The Korean Economic Review
Volume 34, Number 2, Summer 2018, 213-235.

Modeling the Dynamics between Stock Price and Dividend: An Endogenous Regime Switching Approach*

Heejoon Han** · Na Kyeong Lee***

This study considers a new error correction model (ECM) for stock price and dividend, which accommodates nonlinearities in both long- and short-run relationships. First, time-varying coefficient cointegration is adopted to explain the nonlinear long-run relationship between stock price and dividend. Second, the model allows for endogenous regime switching to describe the short-run relationship. The empirical application on the S&P 500 Index and dividend shows that our model fits the data significantly better than existing models and provides estimates with meaningful interpretations. In addition, the linear cointegration is unsuitable to describe the long-run relationship, and the ECM with endogenous regime switching better explains the data than that with conventional Markov switching. An extract latent factor specifically reveals the periods for each regime, and the periods of high-volatility regime include the NBER recession periods and certain periods of financial crisis.

JEL Classification: C22, C32, G12
Keywords: Endogenous Regime Switching, Time-Varying Coefficient Cointegration, Error Correction Model, Stock Price, Dividend

I. Introduction

Research on the relationship between stock price and dividend has been active.

Received: Aug. 14, 2017. Revised: Jan. 28, 2018. Accepted: April 20, 2018.

* We are grateful to two anonymous referees for many useful comments, which improved the quality of the paper. We would like to thank Yoosoon Chang, Cheolbeom Park, Joon Y. Park, and the seminar participants in the 2016 Korean Econometric Society Winter Meeting (Seoul), Time Series Workshop on Macro and Financial Economics (Seoul), and the 17th KEA International Conference (Seoul) for their helpful comments and suggestions. This paper was supported by Sungkyun Research Fund, Sungkyunkwan University, 2014.

** First Author, Department of Economics, Sungkyunkwan University, e-mail: heejoonhan@skku.edu

*** Corresponding Author, Economic Research Institute, Sungkyunkwan University, e-mail: nakyolee@skku.edu

Typically, the present value (PV) model has been adopted to explain the relationship, and its validity has been analyzed in the standard cointegration framework. The PV model, however, is known to be inappropriate for explaining fluctuated stock prices. A series of studies conducted by Shiller (1981a,b) shows evidence against the linear PV model because stock prices are too volatile to accord with the simple PV model. Campbell (1987) also provides evidence against the cointegrating relationship between stock price and dividend.

Many studies on stock return predictability are based on the cointegrating relationship between stock price and dividend. Although stock return predictability is associated with the cointegrating relationship between two series, the empirical evidence has been inconclusive; see Froot and Obstfeld (1989), Craine (1993), and Balke and Wohar (2002), among others. Fama and French (2001) demonstrate that the dividend–price ratio decreased since firms changed the dividend payout policy. Several researchers have attempted to investigate stock return predictability in the situation where the proportion of firms with a traditional dividend payout policy falls; see Lettau and Van Nieuwerburgh (2008) and Kim and Park (2013). Accordingly, recent studies have provided the nonlinear long-run relationship or fractional cointegration of two series; see Gallagher et al. (2001), Kanas (2003), Bohl and Siklos (2004), Kanas (2005), Esteve and Prats (2008), Chen and Shen (2009), Esteve and Prats (2010), and McMillan and Wohar (2010).

We consider a new model to reinvestigate the relationship between stock price and dividend. The model is an error correction model (ECM) accommodating nonlinearities in both long- and short-run relationships. To explain the nonlinear long-run relationship, we adopt the time-varying coefficient (TVC) cointegrating method proposed by Park and Hahn (1999), which becomes useful in modeling the nonlinear cointegrating relationship between stock price and dividend. To model the nonlinear short-run relationship, we allow for the endogenous regime switching mechanism recently proposed by Chang et al. (2017) in the ECM. An important feature of the endogenous regime switching method is that the future transition of states depends on the current state as well as the realization of underlying time series, which plays an important role in properly describing the short-run relationship between stock price and dividend. Studies also adopt error correction models that allow for regime switching to investigate the relationship between stock price and dividend; see Psaradakis et al. (2004) and Hu and Shin (2014). However, they employ a linear cointegrating model to specify the long-run relationship and, moreover, used conventional Markov switching, in which future transition is completely determined by the current state only and does not depend on realizing underlying time series.

We consider the monthly S&P 500 Index and dividend data from January 1974 to June 2017. As the first step, we examine their long-run relationship. Our results show that linear cointegration is unsuitable to describe the long-run relationship between stock price and dividend and that they are nonlinearly cointegrated. This result is in accordance with earlier studies listed previously. We show that the TVC cointegration method by Park and Hahn (1999) is an appropriate way to model the nonlinear long-run relationship between stock price and dividend.

Subsequently, by using the residual of the TVC cointegration model, we consider several error correction models. The usual linear ECM does not show any meaningful relationship between stock price and dividend. In this case, most coefficients are insignificant. However, when we allow for regime switching in the error correction model, the model exhibits reasonable results. Specifically, when we allow for the endogenous regime switching, the model fits the data significantly better than that with conventional Markov switching.

The estimation results of our endogenous regime switching error correction model (RS-ECM) are as follows. First, the latent factor extracted from our model specifically reveals the periods for each regime. We classify the regime with high volatility as the high regime and one with low volatility as the low regime. The high regime includes approximately 18% of the data. Although the average stock return is 0.63% for the entire sample period, it is –1.34% in the high regime and 1.05% in the low regime. The finding corresponds to the commonly observed asymmetric relationship between stock return and volatility. Negative returns are associated with higher volatility than positive returns, and this aspect is called the leverage effect; see Black (1976), Pagan and Schwert (1990), Engle and Ng (1993), and Harvey and Shephard (1996) for a rather incomplete list of related studies.

Second, the error correction coefficient is estimated to be significant in both regimes but insignificant in the linear error correction model. The error correction term in our model can be interpreted as an adjusted ratio of the stock price and dividend as in the study of Kim and Park (2013), and its significance can imply that an adjusted dividend–price ratio predicts the stock return. After controlling for the error correction term, the short-run relationship between stock price and dividend is significant only in the low regime. Stock price decreases by 0.728% when dividend increases by 1%. This result corresponds to early studies by Campbell and Beranek (1955), Miller and Modigliani (1961), and Dasilas (2009), among others.

Third, the transition probability is time varying in our model but constant in the conventional Markov switching model. Our result shows that when a negative event occurs in the stock market, the transition probability from the low regime to the high regime rapidly increases. Additionally, the regimes revealed by the latent factor of our model show that the high regime periods more or less coincide with the NBER recession periods and also contain the most periods of financial crisis.

