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Discretion versus Inflation Targeting in Economies with Relative Habit Persistence

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This paper sets up a canonical new Keynesian model with habit persistence in consumption. The paper estimates key parameters using maximum likelihood and shows that the habit persistence improves the explanatory power of the model over the business cycle, irrespective of habit formation way. If the distortions associated with external habit are not completely eliminated by the fiscal policy, then the remaining external habit entails a gap between the private marginal rate of substitution between consumption and labor and the social marginal rate of substitution, generating an endogenous trade-off between the stabilization of welfare-relevant output gap and inflation. Under this circumstance, discretion, partially taking into account the trade-off between output gap and inflation, can be better than a strict inflation targeting rule in welfare dimension if the fiscal authority does not implement any tax policy to eliminate the distortions associated with external habit. The monetary policy to deal with distortions associated with external habit is less effective in the ratio external habit model than in the difference habit model, resulting in a higher the inflation rate in the ratio external habit model than the inflation rate in the difference habit model.

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I. Introduction

Real rigidities such as internal or external habit formation have been incorporated into sticky price models to capture the hump-shaped, gradual response of spending to monetary shocks.[1] The ways that habit persistence is modeled in the new

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[1] A shock that increases current aggregate consumption leads the household with habit to experience

Keynesian literature are essentially innocuous for business cycle behavior as in Dennis (2009) and Schmitt-Grohé and Uribe (2008). In particular, Dennis (2009) shows that the differences of impulse response functions of output and inflation rate to supply and demand shocks under internal habit and external habit specifications are qualitatively and quantitatively very small.

However, it is important where one discusses the welfare implications of alternative monetary policy rules. Many studies including Amato and Laubach (2004), Chugh (2007), and Woodford (2003) have explored the optimal monetary policy by incorporating internal habit in consumption along the lines of Christiano et al. (2005), Del Negro et al. (2007), and Smets and Wouters (2007). In particular, Woodford (2003) has forcefully shown that the divine coincidence holds in the sticky price model with internal habit in consumption if the fiscal authority can implement a time-invariant subsidy to employment to eliminate the distortion associated with the monopoly power in the goods market: The monetary authority should focus on completely stabilizing inflation every period without concern for the output gap even if there is internal habit persistence.[2] Considering the success of models with habit formation to accurately replicate the hump-shaped response of real spending to exogenous shocks, the monetary policy recommendation warrants a closer look.

When we distinguish internal habit from external habit, the conditions under which the divine coincidence holds in models with external habit in consumption are very complicated. The external habit generates time-varying externality in consumption that should be tackled by the authority. Jung (2015) shows that the divine coincidence holds in the difference external habit formation circumstance where households do not take into account of their behavior on the economy, only if the fiscal authority implements a very simple from of time-varying labor income tax rates to completely eliminate the time-varying distortions associated with external habit. Otherwise, price stability cannot be optimal monetary policy in external habit circumstance.

Although many authors adopt a difference habit formation to discuss important issues related to business cycles and monetary policy,[3] a ratio habit formation rather than a difference habit formation is first introduced to address the relative behavior of consumption over business cycles. As Schmitt-Grohé and Uribe (2009) note, a ratio external habit persistence in Abel (1990) and Furher (2000) which features a

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a higher utility from an additional unit of consumption tomorrow. Therefore, under habit formation, the household gets used to a higher level of consumption, and the marginal utility of consumption gets renormalized at the higher reference level. As a result, the shock propagates consumption and output persistence.

[2] Yun (2005) also shows that the inflation targeting rule responding to changes in the initial relative price distortion is still optimal even if there is initially a substantial relative price distortion.

[3] See Schmitt-Grohe and Uribe (2008) for detail.

relative habit persistence is closely related to Dusenberry (1941)'s relative income hypothesis. To look at the monetary policy implications under a relative income hypothesis, it warrants to explore the welfare ranking of alternative simple monetary policy rules in the ratio external habit model. Furthermore, as the optimal tax policy rules take a very complicated form in a ratio external habit model compared to a very simple form in a difference habit formation habit model, it is practically impossible to implement a time-varying optimal tax on labor income to completely eliminate the time-varying distortions associated with external habit at every state and at every time. To implement an optimal tax policy, the fiscal authority should have complete information about the state of the economy such as conditional covariance of the stochastic discount factor and the future consumption. Under this circumstance, the government is more likely to implement a time-invariant taxation to deal with distortions associated with external habit.

I will first discuss whether habit persistence improves the sticky price model in explaining the business cycle characteristics in terms of second moments of key variables. I will next consider the effect of external habit on the relationship between welfare-relevant output gap and inflation in a new Keynesian Phillips curve and in a welfare function under the assumption that the fiscal authority implements a simple time-invariant tax policy to deal with distortions associated with external habit. Finally, I evaluate the correct welfare ranking of the alternative monetary policy rules such as discretion and inflation targeting rule by employing a second-order approximation as well as a first-order approximation to the structural equilibrium conditions as in Schmitt-Grohé and Uribe (2004), and Woodford (2003). In particular, I will explore whether such a 'leaning against the wind' monetary policy can be better than a strict inflation targeting rule in sticky price model with productivity shocks only and external habit circumstances.

The main findings of this paper can be summarized as follows. First, the optimal discretion can be better than the strict inflation targeting rule in improving the welfare of the household with externality in consumption. The endogenous trade-off resulting from the discrepancy between the private marginal rate of substitution between consumption and labor and the social one due to external habit in consumption calls for monetary authority to deviate from the price stability. Hence, the optimal discretion that partially takes into account the trade-off between output gap and inflation can be better than the inflation targeting rule that disregards it.

Second, the monetary policy to deal with distortions associated with external habit is less effective in the ratio external habit model than in the difference habit model and the inflation rate in the ratio external habit model is higher than the inflation rate in the difference habit model. This result follows from the fact that the consumption ratio does not move much in the ratio external habit model, making the disutility of labor hours dominate the utility of consumption. Hence, the interest rate moves less countercyclically in the ratio external habit model than in the difference external habit model as the marginal utility of consumption moves less in the ratio external model than in the difference external habit model.

Third, the habit persistence improves the explanatory power of the sticky price model over the business cycle, regardless of whether the habit persistence is internal or external. The estimated degree of external habit via maximum likelihood is about 0.66, whose value is comparable to Boldrin, Christiano, and Fisher (2001), Christiano et al. (2005), and Gruber (2004) estimates of b, 0.73, 0.63, and 0.82, respectively.

Finally, the fiscal policy is more important than the monetary policy in welfare dimension since the fiscal policy is relevant to the determination of the efficient natural level of output. The optimal labor income tax rate should be proportional to the degree of externality in consumption as well as the markup size, which restores the efficient equilibrium.

The remainder of the paper is organized as follows. Section 2 presents a canonical sticky price model with the habit persistence and Section 3 estimates key parameters and evaluates the model in time domain. Section 4 delineates the Ramsey optimal monetary policy in the sticky price model with external habit in optimal tax regime. Section 5 explores Ramsey optimal monetary and discretion in the sticky price model with external habit in Ramsey steady-state tax regime. Section 6 compares the welfare of alternative monetary policy rules. Section 7 concludes the paper.

II. A Sticky Price Model with External Habit Formation

2.1. Households

Along the line of Dusenberry (1941), Abel (1990, 1999) and Furher (2000) specified a ratio external habit to consider the equity premium puzzle and monetary policy implications, respectively. In this setup, a representative household derives utility from the level of consumption relative to a time-varying aggregate habit level:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(C_t^d)^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\upsilon}}{1+\upsilon}\right)\right],\quad 0<\beta<1, \tag{1}$$

where β is the household's discount factor, E_t denotes the conditional expectations operator on the information available in period t, and $C_t^d = C(C_t, X_t)$. C_t, N_t, and X_t represent the household's consumption for composite goods, labor hours, and the external habit level of consumption in period t, respectively. A consumption index is introduced as follows:

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2}$$

Here ε measures the elasticity of substitution within each category.

Next X_t summarizes the influence of past consumption levels on today's utility. The household regards the stochastic sequence of habits $\{X_t\}_{t=0}^{\infty}$ as exogenous which is tied to the stochastic sequence of aggregate consumption $\{\tilde{C}_t\}_{t=0}^{\infty}$ as follows.

$$C(C_t, X_t) = \frac{C_t}{\tilde{C}_{t-1}^b}$$

where $\tilde{C}_{t-1}$ is aggregate past consumption and $0 \le b \le 1$ measures the degree of habit persistence. Since there is a representative agent, aggregate consumption equals the household's consumption in equilibrium.

$$\tilde{C}_t = C_t.$$

There exists a complete asset market in the economy. Let B_t denote the nominal payoff of the portfolio purchased in period t and $Q_{t,t+1}$ be the corresponding stochastic discount factor in period t. Then the riskless one-period nominal interest rate in period t is given by $R_t \equiv [E_t Q_{t,t+1}]^{-1}$. In each period, the household chooses decision rules for consumption C_t, labor N_t, and a nominal bonds portfolio B_{t+1} to maximize (1) subject to a sequence of budget constraints:

$$P_t C_t + E_t[Q_{t,t+1} B_{t+1}] \le \Theta_t + TR_t, \tag{3}$$

where TR_t is lump-sum transfers or taxes in period t. Household's wealth Θ_t in the beginning of the period t is given by

$$\Theta_t = B_t + \Pi_t + W_t N_t (1 - \tau_{N,t}), \tag{4}$$

where N_t, Π_t, W_t, and $\tau_{N,t}$ denote the hours worked, the firm's nominal profits, nominal wages, and the tax rate given to the household at time t, respectively. Here it is also assumed that the tax revenues are handed back to the households in a lump-sum fashion as in Ljungqvist and Uhlig (2000).

