

INTERTEMPORAL PRICE DISCRIMINATION UNDER THE MOST-FAVORED-CUSTOMER POLICY*

IN SUNG CHO**

I. INTRODUCTION

When a seller sells his product over time to consumers with different valuations for the product, the seller intends first to charge high price to extract the surplus of high-valuation consumers and then to reduce the price to attract low-valuation consumers. However, a strategic consumer, expecting the price reduction in the future, may delay the purchase, refusing to pay a high price now.

A monopolistic seller facing this situation loses his monopoly power. Considering strategic behavior of consumers, Coase (1972) conjectured¹⁾ that a durable-goods monopolistic seller results in the competitive outcome: a strategic consumer refuses to pay a price greater than the marginal cost, expecting the seller to eventually sell the product at the marginal cost level (or at the minimal valuation of consumers).

One of the ways to avoid the Coase problem is, as Coase suggested, that the monopolist credibly commits himself not to reduce the price (or not to produce any more of the good) in the future.²⁾ This full commitment forces consumers not to wait until next period; if consumers believe it, then the high-valuation consumers will buy the product in the first period as long as the price does not exceed their valuation.

Another way is to use the most-favored-customer (MFC) policy; a guarantee of a seller to reimburse the first-period buyers the difference between the first-pe

* This is a revised version of the paper presented at the annual conference of the Korean Economic Association, February 1996.

** Department of Economics, Kongju University. The author is grateful to anonymous referees for their comments and suggestion.

¹ Coase conjecture was later formally verified in the literature such as Stokey (1981), Bulow (1982), Gul et al. (1986). Bagnoli et al. (1989) show that Coase conjecture and propositions in Stokey (1981) and Bulow (1982) are invalid with discrete demand.

² See Tirole (1988, pp. 83-86) for an overview of the ways of evading the Coase problem.

riod price and the second-period price, if he reduces the price in the second period. Under the MFC policy, a seller tends not to reduce the price. The MFC policy, however, is not a full commitment of the seller not to reduce the price. If he believes that, in the second period, a large number of potential buyers will buy the product for a lower price, he may find it profitable to reduce the price in spite of the refund, he then pays back to the first-period buyers. The MFC policy can be thought as a partial commitment not to reduce the price in the future. By adopting the MFC policy, a seller provides an incentive to the (high-valuation) consumers not to wait until the price reduces.

In some literature the price discrimination is explained in terms of sales behavior of firms. Varian (1980), considering a monopolistically competitive market, interprets the mixed strategy equilibrium explicitly solved as that much of the time firms charge high regular price to extract the surplus of uninformed consumers and from time to time have sales to keep their business, but rarely sell the products at an intermediate price. Lazear (1986) shows that a monopolist who does not know valuation of consumers charges first high price and learns about the demand, and then adjusts the price lower. But it is not their interests to investigate the effect of the MFC policy.

We know that there are sellers who use the MFC policy reduce their prices. This paper is an attempt to explain why a seller has an incentive to reduce his price under the MFC policy. We show that an equilibrium exists in which the seller adopts the MFC policy and reduces the price with positive probability. In a two-period game theoretic model, we consider one seller and a continuum of consumers with a unit demand. There are two types of consumers; type H and type L. The proportion of consumers of a type is selected by nature and no one observes this selection. Every consumer knows his type only. The H-type consumers value the product in the first period more than the L-type consumers. The L-type consumers are perfectly patient and the H-type consumers have some degree of impatience. This means that the preference of the H-type consumers varies over time, while that of the L-type consumers does not change. We consider both cases where, in the second period, an H-type consumer values the product either less or more than the valuation of an L-type consumer. We refer to the former case as the case of a negative correlation of valuation. We refer to the case where an H-type consumer values the product more than an L-type consumer in each of the two periods as the case of a positive correlation of valuation. In each case, the MFC policy will be compared with the ordinary-sale (OS) policy, which refers to a selling policy under which every price paid is final. It will be also compared with the full commitment (FC) policy, under which a seller commits himself not to change the price in the second period.

II. THE MODEL AND PRELIMINARY OBSERVATIONS

2.1 The Model

There are one seller, denoted by S , and a continuum of consumers. The seller has zero cost. The set of consumers is denoted by a unit interval $I = [0, 1]$. There are two periods and each consumer purchases either zero or one unit of the product during two periods. There are two types of consumers. A consumer of type H values the product k in the first period and $1 + \varepsilon$ in the second period (ε may take a negative or a positive value). A consumer of type L values the product 1 in each of the two periods. We assume that $k > \max\{1, 1 + \varepsilon\}$ and $1 + \varepsilon > 0$.³⁾ Observe that, in this model, the preference of an H -type consumer varies with time.⁴⁾ We can interpret ε as the level of patience of a consumer of type H . The seller and the consumers of type L are perfectly patient.

Let us describe now the sequence of moves. At first the seller selects one selling policy, either the MFC policy or the OS policy or the FC policy, and announces it publicly. Next, under each policy chosen, nature selects the type of each consumer in the following way.

Let T be a random variable with a probability density function $f(t)$ over $(0, 1)$. It is assumed that $f(t)$ is continuous and $f(t) > 0$, for all $t \in (0, 1)$. The distribution function of T is denoted by F , namely, $F(t) = \int_0^t f(t)dt$ for $t \in (0, 1)$. Let $F(0)=0$ and $F(1)=1$. Then, F is continuous on $[0, 1]$. For simplicity, we assume that T is uniformly distributed. Nature selects a realization t of T according to $f(t)$. Then every consumer is selected to be of type H with probability t and of type L with probability $1-t$. This is done independently across consumers. Thus we can interpret t as the proportion of the H -type consumers and $1-t$ is the proportion of the L -type consumers. Each consumer is informed of his

³⁾ Let a_1 and $a_2 + \xi$ be the values to the H -type consumers of the product in the first and the second period respectively, and $a_2 > 0$ be that of the L -type consumers in both periods. Assume that $a_1 > \max\{a_2, a_2 + \xi\}$, $a_2 + \xi > 0$, and $a_2 > 0$. Fix $a_2 = 1$. Let $k \equiv a_1/a_2$ and $\varepsilon \equiv \xi/a_2$. We do not lose any generality by this normalization, in the sense that the qualitative results will depend on the ratios of k and ξ to a_2 , but not on the absolute value of a_2 .

