The Banks' Swansong: Banking and the Financial Markets under Asymmetric Information *

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Abstract

Bank runs may serve to communicate information across agents, and thus enhance rather than thwart allocation efficiency by making the fundamentals determine the asset prices. Figuratively speaking, banks die (go bankrupt) singing a swan song (revealing hidden information). In this way bank runs help uninformed agents to achieve efficient allocation under the condition of asymmetric information in the financial markets. The production of information is done efficiently without cost a point which distinguishes between this paper from most other related studies. The efficient bank runs provide new ground for the coexistence of banks and financial markets. Even when all agents deposit their whole endowment of goods with the bank, the markets play their role in allocating resources once efficient bank runs happen. Allowing a run implies that investment in liquidity can be minimized, and the expected utility of uninformed agents thus increased. The role of banks is strengthened when agents have limited access to the markets.

Keywords: Financial Intermediation, Financial Markets, Bank runs, Asset Price, Asymmetric Information, Information Acquisition, Limited Participation, Liquidity
JEL Codes: D4, D5, D8, G1, G2

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"Swans sing before they die - 'twere no bad thing should certain persons die before they sing"

—Samuel Taylor Coleridge

1 Introduction

Consumers participate in asset markets to efficiently allocate their resources over periods of time. They can obtain optimal consumption levels as long a the true asset prices are fully revealed in the markets. However, if some of the consumers (known as uninformed consumers) facing uncertainty have access to only limited information, then the informed consumers might exploit the uninformed and gain profits by utilizing their knowledge and influencing the prices systematically.

The purpose of this paper is to examine whether uninformed consumers can prevent losses arising from asymmetric information by constructing a bank and operating it in a particular way. More specifically, I show that bank runs may serve to communicate information across agents, and thus enhance rather than thwart allocation efficiency. Bank runs can be efficient by revealing new information to uninformed investors and thus preventing informed agents from affecting asset prices. This eventually leads to asset prices being determined by fundamentals. Figuratively speaking, banks die (go bankrupt) singing a swan song (revealing hidden information).

Financial intermediaries[1] play a special role, that of revealing information even without investing resources in order to identify information when the financial markets are imperfect. The role of banks as information producers under information asymmetries has been widely discussed in the literature. While this article shares some common features with the standard models, in that banks manage the problems resulting from asymmetric information,[2] the mechanism of the processing or revealing of information assumed here is quite different from theirs. It is generally assumed, for example, that the production of information in the market "will not be done efficiently or at least cost" [Campbel and Kracaw, 1980, p.881]. Thus, the studies dealing with banks facing information problems focus on the need for those institutions to invest their resources in order to produce

[1]Financial intermediaries are defined as voluntary coalitions of agents in this paper, and as will be described in Sections 4 and 5, informed and uninformed agents form coalitions respectively for their own benefit. To distinguish the coalitions of informed and uninformed agents, I will call the alliances of informed agents 'coalitions' and those of uninformed agents 'banks.'

[2]For example, financial intermediaries can provide efficient lending mechanisms when there exists asymmetry of information between lenders and borrowers. See Diamond [1984].

valuable information. The seminal papers on this issue, by Brealey et al. [1977] and Boyd and Prescott [1986], for example, show that financial intermediaries can produce reliable information which is only known to a borrower ex-ante and thus results in an adverse selection problem.

The striking difference between these studies and mine is that the bank in this model does not produce information deliberately by investing some of its resources. The production of useful information is done without cost throughout bank runs under certain conditions. Let me intuitively explain this result. Suppose that there exist four different states of nature, due to two different types of stochastic shocks: liquidity preference shocks and shocks on illiquid asset returns, both of which are assumed to be known only to informed agents. Since asset prices are affected by random shocks and determined endogenously in the market, there are four different asset prices, depending on the realizations of the shocks. If the informed agents can manipulate an illiquid asset's price in a certain state of nature, by adjusting the supply of the asset, then the uninformed consumers who participate in the market suffer losses due to these distorted prices. To prevent these losses the uninformed form a bank, which makes a specific type of deposit contract with its members, and let it go bankrupt if a particular state of nature occurs. The existence of bank runs itself reveals that a distinct state of nature has occurred (e.g. the fraction of uninformed agents who are subject to preference shocks (call them movers) is revealed through bank runs), which again lets uninformed agents conjecture still unknown information (e.g. the rate of return on illiquid assets) with accuracy. Now that the asymmetric information problem has been resolved, they will be able to trade in the asset market without loss.

More specifically, uninformed agents form an intermediary at date t, and it makes a standard deposit contract promising to return a fixed amount of fiat money to an agent if she turns out to be a mover at date $t+1$. The bank, which maximizes the ex-ante expected utility of the typical uninformed agent, will hold only minimal fiat money hoping that the number of movers will be low. It then goes bankrupt if the fraction of movers is revealed to be higher than expected. Even though the agent's liquidity preference is private information, the occurrence of a bank run itself will disclose the information that the fraction of movers is high. Once the uninformed agents acquire this information, they will be able to identify the only price that can appear in the possible price set of the illiquid asset. They can thus exchange their assets with others without suffering losses caused by asymmetric information, in accordance with their true liquidity preferences. Banks serve as information processors by allowing runs. The financial intermediaries in this model therefore not only play the standard role of liquidity provision as in Bryant [1980] and Diamond and Dybvig [1983], but also have an alternative justification based on information processing when the markets are incomplete.

This paper also provides new ground for the coexistence of banks and financial markets. Even when all agents deposit their whole endowments of goods with a bank, the markets play their role once (efficient) bank runs happen. The lower bound of banking under asymmetric information and limited participation in relation to the financial markets is also considered. Allowing a run implies that the scale of the banking sector is minimized to the lower fraction of consumers who face liquidity shocks, and thus potentially increases welfare.

Gorton and Pennacchi [1990] present a model explaining how financial intermediaries arise endogenously and how security contracts are made when uninformed agents, facing asymmetric information, need to transact. Informed agents can exploit the uninformed by their superiority in information. Facing this problem, uninformed agents form a coalition to protect themselves from losses and let it issue debt and equity simultaneously. Through this arrangement, financial intermediaries “can attract informed agents to hold equity and uninformed agents to hold debt which they then use for trading purposes” [Gorton and Pennacchi, 1990, p.50] if the number of informed agents is sufficiently larger than that of uninformed agents. In this way, “the existence of our intermediary does not rely on providing risk-sharing or resolving inefficient interruption of production. Our notion of liquidity as providing protection from insiders is fundamentally different.” [Gorton and Pennacchi, 1990, p.51] In Gorton and Pennacchi [1990], the institution resolves the problem of asymmetric information by providing different bespoke assets to the uninformed and the informed agents, irrespective of the information structure. In this paper, however, the bank just plays the standard role of providing risk sharing with consumers. The mechanism through which the intermediary reveals the information asymmetries to the uninformed is a simple standard deposit contract allowing bank runs to occur under certain circumstances. Moreover, both banks and financial markets play their roles, unlike in Gorton and Pennacchi [1990].

After that, I study the role of banks when agents have limited access to the markets by extending the results with full participation in the markets. Diamond [1997] studied the effect of introducing limited participation on the relationship between the financial markets and intermediation. Wallace

[1988, p.3] argued that the role of banks in Diamond and Dybvig [1983] may exist only "in an environment in which people are isolated from each other." If there exist full-participation markets, no cross-subsidization among early and late consumers is possible, and no beneficial role of banks exists. Moreover, according to Jacklin [1987],[3] banks are able to enhance their liquidity through demand deposits only when the direct holding of assets is restricted. If markets exist in which equity shares are traded, then this trading mechanism will weakly dominate intermediation. Diamond [1997], faced with this criticism, presents a model of limited participation where there is still a scope for banks. With limited participation, banks create more liquidity than the financial markets and make the market more liquid. Financial intermediaries and the markets coexist, and the amount of cross-subsidization is reduced as more agents participate in the market, but as long as there is a limited-participation market it is not eliminated. This paper, however, provides a different rationale for the coexistence of markets and banks, arising from the possibility of a bank run. Moreover, Diamond [1997] assumes that all assets pay a rate of return that is known with complete certainty, and has thus not considered the effects of uncertainty on the equilibrium and the possibility of bank runs, which are considered important in this paper.

Unlike the traditional costly bank runs,[4] a bank run is considered in this paper to be an efficient phenomenon in certain circumstances. The literature dealing with efficient bank runs deals with information-based runs.[5] The studies focus on the fact that bank runs are reflections of the business cycle, and come from agents' optimizing behavior expecting poor performances of the banks rather than as a result of sunspots. According to Allen and Gale [1998], bank runs can be efficient in the case where a standard deposit contract cannot be made contingent on a 'leading economic indicator,' such as the return on illiquid assets, which is random. A bank run makes consumption contingent on the state of nature. The indicator can be observed with accuracy at the middle period, and thus "the possibility of equilibrium bank runs allows banks to hold the first-best portfolio and produces just the right contingencies to provide first-best risk sharing." [Allen and Gale, 1998, p.1250] No information problem arises at least at times when bank runs actually happen, since it

[3]See Haubrich and King [1990] and Hellwig [1994] in a similar vein.

[4]For the recent studies on costly bank runs, see Martin et al. [2014], Angeloni and Faia [2013], Gertler and Kiyotaki [2015], and Gertler et al. [2016].

[5]Related papers include Chari and Jagannathan [1988], Jacklin and Bhattacharya [1988], Allen and Gale [1998], and Diamond and Rajan [2001], to name just a few.

is assumed that each agent can observe the indicator with precision.[6] In this paper, however, bank runs happen irrespective of whether agents have access to information about the rates of return on risky assets. In other words, bank runs may happen even when a higher rate of return on illiquid assets is expected. Moreover, the run is efficient not because it makes contingent consumption possible, but because it reveals the unknown information to which uninformed agents do not have access.