The remaining parts of the paper are organized as follows. Section 2 introduces the model and explains the TVC cointegration and the endogenous regime switching model. Sections 3 provides the data description and main results of the paper. Section 4 concludes the paper.

II. Econometric Methods and the Model

In this section, we introduce our model for stock price and dividend after explaining two main econometric methods used for our model.

2.1. Cointegrating Regression with TVCs

Park and Hahn (1999) introduced the TVC cointegrating regression, which we use here to demonstrate that stock price and dividend are nonlinearly cointegrated. As we show in Section 3.1, both stock price and dividend can be modeled as unit root processes. When a long-run relationship between integrated time series is nonlinear and evolves over time, a linear cointegration model will be rejected and a nonlinear cointegrating regression will be useful. While several methods for nonlinear cointegrating regression became recently available, we adopt the method by Park and Hahn (1999). This approach exploits the available information efficiently to estimate the parameter of a model and is suitable in our case.[1] Kim and Park (2013) show that stock price and dividend have a nonlinear relationship possibly due to changes in the dividend payout policy by firms. They also adopted the method by Park and Hahn (1999) to model the nonlinear cointegrating relationship between stock price and dividend.

The TVC model is given by

$$y_t = \beta_t x_t + u_t, \tag{1}$$

where u_t is a latent disequilibrium error sequence assumed to be weakly dependent and β_t, which denotes the coefficient to be estimated, is now allowed to change over time in a smooth way. Noth $\{y_t\}$ and $\{x_t\}$ are assumed to be integrated. Specifically, we let

$$\beta_t = \beta\left(\frac{t}{n}\right), \tag{2}$$

where n represents the sample size, $t = 1, 2, \cdots, n$, and β indicates a sufficiently smooth function defined on the unit interval [0, 1]. The time-varying parameter framework in Eq. (2) is widely used in the literature. Related nonparametric inference has received attention for modeling stationary or locally stationary time series data; see Robinson (1989), Orbe et al. (2005), Cai (2007), Li et al. (2011), and Zhang and Wu (2012), among others. However, the literature on this topic for

[1] We discuss this issue in Section 3.2.

integrated time series has been minimal. Exceptions are Park and Hahn (1999) and Phillips et al. (2017). We explain the method by Park and Hahn (1999) in details and briefly discuss the method by Phillips et al. (2017) in Section 3.2.

To estimate β_t, Park and Hahn (1999) adopt a flexible Fourier functional form, which decomposes the function β into a linear combination of a polynomial and pairs of periodic functions. Thus, we assume that the smooth function β can be approximated by the function $\beta_{p,q}$ defined as

$$\beta_{p,q}(r) = \delta_0 + \sum_{j=1}^{p} \delta_j r^j + \sum_{j=1}^{q} (\delta_{p+2j-1}, \delta_{p+2j}) \phi_j(r),$$

where $\phi_j(r) \equiv (\cos 2\pi jr, \sin 2\pi jr)'$ for $r \in [0,1]$. According to Park and Hahn (1999), the function β given in Eq. (2) can be well approximated by $(\beta_{p,q})$ as p and q increase. To attain efficient estimators and a valid inferential basis for the parameters in the TVC model, we use the canonical cointegrating regression (CCR) method proposed by Park (1992). Let $w_t = (u_t, \Delta x_t)$, where (u_t) is the stationary error in the TVC model (Eq. (1)). For the process (ω_t), we further define the long-run covariance matrix as $\Omega = \sum_{k=-\infty}^{\infty} \mathbf{E} w_t w'_{t-k}$, the contemporaneous covariance matrix as $\sum = \mathbf{E} w_0 w'_0$, and the one-sided long-run covariance matrix as $\Gamma = \sum_{k=0}^{\infty} \mathbf{E} w_t w'_{t-k}$. Ω, $\sum$, and Γ are partitioned with the partition of ω_t into cell submatrices Ω_{ij}, $\sum_{ij}$, and Γ_{ij}, for $i, j = 1,2$. By defining $\delta_{p,q} \equiv (\delta_0, \cdots, \delta_{p+2q})'$ and $\Psi_{p,q}(r) \equiv (1, r, \cdots, r^p, \phi'_1(r), \cdots, \phi'_q(r))'$ with $r \in [0,1]$, the CCR-transformed regression of the TVC cointegrating model is given by

$$y_{pqt} = \delta'_{p,q} x_{pqt} + u_{pqt}$$

whose elements are defined by

$$y_{pqt} = y_t - \left(\Psi_{p,q}\left(\frac{t}{n}\right) \otimes \left[\Delta'_{12}\ \Delta'_{22}\right] \Sigma^{-1} w_t \right)' \delta_{p,q} - \left[0\ \ \Omega_{12}\Omega_{22}^{-1}\right] w_t$$

$$x_{pqt} = \Psi_{p,q}\left(\frac{t}{n}\right) \left(x_t - \left[\Delta'_{12}\ \Delta'_{22}\right] \Sigma^{-1} w_t \right)$$

$$u_{pqt} = u_t^* + (\beta - \beta_{p,q})\left(\frac{t}{n}\right) x_t, \text{ where } u_t^* = u_t - \Omega_{12}\Omega_{22}^{-1} \Delta x_t \tag{3}$$

Then, the OLS estimation of the CCR transformed model (Eq. (3)) can be used as CCR estimation yields efficient and optimal estimators demonstrated by Park

(1992).

2.2. Endogenous Regime Switching Model

For conventional Markov switching model, the Markov chain selecting the state of regime is completely independent from all other parts of the model. The future transition between states in Markov switching is completely determined by the current state only and does not depend on the realization of underlying time series. To overcome this shortcoming in conventional Markov switching, Chang et al. (2017) propose an endogenous regime switching model where the future transition between states depends on the realization of underlying time series as well as the current state.[2]

In this approach, the mean or volatility process is switched between two regimes, depending upon whether the underlying autoregressive latent factor ω_t takes values above or below threshold level τ.

