

A Model of Endogenous Party Membership and Platforms with Opportunistic Politicians

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We model endogenous party membership choice and party platform formation when party activists differ not only in their policy preference but also in the intensity with which they pursue policy issues vis-a-vis the spoils from electoral victories. We provide existence results for equilibria in which (i) party activists choose their party affiliation with correct anticipation of the choices by others and (ii) the resulting party platforms are consistent with the affiliation choice. This model offers an explanation for the overlaps in ideology between political parties. It also provides insights into the internal composition of political parties with different sizes.

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I. Introduction

In the tradition of spatial voting theory, political parties have been mostly modeled as unitary actors. In many electoral competition models, parties are just like single individual candidates who choose platforms to maximize their utility as given in a specific form.

This assumption hardly seems viable. In reality, a political party is made up of party members that are heterogeneous in many aspects. First of all, there is a spectrum of ideology within a party. In a liberal party, for example, liberal members and relatively conservative members coexist.¹ Therefore, reaching a party platform is one of the essential political activities that should not be treated trivially. Second,

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¹ Moreover, it is not unusual that there exists an overlap in ideology across parties. That is, a member of a liberal party may be more conservative than a member of a conservative party.

party members may have different priorities. More specifically, some members may value winning highly whereas others may believe maintaining party purity is more important than electoral victories. Thus, even members with the same ideal policy may promote different party platforms.

Roemer (1999) addresses the second issue by explicitly introducing three distinct groups of actors within a party: the militants, the reformists, and the opportunists. The militants want the party to adhere as closely as possible to its principles whereas the opportunists desire only to win office. The reformists are the usual expected payoff maximizers as modeled in Wittman (1983). Although this specification well captures an important aspect of political parties in democracy, some assumptions by Roemer (1999) still seem to lack reality. First, although his assumptions provide a good approximation of reality, it is somewhat arbitrary and ad hoc to categorize party members into three distinct groups. More importantly, it is implicitly assumed that members of a party share the same ideal party platform; it is not because their ideal platforms are different but because they have different priorities that different groups within a party promote different platforms. Therefore, in his model people differ in priorities but not in their ideal policy.

In this paper, we model endogenous party membership choice and party platform formation, assuming that party activists have different priorities as well as different policy preferences. In other words, some people believe winning is more important whereas others want to draw the party platform toward their ideal policy as close as possible. We model this second heterogeneity by introducing relative weights on the utilities from party platforms and the spoils from winning. A party member who puts a large weight on the utility from policy will want to draw the party platform toward his ideal point at the cost of lowering the probability of winning, whereas a member who puts a large weight on victory will do the opposite. We study an equilibrium in which (i) party activists choose their party affiliation with correct anticipation of the choices by all the others and (ii) the resulting party platforms are consistent with the affiliation choice. We obtain general existence results for such equilibria and attempt to characterize them using a simple example.

It is worth mentioning that by explicitly introducing intra-party heterogeneity, this paper sheds light on some issues that the traditional model cannot address. First, it highlights the constraint on the strategic positioning of parties by pointing out the intra-party process to determine party platforms. Second, it provides insights into the internal composition of parties with different prospect of electoral victory. Our analysis shows that a party with a higher (lower) chance of winning exhibits a wider (narrower) spectrum of ideology. It also shows that there typically exists an overlap in ideology spectrum across parties. We believe these constitute a meaningful contribution to the existing literature.

The rest of the paper is organized as follows. We first provide a brief survey of related literature. In Section 2, we describe the environment. In Section 3, we

consider as a preliminary step the membership choice problem for a given pair of platforms. Sections 4 and 5 provide results when party members vote sincerely and strategically in determining platforms, respectively. Section 6 concludes.

1.1. Related Literature

Aldrich (1983) provides a classic study of unidimensional spatial models with party activism. He examines the decisions of individuals as they choose whether or not to become activists in one of the two political parties. In contrast, our model only focuses on the activists and their party affiliation decision. Moreover, Aldrich (1983) assumes exogenously given party positions and so cannot address the issue of platform formation.

As mentioned before, political parties have been mostly modeled as single actors in the existing literature. Some recent studies introduce intra-party heterogeneity and model a political party as a coalition of distinct groups. See Roemer (1999), Medina (2001) and Caillaud and Tirole (1999). In these works, however, heterogeneity is discrete in the sense that they assume several distinct groups within a party that share a common property. In the current paper, we fully generalize heterogeneity by introducing continuous variables that capture the attributes of party members.

Poutvaara (2002) examines a similar issue in a dynamic setting. He investigates how the behavior of voters and potential party activists both determine party membership and the ideological characteristics of party platforms. In his model, however, potential party activists differ only in policy preferences and therefore an overlap in ideology spectrum across parties never occurs.

In this paper, we assume the existence of two parties and do not explore the formation of parties itself. There is, however, growing literature that studies endogenous party formation. Haan (1999) extends the citizen candidate model by allowing for endogenous party formation and shows that, in equilibrium, one left-wing and one right-wing party will be formed. Gomberg et al. (2001) consider endogenous formation of political parties in a highly abstract setting and provide equilibrium conditions.

Conceptually, this paper is closely related to Caplin and Nalebuff (1997) and Roemer (2001). Caplin and Nalebuff (1997) introduce an integrative approach regarding the analysis of group formation. They propose an equilibrium concept in which an institution's policy depends on its membership, and its membership in turn depends on the policies of all the institutions. As will be shown later, this is very similar to the concept we adopt in this paper. Therefore, our work can be regarded as an application of Caplin and Nalebuff (1997) that addresses some important issues in political economy.

Roemer (2001, pp.94~101), in the discussion of endogenous parties, proposes

“Condorcet-Nash equilibrium” to determine the party’s preferences. The equilibrium concept in the current paper is very similar to this equilibrium in that both concepts endogenize party preferences by introducing intra-party political process.

II. Environment

In this section, we describe the political environment including parties, players, and the timing of events.

