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MARGINAL WILLINGNESS-TO-PAY FUNCTIONS IN A DYNAMIC COMPLETE SYSTEM*

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This paper proposes a specification of marginal willingness-to-pay functions in an alternative system, which is called the synthetic inverse demand system (SIDS). It is a new inverse demand system itself and can be used as an alternative hypothesis for a specification test. This paper then extends the system dynamically by incorporating habit formation idea for the first attempt in the inverse demand literature. It will have a contribution to an analysis of a dynamic price formation in a system. Thus, it is hoped that this will enhance its applications to the resource and environmental economics.

I. INTRODUCTION

Many recent papers report that dynamic specification is needed in demand systems to fit the data better and improve forecasting ability.[1] Empirically when we use aggregate data, the factor becomes more important as Blundell et al. (1993) pointed out.[2] However, the existing literature in the subject of inverse demand systems has ignored dynamic factors. By an inverse demand system, we mean a system where variations of marginal valuation, or prices in a competitive market, are explained by variations of quantity demanded.[3] Although dynamic

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[1] See, for example, Chambers (1990). He highlights the importance of dynamics in demand systems.

[2] Blundell et al. (1993) conclude that aggregate models that explain ordinary demands in terms of price and total expenditure variables may exclude many important aggregation factors such as the proportion of total expenditure associated with particular family size, etc. However aggregate models, when aggregate data are used in place of the appropriate micro data, is not necessarily outperformed across all demand equations, once certain aggregation factors as well as trend and seasonal components are included. This would be a defense of our estimated aggregate-based model in empirical part.

[3] Note that the ordinary demand approach inquires into the dependence of the quantities of commodities consumed on the marginal valuations of those commodities.

factors are ignored, inverse demand functions [marginal willingness-to-pay functions in Hicks(1956)' term] are worked by a non-negligible set of authors recently.[4] The most recent advances in modeling willingness-to-pay functions in a complete system include Barten and Bettendorf(1989) and Eales and Unnevehr (1994).

Interests in such models stem from the existence of goods for which the assumption of predetermined prices may not be viable and current supplies may be fixed because of biological, production lags or public good characteristics. Examples of such goods include nonstorable goods and recreational and environmental amenities, the existence of unique environments and endangered species, etc. However, such models have been proposed within a static framework. They are static because a consumer is assumed to adjust instantly to a new equilibrium when expenditure or prices change in an equilibrium. In reality, this assumption is overly restrictive since it ignores some form of endogenous taste formation by past decision or future expectation. As a consequence, static models may yield suboptimal behavior of consumers. To overcome some of these deficiencies, several attempts have been made in the ordinary demand approach to incorporate dynamic structures into static demand systems. However, almost no attempt was made in the inverse demand approach. We start with a brief review of such attempts in the ordinary demand approach. Based on the traditional idea, we develop a dynamic inverse demand system, which is flexible in two senses: it is flexible in that all derivative properties in consumer theory are satisfied in the system and it has more degrees of freedom than traditional inverse demand systems.

There have been essentially three approaches to dynamic behavior, based on state variables and intertemporal optimization i.e., the state variable approach, a dynamic version of the linear expenditure system, and the intertemporal demand approach. The state variable approach was pioneered by Houthakker and Taylor(1970) and applied in various forms by Lee(1970), Mattei(1971), Phlips(1972), Taylor and Weiserbs(1972), El-Safty(1976) and Klevmarken(1981). According to the model of Houthakker and Taylor(1970), the quantity demanded of *ith* good is hypothesized to be a function of physical stock or psychological stock of habits, prices and income. Then, the dynamic demand systems are developed using the functional relation between the reference bundle and past consumption through the use of state variables. The second approach, a dynamic version of the linear expenditure system was developed by Pollak and Wales(1969), Pollak (1970), Philips(1972), and has been applied in Howe, Pollak and Wales(1979),

[4] The terms "inverse demand functions" and "marginal willingness-to-pay functions" are employed interchangeably in this paper. Note, however, that empirical inverse demand functions are marginal willingness-to-pay functions normalized by fixed income.

and Ray(1984, 1985).[5] According to Pollak(1970), the habits are examined within the linear expenditure system (LES) and it is assumed that the LES subsistence (committed consumption) parameters depend on previous period's consumption by incorporating state variables into the subsistence parameters. The third approach consists of generating theoretically plausible dynamic demand systems as the solution of a constrained intertemporal utility maximization problem. It has been developed by Lluch(1974), Phlips(1974), Klijn(1977), Stigler and Becker (1977), Spinnewyn(1981), Phlip and Spinnewyn(1982), Boyer(1983), Becker and Murphy(1988), and Becker et al.(1994). According to Klijn(1977), the allocation problem is cast into a control theory format with the consumer attempting to maximize a discounted utility function subject to wealth and stock constraints. In this framework, the consumer choice problem is to find the time path of consumption $q(t)$ $(0 < t < \infty)$ for a given price over time such that lifetime utility is maximized with wealth constraint.

The primary focus of all the models is on alternative methods to incorporate taste changes into demand systems specifications. Pollak(1970, 1978) has given four reasons why tastes can be considered endogenous: (1) habit formation where taste changes are related to past consumer decisions; (2) interdependent preferences among other consumers; (3) advertising that attempts to affect consumer tastes; (4) prices or snob appeal. As seen above, most approaches consider idea (1) and try to formulate its process, though habits or persistence in consumption patterns may be due to a number of factors in addition to changing tastes. Thus, the differences among those approaches are on how to model the habit formation process.[6] This line of work seems to be reasonable because we believe that people get addicted not only to alcohol, cocaine, and cigarettes but also to work, eating, music, and other activities.[7] Thus, much more behavior than expected may be included into habit forming goods.[8] So we believe that all dynamic demand systems in consumer theory should explain this idea at least in part since tastes are themselves a product of experience. The purpose of this paper is to

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[5] Pollak(1969, 1970) proposed the "linear expenditure system with habit formation" idea and Ray (1984, 1985) generalized that idea in AIDS system and Gorman polar form demand system.

[6] It may be desirable to restrict the concept of habit formation to change in the consumer's indifference map induced by his own past consumption (or possibly past expenditure). A consumer theory taking into account incomplete information and learning could explicitly model behavior under uncertainty, but the case with habit formation will not be the same and the consumer may be assumed to be conscious(rational) or not of its effects(myopic) in his behavior.

[7] See Becker and Murphy(1988) for examples.

[8] For another example, we might think of fish as a habit forming good in the sense that consumers, who learn their utility of consuming one species from experience, generally know about only a few species because experimenting or getting information is costly and time-consuming, and thus may not treat species they have experienced and species they have not experienced as identical even if the species are in fact almost the same in taste. Further all foods might be habit forming goods in this sense.

propose a dynamic inverse demand system based on the framework advanced originally by Anderson and Blundell(1982, 1983). Though they worked in the ordinary demand context, their approach is considerably appealing because it allows disequilibrium or irrationality of consumer behavior in the short-run while at the same time it allows for fully consistent behavior with consumer theory in the long-run. A further advantage of their approach is that short-run and long-run demand functions can be easily obtained from their dynamic demand systems. The system developed in this paper, however, differs from theirs. First, this study extends their model by deriving marginal willingness-to-pay functions instead of ordinary demand functions. Second, this study further extends it to a more general and flexible specification form.[9] A contribution of this paper, therefore, is to develop marginal willingness-to-pay functions in a dynamic system for the first attempt since there is no literature in this subject. In order for dynamic inverse demand systems to make sense, they should be comparable to the static systems in some way and show how dynamic systems nest static systems. Thus, we first propose a general inverse demand system in a static context, which encompasses most of the alternative inverse demand systems in Section Ⅱ. The reason to develop a general system is that economic theory does not give a priori which functional form of marginal willingness-to-pay functions should be chosen. In that matter, our general system has an advantage. It will be another contribution of this paper. Section Ⅲ and Ⅳ develop our model of a dynamic inverse demand system based on the general inverse demand system and applies to U.S. commercial fish demand. Section Ⅴ concludes.

Ⅱ. A DIFFERENTIAL APPROACH FOR MARGINAL WILLINGNESS-TO-PAY FUNCTIONS IN A SYSTEM

In this section, a class of differential marginal willingness-to-pay functions in systems(i.e., Inverse Almost Ideal Demand System, Inverse Rotterdam Demand System, Inverse CBS Demand System, and Inverse NBR Demand System) will be discussed and one alternative system will be newly proposed. Although the proposed system is an inverse demand system itself, it also can be used for a specification test as we shall see later.

1. Functional Specification of Alternative Systems

For good i consumed by a representative consumer at time t, the expenditure share(w_{it}) can be expressed as a function(f_i) of logarithms of quantities($\ln q_{jt}$, $j = 1,\cdots,n$) in a market sense. Then the behavioral equation may be written

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[9] By "more general and flexible", we mean having more degrees of freedom in estimation, i.e., having more sensible variables to nest existing models and fit data.

(1) $w_{it} = f_i(\ln q_{1t},\cdots,\ln q_{nt})$.

