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PUBLIC CHOICE IN AN ECONOMY WITH CAPITALIZATION : A DYNAMIC APPROACH

MAN SOO JOO *

This essay analyzes the behavior of an imperfectly mobile voter in the presence of property value capitalization. It is shown in a dynamic model that the voter's ideal public spending level reflects a blend of his own preferences and those of the future occupant of his property. Although the voter does not seek to maximize the value of the property, his behavior confirms more closely to this principle the nearer is the date of his departure from the community or the greater is the chance of early departure when he is uncertain about the future. Finally, the analysis shows under some circumstances that the voters who are liquidity-constrained are more likely to behave as property-value maximizers than those are unconstrained.

I. INTRODUCTION

In the ideally simple Tiebout world (1956), numerous local governments supply a wide variety of public services, and residents with perfect information locate themselves among communities in an efficient manner by comparing their marginal valuation of a public service to its marginal cost. In equilibrium, residents in a community become homogeneous and the public service provision is Pareto-efficient. This Tiebout hypothesis on efficiency has been long analyzed and tested in various models of the local public economy. While empirical studies have shown mixed results, theoretical analyses have generated general consensus. Acting with entrepreneurial behavior such as profit maximization (Sonstelie and Portney, 1978) or property-value maximization (Brueckner, 1983), local governments can often insure the efficient provision of public services in equilibrium. When local governments are passive, equilibrium is not necessarily efficient. It has been generally recognized that reasons for inefficiency are a heterogeneity in the community and property-tax-induced distortion of the housing market.

One recent trend in local public economics research is to investigate the inter-

* Hanyang University, Department of Economics, 1271 Sa-1-Dong, Ansan-Si Kyunggi-Do, 425-791, Korea. The author would like to thank professor Jan Brueckner and an anonymous refree for their valuable comments. All remaining errors are, if any, mine alone.

action between moving and voting in a single model. "Voting-with-one's-feet" influences property values in a local property market, leading to capitalization of public sector variables into property values. Voters may consider this impact of public sector variables on property value in the voting decision. However, some previous studies assume that all landlords are absentee, so that the role of property value in the voting decision cannot be analyzed explicitly (Rose-Ackerman, 1979). Others limit their analyses to long-run equilibrium where the people moving in have the same preferences as current residents, so that the utility maximization problem turns to a property-value maximization problem (Yinger 1982, 1983).

Wildasin(1979) and Sonstelie and Portney(1980) also argue that a property owner has an incentive to vote for the public good level that maximizes the value of his property. The voter then takes the proceeds and moves to a community where public spending is ideal from a pure consumption point of view. While such property-value maximizing behavior would be appropriate in a frictionless world, voters will behave differently when mobility is realistically constrained by job and family considerations. Since voters in this case must actually consume the public goods they vote for, their own preferences for public spending will play a role in the voting calculus along with property value considerations.

To investigate this blending of motives, this paper extends the Tiebout world to a dynamic context, whose aspect previous studies have ignored.[1] In a simple model of voting with property value capitalization and imperfect mobility, we analyze in a formal way the impact of capitalization on the voting decision. The model includes two generations, current and future residents. A current resident owns a house and votes for a public expenditure level to maximize his utility. His mobility is imperfect so that he must stay in the community for some time. His total financial resources are the present value of his income stream and the value of his property, which he will sell when he leaves the community. The property value depends on the level of public services in the community because the future occupant's bid for the property depends on this level. The current resident takes account of property value impacts in voting on the public service level.

This analysis generates a number of insights that improve our understanding of voting behavior at the local level. It is shown that the voting outcome reflects both voter's preferences and those of the future resident, which help determine

[1] Brueckner and Joo(1991) dealt this problem with a simple two-period model where the resident (voter) lives in a community for two full periods after the vote on the level of public good is taken. At the end of the second period, he sells his house and move away. He tries to maximize his utility stream for the two periods and after moving-away subject to budget constraint including the present values of his income stream and his property. This paper extends the model to a continuous-time context, which is a little more realistic. With the continuous-time model, all BJ's results are confirmed and, in addition, the effect of departure timing on the voter's behavior can be analyzed.

the value of the voter's house. Although the voter does not seek to maximize the value of his house, the analysis shows that he is more likely to behave as a property-value maximizer the nearer is the date of his departure from the community or the greater is the chance of an early departure when he is uncertain about the future. These results are derived under the assumption of a perfect capital market. When a more realistic assumption is imposed, namely that borrowing against future income is impossible, a new force favoring property value maximization emerges. A voter constrained by borrowing limits may vote for a public service level closer to the property-value-maximizing level to extract more financial resources from his house. While this analysis is based on homogeneity in housing stock, the results may hold even under an assumption of housing-stock heterogeneity.

In the next section of the paper, we analyze the basic model with voting and moving in a dynamic context, imposing various assumptions. While section Ⅲ analyzes the effect of departure timing, section Ⅳ extends the basic model by introducing a liquidity constraint in the capital market. In the last two sections, we summarize the results of the analysis and discuss empirical implications and qualifications.