The studies that I have reviewed so far all deal with the economy focusing on real factors in which a role for currency is ignored. Champ et al. [1996], on the other hand, construct a monetary model in which a role for currency is considered and monetary factors play a specific role in banking panics. In Champ et al. [1996], the bank provides insurance to agents who have random needs for liquidity because of the possibility of relocation. The 'relocation shock' takes the place of the 'preference shock' in Diamond and Dybvig [1983]. One of the reasons why Champ et al. [1996] consider the role of currency seriously is because the defining fractures of banking distress involve the currency in a central way. However, except for the assumptions used in the basic set-up of their model (a three-period OLGs model with a relocation shock, where fiat money is used as liquid assets), this paper is not directly related to Champ et al. [1996].

The rest of this paper is organized as follows. The environment is described in Section 2. In Section 3 the process of asset price determination is shown in detail. In Section 4 the asset market equilibriums with and without asymmetric information are described. Then, Section 5 introduces the financial intermediary and considers how the intermediary deals with the problems regarding asymmetric information. In Section 6 I show that financial intermediaries are still needed when we introduce another friction into the financial markets, limited participation, even when each agent has access to perfect information. I then consider an economy having both asymmetric information and limited participation, and study the role of banks. Concluding remarks are presented in Section 7.

[6] In a similar vein, Gorton [1985] provides a model in which agents obtain, in the middle period, some information regarding the rate of return on risky assets the expected values of which are realized in the final period.

2 Environment and Equilibrium

2.1 Agents and Information

There are infinitely many overlapping generations of agents who live for three periods (call them the young, the middle, and the old-aged).[7] The population is divided equally by two distinct locations (islands). There are four types of agents as of date t (see Figure 1). They are divided broadly into two groups: the informed and the uninformed. The measure of the entire set of uninformed agents in each island is normalized to the unit interval $[0, 1]$, and the fraction of informed agents in terms of the scale of the uninformed is denoted as λ_{inf}. Thus, there is a $1 + \lambda_{inf}$ number of agents living in each island.

At the middle of each period stochastic relocation shocks occur and a fraction of movers among the uninformed agents, λ^i, is relocated to the other island at the end of the same period and will spend their final period on that island. This information is privately held and not observable by other agents.[8] λ^i is assumed to take two different values: λ^h, with probability μ_h, or λ^l, with probability μ_l, where $0 < \lambda^l < \lambda^h < 1$. The probability of relocation is assumed to be the same across the islands, so that the population of each island remains constant.[9] There are two different types of assets for savings: capital (k) and fiat money (M). k_t units of investment in capital at date t yields $k_t R_j$ units of output at date $t+2$, and R_j is a random variable taking two different values: R_H, with probability 1/2, or R_L, with probability 1/2, where $R_H > R_L > 0$. Capital is an illiquid asset, for two reasons. Firstly, the capital invested in production needs two periods of time before it is transformed into consumption goods. Secondly, it is assumed to be not transported across islands, and moreover, due to the 'limited communication,' claims against capital are assumed to be useless. In order for movers to consume after being relocated, they need fiat money, which is identical in the two islands and universally accepted as a means of exchange.

[7]The model in this section where only financial markets exist is based on the models used by Champ et al. [1996], Gorton and Pennacchi [1990] and Diamond [1997]. Champ et al. [1996] built an OLGs model with random relocation where agents live for two periods.

[8]If this information can be observed by anyone living on the island, the information problem is removed. Movers deal only with non-movers and non-movers only with movers. Also, with 'open communication,' where the claims against capital can be verified without cost and can be traded without cost, all agents will only hold capital since its rate of return is expected to be greater than that of fiat money.

[9]If there were no aggregate relocation shocks, there were a large number of uninformed agents, and their relocation shocks were assumed to be independent, the fraction of movers would be equal to the probability of being a mover according to the Law of Large Numbers.

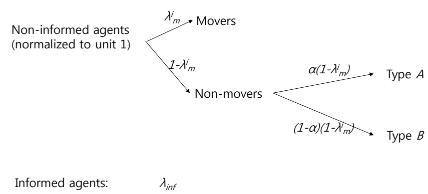

Figure 1: Types of Agents

The informed are assumed to be not subject to the relocation shocks and thus stay on the same islands. Another difference between the informed and the uninformed is that only informed agents have access to the information at date $t+1$ on the precise returns on capital, R_j, and on the proportion of movers, λ^i, which will become known to the uninformed at date $t+2$.

Before they relocate, movers trade their holdings of capital for fiat money in a financial market. They exchange money for consumption goods with the new-born young (or the bank of the young if any) at date $t+2$. Agents who turn out to stay on their home islands exchange some or all of their holdings of money for the capital that movers hold. They consume the return from any capital investment made when young, and from the capital they have bought from the movers when middle-aged, plus consumption goods for which they trade any fiat money holdings (if left) with the newly born young at date $t+2$. The timing of events is illustrated in Figure 2. Non-movers are classified again into two subgroups. A fraction among non-movers $\alpha(1-\lambda^i)$, born at date t and called type A, have access to a financial market at date $t+1$. On the other hand, the fraction of $(1-\alpha)(1-\lambda^i)$ is assumed to not join the market. Thus, $0 \leq \alpha \leq 1$ can be used as an index to show financial market development, as in Diamond [1997]. Except for their possibilities of participating in the market, there are no essential differences between types A and B. The fraction of type B, $(1-\alpha)(1-\lambda^i)$, is assumed to be zero in this section and Sections 4 and 5. The effect of the existence of type B on the role and scale of banking will be discussed in detail in Section 6.

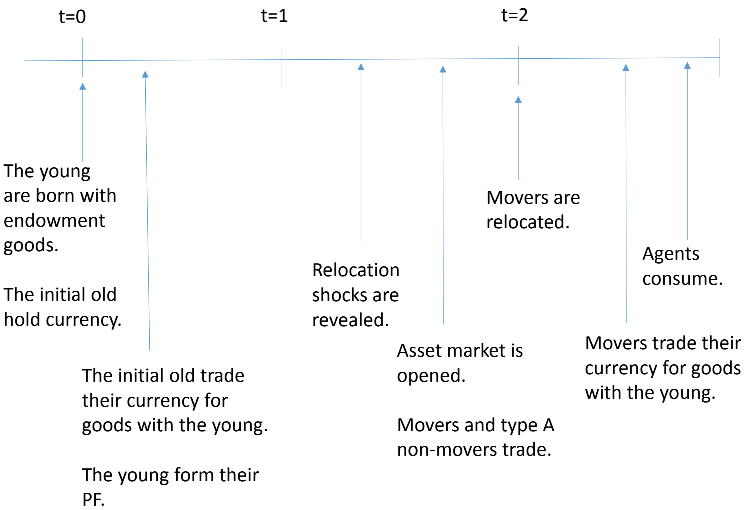

Figure 2: Timing of Events

2.2 Portfolio Decision

Each agent is born with one unit of endowment goods, which can be used both for consumption and investment and cannot be stored from one period to the next. Endowments are used to form a portfolio consisting of two types of assets: fiat money, M, and capital, k. Let v_t be the value of one unit of fiat money in terms of goods at time t. Then $v_t M_t$ ($\equiv q_t$) denotes the real demand for fiat money at date t, and the budget constraint of the uninformed is given as

$$q_t + k_t = 1.$$

No one in the future generations is born with fiat money. In order for young agents to acquire fiat money on each island, they must trade with the initial old living on the same island who are endowed with M_t units of fiat money in total. The initial value of fiat money, v_t, is determined at the beginning of period t by the coincidence of the demand for fiat money by the young and the supply of it by the initial old.[10] The (real) demand for fiat money is q_t, and the supply of fiat money measured in goods coming from the initial old is M_t. Therefore, the value of one unit of fiat

[10] The initial middle-aged agents, holding both currency and capital at date t, are assumed to not participate in this market since the market opens before the relocation shocks are revealed. Under this assumption, the initial middle have no incentives to change their portfolios.

money at date t, v_t is determined such that

$$v_t = \frac{q_t}{M_t}. \tag{1}$$

The money supply is assumed to remain constant, and the two different islands are symmetric.[11] Then, even though v_t and v_{t+1} are determined endogenously within the model, the rate of return on fiat money is just equal to 1, and $E[R_j]$ is assumed to be greater than the two-period rate of return on fiat money, $v_2/v_0 = 1$. It is also assumed that $R_H > v_2/v_0 > R_L$, i.e. the rate of return on capital could be lower than that of fiat money when the random return is low. Uninformed agents will save some of their endowments in the form of currency despite its lower rate of return than its alternative (capital), because some of them are subject to relocation shocks. Thus, the two different forms of savings – fiat money and capital – are not regarded as perfect substitutes for each other.[12]

2.3 Preferences

All agents are risk-neutral and consume in their final period.[13] The preferences of the generation born at time 0 are represented by:

$$U(c_{\tau,2}) = \begin{cases} c_{m,2}, & \text{if movers} \\ c_{n,2}, & \text{if non-movers} \\ c_{inf,2}, & \text{if informed,} \end{cases}$$

where $c_{\tau,2}$ denotes the amount of goods that is consumed in the final period of life by a type of τ born in period 0.

A consumer who turns out to be a mover in her middle age sells her entire holdings of capital

[11] In this paper, I consider stationary and symmetric allocation only, so I drop the time scripts t from now on. The consumption of agents born at date t is written as $c_{\tau,2}$, and the value of fiat money at date $t+s$ is denoted as v_s for all $s \geq 0$. The notations without time subscripts denote the amounts determined at date $t = 0$.

[12] If agents are risk-averse, they will acquire both fiat money and capital even though the rate of return on capital is expected to be higher than that on currency. If agents are assumed to be risk-neutral, all might want to hold only capital. If financial markets open in the middle period of their life, however, at least some should keep some fiat money in order for trades to occur. Some fraction of agents who turn out to be movers will try to sell their holdings of capital. However, since no one holds money, the asset price will go to zero. Thus, there will be at least some agents who might want to hold currency to exploit this arbitrage opportunity. In this paper, agents are assumed to hold both fiat money and currency for this reason, i.e. in equilibrium both $q_t > 0$ and $k_t > 0$.