Consider $\overline{Z}=(\overline{\ln q_1}, \overline{\ln q_2},\cdots,\overline{\ln q_n})$ where the upper bar of ln(q) means $\sum_{t=1} \ln q_t / T$.[10] At $\overline{Z}$, there exists an optimal corresponding price vector normalized by income $\hat{v}=(\ln \hat{v}_1,\cdots,\ln \hat{v}_n)$ and expenditure share $\hat{w}_i = f_i(\overline{Z})(i=1, \cdots, n)$. At $\overline{Z}$, and the corresponding $\hat{v}$, we have:

$$(2)\quad \left(\frac{\partial w_i}{\partial \ln q_j}\right)_{\overline{Z}} = \hat{w}_i(b_{ij}+\delta_{ij}) = \hat{w}_i(b_{ij}^* + \hat{w}_j k_i + \delta_{ij})$$

where b_{ij} is the uncompensated price flexibility, b_{ij}^* is the compensated price flexibility, k_i is the scale elasticity, and δ_{ij} is the kronecker delta ($\delta_{ij}=1$ if $i=j$, $\delta_{ij}=0$ otherwise).[11] Considering Taylor expansion of w_{it} around $\overline{Z}$ and w_{it-1} around $\overline{Z}$ in eq. (1), and subtracting the former from the latter produce

$$(3)\quad \Delta w_{it} = \sum_j \left(\frac{\partial w_i}{\partial \ln q_j}\right)_{\overline{Z}} \Delta \ln q_{jt} + \varepsilon_{it}$$

where an error term involves terms of second and higher order differences. Substituting (2) yields

$$\begin{aligned}(4)\quad \Delta w_{it} &= \sum_j \hat{w}_i(b_{ij}^* + \hat{w}_j k_i + \delta_{ij})\Delta \ln q_{jt} + \varepsilon_{it}\\ &= \sum_j (\hat{w}_i b_{ij}^* + \hat{w}_i \delta_{ij})\Delta \ln q_{jt} + \hat{w}_i k_i \Delta \ln Q_t + \varepsilon_{it}\end{aligned}$$

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[10] Note that we evaluate elasticity at a sample mean of observations in exogenous variables. Thus we consider means of exogenous variables here.

[11] To obtain the required expression, we have used that $w_i = v_i q_i$ and $dw_i = w_i d\ln w_i$. The compensated price flexibility is defined as:

$$b_{ij}^* \equiv \frac{\partial \ln v_i(q,u)}{\partial \ln q_j}$$

and the scale elasticity is similarly defined as:

$$k_i \equiv \frac{\partial \ln v_i(q,u)}{\partial \ln Q}$$

where $\Delta \ln Q = k$ implies all quantities increased by k. The uncompensated and compensated price flexibilities are related through the Antonelli equation analogous to the Slutsky equation. See Appendix for derivation of the Antonelli equation.

where $\Delta \ln Q = \sum \hat{w}_j \Delta \ln q_j$, which is Stone's quantity index showing scale effects analogous to income effects in the ordinary demand system.

As is well known, Working(1943) suggested that a useful form of the Engel curve was given by expressing the budget share of good i as a linear function of the logarithm of total expenditure:

$$(5) \quad w_i = a_i + b_i \log(m).$$

From eq. (5), the marginal shares in the Working model (holding price index constant) may be derived by multiplying m and differentiating with respect to m,

$$(6) \quad \frac{\partial (p_i q_i)}{\partial m} = \frac{\partial w_i}{\partial \log m}\bigg|_{\partial \log P = 0} = \frac{\partial w_i}{\partial \log Q} = a_i + b_i(1 + \log m) = w_i + b_i.$$

where $\log(P)$ and $\log(Q)$ denote price index and quantity index, respectively. If we think of b_i as a constant approximation of $\hat{w}_i k_i$, Working's suggestion leads us to include $\hat{w}_i$ into the scale effect in eq. (4):

$$\frac{\partial w_i}{\partial \log Q} = \hat{w}_i k_i + \hat{w}_i.$$

Thus, adding and subtracting $\hat{w}_i \, d\log Q$ results in the form of a difference approximation of the Inverse Almost Ideal Demand System:

$$(7) \quad \Delta w_{it} = \sum_j [\hat{w}_i b^*_{ij} + \hat{w}_i(\delta_{ij} - \hat{w}_j)] \Delta \ln q_{jt} + (\hat{w}_i k_i + \hat{w}_i) \Delta \ln Q_t + \varepsilon_{it}.$$

As illustrated by Theil(1979), Deaton(1975) and Mountain(1988) in the ordinary demand approach, the Inverse Rotterdam Demand System's coefficients can be viewed as a constant approximation to $\hat{w}_i b^*_{ij}$ and $\hat{w}_i k_i$. Following Mountain (1988), it can be shown that[12)]

$$(8) \quad \Delta w_i = \overline{w}_i \Delta \ln v_i + \overline{w}_i \Delta \ln q_i + \varepsilon_{1i}$$

where $\overline{w}_i$ denotes an average expenditure share for good i and ε_{1i} is an error term involving terms of second and higher order. If we rewrite eq. (8), we have

$$(9) \quad \overline{w}_i \Delta \ln v_i = \Delta w_i - \overline{w}_i \Delta \ln q_i - \varepsilon_{1i}.$$

[12] See Mountain (1988) for details. Note that I drop the time subscripts if confusion does not arise.

Substituting eq. (4) into eq. (9) yields

$$(10)\quad \overline{w}_i \Delta \ln v_i = \sum_j \hat{w}_i b_{ij}^* \,\Delta \ln q_j + \hat{w}_i k_i \Delta \ln Q + \{-\Delta \overline{w}_i \Delta \ln q_i + \varepsilon_i - \varepsilon_{1i}\}$$

where $\Delta \overline{w}_i = \overline{w}_i - \hat{w}_i$. Note that the bracket $\{\cdot\}$ involves second and higher order differences. Thus, we are left with

$$(11)\quad \overline{w}_i \Delta \ln v_i = \sum_j \hat{w}_i b_{ij}^* \,\Delta \ln q_j + \hat{w}_i k_i \Big(\sum_j \overline{w}_j \Delta \ln q_j\Big) + \eta_i$$

where η_i is an error term consisting of terms of second and higher order differences. Eq. (11) is referred to as the Inverse Rotterdam Demand System. Note that the discrete formulation is an approximation in variables. Hence, the estimated elasticities and flexibilities from the Inverse Rotterdam Demand System are also approximations.[13]

The Inverse CBS Demand System follows Working's suggestion but makes use of Inverse Rotterdam Demand System (call it IROT)'s quantity coefficients.[14] It follows from this consideration that

$$(12)\quad \overline{w}_i \Delta \ln v_i = \sum_j \hat{w}_i b_{ij}^* \,\Delta \ln q_j + (\hat{w}_i k_i + \hat{w}_i) \Delta \ln Q + \zeta_i$$

where ζ_i is an error term. Rearranging terms, we obtain the Inverse CBS Demand System (call it ICBS) given by[15]

$$(13)\quad \overline{w}_i \Delta \ln\Big(\frac{P_i}{P}\Big) = \sum_j \hat{w}_i b_{ij}^* \,\Delta \ln q_j + \hat{w}_i k_i \Delta \ln Q + v_i$$

where v_i is an error term consisting of terms of second and higher order differences.

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[13] Eq. (11) would satisfy the following properties of $r_{ij} (= w_i b_{ij}^*)$ and r_i $(= w_i k_i)$:

$\sum_i r_i = -1 \qquad \sum_i r_{ij} = 0$ (adding-up)

$\sum_j r_{ij} = 0$ (homogeneity)

$r_{ij} = r_{ji}$ (Antonelli symmetry)

$\sum_i \sum_j y_i r_{ij} y_j < 0 \;\forall\, y_j \neq 0$ (negativity)

[14] The name CBS was coined by Keller and Driel(1985) acknowledging the support of the Netherlands Central Bureau of Statistics.

[15] In deriving (13), we have used the following facts:

$$\begin{aligned} w_i(d\ln v_i + d\ln Q) &= w_i(d\ln p_i - d\ln m + d\ln Q) \\ &= w_i(d\ln p_i - d\ln P) \\ &= w_i\, d\ln(p_i/P) \end{aligned}$$

and $\Delta \overline{w}_i = \overline{w}_i - \hat{w}_i$ where $d\ln P$ is the Divisia (or Stone's) price index.

The Inverse NBR Demand System uses the Inverse Rotterdam Demand System's scale coefficients but the Inverse Almost Ideal Demand System (call it IAIDS)'s quantity coefficients.[16] Thus, we consider the following:

$$(14)\quad \overline{w}_i \Delta \ln v_i = \sum_j (\hat{w}_i b^*_{ij} + \hat{w}_i \delta_{ij} - \hat{w}_i \hat{w}_j)\Delta \ln q_j + \hat{w}_i k_i \Delta \ln Q + \xi_i$$

where ξ_i is an error term. Rearranging terms in (14), the Inverse NBR Demand System (call it INBR) can be obtained:[17]

$$(15)\quad \Delta w_i - \overline{w}_i \Delta \ln Q = \sum_j \hat{w}_i b^*_{ij}\ \Delta \ln q_j + \hat{w}_i k_i \Delta \ln Q + \xi_i .$$

As noted, these in-between systems arise from the difference between eqs. (4) and (7).