⁴⁾ One may interpret this demand structure as one in which two consumers with different types have different discount factors. In a more general setting, one can assume that an L -type consumer has the discount factor $\delta \equiv \delta_L$ and an H -type consumer δ_H , and that δ_H may or may not be the same as δ . Then the second-period consumption is worth $\delta_H \cdot k$ and $\delta \cdot 1$ to an H -type consumer and an L -type consumer, respectively. Let $\delta_H = \frac{\delta + \varepsilon}{k}$ with $-\delta < \varepsilon \leq k - \delta$. Then, clearly, δ_H and ε have one-to-one relationship each other, for any given k and δ . In the special case where $\varepsilon = \delta(k - 1)$, we have $\delta_H = \delta$. In this paper, for simplicity, we are assuming that $\delta = 1$ and $\delta_H = \frac{1 + \varepsilon}{k}$ with $-1 < \varepsilon < k - 1$. As ε gets sufficiently close to $k - 1$, δ_H approaches to 1.

own type but not of the others'. Neither the seller nor the consumers can observe the realization t of T .

The seller sets a price p_1 in the first period, without knowing the realization t of T . The consumers observe p_1 and decide, simultaneously and independently, whether or not to buy the product. At the end of the first period, the seller observes the proportion, $\tau \in [0, 1]$, of buyers in the first period. No consumer observes τ .

In the second period, the seller sets a price $p_2 = p_2(\tau)$ depending on the proportion of the first-period buyers. The remaining consumers, who did not buy the product in the first period, observe p_2 and then decide, simultaneously and independently, whether or not to buy the product.

We refer to the MFC game as the game with the MFC policy, to the OS game as the game with the OS policy, and to the FC game as the game with the FC policy.

In the MFC game, given the proportion τ of the first-period buyers, the payoff u_s^M of the seller is

$$u_s^M = \tau \min\{p_1, p_2\} + \hat{\tau} p_2,$$

where $\hat{\tau}$ is the proportion of buyers in the second period. The payoff u_H^M of an H-type consumer is

$$u_H^M = \begin{cases} k - \min\{p_1, p_2\} & \text{if H buys in the first period} \\ (1 + \varepsilon) - p_2 & \text{if H buys in the second period} \\ 0 & \text{if H does not buy.} \end{cases}$$

The payoff u_L^M of an L-type consumer is similarly defined except that each of the numbers k and $1 + \varepsilon$ is replaced by 1.

In the OS game, given the proportion τ of the first-period buyers, the payoff u_s^O of the seller is

$$u_s^O = \tau p_1 + \hat{\tau} p_2$$

and the payoff u_H^O of an H-type consumer is

$$u_H^O = \begin{cases} k - p_1 & \text{if H buys in the first period} \\ (1 + \varepsilon) - p_2 & \text{if H buys in the second period} \\ 0 & \text{if H does not buy.} \end{cases}$$

The payoff u_L^O of an L-type consumer is similarly defined except that each of the

numbers k and $1 + \varepsilon$ is replaced by 1.

The payoffs in the FC game are similarly defined as in the OS game, except that p_2 is replaced by p_1 .

We consider symmetric pure strategy equilibria, in which every consumer of the same type takes the same action (to buy or not to buy) in a pure way.

The first-period decision (to buy or to wait) of a consumer depends on p_1 and on his expectation about the second-period price. Thus, to solve for an equilibrium we must consider consumers' expectations about the second-period price. In calculating their expectations, the consumers will be using their posterior probability density of T , depending on their own type.

It is common knowledge that the proportion t of the H-type consumers is selected by nature at random according to the probability density function $f(t)$. Every consumer will update it after he is informed of his type. Let

$$E(T) = \int_0^1 s f(s) ds$$

and

$$f_H(t) = \frac{1}{E(T)} t f(t).$$

This is an H-type consumer's posterior probability density of T . Indeed, let

$$\begin{aligned} F_H(t) &= \Pr(T \leq t | \text{type H}) = \frac{\int_0^t \Pr(H|s) f(s) ds}{\int_0^1 \Pr(H|s) f(s) ds} = \frac{\int_0^t s f(s) ds}{\int_0^1 s f(s) ds} \\ &= \frac{\int_0^t s f(s) ds}{E(T)}. \end{aligned}$$

Consequently, $f_H(t) = F'_H(t) = t f(t) / E(T)$. Similarly, for an L-type consumer,

$$F_L(t) = \Pr(T \leq t | \text{type L}) = \frac{\int_0^t (1-s) f(s) ds}{1 - E(T)},$$

and therefore

$$f_L(t) = \frac{1}{1 - E(T)} (1-t) f(t).$$

Since we are assuming that T is uniformly distributed, $F_H(t) = t^2$ and $F_L(t) = 1 - (1-t)^2 = t(2-t)$.

2.2 Preliminary Observations

We present here some preliminary observations that will be used in our subsequent analysis. The results presented here hold for every ε such that $1 + \varepsilon > 0$.

2.2.1 The OS Game

The following results are straightforward.

Lemma 2.1 *Consider an equilibrium of the OS game. Then,*

- (1) $p_2(\tau)$ is either $1 + \varepsilon$ or 1 for every $\tau \in [0, 1)$.
- (2) If $p_1 > 1$, then no L-type consumer buys the product in the first period.
- (3) If $p_1 \leq 1$, then every H-type consumer buys the product in the first period.

Lemma 2.1(1) enables us to restrict the second period prices of the seller to $1 + \varepsilon$ and 1, under the OS policy.

2.2.2 The MFC Game

Lemma 2.2 (1) *In every equilibrium of the MFC game with nonempty set of second-period buyers, $p_2(\tau)$ is either p_1 or $1 + \varepsilon$ or 1 for every $0 \leq \tau \leq 1$. (2) If no one buys in the second period, then $p_2 \geq p_1$. This case is strategically equivalent to the case where $p_2 = p_1$.*

By Lemma 2.2, in finding the second-period equilibrium prices of the MFC game, it is sufficient to consider the case where p_2 is either p_1 or $1 + \varepsilon$ or 1.

Next, let us consider the actions of the consumers. Under the MFC policy, given a first-period price, the seller is less inclined to reduce the price in the second period. However, the consumers may want to buy the product at a price higher than the one without the MFC policy, in expectation of price reduction.

Lemma 2.3 *Consider an equilibrium of the MFC game.*

- (1) If $p_1 < k$, then every H-type consumer buys the product in the first period.
- (2) If $p_1 > 1$, then no L-type consumer buys the product in the first period.⁵⁾

⁵ In a non-symmetric pure strategy equilibrium, in which the consumers of the same type may use different pure strategy, Lemma 2.3(2) is not trivial. The reason is as follows. If an L-type consumer believes that the second-period price is 1 with probability 1, then, under the MFC policy, his expected payoff is zero if he buys the product either in the first period or in the second period. In this case, some proportion of L-type consumers may buy the product in the first period.