[13] The reason I use a linear utility function is for calculating a closed-form solution of the optimizing agent.

at a price B_1 before she moves. She inelastically supplies her capital holdings, whatever the price is. The total unit of currency that the mover accumulates after the trade at date 1 is represented as:

$$M + \frac{B_1 k}{v_1}. \tag{2}$$

The second term of this equation denotes the amount of fiat money that the mover acquires by selling her capital to non-movers who belongs to the same generation as her. The old-age consumption of a mover is then given as:

$$c_{m,2} = \left(M + \frac{B_1 k}{v_1} \right) v_2 = q + B_1 k. \tag{3}$$

Non-movers must decide to trade either some or all of their holdings of fiat money for illiquid assets (capital) at date 1, by weighing the difference between the expected returns of the two assets. The total supply of fiat money (S^M) by non-movers, which equals the demand for capital, D^K, thus takes the following form:

$$S^M = D^K = (1 - \lambda^i)\gamma M,$$

where γ $(0 \leq \gamma \leq 1)$ is the ratio of cash reserves exchanged with capital. $\gamma = 1$ implies that non-movers trade their entire currency holdings for capital at date 1, and do not carry it over to the next period. The consumption of a non-mover at date 2 is represented as:

$$c_{n,2} = \left(\frac{\gamma q}{B_1} + k \right) R_j + (1 - \gamma) q. \tag{4}$$

Informed agents hold only capital at date 0, since they know that they are not relocated. The consumption of an informed agent at date 2 then is represented as:

$$c_{inf,2} = R_j \tag{5}$$

2.4 Equilibrium

Assume that an asset market at date 1 is opened after the relocation shocks are revealed and before movers are relocated. Let $B_1^{i,j}$ be the (nominal) price of one unit of capital traded in the asset market at date 1 in states of $\{i,j\}$ where $i = \{h,l\}$ and $j = \{H,L\}$. An imperfectly competitive rational expectations equilibrium for agents born at date 0 consists of:

1. Price system (a price vector): $\{B_1^{i,j}\} = (B_1^{h,H}, B_1^{h,L}, B_1^{l,H}, B_1^{l,L})$,
2. Expected consumption of agents:

$$E[C_{\tau,2}] = \begin{cases} E[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\ E[c_{inf,2}], \end{cases}$$

   where $E[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}]$ denotes the expected utility of the uninformed agent,

3. Specification of storage strategies for (market participating) non-movers: γ, where $0 < \gamma < 1$, and
4. A specification of insider coalition strategies : $\lambda_{inf}^{i,j}$ (discussed in detail in Section 4.2),

such that

1. Agents' respective utilities are maximized ($E[c_{\tau,2}]$ maximizes agent τ's expected utility),
2. $\{B_1^{i,j}\}$ and γ clear the asset market in all states of $\{i,j\}$, where $i = \{l,h\}$ and $j = \{L,H\}$, and
3. $\lambda_{inf}^{i,j}$ is self-enforcing.[14]

3 Determination of Asset Prices

This section contains a detailed exposition on how the price of capital, an illiquid and risky asset, is determined when the economy faces uncertainty as to its fundamentals. This discussion is based on the work of Gorton and Pennacchi [1990], Allen and Gale [1994, 2009], and Diamond [1997], which discuss the process of asset price determination in different situations. This section is meaningful in that it draws together the results of the scattered studies into a comprehensive view.

[14]With a self-enforcing agreement, the informed agents will provide capital to the financial markets as precisely as the coalition has predetermined.

3.1 Fundamental Asset Price

We have the following proposition:

Proposition 1

$$B_1^{i,j} = \min\left\{\frac{R_j}{v_2/v_1}, \frac{v_1}{v_0}\frac{\gamma(1-\lambda^i)q}{\lambda^i k}\right\}. \tag{6}$$

Proof. See Appendix A. ■

This proposition tells us that, if the market has enough liquidity (i.e. $\gamma(1-\lambda^i)q$ is big enough), then the asset price will reach its highest possible point, $\frac{R_j}{v_2/v_1}$. However, if the market suffers a liquidity shortage for any reason, the price will be determined by the amount of cash supplied. This is known as 'cash-in-the-market pricing,' in which the asset price is determined as the ratio of total available 'cash' to the amount of assets provided [Allen and Gale, 1994, 2009].

3.2 Asset Prices with Uncertainty

Proposition 2 *Suppose that the random variable R_j takes the values of R_H with a probability of ρ_H and R_L with a probability of ρ_L, where $R_H > R_L > 0$ and all assets are held by consumers. Also assume that the random variable λ^i has the following two-point supports:*

$$\lambda^i = \begin{cases} \lambda^h, & \textit{with probability } \mu_h \\ \lambda^l, & \textit{with probability } \mu_l, \end{cases}$$

where $0 < \lambda^l < \lambda^h < 1$.

Then the capital prices at date 1 in the states of $\{i,j\}$, $B_1^{i,j}$, have the following relationship:

1. *When $R_j = R_H$ the price of capital is determined such that $B_t^{i,H} = \min\left\{\frac{R_H}{v_2/v_1}, \frac{v_1}{v_0}\frac{\gamma^H(1-\lambda^i)q}{\lambda^i k}\right\}$, which is strictly greater than $\frac{v_1}{v_0}$, and when $R_j = R_L$, $B_t^{i,L} = \frac{R_L}{v_2/v_1}$, which is strictly less than $\frac{v_1}{v_0}$.*

2. *γ^H is equal to 1 when $R_j = R_H$, and γ^L is determined such that $R_L = \frac{v_2}{v_0}\frac{\gamma^L(1-\lambda^i)q}{\lambda^i k}$, which implies $\gamma^L = \frac{v_2}{v_0}\frac{\lambda^i k R_L}{(1-\lambda^i)q}$.*

3. *In equilibrium we have*

$$B_1^{l,H} > B_1^{h,H} > v_1/v_0 > B_1^{i,L}. \tag{7}$$

4. Combining the above results, we have the following relationship:

$$R_H \geq \frac{v_2}{v_1} B_1^{l,H} > \frac{v_2}{v_1} B_1^{h,H} > \frac{v_2}{v_0} > \frac{v_2}{v_1} B_1^{h,L} = \frac{v_2}{v_1} B_1^{l,L} = R_L, \tag{8}$$

where $B_t^{i,H} = \min\left\{\frac{R_H}{v_2/v_1}, \frac{v_1}{v_0}\frac{(1-\lambda^i)q}{\lambda^i k}\right\}$.

Proof. See Appendix B. ■

The asset price in this section cannot be higher than the fundamental value, $\frac{R_j}{v_2/v_1}$. The market always undergoes "underpricing" [Allen and Gale, 1994].[15]

Assumption 1 *When the economy has a large proportion of movers, i.e. when $\lambda^i = \lambda^h$, then the asset price is determined by the amount of liquidity in the market.*

When the economy suffers high liquidity needs, which implies less provision of liquidity by a relatively smaller number of non-movers, the asset price is assumed to determined by the 'cash-in-the-market.' This assumption is needed to calculate the exact amount of capital provided to the market by informed agents in order to influence the asset price (see Proposition 3).

Corollary 1 *Directly applying the previous assumption and Proposition 2, we have the following relationship:*

$$B_t^{l,H} = \min\left\{\frac{R_H}{v_2/v_1}, \frac{v_1}{v_0}\frac{(1-\lambda^l)q}{\lambda^l k}\right\} \text{ and } B_t^{h,H} = \frac{v_1}{v_0}\frac{(1-\lambda^h)q}{\lambda^h k}. \tag{9}$$

In a stationary equilibrium where the money supply is constant,[16] all generations face the same decision problem. Therefore, q and thus v have constant values, i.e. $v_{t+s} = v$ for all $s \geq 0$. Therefore, the rate of return on fiat money is written as 1 from now on for notational simplicity.

[15] Asking if the asset price always takes a value less than the fundamental value is questioning whether or not 'bubbles' may exist. This is an interesting question, but is not dealt with in this paper. See Fama et al. [2013] for an introductory description of whether financial assets reflect fundamental values or bubbles exist.

[16] This assumption cannot be sustained in situations where each island faces different idiosyncratic relocation shocks, or where the monetary authorities play an active role to affect the economy by adjusting the money supply. If the two separate islands had different fractions of movers, then the populations of the two islands would be distinct from each other, and would thus have dynamic effects on consumption and the value of money after their relocations. The effects of these incidents on equilibria is a subject for further study.

4 Market Equilibrium

4.1 Full Participation and Complete Information

Let $\widehat{B}_1^{i,j}$ be the full-information prices of an illiquid asset for states $\{i,j\}$, which are determined according to Proposition 2. And letting $E^F[\lambda^i c_{m,2}+(1-\lambda^i)c_{n,2}]$ be the expected utility of uninformed agents with full information and full participation, $E^F[\cdot]$ is then given as:[17]

$$
\begin{aligned}
&E^F[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
&= \frac{\mu_h}{2}\left[\lambda^h\left(q+\widehat{B}_1^{h,H}k\right) + (1-\lambda^h)\left(\frac{q}{\widehat{B}_1^{h,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(q+\widehat{B}_1^{l,H}k\right) + (1-\lambda^l)\left(\frac{q}{\widehat{B}_1^{l,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_h}{2}\left[\lambda^h\left(q+\widehat{B}_1^{h,L}k\right) + (1-\lambda^h)\left(\frac{\gamma^h q}{\widehat{B}_1^{h,L}}R_L + R_L k + (1-\gamma)\frac{v_2}{v_0}q\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(q+\widehat{B}_1^{l,L}k\right) + (1-\lambda^l)\left(\frac{\gamma^l q}{\widehat{B}_1^{l,L}}R_L + R_L k + (1-\gamma)q\right)\right] \\
&= q + E[R_j]k.
\end{aligned} \tag{10}
$$

The second through the fifth lines in (10) represent the consumption level when each state $\{i,j\}$ occurs. In what follows I study how the expected welfare of each type is affected when consumers have asymmetric information.