Parties There are two parties L and R which compete over a one-dimensional policy issue.² The policy space is the unit interval $X \equiv [0, 1]$. Parties L and R each have l and r as their platforms, where $l, r \in X$. We let $l \leq r$ without loss of generality. We will see later how these platforms are determined.

Voters Voters are distributed along X and have single-peaked preferences symmetric about the ideal policy. Thus, the party supported by the median voter wins the election. The location of the median voter is uncertain, however, and the two parties agree on the distribution of the median voter.³ Let $M(\cdot)$ be the cumulative distribution function of the median voter and assume that $M' > 0$ and $M(1/2) = 1/2$. This means that there is no point in X at which the density function (i.e., M') is 0 and that it is equally likely that the median voter lies to the left and to the right of $1/2$, the midpoint of X . Then, given l and r , the probability that L wins the election is given by the function

$$p(l, r) \equiv \begin{cases} M(\frac{l+r}{2}), & l < r \\ \frac{1}{2} & l = r. \end{cases}$$

It follows that R ’s winning probability is $1 - p(l, r)$. We can see that (i) the probability of L ’s winning depends only on the midpoint of l and r when $l \neq r$ and (ii) whoever has a platform closer to $1/2$ has a higher chance of

² We are assuming two exogenously given parties and hence do not explicitly explore the endogenous formation of parties. We could introduce a citizen candidate-type process to endogenize the number of parties, although doing so would not be easy in the current framework. One interpretation of the current setup is that we are focusing on an environment in which only two parties are viable as in the US. However, the nature of the equilibrium in this paper does not depend on the specific number of parties and it is conjectured that most properties of the equilibrium will still hold with more than two parties.

³ This reflects the electoral uncertainty inherent in any electoral competition.

winning.

Party Activists Apart from the general voters, there exists a set of politicians who are to decide which party to join.⁴ Like voters, politicians also have their own favorite policy. In addition to the utility that they derive from the party platform, they also care about the spoils that only the members of the winning party can enjoy. The spoils may include, for example, various perquisites and influences over budget allocation entitled to the winning party. The weight put on the spoils varies with people. Formally, let $x \in X$ be the ideal point of a politician and α be the weight on the utility derived from the platform of the party that he belongs to. Given l and r , his *ex ante* utility if he joins L is defined by

$$u_L(x, \alpha; l, r) \equiv \alpha v(|x - l|) + (1 - \alpha)p(l, r)B,$$

and his *ex ante* utility if he joins R is defined by

$$u_R(x, \alpha; l, r) \equiv \alpha v(|x - r|) + (1 - \alpha)(1 - p(l, r))B,$$

where $v(\cdot)$ is decreasing and concave so that $v' < 0$ and $v'' < 0$, and $B > 0$ is the spoils that only the members of the winning party can enjoy. Note that this specification implies that the utility from the policy issue is determined by the party platform, not by the policy actually implemented by the winning party. The party activists in our model are agents who find it taxing to belong to a party whose platform is far from their ideal policy.⁵ A member with $\alpha = 1$ corresponds to the militants and a member with $\alpha = 0$ corresponds to the opportunists in Roemer (1999). Besides these two extremes, there are intermediate members with $0 < \alpha < 1$ who care about both the utility from the party platform and the spoils. Obviously, a politician will choose the party that will give him the higher utility. Therefore, given l and r , a politician's utility is defined as

$$u(x, \alpha; l, r) \equiv \max\{u_L(x, \alpha; l, r), u_R(x, \alpha; l, r)\}.$$

Politicians are distributed according to the density function $f(x, \alpha)$, where

⁴ Here, we do not explicitly consider the problem of participation. We can think of these politicians as those who derive higher utility by actively participating in politics than by staying out. See Aldrich (1983) for the "calculus of participation."

⁵ Note that this specification implies that the utility of the members of a party is affected by the platform of the other party only to the extent that it affects the probability of winning. We could instead assume that party members also care about the policy that is actually implemented. As can be seen later, such modification would not affect the membership choice problem since a single agent cannot affect the election outcome.

$x \in X$ and $\alpha \in A \equiv [\alpha_{\min}, 1], 0 < \alpha_{\min} < 1$.⁶ We assume that this function is continuous. With a little abuse of notation, define

$$f(x) \equiv \int_{\alpha_{\min}}^1 f(x, \alpha) d\alpha$$

and

$$F(x) \equiv \int_0^x f(x) dx.$$

That is, $f(x)$ is the density of politicians whose ideal point is x and $F(x)$ is the measure of politicians whose ideal policy is no greater than x . We assume that F is strictly increasing so that f has full support and $F(1/2) = 1/2$.

Timing The timing of events is as follows. First, politicians choose party membership with rational expectations of other people's party choice and the resulting platforms. Second, party members simultaneously vote on the party platform and the median voter in each party determines the party platform. Finally, voters vote and the winner is determined accordingly.

As a preliminary step, we will first consider the membership choice problem with given l and r in the next section. Then, in the subsequent sections we will investigate the equilibria under different aggregation rules. This process can be regarded as an application of backward induction to the model.

III. Party Membership Choice with Given Platforms

In this section, we consider a situation in which party platforms have been somehow determined, and potential party activists choose their membership accordingly. We focus on the activists' party membership choice.

The analysis in this section serves two purposes. First, there may be cases where party platforms are fixed at least for a while due to, for example, party tradition or

⁶ Allowing α to be 0 may raise some technical problem regarding the existence of equilibrium. However, an equilibrium may well exist even if $\alpha = 0$. We simply assume away politicians with $\alpha = 0$ to avoid the potential non-existence problem. Note that it is reasonable and realistic to assume that party activists put a positive weight on policy issues; otherwise, it would be hard to justify their active participation in politics in the first place. Also, this assumption may not be as restrictive as it seems because α_{\min} can be arbitrarily close to 0. That is, although we do not deal with perfectly opportunistic politicians, we can deal with 'almost perfectly opportunistic' politicians.

some kind of inertia. This section analyzes how party activists would make their party affiliation choice in this case. Second, this section can be regarded as a preliminary step to our analysis in Sections 4 and 5. In those sections, we will look for equilibria in which everything is consistent in the sense that i) party activists choose which party to join with an expectation of the party platforms and ii) their affiliation choice in fact results in the expected party platforms. Our analysis in the section is used in finding such equilibria. At this stage, either interpretation is acceptable.