Definition: Let e be a real number such that $1 + e \geq 0$ and let $\bar{m} = \max\{1 + e, 1\}$. Define a real-valued function π on (\bar{m}, ∞) by

$$\pi(p) = \int_{\frac{1}{p}}^1 tp f(t) dt + \int_1^{\frac{1}{p}} f(t) dt.$$

Since we are assuming the uniform distribution,

$$\pi(p) = \frac{p^2 + 1}{2p}.$$

The following lemma will be occasionally used in our analysis of the games.

Lemma 2.4 *For every $p > \bar{m}$ and for each given e such that $1 + e \geq 0$, we have $\pi(p) > \max\{(1 + e)E(T), 1\} = \max\left\{\frac{1 + e}{2}, 1\right\}$ and $\pi'(p) > 0$.*

Next, we consider the expected payoff the seller obtains when he charges $p_1 < k$.

Consider an equilibrium of the MFC game. Suppose first that $p_1 \leq 1$. Then by Lemma 2.3 all H-type consumers buy the product in the first period, and hence the seller makes at most 1 from each of the H-type consumers. Since any L-type consumer is not willing to pay more than 1, the seller can obtain at most 1 from each one of the L-type consumers. Thus, by charging $p_1 \leq 1$, the seller obtains at most 1.

Suppose next that the seller charges p_1 such that $1 < p_1 < k$ and observes τ . Since $1 < p_1 < k$, by Lemma 2.3, no L-type consumer will buy the product in the first period, while all H-type consumers will buy the product in the first period. Therefore, if $1 < p_1 < k$, the seller can infer that $t = \tau$. If the seller reduces the second-period price, then he must reduce it to 1 to attract all the remaining consumers, who are all of type L. If the price is lowered to 1 from p_1 , the first-period buyers will get a refund of $p_1 - 1$, and hence they will end up paying 1. In this case the seller makes 1 from each consumer, yielding him the payoff of 1. If the seller does not reduce the price (or equivalently if $p_2 \geq p_1$), there will be no additional buyers at that price as well as no refund, so that the seller obtains τp_1 . Therefore, given p_1 and τ , the seller will reduce the price to 1 only if τp_1 does not exceed 1, namely $\tau \leq \frac{1}{p_1}$. Since, in this case, $t = \tau$, whenever $1 < p_1 < k$, the expected payoff of the seller π^M under the MFC policy is

$$\pi^M(p_1) = \int_{\frac{1}{p_1}}^1 tp_1 f(t) dt + \int_0^{\frac{1}{p_1}} f(t) dt = \frac{p_1^2 + 1}{2p_1} > 1.$$

Since above expression is increasing in p_1 , we have the following.

Lemma 2.5 *Under the MFC policy, if the seller charges p_1 such that $p_1 < k$, then the payoff of the seller is less than $\pi(k) = \frac{k^2 + 1}{2k}$.*

Finally, we show that in equilibrium $\tau > 0$. Suppose to the contrary that $\tau = 0$. Then, the seller learns nothing about the realization t of T . If $\varepsilon > 0$, the seller must charge either $p_1(0) = 1 + \varepsilon$ to sell the product only to the H-type consumers or $p_1(0) = 1$ to sell the product to every consumer in the second period. In this case, the seller obtains $\max\{(1 + \varepsilon)E(T), 1\}$. However, the seller has a better strategy. As discussed above, by charging p_1 such that $\max\{1 + \varepsilon, 1\} < p_1 < k$, the seller obtains $\pi(p_1) > \max\{(1 + \varepsilon)E(T), 1\}$. The inequality follows from Lemma 2.4. If $\varepsilon \leq 0$, the seller gets at most 1. We summarize the above.

Lemma 2.6 *In every equilibrium of the MFC game, $\tau > 0$.*

III. THE ANALYSIS OF THE GAMES

In a symmetric pure strategy equilibrium, consumers of the same type either buy the product in the first period or wait until the second period.

There are three possible kinds of equilibrium outcomes in the first period:

- (i) no consumer buys the product,
- (ii) only H-type consumers buy the product, and
- (iii) all the consumers buy the product in the first period.

Thus, given the proportion t of the H-type consumers, for each first-period price p_1 , the demand for the product is either zero or t or 1.

Notice that the outcome where only L-type consumers buy is not an equilibrium outcome. By Lemma 2.3, L-type consumers buy the product in the first period only if $p_1 \leq 1$, and the H-type consumers buy the product in the first period, if $p_1 < k$. Therefore, if L-type consumers buy the product, then H-type consumers always buy the product.

3.1 The Case of a Positive Correlation of Valuation ($\varepsilon > 0$)

3.1.1 The OS Game

Suppose that $\varepsilon > 0$. We start with the analysis of the actions of the consumers after observing p_1 .

We first claim that in equilibrium $p_1 > 1$. Suppose first that $p_1 < 1$, then clear-

ly every consumer buys the product in the first period and the seller obtains the payoff less than 1. If $p_1 = 1$, then every consumer buys the product and pays 1 (the L-type consumers may buy the product only in the second period). In this case, the seller will obtain 1. We will show that the seller can obtain more than 1 by charging the first-period price $p_1 > 1$.

Suppose that $p_1 > 1$ and that $\tau > 0$, where τ is the proportion of the first-period buyers. By Lemma 2.1, all of the first-period buyers must be of type H. Therefore, the seller can infer that $t = \tau$.⁶ In this case, only the L-type consumers are the potential buyers in the second period, and hence the seller will charge $p_2 = 1$ for any $\tau > 0$. Thus an H-type consumer believes that $p_2 = 1$ with probability 1. Since $p_2 = 1$ for any $\tau > 0$, an H-type consumer obtains the payoff of ε if he buys the product in the second period and obtains $k - p_1$ if he buys the product in the first period. Therefore, the first-period equilibrium price must be $p_1 = k - \varepsilon$ and the payoff of the seller π^0 under the OS policy is

$$\pi^0 = (k - \varepsilon)E(T) + E(1 - T) = (k - \varepsilon + 1) \frac{1}{2} = 1 + (k - (1 + \varepsilon)) \frac{1}{2} > 1.$$

In particular, this implies that $p_1 > 1$ in every equilibrium with $\tau > 0$. Furthermore, if p_1 is slightly below $k - \varepsilon$ then $\tau = t$.

We now consider the case where $\tau = 0$. In this case, p_1 is sufficiently high (for example, $p_1 > k$) so that no one buys the product in the first period. When $\tau = 0$, the seller learns nothing about the realization t of T . In the second period the seller charges either $p_2(0) = 1 + \varepsilon$ to sell the product to only the H-type consumers or $p_2(0) = 1$ to attract every consumer. The seller obtains, on average, $(1 + \varepsilon)/2$ (if he charges $p_2(0) = 1 + \varepsilon$) and 1 (if he charges $p_2(0) = 1$). Thus the seller will charge $p_2(0) = 1 + \varepsilon$ if $\varepsilon > 1$. If $\varepsilon \leq 1$, the seller charges $p_2(0) = 1$ and obtains 1. However, in this case, the seller can obtain more than 1 by reducing the first-period price p_1 to $k - \varepsilon$, as we have seen above. We conclude that if $\tau = 0$ in equilibrium, then it must be that $\varepsilon > 1$. Therefore, the seller obtains $\hat{\pi}^0 \equiv (1 + \varepsilon)E(T) = (1 + \varepsilon)/2$, which is greater than 1. Notice that this case is possible only when $k > 2$, since $\varepsilon < k - 1$.