4.2 Markets with Asymmetric Information

The total amount of capital held by informed agents is then simply λ_{inf}, since informed agents hold only capital at date 0. The informed form a coalition at date 1 after the uncertainty is resolved, and the coalition collectively decides whether to provide the capital holdings of its members in states $\{i,j\}$. Let $\lambda_{inf}^{i,j}$ be the amount (proportion) of capital that the coalition decides to provide to the financial market at date 1, where $0 \le \lambda_{inf}^{i,j} \le \lambda_{inf}$. The question posed in this situation is: will $\lambda_{inf}^{i,j}$ be positive in some information sets $\{i,j\}$ at date 1? The coalition will trade capital for fiat money only when they get additional gains from this trade by manipulating the asset price

[17] See Online Appendix A for computational details.

through providing its holdings of capital. The following proposition[18] shows when that arbitrage opportunity is possible:

Proposition 3 • *Let $\bar{R} = \mu_h' R_H + \mu_l' R_L$ denote the uninformed non-mover's posterior expectation for the rate of return on capital when the state $\{l, L\}$ occurs, and μ_h' and μ_l' are their posterior probabilities on the fraction of movers. Then:*

- *If (i) $\widehat{B}_1^{h,H} = \frac{v_1}{v_0}\frac{(1-\lambda^h)q}{\lambda^h k} \leq \frac{\bar{R}}{v_2/v_1}$, and (ii) the number of informed agents compared to that of the uninformed is equal or greater than $\lambda_{inf} \geq \overline{\lambda}_{inf} = \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h}$, then the coalition of informed agents can mimic the price $\widehat{B}_1^{h,H}$ at date 1 when a state $\{l, L\}$ exists. Letting $\widetilde{B}_1^{l,L}$ be the price which is manipulated by the informed agents when the state $\{l, L\}$ occurs, we have the following:*

$$\widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H} > \widehat{B}_1^{l,L}.$$

  $\overline{\lambda}_{inf}$ is the minimal number of the informed that will influence the asset price. Uninformed non-movers trade their full holdings of fiat money for capital, because they misunderstand that the state $\{h, H\}$ has occurred when they observe $\widetilde{B}_1^{l,L}$, and thus $\gamma^L = 1$.

- *Except for the state of $\{l, L\}$, the true prices are revealed as in Proposition 2, and $\lambda_{inf}^{i,j} = 0$ where $\{i, j\} \neq \{l, L\}$.*

Proof. Refer to Gorton and Pennacchi [1990] to see how the informed agents manipulate the asset price when a state $\{l, L\}$ occurs to make it look like $\widehat{B}_1^{h,H}$.

Let $\widetilde{B}_1^{i,j}$ be the asymmetric-information price for states $\{i, j\}$ when $\lambda_{inf}^{i,j} > 0$, and let us look at how much capital needs to be provided by the coalition of the informed. Let $\lambda_{inf}^{l,L}$ be the amount of capital supplied to the market by the coalition in the state of $\{l, L\}$. The specific $\lambda_{inf}^{l,L}$ value can then be obtained from the market equilibrium condition and using $\widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H}$:

$$\lambda^l k \widetilde{B}_1^{l,L} + \lambda_{inf}^{l,L} \widetilde{B}_1^{l,L} = (1 - \lambda^l) q$$

The left-hand side of this equation denotes the total supply of capital in the state of $\{l, L\}$, and the right-hand side the demand for capital. Substituting $\frac{(1-\lambda^h)q}{\lambda^h k}$ for $\widetilde{B}_1^{l,L} (= \widehat{B}_1^{h,H})$ using Assumption 1

[18] This proposition is a variant of Proposition 1 of Gorton and Pennacchi [1990]. I have modified it for the OLGs model with random relocation shocks and fiat money.

and $v_{t+s} = v, \forall s \geq 0$, we have:

$$\begin{aligned} &\lambda_{inf}^{l,L} \frac{(1-\lambda^h)q}{\lambda^h k} + \lambda^l \frac{(1-\lambda^h)q}{\lambda^h} = (1-\lambda^l)q \\ \Leftrightarrow &\lambda_{inf}^{l,L} = \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h} \equiv \overline{\lambda}_{inf}. \end{aligned} \tag{11}$$

As long as $\lambda_{inf} \geq \overline{\lambda}_{inf} = \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h}$, the coalition of the informed is able to mimic the price $\widehat{B}_1^{h,H}$ when a state $\{l, L\}$ actually occurs. ■

There are a couple of things to note as discussed in Gorton and Pennacchi [1990]. Firstly, since the uninformed agents are rational, they know that when a state $\{l, L\}$ occurs the informed agents make it look like a state $\{h, H\}$ happens. Nevertheless, they will sell their holdings of capital as long as they believe it to be profitable on average. In other words, it will be optimal for the uninformed movers to sell their capital if the following holds:

$$\widetilde{B}_1^{l,L} \leq \bar{R} = \mu_h' R_H + \mu_l' R_L,$$

which would happen when μ_h' is large enough. This condition also acts as a constraint to prevent informed agents from participating in the bank of the uninformed agents.[19] Secondly, λ_{inf} is needed to check whether it is a self-enforcing Nash coalition or not. If any member of the informed agents' coalition sells her capital in a state $\{h, H\}$ privately, then a market price $\widetilde{B}_1^{l,L}$ does not prevail any longer, and the new price due to the deviation will reveal the intention of the informed agents. This inference confirms that λ_{inf} is a self-enforcing Nash coalition.

The preceding proposition implies that when the state $\{l, L\}$ actually occurs, the consumption of the uninformed agents will be:

$$\lambda^l \left(q + \widetilde{B}_1^{l,L} k\right) + (1-\lambda^l) \left(\frac{q}{\widetilde{B}_1^{l,L}} R_L + R_L k\right).$$

Now letting $E^A[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}]$ denote the expected utility of the uninformed agents with

[19]Suppose that informed agents deposited their endowments with the bank as well and the state were $\{l, L\}$. If the informed withdrew all their deposits at date 1 and the number of informed agents were large enough, then the bank would go bankruptcy and the state would look like $\{h, H\}$ in which case the informed agents will extra gains. On the other hand, they will suffer loss if the realized state is $\{h, H\}$ since $R_H > \widehat{B}_1^{h,H}$ (see Proposition 2). The expected loss would overweight the expected gain and the informed would have no incentive to join the uninformed agents' bank as long as the assumption holds.

asymmetric information, $E^A[\cdot]$ is then given as:

$$
\begin{aligned}
&E^A[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
&\quad = \frac{\mu_h}{2}\left[\lambda^h\left(q + \widehat{B}_1^{h,H}k\right) + (1-\lambda^h)\left(\frac{q}{\widehat{B}_1^{h,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(q + \widehat{B}_1^{l,H}k\right) + (1-\lambda^l)\left(\frac{q}{\widehat{B}_1^{l,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_h}{2}\left[\lambda^h\left(q + \widehat{B}_1^{h,L}k\right) + (1-\lambda^h)\left(\frac{\gamma q}{\widehat{B}_1^{h,L}}R_L + R_L k + (1-\gamma)q\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(q + \widetilde{B}_1^{l,L}k\right) + (1-\lambda^l)\left(\frac{q}{\widetilde{B}_1^{l,L}}R_L + R_L k\right)\right].
\end{aligned}
\tag{12}
$$

Since the difference between equations (10) and (12) exists only when the state $\{l, L\}$ occurs, we have the following:

$$
\begin{aligned}
E^F[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] - &E^A[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
&= \frac{\mu_l}{2}\left[\lambda^l k(\widehat{B}_1^{l,L} - \widetilde{B}_1^{l,L}) + (1-\lambda^l)q\left(1 - \frac{R_L}{\widetilde{B}_1^{l,L}}\right)\right] \\
&= \frac{\mu_l}{2}\frac{k(\lambda^h - \lambda^l)}{1-\lambda^h}(\widehat{B}_1^{h,H} - R_L) > 0.
\end{aligned}
\tag{13}
$$

$E^A[\cdot]$ is always lower than $E^F[\cdot]$ when asymmetric information exists.[20]

Finally, let us look at the expected utility of the informed agents. Under full information, where trades occur only at true prices, the expected utility of the informed is just

$$
E^F[c_{inf,2}] = E[R_j].
$$

Under asymmetric information, on the other hand,

$$
\begin{aligned}
E^A[c_{inf,2}] &= \frac{1}{2}R_H + \frac{\mu_h}{2}R_L + \frac{\mu_l}{2}\left(\left[1 - \frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\right]R_L + \frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\frac{\widetilde{B}_1^{l,L}}{v_1}v_2\right) \\
&= E[R_j] + \frac{\mu_l}{2}\frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\frac{(\widehat{B}_1^{h,H} - R_L)}{v_1}v_2 > E[R_j].
\end{aligned}
\tag{14}
$$

The coalition can now exchange for currency $\frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}$ per unit of the capital good at date 1 at a higher

[20]see Online Appendix B for the detailed computation.

price, $\widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H}$, in the state $\{l, L\}$. This gain comes at the expense of the consumption of the non-movers. The trading losses of uninformed agents associated with information asymmetry is equal to the trading gains of the informed agents.[21] The results are summarized in the following proposition:

Proposition 4 *The imperfectly competitive rational expectations equilibrium with asymmetric information and without a coalition of the uninformed young is described as follows:*

1. *Price system:* $\{B_1^{ij}\} = (\widehat{B}_1^{h,H}, \widehat{B}_1^{l,H}, \widehat{B}_1^{h,L}, \widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H})$,

2. *Expected consumption of agents:*

$$E^A[C_{\tau,2}] = \begin{cases} E^A[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] < E^F[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\ E^A[c_{inf,2}] = E[R_j] + \frac{\mu_l}{2}\frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\frac{v_2}{v_1}(\widehat{B}_1^{h,H} - R_L) > E^F[c_{inf,2}], \end{cases}$$

3. *Specification of storage strategies for (market participating) non-movers:*

$$\begin{cases} 0 < \gamma < 1, \ \textit{if } \{i,j\} = \{h, L\} \\ \gamma = 1, \ \textit{otherwise, and} \end{cases}$$

4. *Specification of insider coalition strategies:*

$$\lambda_{inf}^{i,j} = \begin{cases} \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h}, \ \textit{if } \{i,j\} = \{l, L\} \\ 0, \ \textit{otherwise.} \end{cases}$$

5 Role of a Bank

Now we suppose that young, uninformed agents who are born at date t can organize an institution, which is called a bank, and see if this institution can protect them from the losses induced due to the asymmetry of information. These agents deposit some or all of their endowment goods with the bank, and it uses the proceeds to acquire assets on behalf of its members. Free entry into the banking industry forces banks to compete by offering deposit contracts that maximize the expected

[21] Since there are $\lambda_{inf}^{l,L}$ number of informed agents with respect to that of the uninformed agents (which is normalized to 1), the total gains from trade of the informed agents are exactly equal to the total losses of the uninformed agents.

utility of consumers. In dealing with a bank run situation, I follow the assumptions in Allen and Gale [1998] rather than the 'sequential service' assumption in Diamond and Dybvig [1983]. The bank makes a standard deposit contract with its members, promising a nominal amount of fiat money, d, at date 1 if it has enough liquid assets. If the bank does not have enough liquid assets to pay the promised amount, however, it pays out all available liquid assets and capital, divided equally among those withdrawing their deposits.