2. A General Price Formation Model

The four systems discussed above correspond to alternative parameterizations of the budget share differentials. The right hand sides of the four systems contain the same variables. However, the interpretation is not the same because the left-hand sides are different. In this section, a general system will be proposed as an alternative hypothesis, following Barten(1993). The contribution of this section is to extend Barten's approach to the inverse demand systems and show that the proposed system further can be used to test specificational validity. Note, however, that our system can be a new specification of marginal willingness-to-pay functions in its own right.

As noted in section Ⅱ.1, the left-hand sides are different in the four systems while the right-hand sides contain the same variables. Denoting by y_i^{IR}, y_i^{IC}, y_i^{IA}, y_i^{IN} the left-hand sides of the IROT, of the ICBS, of the IAIDS and of the INBR, respectively, we have

$$(16)\quad y_i^{IC} - y_i^{IR} = w_i d \ln\left[\frac{p_i}{P}\right] - w_i d \ln\left[\frac{p_i}{m}\right] = w_i d \ln Q$$

$$(17)\quad y_i^{IA} - y_i^{IC} = w_i d \ln\left[\frac{p_i q_i}{m}\right] - w_i d \ln\left[\frac{p_i}{P}\right] = w_i d \ln\left(\frac{q_i}{Q}\right)$$

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16 The model is named after the National Bureau of Research, where Neves(1987) worked when the model was developed.

17 In deriving (15), we have used the following simple algebra:

$$w_i[d \ln v_i + d \ln\left(\frac{q_i}{Q}\right)] = w_i(d \ln w_i - d \ln Q)$$
$$= dw_i - w_i d \ln Q.$$

$$(18) \quad y_i^{IN} - y_i^{IA} = dw_i - w_i d\ln Q - dw_i = -w_i d\ln Q$$

Barten's key insight, adapted to inverse systems, is that the differences between the left-hand sides of the systems are exogenous because $d(\log Q)$ and $d(\log q_j)$ are exogenous variables in inverse demand systems. This property is exploited to combine systems.

Now we want to show how to combine the models using a simple example of two alternative systems. Suppose that we have two alternative systems in which the right hand side variables are identical but the dependent variable is different:[18]

System 1: $y_t^{IR} = X_t\beta + e_{1t}$
System 2: $y_t^{IC} = X_t\gamma + e_{2t}$

where y is an n-vector of endogenous variables and X is an $n \times (k+1)$ matrix of exogenous variables, viz., $d(\log Q)$ and $d(\log q_1), \cdots, d(\log q_k)$. The β and γ are $(k+1) \times 1$ vectors of coefficients from the IROT and ICBS systems, respectively. The $n \times 1$ disturbance vectors e_{1t} and e_{2t} are assumed to be identically and independently distributed with probability density functions $f(e_{1t})$ and $g(e_{2t})$. For example, it could be postulated that e_{it}'s ($i = 1, 2$) are distributed normally with means (μ_1, μ_2) and variances (Ω_1, Ω_2), so that

$$f(e_{1t}) = (2\pi)^{-n/2} \,|\Omega_1|^{-1/2} \exp\{-\frac{1}{2}[y_{1t} - X_t\beta - \mu_1]'(\Omega_1^{-1})[y_{1t} - X_t\beta - \mu_1]\},$$

$$g(e_{2t}) = (2\pi)^{-n/2} \,|\Omega_2|^{-1/2} \exp\{-\frac{1}{2}[y_{2t} - X_t\gamma - \mu_2]'(\Omega_2^{-1})[y_{2t} - X_t\gamma - \mu_2]\}.$$

Following Atkinson(1970), the general form of the combined p. d. f. (probability density function) is proportional to

$$\{f(e_{1t})\}^{\alpha_1} \{g(e_{2t})\}^{\alpha_2}.$$

In order to have the properties of a density, a normalizing constant is introduced and the combined p. d. f. is written as

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[18] See Alston and Chalfant(1993) for the two alternative ordinary demand system example. Note, however, that they make a scalar combination of dependent variables in two systems and the same right-hand sides in the compounded system. This specification would lead to different interpretations and implications. One might be suspicious of their general model as a demand system in its own right. One might also wonder what is the meaning of a scalar combination of choice variables in the consumer optimization problem. See also Barten(1993) for a similar example of ordinary demand systems to the above.

$$h(\alpha) = \frac{\{f(e_{1t})\}^{\alpha_1}\{g(e_{2t})\}^{\alpha_2}}{\int\int\{f(e_{1t})\}^{\alpha_1}\{g(e_{2t})\}^{\alpha_2}de_1\,de_2}\,.$$

Replacing the expressions for $f(\cdot)$ and $g(\cdot)$ and using well-known results for integration, we obtain

$$h(\alpha) = (2\pi)^{-n/2}\,|\Omega|^{-1/2}\exp\{-\frac{1}{2}[y_t - X_t\pi - \mu]'(\Omega^{-1})[y_t - X_t\pi - \mu]\},$$

where $\Omega = \alpha_1\Omega_1 + \alpha_2\Omega_2$, $y = \alpha_1 y_1 + \alpha_2 y_2$, $\pi = \alpha_1\Omega^{-1}\Omega_1\beta + \alpha_2\Omega^{-1}\Omega_2\gamma$, and $\mu = \alpha_1\Omega^{-1}\Omega_1\mu_1 + \alpha_2\Omega^{-1}\Omega_2\mu_2$. This is the standard normal linear regression model for y_t with X_t and μ as regress variables. Use of the combined p. d. f. thus leads to the compound regression model. In our specific case, the covariance matrices of the models are equal, i. e., $\Omega = \Omega_1 = \Omega_2$ because y^{IR} and y^{IC} have the same random component given that the exogenous variables are non-stochastic [recall that $y^{IC} = y^{IR} +$ (some exogenous variable) by budget share differentiation, see (16)]. Therefore, the combined multivariate regression model can be seen as

$$(19)\quad y_t = \alpha_1 y_t^{IR} + \alpha_2 y_t^{IC} = X_t(\alpha_1\beta + \alpha_1\gamma) + e_t$$

where $e_t = \alpha_1 e_{1t} + \alpha_2 e_{2t}$ and the disturbance vector is distributed normally with mean μ and variance Ω. Letting $\alpha_1 + \alpha_2 = 1$ and rearranging, we obtain[19]

$$(20)\quad y_t^{IR} = w_t d\ln v_t = X_t[(1-\alpha_2)\beta + \alpha_2\gamma] + \alpha_2(y_t^{IR} - y_t^{IC}) + v_t\,.$$

In a scalar form, eq. (20) may be written in the form:

$$(21)\quad w_{it} d\ln v_t = \sum_j \pi_{ij} d\ln q_{jt} + \pi_i d\ln Q_t - \alpha_2 w_{it} d\ln Q_t + v_{it}\,.$$

where π_{ij} and π_i are linear combinations of quantity coefficients and scale coefficients between the IROT and ICBS systems, respectively. As seen above, β and γ are not identified from estimation of eq. (21). However, this is not crucial. What matters is that there is a well defined coefficient vector of X_t in (20) and identified mixing parameter α_2. If $\alpha_2 = 0$, eq. (21) will reduce to the IROT system while if $\alpha_2 = 1$ it reduces to the ICBS system. We may test whether α_2 is significantly different from zero. If it is significantly different from zero, we incorporate the

[19] Note that we allow the data to choose the appropriate parameter sets including mixing parameters (α_1, α_2) unlike the specification test in which we restrict mixing parameters under either system 1 or system 2.

ICBS system into the general system. The estimate of α_2, therefore, reflects the empirical importance of this difference in parameterization.

Extending the previous simple example to four systems discussed in section II. 1 and letting $\alpha_R + \alpha_C + \alpha_A + \alpha_N = 1$, we may write the general model, which nests all four inverse demand systems as

$$(22) \quad y_t^{IR} = X_t\pi + \alpha_C(y_t^{IR} - y_t^{IC}) + \alpha_A(y_t^{IR} - y_t^{IA}) + \alpha_N(y_t^{IR} - y_t^{IN}) + e_t$$

where $\pi = (1 - \alpha_C - \alpha_A - \alpha_N)\beta_R + \alpha_C\beta_C + \alpha_A\beta_A + \alpha_N\beta_N$ and α_i indicates the mixing parameter for the system i. Setting $\alpha_C = 1$ and $\alpha_A = \alpha_N = 0$, we have the ICBS system. Analogous specifications hold for the IAIDS and INBR systems and the IROT system corresponds with all three α's being zero. For the systems considered, however, (16) and (17) imply

$$(23) \quad (y_t^{IR} - y_t^{IC}) - (y_t^{IR} - y_t^{IA}) + (y_t^{IR} - y_t^{IN}) = Z_{Ct} - Z_{At} + Z_{Nt} = 0$$

where $Z_{Ct} = y_t^{IR} - y_t^{IC}$, $Z_{At} = y_t^{IR} - y_t^{IA}$, and $Z_{Nt} = y_t^{IR} - y_t^{IN}$. Thus, we have perfect collinearity in the three extra variables in the general system. Using (23), we can eliminate Z_{At} by substitution of $Z_{At} = Z_{Ct} + Z_{Nt}$ into (22). Hence, eq. (22) can be rewritten as

$$(24) \quad y_t^{IR} = X_t\pi + \theta_1(y_t^{IR} - y_t^{IC}) + \theta_2(y_t^{IR} - y_t^{IN}) + e_t.$$

where $\theta_1 = \alpha_C + \alpha_A$ and $\theta_2 = \alpha_N + \alpha_A$. The scalar form of eq. (24) would be

$$(25) \quad w_{it}d\ln v_t = \sum_j \pi_{ij}d\ln q_{jt} + \pi_i d\ln Q_t - \theta_1 w_i d\ln Q_t - \theta_2 w_i d\ln(q_i/Q)_t + v_{it}.$$

where v_{it} is an error term. This general system of equations may be called the synthetic inverse demand system ('SIDS'). It is a new specification of marginal willingness-to-pay functions in a system. In addition, it can be used as an alternative hypothesis for a specification test. As in the two system example, the α's cannot be identified from θ_1 and θ_2 which are the coefficients to estimate. This is not a problem.