The seller can guarantee to obtain a payoff as close to $\pi^0 = (k - \varepsilon + 1)/2$ as he wishes by charging a price p_1 sufficiently close from below to $k - \varepsilon$. The seller can guarantee to obtain the payoff $\hat{\pi}^0 = (1 + \varepsilon)/2$ by charging p_1 high enough to

⁶ In a non-symmetric pure strategy equilibrium, we are not sure that $t = \tau$. If the H-type consumers are indifferent between buying the product in the first period and waiting until the second period, some proportion of the H-type consumers may wait until next period. In this case, $t > \tau$.

cause $\tau = 0$. Therefore, the seller will compare the two payoffs π^o and $\hat{\pi}^o$ and will choose p_1 accordingly. It is easy to show that if $k > 2$ then

$$\pi^o < \hat{\pi}^o \Leftrightarrow \varepsilon > \frac{k}{2}.$$

Observe that $\frac{k}{2} < k - 1$ iff $k > 2$. Consequently, the equilibrium of the OS game with $\varepsilon > 0$ is one of the followings; the choice of the equilibrium depends on ε , given k .

(E1) The equilibrium where

$$p_1 = k - \varepsilon, \text{ and} \\ p_2(\tau) = 1, \quad \text{for any } \tau > 0.$$

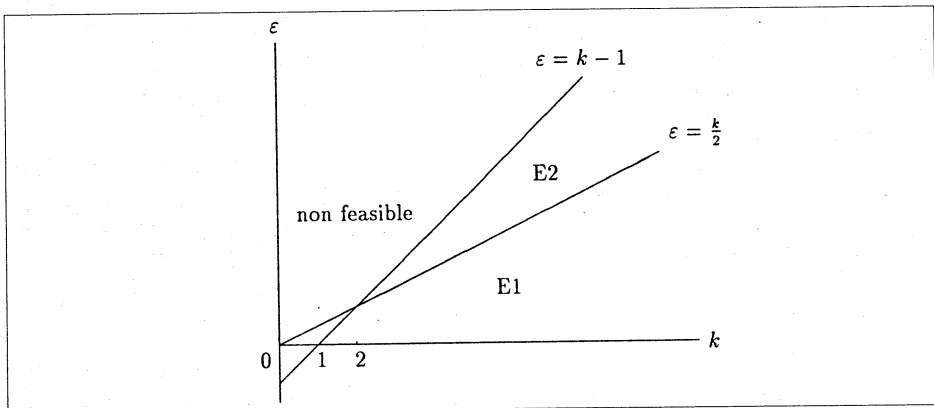
In this equilibrium, every H-type consumer purchases the product in the first period, and every L-type consumer purchases the product in the second period. The seller's payoff is

$$\pi^o = (k - \varepsilon) E(T) + E(1 - T) = (k - \varepsilon + 1)/2.$$

(E2) Any p_1 sufficiently high to cause $\tau = 0$, followed by $p_2(0) = 1 + \varepsilon$. In this equilibrium, every H-type consumer buys the product in the second period, while every L-type consumer does not buy the product in neither one of the two periods. The seller's payoff is

$$\hat{\pi}^o = (1 + \varepsilon) E(T) = (1 + \varepsilon)/2.$$

[Figure 1] The equilibrium of the OS game with $\varepsilon > 0$



Proposition 3.1 *Suppose that $\varepsilon > 0$. If $\varepsilon < \frac{k}{2}$, then (E1) is the equilibrium of the OS game. If $\varepsilon > \frac{k}{2}$ with $k > 2$, then (E2) is the equilibrium of the OS game.*

This result is illustrated in Figure 1.

Proposition 3.1 asserts that there are two types of symmetric pure strategy equilibrium. The first one is the equilibrium where just all the H-type consumers purchase the product in the first period for the price of $k - \varepsilon$. Each buyer obtains a payoff of ε , anticipating the second-period price to drop to 1 to attract the L-type consumers.

The second type of equilibrium is one where no consumers buy the product in the first period. When the monopolist sells his product during two periods, he is in the self-competitive situation. If the seller sells the product in the first period, then he loses the demand in the second period. In other words, a consumer's purchase in the first period is a substitute for his purchase in the second period. Under the OS policy, when an H-type consumer observes a first-period price greater than 1, an H-type consumer expects that the seller will reduce the price to 1 in order to sell the product to the remaining L-type consumers. Thus, an H-type consumer refuses to pay more than $k - \varepsilon$ in the first period, eroding the seller's profit. On the other hand, high value of ε implies that an H-type consumer is willing to pay high price in the second period. Therefore, if ε is sufficiently high the seller finds it more profitable to increase his first-period price to deter any consumer from buying the product in the first period. In this case he will charge in the second period $1 + \varepsilon$ and just the H-type consumers will buy.

3.1.2 The MFC Game

Suppose that $\varepsilon > 0$. By Lemma 2.6, $\tau = 0$ is not an equilibrium outcome. By Lemma 2.5, the seller obtains less than $\frac{k^2 + 1}{2k}$, if he charges $p_1 < k$. We will show that the seller can obtain more than this by charging $p_1 \geq k$.

Suppose that the seller charges $p_1 \geq k$ and that he observes $\tau > 0$. Since $p_1 > 1$, by Lemma 2.3, no L-type consumer buys the product in the first period. That is, any buyer in the first period must be of type H. Hence, in a symmetric equilibrium, all the H-type consumers buy the product at p_1 and the seller infers that $t = \tau$. Therefore, the seller will reduce the price to 1 if $\tau p_1 < 1$, otherwise the seller will not reduce the price. Let

$$\tau_1 = \frac{1}{p_1}.$$

Then, the second-period price can be formally written as

$$p_2(\tau) = \begin{cases} p_1, & \tau \geq \tau_1 \\ 1, & \tau < \tau_1 \end{cases}$$

and the expected payoff of the seller is

$$\pi^M(p_1) = \int_{\tau_1}^1 t p_1 f(t) dt + \int_0^{\tau_1} f(t) dt = \frac{p_1^2 + 1}{2p_1}. \quad (1)$$

By Lemma 2.4 this payoff is increasing in p_1 . Since $p_1 \geq k$, the payoff is at least $\pi^M(k)$. In particular, this together with Lemma 2.5 implies that a first-period price p_1 such that $p_1 < k$ is inferior for the seller to the price $p_1 = k$.