Ⅱ. PUBLIC CHOICE IN A ECONOMY WITH PROPERTY CAPITALIZATION

2.1 The Basic Model

To investigate how voters consider the impact of public expenditure on their property value in the voting decision, we build a model with both voting and moving. We assume that a community has two generations of residency, current and future residents, one after the other. Each current resident already owns a house and has a right to vote for the public service level. Future residents treat the public service level as given and bid for houses when the current residents move out. Their bids depend on the public service level in the community.

The current resident utility function is assumed to be $u(x, z)$, where x is consumption of a composite private good with a price of unity and z is consumption of a local public good. The function u is assumed to be concave[2] and twice differentiable. We also assume that there are no externalities among communities associated with the public good and no congestion in consuming the public good. The discounted sum of utilities over the residency period in a community, [0, T], constitutes the resident's objective function.[3] This function is

[2] Quasi-concavity of u is not sufficient to guarantee satisfaction of the second-order condition for the optimization problem analyzed below.

[3] We also assume that the resident ignores the period after T. While this assumption is very strong, we can consider the period [0, T] to be his lifespan.

$$\int_0^T u(x(t), z)e^{-\theta t}\, dt \tag{1}$$

where θ denotes the subjective discount rate and t is time. Note in (1) that while x depends on t, z is constant over time. This reflects the assumption that durable public facilities such as schools, roads, and parks from which public services come are built at $t = 0$ and do not decay with the age of the facilities. Since housing sizes in a community are assumed to be fixed, housing consumption is suppressed as an argument of utility function.

To produce the public good, the government borrows total production cost from outside of the community and pays it back with local revenue, all of which comes from the property taxation levied on n houses in a community. Therefore, the local government budget constraint is

$$n\tau(z) = c(z), \tag{2}$$

where $\tau(z)$ is a property tax per house, n is the number of houses, and $c(z)$ is the current cost of public service level, z. The cost function c is assumed to be convex. Thus, $\tau(z) = c(z)/n$, with $\tau_z > 0$ and $\tau_{zz} > 0$. The cost can be thought of as the sum of interest on the debt used to finance construction of public facilities and the variable cost per period of operating them. Note that the property tax, τ, depends on z but not on t.

The resident finances his consumption of x and z using his income stream and a given wealth. His wealth is housing value at time T, discounted back to time 0. The property value is in turn equal to the present value of the net income stream generated by the house. That is,

$$W = (R - \tau)/r, \tag{3}$$

where R denotes housing rent resulting from the future occupant's bidding on the house and r is the interest rate. The effect of the fiscal variable, z, on house value can be explained by capitalization. We imagine that the house is eventually bought by a family whose next best opportunity is to occupy a house in some other community. Unlike the standard consumer theory where consumers choose an optimal consumption bundle given exogenous prices so as to reach the maximum possible utility level, rents must adjust to insure that the utility level of a future resident equals some fixed value.[4] Letting $v(x, z)$ denote the future resi-

[4] The fixed value is the highest utility level which a future resident could reach if he would choose some other community. In order for a community to attract the future resident, the community has to insure the utility level by adjusting (lowering) rents. This is a usual analytic method in locational equilibrium literature(for example, see Brueckner(1987)).

dent's utility function, it must be true that $v^* = v(x, z)$ where v^* is an exogenously given utility level. Since all houses in a community are assumed to be identical, the utility function does not depend explicitly on housing services. The future resident's budget constraint is $x = y - R$ where y is his income[5] and R is the amount that he would be willing to pay to occupy the house. Substituting in v yields $v(y - R, z) = v^*$, and differentiating with respect to z gives

$$R_z = v_z / v_x. \tag{4}$$

Strict quasi-concavity of v further implies that $R_{zz} \langle 0$. Equation (4) shows that rent increases with z at a rate equal to marginal rate of substitution (MRS).

The effect on property value of a change in z is found by differentiating (3), which yields $W_z = (R_z - \tau_z)/r$. Since W is concave given $R_{zz} \langle 0$ and $\tau_{zz} \rangle 0$, property value is maximized when $R_z = \tau_z$. Letting z_m denote the z that maximizes property value, it then follows that

$$W_z \gtreqless 0 \qquad \text{as} \qquad z \lesseqgtr z_m \qquad \text{or as} \quad v_z/v_x \gtreqless \tau_z \ (=c_z/n). \tag{5}$$

This shows that an increase in the public good level raises the value of the house when the MRS of the future resident is greater than marginal cost, a relationship that holds when z is relatively low. When the relationship is reversed (when z is high), an increase in z lowers the value.

A current resident is assumed to have perfect information about the housing market and to know the exact form of $W(z)$, which might be unrealistic. When the capital market is perfect, the resident can borrow against future income. He can also borrow We^{-rT} at time 0 and pay it back with interest at time T when he sells the house. His budget constraint during residency in the community is therefore

$$\int_0^T x(t)e^{-rt}\,dt = \int_0^T y(t)e^{-rt}\,dt - \int_0^T \tau(z)e^{-rt}\,dt + W(z)e^{-rT} \tag{6}$$

where $y(t)$ denotes the resident's income stream, which is known with certainty. The constraint says that the present value of expenditure on x equals the present values of receipts after paying taxes. The resident has no nonhousing wealth but

[5] Note that since the future resident is able to borrow against future receipts, the income variable y in the constraint $v(y-\mathrm{R}, z) = v^*$ should be interpreted as embodying the proceeds of any such borrowing. Futrue receipts may include capital gain from resale of the house. If such gains were to depend on the current level of z, then the level of z would affect y. In order for relationship (3) to hold, such dependency should be ruled out. This can be guaranteed by assuming that z changes prior to resale of the house as a result of a new referendum.

owns his house outright so that no mortgage payment or other costs appear in (6).