Applied welfare economics usually focuses on elasticities to evaluate policy on the consumer side. Applied economists have built up intuition based largely on elasticities. In order to derive the scale and cross-price flexibilities from our new system, we may write eq. (25) in the general form:

$$(26) \quad w_{it}d\ln v_{it} = \sum_j [\pi_{ij} - \theta_2 w_i \delta_{ij} + \theta_2 w_i w_j]d\ln q_{jt} + [\pi_i - \theta_1 w_i]d\ln Q_t$$

where θ_i's are system mixing parameters to be estimated and tested. Letting $[\pi_{ij} -$

$\theta_2 w_i \delta_{ij} + \theta_2 w_i w_j] = b(\pi_{ij};\ \theta_2)$ and $[\pi_i - \theta_1 w_i] = c(\pi_i;\ \theta_1)$, eq. (26) can be expressed as a simple form:

$$(27)\quad w_{it} d\ln v_{it} = \sum_j b(\pi_{ij};\ \theta_2) d\ln q_{jt} + c(\pi_i;\ \theta_1) d\ln Q_t.$$

There are two sets of restrictions on the parameters of (26). The first set of weak restrictions on consumer demand, following from the budget constraint, are

$$(28)\quad \sum_i [\pi_{ij} - \theta_2 w_i \delta_{ij} + \theta_2 w_i w_j] = \sum_i \pi_{ij} = 0 \qquad \text{(adding-up)}$$

$$(29)\quad \sum_i [\pi_i - \theta_1 w_i] = -1 \qquad \text{(adding-up)}$$

$$(30)\quad \sum_j [\pi_{ij} - \theta_2 w_i \delta_{ij} + \theta_2 w_i w_j] = \sum_j \pi_{ij} = 0 \qquad \text{(homogeneity).}$$

For strong restrictions on consumer demand, following from the utility maximization, are

$$(31)\quad \pi_{ij} = \pi_{ji} \qquad \text{(symmetry).}$$

The scale elasticity and price flexibility are derived easily by dividing w_i through eq. (26). The scale elasticity is then given by

$$(32)\quad k_i = \pi_i / w_i - \theta_1$$

where k_i indicates the scale elasticity of good i. The cross-price flexibility is

$$(33)\quad b^*_{ij} = \pi_{ij} / w_i + \theta_2 w_j,$$

and the own price flexibility may be written

$$(34)\quad b^*_{ii} = \pi_{ii} / w_i - \theta_2 + \theta_2 w_j$$

where b^*_{ij} is the compensated price flexibility of good i with respect to good j. The uncompensated price flexibility can be derived from the Antonelli equation, i. e., $b^*_{ij} = b_{ij} - w_j k_i$, where the "*" indicates "compensated". Note that in eq. (32), if $\theta_1 = 1$, it becomes the IAIDS scale elasticity. Note also that if $\theta_2 = 1$, it would be the IAIDS price flexibility in eqs. (33) and (34), when the IAIDS uses Stone's quantity index as an approximation.

In this section, we have examined the way to parameterize an inverse demand system and proposed a general inverse demand system, a new specification of marginal willingness-to-pay functions in a system. The proposed system further can be used as an alternative system for the purpose of a specification test in the static context. Whatever inverse demand system specification tests validate, the model is static and this type of static model assumes that adjustment toward equilibrium is instantaneous, thereby leaving no room for price to diverge from long-run equilibrium levels. However, in reality, the price response to a change in quantity demanded may be dispersed over more than one period and thus some dynamic aspect such as taste changes may improve the estimates of the static inverse demand system because discrepancy between outcome and postulated equilibrium may contain useful information omitted in the static inverse demand. Another advantage of dynamic specification of the model is that it makes easier to distinguish between the short-run and long-run marginal willingness-to-pay functions. In the next section, we develop such a dynamic inverse demand system.

Ⅲ. A DYNAMIC GENERALIZATION OF MARGINAL WILLINGNESS-TO-PAY FUNCTIONS IN A SYSTEM

A dynamic version of consumer theory may be obtained by introducing a variable related to past behavior into the utility function. This new variable would include the influence that the behavior in past period exerts on the present, which is related to the habit strength, psychological or physical stock of good i, H_i. As is well known, the introduction of state variables into the utility function is a way of taking adjustment costs, whether psychological or not, into account.

Following El-Safty(1976), it is assumed that the consumer's ordinary utility function be of the form:

$$(35)\quad U(\cdot) = U(\phi_1, \phi_2, \cdots, \phi_n)$$

where $(\phi_1, \phi_2, \cdots, \phi_n)$ are the current services provided by the purchases of the n-goods, $(q_1, q_2, \cdots, q_n)$. The service function ϕ_i is assumed to depend on current purchases of good i and the psychological stock of habits for good i, H_i. That is

$$(36)\quad \phi_i = \phi_i(q_i, H_i),\ i = 1, 2, \cdots, n$$

where ϕ_i is assumed to be a strictly increasing function of q_i. For the special case, Fisher and Shell (1968) have treated the case when $\phi_i = f(H_i)q_i$. Houthakker and Taylor(1970) have studied the case when $U(\cdot)$ is quadratic and $\phi_i = q_i - \alpha_i H_i$.

We shall assume that H changes according to

$$(37)\quad \ln H_{it} = (1-\delta_i)\ln H_{it-1} + \ln q_{it-1} \qquad 0 < \delta_i \leq 1$$

which implies $\ln H_{it} = \ln q_{it-1} + (1-\delta_i)\ln q_{it-2} + (1-\delta_i)^2 \ln q_{it-3} + \cdots\cdots$. Although the consumer anticipates changes in H, we assume that he is not aware of the underlying mechanisms by which these changes take place. Then the consumer's problem will be

$$(38)\quad \text{Max. } U[\phi_1(q_1, H_1), \phi_2(q_2, H_2), \cdots, \phi_n(q_n, H_n)] \quad \text{s.t.} \quad \sum_i p_i q_i = m.$$

The first order conditions after normalizing prices by income are

$$(39)\quad \phi_q U_\phi - \lambda v = 0, \quad \sum_i v_i q_i - 1 = 0$$

where ϕ_q and U_ϕ denote the derivatives of ϕ and U with respect to q and ϕ, respectively, λ is a Lagrangian multiplier, and v is a vector of normalized prices of n-goods. In order to simplify the analysis, we assume that $\phi_i(q_i, H_i) = \phi(q_i, H_i)$ where ϕ is a monotonically increasing function[20] and is the same for all i's. Since $U[\phi(\cdot)]$ is a monotonic transformation of ϕ, $U[\phi(\cdot)]$ ranks the order of preferences in the same way as $\phi(\cdot)$. Letting $U[\phi(\cdot)] = \phi(\cdot) = U(\cdot)$, we might rewrite the above consumer's problem simply as

$$(40)\quad \text{Max. } U(q, H) \quad \text{s.t.} \quad \sum_i v_i q_i = 1$$

where q is a vector of quantities and H is a vector of psychological stock of habits. The first order conditions would then be

$$(41)\quad (a)\ U_q(q, H) - \lambda v = 0 \qquad (b)\ v'q - 1 = 0.$$

These conditions can be solved for v:

$$(42)\quad v = \left(\frac{1}{\lambda}\right) U_q = \left(\frac{1}{q'U_q}\right) U_q.$$

Noting that $dU_q = U_{qq}dq + U_{qH}dH$ where $U_{qq} = \partial^2 U/\partial q \partial q'$, we take the total differentiation for (42) and obtain

[20] It is defined as a function with the property that successively larger values of the independent variable always lead to successively larger values of the function, that is, $q^1 > q^2$ implies $\phi(q^1) > \phi(q^2)$.

$$(43)\quad dv = (I - vq')tU_{qq'}(I - qv')dq - [v - (I - vq')tU_{qq'}q]d\ln Q + (I - vq')tU_{qH}dH$$

where $t = 1/q'U_q$, I denotes an identity matrix and all other variables are defined as before. Letting R be the coefficient of change in H, then R can be written as

$$(44)\quad R = \frac{1}{\lambda} AU_{qq}^{-1}U_{qH}$$

where $\lambda = q'U_q$; A is the Antonelli matrix; and in computation, we utilize the relation $mSA = I - qv'$ and $mASA = A$ where S is the symmetric Slutsky matrix.[21] Thus, it can be written in the scalar form:

$$(45)\quad \frac{dv_i}{dH_j} = \frac{1}{\lambda}\sum_r a_{ir}\left[\frac{\partial^2 U}{\partial q_i \partial q_r}\right]^{-1}\frac{\partial^2 U}{\partial q_r \partial H_j} \qquad (i, j = 1, \cdots, n)$$

As seen above the stock effect of habits has two effects, i. e., direct and indirect effects. To see the direct effect, it is sufficient to know the influence of a variation in the stock on the marginal utility since a_{ii} is always negative and the Hessian of the utility function is negative by concavity. If $U_{qH} > 0$, then $dv_i/dH_i > 0$ which implies that as the stock of habits increase, marginal willingness to pay (or marginal valuation) would increase as well. If good i and j are q-substitutes but neutral to other goods, then $a_{ij} < 0$ which implies that marginal valuation of good i would increase as the stock of habits for good j increases. In this case, good i may be said to be a q-habit substitutes for good j. For complementary goods, the effect would be the opposite.