Next we examine the first-period action of an H-type consumer after observing p_1 . Consider the subjective probability of an H-type consumer that the second-period price will be reduced to 1. Since in equilibrium $t = \tau$, it is $\Pr(T \leq \tau_1 \mid \text{type H}) = F_H(\tau_1)$, where $\tau_1 = 1/p_1$. Recall that $F_H(\tau_1) = \tau_1^2$, since we are assuming the uniform distribution.

A consumer of type H will buy the product at p_1 only if

$$(1 - F_H(\tau_1))(k - p_1) + F_H(\tau_1)(k - 1) \geq F_H(\tau_1)\varepsilon, \quad (2)$$

which means that the payoff the consumer expects to obtain by purchasing the product in the first period, is at least as high as the maximum payoff he expects to obtain by either purchasing the product in the second period or not purchasing the product at all.

From (2) and from the assumption that $p_1 \geq k$, we have

$$k \leq p_1 \leq \frac{k - F_H(\tau_1)(1 + \varepsilon)}{1 - F_H(\tau_1)}, \quad (3)$$

where $\tau_1 = \frac{1}{p_1}$.

The seller's problem is to choose a first-period price p_1 , in the region defined by (3), as to maximize his payoff given in (1).

Since the payoff $\pi^M(p_1)$ in (1) is increasing in p_1 , the seller must choose a price p_1 such that

$$p_1 = \frac{k - F_H\left(\frac{1}{p_1}\right) (1 + \varepsilon)}{1 - F_H\left(\frac{1}{p_1}\right)}. \quad (4)$$

Notice that $p_1 > k$ and that $0 < \frac{1}{p_1} < 1$. This means that a consumer of type H is willing to pay more than his first-period reservation price (unless $F_H(1/p_1) = 0$, in which case he is sure that the price will not be reduced, and pays exactly k).

Every solution of equation (4) is a first-period equilibrium price outcome. The following lemma shows the first-period equilibrium price is uniquely determined.

Lemma 3.1 *Let e be a real number such that $k > 1 + e$ and $e \geq 0$. Define a real valued function ϕ on $[0, 1]$ by*

$$\phi(t) = \frac{1 - F_H(t)}{k - (1 + e) F_H(t)}.$$

Then, for any k and e , there exists a unique number $t_1(k, e) \in (0, 1)$ such that $t_1 = \phi(t_1)$. Moreover, $t < \phi(t)$ for $t < t_1(k, e)$ and $t > \phi(t)$ for $t > t_1(k, e)$.

The equation (4) can be written as $\frac{1}{p_1} = \phi\left(\frac{1}{p_1}\right)$. Since $k > 1 + \varepsilon$ and $\varepsilon > 0$, by Lemma 3.1, this equation has a unique solution $\frac{1}{p_1(k, \varepsilon)}$ for any k and ε . Further, $p_1(k, \varepsilon)$ is the solution of (4) and hence it is the unique first-period equilibrium price.

We summarize the above.

Proposition 3.2 *Consider an equilibrium of the MFC game with $\varepsilon > 0$. The first-period equilibrium price is the unique solution of the equation*

$$p_1 = \frac{k - \left(\frac{1}{p_1}\right)^2 (1 + \varepsilon)}{1 - \left(\frac{1}{p_1}\right)^2}.$$

The second-period equilibrium price is

$$p_2(\tau) = \begin{cases} p_1, & \tau \geq \frac{1}{p_1} \\ 1, & \tau < \frac{1}{p_1}. \end{cases}$$

In this equilibrium, every H-type consumer buys the product in the first period and every L-type consumer buys the product in the second period only if $p_2 = 1$. The seller's payoff π^M is

$$\pi^M = \int_{\frac{1}{p_1}}^1 t p_1 f(t) dt + \int_1^{\frac{1}{p_1}} f(t) dt = \frac{p_1^2 + 1}{2p_1}.$$

We give an intuitive explanation of this result. The equilibrium outcome for an H-type consumer is the following: The H-type consumer obtains the product for sure in the first period for the price p_1 and he will be reimbursed $p_1 - 1$ if $t < \tau_1$, where $\tau_1 = \frac{1}{p_1}$. He gives up the option to wait until the next period and obtain the product for the price $p_2 = 1$ if the event $t < \tau_1$ occurs. The price p_1 is determined so that the H-type consumer is indifferent between buying the product in the first period and waiting until the second period. Hence the ex-ante value of this outcome to the H-type consumer is k —expected value to H of waiting one period, which is $k - F_H(\tau_1)\varepsilon$.

The outcome for an L-type consumer is that he buys the product only in the second period and only if $t \leq \tau_1$ and he pays the price $p_2 = 1$. The equivalent ex-ante value of this outcome to the L-type consumer is $F_L(\tau_1) \cdot 1$, where $F_L(\tau_1)$ is his subjective probability that the second-period price will be reduced to 1. Recall that $F_L(\tau_1) = \tau_1(2 - \tau_1)$.

Therefore, the expected payoff of the seller is

$$E(T)[k - F_H(\tau_1)\varepsilon] + E(1 - T)F_L(\tau_1).$$

This is, in fact, the equilibrium payoff of the MFC game, since

$$E(T)[k - F_H(1/p_1)\varepsilon] + E(1 - T)F_L(1/p_1) = \frac{p_1^2 + 1}{2p_1}.$$

Next, let us see how equilibrium price and equilibrium payoff change when the parameters, k and ε , change.

Proposition 3.3 Consider the equilibrium of the MFC game with $\varepsilon > 0$. The first-period equilibrium price, $p(k, \varepsilon)$, is strictly increasing in k and strictly decreasing in ε , so is the equilibrium payoff, $\pi^M(p(k, \varepsilon))$, of the seller.

Proposition 3.3 asserts that the more an H-type consumer is patient, the less is he willing to pay in the first period and the higher his valuation is, the more he pays in the first period. Notice also that $\lim_{\varepsilon \rightarrow k-1} p_1 = k$. Thus, if an H-type consumer is perfectly patient, he pays exactly k . By Lemma 2.4, the seller's payoff is strictly increasing in the first-period price. Thus, the seller's payoff decreases as the H-type consumer's patience increases, and it increases as the first-period valuation of an H-type consumer increases.