5.1 Model

At the beginning of time 0, the uninformed young deposit their endowment goods with the bank in exchange for a promised amount of fiat money at date 1 if they turn out to be movers. If they are non-movers they become residual claimants and share whatever is left over equally with the other non-movers. The bank's problem is then to choose how much capital and fiat money to acquire to maximize the ex-ante expected utility of the typical uninformed agent, and the standard deposit contract is made in the process.[22]

$$\max_{c_{m,2},c_{n,2}} \Psi = E[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \tag{15}$$

$$\begin{aligned} s.t. \quad (i) \quad & q^i + k^i \le 1, \\ (ii) \quad & \lambda^i d \le \frac{q^i}{v_0}, \\ (iii) \quad & (1-\lambda^i)c_{n,2} \le v_2\left(q^i - \lambda^i v_0 d\right) + R_j k^i, \\ (iv) \quad & c_{n,2} \ge c_{m,2} = v_2 d \end{aligned} \tag{16}$$

The first constraint is the budget constraint facing the bank.[23] The bank determines how to divide its member's endowment goods between liquid and illiquid assets. The second decision made by the bank is how much it should pay to movers who must withdraw at date 1. The bank makes a standard

[22] To make the transaction sequence as simple as possible, I assume that the initial old at date 0 and the old movers after date 1 consume after the bank accepts deposits from the young uninformed.

[23] In this section I suppose that each uninformed young agent deposits the entirety of her endowment goods when she is young. However, we do not necessarily need to assume this way. For example, agents may decide to hold capital directly and deposit only parts of their endowment goods in the bank. Diamond [1997] explains this type of deposit agreement.

deposit contract with its members, which promises to give a fixed amount of fiat money, d, at date 1[24] by paying out all available liquid assets, divided equally among those making withdrawals. Movers will withdraw their deposits in the form of fiat money from the bank in their middle ages before they are relocated, and then take the money to the bank in the foreign island and trade it for consumption goods. Each mover will consume $v_2 d = \frac{v_2}{v_0}\frac{q^i}{\lambda^i}$. If the bank does not have enough liquid assets to pay the promised amount to the movers at date 1, it will go bankrupt. Constraint (ii) says that the bank's holding of fiat money must be sufficient to provide the movers with d in accord with their deposit contracts. Non-movers are paid whatever is still available in their third period, as is shown in constraint (iii). Constraint (iii) says that the consumption of non-movers is limited by the total value of the risky asset, capital, plus the amount of fiat money left over, if any, after the movers are paid off.

Constraint (iv), the incentive compatibility constraint, says that the consumption of the non-movers must be at least as much as that of the movers. Non-movers will get whatever is left over in a bank when they are old. Since whether a certain agent is relocated or not is private information, and the bank can thus not identify who is a mover and who is a non-mover, non-movers have an incentive to pretend to be movers unless this constraint holds. The incentive constraint tells us that non-movers do not have any incentive to pretend to be movers as long as they will get a higher level of the consumption good in their final period.

In the optimum situation, $\lambda^i d$ must be equal to q^i/v_0; otherwise (i.e. $\lambda^i d < q^i/v_0$) the bank could increase expected utility by reducing q since the rate of return on capital is expected to be higher than that on the fiat money. Note that it is not possible to make a contingent deposit contract, because the portfolio decision is made before the relocation shocks are revealed. Since there are two possible states for the fractions of movers, either of the following two relationships will hold depending upon how the bank chooses q^i:

$$\lambda^h d = \frac{q^h}{v_0} > \lambda^l d \tag{17}$$

or,

$$\lambda^l d = \frac{q^l}{v_0} < \lambda^h d. \tag{18}$$

[24] q^i/v_0 denotes the amount of fiat money that the bank holds at date 1.

If the bank chooses to spend q^h instead of q^l, and if a state l actually occurs, then the bank will have excess liquidity which is carried over to the next period. If on the other hand the real money demand is equal to q^l rather than q^h, and the state h occurs, then the bank will experience a shortage of liquidity and suffer bankruptcy.

5.2 Equilibrium

As noted in the preceding subsection, the optimal amount of q is not determined as a fixed amount, since the optimal q does not take a single value but depends on how many agents are subject to the relocation shocks. Therefore, regarding the bank's choice of how much to pay to the movers, there are two possible choices between q^l and q^h.[25] Whether q^i is the optimal amount of liquidity that the bank holds depends on the realization of λ^i. Now let us consider the effects of the bank's two different choices on its members' utility. Analysis shows that the uninformed young will enjoy higher consumption levels when the bank spends a minimum on purchasing liquid assets at date 0. In other words, the bank will always choose q^l instead of q^h. Note, however, that in any case the welfare of the uninformed is increased when they form a bank over their welfare otherwise.

5.2.1 Equilibrium with Bankruptcy

Suppose that the bank makes a contract with its depositors expecting that state l will occur and holds liquid assets such that $q^l = \lambda^l$. There is a good reason for choosing $q^l = \lambda^l$. In fact, with the linear utility function the expected utility of the uninformed agents will be greater the less that q is. However, if $q^l < \lambda^l$ then the bank always suffers bankruptcy, and so the bankruptcy's unique role of revealing unknown states of nature is no longer being performed.[26]

The bank's choice of how much to spend to retain liquid assets has two consequences, depending upon how the actual state of nature i is realized. If the state l happens at date 1, then the bank will hold a sufficient amount of fiat money to distribute to the movers. The expected consumption

[25] In a situation where bank runs are costly, in that physical liquidation incurs costs [Allen and Gale, 2000], or individuals do not have access to the financial markets, not allowing bank runs by holding sufficient liquidity can be optimal. Allen et al. [2009] take this position.

[26] In the standard Diamond and Dybvig type model with a log utility, the optimal amount of q is just equal to λ. Another reason why banks must have at least λ^l in liquidity is that there might be liquidity regulation by the central bank. However, the issue of central bank regulation of liquidity or capital is not a subject of this paper, and so I will not discuss it further.

levels of the different agents are then is given as:

$$\begin{aligned} \text{movers} \quad & c_{m,2} \le v_2 d = \frac{v_2}{v_0}\frac{q^l}{\lambda^l}, \\ \text{non-movers} \quad & c_{n,2} \le \frac{1}{1-\lambda^l}\left(v_2\left(q^l - \lambda^l v_0 d\right) + k^l R_j\right). \end{aligned}$$

Since carrying over the fiat money to date 2, i.e. $c_{m,2} < v_2 d$, is not optimal, in the optimal plan the bank always chooses q and thus d such that

$$c_{m,2} = v_2 d = \frac{q^l}{\lambda^l}, \tag{19}$$

which determines the consumption of the non-movers as

$$c_{n,2} = \frac{k^l R_j}{1-\lambda^l}. \tag{20}$$

If the state h occurs at date 1, on the other hand, then the bank will not hold enough liquidity to provide to the movers, since $\lambda^h d > \frac{q^l}{v_0} = \lambda^l d$. The bank inevitably goes bankrupt (a bank run happens). Movers and non-movers equally divide the fiat money and capital that the bank holds, $q^l + k^l$, and movers and non-movers trade these assets with one another in the financial markets.

From the fact that a bank run happens, the agents can now realize that the state h has occurred instead of the state l. From this newly acquired knowledge they can confine the remaining possible asset prices into two distinct cases. In other words, they know that the only viable remaining states of nature will be either $\{h, H\}$ or $\{h, L\}$, in both of which cases the true values of the asset prices are expected to be revealed (see Proposition 3). Non-movers can thus now trade their holdings of fiat money for capital without any risk caused by information asymmetry.

$\widehat{B}_1^{h,L}$ is always equal to R_L. Note, however, that the asset price in the state of $\{h, H\}$ takes a different form from that in Corollary 1. The bank has chosen q^l, but since a state h occurs, the actual asset price in the state of $\{h, H\}$ is expressed as:

$$\bar{B}_1^{h,H} = \frac{(1-\lambda^h)q^l}{\lambda^h k^l}. \tag{21}$$

The different asset price in the state $\{h, H\}$ does not affect the expected utility of the uninformed agents, since they are assumed to be risk-neutral. The consumptions of a mover in the two states can then be represented as:

$$
\begin{aligned}
&\{h, H\} \quad c_{m,2} = q^l + \bar{B}_1^{h,H} k_0^l, \text{ and} \\
&\{h, L\} \quad c_{m,2} = q^l + \widehat{B}_1^{h,L} k_0^l,
\end{aligned}
\tag{22}
$$

and the consumptions of a non-mover as:

$$
\begin{aligned}
&\{h, H\} \quad c_{n,2} = \left(\frac{q^l}{\bar{B}_1^{h,H}} + k_0^l \right) R_H, \text{ and} \\
&\{h, L\} \quad c_{n,2} = \left(\frac{\gamma q^l}{\widehat{B}_1^{h,L}} + k_0^l \right) R_L + (1-\gamma) q^l.
\end{aligned}
\tag{23}
$$

Substituting (19), (20), (22) and (23) into the objective function (15), we can calculate the expected utility of the uninformed agents:

$$
\begin{aligned}
&E^l[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}] \\
&\quad = \mu^l \left[q^l + k^l E[R_j] \right] \\
&\qquad + \frac{\mu_h}{2} \left[\lambda^h \left(q^l + \bar{B}_1^{h,H} k^l \right) + (1-\lambda^h) \left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l \right) R_H \right] \\
&\qquad + \frac{\mu_h}{2} \left[\lambda^h \left(q^l + \widehat{B}_1^{h,L} k^l \right) + (1-\lambda^h) \left\{ \left(\frac{\gamma q^l}{\widehat{B}_1^{h,L}} + k^l \right) R_L + (1-\gamma) q^l \right\} \right] \\
&\quad = q^l + E[R_j] k^l,
\end{aligned}
\tag{24}
$$

where $E^l[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}]$ is the expected utility of the uninformed agents when they organize a bank which chooses q^l. The losses that normally occur under asymmetric information do not occur in this situation,[27] and the uninformed agents thus have higher expected utility than under normal conditions of asymmetric information.[28]

5.2.2 Equilibrium without Bankruptcy

If a bank makes an alternative choice, expecting that the state h will occur and holds $q^h = \lambda^h$, then there exist two possible scenarios depending upon the realization of state i, as in the previous subsection. Even if a state h occurs at date 1, unlike in the previous subsection bank runs do not

[27] See Online Appendix C for the computational details.