2.2. Firms

Suppose that there is a continuum of firms producing differentiated goods, and each firm indexed by $j \in [0,1]$ produces its product with constant returns to scale technology.

$$Y_t(j) = A_t N_t(j), \tag{5}$$

where A_t is the technology process at period t, and $Y_t(j)$ and $N_t(j)$ are the output and total labor input of the *j*th firm, respectively. I assume that the (log) technology shock follows an $AR(1)$ process as $a_t = \rho_a a_t + \xi_{At}$, $-1 < \rho_A < 1$, where $a_t \equiv \ln A_t$, $E(\xi_{At}) = 0$ and ξ_{At} is i.i.d. over time. Each firm j takes P_t and the aggregate demand as given, and chooses its own product price $P_{t,t}(j)$. Cost minimization leads to the following labor demand

$$W_t = MC_t(j) A_t. \tag{6}$$

The marginal cost of each firm is equal, i.e. $MC_t(j) = MC_t$ for each j as the production function is constant returns to scale:

Next, the monopolistically competitive firms in the product markets set their own prices in advance by maximizing the present discounted value of profits. Only the fraction $(1-\alpha)$ of the firms sets the new price, $P_{t,t}$, and the other fraction of firms, α, sets its current price at its previous price level à la Calvo (1983) and Yun (2005). Let $P_{t,t+k}$ denote the price at period $t+k$ that is predetermined at period t. The firm's maximization problem can be written as follows.

$$\max E_t \left\{ \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} [P_{t,t+k} Y_{t,t+k}(j) - MC_{t+k} Y_{t,t+k}(j)] \right\}, \tag{7}$$

subject to the sequence of demand constraints

$$Y_{t,t+k}(j) \leq \left(\frac{P_{t,t}}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}.$$

2.3. The Government

The fiscal authority collects lump-sum taxes $T_t \equiv TR_t + W_t N_t \tau_{N,t}$ to offset the distortions in goods market and consumption, and then it hands back the tax revenues to the households in a lump-sum fashion as in Ljungqvist and Uhlig

(2000). The monetary authority implements one of the monetary policy rules such as inflation targeting, discretion, and Taylor rule.

2.4. Equilibrium

I will focus on the symmetric equilibrium in which all agents make the same decisions in what follows.

2.4.1. First-Order Conditions

The first-order conditions for the household can be summarized as follows:

$$N_t^{v}\left(\frac{C_t}{C_{t-1}^{b}}\right)^{\sigma} C_{t-1}^{b} = w_t(1-\tau_{Nt}), \tag{8}$$

$$Q_{t,t+1} = \beta\frac{P_t}{P_{t+1}}\frac{(C_{t+1}/C_t^{b})^{1-\sigma}C_t}{(C_t/C_{t-1}^{b})^{1-\sigma}C_{t+1}}, \tag{9}$$

and the budget constraint (3). Here $w_t \equiv \frac{W_t}{P_t}$ is the real wage in period t. If $\tilde{R}_t$ represents the risk-free gross real interest rate, equation (8) and equation (9) gives the following Euler equation:

$$\beta\tilde{R}_t E_t\left[\frac{(C_{t+1}/C_t^{b})^{1-\sigma}C_t}{(C_t/C_{t-1}^{b})^{1-\sigma}C_{t+1}}\right] = 1. \tag{10}$$

If the households have internal habit in consumption rather than external habit in consumption, the first order conditions for consumption and labor hours are replaced by

$$[(C_t/C_{t-1}^{b})^{1-\sigma}C_t^{-1} - b\beta E_t(C_{t+1}/C_t^{b})^{1-\sigma}C_t^{-1}] = \Lambda_t \tag{11}$$

$$N_t^{v}[(C_t/C_{t-1}^{b})^{1-\sigma}C_t^{-1} - b\beta E_t(C_{t+1}/C_t^{b})^{1-\sigma}C_t^{-1}]^{-1} = w_t(1-\tau_{Nt}), \tag{12}$$

where Λ_t is the Lagrange multiplier on the budget constraint. Combining (6) and (8), the private marginal rate of substitution (MRS) between consumption and labor equals the marginal product of labor multiplied by the net real marginal cost:

$$N_t^{v}\left(\frac{C_t}{C_{t-1}^{b}}\right)^{\sigma} C_{t-1}^{b} = A_t mc_t(1-\tau_{Nt}), \tag{13}$$

where $mc_t \equiv \frac{MC_t}{P_t}$ is the real marginal cost or the inverse of the markup in period

t.

Next, notice that the optimal price setting equation in the Calvo-type model can be rewritten as

$$E_t \left\{ \sum_{k=0}^{\infty} (\alpha\beta)^k \Lambda_{t+k} \left(\frac{P_{t,t}}{P_{t+k}} \right)^{-1-\varepsilon} Y_{t+k} \left[mc_{t+k} - \frac{\varepsilon - 1}{\varepsilon} \frac{P_{t,t}}{P_{t+k}} \right] = 0 \right. . \tag{14}$$

The optimal price setting equation can be expressed as a recursive form as in Schmitt-Grohé and Uribe (2004) and Yun (2005):

$$\frac{\varepsilon}{\varepsilon - 1} S_{1,t} = S_{2,t} , \tag{15}$$

where

$$S_{1,t} = \tilde{p}_t^{-1-\varepsilon} Y_t mc_t + \alpha\beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \pi_{t+1}^{\varepsilon} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-1-\varepsilon} S_{1,t+1} \right], \tag{16}$$

and

$$S_{2,t} = \tilde{p}_t^{-\varepsilon} Y_t + \alpha\beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \pi_{t+1}^{\varepsilon-1} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\varepsilon} S_{2,t+1} \right]. \tag{17}$$

Here $\tilde{p}_t \equiv \frac{P_{t,t}}{P_t}$ is the relative price of any good whose price was adjusted in period t.

2.4.2. Aggregation

Aggregating individual output across firms, one finds a wedge between the aggregate output and aggregate factor input

$$Y_t = \frac{A_t N_t}{\Delta_t} , \tag{18}$$

where

$$\Delta_t = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj \tag{19}$$

is the relative price dispersion in period t. The price level that satisfies the recursive form can be rewritten as

$$1 = (1-\alpha)\tilde{p}_t^{1-\varepsilon} + \alpha(1+\pi_t)^{\varepsilon-1} . \tag{20}$$

The relative price distortion Δ_t that results from the firms' staggered price setting practice in the Calvo-type model can be rewritten as a recursive form:

$$\Delta_t = (1-\alpha)\tilde{p}_t^{-\varepsilon} + \alpha(1+\pi_t)^{\varepsilon}\Delta_{t-1}, \tag{21}$$

with Δ_{-1} given. This is a source of welfare losses from inflation or deflation. When the non-stochastic inflation rate is nil, i.e. when $\pi = 0$, the state variable Δ_t follows a deterministic autoregressive process and the dynamics of Δ_t can be ignored up to the first-order. However, one should take into account the dynamics of the relative price distortion, i.e. (21) if one is interested in higher-order approximation to the equilibrium conditions or if the inflation rate is not nil.[4] In the welfare analysis with the second-order approximations, the dynamics of the relative price distortion Δ_t should be incorporated into the equilibrium conditions as in Yun (2005) and Schmitt-Grohé and Uribe (2004).

The symmetric equilibrium conditions consist of the efficiency conditions and the budget constraint of the households, and firms and equilibrium conditions of each goods market, capital rental market, labor market, money, and bond market. Specifically, a symmetric equilibrium is an allocation of households $\{C_t, N_t, Y_t\}_{t=0}^{\infty}$, a sequence of prices and costate variables $\{P_{t,t}, P_t, W_t, \tilde{p}_t, MC_t, R_t, \Delta_t, S_{1,t}, S_{2,t}\}_{t=0}^{\infty}$ satisfying equilibrium conditions (6), (8), (10), (15), (16), (17), (18), (20), (21), and $\tilde{p} \equiv \frac{P_{t,t}}{P_t}$, the resource constraint $Y_t = C_t$ with monetary policy rules, given the initial conditions for the variables for C_{-1}, Δ_{-1} and the exogenous stochastic processes $\{\xi_{At}\}_{t=0}^{\infty}$.

Before turning to the analysis of the welfare ranking of alternative monetary policy rules in the model with external habit, I will explore the role of external habit in the explanation of output dynamics over business cycles and some intuitions about the effect of habit on welfare in next section.

III. Habit and Business Cycles

3.1. Estimates and Parameter Values

For an empirical analysis, I will close the model by assuming that the monetary authority conducts monetary policy according to a Taylor rule as in Ireland (2001). Then the log-linearized equilibrium conditions of external habit model can be represented in the form of a state-space econometric model, driven by the three exogenous shocks, a_t, u_t, and ξ_{rt}:

[4] Yun (2005) shows that a nonlinear relation between Δ_t and π_t requires to take into account this equation when one discuss the effect of monetary policy on welfare.

$$\tilde{y}_t = \frac{\sigma}{\sigma + b(\sigma - 1)} E_t[\tilde{y}_{t+1}] - \frac{b(1-\sigma)}{\sigma + b(\sigma - 1)} \tilde{y}_{t-1} + \frac{1}{\sigma + b(\sigma - 1)} [r_t - E_t \pi_{t+1} - r_{nt}], \quad (22)$$

$$\pi_t = \frac{\beta}{1 + \zeta\beta} E_t \pi_{t+1} + \frac{\zeta}{1 + \zeta\beta} \pi_{t-1} + \frac{\gamma}{1 + \zeta\beta} \tilde{y}_t + u_t, \quad (23)$$

$$r_t = \rho_r r_{t-1} + a_y \tilde{y}_t + a_\pi \pi_t + \xi_{rt}, \quad (24)$$

$$u_t = \rho_u u_{t-1} + \xi_{ut}, \quad (25)$$

where $\gamma \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ and r_{nt} is the natural real interest rate which is a function of the technology shock. Note that a partial-indexation specification with a markup shock u_t is employed as in Smets and Wouters (2007). Here (24) is Taylor rule and the zero mean, serially uncorrelated innovation ξ_{rt} is normally distributed with standard deviation σ_r. As in Christiano et al. (2005), I assume that firms that do not reoptimize their prices update their indexation factor by multiplying the previous period's inflation to their previous period's indexation factor ζ.[5] The model has nine parameters to be estimated, $\Xi \equiv [b, \rho_r, a_y, a_\pi, \rho_a, \rho_u, \sigma_a, \sigma_u, \sigma_r]'$ whose values are estimated via maximum likelihood, using the methods outlined by Hamilton (1994) and Ireland (2001), with data on three variables, the US GDP, the US GDP deflator and the federal fund rate. All of these data, except the interest rate, are seasonally adjusted. The series for GDP is expressed in per-capita terms by dividing the civilian noninstitutional population, age 16 and over. The data are quarterly and run from 1959:1 through 2006:4 and the linearly detrended series for the GDP is used to estimate the model as in Ireland (2001).