We can solve this problem by tracking down the agents that are indifferent between the two parties.⁷ For a given x , let $\alpha_c(x; l, r)$ be the agent who is indifferent between L and R . Alternatively, we could let $x_c(\alpha; l, r)$ denote the agent who is indifferent between L and R for a given α . Consider

$$\alpha v(|x - l|) + (1 - \alpha)p(l, r)B = \alpha v(|x - r|) + (1 - \alpha)(1 - p(l, r))B, \quad (1)$$

or equivalently

$$v(|x - l|) - v(|x - r|) = \frac{1 - \alpha}{\alpha}(1 - 2p(l, r))B. \quad (2)$$

First consider a special case in which l and r are symmetric about $1/2$. Since $p(l, r) = 1/2$ when $l + r = 1$, (1) reduces to

$$v(|x - l|) = v(|x - r|),$$

and therefore the line $x_c(\alpha; l, r) = 1/2$ for all α characterizes the decision. Since the probability of winning is the same for either party, politicians will be sorted solely by their ideological preferences.

Now consider a more general case in which $l + r \neq 1$. Without loss of generality, let $l + r < 1$ and hence R has a higher chance of winning.⁸ Note first that (2) may not be satisfied if α is too small. More specifically, let

$$\bar{v} \equiv \max_{x < \frac{l+r}{2}} \{v(|x - l|) - v(|x - r|)\}.$$

It follows that $\bar{v} = v(l) - v(r)$ at $x = 0$ since $v(|x - l|) - v(|x - r|)$ is decreasing

⁷ Notice that the activists' problem in this section does not involve any strategic thinking as the platforms are fixed and not affected by other activists' decision. The utility an activist gets only depends on which party he joins, and hence he simply compares two utility levels and chooses the party that gives higher utility.

⁸ The case of $l + r > 1$ can be analyzed analogously.

in x . Let $\underline{\alpha}(l, r)$ be such that

$$\bar{v} = \frac{1 - \underline{\alpha}(l, r)}{\underline{\alpha}(l, r)} |1 - 2p(l, r)| B.$$

Then, (1) cannot be satisfied for $\alpha < \underline{\alpha}(l, r)$ since the right hand side will be bigger than the left hand side for any x . This means that if a politician is sufficiently opportunistic, he will choose a party that has a higher chance of winning, R in this case, regardless of his ideal policy. For $\alpha \geq \underline{\alpha}(l, r)$, the cutoff agent $\alpha_c(x; l, r)$ is given as

$$\alpha_c(x; l, r) = \frac{(1 - 2p(l, r))B}{v(|x - l|) - v(|x - r|) + (1 - 2p(l, r))B}$$

from (1). It follows that

$$\frac{\partial \alpha_c(x; l, r)}{\partial x} = \frac{(1 - 2p(l, r))B}{\{v(|x - l|) - v(|x - r|) + (1 - 2p(l, r))B\}^2} \left(-\frac{\partial v(|x - l|)}{\partial x} + \frac{\partial v(|x - r|)}{\partial x} \right) > 0.$$

Therefore, $\alpha_c(x; l, r)$ is strictly increasing in x . Now, define $\alpha(x; l, r)$ as

$$\alpha(x; l, r) \equiv \max\{\alpha_{\min}, \alpha_c(x; l, r)\},$$

where $x \leq (l + r)/2$. Then $\alpha(x; l, r)$ characterizes politicians' party choice: given x , those to the left of $\alpha(x; l, r)$ join L and those to the right join R .⁹ See Figure 1 for an illustration.

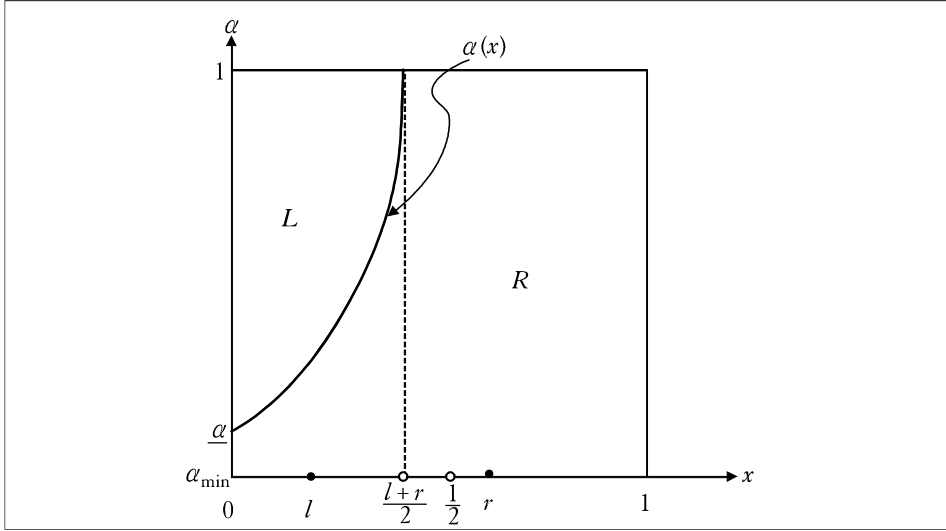
The following observations can be made from Figure 1. First, the cutoff x is increasing in α . People with a bigger α care more about policy and, in extreme, politicians with $\alpha = 1$ will only care about the policy and so the cutoff will be $(l + r)/2$, the midpoint of l and r . In contrast, highly opportunistic politicians mostly care about the spoils and hence will likely join the party with higher chance of winning.