In order to derive a dynamic inverse demand system based on equation (43), we multiply q through equation (43). The resulting equation can be written as

$$(46)\quad wd\ln v = q(I - vq')tU_{qq'}(I - qv')q'd\ln q - q[v - (I - vq')tU_{qq'}q]d\ln Q + q(I - vq')tU_{qH}H'd\ln H$$

where w denotes a diagonal matrix with elements w_{it}, $\ln q$ is an $(n \times 1)$ vector of exogenous variables, and $\ln v$ is an $(n \times 1)$ vector of endogenous variables. Assume that we have the following form of a structural demand function:[22]

---

[21] See Deaton and Muellbauer(1980), and Stern(1986) for the derivation of this relation.

[22] Recall the behavioral equation in section Ⅱ. 1, $w_{it} = f(q_{1t}, \cdots, q_{nt})$. Considering habit effects, we now have the behavioral equation as $w_{it} = f(q_{1t}, \cdots, q_{nt}, H_{1t}, \cdots, H_{nt})$. The structural demand function follows from this. Note that this is different from the long-run equilibrium inverse demand function since it includes H_t. In the long-run, $\Delta \log H_t = 0$.

(47) $w_{it} = \alpha_{0i} + \sum_{j} \alpha_{ij} \ln q_{jt} + \alpha_i \ln Q_t + \alpha_{hi} \ln H_{it}.$

Using (47) as a structural demand function, ln H_t can be expressed, in a vector form, as

(48) $\ln H_t = A^{*-1}_3 [w^*_t - A^*_0 - A^*_1 \ln q_t - A^*_2 \ln Q_t]$,

where A^*_1 and A^*_3 denote the matrix $[\alpha_{ij}]$, and a diagonal matrix with elements α_{hi} $A^*_2 = (\alpha_1, \cdots, \alpha_n)'$, $A^*_0 = (\alpha_{01}, \cdots, \alpha_{0n})'$, $\ln H_t = (\ln H_{1t}, \cdots, \ln H_{nt})'$, $w^*_t = (w_{1t}, \cdots, w_{nt})'$, and $\ln q_t = (\ln q_{1t}, \cdots, \ln q_{nt})'$. Lagging one period in (48), and plugging the result into (37) and (46) after approximating an infinitesimal differential by a finite difference, we obtain

(49) $w_t \Delta \ln v_t = G \Delta \ln q_t + g \Delta \ln Q_t - \gamma(w_{t-1} - A_0 - A_1 \ln q_{t-1} - A_2 \ln Q_{t-1})$,

where $G = q(I - vq')tU_{qq'}(I - qv')q'$, $g = -q[v - (I - vq')tU_{qq'}q]$, $\gamma = qRH'\delta A^{*-1}_3$, $A_0 = A^*_0$, $A_2 = A^*_2$, and $A_1 = A^*_1 + \delta^{-1} A^*_2$. This is the form similar to an error-correction model used extensively throughout the recent time-series econometrics literature, where the last term in parenthesis letting U_{t-1},

$$U_{t-1} = w_{t-1} - A_0 - A_1 \ln q_{t-1} - A_2 \ln Q_{t-1}$$

plays a role as an error correction term.[23] In (49), the error correction term is the residual of the IAIDS system. Note that the error-correction term crucially depends upon the form of the structural equation and gives an expression for long-run elasticities directly. For a single equation model, the error correction term has a natural interpretation even in the inverse demand context: if w_t rises above an equilibrium level by quantity changes, then U_{t-1} would be positive but $-A_1 < 0$ makes $\Delta \ln v_t$ lower toward its steady state path. A similar interpretation can be extended to the multi-equation model.

This adaptive adjustment mechanism has been rationalized as the optimal reaction of an agent to the adjustment costs of implementing a consumption/production plan. For a consumer, it suffices to interpret adaptive adjustment as a trade-off between the costs of not altering the utility maximizing solution and the costs of adjusting to the new position. The costs of change are a function of past behavior, which may be summarized in an adjustment coefficient or the state variables.

Based on (49), we can construct a dynamic general inverse demand system

[23] Error-correction terms were used by Sargan(1964), Hendry and Anderson(1977), and Davidson et al.(1978).

specific to our purpose. Noting $q=sq^*$($s=$ scale variable, $q^*=$ initial consumption bundle) and taking a finite approximation, we may rewrite the *ith* equation of (49) as[24)]

$$(50) \quad w_{it}d\ln v_{it} = \sum_j b(\pi_{ij};\theta_2)d\ln q_{jt} + c(\pi_i;\theta_1)d\ln Q_t + \lambda_i[w_{it-1} - \alpha_{0i} - \sum_j a_{ij}\ln q_{jt-1} - a_i\ln Q_{t-1}] + \varepsilon_{it},$$

where $b(\cdot)=\pi_{ij}-\theta_2 w_i\delta_{ij}+\theta_2 w_i w_j$, $c(\cdot)=\pi_i-\theta_1 w_i$, and $\ln q_{t-1}$ is a vector of quantities demanded at time $t-1$. Equation (50) will be the maintained framework of a dynamic inverse demand system in this paper.

In the specification of eq. (50), the long-run scale elasticity and long-run price flexibility can be derived and defined as

$$(51) \quad k_i^L = -1 + \frac{a_i}{w_i},$$

$$(52) \quad b_{ij}^L = \left[\frac{a_{ij}+a_i w_i}{w_i}\right],$$

where k_i^L and b_{ij}^L are the long-run scale elasticity and price flexibility, respectively. Some restrictions on the long term part of the model are discussed later in the empirical section.

Ⅳ. AN APPLICATION TO U. S. FISH DEMAND

1. Data

In this section, the dynamic inverse demand system discussed are applied to the demand for fish in the United States, especially Southeast region, from 1977-1992. The data used are seasonally unadjusted monthly time series on prices and landings per capita converted from the data collected by the National Marine Fisheries Service (NMFS). As is well known, seasonally adjusted data can distort the underlying relationship between variables.[25)] Thus, use of seasonally unadjusted data will show better structural relationship between variables.

The present study used data set related to six broad types of commercial fi-

[24] Note that it can be shown that ln s=ln Q, i. e., scale variable which plays the same role as real income variable in the ordinary demand system. Note also that the matrix $[a_{ij}]=A_1$ and the vector $[a_i]=A_2$ in (49).

[25] See Wallis(1974) for details.

[Table 1] Main Six Fish Types, Shares in Expenditure, Variation in Quantities in the Southeast Region (Jan. 1977-Dec. 1992)

Type of fish	Sample average share in total monthly expenditure	Average share in 1992	Minimum quantity (lbs)	Maximum quantity (lbs)
Groupers	42.22	48.02	312,028	2,237,368
Snappers	41.75	32.69	320,480	1,800,177
Porgies	5.16	4.93	75,275	1,178,508
Jacks	3.79	7.01	30,699	2,594,571
Tilefishes	3.63	4.46	1,736	505,367
Sea Basses	3.45	2.88	6,163	620,923

shes for the United States. The groupings are (1) Groupers, (2) Porgies, (3) Snappers, (4) Jacks, (5) Tilefishes, and (6) Sea basses.[26] Table 1 shows the group species and the average share of total expenditure on these fish over the sample period. There are wide ranges of the landed quantities of each type of fish. Part of the variation may be explained by seasonality and part of it may be explained by trend.

As seen in Table 1, Groupers and Snappers are the dominant groups of fish in the Southeast region. Groupers and Snappers remain the prime fish over sample periods and the landed quantities display a wider range than the prices of the various types of fish. The strong seasonal variation in the landed amounts suggests that it is essential to work out on the price formation by the quantity effects.

2. A Specification Test for Price Formation Models

Using (27) as an alternative model, we test the null models presented earlier. Our approach is proposed that can deal with non-nested models with different dependent variables. In fact we test whether matrix weights (θ_1, θ_2) to combine the null models are significantly different from zero. If these matrix weights are not zero statistically, the basic null model falls short in explaining reality on its

[26] The original data have 76 individual species in the Southeast region which were then aggregated into 9 groups, i. e., Groupers, Snappers, Porgies, Jacks, Tilefishes, Sea basses, Wrasses, Grunts, and Triggerfishes. The reason for aggregating to larger categories is that some specific individual species are reclassified at some year and so there are big jump or drop in quantity series of some species. For example, the grouper with fish code 1410 has a big drop in series of quantity because of reclassification into other coded groupers around 1986. Thus, aggregation to larger categories, using family name, deals with this issue. Another reason for this may be that there are similarities in physical characteristics and tastes among those individual species under the common family name of fishes. The reason for analyzing the above six types of fish is that they have non-negligible shares in expenditure.