3.1.3 MFC versus OS Policy

We now compare two policies, the MFC policy and the OS policy. What we have to do is to compare the seller's equilibrium payoff under the OS policy, either π^0 or $\hat{\pi}^0$ given in Proposition 3.1, with the payoff under the MFC policy, π^M given in Proposition 3.2. If ε is sufficiently high (i.e., close to $k-1$), then $\hat{\pi}^0 = \frac{1+\varepsilon}{2}$ is the equilibrium payoff under the OS policy. Lemma 2.4 implies that this payoff is less than the equilibrium payoff π^M under the MFC policy. Thus, we are left to compare the payoff π^0 with π^M . We want to find the necessary and sufficient condition on the parameter ε for which $\pi^M > \pi^0$, for any given $k > 1$.

Lemma 3.2 For each $k > 1$, define a function $h_k: [0, \min\{1, k-1\}] \rightarrow \mathcal{R}$ by

$$h_k(\varepsilon) = \varepsilon^3 - (3+k)\varepsilon^2 - \varepsilon + (k-1).$$

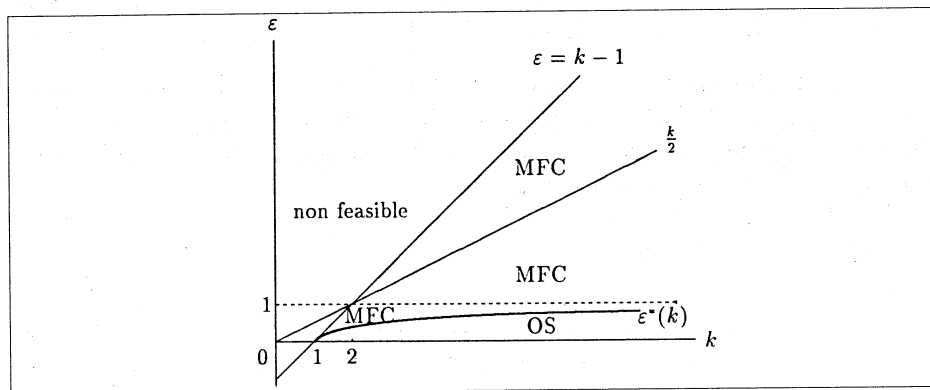
Then, for each $k > 1$, there exists a unique number $\varepsilon^*(k)$ such that $0 < \varepsilon^*(k) < \min\{1, k-1\}$ and $h_k(\varepsilon^*) = 0$. Moreover, $h_k(\varepsilon) < 0$ for $\varepsilon > \varepsilon^*(k)$ and $h_k(\varepsilon) > 0$ for $\varepsilon < \varepsilon^*(k)$, for any given $k > 1$.

With some efforts, one can verify that the number $\varepsilon^*(k)$ serves as the critical point to determine which policy is better than the other. We now present our result.

Proposition 3.4 Suppose that $\varepsilon > 0$. For each $k > 1$, let $\varepsilon^*(k)$ be a number defined in Lemma 3.2. Then, the MFC policy is better for the seller than the OS policy if and only if $\varepsilon > \varepsilon^*(k)$.

Proposition 3.4 asserts that if the H-type consumers are relatively patient (sufficiently high ε), then the seller prefers the MFC policy to the OS policy. Proposition 3.4 also implies that if the H-type consumers are relatively impatient, then the seller prefers the OS policy to the MFC policy. This result is illustrated in Figure 2. In the figure, $\varepsilon^*(k)$ is drawn with positive slope. One can easily verify

[Figure 2] MFC policy versus OS policy



that this is true, using the definition of $\varepsilon^*(k)$. Notice that when ε is sufficiently close to $k - 1$, the MFC policy is better for the seller than the OS policy.

3.2 The Case of a Negative Correlation of Valuation ($\varepsilon \leq 0$)

In this section, we analyze the games with $\varepsilon \leq 0$. The analysis that follows is similar to the analysis of the games with $\varepsilon > 0$. However, there are some different aspects to consider. For example, if $\varepsilon > 0$ then an H-type consumer can enjoy some positive payoff by waiting if the second-period price is reduced to 1, while if $\varepsilon \leq 0$ then an H-type consumer obtains zero by waiting. Therefore, under either policy, when $\varepsilon \leq 0$ an H-type consumer, in the first period, is willing to pay more than he is when $\varepsilon > 0$.

3.2.1 The OS Game with $\varepsilon \leq 0$

As we have seen in the case where $\varepsilon > 0$, we first claim that in equilibrium $p_1 > 1$. Suppose that $p_1 \leq 1$. By Lemma 2.1, the second-period price is either 1 or $1 - |\varepsilon|$. Since $p_1 \leq 1$ and since $p_2(\tau) \leq 1$ for any proportion τ of the first-period buyers, the seller will obtain at most 1. However, it is shown below that the seller can obtain more than 1 by charging a price with $p_1 > 1$.

Suppose that $p_1 > 1$ and that $\tau > 0$. Since $p_1 > 1$, by Lemma 2.1, no L-type consumer will buy the product in the first period, and hence the seller can infer that $t = \tau$. Since only L-type consumers remain in the second period, the seller will charge $p_2(\tau) = 1$ for any $\tau > 0$.

A consumer of type H obtains $k - p_1$ if he buys the product in the first period, while he obtains at most zero if he waits until the second period, since $\varepsilon \leq 0$. Thus, an H-type consumer buys the product in the first period only if $p_1 \leq k$.

Since we are assuming $p_1 > 1$ and $\tau > 0$, p_1 must satisfy $1 < p_1 \leq k$.

In this case, the expected payoff of the seller is $\int_0^1 [tp_1 + (1-t)]f(t)dt = (p_1 + 1)/2$. The seller will choose $p_1 = k$ in the region of p_1 such that $1 < p_1 \leq k$ as to maximize his payoff. Thus the payoff of the seller, in this case, is

$$kE(T) + E(1 - T) = (k + 1)/2 > 1.$$

Particularly, this implies that $p_1 > 1$ in every equilibrium with $\tau > 0$. Furthermore, if p_1 is slightly below k then $\tau = t$.

Finally, let us examine the case where $\tau = 0$. It is clear that this outcome occurs if and only if $p_1 > k$. In this case, the seller learns nothing about the realization t of T . Thus, the seller will charge in the second period either 1 or $1 - |\varepsilon|$, and hence the seller's payoff does not exceed 1, and hence a price p_1 such that $p_1 > k$ cannot be an equilibrium price.

Consequently, $p_1 = k$ is the first-period equilibrium price of the OS game with $\varepsilon \leq 0$, and the seller can guarantee himself to obtain a payoff as close to $\pi^0 = (k + 1)/2$ as he wishes by charging a price p_1 sufficiently close from below to k . We summarize the above.