The optimization problem for the current resident is to choose consumption levels of x and z to maximize (1) subject to (6). The problem can be converted into a problem in optimal control by creating a state variable

$$q(t) = \int_t^T x(\hat{t}) e^{-r\hat{t}} d\hat{t}. \quad (7)$$

Differentiating (7), the state equation for the problem becomes

$$\dot{q}(t) = -xe^{-rt}. \quad (8)$$

Using (8), the constraint (6) is replaced by the condition

$$q(0) = F(z), \quad (9)$$

where $F(z)$ represents the right-hand side of the equation (6). The requirement

$$q(T) = 0 \quad (10)$$

also must be satisfied.

For a given value of z, the Hamiltonian for the problem of choosing x optimally is

$$u(x, z)e^{-\theta t} - \alpha x e^{-rt}. \quad (11)$$

where α is the Lagrangian multiplier. The necessary conditions for an optimum are (8), (9), (10), and

$$u_x e^{-\theta t} - \alpha e^{-rt} = 0 \quad (12)$$

$$\dot{\alpha} = 0. \quad (13)$$

The necessary conditions are also sufficient for optimality since $u_{xx} < 0$.

The condition (13) implies that α is constant over time. The time path of consumption of x can be derived by differentiating (12) with respect to t and rearranging, which yields

$$\dot{x}(t) = \frac{(\theta - r)u_x}{u_{xx}}. \quad (14)$$

When the discount rate equals the interest rate ($\theta = r$), the resident consumes a constant amount of the private good over [0, T], but when $\theta > r$ ($\theta < r$), the resident decreases (increases) the consumption of x over time.

The next problem is to choose optimally the consumption of the public good. Letting $x^*(t, z)$ denote the optimal $x(t)$ solving the set of necessary conditions of the previous maximization problem, the resident votes for z at $t = 0$ to maximize his utility,

$$\int_0^T u(x^*(t, z), z)e^{-\theta t}\, dt. \tag{15}$$

Differentiating (15) with respect to z yields the necessary condition

$$G \equiv \int_0^T \left(u_x \frac{\partial x^*}{\partial z} + u_z \right) e^{-\theta t}\, dt = 0. \tag{16}$$

To investigate the implications of the condition (16), we use

Lemma 1[6]: The costate variable, $\alpha(t)$, is equal to the derivative of the objective functional with respect to the state variable, $q(t)$ at $t < T$.

Using the lemma 1, it follows that

$$\alpha(0) = \int_0^T u_x \frac{\partial x^*}{\partial q(0)} e^{-\theta t}\, dt, \tag{17}$$

which means the constant costate value α measures the sensitivity of the objective functional to an increase in $q(0)$. Applying (17), the condition (16) can be rewritten

$$G = \int_0^T u_x \frac{\partial x^*}{\partial q(0)} \frac{\partial q(0)}{\partial z} e^{-\theta t}\, dt + \int_0^T u_z e^{-\theta t}\, dt = 0. \tag{18}$$

Using (9) and (17), the necessary condition (18) reduces to

$$F_z = -\int_0^T \frac{u_z}{\alpha(0)} e^{-\theta t}\, dt. \tag{19}$$

By differentiating the right-hand side of (6) with respect to z, we obtain

$$F_z = W_z e^{-rT} - \tau_z(1 - e^{-rT})/r \tag{20}$$

[6] See Kamien and Schwartz(1981). For an application, see Brueckner(1984).

Since we know that $\alpha(0) = \alpha(t) = u_x e^{-(\theta - r)t}$ from (12) and (13), combining (19) and (20) yields

$$\int_0^T \frac{u_z}{u_x} e^{-rt}\, dt + W_z e^{-rT} = \tau_z (1 - e^{-rT})/r \tag{21}$$

To obtain some intuition from the utility maximization condition (21), it is helpful to investigate a special case where the voter's discount rate θ is equal to the interest rate r.[7] Since x is constant when $\theta = r$, the marginal rate of substitution (MRS) does not depend on t and (21) turns into

$$\frac{u_z}{u_x}(1 - e^{-rT}) + \frac{v_z}{v_x} e^{-rT} = \frac{c_z}{n}, \tag{21'}$$

making use of $W_z = (R_z - \tau_z)/r$, $R_z = v_z/v_x$, and $\tau_z = c_z/n$. It is useful to rearrange (21') slightly so that it reads

$$\left(\frac{u_z}{u_x} - \frac{c_z}{n}\right)(1 - e^{-rT}) + \left(\frac{v_z}{v_x} - \frac{c_z}{n}\right) e^{-rT} = 0. \tag{22}$$

These conditions show that the preferences of the future resident are considered in the voting decision. The reason is that while a current resident votes for z to maximize his utility subject to his budget constraint, his wealth includes the value of his house. The housing value is determined by the future resident's bid, which reflects his preferences. Therefore, the future resident's preferences enter the voting decision. The condition (21') indicates that in maximizing his own utility, the voter equates a weighted average of his own MRS and future occupant's MRS to the per capita marginal cost of the public goods.[8]

As a concrete example of such behavior, consider the voting behavior of a middle-aged couple in a school funding referendum. With no children in school, the couple's own preferences favor low spending. However, in anticipation of retirement to a warmer climate, the couple may cast a favorable vote in the referendum in order to enhance the value of their house to a prospective buyer with school-aged children. The preferences of such a buyer are thus reflected in the couple's voting behavior.