[Table 1] Parameter Values

Parameter	Values	Description and definitions
b	[0.6, 0.9]	Degree of externality in consumption
α	3/4	Fraction of firms that do not change their prices in a given period
ε	6	Elasticity of demand for a good with respect to its own price
σ	2	Relative risk aversion
ν	1	Inverse of elasticity of labor supply
r	0.016	Steady state real interest rate

Several of the model's parameters are fixed prior to estimation, presented in Table 1. First, I set the intertemporal elasticity of substitution σ^{-1} to 2 and the intratemporal elasticity of labor supply, $\varepsilon_n (\equiv \nu^{-1})$ to 1 in the benchmark model. I also set the subjective discount factor to $1.04^{-1/4}$, which is consistent with an annual real rate of interest of 4 percent. The nominal rigidity parameter value α is set to 3/4. I also set the elasticity of substitution among varieties ε to 6, implying

[5] In the literature, $\zeta \in [0,1]$ is introduced to generate a hump-shaped and delayed response of inflation rate to the monetary shock.

the average size of markup, μ to be 1.2 as in Galí (2008).

[Table 2] Maximum Likelihood Estimates and Standard Errors

Parameter	Estimate	Standard Error
b	0.6613	0.0182
ρ_r	0.8251	0.0236
a_y	0.0039	0.0024
a_π	0.3021	0.0001
ρ_u	0.8033	0.0002
ρ_e	0.7079	0.0110
σ_u	0.0185	0.0009
σ_e	0.0449	0.0023
σ_r	0.0025	0.0001
L^*	2269.5	

Note: L^* denotes the maximized value of the model's log-likelihood function.

Table 2 presents maximum likelihood estimates of the model's remaining parameter values. The standard errors are computed by taking the square roots of the diagonal elements of minus one times the inverted matrix of second derivatives of the maximum log-likelihood function. In the interest rate rule, lagged interest rate and inflation rate enter significantly as determinants of the current interest rate, while output does not. The estimates of exogenous shock processes are comparable to those of Ireland (2001) and the high estimates of ρ_a and ρ_u imply that the model's exogenous shocks are highly persistent. Finally, note that the degree of external habit b is estimated about 0.66, whose value is comparable to Boldrin, Christiano, and Fisher (2001), Christiano, Eichenbaum, and Evans (2005), and Gruber (2004) estimates of b, 0.73, 0.63 and 0.82, respectively.

IV. Optimal Monetary Policy under Time-varying Tax Regime

In this section, I will turn to the specification of the optimal policy problem in a dynamic context. The optimal monetary policy can be defined as the process $\{R_t\}_{t=0}^{\infty}$ associated with the competitive equilibrium that yields the highest utility to the representative household, given a tax policy $\{\tau_{Nt}\}_{t=0}^{\infty}$. The optimal monetary policy prescription takes a different form in external habit circumstances, depending upon the available tax instruments to deal with the time-varying distortions associated with external habit.

4.1. A Simplification of the Ramsey Problem

Given distortions associated with external habit and monopoly power in goods market, the Ramsey planner who internalizes the external habit in consumption chooses optimal time-varying tax and monetary policy prescriptions for $\{\tau_{Nt}, R_t\}_{t=0}^{\infty}$ as well as plans for $\{C_t, N_t, Y_t, P_t, P_{t,t}, \pi_t, mc_t, \Delta_t\}_{t=0}^{\infty}$ to maximize the welfare of the representative household satisfying the competitive equilibrium conditions.

Since the block of variables, R_t, τ_{Nt}, mc_t, $S_{1,t}$, $S_{2,t}$ do not enter the Ramsey problem anywhere else, the Ramsey problem can be set without the variables and the equations associated with them as in Yun (2005) and Christiano et al. (2010). Let $\mathbf{V}(C_{-1}, \Delta_{-1}, A_t)$ represent the value function in the Bellman equation for the optimal policy problem with time-varying tax in period t. Then, the Ramsey planner solves the following maximization problem:

$$\mathbf{V}(C_{t-1}, \Delta_{t-1}, A_t) = \max._{\{C_t, N_t, \tilde{p}_t, \pi_t, \Delta_t\}} \left[\frac{(C_t / C_{t-1}^b)^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu} + \beta E_t \mathbf{V}(C_t, \Delta_t, A_{t+1}) \right], \quad (26)$$

subject to

$$C_t = \frac{A_t N_t}{\Delta_t}, \quad (27)$$

$$1 = (1-\alpha)\tilde{p}_t^{1-\varepsilon} + \alpha(1+\pi_t)^{\varepsilon-1}, \quad (28)$$

$$1 = (1-\alpha)\tilde{p}_t^{-\varepsilon}\Delta_t^{-1} + \alpha(1+\pi)^{\varepsilon}\Delta_{t-1}\Delta_t^{-1}, \quad (29)$$

with the exogenous technology shock process $\{A_t\}_{t=0}^{\infty}$, an initial consumption C_{-1}, relative price distortion Δ_{-1}, and the optimal tax rate given. In the Ramsey problem specified above, the required tax rate τ_{Nt} to completely eliminate time-varying distortions associated with external habit and monopoly power in goods market must satisfy the efficiency condition of the Ramsey planner who internalizes the external habit of the agent to maximize the welfare.

The optimization condition is given by

$$N_t^{\nu}\left[\left(\frac{C_t}{C_{t-1}^b}\right)^{1-\sigma}\frac{1}{C_t} - b\beta E_t\left[\left(\frac{C_{t+1}}{C_t^b}\right)^{1-\sigma}\frac{1}{C_t}\right]\right] = w_t(1-\tau_{Nt}) \quad (30)$$

As (30) shows, the benevolent government must fully take into account the relative consumption C_{t+1}/C_t^b which has been neglected in the household's decision rules. This fiscal policy prescription can make the decision rules of households with external habit to be compatible with the efficient ones in the internal ratio model.

4.2. Relative Habit Persistence and Time-Varying Tax

The way how current consumption enters into the household's utility also affects the form of optimal tax rates in the two variants of external habit. In the ratio external habit model, the benevolent government utilizes the information that it is the consumption ratio, not the consumption difference that matters in the welfare of the external habit household in the design of optimal fiscal policy to eliminate completely the externality in consumption. Specifically, an optimal fiscal policy prescription takes a form of labor income tax rate that is proportional to the present discount value of future expected consumption relative to current consumption and the degree of external habit in consumption as in Proposition 1.

Proposition 1 *Suppose that the degree of distortion associated with imperfect competition in goods market and the degree of externality in consumption are equal to* $\mathcal{M}$ *and* b*, respectively in the ratio external habit model. Then the optimal fiscal policy is to implement a labor income tax rate* τ_{Nt} *equal to* $1-\frac{1}{mc_t\Delta_t}[1-bE_t[\tilde{Q}_{t,t+1}\frac{C_{t+1}}{C_t^b}]]$ *to completely eliminate both the externality in consumption and distortions in the goods market due to monopolistic competition, where* $\tilde{Q}_{t,t+1}\equiv\beta\frac{\Lambda_{t+1}}{\Lambda_t}$.

Proof: Please refer to the appendix.

Since $E_t[\tilde{Q}_{t,t+1}\frac{C_{t+1}}{C_t^b}]=\tilde{R}_t^{-1}E_t(\frac{C_{t+1}}{C_t^b})+\mathrm{cov}_t(\tilde{Q}_{t,t+1},\frac{C_{t+1}}{C_t^b})$, the optimal labor income tax rate should be inversely related to the riskless interest rate. Furthermore, the government needs to have complete information about a conditional covariance of the stochastic discount factor and the future consumption to completely eliminate the distortions associated with a ratio external habit.

4.2.1. Optimal Monetary Policy

With the implemented optimal tax on employment taking into account the degree of externality in consumption and the size of markup, the optimal monetary policy prescription that satis.es (31) and (32) can be determined as in Yun (2005).

Optimal labor income tax policy in place, nominal variables, π_t and Δ_t are determined by the following two equations

$$\pi_t=\frac{\Delta_t}{\Delta_{t+1}}-1, \tag{31}$$

$$\Delta_t=\Delta_{t-1}(\alpha+(1-\alpha)\Delta_{t-1}^{\varepsilon-1})^{\frac{1}{1-\varepsilon}}. \tag{32}$$

From the equation (31), the aggregate price ratio between period $t+k$ and t is given by

$$\frac{P_{t+k}}{P_t} = \frac{\Delta_{t+k}}{\Delta_t}. \tag{33}$$

Note that $(C_t / C_{t-1}^b)^{-\sigma} C_{t-1}^b [1 - b\beta E_t[(C_{t+1} / C_t^b)^{-\sigma} C_{t+1} C_t^{-b}] = N_t^\nu \frac{\Delta_t}{A_t}$ in the Ramsey problem, and $(C_t / C_{t-1}^b)^{-\sigma} C_{t-1}^{-b} [1 - b\beta E_t[(C_{t+1} / C_t^b)^{-\sigma} C_{t+1} C_t^{-b}]^{-1} N_t^\nu = w_t$ in the competitive equilibrium with optimal tax on labor income. Therefore, the labor demand relation, $w_t = mc_t A_t$ implies that the real marginal cost associated with the optimal fiscal and monetary policy is given by

$$mc_t = \frac{1}{\Delta_t}. \tag{34}$$

Next, substituting (33) and (34) into (14), one finds that the relative price of the new price set by firms in period t is given by

$$\frac{P_{t,t}}{P_t} = \frac{1}{\Delta_t}. \tag{35}$$

This is the so-called inflation targeting rule responding to the relative price distortion in Yun (2005). The optimal monetary policy rule supporting (35) then can be found by substituting the relation between the real marginal cost and the relative price distortion into the Euler equation (10):

$$R_t = \left(\frac{\Delta_{t+1}}{\Delta_t} \right)^{\frac{(1-\sigma)\nu}{\sigma+\nu}} \tilde{R}_t.$$

Table 3 and 4 report some sampling moments of the key macroeconomic variables associated with internal and external ratio habit persistence under the Ramsey fiscal and monetary policy rules. First, note that comparison of the first columns in Table 3 and Table 4 shows that the conditional welfare in the internal habit formation is the same as the one in external habit formation. Moreover, there is no difference between sampling moments of output and inflation rate associated with the ratio internal habit model and the ones with the ratio external habit model.