Second, there is an overlap in the ideological spectrum. There exist L members who are more conservative than some R members and vice versa. Party activists will be aligned by ideology only when the two platforms are symmetric and hence both parties have the same chance of winning. It is also notable that the most extreme members in each party whose ideal point is near the platform of the other party possess the opposite characteristics: the most conservative L members are

⁹ Indifference can be resolved in either way and does not affect the result.

the most ideological ones with $\alpha = 1$ whereas the most liberal R members are the most opportunistic ones with small α .

[Figure 1] Party choice with fixed platforms ($l + r < 1$)



Third, if the distribution of party activists is more or less uniform, then the party with a higher probability of winning induces more members. Since we assumed $F(1/2) = 1/2$, it follows that R in Figure 1 ends up with strictly more members than L . It also follows that the members of the major party display a wider range of ideological spectrum.

Lastly, consider the internal composition of the parties. It turns out that if the distribution of the party members is more or less uniform, the minor party attracts relatively more ideological politicians and the major party attracts relatively more opportunistic politicians. A minor party in our setting is the one with a lower winning probability and hence its members are those who care relatively more about the platform and less about spoils.

In this section, we analyzed party membership choice when party platforms are exogenously given. It is true that party platforms are fixed in the short run and hence can be treated as exogenously given. However, they cannot be truly exogenous; it is more reasonable to view them as an outcome of the interaction among the heterogeneous party members within each party. In this sense, party platforms are better viewed as endogenous variables. In the following sections, we study the party membership choice when platforms are determined endogenously by party members.

There could be numerous ways in which the composition of party members is mapped to party platforms. In this paper we will adopt the median voter theorem,

the most widely used aggregation rule in political economy, and assume that the median party activist determines the party platform. In Section 4, we investigate a case in which politicians' ideal points are observable and hence each agent votes for his ideal policy. In Section 5, we will investigate a case in which people vote strategically.

IV. Endogenous Party Platforms: Sincere Voting

Consider a situation in which after the party membership choice has been made, each party activist votes for a policy for a party platform. In this section, we consider a case in which each party activist votes according to his policy preference.¹⁰ Then, the policy supported by the median activist is chosen as the party platform.

We are looking for a policy pair (l, r) that is consistent with the activists' membership choice decision. Given (l, r) , the party membership choice is characterized by $\alpha(x; l, r)$ that we defined in Section 3. For consistency, then, l and r should be in fact the median policy in each party. That is, l should be the median policy among L members and r should be the median policy among R members. We define the following.

Definition 1 *A policy pair (l, r) is consistent with the party choice decision with sincere voting if*

- i) party activists' choice is characterized by $\alpha(x; l, r)$,*
- ii) party activists vote sincerely in electing party platforms and the median policy in each party becomes the party platform, and*
- iii) (l, r) coincides with the party platforms obtained in ii).*

Then, we have the following result on existence.

Proposition 1 *There exists a policy pair (l, r) that is consistent with the party choice decision with sincere voting.*

Proof. Take a platform pair (l, r) . Suppose $l + r < 1$. For (l, r) to be an equilibrium, it should satisfy

¹⁰ This will be the case when the activists' policy preferences are publicly known. This assumption justifies the use of the median voter theorem. Since a party activist's utility is affected by the chance of winning, however, he may find it in his interest to vote for a policy different from his ideal policy. We will investigate this case in the next section.

$$\int_0^l \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx = \frac{1}{2} \int_0^{\frac{l+r}{2}} \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx,$$

$$F(r) - \int_0^{\frac{l+r}{2}} \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx = 1 - F(r).$$

If $l+r > 1$, it should satisfy

$$F(l) = 1 - F(l) - \int_{\frac{l+r}{2}}^1 \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx,$$

$$\int_{\frac{l+r}{2}}^r \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx = \frac{1}{2} \int_{\frac{l+r}{2}}^1 \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx.$$

If $l+r = 1$, it should satisfy

$$F(l) = F\left(\frac{1}{2}\right) - F(l),$$

$$F(r) - F\left(\frac{1}{2}\right) = 1 - F(r).$$

Define a function G_I as $G_I(l,r) = (l',r')$ such that l' and r' satisfy

$$\int_0^{l'} \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx = \frac{1}{2} \int_0^{\frac{l+r}{2}} \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx,$$

$$F(r') - \int_0^{\frac{l+r}{2}} \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx = 1 - F(r'),$$

Similarly, define G_{II} as $G_{II}(l,r) = (l',r')$ such that l' and r' satisfy

$$F(l') = 1 - F(l') - \int_{\frac{l+r}{2}}^1 \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx,$$

$$\int_{\frac{l+r}{2}}^{r'} \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx = \frac{1}{2} \int_{\frac{l+r}{2}}^1 \int_{\alpha(x;l,r)}^1 f(x,\alpha) d\alpha dx$$