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[7] When we consider a case of $\theta = r$, the weak inequality of u_{xx} is enough to satisfy the sufficient condition because the allocation of x over time is indeterminate.

[8] It can be shown that (21') is the condition for a Pareto-efficient allocation in the case where the voter's life ends at time T, where the future resident remains in the house forever, and where z is fixed forever. Since these conditions are implausible, this efficiency result is not of much interest.

Combining (5) and (22), we can compare the optimal z with the property-value maximizing level, z_m. Whether z at the optimum is greater or smaller than z_m depends on the relation between current and future residents' preferences. From (22),

$$z \gtreqless z_m \quad \text{as} \quad u_z/u_x \gtreqless v_z/v_x, \tag{5'}$$

where the MRS expressions are evaluated at the optimal z. If the current resident's demand for public spending (as reflected in his MRS) is high relative to the future resident's, then his optimal z is above z_m. Conversely, if the current resident is a low demander relative to the future resident, then the optimal z is below z_m. The latter case describes the middle-aged couple considered above, whose preferred school expenditure is below the level that maximizes the value of their house.

2.2 Identical Preferences and the Voting Outcome

The above discussion shows that the property-value-maximization principle advanced by Wildasin(1979) and Sonstelie and Portney(1980) is invalid in a world with imperfect mobility when the public good demands of the current and future residents diverge. It is interesting to note that such a divergence may occur even when their preferences are identical provided that income effects are present. Then, if life-cycle factors put the current and future residents on different indifference curves, their MRS's will diverge, leading to a difference between the voter's preferred z level and property-value maximizing level.

Suppose first that the preferences of current and future residents are identical and their utility functions have no income effect on z. This implies that both $u(x, z)$ and $v(x, z)$ have the form $x + f(z)$. In this case, both u_z/u_x and v_z/v_x equal f_z and (21′) implies

$$u_z/u_x = v_z/v_x = c_z/n \tag{23}$$

which is identical to the Samuelson condition for n identical residents.[9] At the same time, the voting outcome maximizes the property value. That is, a voter can maximize his utility by maximizing his housing value. This outcome can be also obtained if both current and future residents have the same preferences and

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[9] The samuelson conditions says that at the optimum, the marginal cost of supplying the last unit of a public good in terms of a private good foregone(c_z) just equals the sum of the marginal benefits that all users of the increment of the public good simultaneously obtain in terms of the private good ($n(u_z/u_x)$ or $n(v_z/v_x)$ for n identical users) (Samuelson(1954)).

same budget condition,[10] as in Yinger(1982).

If preferences are identical but there is a positive income effect on z, is the voting outcome on z still equal to property-value maximizing level? Because the current residents may be assumed to be older than the future residents at time T, it is reasonable to assume that the current residents reach a higher indifference curve. Since a positive income effect will then make the MRS of the current resident greater than that of the future resident, it follows from (22) that $v_z/v_x - c_z/n < 0$. Therefore, the voter's preferred z is greater than the property-value maximizing level. If there is a negative income effect on z, or if the future residents have higher income, then the opposite result holds.

III. THE EFFECT OF DEPARTURE TIMING

3.1 The Effect of Time-Until-Move

While in the basic model, time-until-move, T, is assumed to be fixed, a change in T has an effect on the voter's preferred z. As T approaches zero, which means the current resident will sell his house and move out soon, (21′) becomes $v_z/v_x = c_z/n$ and the property-value maximizing z level is optimal. The reason is obvious. Because he will sell his house immediately, the resident tries to maximize his property value for the largest consumption. On the other hand, (21′) becomes $u_z/u_x = c_z/n$ as T approaches infinity. The resident does not consider his property value in voting on z because its current value shrinks to zero as T goes to infinity. This outcome is identical to the original Samuelson condition.

The effect of T on z can be found by differentiating (21′) with respect to T. When this is done, two forces emerge: an increase in T alters the marginal condition (21′) by changing the weights on the current and future residents' preferences; the higher T also changes the current resident's purchasing power by decreasing the present values of his property. While the second force makes the net effect ambiguous (the problem is that the resulting change in x affects u_z/u_x), the outcome is simple when the demand for z exhibits no income effect so that u_z/u_x is independent of x. In this case, the purchasing power effect disappears and the outcome is solely the result of the changed weights on the preferences. The effect on z of T is then

$$\frac{\partial z}{\partial T} = \frac{r}{\Omega e^{rT}}(u_z/u_x - v_z/v_x) \gtrless 0 \qquad \text{as} \quad u_z/u_x - v_z/v_x \gtrless 0 \tag{24}$$

where Ω is the negative of the total derivative of the left-hand-side of (22) with