The optimal labor income tax rate in internal habit persistence is a time-invariant subsidy as in Woodford (2003), while it moves procyclically in the external habit circumstance. Moreover, the interest rate in the ratio external habit model does not move exactly in the opposite direction as the interest rate in the difference external

[Table 3] Dynamic Properties of the Ramsey Allocation in the Ratio Internal Habit Model (Linear approximation)

Variable	Mean	Std. Dev.	Auto. Corr	$Corr(x,y)$
$\mathcal{W}=$ -143.3005	$(b=0.6)$			
τ_N	-20.0000	0	-	-
π	0	0	-	-
R	3.7039	1.2456	0.6023	-0.9565
y	0.6871	0.0183	0.8864	1
$\mathcal{W}=$ -130.1598	$(b=0.8)$			
τ_N	-20.0000	0	-	-
π	0	0	-	-
R	3.7028	0.4840	0.9025	-0.9993
y	0.4897	0.0121	0.9231	1
$\mathcal{W}=$ -102.9890	$(b=1)$			
τ_N	-20.0000	0	-	-
π	0	0	-	-
R	3.5795	1.4984	0.7813	-0.6212
y	0.0987	0.0011	0.9823	1

Note: τ, π, and R are expressed in annual percentage points and y, h, and c in levels. The parameter values are $\beta=(1.04)^{-1/4}$, $\sigma=2$, $\nu=1$, $T=200$, and $J=1000$.

[Table 4] Dynamic Properties of the Ramsey Allocation in the Ratio External Habit Model (Linear approximation)

Variable	Mean	Std. Dev.	Auto. Corr	$Corr(x,y)$
$\mathcal{W}=$ -143.3005	$(b=0.6)$			
τ_N	51.3055	0.4591	0.39064	0.7187
π	0	0	-	-
R	3.8186	0.7560	0.8025	-0.9833
y	0.6872	0.0192	0.8861	1
$\mathcal{W}=$ -130.5198	$(b=0.8)$			
τ_N	75.06970	0.5048	0.4755	0.6334
π	0	0	-	-
R	3.8695	0.5138	0.9164	-0.9993
y	0.4896	0.0132	0.9237	1
$\mathcal{W}=$ -102.9890	$(b=1)$			
τ_N	98.8297	0.0720	0.7229	0.4738
π	0	0	-	-
R	3.9357	0.0927	0.7705	-0.4327
y	0.0987	0.0011	0.9826	1

Note: τ, π, and R are expressed in annual percentage points and y, h, and c in levels. The parameter values are $\beta=(1.04)^{-1/4}$, $\sigma=2$, $\nu=1$, $T=200$, and $J=1000$.

habit model. This difference has been expected because the comovement of the stochastic discount factor and future consumption affects the tax rate in the ratio external habit model.

To restore the efficient natural level of output at every state and every period by implementing the optimal labor income tax rate and the inflation targeting rule, the benevolent government should utilize the stochastic discount factor to evaluate the future consumption relative to current consumption that is ignored in the decision rules of the external habit household in the design of optimal fiscal policy. It is impractical to implement the optimal time-varying fiscal policy by incorporating the precise and exact complicated information about the stochastic discount factor into the policy rules in the ratio external habit model.

In such an environment, it is natural to ask what is the optimal and simple rules among the available alternative rules. Intuitively, if the government does not implement the optimal, time-varying labor income tax rates, then there remain externalities in consumption, making the monetary authority to face endogenous trade-off between output gap stabilization and inflation stabilization. Under this circumstance, price level stability ceases to be optimal monetary policy prescription.

V. Optimal Monetary Policy under Time-invariant Tax Regime

When the fiscal authority cannot implement an optimal tax policy to completely eliminate the externality in consumption, it will try to find feasible fiscal and monetary policy rules to maximize the welfare. In this section, it is assumed that the fiscal authority levies a time-invariant steady state tax rate to attain the efficient steadt-state.

I will first derive a discretion as well as an optimal monetary policy in a linear form by employing the linear-quadratic approach. And then I will evaluate alternative monetary policy rules such as a discretion and inflation targeting in welfare dimension as in Woodford (2003).

5.1. Time-invariant Tax and Natural Level of Output

Before turning to explore the endogenous trade-off between the welfare-relevant output gap and inflation that comes from the externality in consumption and its implications on monetary policy, I will first look at the relation between an inefficient natural level of output as well as the efficient one in the presence of external habit in consumption.

The efficient level of output Y_{et} and its steady state value Y_e are determined by the following efficient allocation conditions:

$$\frac{\Delta_t^{\nu} Y_{et}^{\nu}}{A_t^{1+\nu}}\left[\left[\frac{Y_{et}}{Y_{et-1}^{b}}\right]^{-\sigma}\frac{1}{Y_{et-1}^{b}}-b\beta E_t\left[\frac{Y_{et+1}}{Y_{et}^{b}}\right]^{1-\sigma}\frac{1}{Y_{et}}\right]^{-1}=1. \tag{36}$$

$$\bar{Y}_e=(1-b\beta)^{\frac{1}{\Upsilon}}\bar{\Delta}^{\frac{-\nu}{\Upsilon}}\bar{A}^{\frac{1+\nu}{\Upsilon}}, \tag{37}$$

where $\Upsilon \equiv b(1-\sigma)+\sigma+\nu>0$ and $\bar{\Delta}$ is the steady-state relative price distortion.[6] The efficient steady state output, Y_e is decreasing in the degree of habit b as in Table 4 and 5.

In the external habit model, the natural level of output Y_{nt} and its steady-state value $\bar{Y}_e$ are determined given by

$$\left[\frac{Y_{nt}}{Y_{nt-1}^{b}}\right]^{\sigma} Y_{nt-1}^{b}\Delta_t^{\nu}Y_{nt}^{\nu}=\frac{A_t^{1+\nu}(1-\tau_N)}{\mathcal{M}},$$

$$\bar{Y}_n=\bar{\Delta}^{\frac{-\nu}{\Upsilon}}\bar{A}^{\frac{1+\nu}{\Upsilon}}(1-\tau_N)^{\frac{1}{\Upsilon}}\mathcal{M}^{\frac{-1}{\Upsilon}}. \tag{38}$$

Substituting (37) into (38), one can see that $Y_n/\bar{Y}_e$ is increasing in the degree of external habit formation in consumption, while it is decreasing in the markup:

$$\frac{Y_n}{\bar{Y}_e}=(1-b\beta)^{\frac{-1}{\Upsilon}}\left(\frac{1-\tau_N}{\mathcal{M}}\right)^{\frac{1}{\Upsilon}}. \tag{39}$$

In (39), the labor income tax rate equal to $1-(1-b\beta)\mathcal{M}$ restores the efficient steady-state output level $\bar{Y}_e$. (39) shows that households with external habit work more than necessary to catch up with the Joneses. The wedge between efficient output and inefficient output increases with the degree of habit in consumption (b). Hence, a higher tax rate is required to induce households with external habit formation in consumption to work less and moderate the fluctuation of the economy.

5.2. New Keynesian Phillips Curve

Since the optimal fiscal/tax policy that guarantees the efficient natural level of output takes a very complicated form in the ratio external habit model, it is not likely to provide a practical guidance for the government. These practical

[6] In the efficient steady state, there is no relative price distortion, requiring that $\bar{\Delta}$ be one. This corresponds to π being zero.

shortcomings of the optimal fiscal policy lead the government to implement a simple labor income tax policy rule $\tau_N = 1-(1-b\beta)\mathcal{M}$ to deal with distortions associated with external habit and monopolistically competitive goods market.

To see how the trade-off between the output-gap and inflation affect the new Keynesian Phillips curve under this circumstance, rewrite the percentage deviation of markup from the steady state, $\hat{\mu}_t$ in terms of the efficient output gap as follows:

$$\hat{\mu}_t = -(\sigma+\nu)\sum_{i=0}^{\infty}\eta^i[x_{et-i}-\eta x_{et-i-1}] - b(1-\sigma)\sum_{i=0}^{\infty}\eta^i[x_{et-i-1}-\eta x_{et-i-2}] + \bar{\Gamma}(\mathbf{Y}_{et}-\mathbf{Y}_{nt}) . \quad (40)$$

Here $x_{et} \equiv y_t - y_{et}$ yet is the welfare-relevant output gap, and $\bar{\Gamma}(\mathbf{Y}_{et}-\mathbf{Y}_{nt})$ represents the difference terms between the efficient and inefficient natural output, $\{y_{et-i}-y_{nt-i}\}_{i=0}^{\infty}$ that are independent of the monetary policy. Also $0 \le \eta \le b$ is the smaller root of the quadratic equation

$$(1+\beta\eta^2)(\sigma-1)b = \eta\Psi ,$$

where $\Psi \equiv \sigma+\nu-b\beta-b^2\beta+b^2\beta\sigma-b\beta\nu > 0$. With the reformulated markup, (40), the new Keynesian Phillips curve can be expressed in terms of the welfare-relevant output gap:

$$\begin{aligned}\pi_t-\eta\pi_{t-1} &= \beta(E_t\pi_{t+1}-\eta\pi_t)+\gamma(\sigma+\nu)[x_{et}-\eta x_{et-1}]\\ &\quad +\gamma b(1-\sigma)(x_{et-1}-\eta x_{et-2})+\Gamma(\mathbf{Y}_{et}-\mathbf{Y}_{nt}), \end{aligned} \quad (41)$$

where $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} > 0$, $\Gamma(\mathbf{Y}_{et}-\mathbf{Y}_{nt}) = (1-\eta L)\gamma\bar{\Gamma}(\mathbf{Y}_{et}-\mathbf{Y}_{nt})$, and L is a lag operator.

Note that $\Gamma(\mathbf{Y}_{et}-\mathbf{Y}_{nt})$ in (41) that plays a similar role of the cost-push shock in the typical new Keynesian Phillips curve generates an endogenous trade-off between welfare-relevant output gap and inflation in the ratio external habit model.[7] Moreover, the new Keynesian Phillips curve has an additional price indexation term, $\eta\pi_{t-1}$ as the Phillips curve with price indexation model when it is expressed in terms of welfare-relevant output gap. The size of price indexation, however, is not determined by firms as in Christiano et al. (2005), but by the degree of externality in consumption η . Hence, the current inflation rate depends not only on the expected inflation rate, but also on the previous inflation rate even if firms that do not reoptimize their prices set prices without indexation to previous inflation rate. Also, the inflation rate depends on the current and previous period welfare-relevant output gap.

[7] One can show that the same form of endogenous trade-offs between the welfare-relevant output gap and inflation is derived under the external difference habit assumption.

Equation (41) shows why it is impossible to simultaneously stabilize both inflation and the welfare-relevant output gap when distortions associated with external habit is not completely eliminated: The complete stabilization of inflation cannot be optimal because the monetary policy rule stabilizes output at the inefficient natural level of output, not at the efficient natural level of output. If the monetary authority implements an inflation targeting rule, the welfare-relevant output gap x_{et} cannot be zero as in the internal habit model. Only the inefficient output gap x_{nt} is zero with the implementation of an inflation targeting rule. The monetary policy geared toward to zero inflation rate does not make the welfare-relevant output gap zero. Hence, it is impossible to stabilize both inflation rate and output gap around the efficient level without resort to optimal fiscal policy to completely eliminate the externality in consumption.