The endogenous regime switching model can be generally expressed as

$$y_t = m(x_t, \omega_t) + \sigma(\omega_t)u_t = m(x_t, s_t) + \sigma(s_t)u_t, \tag{4}$$

where m and σ denotes the mean and volatility functions, respectively, and x_t is a regressor. Let a series (ω_t) follow a first-order autoregressive process as follows:

$$\omega_t = \alpha\omega_{t-1} + v_t \tag{5}$$

for $t = 1, 2, \cdots$ with parameter $\alpha \in (-1, 1]$ and i.i.d. standard normal innovations (v_t). Considering the realized value of the latent factor ω_t and the threshold level τ, we interpret two events, namely, $\{\omega_t < \tau\}$ and $\{\omega_t \geq \tau\}$, as two regimes that are switched. The state process (s_t) represents a low or high state depending upon whether it takes a value of 0 or 1.

$$s_t = I\{\omega_t \geq \tau\}, \tag{6}$$

where $I\{\cdot\}$ is an indicator function. The latent factor (ω_t) is assumed to be correlated with the previous innovation in the model. Specifically, (u_t) and (v_t)

[2] Kim et al. (2008) propose the regime switching model allowing for endogeneity as well. One of the primary differences between the model of Chang et al. (2017) and their model is that Kim et al. (2008) postulated the presence of contemporaneous correlation between the state variable and the innovation, whereas the innovation in Chang et al. (2017) is assumed to be correlated with the state variable in the next period. Furthermore, Chang et al. (2017) provide a general class of processes by allowing for nonstationary transition, whereas Kim et al. (2008) impose stationarity in transition. For a further detailed discussion, see Chang et al. (2017).

are jointly i.i.d. as

$$\begin{pmatrix} u_t \\ v_{t+1} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right). \tag{7}$$

For $\rho \neq 0$, as the autoregressive latent factor ω_{t+1} is correlated with the observed time series y_t, the future transition between states is endogenously affected by underlying time series. This phenomenon is why Chang et al. (2017) called it endogenous regime switching. However, if $\rho = 0$, then it is an exogenous regime switching model because the future transition between states now does not depend on y_t. Chang et al. (2017) show that if $\rho = 0$ together with $|\alpha| < 1$, then the endogenous regime switching model reduces to the conventional Markov switching model. See Section 2.2 in Chang et al. (2017) for details on the relationship between the endogenous regime and conventional Markov switching models.

The use of the endogenous regime switching model seems desirable for the following reasons: (a) The model allows for endogeneity in regime switching, and, therefore, the dynamics of mean and volatility can be further explained. (b) The model becomes observationally equivalent to the conventional Markov switching model when the autoregressive latent factor is exogenous ($\rho = 0$). The endogenous regime switching model is regarded as an extended Markov switching model. (c) The model allows the transition of the state process to be persistent. Recently, studies have been providing interesting findings by adopting the model. Chang and Kwak (2018) investigate the US monetary and fiscal policy regime interactions, and Cho et al. (2018) considered the profitability of carry trades in the foreign exchange market. These works conducted a further analysis by using an extracted latent factor, which is not possible in the conventional Markov switching framework. Particularly, Chang and Kwak (2018) show that latent policy regime factors exhibit patterns of correlation with macroeconomic time series.

In the endogenous regime switching model, we need to use the modified Markov switching filter developed by Chang et al. (2017) as the state process (s_t) defined in Eq. (6) is not a Markov chain unless $\rho = 0$. As a result, the conventional Markov switching filter is inapplicable. To develop the modified filter, a newly introduced transition probability is considered accordingly as follows:

$$\mathbb{P}(s_t \mid s_{t-1}, y_{t-1}) = (1 - s_t)\omega(s_{t-1}, y_{t-1}) + s_t[1 - \omega(s_{t-1}, y_{t-1})],$$

where ω is the transition probability of the endogenous state process (s_t) to a low state. The state process (s_t) is defined in Eq. (6). If $|\alpha| < 1$ and $|\rho| < 1$, then ω is given by

$$\omega(s_{t-1}, y_{t-1}) = \frac{\left[(1-s_{t-1})\int_{-\infty}^{\tau\sqrt{1-\alpha^2}} + s_{t-1}\int_{\tau\sqrt{1-\alpha^2}}^{\infty}\right]\Phi\left(\left(\tau-\rho u_{t-1}-\frac{\alpha x}{\sqrt{1-\alpha^2}}\right)/\sqrt{1-\rho^2}\right)\psi(x)dx}{(1-s_{t-1})\Phi(\tau\sqrt{1-\alpha^2})+s_{t-1}[1-\Phi(\tau\sqrt{1-\alpha^2})]}$$

As shown previously, the endogeneity of regime switching has an important effect on the performance of transition probabilities. Specifically, a negative correlation, i.e., $\rho < 0$, implies that a negative shock to (y_t) in the current period decreases the probability of staying in the low regime in the next period, whereas a positive realization of u_t increases the probability of having low regime at $t+1$.

2.3. Model

Psaradakis et al. (2004) and Hu and Shin (2014) consider error correction models with regime switching for the relationship between stock price and dividend. Their models allow for nonlinear adjustment to equilibrium driven by the conventional Markov switching. A regime switching ECM is well suited to situations where variables are unlikely to follow a linear adjustment to the long-run equilibrium.

As in Psaradakis et al. (2004) and Hu and Shin (2014), we adopt an ECM with regime switching. However, the distinct feature of our model is that we adopt the TVC model for the long-run relationship between stock price and dividend and, more importantly, the endogenous regime switching in the ECM. Hence, we call our model as the endogenous RS-ECM.

We denote the stock price and dividend as p_t and d_t, respectively. Both variables are in logarithm. First, we specify the long-run relationship between stock price and dividend as

$$p_t = \beta_t d_t + \varepsilon_t, \tag{8}$$

where β_t is allowed to change over time in a smooth way. Second, our endogenous RS-ECM has the following specification:

$$\Delta p_t = \lambda_0(s_t) + \lambda_1(s_t)\hat{\varepsilon}_{t-1} + \gamma(s_t)\Delta d_t + \sigma(s_t)u_t, \tag{9}$$

where the state process (s_t) is defined as in Eq. (6), $\hat{\varepsilon}_{t-1}$ indicates the lagged residual from the TVC model of p_t and d_t as described in Eq. (8), and Δ refers to the differencing operator defined by $\Delta p_t = p_t - p_{t-1}$. The state dependent parameters, namely, λ_0, λ_1, γ, and σ, are switched between two regimes such that $\lambda_j(s_t) = \lambda_j^l(1-s_t) + \lambda_j^h s_t$ for $j \in \{0,1\}$, $\gamma(s_t) = \gamma^l(1-s_t) + \gamma^h s_t$, and $\sigma(s_t) =$

$\sigma^l(1-s_t)+\sigma^h s_t$. The latent factor ω_t is defined as in Eq. (5). We assume that (u_t) and (v_{t+1}) are jointly i.i.d. as in Eq. (7) with $\rho \neq 0$. Subsequently, we estimate endogenous switching ECM by using the maximum likelihood method considering the modified filter entailing suggested transition probabilities. For the detail description, see Chang et al. (2017). As explained in the previous subsection, this result implies that the future transition between states is endogenously affected by underlying time series. For comparison, we also estimate the conventional Markov switching ECM (MS-ECM).