[10] The same budgetary condition does not mean that they have the same income but does that they have just enough financial sources to consume exactly the same amounts of x and z.

respect to z, a quantity which must be positive by the second-order condition. Combining (24) with (5′) yields

$$\partial z/\partial T \gtreqless 0 \qquad \text{as} \qquad z \gtreqless z_m \tag{24′}$$

Equation (24′) shows that if the voter is a higher demander of z than the future resident, so that z is above z_m, then moving the date of departure closer in time lowers the preferred level of z. The reason, of course, is that the future resident's lower demand counts for more when the time of depature is closer. Conversely, when the future resident is a higher demander than the current resident, a closer departure date pushes z up toward z_m. In both cases, a closer date of departure moves the preferred z nearer to the property-value-maximizing level. This shows that as the near-term mobility of the voter rises, the property-value-maximization principle becomes more accurate as a description of behavior.

The preceding discussion has been based on the simplifying assumption that the voter's discount rate is equal to the interest rate. Although the analytics are not as clean, the principles elucidated above can be generalized easily to the case where $\theta \neq r$.

3.2 The Effect of Uncertainty about Departure Timing

So far, the voter knows with certainty that he will leave the community at time T, an assumption that may be unrealistic in some cases. For example, the voter might expect a job transfer in the future without knowing exactly when it will occur. To introduce this kind of uncertainty, suppose that the voter's transfer at t comes with probability $\pi(t)$. The voter must vote in the spending referendum (which occurs at $t=0$) before he knows exactly when he will leave the community. Suppose, however, that x consumption over time is chosen after the departure decision is known.[11] This timing sequence allows the voter to choose x optimally conditional on z and the departure decision, with z then chosen to maximize expected utility using the departure probabilities. In other words, letting $L(z;\, t)$ denote maximized utility conditional on z when the departure occurs at time t, z is chosen to maximize

$$\int_0^T \pi(t)\ L(z;\, t)\, dt, \tag{25}$$

satisfying the first-order condition

[11] The alternative case where x must be chosen before the transfer decision is known does not yield useful results.

$$\int_0^T \pi(t)\ L_z(z;t)\,dt = 0. \tag{26}$$

While the function $L(z;\ T)$ has played a role in the previous analysis, we need a general function $L(z;\ t)$ here, which gives maximized utility when departure occurs at a time t in the period [0, T]. This function, $L(z;\ t)$, is equal to

$$\max_{\{x\}} \int_0^t u(x(\hat{t}), z)\, e^{-\theta\hat{t}}\, d\hat{t}.$$

subject to[12]

$$\int_0^t x(\hat{t})e^{-r\hat{t}}\, d\hat{t} = \int_0^t ye^{-r\hat{t}}\, d\hat{t} - \int_0^t \tau(z)e^{-r\hat{t}}\, d\hat{t} + W(z)e^{-rt}. \tag{27}$$

Converting the problem (27) into an optimal control problem and following the same steps as those in the basic model, we obtain

$$L_z(z;\ t) = \int_0^t \frac{u_z}{u_x} e^{-r\hat{t}}\, d\hat{t} + W_z e^{-rt} - \tau_z(1-e^{-rt})/r. \tag{28}$$

Our goal is to analyze the effect of a change in the probability (for example, an increase of the chance of departure in the early part of the period [0, T]) on the voter's preferred z. In particular, we wish to know whether a larger chance of early departure makes the voter behave more like a property-value maximizer (this seems plausible, as wealth motives should be more important when early departure is more likely).

To address this issue, note first that the preferred z from (26) will be a probability-weighted average of the solutions to $L_z(z;\ t) = 0$ over [0, T] (that is, a weighted average of z's that are optimal with departure at time t). We assume a linear probability density function, $\pi(t) = a_1 t + a_2$ where $a_2 = 1/T - a_1 T/2$,[13] for simplicity of the analysis. Note that whatever the value of a_1 is, a decrease in a_1 means a relative increase in a probability of departure at an earlier stage of the period. As a result, when a_1 decreases, the weighted average used to compute the optimal z is altered in favor of z's with earlier departure, moving the preferred z closer to these values.

With the assumptions that income effects are absent[14] and $\theta = r$, the condition $L_z(z;\ t) = 0$ becomes

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[12] Note that this constraint embodies a stationarity assumption in that the functin W applies to any t.

[13] This value for a_2 comes from the requirement that the density function integrate to one over [0, T] ($\int(a_1 t + a_2)dt = 1$).

[14] Recall that the MRS equals f_z in the zero-income-effect case.

$$(f_z - \tau_z)\int_0^{t} e^{-rt}\,d\hat{t} + W_z e^{-rt} = 0. \tag{29}$$

By applying a similar argument with one used in the liquidity-constrained analysis, it can be shown that when $t_1 \langle t_2$, z_{t1} lies between z_{t2} and z_m ($z_{t2} \langle z_{t1} \langle z_m$ when $z_{t1} \langle z_m$ and $z_{t2} \rangle z_{t1} \rangle z_m$ when $z_{t1} \rangle z_m$), where z_{ti}'s are optimal with departure at t_i. That is, the earlier-departure z's are closer to z_m than are the later-departure z's. With the weighted average increasingly favoring the earlier-departure values as a_1 decreases, it then follows (see the appendix) that

$$\partial z/\partial a_1 \gtrless 0 \qquad \text{as} \qquad z \gtrless z_m. \tag{30}$$