5.3. Alternative Monetary Policy Rules and Welfare

To intuitively address the effects of external habit on welfare under alternative monetary policy rules with a time-invariant labor income tax, I will employ traditional linear-quadratic frameworks that have been extensively used in the sticky price literature as in Woodford (2003). First, note that a quadratic approximation of the welfare function can be expressed as

$$\mathbf{W} = \frac{\Psi}{2(1+\beta\eta^2)} \sum_{t=0}^{\infty} \beta^t E_0 \left[(x_{et} - \eta x_{et-1})^2 - \eta^2 x_{e,-1}^2 + \frac{\varepsilon}{\kappa} \pi_t^2 \right] + t.i.p. + O(\|\xi\|^3), \quad (42)$$

where $\kappa \equiv \frac{\lambda\Psi}{(1+\beta\eta^2)(1-b\beta)}$ and $t.i.p.$ denotes the terms independent of monetary policy. When there is no external habit in consumption, for example when $b = 0$, the terms with x_{et-1} in the loss function drop out. Taking unconditional expectation, welfare loss can be rewritten as a fraction of steady state consumption:

$$\hat{\mathbf{W}} = -\frac{\Psi}{2(1+\beta\eta^2)} \left[V(x_{et} - \eta x_{et-1}) + \frac{\varepsilon}{\kappa} V(\pi_t) \right], \quad (43)$$

where the measure of variability for any variable z is defined by

$$V(z) \equiv (1-\beta) E\left[\sum_{t=0}^{\infty} \beta^t E_0 z_t^2 \right].$$

5.3.1. Commitment

Suppose that the monetary authority implements an optimal monetary policy taking into account the efficient natural level of output. The monetary authority precommits to a path for current and future inflation and the output gap to maximize the welfare subject to the expectational IS curve and the new Keynesian Phillips curve. Let λ_{pt} denote the Lagrangian multiplier associated with the Keynesian Phillips curve. Then the monetary authority chooses π_t and x_{et} to minimize

$$\begin{aligned}\mathcal{L} = -\frac{\Psi}{2(1+\beta\eta^2)} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \{[(x_{et} - \eta x_{et-1})^2 - \eta^2 x_{e-1}^2] + \frac{\varepsilon}{\kappa}\pi_t^2 \\ & + 2\lambda_{pt}\{(1+\beta\eta)\pi_t - \beta E_t \pi_{t+1} - \eta\pi_{t-1} - \gamma(\sigma+\nu)(x_{et} - \eta x_{et-1}) \\ & - \gamma b(1-\sigma)(x_{et-1} - \eta x_{et-2}) - \Gamma(\mathbf{Y}_{et} - \mathbf{Y}_{et})\}\,,\end{aligned}$$

The first order conditions are given by

$$\frac{\varepsilon(1+\beta\eta^2)(1-b\beta)}{\kappa}\pi_t + \lambda_{pt} - \beta\eta E_t \lambda_{pt+1} = 0 \quad \text{for} \quad t = 0\,, \tag{44}$$

$$\frac{\varepsilon(1+\beta\eta^2)(1-b\beta)}{\kappa}\pi_t + (1+\beta\eta)\lambda_{pt} - \lambda_{pt-1} - \beta\eta E_t \lambda_{pt+1} = 0\,, \text{for} \quad t \geq 1 \tag{45}$$

$$(x_{et} - \eta x_{et-1}) - \gamma(\nu+\sigma)\lambda_{pt} - \beta\gamma b(1-\sigma)E_t \lambda_{pt+1} = 0\,. \tag{46}$$

Equations (44) and (45) show that the optimal monetary policy is not time consistent. Following Woodford (2003)'s timeless perspective, the Ramsey optimal monetary policy can be characterized by the following condition:

$$\beta\eta(E_t x_{et+1} - (1+\eta)x_{et} + \eta x_{et-1}) - (x_{et} - (1+\eta)x_{et-1} + \eta x_{et-2}) = \Theta(\pi_t - \tilde{\beta} E_t \pi_{t+1})\,, \tag{47}$$

where $\tilde{\beta} \equiv \frac{b\beta(\sigma-1)}{\nu+\sigma}$ and $\Theta \equiv \varepsilon(\nu+\sigma)(1+\beta\eta^2)(1-b\beta)/\Psi > 0$. Equation (47) shows that the future expected output gap and the future expected inflation rate as well as the past output gap do matter for the monetary authority's judgement about which inflation and output are acceptable in the external habit. Moreover, the degree of external habit persistence in consumption should be taken into account in the design of the optimal criteria. This is in contrast with Woodford (2003)'s robustly optimal target criteria or the flexible inflation targeting rule in the internal habit persistence given by

$$\pi_t + \varepsilon(x_{et} - x_{et-1}) = 0\,. \tag{48}$$

In (48), the degree of internal habit persistence and the future expected output gap and inflation do not matter for the monetary authority's judgement about which levels of inflation and output are acceptable. The difference between the optimal target criteria in (47) and the so-called robustly optimal target criteria (48) is manifest if one rewrite (47) as following

$$(1-\eta L)(x_{et}-x_{et-1}) = \Theta\sum_{j=0}^{\infty}(\beta\eta)^{j}E_{t}(\pi_{t+j}-\tilde{\beta}\pi_{t+j+1})\,. \tag{49}$$

Equation (49) shows that the so-called 'divine coincidence' does not holds in the model with external habit if the distortions associated with external habit in consumption is not completely eliminated by optimal fiscal policy. The expected entire future path of the inflation rate needs to be taken into account in the optimal target criteria when the agents have external habit rather than internal habit in consumption.

[Table 5] Dynamic Properties of Commitment with Time-invariant Tax in Ratio External Habit Model ($\sigma = 2, \nu = 1, \varepsilon = 6$)

Variable	Mean	Std. Dev.	Auto. Corr	$Corr(x,y)$
$\mathcal{W}=$ -143.3010	$(b=0.6)$			
π	0.0010	0.0355	-0.0885	0.0655
R	3.7731	0.8529	0.7963	-0.9820
y	0.6868	0.0207	0.8794	1
$\mathcal{W}=$ -130.5214	$(b=0.8)$			
π	0.0056	0.0905	0.0287	0.0276
R	3.8593	0.6657	0.9067	-0.9968
y	0.4898	0.0154	0.9125	1
$\mathcal{W}=$ -102.9913	$(b=1)$			
π	0.2385	0.5855	0.4667	0.2182
R	4.1749	0.2404	0.9087	-0.1449
y	0.0986	0.0024	0.9696	1

Note: τ, π, and R are expressed in annual percentage points and y, h, and c in levels. The parameter values are $\beta = (1.04)^{-1/4}$, $\sigma = 2$, $\nu = 1$, $T = 200$, and $J = 1000$.

Table 5 and 6 report some sampling moments of the key macroeconomic variables under the Ramsey monetary policy rule with a Ramsey steady-state tax on labor income to eliminate the first-order distortions associated with a ratio external habit and a difference external habit. Intuitively, the Ramsey monetary policy rule or commitment, (47), in the sticky price model with external habit shows that the expected entire future path of the inflation rate needs to be taken into account in the optimal target criteria when the agents have external habit rather than internal habit in consumption. The figures in the Table 5 show that the Ramsey planner prescribes a procyclical inflation rate to smooth the fluctuation of the economy with external habit, while it permits a countercyclical interest rate. That is, the so-called 'divine coincidence' does not hold.

[Table 6] Dynamic Properties of Commitment with Time-invariant Tax in Difference External Habit Model ($\sigma = 2, \nu = 1, \varepsilon = 6$)

Variable	Mean	Std. Dev.	Auto. Corr	$Corr(x, y)$
$\mathcal{W} = -244.0651$	$(b = 0.5)$			
π	0.0000	0.0042	0.0035	0.0127
R	3.9656	0.1966	0.7309	-0.8286
y	1.2644	0.0157	0.9676	1
$\mathcal{W} = -283.2186$	$(b = 0.6)$			
π	0.0001	0.0063	0.0610	0.0482
R	3.9700	0.2056	0.7891	-0.8083
y	1.3641	0.0171	0.9758	1
$\mathcal{W} = -343.1108$	$(b = 0.7)$			
π	0.0001	0.0092	0.1333	0.0850
R	3.9797	0.2092	0.8554	-0.7998
y	1.5051	0.0184	0.9820	1
$\mathcal{W} = -449.6617$	$(b = 0.8)$			
π	0.0003	0.0136	0.2029	0.1242
R	3.9786	0.2062	0.9213	-0.7918
y	1.7320	0.0199	0.9875	1

Note: τ, π, and R are expressed in annual percentage points and y, h, and c in levels. The parameter values are $\beta = (1.04)^{-1/4}$, $\sigma = 2$, $\nu = 1$, $T = 200$, and $J = 1000$.

Inspection of Table 5 and 6 shows that labor hours and output decrease with the degree of habit in the ratio external habit, while they increase with the degree of habit in difference habit as in (37). This opposite behavior is reflected in the way how the habit is introduced into the model. In the ratio external habit model, $E[\frac{C_t}{C_{t-1}^b}] \approx 1$, making the disutility of labor hours dominate the utility of consumption. Hence, the welfare increases with the degree of habit in the ratio habit model contrary to the difference external habit model. Moreover, the interest rate moves less countercyclically in the ratio external habit model than in the difference external habit model since the marginal utility of consumption moves less in the ratio external model than in the difference external habit model. As the monetary policy to deal with distortions associated with external habit is less effective in the ratio external habit model than in the difference habit model, the inflation rate in the ratio external habit model is higher than the inflation rate in the difference habit model.

5.3.2. Discretion

Suppose that the monetary authority optimizes sequentially, that is, it makes an optimal decision every period without committing itself to any future action. Because the current decisions of the monetary authority do not bind it in any future periods and cannot affect the private sector's expectations about future inflation, the monetary authority faces a single period problem.

The first order condition for the discretion that speci.es the acceptable output gap and inflation is given by

$$\Theta\pi_t + [x_{et} - \eta x_{et-1}] = 0 . \tag{50}$$

Discretion, (50) takes a similar form as the flexible inflation targeting rule (48) which is the optimal target criteria in the internal habit persistence. The monetary authorities take into account a trade-off between inflation stabilization and output gap stabilization due to external habit persistence in consumption when they implement the discretionary policy. However, the discretionary prescription in the external habit (50) is different from the robustly optimal target criteria condition in the internal habit (48) in that the degree of external habit persistence matters in the determination of acceptable output and inflation paths via Θ.