Proposition 3.5 Suppose that $\varepsilon \leq 0$. The equilibrium prices of the OS game are

$$\begin{aligned} p_1 &= k, \\ p_2(\tau) &= 1, \text{ for any } \tau. \end{aligned}$$

In this equilibrium, every *H*-type consumer purchases the product in the first period, and every *L*-type consumer purchases the product in the second period. The payoff of the seller is

$$\pi^0 = kE(T) + E(1 - T) = \frac{k + 1}{2}.$$

This is a striking result. The monopolistic seller selling his product over time to a continuum of strategic consumers enjoys the complete monopoly power; he extracts all the consumer surplus even though the valuations of consumers are private information of the consumers. This phenomenon is the opposite to the Coase conjecture. A similar counterexample to the Coase conjecture is given in Bagnoli et al. (1989) in a different model.⁷ They provide counterexamples show-

⁷ In Bagnoli et al. (1989), it is assumed that the reservation price of each consumer is common knowledge. In our model, however, the reservation price of each consumer is private information. In this paper as well as Bagnoli et al. (1989), finite types of consumers are considered.

ing that Coase conjecture is not correct, when the assumption of a continuum of consumers is replaced by a finite number of consumers. But our result opposite to the Coase conjecture is derived with a continuum of consumers. Here, the critical factor to derive the opposite case to the Coase conjecture is a negative correlation of valuation ($\varepsilon \leq 0$).

3.2.2 The MFC Game with $\varepsilon \leq 0$

By Lemma 2.6, $\tau=0$ is not an equilibrium outcome. By Lemma 2.5, the seller obtains less than $\pi(k) = \frac{k^2+1}{2k}$, by charging $p_1 < k$. This case will be ruled out in equilibrium.

Suppose that $p_1 \geq k$ and that $\tau > 0$. Since $p_1 > 1$, no L-type consumer buys the product in the first period and all the consumers are of type H, in a symmetric pure strategy equilibrium. Thus, in equilibrium, the seller can infer that $t=\tau$. Since only the L-type consumers are left in the second period, the seller will charge either p_1 to avoid refund or 1 to sell to the L-type consumers at the cost of refunding $p_1 - 1$ to the H-type consumers who bought the product in the first period. The seller will choose $p_2(\tau) = p_1$ only if $\tau p_1 \geq 1$, otherwise it will reduce the price to 1. Thus the second-period price can be written as

$$p_2(\tau) = \begin{cases} p_1, & \tau \geq \tau_1 \\ 1, & \tau < \tau_1 \end{cases}$$

where $\tau_1 = \frac{1}{p_1}$.

Next, we examine the behavior of an H-type consumer in the second period. Since $\varepsilon \leq 0$, an H-type consumer obtains zero if he waits until the second period. Thus an H-type consumer will buy the product in the first period only if

$$[1 - F_H(\tau_1)](k - p_1) + F_H(\tau_1)(k - 1) \geq 0.$$

Thus, p_1 must satisfy

$$k \leq p_1 \leq \frac{k - F_H(\tau_1)}{1 - F_H(\tau_1)}, \quad (5)$$

where $\tau_1 = \frac{1}{p_1}$. Notice that the right-hand side of (5) is greater than k .

Since in equilibrium $t = \tau$, the payoff of the seller is

$$\pi^M(p_1) = \int_{1/p_1}^1 t p_1 f(t) dt + \int_1^{1/p_1} f(t) dt.$$

Since $p_1 \geq k$, by Lemma 2.4, this payoff is at least $\pi(k)$. In particular, this implies that a price p_1 such that $p_1 < k$ is inferior to a price p_1 satisfying (5).

The seller's problem is to choose p_1 as to maximize his payoff, in the region defined in (5). Since the payoff is increasing in p_1 , the seller will choose p_1 such that

$$p_1 = \frac{k - F_H\left(\frac{1}{p_1}\right)}{1 - F_H\left(\frac{1}{p_1}\right)}. \quad (6)$$

The equation (6) can be written as $\frac{1}{p_1} = \phi\left(\frac{1}{p_1}\right)$ with $e=0$. By Lemma 3.1 (with $e=0$), there is a unique solution $\frac{1}{p_1}$. This implies that p_1 is the first-period equilibrium price. We summarize the above.

Proposition 3.6 *Consider an equilibrium of the MFC game with $\varepsilon \leq 0$. The first-period equilibrium price is the unique solution of the equation*

$$p_1 = \frac{k - F_H\left(\frac{1}{p_1}\right)}{1 - F_H\left(\frac{1}{p_1}\right)}.$$

The second-period equilibrium price is

$$p_2(\tau) = \begin{cases} p_1, & \tau \geq \frac{1}{p_1} \\ 1, & \tau < \frac{1}{p_1}. \end{cases}$$

In this equilibrium, every H-type consumer buys the product in the first period and every L-type consumer buys the product in the second period only if $p_2=1$. The seller's payoff π^M is

$$\pi^M = \int_{\frac{1}{p_1}}^1 t p_1 f(t) dt + \int_0^{\frac{1}{p_1}} f(t) dt = \frac{p_1^2 + 1}{2p_1}.$$

3.2.3 MFC versus OS Policy

As we discussed earlier, when $\varepsilon \leq 0$, the seller extracts all the consumer surplus under the OS policy. Thus we can conclude that the OS policy is optimal for the seller. We give another intuitive explanation. The equilibrium price under the MFC policy must satisfy (6). From (6), we have

$$[1 - F_H(1/p_1)]p_1 + F_H(1/p_1) \cdot 1 = k,$$

which means that the expected value of the price that the H-type consumers end up paying is k . That is, under the MFC policy, the seller obtains on average k from every H-type consumer. This is also the case under the OS policy. But, under the MFC policy, the seller loses the L-type consumers with positive probability, because the seller does not reduce the price in the second period with positive probability. On the other hand, under the OS policy, the seller for sure sells the product to the L-type consumers. Therefore, the OS policy is better for the seller than the MFC policy.

This can be rigorously proved by showing that

$$\pi^M = E(T)k + F_L(1/p_1)E(1-T) < E(T)k + E(1-T) = \pi^O.$$

Proposition 3.7 Suppose that $\varepsilon \leq 0$. Then, the OS policy is better for the seller than the MFC policy.⁸⁾

3.3 The FC Game

The analysis of the FC game is obvious. Under the FC policy, the seller uses one price over the two periods. That is, $p_2 = p_1$. In this case, no consumer has incentive to wait until the next period. This implies that in the first period the consumers are willing to pay up to their reservation price. Therefore, it is clear that, in the first-period, the seller will charge either k (to attract only H-type consumers) or 1 (to attract all the consumers).

If $p_1 = k$, every H-type consumer buys the product and no L-type consumer buys the product, and hence the seller obtains $E(T)k = \frac{k}{2}$. If $p_1 = 1$, every consumer buys the product and hence the seller obtains 1. Therefore, if $k > 2$, then the seller will choose $p_1 = k$ and if $k < \frac{1}{E(T)}$, then the seller will choose $p_1 = 1$.