Therefore, it follows that when income effects are absent, a higher probability of early departure from the community pushes the voter's preferred z toward the property-value maximizing level. That is, as a_i decreases, the voter behaves more like a property-value maximizer. As in the liquidity-constrained analysis, this result need not hold in the presence of income effects. The problem is that (intuition notwithstanding), the early departure z need not be closer to z_m when income effects are present, so that the above argument cannot be made.[15]

IV. THE EFFECT OF A LIQUIDITY CONSTRAINT

In the basic model, the resident's optimization problem is formulated under the assumption that the capital market is perfect so that a voter can borrow freely against future income and the resale value of property. In the real world, however, unsecured borrowing is somewhat restricted. This section investigates the effect on the voting outcome of a liquidity constraint, which rules out borrowing against future income while permitting the voter borrow against his property value.

Before introducing the constraint, it is convenient to reformulate the basic model. Letting $B(t)$ denote borrowing at t and $TB(t)$ the total balance at t, the previous optimization problem can be rewritten as one of maximizing the objective function (1) subject to

$$x(t) = y + B(t) - \tau(z) \tag{31}$$

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[15] The problem is that the MRS's in the conditions $L_z(z; t) = 0$ may be based on different x values as the time of departure changes (conditional on z, the optimal x values over $[0, t_1]$ may be different from the optimal x values over $[0, t_2]$). When income effects are present, this means that the MRS's in different departure times are not necessarily equal at a given z value. This in turn rules out the argument used above to conclude that the early-departure z is closer to z_m. A similar difficulty arises in the liquidity-constrained case when income effects are present.

$$TB(T) = 0 \tag{32}$$

where

$$TB(t) = W(z)e^{-r(T-t)} - \int_0^t B(\hat{t})e^{-r(\hat{t}-t)}\, d\hat{t}. \tag{33}$$

The constraint (31) means that the resident finances consumption of x and tax payments from current income, assumed here constant over time, and/or borrowing against future income or property. Note that the borrowing can be negative, which implies that saving from income in a period is possible. The total balance at t equals the house value less total borrowing over [0, t]. Constraint (32) means that the total balance is zero at the end of the period.

Introducing the liquidity constraint adds to this formulation the inequality,

$$TB(t) \geq 0 \tag{34}$$

which implies that total borrowing over [0, t] for any $t \leq T$ cannot exceed the house value. The liquidity constraint is likely to have no effect on residents who have a high early income or a low discount rate. Other residents, however, may wish to transfer more purchasing power to the early part of the period than the constraint allows.

The optimization problem with the liquidity constraint is to maximize (1) subject to constraints (31), (32), and (34). In order to convert the problem into an optimal control problem, a state variable is needed. This variable is

$$s(t) = \int_t^T B(\hat{t})e^{-r(\hat{t}-t)}\, d\hat{t}, \tag{35}$$

which equals total borrowing over [t, T] evaluated at t. Since, from (32) and (33),

$$W(z) = \int_0^T B(\hat{t})e^{-r(\hat{t}-T)}\, d\hat{t}, \tag{36}$$

combining (33) with (35) and (36) yields

$$TB(t) = s(t). \tag{37}$$

By using (37), the constraints (32), (33), and (34) are replaced by the set of constraints

$$s(T) = 0 \tag{38}$$

$$s(0) = W(z)e^{-rT} \tag{39}$$

$$s(t) \geq 0. \tag{40}$$

Note the constraint (39) implies that a current resident's initial balance equals the present value of his property at $t = 0$. Finally, the state equation for the problem is obtained by differentiating (35) with respect to t,

$$\dot{s}(t) = -B(t) + rs. \tag{41}$$

When the liquidity constraint is introduced into the problem, the Hamiltonian for choosing B optimally for a given value of z becomes

$$u(y + B(t) - \tau(z), z)e^{-\theta t} + \alpha(rs(t) - B(t)) + \mu s(t) \tag{42}$$

where α and μ are the Lagrangian multipliers associated with the constraints. The necessary conditions for the optimum are (38), (39), (40), (41), and

$$u_x e^{-\theta t} - \alpha = 0 \tag{43}$$

$$\dot{\alpha} = -\alpha r - \mu \tag{44}$$

$$\mu \geq 0 \tag{45}$$

$$\mu s(t) = 0 \tag{46}$$

From (44), α is no longer constant over time. Instead, solving for $\alpha(t)$ from the necessary conditions yields

$$\alpha(t) = e^{-rt}\left(\alpha(0) - \int_0^t \mu(\hat{t})e^{r\hat{t}}\, d\hat{t}\right). \tag{47}$$

Plugging (43) into (47) and solving for $\alpha(0)$, we obtain

$$\alpha(0) = u_x e^{-(\theta - r)t} + \int_0^t \mu(\hat{t})e^{r\hat{t}}\, d\hat{t}. \tag{48}$$

The necessary conditions can be solved for $B^*(t, z)$, an optimal $B(t)$ for a given z. The resident tries to maximize his utility by choosing z as well as B optimally. The necessary condition for the optimal z satisfies

$$\int_0^T \left(u_x \frac{\partial B^*}{\partial z} - u_x \tau_z + u_z \right) e^{-\theta t}\, dt = 0 \tag{49}$$

