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# THE WELFARE COSTS OF INFLATION WITH TWO ALTERNATIVE MEANS OF PAYMENT: MONEY AND TRADE CREDIT*

JOONWON KIM**

*This paper quantifies the magnitude of welfare losses that result in steady state due to a moderate to high inflation tax. In the economy, both money and trade credit provide transactions services and the mix of two alternative means of payment is endogenous. With an explicit transactions technology parameterized to mimic the way U.S. households use cash and other means of payment in making their transactions, a welfare cost of 2.89% of output results from a 10% annual inflation rate relative to the Friedman (1969) rule. The estimate of welfare costs of inflation captures the fact that the consumer inefficiently economizes on her holdings of real money balances in the face of positive inflation tax by purchasing a wider range of goods with trade credit that uses up real resources as well as the fact that moderate inflation rates cause people to inefficiently substitute away from goods for leisure. In addition to this substitution effect, however, there is a dominant negative wealth effect associated with higher inflation rates. The result strengthens the view that regards price stability as the most widely-cited objective for monetary policy.*

## I. INTRODUCTION

This paper takes a general equilibrium approach to providing estimates of the welfare costs of inflation. With the model which extends the work of Lucas and Stokey (1983) by formally specifying the transactions costs associated with trade credit and endogenizing the choice between two alternative means of payment, money and trade credit, we quantify the magnitude of welfare losses that result in

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** Assistant Professor, Department of Economics, Sogang University, C.P.O. Box 1142, Seoul, Korea 121-742. I am grateful to Costas Azariadis and *Mariano Tommasi for helpful comments and suggestions.* I also thank two anonymous referees for their helpful comments in revising the paper. All errors are mine.

steady state due to a moderate to high inflation tax. By allowing trade credit in addition to money to provide transactions services, we capture the fact that trade credit is used as a means of payment in virtually all business transactions; in the fourth quarter of 1993, all U.S. manufacturing firms' accounts receivable (trade accounts and trade notes receivable) totaled $365 billion, or 36% of total current assets, while accounts payable (trade accounts and trade notes payable) were $222 billion, or 31% of total current liabilities.[1]

The widely-held view that policy maker should commit to an objective for price stability as a means of achieving maximum sustainable long-run growth necessarily requires an accurate assessment of the welfare costs of inflation. Thus, economists have long come to grips with this problem. The traditional approach developed by Bailey (1956) measures the cost of inflation as a (welfare) triangle under the money demand function. The Bailey measure reflects real resource cost of avoiding inflation tax in alternative means of exchange. Fischer (1981) and Lucas (1981) update the Bailey measure and find that a 10% annual inflation rate results in a welfare cost of 0.3% and 0.45%, respectively, of output relative to a 0% inflation rate. It has been argued, however, that these estimates may underestimate the true cost of inflation for the Bailey measure is based on partial equilibrium.[2] Fischer (1981) himself suggests that welfare costs of inflation tax could be as high as 2~3% of output at a 10% inflation rate if tax-cum-inflation related distortions were analyzed fully.

An early attempt to assess the welfare costs of inflation in a general equilibrium model is made by Cooley and Hansen (1989). Using a cash-only economy, they report that a 10% annual inflation rate gives rise to a welfare cost of 0.387% of output relative to the Friedman (1969) rule, under which the money supply is contracted at the rate of time preference so as to make the nominal interest rate equal to zero. Their estimate is also lower to justify the universal aversion for even moderate inflation because it captures only the fact that inflation causes the economic agent to inefficiently substitute out of market activity which requires money and into leisure which does not require cash rather than any Bailey-type real resource cost.

To estimate the steady state welfare costs of inflation fully, we model an economy and discuss the qualitative properties of equilibrium in sections II and III. Both money and trade credit provide transactions services and the mix of two alternative means of payment is endogenous. In the model, fluctuations in the opportunity cost of money will alter the mix of exchange media used. The equilibrium mix depends on the relative costs of cash and trade credit. Cash use re-

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[1] Data are from the U.S. Commerce Department, Bureau of the Census, *Quarterly Financial Report* for the fourth quarter of 1993, p. 4.

[2] Imrohoroglu and Prescott (1991) and Tommasi (1993), among others, show that the Bailey analysis captures only a fraction of the total cost of inflation.

sults in no real resource costs but an opportunity cost of foregone interest, while trade credit use permits individuals to avoid the opportunity cost of holding cash but instead imposes a real resource cost. Individuals balance the resource cost of trade credit against the opportunity cost of cash at the margin to determine the use of both transaction media.

Section IV contains the quantitative properties of equilibrium. In an economy with multiple means of payment, an explicit transactions technology is parameterized to mimic the way U.S. households use cash and other means of payment in making their transactions. In this economy, a welfare cost of 2.89% of output results from a 10% annual inflation rate relative to the Friedman rule. While a (nested) cash-only economy in which the transactions cost of trade credit becomes prohibitively high yields much smaller estimate of 0.86% of output.