[28] As long as q^l is less than or equal to $E[R_j]k^l$, the incentive constraint is satisfied.

occur. The expected consumption levels of the different agents are given as:

$$\begin{aligned} \text{movers} \quad c_{m,2} &= v_2 d = \frac{q^h}{\lambda^h}, \text{ and} \\ \text{non-movers} \quad c_{n,2} &= \frac{k^h R_j}{1-\lambda^h}. \end{aligned} \tag{25}$$

If the state l occurs at date 1, then the bank holds excessive liquidity (which is not intended when the bank first chooses its portfolio at date 0) amounting to $(q^h - \lambda^l v_0 d)$.[29] The expected consumption levels of the agents when state l occurs are given as:

$$\begin{aligned} \text{movers} \quad c_{m,2} &= \frac{v_2}{v_0}\frac{q^h}{\lambda^h}, \text{ and} \\ \text{non-movers} \quad c_{n,2} &= \frac{v_2\big[d(\lambda^h - \lambda^l)\big] + R_j k^h}{1-\lambda^l} \\ &= \frac{\big(\lambda^h - \lambda^l\big) + R_j k^h}{1-\lambda^l} < \frac{R_j k^l}{1-\lambda^l}. \end{aligned} \tag{26}$$

The first term on the right-hand side of $c_{n,2}$, $v_2\big[d(\lambda^h - \lambda^l)\big]$, represents the increased amount of consumption due to the fiat money carried over from date 1 to date 2. The second term, $R_j k^h$, is the consumption from illiquid assets. Note that the expected consumption of non-movers is lower than that in (20), because the bank holds more fiat money than necessary and less capital (note that $k^h = 1 - \lambda^h < 1 - \lambda^l = k^l$). This implies that inefficiency is caused by the bank's holding of excessive liquidity at date 0. This inefficiency arises because the decision by a bank is made at date 0, in expectation that the specific state h will occur, but a state l actually occurs at date 1. Therefore, the bank holding excessive liquidity at date 1 may wish to trade this excessive liquidity for capital with the informed if $\frac{E[R_j]}{B_1^{h,j}} > \frac{v_2}{v_1}$ is expected. Does it buy additional capital from the informed at date 1? It turns out that it will never end up trading with the informed, for the following reason:

By the fact that the bank holds excessive liquidity after the movers' withdrawals, it now gets a new information that state l occurred instead of state h. It still does not know, however, the exact return on capital that will be realized at date 2, which is known only to the informed. However, it does know that the possible asset prices will be confined to only two distinct cases: $\{i, j\} = \{l, H\}$ or $\{l, L\}$. It also knows that the informed agents will try to sell their capital only when the returns

[29] This is because $\lambda^h d = \frac{q^h}{v_0} > \lambda^l d$.

on capital are low, i.e. only in the state of $\{l, L\}$ (see Proposition 3). The financial market now can be seen as the "market for lemons" [Akerlof, 1970]. The bank chooses to keep its currency until the next period, and trades in the financial market are therefore not made at date 1. The uninformed agents will consume according to (25) and (26). Substituting (25) and (26) into the objective function (15), and letting $E^h[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}]$ denote the expected utility of the uninformed agents when the bank chooses q^h, we have

$$\begin{aligned}
&E^h[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
&=\mu_h \left[q^h + k^h E[R_j]\right] + \mu_l \left[\frac{\lambda^l q^h}{\lambda^h} + (\lambda^h - \lambda^l) + k^h E[R_j]\right] \\
&=\mu_h \left[q^h + k^h E[R_j]\right] + \mu_l \left[q^h + k^h E[R_j]\right] \\
&=q^h + k^h E[R_j].
\end{aligned} \tag{27}$$

The second term in the third line of this equation is calculated using $\lambda^l = q^l$ and $\lambda^h = q^h$. The losses that occur under asymmetric information do not occur when the bank chooses q^h as well. However, the expected utility with q^l is greater than that with q^h, because from (24) and (27)

$$\begin{aligned}
&E^l[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] - E^h[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
&\quad = (q^h - q^l)(E[R_j] - 1) > 0.
\end{aligned} \tag{28}$$

Therefore, the bank will always choose $q = \lambda^l$ rather than $q = \lambda^h$, and it can effectively eliminate the losses due to asymmetric information by allowing a run and thus discovering the true prices of capital. Also note that the financial markets and banks coexist if a bank run occurs.

Until now it has been assumed that all uninformed young agents deposit their whole endowments with the bank at date 0. However, this assumption is not necessarily needed. Uninformed agents can also deposit only parts of their endowments with the bank and hold capital directly. Let the scale of banking be β, the fraction of endowment goods deposited with the bank. The lower bound on the date 0 scale of banking under asymmetric information, β^{AI}, is then the amount of goods needed to buy fiat money, $\beta \geq \beta^{AI} = \lambda^l$. When $\beta = \lambda^l$, each agent directly holds the capital amounting to $1 - \beta$. A summing up of all of the findings until now can be summarized as the following proposition:

Proposition 5 *The imperfectly competitive rational expectations equilibrium when the uninformed young form a bank is given as*

1. *Price system:* $\{B_1^{ij}\} = (\bar{B}_1^{h,H}, \widehat{B}_1^{l,H}, \widehat{B}_1^{h,L}, \widehat{B}_1^{l,L})$, *where* $\bar{B}_1^{h,H} = \frac{v_1}{v_0}\frac{(1-\lambda^h)q^l}{\lambda^h k^l}$,

2. *Expected consumption of agents:*

$$E[C_{\tau,2}] = \begin{cases} E^l[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] = q^l + k^l E[R_j] \\ E^l[c_{inf}] = E[R_j], \end{cases}$$

   where $q^l = \lambda^l$ *and* $k^l = 1 - \lambda^l$,

3. *Specification of storage strategies for (market participating) non-movers:*

$$\begin{cases} \gamma < 1, \text{ in states of } \{i, L\} \\ \gamma = 1, \text{ in states of } \{i, H\}, \text{ where } i = \{h, l\}, \end{cases}$$

4. *Specification of insider coalition strategies:* $\lambda_{inf}^{i,j} = 0$, *and*

5. *Lower bound on the date* 0 *scale of banks:* $\beta \geq \beta^{AI} = \lambda^l$.

Bank runs are inevitable and efficient when state $\{h, j\}$ *happens, where* $j = \{H, L\}$. *The financial markets and banks coexist when the states* $\{h, j\}$ *occur even when* $\beta = 1$ *(when uninformed agents deposit their whole endowments with the bank).*

6 Limited Participation in Markets, and Imperfect Information

Until this point the analysis has assumed that uninformed agents have access to the market without limitations. This section analyzes an incomplete asset market caused by asymmetric information and limited participation at the same time. Let $\breve{B}_1^{i,j}$ be the price of an illiquid asset that would prevail with limited participation and complete information. Then asset prices take the following relations:

$$R_H > \breve{B}_1^{l,H} > \breve{B}_1^{h,H} > 1 > R_L = \breve{B}^{l,L} = \breve{B}^{h,L}, \tag{29}$$

where $\breve{B}_t^{l,H} = \min\left\{R_H, \frac{\alpha(1-\lambda^l)q}{\lambda^l k}\right\}$ and $\breve{B}_t^{h,H} = \frac{\alpha(1-\lambda^h)q}{\lambda^h k}$ (see Corollary 1). Asset prices under limited participation, $\breve{B}_t^{i,j}$, are always lower than or equal to those prices under full participation, $\widehat{B}_t^{i,j}$, due to the lower supply of liquidity from non-movers.[30] As before, $\breve{B}_1^{l,H}$ and $\breve{B}_1^{h,H}$ are equated through the adjustment of γ^L, such that[31]

$$\gamma^L = \frac{R_L \lambda^i k}{\alpha(1-\lambda^i)q}.$$

Now look at the consumption of a mover at date 2:

$$c_{m,2} = q + \breve{B}_1 k.$$

The consumptions of types A and B non-movers at date 2 are given as:

$$c_{nA,2} = \left(\frac{\gamma^j q}{\breve{B}_1} + k\right) R_j + (1-\gamma^j)q, \text{ and}$$
$$c_{nB,2} = v_2 M + kR_j = q + kR_j.$$

Direct calculation shows that the following holds:[32]

$$E^{F'}[c_{m,2} + c_{A,2} + c_{nB,2}] < q + E[R_j]k = E^F[c_{m,2} + c_{n,2}], \tag{30}$$

where $E^{F'}[\cdot]$ denotes the expected utility under full information and limited participation. The uninformed young will still like to form a coalition at date 0 with full-information, but some of their members are isolated from the market. Although the calculation process is messy, the economic intuition behind this result is straightforward. Limited participation in the markets implies that the type B non-movers end up holding inefficient liquidity assets, which are carried over from period 1 to period 2 for consumption. The economy will thus eventually keep excessive liquidity.

[30] Note that the price in the state of $\{h, H\}$, under full participation, $\widehat{B}_1^{h,H}$, can be higher than that in the state of $\{l, H\}$ under limited participation, $\breve{B}_1^{l,H}$, if the fraction of non-movers who have access to the market, α, is lower than some point.