Intuitively, the discretionary monetary policy that maximizes the period by period welfare function can be better than the inflation targeting because the former taking into account the output deviation from the efficient output level as well as the inflation fluctuation can partially affect the future expected inflation rate in the economy with NKPC given by (41). The relative impact of external habit in consumption on welfare-relevant output gap and inflation in the welfare function will determine the welfare ranking of alternative monetary policy rules.

VI. Numerical Evaluation

The conditional expected discounted utility (1) of the representative household is used as the welfare metric in the numerical analysis. For numerical evaluation of alternative monetary policy rules, I compute a second-order approximation of the equilibrium conditions around the long-run deterministic steady-state implied by each policy regime, assuming that the economy is subject to a stationary distribution of a productivity shock only.[8]

6.1. Simple, Optimal, and Implementable Monetary Policy Rules

In addition to the simple rules such as commitment, discretion, and a strict inflation targeting rule, I also consider the so-called Taylor rule to compare the

[8] For numerical evaluation, I employ the Matlab code compiled by Schmitt-Grohé and Uribe (2006). I compute first, second moments, and the implied discounted utility for artificial time series of length $T = 200$, by iterating the computation $J = 1000$ times and averaging across experiments as in Schmitt-Grohé and Uribe (2006).

welfare rankings of alternative monetary policy rules as in Schmitt-Grohé and Uribe (2007). Assume that the fiscal authority implements a Ramsey steady-state labor income tax to deal with external habit:

$$\ln(R_t / R_e) = a_r \ln(R_{t-1} / R_e) + a_\pi \ln(\pi_t) + a_y \ln(Y_t / Y_{e,t}), \tag{51}$$

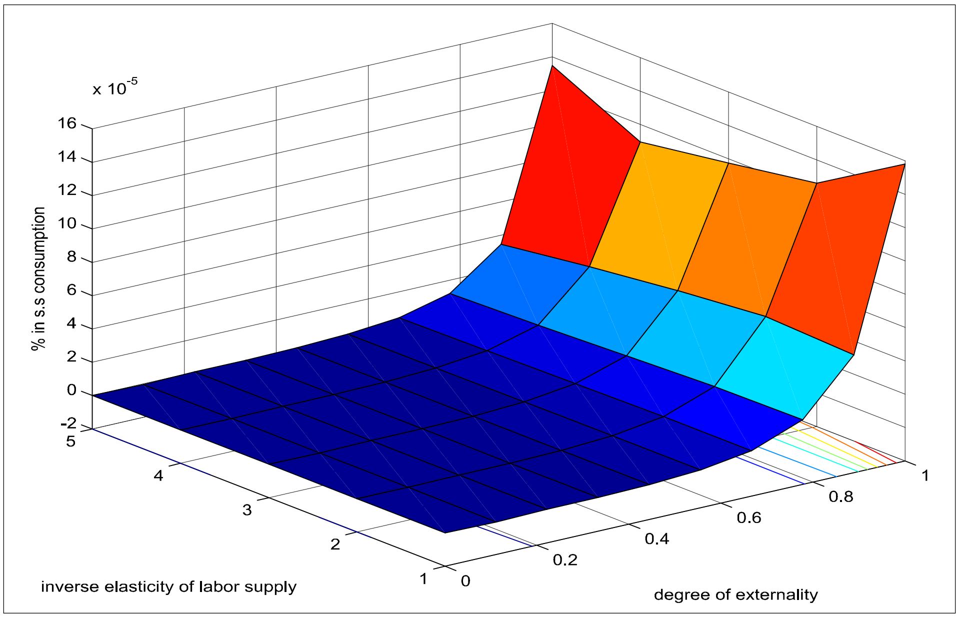
where the target variable R_e is assumed to be the Ramsey steady-state values of their associated endogenous variables, while the target variable $Y_{e,t}$ is assumed to be the time-varying, efficient output at time t.

I will characterize values of a_π, a_y, and a_r that are associated with the highest welfare of the representative households within the family of the interest rate feedback rules of the form, (51), that respond to inflation gap as well as output gap as in Schmitt-Grohé and Uribe (2007). In the optimized rules, the policy parameters a_y, and a_r are restricted to lie in the interval [0, 3], while a_π is restricted to [2, 10].

6.2. Welfare Cost of Alternative Monetary Policy Rules

In this subsection, I will compare the welfare cost or gain of an inflation targeting rule relative to alternative monetary policies such as commitment, optimal, simple, implementable Taylor rule, and a discretionary policy.

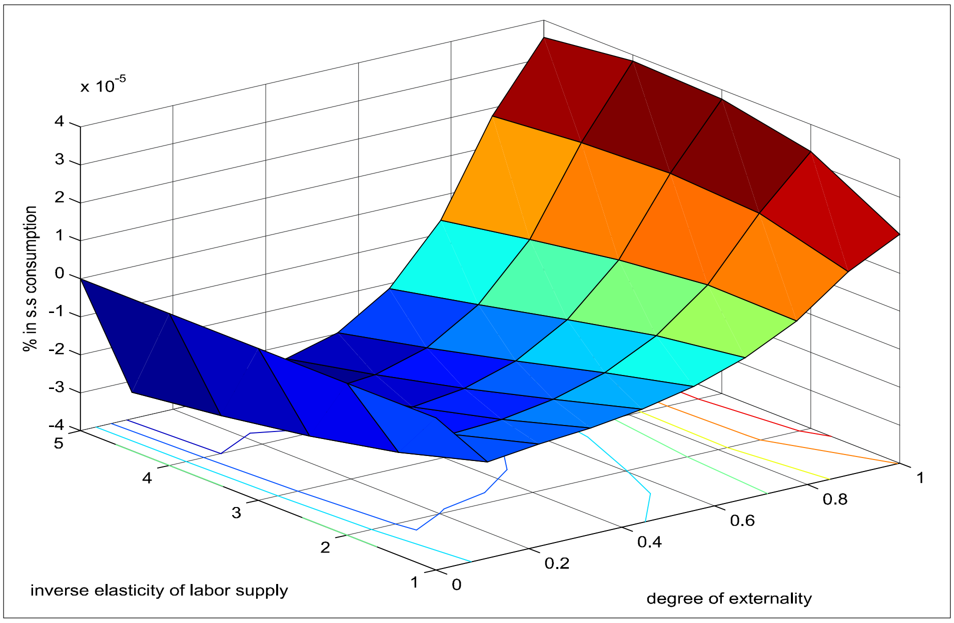
[Figure 1] Welfare Gain from Commitment relative to CPI Targeting Rule under Time-invaraint Tax Regime



First, Figure 1 shows that the gap between the welfare associated with the Ramsey monetary policy and the one associated with a strict inflation targeting rule increases as the degree of external habit in consumption increases with the distortions associated with external habit on the economy being amplified.

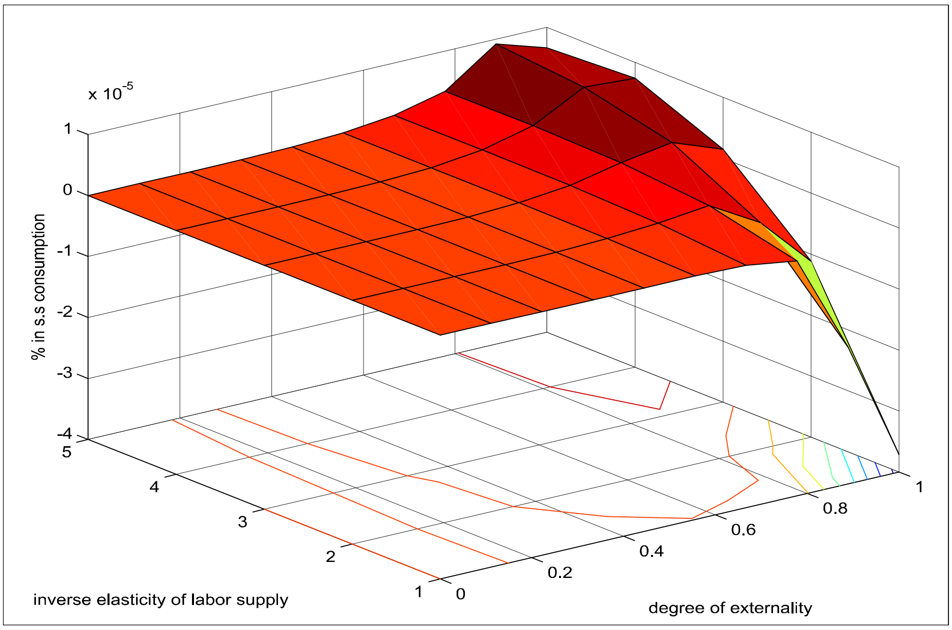
Second, Figure 2 shows that the gap between the welfare associated with an optimized, simple, Taylor rule and the one associated with a strict inflation targeting rule. Because the negative effect of external habit that is not eliminated by the time-invariant labor income tax increases with the degree of external habit, the difference between the welfare associated with the optimized interest rate rule and the welfare with a strict inflation targeting turns into positive value from the negative one with the degree of external habit.

[Figure 2] Welfare Loss from Taylor Rule relative to CPI Targeting Rule under Time-invariant Tax Regime



Next, Figure 3 presents the conditional expected welfare gains from a simple discretion relative to a strict inflation targeting rule in the Calvo-type staggered price model with externality in consumption as the elasticity of labor supply (ν^{-1}) and the degree of habit vary, assuming that the fiscal authority implements time-invariant tax on labor income to eliminate the distortions associated with monopolistic competition in goods market and external habit in consumption. They show that a strict inflation targeting generates a higher welfare than a discretionary policy when the degree of externality in consumption is relatively small. It does not matter much to disregard the trade-off between output gap and inflation when the externality in consumption is small. However, the welfare ranking reverses when the degree of externality increases, thereby making the trade-off between inflation and output gap significant in welfare dimension. The welfare difference from a monetary discretion relative to the inflation targeting turns into positive as the degree of externality in consumption (b) increases over 0.4. The maximum welfare gain from the simple discretion amounts to about 10^{-5} percent of the steady-state consumption when the Frisch labor supply elasticity equals 1/3. This reversal of welfare reflects the fact that a complete stabilization of inflation does not necessarily lead to a stabilization of output to the efficient level when there is a substantial externality in consumption due to catching up with the Joneses.