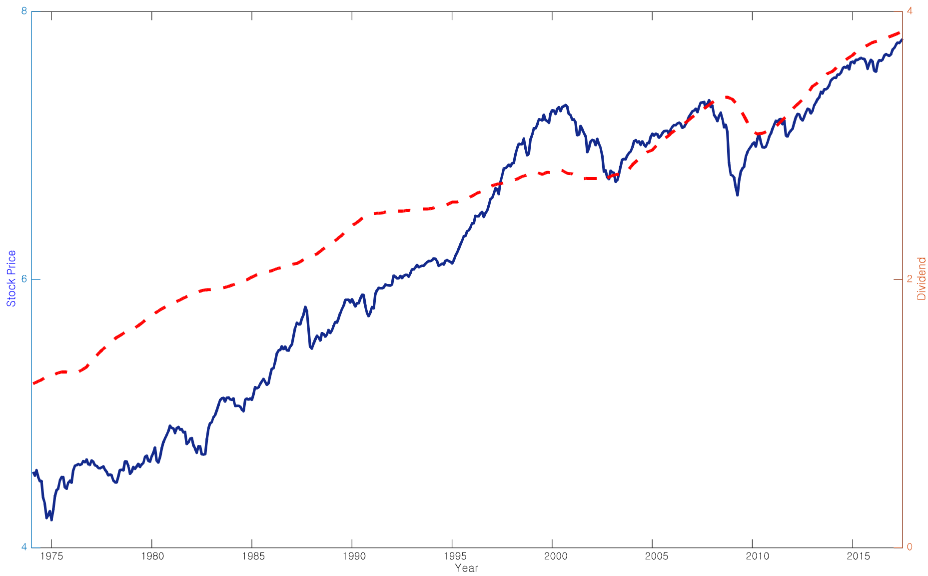
III. Main Results

3.1. Data Description

For our analysis, we employ the monthly S&P composite stock price and dividend data covering the time period January 1974 to June 2017. Following Hu and Shin (2014), we select year 1974 as the starting period for the study. We are referring to the data series provided by Robert Shiller. Figure 1 displays a pair of the log of stock price (solid line) and log of dividend (dotted line) in the US for the sample period. Stock price and dividend tend to move together. However, for certain short periods in the sample, such as around 1975, the beginning of the 2000s, and the end of the 2000s, the stock price behaved relatively differently from the dividend. Thus, short-run deviations from the long-run equilibrium are substantial.

Table 1 shows the results of unit root tests for the series. We consider two alternative autoregressive specifications for the series: with and without a linear deterministic trend. The test results strongly support the presence of a unit root in each series. For the stock price, the estimated autoregressive coefficients are close to unity, the augmented Dickey–Fuller (ADF) tests cannot reject the null hypothesis of a unit root, and the KPSS tests reject the null hypothesis of stationarity at 1% significance level. The results for dividend are similar to those for stock price except for one ADF test. Overall, the results show that both series can be modeled as unit root processes.

[Figure 1] Stock Prices and Dividends



Notes: Figure 1 shows a pair of the log of stock price (solid line) and log of dividend (dashed line) in the US from January 1974 to June 2017.

[Table 1] Unit Root Test

		With intercept	With intercept and trend
p_t	AR coefficient	0.999	1.001
	ADF test	–0.508	–1.507
	KPSS test	49.939	7.929
d_t	AR coefficient	1.000	0.997
	ADF test	–0.721	–1.063
	KPSS test	49.746	4.225

Notes: p_t and d_t stand for log stock price and log dividend, respectively. The critical values for ADF test are –3.44 (1%), –2.87 (5%), and –2.57 (10%) with intercept and –3.98 (1%), –3.42 (5%), and –3.13 (10%) with intercept and linear time trend. The KPSS test has critical values of 0.739 (1%) with intercept and 0.216 (1%) with intercept and linear time trend.

3.2. Long-Run Relationship

US stock prices (p_t) and dividends (d_t) have a linear long-run relationship (Campbell and Shiller, 1988). By contrast, Froot and Obstfeld (1989) find a strong evidence of the nonlinear relationship between stock prices and dividends in the presence of an intrinsic bubble.[3] Kim and Park (2013) analyze and find a nonlinear

[3] Froot and Obstfeld (1989) suggest the model of stock overreaction behavior considering that the

relationship between stock prices and dividends due to changes in the dividend payout policy by firms. Therefore, the long-term relationship between two series, namely, (p_t) and (d_t), may not necessarily be linear. To find the cointegrating relationship between two non-stationary variables, namely, (p_t) and (d_t), we begin with a simple regression model of $p_t = \theta_0 + \theta_i d_t + v_t$. Then, we use Phillips–Ouliaris residual-based tests for cointegration. Table 2 reports that the tests cannot reject the null hypothesis of no cointegration at the 1% significance level. Consequently, we are not able to support the linear cointegrating relationship between the two series. This result corresponds to early studies showing the nonlinear cointegrating relationship between the stock prices and dividends. See Kanas (2003), Bohl and Siklos (2004), Kanas (2005), Chen and Shen (2009), Esteve and Prats (2010), and Kim and Park (2013), among others.

[Table 2] Test for Linear Cointegration

	Value	p-value
Phillips–Ouliaris τ-statistic	–1.9879	0.5348
Phillips–Ouliaris z-statistic	–7.7218	0.5149

Notes: The null hypothesis is that two series are not cointegrated.

To search for a nonlinear cointegrating relationship between stock prices and dividends, we adopt the TVC model by Park and Hahn (1999) because this model (Eq. (1)) is useful for exploring complicated nonlinear interactions between two variables. We let the parameter to evolve over time and accordingly specify the model as Eq. (8). Hence, β_t can be approximated by a linear combination of polynomial and/or trigonometric functions on $[0,1]$. To determine the number of the trigonometric pairs and the degree of a polynomial to be used for the estimation, we consider Bayesian information criterion (BIC). Table 3 shows that a constant, a linear time trend, and three pairs of trigonometric terms $\{\sin(2j\pi t), \cos(2j\pi t)\}_{j=1,2,3}$ are selected. The resulting TVC model is estimated by using the CCR method developed by Park (1992) and extended by Park and Hahn (1999).