[31] As long as $\alpha < 1$, and all other things being equal, the money holding ratio γ tends to be bigger than under the case of full participation. Due to the reduced demand for capital (because of the limited participation) the asset prices in the state $\{i, L\}$ could be lower than R_L. The lower asset prices will induce type B non-movers, who would hold more currency if all agents had access to the markets, to want to trade more of their currency holdings for capital.

[32] see Online Appendix D for the detailed computation

The total losses from this limited participation constraint are always greater than 0 as long as $\alpha < 1$. This loss is a decreasing function of α, and the maximum loss incurred from limited participation happens when $\alpha = 0$.[33] A bank is necessary in order to avoid the inefficient holdings of fiat money by type B non-movers at date 1. The bank will minimize its holding of fiat money as a fraction of the movers λ^i.[34]

In the following, I will consider the effects of the characteristics of asymmetric information and limited participation on equilibria in two different cases. The first deals with the resource allocation problem when there is only the asset market, and the second analyzes the case where the asset market and the bank exist together. Note that the results will differ quite a bit depending on whether α is known to the uninformed agents or not. Moreover, the asset prices, which the informed agents can mimic when a low rate of return on capital is expected, depend on the magnitude of α. When α is small the coalition of the informed can manipulate any prices as it wishes. Therefore, the optimal level of q cannot be given as a single value if there exists a limitation on market participation. Rather, it will depend on the number of non-movers who have access to the market, α. The results will also be different from those in Section 5, where the optimal deposit contract is made expecting the state l to occur and choosing to give the movers the amount of fiat money, d, by holding a minimal amount of fiat money. From now on, however, I assume that the fraction of α is known to all agents and α is not low enough for the coalition to mimic any price, and that price manipulation is possible only in the state of $\{h, H\}$ as in Section 5.

6.1 Market Equilibrium

As discussed in Section 4.2, type A non-movers will not keep fiat money for consumption at date 2 when $R_j = R_H$, and some storage happens when $R_j = R_L$ irrespective of the state of i. Let $0 \leq \breve{\lambda}^{i,j}_{inf} \leq \lambda_{inf}$ be the amount (proportion) of capital that the coalition decides to provide to the financial market at date 1 when the markets are imperfect. In the state $\{l, L\}$, the coalition of the informed can mimic state $\{h, H\}$ by selling their capital holdings and thus increasing the total

[33] See Online Appendix D for the computational details.
[34] Since complete information is assumed here, λ^i is not a random variable.

supply of illiquid assets to equate the prices in the two states such that

$$\breve{B}_1^{h,H} = \frac{\alpha(1-\lambda^h)q}{\lambda^h k} = \widetilde{B}^{l,L} = \frac{\alpha(1-\lambda^l)q}{\lambda^l k + \breve{\lambda}_{inf}^{l,L}} > \breve{B}_1^{l,L}. \tag{31}$$

This happens when all agents including the uninformed agents know the value of α, the fraction of those who have access to the markets. The informed agents can mimic the price $\breve{B}_1^{h,H}$ $(> \breve{B}_1^{l,L})$ at date 1 when the state $\{l, L\}$ occurs, as in Proposition 3, and $\breve{\lambda}_{inf}^{l,L}$ can be obtained from the market equilibrium condition. Using $\widetilde{B}_1^{l,L} = \breve{B}_1^{h,H} = \frac{\alpha(1-\lambda^h)q}{\lambda^h k}$, we have

$$\lambda^l k \widetilde{B}_1^{l,L} + \breve{\lambda}_{inf}^{l,L} \widetilde{B}_1^{l,L} = \alpha(1-\lambda^l)q.$$

The minimum fraction of informed agents compared to the uninformed that is needed to affect the price is given by substituting $\frac{\alpha(1-\lambda^h)q}{\lambda^h k}$ for $\widetilde{B}_1^{l,L}$:

$$\begin{aligned} &\breve{\lambda}_{inf}^{l,L} \frac{\alpha(1-\lambda^h)q}{\lambda^h k} + \lambda^l \frac{\alpha(1-\lambda^h)q}{\lambda^h} = \alpha(1-\lambda^l)q \\ \Leftrightarrow &\breve{\lambda}_{inf}^{l,L} = \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h} = \widetilde{\lambda}_{inf}^{l,L} \end{aligned} \tag{32}$$

Note that $\breve{\lambda}_{inf}^{l,L}$ is the same as $\lambda_{inf}^{l,L}$ in (11). The minimum number of informed agents needed to manipulate the price is the same as in the full participation case.[35] As discussed in Section 4.2, the welfare of the informed agents becomes higher at the expense of the uninformed agents' welfare.

6.2 Bank Equilibrium

As analyzed in Section 5, the bank faces two possible choices concerning how much to invest for real money balances, q, at date 1. Following the same approach as used in Section 5, let us consider the effect on its members' utility of the bank's two different choices about q. The analysis is summarized as the following proposition:

Proposition 6 *Proposition 5 holds with limited participation when agents have a linear utility function.*

[35] As mentioned earlier, this fraction can change considerably depending on the assumption made regarding α, and under certain conditions the existence of the informed agents might result in Pareto improvement. However, these issues are not discussed in this paper.

Proof. To prove this proposition I analyze two different possible cases of a bank, and see which behavior will give a higher consumption profile of the uninformed as in Section 5.

6.2.1 Equilibrium with Bankruptcy

The bank makes a deposit contract expecting the state l to occur and chooses to invest $q^l = \lambda^l$ in liquid assets and promises to give $d = \frac{q^l}{v_0 \lambda^l}$ to any person who withdraws at date 1. This implies that the bank determines to sell the minimum amount of goods for fiat money, and to hold as much as $k^l = 1 - \lambda^l$. If the state l occurs at date 1, then the expected consumption levels of the agents are given as

$$\begin{aligned} &\text{movers} \quad c_{m,2} = v_2 d = \frac{q^l}{\lambda^l}, \text{ and} \\ &\text{non-movers} \quad c_{nA,2} = c_{nB,2} = \frac{k^l R_j}{1-\lambda^l}. \end{aligned} \tag{33}$$

If the state h occurs at date 1, however, the bank goes bankrupt (a bank run happens) because $\lambda^h d > \frac{q^l}{v_0} = \lambda^l d$. If a run occurs, then the movers and the non-movers equally divide the fiat money and capital that the bank is holding, $q^l + k^l$. Movers and type A non-movers will trade these assets in a financial market with one another. The bank run itself instructs the uninformed agents that the total fraction of movers is λ^h. This information will confine the possible remaining states of nature to two distinct cases: $\{i, j\}$ is either $\{h, H\}$ or $\{h, L\}$. We have already shown that the true asset prices are expected to be revealed in both cases.

Let $\bar{B}_1^{h,H}$ be the asset price in state $\{h, H\}$ when the bank chooses q^l. Then $\bar{B}_1^{h,H}$ is expressed as

$$\bar{B}_1^{h,H} = \frac{\alpha(1-\lambda^h)q^l}{\lambda^h k^l}. \tag{34}$$

The consumption of a mover, then, can be represented depending upon the realized state of nature as:

$$\begin{aligned} &\{h, H\} \quad c_{m,2} = q^l + \bar{B}_1^{h,H} k^l, \text{ or} \\ &\{h, L\} \quad c_{m,2} = q^l + \breve{B}_1^{h,L} k^l. \end{aligned} \tag{35}$$

The consumption of a type A non-mover is:

$$\begin{aligned} \{h,H\} \quad c_{nB,2} &= \left(\frac{q^l}{\bar{B}_1^{h,H}} + k_0^l \right) R_H, \text{ or} \\ \{h,L\} \quad c_{nB,2} &= \left(\frac{\gamma q^l}{\breve{B}_1^{h,L}} + k_0^l \right) R_L + (1-\gamma) q^l. \end{aligned} \tag{36}$$

And the consumption of a type B non-mover:

$$\{h,j\} \quad c_{nB,2} = q^l + k^l E[R_j]. \tag{37}$$

Type B non-movers are not affected by the state of j, since they do not have access to the financial market. Substituting (33), (35), (36) and (37) into the objective function (15), and assuming a constant money supply, we can calculate the expected utility of the uninformed agents as follows:[36]

$$\begin{aligned} &E^{l'}[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}] \\ =&\mu^l \left[q^l + k^l E[R_j \right]] + \frac{\mu_h}{2} \left[\lambda^h \left(q^l + \bar{B}_1^{h,H} k^l \right) \right. \\ &\left. + (1-\lambda^h) \left\{ \alpha \left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l \right) R_H + (1-\alpha) \left(q^l + k^l R_H \right) \right\} \right] \\ &+ \frac{\mu_h}{2} \left[\lambda^h \left(q^l + \breve{B}_1^{h,L} k^l \right) \right. \\ &\left. + (1-\lambda^h) \left(\alpha \left\{ \left(\frac{\gamma q^l}{\breve{B}_1^{h,L}} + k^l \right) R_L + (1-\gamma) q^l \right\} + (1-\alpha) \left\{ q^l + k^l R_L \right\} \right) \right] \\ =&q^l + E[R_j] k^l = E^l[c_{m,2} + c_{n,2}], \end{aligned} \tag{38}$$

where $E^{l'}[c_{m,2} + c_{n,2}]$ denotes the uninformed agents' expected utility with limited market participation and asymmetric information when they have formed a bank. This equation shows that, if they can form a bank, the restriction on market participation does not affect the uninformed agents' welfare without reference to how big the parameter α is. This rather weird result comes because of the assumption that the agents are all risk-neutral. Since the portfolio decision by the bank is made at date 0, their expected utility is the same as long as no loss occurs in trading in the financial markets. Once we assume a standard, risk-averse preference, however, the result might

[36]See Online Appendix E for the computational details.

be different.