[Figure 3] Welfare Gain from Discretionary Policy relative to CPI Targeting Rule under Time-Invariant Tax Regime



Intuitively, even though the existing discrepancy between the private MRS and the social MRS is not eliminated at all by a fiscal policy, the discretionary policy works to reduce the wedge. Since the discretion gives penalties to the output deviation from the efficient natural level of output as well as the inflation, it can be marginally better than the conditional inflation targeting in terms of welfare. When the monetary authority chooses an inflation targeting rule conditional on the inefficient natural output without taking into account the adverse effect of external habit on the economy, it makes output to be stuck to the inefficient level of output. This monetary policy is not desirable because the inflation targeting rule, not taking into account the household's desire to catch up with the Joneses, makes the economy more vulnerable to the shock.

VII. Conclusion

This paper incorporates the idea of catching up with the Joneses into a sticky price model that helps to improve the performance of small-scale macroeconomic models and explores the welfare ranking of alternative monetary policy rules. The paper shows that the external habit in consumption entails a divergence between the private MRS between consumption and labor and the social MRS between consumption and labor, thereby generating a trade-off between the output gap and inflation, making a inflation targeting rule suboptimal under a fiscal policy regime with a steady state tax rate on labor income to eliminate the distortions associated with external habit in consumption and imperfect competition in goods market.

The paper also shows that the monetary discretion, partially taking into account the trade-off between output stabilization and price stabilization, can be better than a strict inflation targeting rule that ignores the trade-off as long as there are substantial degree of external habit in consumption. The difference between the welfare associated to the discretionary monetary policy and the welfare associated to a inflation targeting rule increases as the Frisch labor supply elasticity increases.

1. Proof of Proposition

Proof of Proposition 6.1.

The Ramsey problem can be formulated in terms of Lagrangian function as follows:

$$\begin{aligned}
\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^{t+i} \Bigg\{ & \left(\frac{(C_{t+i} / C_{t+i-1}^{b})^{1-\sigma}}{1-\sigma} - \frac{N_{t+i}^{1+v}}{1+v} \right) \\
& + \lambda_{1,t+i} \left[\frac{A_{t+i} N_{t+i}}{\Delta_{t+i}} - C_{t+i} \right] \\
& + \lambda_{2,t+i} [\Delta_{t+i} - (1-\alpha) \tilde{p}_{t+i}^{-\varepsilon} - \alpha (1+\pi_{t+i})^{\varepsilon} \Delta_{t+i-1}] \\
& + \lambda_{3,t+i} [1 - (1-\alpha) \tilde{p}_{t+i}^{1-\varepsilon} - \alpha (1+\pi_{t+i})^{\varepsilon-1}] \\
& + \lambda_{4,t+i} [A_{t+i} (1-\tau_{Nt+i}) mc_{t+i} - N_{t+i}^{v} (C_{t+i} - bC_{t+i-1})^{\sigma}] \\
& + \lambda_{5,t+i} \left[S_{1,t+i} - \tilde{p}_{t+i}^{-1-\varepsilon} C_{t+i} mc_{t+i} - \alpha\beta \left[\frac{\Lambda_{t+i+1}}{\Lambda_{t+i}} (1+\pi_{t+i+1})^{\varepsilon} \left(\frac{\tilde{p}_{t+i}}{\tilde{p}_{t+i+1}} \right)^{-1-\varepsilon} S_{1,t+i+1} \right] \right. \\
& + \lambda_{6,t+i} \left[S_{2,t+i} - \tilde{p}_{t+i}^{-\varepsilon} C_{t+i} - \alpha\beta \frac{\Lambda_{t+i+1}}{\Lambda_{t+i}} (1+\pi_{t+i+1})^{\varepsilon-1} \left(\frac{\tilde{p}_{t+i}}{\tilde{p}_{t+i+1}} \right)^{-\varepsilon} S_{2,t+i+1} \right] \\
& + \lambda_{7,t+i} \left[S_{2,t+i} - \frac{\varepsilon}{\varepsilon-1} S_{1,t+i} \right] \\
& + \lambda_{8,t+i} \left[1 - \frac{\beta R_{t+i}}{1+\pi_{t+i+1}} \left(\frac{C_{t+i+1} / C_{t+i}^{b}}{C_{t+i} / C_{t+i-1}^{b}} \right)^{-\sigma} \right] \Bigg\}
\end{aligned}$$

First, note that the first order conditions with respect to R_t and τ_{Nt} imply that

$$\lambda_{4,t} = \lambda_{8,t} = 0 . \tag{A1}$$

Next, the first order condition with respect to mc_t and (A1) imply that

$$\lambda_{5,t} = 0 . \tag{A2}$$

Finally, the first order conditions with respect to $S_{1,t}$, $S_{2,t}$ and (A2) imply that

$$\lambda_{6,t} = \lambda_{7,t} = 0 .$$

hence, the Ramsey problem can be simplified as

$$
\begin{aligned}
\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^{t+i} \Bigg\{ & \left(\frac{(C_{t+i} / C_{t+i-1}^b)^{1-\sigma}}{1-\sigma} - \frac{N_{t+i}^{1-\nu}}{1+\nu} \right) \\
& + \lambda_{1,t+i} \left[\frac{A_{t+i} N_{t+i}}{\Delta_{t+i}} - C_{t+i} \right] \\
& + \lambda_{2,t+i} [\Delta_{t+i} - (1-\alpha) \tilde{p}_{t+i}^{-\varepsilon} - \alpha (1+\pi_{t+i})^{\varepsilon} \Delta_{t+i-1}] \\
& + \lambda_{3,t+i} [1 - (1-\alpha) \tilde{p}_{t+i}^{1-\varepsilon} - \alpha (1+\pi_{t+i})^{\varepsilon-1}].
\end{aligned}
$$

The first order conditions with respect to consumption and labor hours can be written as

$$
(C_t / C_{t-1}^b)^{-\sigma} C_{t-1}^b [1 - b\beta E_t [(C_{t+1} / C_t^b)^{-\sigma} C_{t+1} C_t^{-b}] = N_t^{\nu} \frac{\Delta_t}{A_t}. \tag{A3}
$$

From the equation the market equilibrium condition (13) and (A3), the optimal labor income tax rate is given by

$$
\begin{aligned}
& N_t^{\nu} \left(\frac{C_t}{C_{t-1}^b} \right)^{\sigma} C_{t-1}^b = A_t mc_t (1 - \tau_{Nt}) \\
& \tau_{Nt} = 1 - \frac{1}{mc_t \Delta_t} [1 - bE_t \left[Q_{t,t+1} \frac{C_{t+1}}{C_t^b} \right].
\end{aligned}
$$

QED. ■

2. External Habit and the New Keynesian Phillips Curve

To see the trade-off between output gap and inflation, one can express the percentage deviation of markup from the efficient steady state $\hat{\mu}_t$ as

$$
\hat{\mu}_t = -(\sigma - \nu) x_{nt} - b(1-\sigma) x_{nt-1},
$$

where $x_{nt} \equiv y_t - y_{nt}, y_{nt} \equiv \log(\frac{Y_{nt}}{\bar{Y}})$, and Y_{nt} is the inefficient natural level of output in the presence of externality in consumption in period t. Rewrite the markup deviation in terms of the efficient level of natural output as

$$
\begin{aligned}
\hat{\mu}_t = & -(\nu + \sigma) [x_{et} - \eta x_{et-1}] - (1-\sigma) b (x_{et-1} - \eta x_{et-2}) \\
& + (\nu + \sigma) [(x_{et} - x_{nt}) - \eta x_{et-1}]
\end{aligned}
$$

$$
\begin{aligned}
&+(1-\sigma)b[(x_{et-1}-x_{nt-1})-\eta x_{et-2}] \\
&=\ldots \\
&=-(\nu+\sigma)\sum_{i=0}^{\infty}\eta^{i}[x_{et}-\eta x_{et-1}] \\
&\quad -(1-\sigma)b\sum_{i=0}^{\infty}\eta^{i}(x_{et-2}-\eta x_{et-3})+\Gamma(\mathbf{Y}_{et}-\mathbf{Y}_{nt})\,.
\end{aligned}
$$

That is,

$$
\begin{aligned}
\hat{\mu}_t &= -(\nu+\sigma)(1-\eta L)^{-1}[x_{et}-\eta x_{et-1}] \\
&\quad +(\sigma-1)b(1-\eta L)^{-1}[x_{et-1}-\eta x_{et-2}]+\overline{\Gamma}(\mathbf{Y}_{et}-\mathbf{Y}_{nt})\,. \qquad \text{(A4)}
\end{aligned}
$$

Note that terms of $\{x_{et-i}-x_{nt-i}\}_{i=0}^{\infty}$ are independent of monetary policy because

$$
\begin{aligned}
&x_{et}-x_{nt} \\
&=\ln(y_t/y_{et})-\ln(y_t/y_{nt}) \\
&=\ln(y_{nt}/y_{et})\,.
\end{aligned}
$$

Finally, plugging (A4) into NKPC

$$\pi_t=\beta E_t\pi_{t+1}-\gamma\hat{\mu}_t$$

leads to equation (41):

$$
\begin{aligned}
\pi_t(1-\eta L) &= \beta(1-\eta L)E_t\pi_{t+1}+\gamma(\nu+\sigma)[x_{et}-\eta x_{et-1}] \\
&\quad +\gamma(1-\sigma)b[x_{et-1}-\eta x_{et-2}]+\Gamma(\mathbf{Y}_{et}-\mathbf{Y}_{nt})
\end{aligned}
$$

3. LQ Approximation of the Welfare Function

The utility flow of the representative household is given by

$$U(C_t,X_t)-V(N_t)=\frac{(C_t/X_t)^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\nu}}{1+\nu}\,. \qquad \text{(A5)}$$