[Table 2] Test Results for the various Null Models

Null models	θ_1	θ_2	F-statistics
IROT	0	0	319.455**
ICBS	1	0	0.3359
INBR	0	1	5354.928**
IAIDS	1	1	4967.777**

** significant at 5% significance level.

own and could employ some of the information contained in the other models in the linear combination. Therefore, the matrix weights reflect the empirical performance of the different parameterization in different null models.

The equation (26) has been estimated by Zeller's Seemingly Unrelated Regression Method after removing autocorrelation in residuals by the Cochrane-Orcutt method. The test statistics for the estimated matrix weights are presented in Table 2. From Table 2, the artificial nesting procedure is clear since the SIDS system (26) shows that θ_1 and θ_2 parametrize the differences between the null models. The last column gives the test statistics for the null model and (26) as the alternative model.

The test statistics shows that the sample favors the IAIDS type of scale coefficients and the IROT type of quantity coefficients. Accordingly, the inverse CBS demand model (ICBS) appears to strongly support data generating process. Although the ICBS has empirically strong performance, we will use our SIDS system as a basic differential marginal valuation model. This is because the SIDS itself is a general system reflecting all these matters into its coefficients.

3. Estimation of a First Differenced Dynamic Inverse Demand Model

In order to estimate the parameters, we modify the model in several respects. First, we add the disturbance terms to estimate eq. (50). Second, to account for seasonality of demand, eq. (50) is augmented with 11 seasonal dummy variables; D_k $(k=2, \cdots, 12)$; whose associated coefficients must sum to zero over i for adding up. Finally, the finite differences are used for the differentials as an approximation. The resulting equation would then be in the form:

$$\begin{aligned}(53)\quad \overline{w}_{it}\Delta\ln v_{it} = {} & \alpha_i + \sum_{k=2}\phi_{ik}D_{kt} + \sum_j \pi_{ij}\Delta\ln q_{jt} + \pi_i\Delta\ln Q_t - \theta_2\overline{w}_{it}\Delta\ln q_{jt} \\ & - \theta_1\overline{w}_{it}\Delta\ln(q_{it}/Q_t) + \lambda_i[w_{it-1} - \alpha_{i0} - \sum_j a_{ij}\ln q_{jt-1} - a_i\ln Q_{t-1}] + \varepsilon_{it}\end{aligned}$$

where $\ln Q_t$ is given by $\sum_j (\overline{w}_j + \sum_k \phi_{jk}D_k) \ln q_{jt}$

and $\overline{w}_{it} = \frac{w_{it} + w_{it-1}}{2}$. As Anderson and Blundell(1983) suggest, we are not assuming that agents are in equilibrium in the short-run and thus there seems no reason why short-run behavior should satisfy any demand restriction. By this reason, the restrictions suggested by economic theory are imposed only on the long-run structure.

An assumption that is necessary to yield reliable estimates of the demand parameters is that the error-correction term (residual of long-run inverse demand) in eq. (53) should be stationary. If the levels of the variables are nonstationary, the stationarity of error-correction terms requires that these nonstationary variables be cointegrated as discussed in Engel and Granger(1987). If not cointegrated, then the regression equations in (53) are subject to the spurious regression phenomenon and the first difference regression without error-correction terms is appropriate.[27] The test for cointegration consists of two steps:

Step 1: Unit root test for the variables.
In order to test for the presence of unit roots, the following augmented Dickey-Fuller (ADF) regressions were run:

$$(54) \quad \Delta y_t = \alpha + \beta t + (\rho - 1) y_{t-1} + \sum_{s=1}^{m} d_s \Delta y_{t-s} + \varepsilon_t$$

where y is the variable under consideration; m is the number of lags that ensures that the error term is white noise (for the monthly data, the usual maximum lag is 12). If the null that $\rho = 1$ is rejected, the series is stationary in levels.

Step 2: Unit root test for the residuals of the levels regressions estimated using the nonstationary variables.
If the residuals (U_t) do not have a unit root, then the nonstationary variables are said to be cointegrated. This step has two regressions. The first regression is the cointegrating regression of the form:

$$(55) \quad x_{1t} = a + bx_{2t} + U_t$$

where x_{1t} and x_{2t} are the $\{w_t\}$ and $\{\log q_t, \log Q_t\}$, respectively. The second regression tests for a unit root in the residuals and is of the form:

$$(56) \quad \Delta U_t = d \cdot U_{t-1} + \sum_{s=1}^{m} g_s \Delta U_{t-s}$$

[27] If not cointegrated, the first difference regressions are misspecified because they omit the relevant variable, dynamic element. Deaton and Muellbauer(1980) explained the reason for autocorrelation found in the residuals of the demand equations by the exclusion of dynamic element, i. e. habit formation.

[Table 3] Tests for Unit Roots and Cointegration

Variable	ADF statistic	AR coefficient	Lag order
w_1	−3.546	0.146	1
w_2	−7.652	0.505	1
w_3	−3.561	0.159	1
w_4	−5.339	0.328	1
w_5	−2.950	0.120	2
w_6	−6.986	0.391	1
$\ln q_1$	−4.606	0.219	1
$\ln q_2$	−6.325	0.335	1
$\ln q_3$	−7.498	0.610	1
$\ln q_4$	−7.160	0.440	1
$\ln q_5$	−3.765	0.117	1
$\ln q_6$	−6.819	0.320	1
$\ln Q$	−5.350	0.346	1
Commodity	ADF statistic	AR coefficient	Lag order
1.	−5.353	0.367	1
2.	−7.707	0.638	1
3.	−7.295	0.520	1
4.	−7.110	0.602	1
5.	−5.799	0.418	1
6.	−8.009	0.719	1

Note: Lag order for augmented DF test chosen by using the Akaike Information Criterion Test (AIC); Critical value: −2.58.

where m is the number of the lags chosen by Akaike's final prediction error criterion.

The ADF statistics, along with the estimates of coefficients of y_{t-1}, are presented in Table 3. Using a 5% significance level, the null hypothesis of a unit root is rejected for any variable in the study. Testing cointegration is also conducted by checking the stationarity of the residuals of long-run inverse demand, presented in the bottom half of Table 3. The presence of unit roots is again rejected as expected.

Given that the variables are cointegrated, the three stage least square estimation (3SLS) may be used to estimate the parameters of the system. For the convenient specification, eq. (53) may be written in terms of vectors and matrices:

$$(57)\quad \overline{w}_t \Delta_1 y_t = s_t + \Gamma \Delta_1 c_t + \theta_1 \Delta_1 g_{ct} + \theta_2 \Delta_1 g_{nt} + \lambda(w_{t-1} - Ac_{t-1})$$

where

[Table 4] Estimated Quantity Adjustment Parameters

Adjustment parameters in a system	Coefficients of error-correction
λ_1	-0.03 (0.55)
λ_2	-0.32 (4.75)
λ_3	-0.01 (0.15)
λ_4	-0.01 (0.37)
λ_5	-0.04 (1.62)
λ_6	-0.08 (2.15)

Note: Values in parentheses are t-statistics.

$$y_t = [\log v_{it}],\quad \Delta_1 c_t = [\Delta_1 \log(q_{it}/pop_t),\ \Delta_1 \log Q_t],$$
$$\Delta_1 g_{ct} = [-\overline{w}_{it}\Delta_1 \log Q_t],\quad \Delta_1 g_{nt} = [-\overline{w}_{it}\Delta_1 \log(q_{it}/Q_t)],\quad s_t = [\alpha_i + \sum_{k=2} \phi_{ik} D_{kt}],$$

and Γ and A are coefficient matrices for $\Delta_1 c_t$ and c_{t-1}, respectively, while $\overline{w}_t$ is a diagonal matrix with elements $\overline{w}_{it}$ and 'pop' indicates population. In the procedure, the predicted value of w_{t-1} is first found from an ordinary least-squares (OLS) estimate of w_{t-1} on lags of instrumental variables which consist of all past exogenous variables. Next w_{t-1} is replaced by the predicted value and then Zeller's Seemingly Unrelated Regression is used to estimate the system with long-run restrictions and mixing parameter restrictions ($\theta_1 = 1$ and $\theta_2 = 0$) imposed. Note that as mentioned before, since disturbances of this type of equation (57) are usually autocorrelated, we estimate the transformed model by the Cochrane-Orcutt procedure.

The estimated coefficients of the error correction terms are shown in Table 4. It shows that three of six coefficients are statistically different from zero and thus the system may not be expressed in terms of differenced variables alone. The coefficient estimates for the model augmented with seasonal dummies may be difficult to interpret directly. Thus, instead of reporting the coefficient estimates, price flexibilities and scale elasticities are derived using eqs. (51) and (52) and presented in Table 5. Looking at the results shown in Table 5, it appears that all the estimates of the scale elasticities are negative and relatively large with respect to their standard errors, thus giving high t-ratios. Since the normalized price goes down as all quantities increase assuming the absolute prices unchanged, this is what we expect. We may also note that the estimated scale coefficients are rather close to minus the average share of fish i of Table 1, which suggests that preferences are homothetic.