There will be no gap in consumption between types A and B non-movers when the state $\{h, L\}$ occurs. They all consume $q^l + k^l R_L$. However, when the state $\{h, H\}$ occurs the difference in consumption betweens types A and B is:

$$\left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l\right) R_H - \left(q^l + k^l R_H\right) = \left(\frac{R_H}{\bar{B}_1^{h,H}} - 1\right) q^l > 0.$$

The first term on the left-hand side is the consumption of a type A non-mover, and the second term is a type B non-mover's consumption in the state of $\{h, H\}$. Since $R_H > \bar{B}_1^{h,H}$ the difference is always positive.[37]

6.2.2 Equilibrium without Bankruptcy

The results are the same as in Sec 5.2. Thus,

$$E^{h'}[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] < E^{l'}[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}].$$

Therefore, as long as the agents are regarded as risk-neutral, the limitation on market participation does not affect the optimal behavior of banks. ■

7 Conclusions

Given asymmetric information and restrictions on market participation, financial intermediaries can achieve the revealing of unknown information without needing to invest their resources to identify it, by just allowing bank runs to occur. Uninformed agents can then make transactions with each other knowing that the true asset prices are available. In this way financial intermediaries and the markets coexist after banks go bankrupt. This mechanism, of 'a bank run first' and 'transactions in the market second', thus, enhances the expected utility of the uninformed, who otherwise face lower expected consumption generated by information asymmetries. The arrangement for a bank's

[37] if we assume a risk-averse preference, then the expected utility of the non-movers may be lower when only a limited fraction of them has access to the markets. The loss from limited participation is a decreasing function of α. If α were very low, then allowing bank runs might not be efficient. Investing in more liquid assets in order to avoid bank runs might then enhance the uninformed agents' welfare, which I do not discuss further.

achieving of this goal is made through a standard deposit contract. Without restrictions on market participation, i.e. when all uninformed agents have access to the financial markets, the intermediary would hold the minimal amount of fiat money, expecting a lower fraction of movers to be realized. Thus the scale of the banking sector is minimized to the lower fraction of movers, λ^l. Even though there exist restrictions on accessing the market among the uninformed, the lower bound of the banking scale remains the same as long as agents have risk-neutral preferences and the parameter indicating the level of financial market development, α, is common knowledge.

This conclusion, however, might be limited when α is unknown to the uninformed agents or when α has a very low value. Also, bank runs might be costly if we assume the standard, strictly concave utility function, because the type B non-movers' losses due to the limited participation can be critical. Then the optimal spending on liquid assets would be increased to the level where bank runs do not happen, and the spending would be a function of α. If the value is lower than the critical point, then the intermediary will not allow bank runs to occur by holding more liquid assets amounting to λ^h, even though it incurs some cost due to a lower rate of return on fiat money. The lower bound of the banking scale is higher than under the case of full participation.

Moreover, this paper is concerned only with a stationary allocation and a constant money supply, and thus assumes that the rate of return on fiat money is constant each period. If a central bank having authority to print money exists, the rate of return on fiat money will then be affected by its monetary policy, and this will potentially change the bank's portfolio decision and thus the behavior of the agents and the intermediary. Another interesting question would be to ask whether the central bank is able to remove the inefficiency induced by asymmetric information.

I also considered the symmetric allocation between islands. Each island faces the same stochastic shocks, so that the population of each island remains constant. What would happen if the relocation shocks were idiosyncratic across each island, so that the population of each region changes as each region sends a small number of movers and takes a large number of movers? Would there be contagion or systemic risk in the interbank markets? It is also important to know what would happen to the mutual relations between the two regions if they have different financial market structures or levels of financial market development, and whether financial exchange between these two regions can bring systematic risks or contagion. These questions ask us to consider the interbank markets between islands and to solve a general equilibrium problem.

Appendix

A Proof of Proposition 1

Proof. The supply of capital comes from the movers. They inelastically supply their holdings of illiquid assets, whatever the price is. There is thus a vertical supply curve, and the quantity supplied (the demand for money) is:

$$S^K(= D^M) = \lambda^i k.$$

The demand for illiquid assets comes from the non-movers. Non-movers need to decide whether to hold fiat money and carry it over to the next period. For this decision they compare the expected rate of return on fiat money with that of capital. Let γ be the ratio of money exchanged for capital at date 1. Then a fraction of $(1-\gamma)$ among their money holdings (q/v_0) is carried over to the date 1. γ takes a value between 0 and 1, and $\gamma = 1$ implies that non-movers inelastically supply all of their fiat money to the market. However, $\gamma = 0$ (storage of all fiat money) will not occur in an equilibrium since as $\gamma \to 0$, $B_t^{i,j} \to 0$. So as long as $R_j > 0$ there exists a fiat money provider to trade for illiquid assets.

One unit of fiat money will purchase $1/B_1^{i,j}$ units of claim on the consumption goods at date 1, and this will produce $R_j/B_1^{i,j}$ units of consumption goods at date 2. One unit of the goods is traded for $1/v_1$ units of fiat money at date 1, and this will purchase v_2/v_1 units of consumption goods at date 2.

If $R_j < \frac{v_2}{v1} B_1^{i,j}$, then no non-movers will hold capital from date 1 to 2, which implies $\gamma = 0$. If $R_j = \frac{v_2}{v1} B_1^{i,j}$, then the two assets are perfect substitutes to the non-movers. Thus any γ between $0 < \gamma \leq 1$ is possible. If $R_j > \frac{v_2}{v1} B_1^{i,j}$, no non-movers will hold fiat money from date 1 to 2, which implies $\gamma = 1$. Then, in order for the non-movers to trade fiat money for capital it is required that $B_1^{i,j} \leq \frac{R_j}{v_2/v_1}$. The maximum price of the asset is $B_1^{i,j} = \frac{v_2}{v_1} R_j$.

The aggregate demand for capital (supply of money) of the non-movers is

$$D^K(= S^M) = (1-\lambda^i)\gamma M v_1 = \frac{v_1}{v_0}(1-\lambda^i)\gamma q.$$

The asset market clearing condition requires that

$$\begin{aligned} \lambda^i k B_1^{i,j} &\leq \frac{v_1}{v_0}\gamma(1-\lambda^i)q \\ \Leftrightarrow \quad B_1^{i,j} &\leq \frac{v_1}{v_0}\frac{\gamma(1-\lambda^i)q}{\lambda^i k}. \end{aligned} \tag{39}$$

If the market has enough liquidity, then the asset price will reach its highest possible point, $\frac{R_j}{v_2/v_1}$. However, if the market suffers a shortage of liquidity for any reason, the price will be determined by the amount of cash supplied. This is known as 'cash-in-the-market pricing,' in which the asset price is determined as the ratio of total available 'cash' to the amount of assets provided [Allen and Gale, 1994, 2009]. ■

B Proof of Proposition 2

Proof. No fiat money is passed over to the date 2, i.e. $\gamma = 1$, if and only if the rate of return on capital is greater than or equal to that on fiat money, i.e.

$$R_j \geq \frac{v_2}{v_1}B_1^{i,j}$$

This implies that when the rate of return on capital is high, agents will want to hold only capital from period 1 to period 2. Since the rate of return on capital takes two values, it is assumed without loss of generality that the preceding relation holds only when $R_j = R_H$. The value of γ with R_H, γ^H, becomes 1.

On the other hand, there is some fiat money stored for consumption at date 2, i.e. $0 < \gamma < 1$, only when

$$\frac{R_j}{B_1^{i,j}} = \frac{v_2}{v_1}.$$

In other words, agents will hold some currency and carry it over to the next period only when the rate of return on capital is low. I therefore assume that $0 < \gamma^L < 1$ when $R_j = R_L$.

Since $\frac{(1-\lambda^l)q}{\lambda^l k} > \frac{(1-\lambda^h)q}{\lambda^h k}$,[38] the price of capital in the state l, $B_t^{l,j}$, is higher than or equal to that

[38] $\left[\frac{(1-\lambda^i)q}{\lambda^i k}\right]$ decreases when λ^i increases, i.e. $\frac{\partial}{\partial \lambda^i}\left[\frac{(1-\lambda^i)q}{\lambda^i k}\right] < 0$.

in the state of h, $B_t^{h,j}$, we have

$$\frac{R_j}{v_2/v_1} \geq B_1^{l,j} \geq B_t^{h,j}, \quad \left(=, \text{ if } B_t^{l,j} = B_t^{h,j} = \frac{R_j}{v_2/v_1}\right). \tag{40}$$

Note, however, that B_1^l cannot be equal to B_1^h. If $B_1^l = B_1^h$, then we have $B_1^l = B_1^h = \frac{R_j}{v_2/v_1}$, [39] which implies that capital dominates fiat money. If agents hold only capital, i.e. $k = 1$, then the movers sell capital at a price of $B_1^{i,j}$ and consume $\frac{v_2}{v_1} B_1^{i,j} k = \frac{v_2}{v_1} B_1^{i,j} = E[R_j] > \frac{v_2}{v_0}$. No one would hold fiat money in the first place, therefore, which cannot be an equilibrium. We should therefore have the relationship: $\frac{R_j}{v_2/v_1} \geq B_t^l > B_t^h$.

An additional condition, $B_1^{h,j} < v_1/v_0 < B_1^{l,j}$, is required because otherwise fiat money is (weakly) dominated by capital as well. If $B_1^{l,j} > B_1^{h,j} \geq v_1/v_0$, then a consumer holding only capital sells it at a price $B_1^{i,j}$ and consumes $\frac{v_2}{v_1} B_1^{l,j} > \frac{v_2}{v_1} B_1^{h,j} \geq \frac{v_2}{v_0}$. Also, if $B_1^{h,j} < B_1^{l,j} \leq v_1/v_0$, then capital is weakly dominated by fiat money since $\frac{v_2}{v_1} B_1^{h,j} < \frac{v_2}{v_1} B_1^{l,j} \leq \frac{v_2}{v_0}$. Therefore we have $B_1^{h,j} < v_1/v_0 < B_1^{l,j}$. All of these explanations yield the conclusion that $\frac{B_t^1}{p_{t+1}} < \frac{p_t}{p_{t+1}} < \frac{B_t^0}{p_{t+1}} \leq R$.

$$R_L = \frac{v_2}{v_1} B_1^{i,L} < \frac{v_2}{v_0} < \frac{v_2}{v_1} B_1^{i,H} \leq R_H,$$

where $i = \{l, h\}$. γ^L takes on any value between $0 < \gamma^L < 1$ depending upon the parameter values.

■

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[39]Because $\frac{(1-\lambda^l)q}{\lambda^l k} \neq \frac{(1-\lambda^h)q}{\lambda^h k}$, the only way $B_1^l = B_1^h$ is if both B_1^l and B_1^h have the fundamental value.

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