and define $G_{III}(l,r) = (l',r')$ such that l' and r' satisfy

$$F(l') = F\left(\frac{1}{2}\right) - F(l'),$$

$$F(r') - F\left(\frac{1}{2}\right) = 1 - F(r').$$

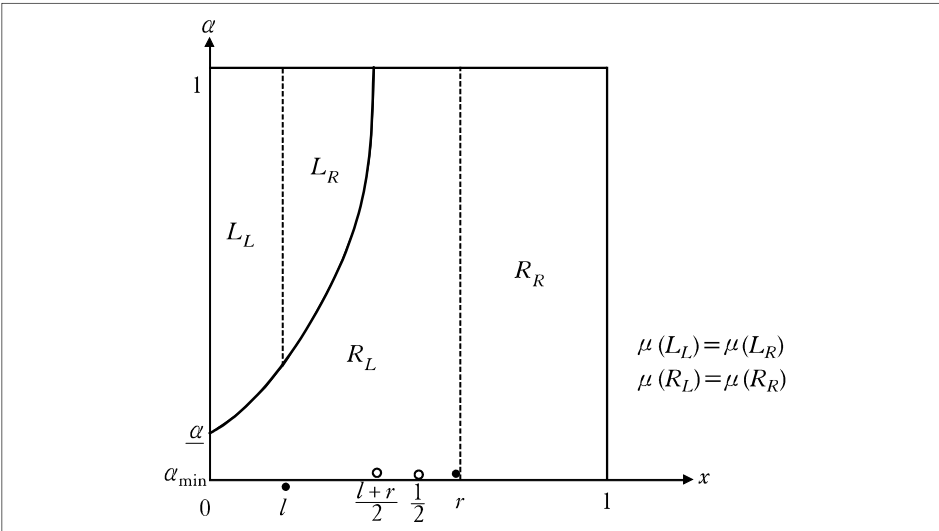
Notice that l' and r' are each the median of L and R induced by the initial pair (l, r) . Now define G as

$$G(l, r) = \begin{cases} G_I(l, r), & \text{if } l + r < 1 \\ G_{II}(l, r), & \text{if } l + r > 1 \\ G_{III}(l, r), & \text{if } l + r = 1. \end{cases}$$

First, it is obvious that in equilibrium, it cannot be that $l < r < \frac{1}{2}$ or $\frac{1}{2} < l < r$. Therefore, we can restrict the domain of G to $\chi \equiv [0, \frac{1}{2}] \times [\frac{1}{2}, 1]$. This set is closed, bounded and convex. Second, G is a function from χ to itself. For any $(l, r) \in \chi$, it is not possible to have $G(l, r) \notin \chi$. Third, since $\alpha(x; l, r)$ and $f(x, \alpha)$ are continuous, G_I , G_{II} , and G_{III} are also continuous. Moreover, $\lim_{\varepsilon \rightarrow 0} G_I(l, 1-l-\varepsilon) = G_{III}(l, 1-l)$ and $\lim_{\varepsilon \rightarrow 0} G_{II}(l, 1-l+\varepsilon) = G_{III}(l, 1-l)$ for any $l < \frac{1}{2}$. Hence, G is continuous. Therefore, by Brouwer's Fixed Point Theorem, there exists a pair $(l, r) \in \chi$ for which $G(l, r) = (l, r)$. ■

Proposition 1 guarantees the existence of equilibrium. Figure 2 illustrates an equilibrium with $l + r < 1$.

[Figure 2] Sincere voting ($l + r < 1$)



In a special case where $f(x)$ is symmetric around $1/2$, we have the following result whose proof is straightforward and hence omitted.

Corollary 1 *If $f(x)$ is symmetric, there always exists a symmetric equilibrium, in*

which l and r satisfy

$$F(l) = F\left(\frac{1}{2}\right) - F(l),$$

$$F(r) - F\left(\frac{1}{2}\right) = F(l) - F(r).$$

In this special case, l is the median of the politicians with $x \leq 1/2$ and r is the median of the politicians with $x \geq 1/2$. Also, l and r are symmetric so that $l + r = 1$. In this case, politicians are sorted solely by the ideal point and an overlap in ideology does not occur.

It is hard to characterize the equilibrium generally since the relevant parameter in the model is the distribution function $f(x, \alpha)$ itself, not individual variables. In the following subsection, we use an example with a specific distribution function to address this issue.

4.1. Example

We consider a case in which measure q politicians have $\alpha = 1$, and measure $1 - q$ politicians have $\alpha = a < 1$, where $q \in (0, 1)$. For each $\alpha \in \{a, 1\}$, x is uniformly distributed on X . Also, $v(x) = -(x - k)^2$ for $k \in (l, r)$ and $p(l, r) = (l + r)/2$ for $l \neq r$ and $p(l, r) = 1/2$ for $l = r$. Note that this specification satisfies all the assumptions of the model. Without loss of generality, let $l < r$ and $l + r \leq 1$.¹¹

For $\alpha = 1$, the cutoff point is obviously $x(1) = (l + r)/2$. For $\alpha = a$, solve

$$-a(x - l)^2 + (1 - a)\frac{l + r}{2}B = -a(x - r)^2 + (1 - a)\left(1 - \frac{l + r}{2}\right)B$$

to get

$$x(a) = \frac{l + r}{2} - \frac{1 - a}{2a} \frac{1 - (l + r)}{r - l} B.$$

It is straightforward to see that

$$\frac{\partial}{\partial a} x(a) = \frac{1 - (l + r)}{r - l} B \frac{1}{2a^2} > 0.$$

¹¹ Since $f(x)$ is symmetric, if (l, r) is an equilibrium pair for $l + r < 1$, $(1 - r, 1 - l)$ is an equilibrium pair for $l + r > 1$.

That is, the more ideological an agent is, the closer to the median the cutoff is. In extreme, $x(a)$ becomes $(l+r)/2$ for $a=1$.

Let (l, r) be an equilibrium platform pair. There are two cases to check. First, suppose $x(a) < l$. Then, for (l, r) to be an equilibrium, l and r should satisfy

$$ql + (1-q)x(a) = q\left(\frac{l+r}{2} - l\right),$$

$$q\left(r - \frac{l+r}{2}\right) + (1-q)(r - x(a)) = 1 - r.$$

Solving the two equations yields

$$l^* = \frac{2q-1}{2(q+1)} + \frac{(2-q)(1-q)^2(1-a)}{2(q+1)a}B,$$

$$r^* = \frac{2q+1}{2(q+1)} - \frac{q(1-q)^2(1-a)}{2(q+1)a}B.$$

This is the equilibrium platform pair for $l+r < 1$.¹² It follows that

$$\frac{\partial}{\partial a}l = \frac{1}{2} \frac{(q-2)(q-1)^2}{(q+1)a^2}B < 0,$$

$$\frac{\partial}{\partial a}r = \frac{1}{2} \frac{q(q-1)^2}{(q+1)a^2}B > 0.$$

Therefore, l and r become more extreme as a increases. That is, the equilibrium platforms become more extreme as the activists become more ideological.