A second-order approximation of the utility function with respect to consumption around the efficient flexible-price equilibrium yields:

$$U(C_t,X_t)=U(\overline{C}_t,\overline{X}_t)+U_C\overline{C}\left(c_t+\frac{1}{2}c_t^2\right)+U_X\overline{X}\left(x_t+\frac{1}{2}x_t^2\right)$$

$$+\frac{1}{2}U_{CC}\left[\bar{C}(c_t+\frac{1}{2}c_t^2)\right]^2+U_{CX}\left[\bar{C}(c_t+\frac{1}{2}c_t^2)\right]\left[\bar{X}(x_t+\frac{1}{2}x_t^2\right]$$
$$+\frac{1}{2}U_{XX}\left[\bar{X}\left(x_t+\frac{1}{2}x_t^2\right)\right]^2+O(\|\xi\|^3),$$

where $c_t \equiv \ln(C_t/\bar{C})$, $x_t \equiv \ln(X_t/\bar{C})$, and $\bar{C}$ is the consumption level in the efficient steady state. Since $U_C\bar{C}=\bar{C}^{1-\sigma}\bar{C}^{b(\sigma-1)}=\bar{C}^{(b-1)(\sigma-1)}\equiv\tilde{C}$, $U_xX=-b\tilde{C}$, $U_{CX}\overline{CX}=-(1-\sigma)b\tilde{C}$, $U_{CC}\bar{C}^2=-\sigma\tilde{C}$, $U_{XX}\bar{X}^2=-b(b(\sigma-1)-1)\tilde{C}$, and $\frac{U_{CC}C}{U_C}=-\sigma$, $\frac{U_{XX}X}{U_X}=-b(b(\sigma-1)-1)$, the approximation of the temporal utility function can be rewritten as

$$U(C_t,X_t)=U(\bar{C},\bar{X})+\tilde{C}\left((c_t-bx_t)+\frac{1-\sigma}{2}(c_t^2+b^2x_t^2-2bc_tx_t\right)+O(\|\xi\|^3).$$
$$\sum_{t=0}^{\infty}\beta^tE_t[U(C_t,X_t)]=\frac{U(\bar{C},\bar{X})}{1-\beta}+\tilde{C}\sum_{t=0}^{\infty}\beta^tE_t\left((1-b\beta)c_t+\frac{1-\sigma}{2}(c_t^2+b^2x_t^2-2bc_tx_t\right)+O(\|\xi\|^3). \quad \text{(A6)}$$

Next, consider the temporal utility function for labor supply.

$$V(N_t)=V(\bar{N})+V_N\bar{N}\left[n_t+\frac{1}{2}(1-\nu)n_t^2\right]+O(\|\xi\|^3)$$
$$=V(\bar{N})+(1-b\beta)\tilde{C}\left[(y_t-a_t+\delta_t)+\frac{1}{2}(1-\nu)(y_t-a_t)^2\right]+O(\|\xi\|^3), \quad \text{(A7)}$$

where $\delta_t \equiv \int_0^1(\frac{P_t(i)}{P_t})^{-\varepsilon}di$.

Hence,

$$\tilde{\mathbf{W}}=\frac{U(\bar{C},\bar{X})-V(N)}{1-\beta}$$
$$+U_C\bar{C}\sum_{t=0}^{\infty}\beta^tE_t\left\{(1-b\beta)y_t+\frac{1-\sigma}{2}(y_t^2+b^2y_{t-1}^2-2by_ty_{t-1})\right.$$
$$\left.-(1-b\beta)U_C\bar{C}\left[(y_t-a_t+\delta_t)+\frac{1}{2}(1-\nu)(y_t-a_t)^2\right]\right\}+O(\|\xi\|^3)$$
$$=\frac{U(\bar{C},\bar{X})-V(N)}{1-\beta}+U_C\bar{C}\sum_{t=0}^{\infty}\beta^tE_t\left\{\frac{(1-\sigma)}{2}((1+b\beta^2)y_t^2-2by_ty_{t-1})\right.$$
$$\left.-\frac{(1-b\beta)(1+\nu)}{2}(y_t^2-2y_ta_t+a_t^2)-(1-b\beta)\delta_t)\right\}+O(\|\xi\|^3)$$

Therefore,

$$\mathbf{W} = \sum_{t=0}^{\infty} \beta^t E_t \left\{ \frac{-(\sigma + \nu - b\beta - b\beta^2 + b\beta^2\sigma - b\beta\nu)}{2} y_t^2 \right.$$
$$\left. -(1-\sigma) b y_t y_{t-1} + \frac{(1-b\beta)(1+\nu)}{2}(2y_t a_t - a_t^2) - (1-b\beta)\delta_t) \right\} + O(\|\xi\|^3), \quad \text{(A8)}$$

where $\mathbf{W} \equiv \frac{\tilde{\mathbf{W}} - \bar{W}}{U_C \bar{C}} \bar{\mathbf{W}} = \frac{U(\bar{C}, \bar{X}) - V(N)}{1-\beta}$. Let $\Psi \equiv \sigma + \nu b\beta - b\beta^2 + b\beta^2\sigma - b\beta\nu$.

Next, notice that the efficient natural level of output from the efficient steady state output is given by

$$E_t[b\beta(1-\sigma)y_{et+1} + \Psi y_{et} + b(1-\sigma)y_{et-1}] = (1+\nu)(1-b\beta)a_t. \quad \text{(A9)}$$

Finally, plugging (A9) into (A8), the welfare loss can be expressed as follows:

$$\begin{aligned}
\mathbf{W} &= \frac{1}{2}\sum_{t=0}^{\infty} \beta^t E_t \{-\Psi y_t^2 - 2(1-\sigma) b y_t y_{t-1} + 2(1-b\beta)(1+\nu)(y_t a_t - a_t^2) \\
&\quad -2(1-b\beta)\delta_t\} + O(\|\xi\|^3) \\
&= -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t E_t \left[\Psi x_{et}^2 + 2(1-\sigma) b x_{et} x_{et-1} + (1-b\beta)\frac{\varepsilon}{\gamma}\pi_t^2 \right] + t.i.p. + O(\|\xi\|^3) \\
&= -\frac{\Psi}{2}\sum_{t=0}^{\infty} \beta^t E_t \left[x_{et}^2 + \frac{2(1-\sigma)b}{\Psi} x_{et} x_{et-1} + \frac{(1-b\beta)}{\Psi}\frac{\varepsilon}{\gamma}\pi_t^2 \right] + t.i.p. + O(\|\xi\|^3).
\end{aligned}$$

The welfare loss function can be further simplified in terms of output gap and inflation with the new Keynesian Phillips curve as following

$$\hat{\mathbf{W}} = -\Omega \sum_{t=0}^{\infty} \beta^t E_t \left[(x_{et} - \eta x_{et-1})^2 - \eta^2 x_{e,-1}^2 + \frac{\varepsilon}{\kappa}\pi_t^2 \right] + t.i.p. + O(\|\xi\|^3).$$

where $\Omega \equiv \frac{\Psi}{2(1+\beta\eta^2)}$, $\kappa \equiv \frac{\Psi\gamma}{(1+\beta\eta^2)(1-b\beta)}$ and η is a solution of

$$(1+\beta\eta^2)(\sigma-1)b = \eta\Psi.$$

References

Abel, Andrew. B. (1990), "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic review Papers and Proceedings*, 80 (2), 38-42.

_______________ (1999), "Risk Premia and Term Premia in General Equilibrium," *Journal of Monetary Economics*, 43 (1), 3-33.

Amato, Jeffrey D., and Thomas Laubach (2004), "Implications of Habit Formation for Optimal Monetary Policy," *Journal of Monetary Economics*, 51(2), 305-325.

Boldrin, Michel, Lawrence J. Christiano, and Joan Fisher (2001), "Habit Persistence, Asset Returns and the Business Cycle," *American Economic Review*, 91 (1), 149-166.

Calvo, Guillermo A. (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12 (3), 383-398.

Christiano, Lawrence J., Martin S. Eichenbaum, and Charles L. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 115 (1), 1-45.

Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin (2010), "DSGE Models for Monetary Policy," in Friedman, Benjamin, M. and Michael Woodford eds. *Handbook on Monetary Economics Vol. III*, Chapter 7, 285-367, Amsterdam; New York and Oxford, Elsevier Science, North-Holland,.

Chugh, Sanjay, K. (2007), "Optimal Inflation Persistence: Ramsey Taxation with Capital and Habits," *Journal of Monetary Economics*, 54 (6), 1809-1836.

Del Negro, Macro, Frank Schorfheide, Frank Smets, and Raf Wouters (2007), "On the Fit of New Keynesian Models," *Journal of Business and Economic Statistics*, 25(2), 123-143.

Dennis, Richard (2009), "Consumption Habits in a New Keynesian Business Cycle Model," *Journal of Money, Credit and Banking*, 41(5), 1015-1030.

Dixit, Avinash K., and Joseph E. Stiglitz (1977), "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67 (3), 297-308.

Dusenberry, James (1949), *Income, Saving, and the Theory of Consumer Behavior*, Cambridge, MA: Harvard University Press.

Fuhrer, Jeffrey C. (2000), "Habit Formation in Consumption and its Implications for Monetary-Policy Models," *American Economic Review*, 90 (3), 367-390.

Galí, Jordi (2007), *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press, Princeton, New Jersey.

Gruber, Joseph (2004), "A Present Value Test of Habits and the Current Account," *Journal of Monetary Economics*, 51(7), 1495-1507.

Hamilton, James D. (2004), *Time Series Analysis*, Princeton University Press, Princeton, New Jersey.

Ireland, Peter (2001), "Sticky-price models of the business cycle: Specification and stability," *Journal of Monetary Economics*, 47(1), 3-18.

Jung, Yongseung (2015), "Price Stability in Economies with Habit Persistence," *Journal of Money, Credit and Banking*, 47(4), 517-549.

King, Robert, G. and Mark W. Watson (1996), "Money, Prices, Interest Rates and the

Business Cycle," *Review of Economics and Statistics*, 78 (1), 33-53.

Ljungqvist, Lars and Harold, Uhlig (2000), "Tax Policy and Aggregate Demand Management under Catching Up with the Joneses," *American Economic Review*, 90 (3), 356-366.

Smets, Frank, and Raf Wouters (2007), "Sources of Business Cycle Fluctuation in the U.S.: A Bayesian DSGE Approach," *American Economic Review*, 97 (3), 586-606.

Stephanie, Schmitt-Grohé and Martin Uribe (2004), "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28 (1), 755-775.

__________(2007), "Optimal Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics*, 54 (6), 1702-1725.

__________(2008), "Habit Persistence," *The New Palgrave Dictionary of Economics*, Second Edition, edited by S. Durlauf and L. Blume.

Stock, James. H., and Mark W. Watson (1999), "Business Cycle Fluctuations in US Macroeconomic Time Series," In *Handbook of Macroeconomic*, Vol. 1c, edited by Taylor J. B. and Woodford, M., 3-64, Amsterdam, North-Holland.

Woodford, Michael (2003), *Interest and Prices*, Princeton University Press.

Yun, Tack (2005), "Optimal Monetary Policy with Relative Price Distortions," *American Economic Review March*, 95 (1), 89-108.