To confirm whether our proposed TVC model is well suited or not, we consider the following test statistic suggested by Park and Hahn (1999):

$$W_T = \frac{RSS_{FC} - RSS_{TVC}}{\hat{\varpi}^{2*}},$$

where RSS_{FC} and RSS_{TVC} are the sums of squared residuals from CCR-transformed regression, respectively, with fixed coefficients and TVC. $\hat{\varpi}^{2*}$ is the long-run variance of estimates noted by Eq. (3). This test is designed

bubble is a nonlinear function of dividends.

[Table 3] TVC Model Estimates

	Model 1		Model 2		Model 3	
	Est.	SE	Est.	SE	Est.	SE
β_0	2.3490	0.0529	2.4068	0.0330	2.4162	0.0298
$\beta_1 : \frac{t}{T}$						
$\beta_2 : \cos(2\pi \frac{t}{T})$	–0.2625	0.0699	–0.1305	0.0450	–0.1082	0.0416
$\beta_3 : \sin(2\pi \frac{t}{T})$	0.0744	0.0762	0.1380	0.0474	0.1511	0.0425
$\beta_4 : \cos(4\pi \frac{t}{T})$			–0.0440	0.0429	–0.0340	0.0403
$\beta_5 : \sin(4\pi \frac{t}{T})$			0.1938	0.0465	0.2138	0.0428
$\beta_6 : \cos(6\pi \frac{t}{T})$					–0.0674	0.0391
$\beta_7 : \sin(6\pi \frac{t}{T})$					0.0226	0.0410
BIC:	–441.7		–729.9		–728.0	
	Model 4		Model 5		Model 6	
	Est.	SE	Est.	SE	Est.	SE
β_0	3.1246	0.0318	3.1644	0.0148	3.3496	0.0069
$\beta_1 : \frac{t}{T}$	–1.2755	0.0536	–1.3432	0.0263	–1.7044	0.0126
$\beta_2 : \cos(2\pi \frac{t}{T})$	0.0467	0.0135	0.0695	0.0052	0.0742	0.0021
$\beta_3 : \sin(2\pi \frac{t}{T})$	–0.1964	0.0190	–0.2231	0.0092	–0.3450	0.0043
$\beta_4 : \cos(4\pi \frac{t}{T})$			0.0788	0.0049	0.0878	0.0020
$\beta_5 : \sin(4\pi \frac{t}{T})$			0.0474	0.0055	–0.0237	0.0026
$\beta_6 : \cos(6\pi \frac{t}{T})$					0.0887	0.0019
$\beta_7 : \sin(6\pi \frac{t}{T})$					–0.0680	0.0020
BIC:	–1302.2		–1506.8		–1957.0	

Notes: Table 3 includes coefficient estimates and standard error (SE) for the TVC model in Eq. (8) using CCR estimation. BIC is used to determine the degrees of polynomials and pairs of trigonometric terms of the FFF form approximation.

for the null hypothesis of fixed coefficient cointegration and the alternative of TVC cointegration. The value of the test statistic is 6477.18, whereas the 1% critical value is 18.48 for the chi-square distribution with degree of freedom $7(p+2q=7)$. This result shows that we strongly reject the null hypothesis of a fixed coefficient cointegrating relationship in favor of a time-varying cointegrating relationship. When we consider τ_1 and τ test statistics suggested by Park and Hahn (1999), both tests reject the null hypothesis of cointegration. However, various unit root tests for the residuals show that the residual from the TVC model is stationary, whereas that from the fixed coefficient model is nonstationary. These results support the nonlinear cointegration modeled by the TVC model.

Although we confirm that adopting the method by Park and Hahn (1999) is suitable to fit the nonlinear cointegrating relationship between stock price and dividend, other approaches for nonlinear cointegrating regression are also available and we briefly discuss them. Alternatively, one can adopt a kernel smoothing method to estimate the function β_t given in Eq. (8). Phillips et al. (2017) recently established related asymptotic theories in a nonstationary time series setting. For fixed $\delta \in (0,1)$, the kernel smoothing estimator of β_t in Eq. (8) is as follows:

$$\hat{\beta}_n(\delta) = \left[\sum_{t=1}^{n} d_t^2 K_{th}(\delta) \right]^{-1} \left[\sum_{t=1}^{n} d_t p_t K_{th}(\delta) \right], \tag{10}$$

where

$$K_{th}(\delta) = \frac{1}{h} K\left(\frac{t - n\delta}{nh} \right).$$

When we estimated β_t, we follow Phillips et al. (2017) and select $K(x) = \frac{1}{2} I\{-1 \le x \le 1\}$, with $I\{\cdot\}$ being the indicator function and the cross-validation bandwidth. See Eqs. (5) and (10) in Phillips et al. (2017). This method also provides similar estimation results to our results. The fitted β_t and the estimation results of the endogenous RS-ECM are also similar to our results.

Another alternative approach is to consider

$$p_t = m(d_t) + \varepsilon_t$$

and estimate the unknown function $m(\cdot)$ by using the kernel estimation method. Wang and Phillips (2009a,b) establish related asymptotic theories in a nonstationary time series setting. The kernel estimator of $m(\cdot)$ evaluated at $d \in \mathbb{R}$ is denoted by

$$\hat{m}(d) = \frac{\sum_{t=1}^{n} p_t K_{d,h}(d_t)}{\sum_{t=1}^{n} K_{d,h}(d_t)},$$

where $K_{d,h}(d_t) = (1/h) K((d_t - d)/h)$. However, when we apply this method and estimate the endogenous RS-ECM, it fits the data worse than our method. To sum up, these alternative methods do not provide better results than the method by Park and Hahn (1999).

3.3. Short-Run Relationship

By using the residual of the TVC model $\hat{\varepsilon}_t$ in the previous subsection, we estimate several error correction models. First, we estimate the linear ECM given as

$$\Delta p_t = \lambda_0 + \lambda_1 \hat{\varepsilon}_{t-1} + \gamma \Delta d_t + \sigma u_t \quad \text{where} \quad u_t \sim \mathcal{N}(0,1). \tag{11}$$

This model does not allow for regime switching. The term $\hat{\varepsilon}_{t-1}$ in the model is the lagged equilibrium error, which represents the deviation of the stock price from the long-term equilibrium. The parameters of the ECM in Eq. (11) are associated with two distinct effects: the short- and long-run effects. The parameter γ is associated with the short-run effect: how the stock price (p_t) changes immediately in reaction to a contemporary change of the dividend (d_t). The long-run effect is associated with the parameter λ_1, which is commonly called as the error correction coefficient. A constant fraction of λ_1 of the lagged equilibrium error is eliminated each month.