Using lemma 1 and (48), the appendix shows that the condition (49) can be rewritten as

$$W_z e^{-rT} + \int_0^T \left(\frac{u_z}{u_x} - \tau_z \right) e^{-rt} H(z, t) dt = 0 \tag{50}$$

where

$$H(z, t) = \frac{u_x e^{-(\theta - r)t}}{u_x e^{-(\theta - r)t} + \int_0^t \mu(\hat{t}) e^{-r\hat{t}} d\hat{t}} \tag{51}$$

and $0 < H(z, t) \leq 1$ for all $0 \leq t \leq T$ but $H(z, t) < 1$ over some interval in [0, T]. Note that if borrowing against income is possible, then $\mu = 0$ and $H(z, t) = 1$ so that (50) is identical to (21).

How does the liquidity constraint affect the voting outcome? Intuitively, there are two ways for the resident to lessen the impact of the constraint when it is binding. First, the resident can extract more financial resources from the house by setting z closer to the property-value maximizing level, z_m. This suggests that the optimal z value when the constraint is binding (z_b) should be closer to z_m than the optimal value when the constraint is not binding (z_{nb}). The second way to overcome the binding constraint is to reallocate resources between x and z, assuming that property value does not change as z changes. That the constraint is binding means that a liquidity-constrained resident wants to consume x at the early stage of residency rather than late. He can do this only by decreasing z, because resources are assumed fixed. This argument suggests that z_b should be smaller than z_{nb} regardless of whether z_{nb} is below or above z_m. If these two arguments are combined, it follows that z_b is lower than z_{nb} when z_b is larger than z_m. The relation between z_b and z_{nb} is unclear, however, when $z_b < z_m$.

Analytically, it is hard to deduce an implication from (50) because the MRS of the current resident depends on time even when $\theta = r$.[16] Suppose that the income effect on public services is zero so that the utility functions have the form of $x + f(z)$. If the discount rate θ is larger than the interest rate r, the liquidity constraint is binding because the voter wants to consume as much x as possible at $t = 0$. Then, as shown in the appendix, the necessary condition analogous to (50) is

$$(f_z - \tau_z) \int_0^T e^{-\theta t} dt + W_z e^{-rT} = 0. \tag{52}$$

[16] In an unrealistic case when $\theta = r + \mu e^{\theta t} / u_x$ so that x is constant over [0, T], the MRS of the current residents is free from time. Differentiatin (48) with respect to t and solving for $\dot{x}$ gives us $\dot{x} = ((\theta - r) u_x - \mu e^{\theta t}) / u_{xx}$. Therefore, $\dot{x} = 0$ when $\theta = r + \mu e^{\theta t} / u_x$.

In the case when $\theta = r$, the constraint is not binding because any allocation of x over the period [0, T] is equivalent for the voter. The optimal condition for this case is identical to (21). With the special utility functional form, this condition is

$$(f_z - \tau_z)\int_0^T e^{-rt}\,dt + W_z e^{-rT} = 0. \tag{53}$$

Suppose that z_b, which solves (52), is larger than the property-value maximizing level. Then $W_z < 0$ so that $f_z - \tau_z > 0$ follows from (52). Thus, setting $z = z_b$, it follows that

$$(f_z - \tau_z)\int_0^T e^{-\theta t}\,dt < (f_z - \tau_z)\int_0^T e^{-rt}\,dt \tag{54}$$

since $\theta > r$. But since (54) implies that (53) is positive at $z = z_b$, it follows from the second-order condition that z_{nb} (which solves (53)) must satisfy $z_{nb} > z_b$. This shows that the voting outcome z_{nb} is larger than z_b when z_b is larger than the property-value-maximizing level. The opposite is true when z_b is below the property-value-maximizing level. This result implies that when the constraint is binding, the voter prefers a z closer to z_m in order to extract more financial resources from the property. This is an important result because it says that, in the presence of the constraint, a voter who wants his x early rather than late will put more weight on the future resident's preferences in voting on z than will a more patient voter. Note that when $\theta = r$, there is no possibility of using the second intuitive argument to overcome the binding constraint because all allocations of x are equivalent.[17]

V. EMPIRICAL IMPLICATIONS[18]

While clarifying the motives of property owners in voting booth, the analysis also suggests a possible need for reinterpretation of recent empirical estimates of the demand for public goods. In particular, estimates based on survey results, such as those presented in Bergstrom, Rubinfeld, and Shapiro's (BRS; 1982) study of education demand, may be contaminated by the blending effect identified in this paper. That is, a voter's response when questioned about the adequacy of current public spending in his community may reflect his own preferences as well as the preferences of future residents who will potentially buy his property. When

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[17] If $\theta < r$, then the constraint $x(t) \geq 0$ over [0, T] must be added to keep $x(t)$ from becoming negative. Since (50) and (52) are invalid when this additional constraint is binding, the $\theta < r$ case cannot be handled without further analysis.