Now, consider the measure of L members

$$\mu(L) \equiv q \frac{l+r}{2} + (1-q)x(a) \Big|_{(l,r)=(l^*,r^*)},$$

where l , r , and $x(a)$ are as given above. It follows that

¹² Of course, this is true only to the extent that $0 < l < \frac{1}{2} < r < 1$, $l+r < 1$, and $0 \leq x(a) < l$. This puts restrictions on B in relation to q and a . Algebra shows that the set of (B, q, a) that satisfies the above conditions is nonempty.

$$\frac{\partial}{\partial a} \mu(L) = \frac{(q-1)^2(1+q^2)}{2(q+1)a^2} B > 0.$$

Hence, the higher the value of a , the higher the number of people who join L .

Second, suppose $x(a) > l$. For (l, r) to be an equilibrium, (l, r) should satisfy

$$\begin{aligned} l &= q \left(\frac{l+r}{2} - l \right) + (1-q)(x(a) - l), \\ a \left(r - \frac{l+r}{2} \right) + (1-q)(r - x(a)) &= 1 - r. \end{aligned}$$

Solving the two equations gives

$$l^{**} = \frac{1}{4}, \quad r^{**} = \frac{3}{4}.$$

This symmetric equilibrium is natural since $f(x)$ is symmetric. Moreover, each platform is the midpoint of the left and right wings of X since $f(x)$ is uniform. Unlike (l^*, r^*) , the pair (l^{**}, r^{**}) is robust to changes in any parameter value.

V. Endogenous Party Platforms: Strategic Voting

In the previous section, party members were only modelled to vote for their ideal policy. In this section, we allow them to vote strategically. More specifically, agents are assumed to vote for a policy that maximizes the utility given the platform of the other party.¹³ Everything else remains the same.

We consider the problem of L members.¹⁴ Given r , each L member has his best response. We require that the policy l supported by the median voter be the best response to r , which in return should be the best response to l . First note that $\alpha(x; l, r)$ will continue to characterize the party choice for a given (l, r) ; party activists vote strategically in the voting stage but once the platform pair is set, they will choose the party that gives a higher utility as in Section 3. We define the following.

¹³ We use the term “strategic” in the sense that agents take into account the platform of the other party. We could assume that agents take into account the decision of his ‘comrades’ as well. Since there are a continuum of activists, however, a single vote doesn’t change the outcome and the main result of the paper will still hold.

¹⁴ The problem of R members can be analyzed analogously.

Definition 2 A policy pair (l, r) is consistent with the party choice decision with strategic voting if

- i) party activists' choice is characterized by $\alpha(x; l, r)$,
- ii) party activists vote strategically in electing party platforms and the median policy in each party becomes the party platform, and
- iii) (l, r) coincides with the party platforms obtained in ii).

Now consider an L member's voting. Formally, given $r > 1/2$, he solves

$$\max_{l < r} \alpha v(|x - l|) + (1 - \alpha) p(l, r) B.$$

We have the constraint $l < r$ since we can increase the utility by decreasing l if $l \geq r$.¹⁵ For people with $\alpha = 1$, the solution is obviously x . We consider people with $\alpha < 1$. Assume that the solution is unique and characterized by the FOC.¹⁶ The FOC for the interior solution is

$$\alpha \frac{\partial v(|x - l|)}{\partial l} + (1 - \alpha) \frac{\partial p(l, r)}{\partial l} B = 0.$$

Denote the solution by $l(x, \alpha; r)$. Since $\frac{\partial p(l, r)}{\partial l} > 0$ for $l < r$, it follows that $\frac{\partial v(|x - l|)}{\partial l} < 0$, which implies $l(x, \alpha; r) > x$. That is, people vote for a policy that is higher than their ideal point.

We now consider the properties of the interior solution $l = l(x, \alpha; r)$. By the implicit function theorem, we get

$$\frac{d\alpha}{dx} = - \frac{\frac{\partial}{\partial x} l(x, \alpha; r)}{\frac{\partial}{\partial \alpha} l(x, \alpha; r)}.$$

From the FOC, we have¹⁷

$$\frac{\partial l(x, \alpha; r)}{\partial x} = - \frac{\alpha v_{lx}}{\alpha v_{ll} + (1 - \alpha) p_{ll} B} > 0,$$

¹⁵ Note that r cannot be a solution since $p(l, r)$ drops discontinuously as l increases from $r - \varepsilon$ to r , where $\varepsilon \approx 0$.

¹⁶ This assumption justifies the use of the median voter theorem. One sufficient condition for the unique solution is $\frac{\partial^2}{\partial l^2} p(l, r) \leq 0$.

¹⁷ Note that $v(|x - l|) = v(x - l)$ if $x \geq l$ and $v(|x - l|) = v(l - x)$ if $x < l$. In either case, $v_{lx} = -v'' > 0$. Also note that $v_l - p_l B = \frac{1 - \alpha}{\alpha} p_l B - p_l B < 0$ from the FOC.

$$\frac{\partial l(x, \alpha; r)}{\partial \alpha} = -\frac{v_l - p_l B}{\alpha v_{ll} + (1 - \alpha) p_{ll} B} < 0.$$

Therefore,

$$\frac{d\alpha}{dx} = -\frac{\alpha v_{lx}}{v_l - p_l B} > 0.$$

The above algebra shows that (i) fixing α , the best response is increasing in x , (ii) fixing x , the best response is decreasing in α and (iii) the curve that connects the agents with the same best response is upward sloping.

Recall that the best response of people with $\alpha = 1$ is independent of r and coincides with their ideal policy. Thus, the best response of an L member is the x -coordinate of the point where the ‘iso-best response curve’ that goes through that agent intersects the horizontal line $\alpha = 1$.