Table 4 reports the estimation results of the linear ECM. The estimation result shows that the linear specification is inappropriate to model the relationship between stock price and dividend. The error correction coefficient λ_1 is not significantly different from zero. This result implies that stock prices in the US have not adjusted to any long-run disparity between (p_t) and (d_t) for the period from January 1974 to June 2017. In addition, the short-run effect of dividend appears to be insignificant.

As we confirm that the linear ECM is inappropriate, we allow for regime switching in the error correction model. We focus on the endogenous RS-ECM described in Section 2.3 and also compare it with the MS-ECM. We suppose that the stock return with high volatility belongs to the high regime $(s_t = 1)$ and the return with low volatility belongs to the low regime $(s_t = 0)$.

[Table 4] ECM using the TVC model

Parameter	Est.	SE
λ_0	0.0068***	0.0021
λ_1	–0.0135	0.0109
γ	–0.0989	0.2684
σ	0.0366***	0.0011
Log-likelihood	982.6784	

Notes: Table 4 reports estimated coefficients from the regression of the form (Eq. (11)). The coefficient λ_1 appears on the lagged error correction residual in Eq. (11). The error correction term is insignificantly different from zero. *** denotes the level of significance at 1%.

Interpretation of Estimates

Table 5 reports the estimation results of both endogenous RS-ECM and MS-ECM for the sample period. First of all, the endogenous RS-ECM fits the data better than the MS-ECM. The endogenous RS-ECM exhibits a higher log-likelihood value and lower AIC than the counterparts of the MS-ECM. Moreover, the likelihood ratio (LR) test shows that the endogenous RS-ECM is significantly better than MS-ECM. When we estimate the endogenous RS-ECM with the restriction of $\rho = 0$, the estimation results are almost identical to those of the MS-ECM. This confirms what Chang et al. (2017) showed. That is, if $\rho = 0$ and $|\alpha| < 1$, then the endogenous regime switching model reduces to the conventional Markov switching model. Hence, we conduct the LR test for $H_0 : \rho = 0$ and $H_1 : \rho \neq 0$. Under the null hypothesis, the difference of the goodness of fit between the endogenous RS-ECM and MS-ECM is not statistically significant. Table 5 shows that the LR test rejects the null hypothesis at the 1% significance level, which indicates that the endogenous RS-ECM fits the data significantly better than the MS-ECM.

The error correction coefficient λ_1 provides the rate at which the model re-equilibrates, i.e., the speed at which it returns to its equilibrium level. For the endogenous RS-ECM, λ_1^l and λ_1^h are estimated to be –0.0253 and –0.1165, respectively. As they are significantly different from zero at the 1% level, the result supports the existence of an error correction mechanism. The result depicts the stability of the system and convergence toward the equilibrium path in case of any disturbance in the system for both regimes. Moreover, the result implies that the high regime is associated with the fast disequilibrium adjustment, whereas the low regime is associated with the slow adjustment. That is, in the low regime, approximately 2.5% of any disequilibrium is absorbed in the next month, whereas the correction is around 11.7% in the high regime. For the MS-ECM, the error correction coefficient is significant only at the 10% level in the high regime but is significant at the 5% level in the low regime.

Comparing our results with those of Kim and Park (2013) is interesting. They show that an adjusted dividend–price ratio provides a strong evidence of predictability for cumulative stock excess return[4] but only a weak evidence of predictability of the conventional dividend–price ratio, $d_t - p_t$. The adjusted dividend–price ratio is defined as $\hat{\beta}_t d_t - p_t$ in Eq. (8), which is identical to $-\hat{\varepsilon}_t$ in the error correction models (Eqs. (9) and (11)). Considering that λ_1 is estimated to be insignificant in Eq. (11), the adjusted dividend–price ratio does not provide any evidence of predictability when the linear model is adopted. However, λ_1^l and λ_1^h are estimated to be significant in Eq. (9), which indicates a strong evidence of predictability as regime switching is allowed in the model. This result

[4] They consider cumulative excess returns of 12, 24, 36, 48, 60, and 72 months.

may imply that properly capturing a nonlinear relationship between stock return and dividend–price ratio in the short run is also important.

[Table 5] Endogenous RS-ECM between January 1974 and June 2017

$\Delta p_t = \lambda_0(s_t) + \lambda_1(s_t)\hat{\varepsilon}_{t-1} + \gamma(s_t)\Delta d_t + \sigma(s_t)u_t$, where $p_t = \beta_t d_t + \varepsilon_t$

	Endogenous RS-ECM		MS-ECM	
Parameter	Est.	SE	Est.	SE
λ_0^l	0.0158***	0.0018	0.0171***	0.0020
λ_0^h	–0.0242***	0.0102	–0.0203*	0.0093
λ_1^l	–0.0253***	0.0094	–0.0244**	0.0108
λ_1^h	–0.1165***	0.0429	–0.0777*	0.0431
γ^l	–0.7284***	0.2095	–0.7155***	0.2242
γ^h	1.7535	1.2093	0.0633	0.9290
σ^l	0.0243***	0.0011	0.0241***	0.0001
σ^h	0.0548***	0.0039	0.0552***	0.0004
α	0.9417***	0.0279		
τ	2.0607***	0.6820		
ρ	–0.9999***	0.0006		
$\mathbb{P}\{s_t = 1 \mid s_{t-1} = 1\}$	Time varying		0.96	
$\mathbb{P}\{s_t = 0 \mid s_{t-1} = 0\}$	Time varying		0.86	
Likelihood	1052.777		1046.350	
AIC	–2.0791e + 03		–2.0727e + 03	
BIC	–2.0239e + 03		–2.0302e + 03	
LR test	9.5160***			

Notes: Table 5 reports the maximum likelihood estimated coefficients for both endogenous RS-ECM and MS-ECMs for the sample period (January 1974 to June 2017). The second and third columns are estimated values of parameters and the associated asymptotic SEs of endogenous RS-ECM in Eq. (9), respectively. The last two columns of the table show the maximum likelihood estimates of the parameters of MS-ECM.
The LR test is for $H_0 : \rho = 0$ and $H_1 : \rho \neq 0$. The 1% critical value is 6.635.
*** denotes the level of significance at 1%, ** indicates the level of significance at 5%, and * represents the level of significance at 10%.