[18] This section comes from Brueckner and Joo(1991).

the preferences of the current and future residents are different for life-cycle or other reasons, the voter's response may seriously over- or understate his own demand for public spending. BRS's results, for example, shows a significantly positive coefficient for a dummy variable indicating whether the voter is over sixty-five years of age. The result, which indicates that elderly voters have higher demands for education spending than otherwise similar individuals, probably gives a false picture of the personal preferences of the elderly. It is more likely that the reported high demand reflects the close anticipated sale date of the property belonging to the typical elderly voter (a typical buyer is likely to have school-age children). With a property sale not so imminent, a similarly-situated middle-aged voter would probably answer the survey in a way that more accurately reflects his personal preferences.

Although the blending problem is most obvious in the elderly case, its effects could be pervasive. For example, since an educated voter is probably more mobile than one with little education, such a voter may pay more attention to the resale value of his house in expressing a demand for education spending. BRS's college-graduate dummy coefficient, which is significantly positive,[19] may thus be contaminated by a mobility effect unrelated to personal preferences. To net out this effect, one possibility would be to include in the estimating equation a proxy for mobility such as the date of entry to the community (a recent move often indicates a higher propensity to move again). Better yet, the survey could include a question regarding expected duration of residence. The presence of such variables holds the anticipated date of sale constant, allowing the pure effects of elderly-status or education level on the voter's demand to be measured.

VI. CONCLUDING REMARKS

This paper has analyzed the behavior of an imperfectly mobile voter in a dynamic context with property value capitalization. We have seen that the voter's optimal public spending level reflects a blend of his own preferences and those of the future resident who will eventually occupy his house. The voter does not seek to maximize the value of his property, although his behavior conforms more closely to this principle the nearer is the date of his departure from the community or the greater is the chance of an early departure when he is uncertain about the future. In addition, voters who are liquidity-constrained are more likely to behave like property-value-maximizers than those who are unconstrained. It was also argued that the blending effect identified in the paper may require reinterpretation of empirical estimates of the demand for public goods.

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[19] This result is confirmed in another research(Rubinfeld and Shapiro, 1989), although the coefficient is not strongly significant.

Although the analysis clarifies the motives of a representative voter, it is silent about the ultimate voting equilibrium. This gap, however, is readily filled once it is recognized that preferred public good levels will vary among voters in the community in the usual fashion. Preferred level of public goods (z) will differ as a result of differences in income streams, preferences, departure times, or house sizes. As usual, the z level chosen through the voting process will be the median of these preferred values.

The model could be closed by considering the complete residential cycle of a community resident under steady state conditions with perfect foresight. In such a model, a new resident would enter the community with the (correct) expectations that the public good level will remain constant during his residence and that the resale value of his house upon exit will equal its original purchase price. Given that public spending referenda are infrequent, we could imagine that the resident has the opportunity to vote just once during his time in the community. With the preferences and incomes of successive generations of residents identical over time, the voter would find that his preferred spending level maximizes the value of his property. The property-value-maximizing level of public goods from (23) is thus the steady-state equilibrium spending level. New entrants take this level as given, and they have no incentive to alter it when they eventually enter the voting booth.

APPENDIX

1) Procedure for (30):

With the assumptions that income effects are absent and $\theta = r$, we can reduce (26) to

$$(f_z - \tau_z) \int_0^T \pi(t)(1 - e^{-rt})/r \, dt + W_z \int_0^T \pi(t) e^{-rt} \, dt = 0. \tag{a}$$

Substituting $\pi(t) = a_{1t} + 1/T - a_1 T/2$ and totally differentiating (a) yields

$$\partial z / \partial a_1 = \Phi^{-1} B / r(f_z - \tau_z - r W_z), \tag{b}$$

where $\Phi \langle 0$ is the z-derivative of (29) and

$$B \equiv \int_0^T (t - T/2) e^{-rt} \, dt \langle 0.$$

To sign (30), note that $f_z - \tau_z$ and W_z have opposite signs by (a) so that $\partial z / \partial a_1$ and W_z have also opposite signs. Then, we can obtain (30).

2) Procedure for (50):

From (49),

$$\int_0^T \left(u_x \frac{\partial B^*}{\partial s(0)} \frac{\partial s(0)}{\partial z} - u_x \tau_z + u_z \right) e^{-\theta t} \, dt = 0. \tag{c}$$

Since $\partial s(0) / \partial z = W_z e^{-rT}$ from (39) and the derivative of the objective functional with respect to $s(0)$ is equal to $\alpha(0)$ from lemma 1, the condition (c) can be re-written as

$$W_z e^{-rt} + \int_0^T (u_z - u_x \tau_z) e^{-\theta t} / \alpha(0) \, dt = 0. \tag{d}$$

Plugging (48) into (d) for $\alpha(0)$ and rearranging it by dividing by u_x yields (50).

3) Procedure for (52):

When $\theta \rangle r$, the voter wants to consume as much x as he can at $t = 0$, so that liquidity constraint is binding. This means that he consumes all financial resources from his wealth at $t = 0$. That is, $x = We^{-rT} + y - c(z)$ at $t = 0$ and $x = y - c(z)$ over (0, T]. His objective function to maximize is

$$\int_0^T (y - \tau(z) + f(z))e^{-\theta t}\,dt + W(z)e^{-rT} \tag{e}$$

By differentiating (e) with respect to z, we obtain (52).

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