Looking at the estimated elasticity form of the Antonelli substitution matrix of Table 5, All the diagonal elements of the Antonelli matrix, i. e., the own-price flexibilities, have been estimated negatively with a high degree of precision except Tilefishes and Seabasses in short-run price formation and the magnitude of the

[Table 5] Estimation Result of a First-differenced Dynamic Model with Seasonal Dummies

Item	Price flexibility						Scale elasticity	Long-run own-price flexibility	Long-run Scale elasticity	R^2/DW
	GP	PG	SP	JK	TL	SB				
1	−0.0702	0.0212	0.0146	0.0071	−0.0002	0.0093	−0.964	−0.133	−0.905	0.98/1.9
	(4.38)	(3.86)	(1.09)	(1.73)	(0.06)	(2.15)	(15.82)	(7.28)	(37.66)	
2	0.1738	−0.1254	0.0169	−0.0052	−0.0084	−0.0450	−1.003	−0.180	−1.075	0.85/2.1
	(3.86)	(3.58)	(0.40)	(0.87)	(0.49)	(2.40)	(12.49)	(2.30)	(10.19)	
3	0.0147	0.0021	−0.0357	0.0019	0.0045	0.0015	−0.991	−0.021	−1.038	0.98/2.1
	(1.09)	(0.40)	(2.60)	(0.52)	(1.41)	(0.39)	(41.48)	(1.60)	(47.0)	
4	0.0790	−0.0071	0.0214	−0.0833	0.0009	0.0357	−1.115	−0.192	−0.965	0.79/2.0
	(1.73)	(0.31)	(0.52)	(3.28)	(0.06)	(1.94)	(14.21)	(4.09)	(6.71)	
5	−0.0024	−0.0119	0.0517	0.0009	−0.0091	0.0028	−1.075	−0.262	−1.050	0.84/2.0
	(0.06)	(0.53)	(1.41)	(0.06)	(0.42)	(0.17)	(15.91)	(9.27)	(8.89)	
6	0.1135	−0.0674	−0.0181	0.0392	0.0030	−0.0427	−1.172	−0.172	−1.584	0.83/2.1
	(2.15)	(2.40)	(0.39)	(1.94)	(0.17)	(1.39)	(13.50)	(2.05)	(9.27)	

Note: Item 1 = groupers (GP); item 2 = porgies (PG); item 3 = snappers (SP); item 4 = jacks (JK); item 5 = tilefishes (TL); item 6 = seabasses (SB); values in the brackets are t-statistics; the last column shows R^2 (the coefficient of determination) and Durbin-Watson statistics. System weighted $R^2 = 0.9926$.

The model transformed by Cochrane-Orcutt procedure is:

Grouper: $y_{1t} - \Pi x_{1t} = (1 - \underset{(2.92)}{0.22L} - \underset{(1.97)}{0.14L^2} - \underset{(1.77)}{0.12L^3} - \underset{(1.66)}{0.11L^4} - \underset{(1.95)}{0.11L^{11}} - \underset{(2.06)}{0.11L^{12}})^{-1} e_{1t}.$

Porgy: $y_{2t} - \Pi x_{2t} = (1 - \underset{(1.93)}{0.17L} - \underset{(1.70)}{0.12L^3} - \underset{(1.95)}{0.13L^9} - \underset{(2.89)}{0.18L^{12}})^{-1} e_{2t}.$

Snapper: $y_{3t} - \Pi x_{3t} = (1 - \underset{(3.49)}{0.25L} + \underset{(2.05)}{0.14L^2} - \underset{(2.33)}{0.15L^3} - \underset{(2.79)}{0.18L^4} - \underset{(2.09)}{0.12L^{12}})^{-1} e_{3t}.$

Jack: $y_{4t} - \Pi x_{4t} = (1 - \underset{(3.97)}{0.29L} - \underset{(3.65)}{0.27L^2} - \underset{(3.24)}{0.25L^3} - \underset{(2.49)}{0.20L^4} - \underset{(1.86)}{0.17L^6} - \underset{(2.46)}{0.21L^7} - \underset{(1.77)}{0.15L^{10}})^{-1} e_{4t}.$

Tilefish: $y_{5t} - \Pi x_{5t} = (1 - \underset{(3.94)}{0.37L} - \underset{(3.20)}{0.26L^4})^{-1} e_{5t}.$

Seabass: $y_{6t} - \Pi x_{6t} = (1 - \underset{(3.27)}{0.26L} - \underset{(1.95)}{0.12L^{10}} + \underset{(2.80)}{0.18L^{11}})^{-1} e_{6t}.$

where numbers in parentheses are t-statistics and lag operator is denoted by L.

short-run response of marginal willingness-to-pay is smaller than the long-run response of that except for Snappers. For the off-diagonal elements of the Antonelli matrix representing cross substitution in the short-run, only eight of the thirty different cross effects are negative and ten among thirty are significantly different from zero, thus showing complementarity bias as usual in the price formation models.

[Table 6] Tests for Seasonal Unit Roots

Series	Frequency							
	0	π	$\pi/2$	$\pm 5\pi/6$	$\pm\pi/6$	$\pm\pi/3$	$\pm 2\pi/3$	All seasonal
w_1	−1.81	−3.25*	18.87*	19.25*	8.88*	14.68*	17.39*	24.95*
w_2	−1.88	−2.98*	8.88*	6.55*	2.15	8.97*	3.75$^+$	7.01*
w_3	−1.90	−3.06*	14.47*	15.90*	4.64*	8.11*	14.82*	13.47*
w_4	−0.71	−4.35*	16.32*	9.62*	8.04*	10.09*	8.66*	12.14*
w_5	−2.26	−4.39*	38.58*	7.92*	7.58*	11.53*	27.78*	40.98*
w_6	−2.01	−3.27*	23.02*	5.30*	3.50$^+$	18.77*	13.82*	13.41*
$\ln q_1$	−2.54	−2.78*	27.59*	9.48*	11.60*	15.39*	13.26*	29.09*
$\ln q_2$	−3.51	−3.34*	12.07*	10.05*	4.45*	13.32*	7.10*	10.57*
$\ln q_3$	−1.34	−3.21*	9.37*	8.87*	5.87*	10.28*	11.97*	7.54*
$\ln q_4$	−1.14	−4.28*	18.55*	16.93*	5.88*	10.48*	4.94*	13.24*
$\ln q_5$	−2.26	−3.74*	32.63*	9.14*	14.00*	15.12*	15.87*	42.30*
$\ln q_6$	−2.38	−5.12*	23.14*	9.82*	6.54*	13.33*	11.63*	20.48*
$\ln Q$	−3.84	−2.91*	25.48*	9.47*	21.05*	22.46*	17.68*	17.45*

Note: The test procedure uses the parameterization of the test regression adopted in Franses(1991, eq.(5)).
The test regressions include a constant, seasonal dummies.
* Test statistic significant at the 10% level.
\+ Test statistic significant at the 20 % level.

4. Estimation of a Seasonal Differenced Dynamic Inverse Demand Model

As Davidson et al.(1978) suggest, the closest equivalent of a transformation of the form $\Delta_1 y_t^a = y_t^a - y_{t-1}^a$ (in seasonally adjusted data) is $\Delta_{12} y_t = y_t - y_{t-12}$ (in raw data), since both transformed variables represent changes net of seasonal factors. Following their suggestion, we transform the model (57) to be estimated as

$$(58)\quad \overline{w}_t \Delta_{12} y_t = \Gamma \Delta_{12} c_t + \theta_1 \Delta_{12} g_{ct} + \theta_2 \Delta_{12} g_{nt} + \lambda(w_{t-12} - Ac_{t-12}).$$

The relationship above may be approximately interpreted as follows: marginal valuation of consumers in each month of a year is the same as that in the month of the previous year modified by a proportion of their annual change in quantity demanded; these together determine a short-run marginal valuation decision, which is altered by $\lambda(w_{t-12} - Ac_{t-12})$ to ensure coherence with the long-run equilibrium. In this point, the use of transformed variables like $\Delta_{12} y_t$, etc., seems to be not because we want to seasonally adjust and achieve stationary but because $\Delta_{12} y_t$ represents a sensible decision variable when different goods are being purchased in different months of the year. Thus, we may prefer estimating eq. (58).

[Table 7] Estimation Result of a Seasonal-differenced Dynamic Model

Item	Price flexibility						Scale elasticity	Long-run own-price flexibility	Long-run Scale elasticity	R^2/DW
	GP	PG	SP	JK	TL	SB				
1	−0.0775	0.0052	0.0215	0.0166	0.0131	0.0089	−1.014	−0.103	−0.945	0.97/1.6
	(4.74)	(0.96)	(1.65)	(5.11)	(3.62)	(2.43)	(29.85)	(5.04)	(13.62)	
2	0.0429	−0.1785	0.0407	0.0024	−0.0007	0.0471	−0.813	−0.099	−0.889	0.77/2.1
	(0.96)	(4.45)	(0.28)	(0.63)	(1.16)	(3.45)	(9.22)	(1.81)	(8.48)	
3	0.0217	0.0050	−0.0425	−0.0024	−0.0003	−0.0015	−0.953	−0.059	−1.049	0.97/2.2
	(1.65)	(0.28)	(2.54)	(0.71)	(0.55)	(0.48)	(31.08)	(3.53)	(42.7)	
4	0.1843	0.0003	−0.0260	−0.1144	−0.0307	0.0018	−1.126	−0.181	−0.700	0.71/2.1
	(5.11)	(0.63)	(0.71)	(3.80)	(2.35)	(0.17)	(12.96)	(3.68)	(4.42)	
5	0.1527	−0.0010	−0.0035	−0.0321	−0.0001	−0.0011	−1.029	−0.228	−0.957	0.69/2.1
	(3.33)	(1.16)	(0.55)	(2.35)	(1.31)	(1.20)	(14.35)	(7.06)	(6.56)	
6	0.1088	0.0704	−0.0186	0.0019	−0.0012	−0.1044	−1.287	−0.188	−1.632	0.86/2.0
	(2.43)	(3.45)	(0.48)	(0.17)	(1.20)	(4.89)	(15.49)	(3.31)	(9.79)	

Note: Values in parentheses are t-statistics. System weighted $R^2 = 0.9886$. The model transformed by the Cochrane-Orcutt procedure is:

Grouper: $y_{1t} - \Pi x_{1t} = (1 + \underset{(2.58)}{0.53}L)^{-1} e_{1t}$.