The coefficient of Δd_t, $\gamma(s_t)$, captures the short-run relationship between stock price and dividend. For the endogenous RS-ECM, γ^l is estimated to be –0.728 in the low regime and is significant. Thus, in the low regime, the stock price p_t decreases by 0.728% when the dividend d_t increases by 1%. The finding is consistent with the previous studies that demonstrate an inverse performance between stock prices and dividends (Campbell and Beranek, 1955; Miller and Modigliani, 1961; Dasilas, 2009). Campbell and Beranek (1955) and Dasilas (2009) pointed out that stock prices decrease on ex-dividend days by an amount that is less than the dividend. On the other hand, γ^h is estimated to be insignificant in the high regime. This result implies that the short-term effect does not appear in the high regime in which the stock return is more volatile. Notably, the short-run relationship between stock price and dividend significantly appears only for the low regime. For the MS-ECM, the results for $\gamma(s_t)$ are similar to those for the endogenous RS-ECM.

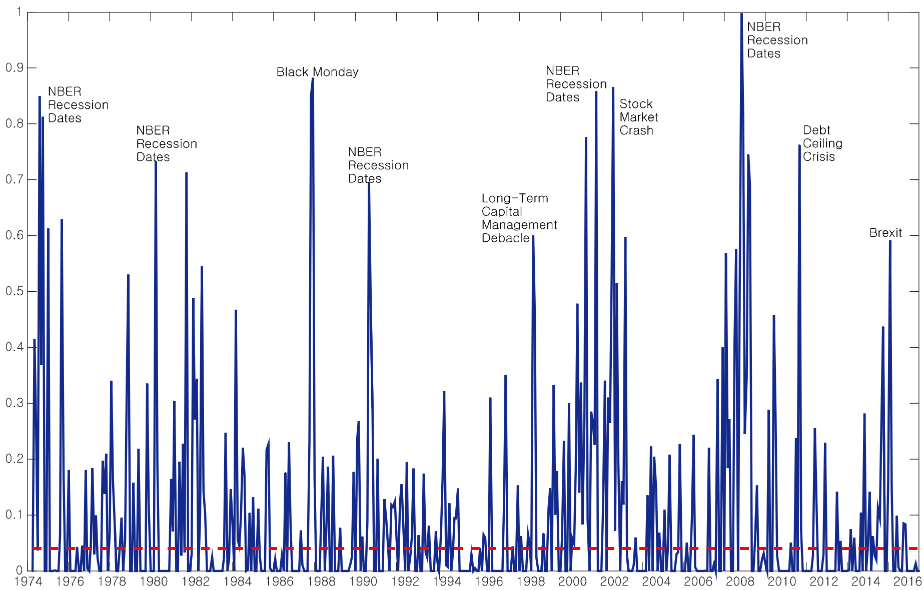
In the endogenous regime switching model, the latent factor is assumed to be correlated with the previous innovation in the model. Thus, the correlation coefficient ρ between the current (u_t) and next period (v_{t+1}) innovations in Eq. (7) measures the degree of endogeneity of regime changes. In Table 5, the estimate for the endogeneity parameter ρ is relatively substantial, that is, –0.9999, and we reject the null of no endogeneity at 1% significance level. Considering the strongly negative estimated value of the correlation, a positive shock to Δp_t at time t in Eq. (9) increases the probability of having a low regime at time $t+1$. By contrast, a negative shock to Δp_t increases the probability of having a high regime at time $t+1$. In the volatility part, a negative shock to Δp_t at time t results in an increase in volatility at time $t+1$, whereas a positive shock to Δp_t attempts to decrease the volatility in the following period. This result corresponds to the leverage effect describing that negative returns are associated with higher volatility than positive returns.

Two events, namely, $\{\omega_t < \tau\}$ and $\{\omega_t \geq \tau\}$, regarded as two regimes are switched by the realized value of the latent factor ω_t and the threshold level τ. The extracted latent factor represents unobserved economic fundamentals, and the latent factor and threshold level determine regimes. As long as the latent factor remains above the threshold level, the regime is classified as a high regime. The threshold level is a certain level of the latent factor by which the regime (or status of economic fundamentals) switches. The estimation results show that 18.11% of the data remain in the high regime. For the entire sample period, the average of stock returns is 0.63%. Dividing it into two regimes by the extracted latent factor, the averages of stock returns are –1.34% and 1.05% in the high and low regimes. This result corresponds to the commonly observed asymmetric relationship between stock return and volatility. Volatility is high when the stock return is negative. The autoregressive coefficient α of the latent factor is estimated to be 0.942 in the endogenous RS-ECM. This result shows that the latent factor is persistent; therefore, the transition of the state process is also persistent for the data.

Transition Probability and Revealed Regimes

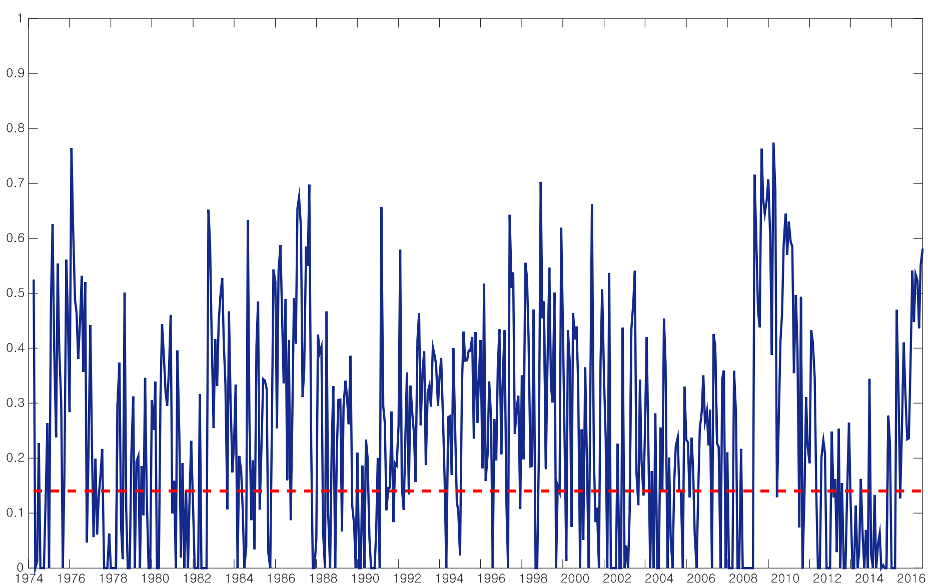
Both graphs of Figures 2 and 3 clearly show the difference in the time series plots of the transition probabilities estimated from the endogenous RS-ECM and MS-ECM. The estimated transition probability by the endogenous RS-ECM (real line) varies over time as the probability depends upon the previous state (s_{t-1}) as well as

[Figure 2] Estimated Transition Probability from Low to High State



Notes: The figure indicates the transition probabilities from low to high regime. The blue solid line refers to $\mathbb{P}(s_t = 1 \mid s_{t-1} = 0)$ in the endogenous RS-ECM, whereas the red dashed line represents $\mathbb{P}(s_t = 1 \mid s_{t-1} = 0) = 0.04$ in MS-ECM.