The relatively larger welfare estimate in the economy with multiple means of payment stems from the following two distortions associated with the inflation tax. First, as in Cooley and Hansen (1989), moderate inflation lowers welfare by causing people to inefficiently substitute away from goods for leisure. One interesting feature of our model is that the wealth effect which is dominated by this substitution effect at moderate inflation rates becomes dominant as the inflation rate goes beyond a certain level.[3] Even though higher inflation rates cause people to work more as negative wealth effect dominates this substitution effect, the welfare costs of inflation rise monotonically with inflation rates because the wedge between output and consumption widens.

Second, inflation brings about additional welfare cost as the consumer dissipates real resources to avoid inflation as in Bailey (1956). Having multiple means of payment, the consumer faces higher welfare costs in comparison to cash-only economy because the consumer inefficiently economizes on her cash balances in the face of a positive inflation tax by purchasing a wider range of goods with trade credit that uses up real resources.[4] And since real interest rate in the model is determined in part by the path of nominal rates, this substitution out of cash into trade credit results in a greater waste of resources as the inflation rate rises by distorting the intertemporal choice of consumption. These results strengthen the view that regards price stability as the most widely-cited objective for monet-

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[3] Recall that in the typical cash-in-advance models like Cooley and Hansen (1989), the representative worker always substitutes out of market activity into leisure when the inflation rate rises. However, in our model, as in Cole and Stockman (1992), higher inflation rates cause people to work more for there is a dominationg negative wealth effect associated with a rate of inflation which is beyond a critical level. According to our method of parameterization, the critical inflation rate is about 15% annually.

[4] In the same spirit but with different frameworks, Gillman (1993) and Dotsey and Ireland (1994) also assess the welfare costs of inflation. Making credit costly in *time*, Gillman (1993) suggests that a sustained 10% inflation relative to a 0% inflation costs the economy the equivalent of 2.19% of output per year, while Dotsey and Ireland (1994) report 1.73% figure as a welfare cost of inflation.

ary policy. Section V ends the paper.

## Ⅱ. THE THEORETICAL MODEL

The model is built on the assumptions presented below.

### 2.1. Economic Environment

We study an economy with many goods and many agents. There is a continuum of markets, indexed by $z \in [0, 1]$, arranged on the boundary of a circle with unit circumference. In each location $z$, a distinct, perishable consumption good is produced and traded in each period $t = 0, 1, \cdots$. Hence, consumption goods are also indexed by $z \in [0, 1]$, where good $z$ is sold in market $z$. Enough symmetry[5] is imposed so that the analysis considers without loss of generality the behavior of representative agent. The representative agent lives at location 0, so that the index $z$ measures the distance of market $z$ from her home. Households at location $z$ are capable of producing only type-$z$ good but each household desires consumption of goods of all types.

To formally specify the model and to focus on the substitution between the use of cash and credit in exchange, we assume that household preferences are Leontieff across goods at a given date, effectively requiring that they be consumed in equal amounts. This allows a particularly simple aggregation over consumption of goods within periods. Our results would carry over to environments with more general preferences that allow substitution among consumption of different goods as well as substitution in the means of payment, although deriving the results in such environments would be considerably more complex.

In our economy, there is no uncertainty and all agents have perfect foresight. Each worker is endowed with fixed units of leisure time in each period $t$[6] and she devotes $n_t$ units of labor time to production, resulting in $n_t$ units of consumption good.

### 2.2. Securities Market

Within each period, exchange takes place sequentially. First a securities market opens at each location. During securities trading at date $t$, the household uses the end-of-period currency, $\widetilde{m}_t$, to purchase $a_t$ units of the claim which has the price of $q_t$ and provides $\eta_{t+1}$ units of currency in period $t+1$, and $b_t$ units of no-

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[5] In effect, perfect competition and identical preferences, endowments and transactions technology in each market are assumed.

[6] The amount of total time endowment is normalized to one.

minal one-period government bonds which pay a nominal interest rate $i_t$. The remaining currency, $m_t$, is held for use in the period $t$ goods market which opens after securities trading ends.

A positive quantity $B_t$ of nominal one-period government bond is outstanding from period $t$ to period $t+1$, bearing interest at rate $i_t$. At date $t+1$ the maturing obligation, $(1+i_t)B_t$, is funded by a combination of new bond issue, $B_{t+1}$, and new money issue, $M_{t+1}-M_t$. Thus, the government budget constraint is:

$$M_{t+1}+B_{t+1}=M_t+(1+i_t)B_t. \tag{1}$$

We parameterize the monetary policy by the rate of money growth, $g_t$, which is defined as:

$$M_{t+1}\equiv(1+g_t)M_t. \tag{2}$$

The monetary authority announces the complete sequence $\{g_t\}_{t=0}^{\infty}$ of money growth rates at the beginning of period $t=0$.

