p^T(E[\lambda|s^G])$. Note that unless the policymaker's estimate is unbiased, the asset price will not match precisely with the announced target price, $p(\lambda, s^G) \neq p^T(\lambda)$.

In such a case, the private sector takes $p(\lambda, s^G)$, instead of $p^T(\lambda)$, as given when making an information choice. The policymaker intervenes at date 1 to implement the price target given s^G, $p^T(E[\lambda|s^G])$, which is consistent with the private sector's belief $p(\lambda, s^G)$. This incurs some intervention costs, which can be increasing in the estimate bias, $|p(\lambda, s^G) - p^T(\lambda)|$. Therefore, the public announcement is less likely to be effective if the policymaker has less precise information on the current state. Notice that the volatility of the asset price can be increasing in the variance of s^G; the government intervention with the imprecise knowledge on the current state may lead to financial instability in this case.

[40] Since this policy does not require the government to have better knowledge than the private sector has, the government can make a credible announcement in this regard.