[Figure 3] Estimated Transition Probability from High to Low State



Notes: The figure indicates the transition probabilities from low to high regime. The blue solid line refers to $\mathbb{P}(s_t = 0 \mid s_{t-1} = 1)$ in the endogenous RS-ECM, whereas the red dashed line represents $\mathbb{P}(s_t = 0 \mid s_{t-1} = 1) = 0.14$ in MS-ECM.

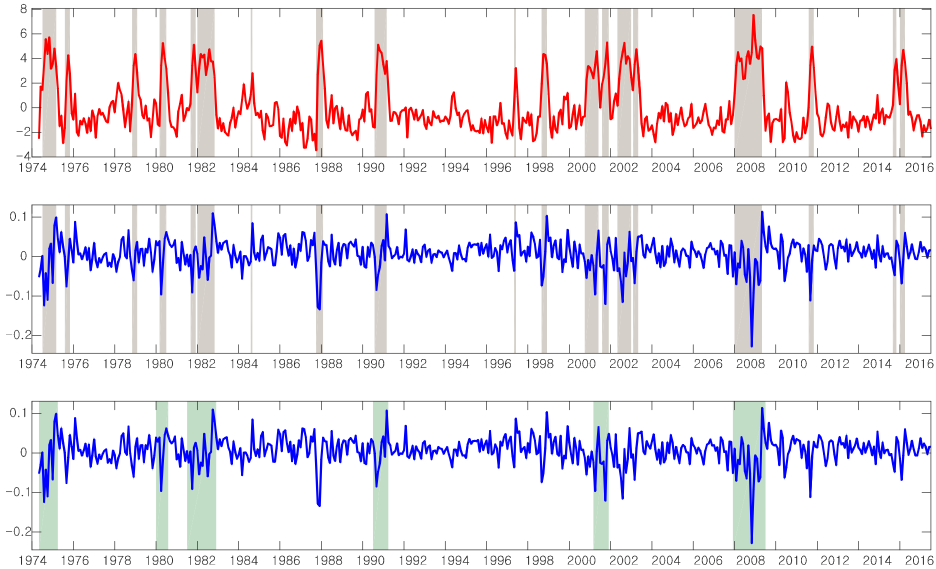
the realized value of the lagged stock return Δp_{t-1}. These results are consistent with the study by Chang et al. (2017). On the other hand, the transition probability estimated by the MS-ECM (dotted line) is constant for the entire sample period as the future transition between states is completely determined by the current state and does not depend on the realizations of the underlying time series.

Figure 2 presents the transition probability from the low regime at $t-1$ to the high regime at t estimated by the endogenous RS-ECM and MS-ECM. For the sample period, this low to high transition probability is estimated to be 4.0% by the MS-ECM, whereas the estimated probability from the endogenous RS-ECM is time varying. For the endogenous RS-ECM, the transition probability exhibits spikes when a seriously negative event in the market occurs. Therefore, the transition probability from the endogenous RS-ECM is more realistic than that from MS-ECM, and this feature cannot be accommodated by the MS-ECM. The estimated transition probability from the low to high state reaches as high as 99.81% on September 2008 when the Lehman Brothers filed for bankruptcy. This result indicates that the maximum estimated transition probability from the low to high regime by using the endogenous RS-ECM is 24.95 (99.81/4.00) times larger than that by using the MS-ECM. As illustrated in Figure 2, we similarly demonstrate the same point with the estimated transition probabilities from the high state at $t-1$ to the low state at t by using two models.

We extract the latent factor that determines the states from the endogenous RS-ECM and compare it with NBER-defined recession periods.[5] In the first graph of Figure 4, the extracted latent factor is presented, and the shaded areas indicate the high regime where the latent factor is larger than the threshold value $\hat{\tau} = 2.061$. In the second graph of Figure 4, the stock return series is presented, whereas the shaded areas indicate the high regime. As shown, stock returns are more volatile and largely negative in the high regime. Finally, in the third graph of Figure 4, the shades now represent the NBER recession periods during the sample period. We can clearly identify that the high regime defined by the extracted latent factor more or less coincides with NBER recession periods. As shown in the first graph of Figure 4, the shaded areas other than the NBER recession periods are considered to be the financial crisis in the US, such as Black Monday (October 19, 1987), Asian financial crisis (1997), collapse of Long-Term Capital Management (LTCM) (1998), stock market crash (2002), debt-ceiling crisis (2011), and Brexit (2016). Therefore, our extracted latent factor from endogenous RS-ECM can be used for a potential indicator for recession as well as for financial crisis.

[5] NBER recession dates are available online at www.nber.org.

[Figure 4] Extracted Latent Factor, Stock Return and NBER Recession Periods



Notes: Figure 4 shows the extracted latent factor, stock return, high-regime periods, and NBER recession dates during the sample period. In the upper graph, the solid line represents the latent factor obtained from endogenous RS-ECM, whereas the shaded areas indicate the high regime. In the middle graph, the stock return (solid line) and high regime identified from the extracted latent factor are presented. The lower graph of Figure 4 presents the stock return (solid line) and NBER recession dates (shades) during the sample period. Shaded areas (upper graph) other than the areas corresponding to NBER recession periods (lower graph) are regarded as the financial crisis, including Black Monday (1987), LTCM debacle (1998), stock market crash (2002), debt-ceiling crisis (2011), and Brexit (2016).

IV. Conclusion

We have shown that the ECM with both TVC cointegration and endogenous regime switching better explains the long- and short-run relationships between stock price and dividend than the existing models with linear cointegration or conventional Markov switching. The latent factor extracted from our model specifically reveals the periods for each regime. Moreover, the periods of high regime (with high volatility) more or less coincide with the NBER recession periods and contain certain periods of financial crisis. Our results show that considering the nonlinearity in both long- and short-run relationships between stock price and dividend is important. This case implies that accommodating these nonlinearities can be important in investigating whether the dividend–price ratio predicts the excess stock return or not.

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