Porgy : $y_{2t} - \Pi x_{2t} = (1 + \underset{(8.40)}{0.46}L)^{-1} e_{2t}$.

Snapper: $y_{3t} - \Pi x_{3t} = (1 + \underset{(10.63)}{0.55}L)^{-1} e_{3t}$.

Jack: $y_{4t} - \Pi x_{4t} = (1 + \underset{(6.25)}{0.42}L)^{-1} e_{4t}$.

Tilefish: $y_{5t} - \Pi x_{5t} = (1 + \underset{(8.56)}{0.57}L)^{-1} e_{5t}$.

Seabass: $y_{6t} - \Pi x_{6t} = (1 + \underset{(5.81)}{0.39}L)^{-1} e_{6t}$.

where numbers in parentheses are t-statistics and lag operator is denoted by L.

[Table 8] Estimated Quantity Adjustment Parameters

Adjustment parameters in a system	Coefficients of error-correction
λ_1	−0.18 (3.75)
λ_2	−0.17 (3.04)
λ_3	−0.20 (4.21)
λ_4	−0.21 (4.17)
λ_5	−0.11 (2.73)
λ_6	−0.05 (1.07)

Note: Values in parentheses are t-statistics

Another version of eq. (58) may be possible, based on the transfer function methodology proposed by Box and Jenkins(1970). We include $\Delta_{12}c_{t-1}$, $\Delta_{12}g_{c,t-1}$, and $\Delta_{12}g_{n,t-1}$ into eq. (58) by purely ad hoc consideration (possibly, better fitting data). We then obtain

$$(59)\quad \overline{w}_t\Delta_{12}y_t = (\Gamma^1+\Gamma^2)\Delta_{12}c_t - \Gamma^2\Delta_1\Delta_{12}c_t + (\theta_1^1+\theta_1^2)\Delta_{12}g_{ct} - \theta_1^2\Delta_1\Delta_{12}g_{ct} + (\theta_2^1+\theta_2^2)\Delta_{12}g_{nt} - \theta_2^2\Delta_1\Delta_{12}g_{nt} + \lambda(w_{t-12} - Ac_{t-12})$$

by using the relation that $\Delta_{12}x_{t-1} = \Delta_{12}x_t - \Delta_1\Delta_{12}x_t$, and the superscripts 1 and 2 indicate the coefficients of the original variables and those of the new included variables, respectively.

As seen in the first differenced model, the error-correction term (residual of long-run inverse demand) in eq. (58) should be stationary to yield reliable estimates of the demand parameters. The seasonal unit root testing procedure developed in Hylleberg et al.(1990) and extended to the case of monthly data by Franses(1991) and Beaulieu and Miron(1993) is used in this section. Table 6 reports the outcome of the seasonal unit root tests for the variables in the error-correction term. Note that significance at all frequencies but zero frequency implies no seasonal unit roots. Using the critical values in Franses(1991), seasonal unit roots are rejected although the evidence for w_2 is not overwhelming. Since non-stationary stochastic seasonality is not important feature, the system will be stable.

Next we estimate eq. (58) and the estimated result is shown in Table 7. According to the table, 5 of 6 own price flexibilities are estimated negatively and with a high degree of precision. For the off-diagonal elements, only 12 of 30 are significantly different from zero and 12 among 30 are substitutes, which is more than that of the seasonal dummy model. Considering the significant coefficients, it appears that four groups of fishes are substitutes, i. e., Jacks and Tilefishes; Tilefishes and Seabasses; and that 12 groups are complements - Groupers with Snappers, Jacks, Tilefishes, and Seabasses; Porgies with Seabasses. Note also that the estimated coefficients of the error correction terms are shown in Table 8, which shows that five of six coefficients are statistically different from zero while three of six were significant in the first-differenced model. Thus, the system appears not to be expressed in terms of differenced variables alone. It implies that dynamic factors are important.

V. CONCLUDING REMARK

The paper proposed a new specification of marginal willingness-to-pay functions in a complete system, that is, the synthetic inverse demand system (SIDS). This system was be used for a specification test that can deal with non-nested models with different dependent variables. In fact, it tests whether mixing parameters to combine the null models are jointly and significantly different from zero. Among various models, the inverse CBS demand system shows the strongest empirical performance.

Using the SIDS system as a static differential price formation model, the paper generalized it dynamically incorporating habit formation. The structure of the model is similar to that of Anderson and Blundell(1983). However, it is different from theirs in two respects. One is that this paper derives marginal willingness-to-pay functions while they use ordinary demand functions. The other is that this study develops a general system in the static and dynamic context while they take the IAIDS system arbitrarily. The empirical results shows strong evidence of importance of dynamic factor, especially for the seasonally differenced dynamic model.

Our framework and approach should give an interest to policymakers because environmental quality change or natural resource regulation is done by change in quantity, which is exogenous in our model and now a policy variable. Thus, change in consumer welfare can be easily measured and analyzed by policymakers. However, the paper didn't deal with forecasting performance and welfare measurement of its proposed model, leaving it to the future agenda.

APPENDIX

The dual problem of minimizing indirect utility with respect to prices subject to budget constraint yields inverse demand functions. The first order conditions for this problem are:

$$\text{(A1)} \quad V_p = \lambda q,$$

where λ is a positive Lagrangian multiplier and V denotes an indirect utility function. Together with budget constraint, these are solved for λ and p. The latter solutions give the uncompensated inverse demand functions

$$\text{(A2)} \quad p_i = g_i(q_1, \cdots, q_n, m), \quad i = 1, \cdots, n,$$

or using the fact that g_i is linearly homogeneous in m,

$$\text{(A3)} \quad v_i = g_i(q_1, \cdots, q_n), \quad i = 1, \cdots, n,$$

where v_i is a normalized price of commodity i, i. e., $v_i = p_i/m$. A simpler primal method to obtain this result is to apply the Hotelling-Wold identity which is analogous to Roy's identity in ordinary demand systems:

$$\text{(A4)} \quad p_i = g_i(q_1, \cdots, q_n, m) = \left(\frac{\dfrac{\partial U(q)}{\partial q_i}}{\sum_j q_j \cdot \dfrac{\partial U(q)}{\partial q_j}} \right) m,$$

which is an explicit representation of (A2) in terms of the direct utility function.

In order to derive the Antonelli equation, analogous to the Slutsky equation, we let s be a scale variable such that we obtain a reference quantity vector $q^* = q/s$ using the scale variable (s) to deflate a quantity vector q. Accordingly, the Hotelling-Wold identity can be expressed as a function of q^* and s such that

$$\text{(A5)} \quad v = g(q) = h(s, q^*),$$

where v is an $(n \times 1)$ vector of normalized prices and q^* is an $(n \times 1)$ vector of reference quantities. Its differential form can then be written as

$$\text{(A6)} \quad dv_i = \sum_j h_{ij} dq_j^* + h_{is} ds,$$

where $h_{ij} = \partial h_i / \partial q_j$ and $h_{is} = \partial h_i / \partial s$. It appears that change in s must compensate for change in q_j^* so as to achieve the same utility level. Thus, totally differentiating the direct utility function $U(sq^*)$, we can write:

$$\text{(A7)} \quad dU = \sum_i \left[\left(\frac{\partial U}{\partial q_i} \right) q_i^* ds + \left(\frac{\partial U}{\partial q_i} \right) sdq_i^* \right].$$

In order to find the change in scale, ds, to compensate for an arbitrary change in q_j, dq_j, we set $dU = 0$ and $dq_i = 0$ for $i \neq j$, to obtain:

$$\text{(A8)} \quad ds = -\left[\frac{\dfrac{\partial U}{\partial q_j}}{\sum_i q_i \left(\dfrac{\partial U}{\partial q_i} \right)} \right] sdq_j^* = -v_j sdq_j^*.$$

Letting $s = 1$ and inserting (A8) into (A6), we obtain

$$\text{(A9)} \quad \left(\frac{dv_i}{dq_j} \right)_U = h_{ij} - h_{is} v_j,$$

which is called the Antonelli equation, analogous to the Slutsky equation in the ordinary demand system. In flexibility terms, we can rewrite

$$\text{(A10)} \quad b_{ij}^* = b_{ij} - k_i w_j,$$

where b_{ij} is the uncompensated price flexibility of good i for good j, k_i the scale elasticity of good i, and w_j the share of expenditure on good j.

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