⁸⁾ Proposition 3.7 holds for any distribution of T . This implies that even when the proportion of the H-type consumers is extremely likely to be large and every H-type consumer is willing to pay more than k , the MFC policy is never better than the OS policy, if $\varepsilon \leq 0$.

The seller can guarantee to obtain a payoff sufficiently close to $\frac{k}{2}$ by charging a price slightly below k . Also, the seller can guarantee to obtain a payoff sufficiently close to 1, by charging a price slightly below 1.

Notice that the above argument does not depend on ε . We summarize the above.

Proposition 3.8 *Consider an equilibrium of the FC game. The equilibrium payoff of the seller is $\pi^c = \max\{k E(T), 1\} = \max\{\frac{k}{2}, 1\}$.⁹⁾*

3.4 The Comparison of the Three Policies

We now compare the three policies: the MFC policy, the OS policy, and the FC policy. Under the MFC policy, the equilibrium payoff is of the form $\frac{p_1^2 + 1}{2p_1}$ and the equilibrium price p_1 is at least k . Thus, clearly, the following result holds (by Lemma 2.4 and Proposition 3.8).

Proposition 3.9 *The MFC policy is better for the seller than the FC policy.*

Finally, we characterize the optimal policy among the three policies.

Corollary 3.10 *If $\varepsilon \leq 0$, then the OS policy is optimal for the seller.*

This is the consequence of Proposition 3.7 and Proposition 3.9.

Corollary 3.11 *For each $k > 1$, let $\varepsilon^*(k)$ be the number defined in Lemma 3.2. Then,*

- (1) *The MFC policy is optimal for the seller if and only if $\varepsilon > \varepsilon^*(k)$.*
- (2) *The OS policy is optimal for the seller if and only if $\varepsilon < \varepsilon^*(k)$.*

This corollary follows from Proposition 3.4 and Corollary 3.10.

IV. CONCLUSION

In a two-period game theoretic model, where one seller and a continuum of consumers are considered, we compared three selling policies of the seller: the MFC, the OS, and the FC policy. Each consumer is either of type H or of type L. An H-type consumer values the product more than an L-type consumer in the

⁹ The superscript C of π^C stands for 'commitment'.

first period. An L-type consumer is perfectly patient and an H-type consumer has some degree of impatience. We examined the two cases where every H-type consumer has either higher or lower second-period valuation than an L-type consumer.

Our main interest was to investigate if and when it is optimal for the seller to adopt the MFC policy and if the seller really reduces the price under the MFC policy despite of refund. We obtained the following results. Under the OS policy, if the H-type consumers are relatively patient (ε is sufficiently high), the seller's monopoly power is eroded. The extreme case is the case where the H-type consumers are perfectly patient ($k = 1 + \varepsilon$), which is a limit case of our model. In this case, the seller loses all the monopoly power, as Coase conjectured. On the other hand, it was shown that if the H-type consumers are relatively patient the MFC policy is best to the seller; (the Coase problem could be avoided by adopting the MFC policy). As the level, ε , of the patience drops below a critical value, the OS policy becomes the best policy among the three. The FC policy is always inferior to either the MFC policy or to the OS policy. A striking result is that the seller enjoys a complete monopoly power (with respect to the full information case) under the OS policy if and only if there is a negative correlation of valuation ($\varepsilon \leq 0$). In this case, the seller extracts all the consumer surplus and hence the OS policy is optimal. This is the opposite case to the Coase conjecture.

We find that under the OS policy, every consumer pays at most his reservation price. On the other hand, under the MFC policy, the first-period price is always higher than the reservation price, k , of the H-type consumers and it is decreasing with ε . If, as a limit case, the H-type consumers are perfectly patient, then the first period price is exactly k . Our findings also include that under the MFC policy the price is reduced in the second period with positive probability. This means that the MFC policy materializes with positive probability.

We should mention a very closely related paper by Png (1991). Png compares the OS policy with the MFC policy. There is a main difference between Png's approach and our approach. Png deals only with perfectly patient consumers, which is a very significant simplification of our model. Png's finding fits our special limit case where the consumers are perfectly patient. Png assumes that the capacity of the seller is limited and never exceeds the total demand, and its value is common knowledge to every participant. In this paper we assume that there is no capacity constraint and that this is common knowledge.

We have assumed that the seller's production cost is zero. However the results will essentially remain the same under a constant marginal cost.

The MFC policy has also been investigated as a practice to facilitate tacit collusion, in the literature such as Hay (1982), Salop (1986), Cooper (1986), and Schnitzer (1994). There, the MFC policy plays a role to mitigate the competition among the sellers. However, the MFC policy could be used as a means of competition among oligopolistic sellers. That is, consumers may prefer to buy the

product from the seller who provides the MFC policy. In this case, the MFC policy might play a role to deepen the competition among sellers. In the context of intensifying competition among sellers, the examination the MFC policy in a duopolistic market is left for further work.

REFERENCES

- Bagnoli, Mark, Stephen W. Salant, and Joseph E. Swierzbinski (1989), "Durable-Goods Monopoly with Discrete Demand," *Journal of Political Economy*, 97, 1459-1478.
- Bulow, Jeremy I. (1982), "Durable-Goods Monopolists," *Journal of Political Economy*, 90, 314-332.
- Coase, Ronald H. (1972), "Durability and Monopoly," *Journal of Law and Economics*, 15, 143-149.
- Cooper, Thomas E. (1986), "Most-favored-customer pricing and tacit collusion," *Rand Journal of Economics*, 17, 377-388.
- Gul, Faruk, Hugo Sonnenschein, and Robert Wilson (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155-190.
- Hay, George A. (1982), "Oligopoly, Shared Monopoly, and Antitrust Law," *Cornell Law Review*, 67, 439-481.
- Lazear, Edward P. (1986), "Retail Pricing and Clearance Sales," *American Economic Review*, 76, 14-32.
- Png, I.P.L. (1991), "Most-Favored-Customer Protection versus Price Discrimination over Time," *Journal of Political Economy*, 99, 1010-1028.
- Salop, Steven C. (1986), "Practices that (Credibly) Facilitate Oligopoly Coordination," in *New Developments in the Analysis of Market Structure*, eds., Joseph E. Stiglitz and G. Frank Mathewson, Cambridge, Mass: MIT press.
- Schnitzer, Monika (1994), "Dynamic duopoly with best-price policy," *Rand Journal of Economics*, 25, 186-196.
- Stokey, Nancy L. (1981), "Rational expectations and durable goods pricing," *Bell Journal of Economics*, 12, 112-128.
- Tirole, Jean (1988), *The Theory of Industrial Organization*, Cambridge, Mass: MIT Press.
- Varian, Hal R. (1980), "A Model of Sale," *American Economic Review*, 70, 24-41.