To proceed, we define

$$\begin{aligned} S_L(l, r) &= \{(x, \alpha) \in X \times A : u_L(x, \alpha; l, r) \geq u_R(x, \alpha; l, r)\}, \\ S_R(l, r) &= \{(x, \alpha) \in X \times A : u_L(x, \alpha; l, r) < u_R(x, \alpha; l, r)\}. \end{aligned}$$

The set $S_L(l, r)$ is the set of agents who prefer L to R given (l, r) and $S_R(l, r)$ is similarly defined. Also define

$$\begin{aligned} BR_L(l'; l, r) &= \{(x, \alpha) \in S_L(l, r) : l(x, \alpha; r) \leq l'\}, \\ BR_R(r'; l, r) &= \{(x, \alpha) \in S_R(l, r) : r(x, \alpha; r) \leq r'\}. \end{aligned}$$

The set $BR_L(l'; l, r)$ is the set of L members whose best response is not bigger than l' . The set $BR_R(r'; l, r)$ is defined analogously.

Define a function H as $H(l, r) = (l', r')$, where (l', r') satisfies

$$\begin{aligned} \mu(BR_L(l'; l, r)) &= \frac{1}{2} \mu(S_L(l, r)), \\ \mu(BR_R(r'; l, r)) &= \frac{1}{2} \mu(S_R(l, r)), \end{aligned}$$

where $\mu(\cdot)$ is the measure function. Given (l, r) , the pair (l', r') is the policy pair supported by the median voter in each party. We are looking for a pair (l, r) that satisfies $H(l, r) = (l, r)$. Recalling $\mathcal{X} = [0, \frac{1}{2}] \times [\frac{1}{2}, 1]$, assume the following.

Assumption 1

$$\mu\left(BR_L\left(\frac{1}{2};l,r\right)\right)\geq\frac{1}{2}\mu(S_L(l,r)),\forall(l,r)\in\chi,$$

$$\mu\left(BR_R\left(\frac{1}{2};l,r\right)\right)\leq\frac{1}{2}\mu(S_R(l,r)),\forall(l,r)\in\chi.$$

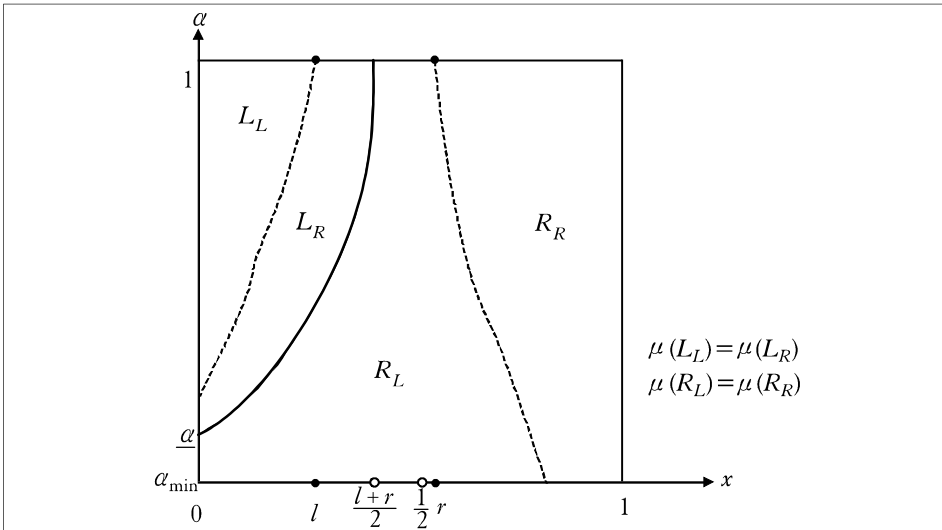
This assumption guarantees that for any (l,r) , the policy that is supported by the median voter in L is never bigger than $1/2$ and similarly, the policy that is supported by the median voter in R is never smaller than $1/2$. Now, we have the following results on the existence of equilibria.

Proposition 2 *There exists a policy pair (l,r) that is consistent with the party choice decision with strategic voting.*

Proof. By Assumption 1, H is a function from χ to itself. Moreover, it is continuous. Hence, by Brouwer's Fixed Point Theorem, there exists a pair (l,r) that satisfies $H(l,r)=(l,r)$. ■

Figure 3 illustrates an equilibrium.

[Figure 3] Strategic voting ($l+r<1$)



Again, it is not easy to characterize the equilibrium. We use the example used in Section 4 to address this issue.

5.1. Example

We use the same example that we examined in the previous section. Consider the problem of an L member given r . For those who have $\alpha = 1$, obviously $l = x$. Consider the problem of an L member with $\alpha = a$. He solves

$$\max_l -a(x-l)^2 + (1-a)\frac{l+r}{2}B.$$

Solving this yields

$$l(x) = x + \frac{1-a}{4a}B.$$

Likewise, for an R member with $\alpha = a$, we get

$$r(x) = x - \frac{1-a}{4a}B.$$

Note that for any (l, r) , the cutoff agent with $\alpha = 1$ is $(l+r)/2$ and the cutoff agent with $\alpha = a$ is given by

$$x(a) = \frac{l+r}{2} - \frac{1-a}{2a} \frac{1-(l+r)}{r-l}B$$

as before. Thus, among $\alpha = a$ agents, $x \in [0, x(a)]$ joins L and $x \in [x(a), 1]$ joins R . When they vote, however, L members vote for a policy that is higher than their ideal policy (i.e., $x + \frac{1-a}{4a}B$), and R members vote for a policy that is lower than their ideal policy (i.e., $x - \frac{1-a}{4a}B$). Therefore, the set of points supported by L members with $\alpha = a$ is $[\frac{1-a}{4a}B, x(a) + \frac{1-a}{4a}B]$ and that by R members is $[x(a) - \frac{1-a}{4a}B, 1 - \frac{1-a}{4a}B]$.

To solve for equilibria, we have to consider all possible cases. First, suppose $x(a) + \frac{1-a}{4a}B < l$. Then, (l, r) should satisfy

$$\begin{aligned} ql + (1-q)x(a) &= q\left(\frac{l+r}{2} - l\right), \\ q\left(r - \frac{l+r}{2}\right) + (1-q)\left(r - x(a) + \frac{1-a}{4a}B\right) &= q(1-r) + (1-q)\left(1 - \frac{1-a}{4a}B - r\right), \end{aligned}$$

but algebra shows that there exists no (l, r) that satisfies these equations.

Second, suppose $\frac{1-a}{4a}B < l < x(a) + \frac{1-a}{4a}B$. Then, (l, r) should satisfy

$$\begin{aligned} ql + (1-q)\left(l - \frac{1-a}{4a}B\right) &= q\left(\frac{l+r}{2} - l\right) + (1-q)\left(x(a) + \frac{1-a}{4a}B - l\right), \\ q\left(r - \frac{l+r}{2}\right) + (1-q)\left(r - x(a) + \frac{1-a}{4a}B\right) &= q(1-r) + (1-q)\left(1 - \frac{1-a}{4a}B - r\right). \end{aligned}$$

Solving this, we get

$$\begin{aligned} l^* &= \frac{1}{4} + \frac{(1-q)(1-a)}{4a}B, \\ r^* &= \frac{3}{4} - \frac{(1-q)(1-a)}{4a}B. \end{aligned}$$

Plugging these back to the condition $\frac{1-a}{4a}B < l < x(a) + \frac{1-a}{4a}B$ yields

$$q < \frac{a}{(1-a)B}.$$

We can see that

$$\frac{\partial}{\partial a}l^* = -\frac{1}{4} \frac{1-q}{a^2}B < 0, \quad \frac{\partial}{\partial a}r^* = \frac{1}{4} \frac{1-q}{a^2}B > 0$$

and

$$\frac{\partial}{\partial q}l^* = -\frac{1-a}{4a}B < 0, \quad \frac{\partial}{\partial q}r^* = \frac{1-a}{4a}B > 0.$$

Since (l^*, r^*) is symmetric, $\mu(L) = \mu(R) = 1/2$ and this value is robust to changes in parameter values.

Third, suppose $l < \frac{1-a}{4a}B < 1-r$. For (l, r) to be an equilibrium, we should have

$$\begin{aligned} ql &= q\left(\frac{l+r}{2} - l\right) + (1-q)x(a), \\ q\left(r - \frac{l+r}{2}\right) + (1-q)\left(r - x(a) + \frac{1-a}{4a}B\right) &= q(1-r) + (1-q)\left(1 - \frac{1-a}{4a}B - r\right), \end{aligned}$$

but it can be shown that such (l, r) does not exist.

Finally, suppose $\frac{1-a}{4a}B > 1-r$. Then, (l, r) should satisfy

$$\begin{aligned} ql &= q \left(\frac{l+r}{2} - l \right) + (1-q)x(a), \\ q \left(r - \frac{l+r}{2} \right) + (1-q)(1-x(a)) &= q(1-r). \end{aligned}$$

Solving this yields

$$\begin{aligned} l^{**} &= \frac{1}{4q}, \\ r^{**} &= 1 - \frac{1}{4q}. \end{aligned}$$

Plugging these back to the condition $\frac{1-a}{4a}B > 1-r$, we get

$$q > \frac{a}{(1-a)B}.$$

It straightforwardly follows that $\frac{\partial}{\partial q} l^{**} < 0$ and $\frac{\partial}{\partial q} r^{**} > 0$. As in the second case, this equilibrium is symmetric and hence $\mu(L) = \mu(R) = 1/2$.

In summary, the equilibrium is (l^*, r^*) if $q < \frac{a}{(1-a)B}$ and (l^{**}, r^{**}) if $q > \frac{a}{(1-a)B}$. Both equilibria are symmetric and in either case, the two platforms diverge as q increases.

VI. Concluding Remarks

In most literature on political competition, political parties have been modeled as a single actor. This simplification is very useful when we analyze electoral competition but misses a very important aspect of the political landscape: a political party is the set of heterogeneous agents. Recent works that introduce such heterogeneity still have limitations in the sense that they arbitrarily assume a few distinct factions within a party.

In this paper, we presented a model with endogenous party membership choice and party platform formation when party activists are heterogeneous in two respects: policy preferences and priority of goals. We provided existence results and gained

some insights regarding the size and composition of political parties.

This paper is based on theoretic modeling and the result of the paper is somewhat hard to test empirically; it would be hard to find the data that capture party activists' ideal points and weights on spoils. However, the result of the paper does match some notable stylized facts in the political arena: a political party exhibits a spectrum of ideology with big (small) parties usually having wider (narrower) ideology spectrum and there exists an overlap in ideology between parties. Although not the unique answer, this paper provides a framework to look at these issues in a systematic way.

There are some directions in which the model can be further developed. First, the characterization of the equilibrium in this paper is limited; we could gain more insights regarding the equilibrium platforms by, for example, running a numerical simulation based on some well-known distribution functions and conducting comparative static analyses. Second, in our model a party itself plays no active role. People somehow join a party and the party platform is determined by the median member. We can explicitly model active roles played by parties. For instance, we can establish the party constitution that maximizes the welfare of party members. This constitution may then include entry barriers to potential entrants to prevent welfare loss. Third, our model is basically a static model; we describe an equilibrium in which everybody's affiliation choice is consistent with the party platforms and the platforms are in turn consistent with membership choice. We can extend our setting to a dynamic one. In particular, if the spoils in the model include the party activists' prospect of winning in, say, the legislative election, dynamics should matter. So, if being a member of the ruling party significantly increases the probability of reelection, a party member may even want to change his party membership.¹⁸ Lastly, we can also introduce changes in voter preferences and see how it affects the formation of party platforms.

¹⁸ This may explain the reshuffling of lawmakers in Korea that used to occur immediately before or after the presidential election in the past decades.

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