### 2.3. Goods Market

In the goods market, the representative shopper travels around the circle in *either* direction to purchase goods for her household's consumption. As in Lacker and Schreft (1991), among others, the shopper chooses between two alternative means of payment, money and trade credit.[7] From the perspective of a shopper at a distance of $z$ from home, the opportunity cost of using cash rather than trade credit to make a purchase is the cost of borrowing (or the cost in foregone interest of not lending) the cash in the period $t$ securities market at the nominal interest rate $i_t$. Since competition equates the nominal price $p_t$ of consumption goods across markets, the cash-using shopper acquires one unit of good at $(1+i_t)p_t$ in period $t$.

### 2.4. Transactions Cost of Trade Credit

We model the transactions cost of trade credit to be proportional to the distance $z$ as well as the size of the purchase, so that a real resource cost of $kz$, where $k$ is a strictly positive constant, units of output is incurred whenever the representative shopper acquires one unit of consumption good on credit. Indiv-

[7] In Den Haan (1990), Gillman (1993), and Dotsey and Ireland (1994), costly credit also provides transactions services. However, they require time for credit exchange.

iduals balance the opportunity cost of cash against the resource cost of trade credit at the margin to determine the use of both transaction media; an optimizing agent chooses the least costly method of financing a purchase and, therefore, buys on credit if $z$ satisfies:

$$(1 + kz_t)\, p_t \leq (1 + i_t)\, p_t. \tag{3}$$

Let $z_t^*$ be the distance from home at which a shopper is indifferent between the use of cash and trade credit at date $t$. Then,

$$z_t^* = \min \left[ \frac{i_t}{k}, \frac{1}{2} \right]. \tag{4}$$

That is, trade credit is used for purchases at or within a distance of $z_t^*$ from home in *either* direction, and cash is used elsewhere. Note that the shopper uses trade credit close to home and cash far from home since transactions cost of credit increases with distance. Note further that all purchases are made on credit if $i_t \geq k/2$ and that only cash purchases are made as $k \to \infty$. In what follows I assume that $k > 2i_t$, $\forall t$ in order to exclude the case of a pure credit economy.[8]

### 2.5. Preferences

Household preferences are represented by:

$$\sum_{t=0}^{\infty} \beta^t \{V[W(c_t(z))] + \alpha(1 - n_t)\}, \ \beta \in (0, 1), \ \alpha > 0, \tag{5}$$

where $\beta$ and $\alpha$ represent, repectively, the subjective time discount factor on future utility and the consumer's relative liking for leisure, and $c_t(z)$ denotes the agent's consumption of good $z$ in period $t$. In (5), $W(c_t(z)) \equiv \inf_{z \in [0,\, 1]} \{c_t(z)\}$. $V(\cdot)$ is twice continuously differentiable, strictly increasing and strictly concave. We will eventually assume that $V(\cdot)$ is logarithmic. Our assumption about $W(\cdot)$ eliminates substitution among consumption of goods of various types and allows us to focus attention on the choice between the use of cash and trade credit. Specifically, if the opportunity cost of acquiring consumption goods is positive for all goods, as is true in all equilibria considered, then households will buy the same amount

[8] In all $z_t^* = i_t/k < 1/2$, in equilibrium. This value of $z_t$ makes both the shopper and the seller indifferent between the use of cash and trade credit, since (3) holds with equality when $z_t = z_t^*$.

of each good. We have, then, that $c_t(z) = c_t$, for all $z \in [0, 1]$.[9]

In sum, every period, the representative household at market 0 maximizes:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t), \tag{5'}$$

where $U(c, 1 - n) = \ln c + \alpha(1 - n)$.

### 2.6. Trading Opportunities

The household begins period $t$ with $b_{t-1}$, $a_{t-1}$ and $\tilde{m}_t$ units of currency held over from the previous period. In addition, the household has trade credit receivable of $(1 + kz_{t-1})p_{t-1}\zeta_{t-1}$, where $\zeta_{t-1}$ is the amount of the consumption good sold on credit during the previous period.[10] Since only shoppers located at or within a distance $z^*_{t-1}$ in either direction used credit, total trade credit receivable is:

$$2\int_0^{z_{t-1}^*}(1 + kz_{t-1})p_{t-1}\zeta_{t-1} = (2z^*_{t-1} + kz^{*2}_{t-1})p_{t-1}\zeta_{t-1}. \tag{6}$$

Similarly, the household's total trade credit payable is $(2z^*_{t-1} + kz^{*2}_{t-1})p_{t-1}c_{t-1}$, due in the period $t$ securities market. Summarizing then, the household faces the following constraint on the sources and uses of currency in the period $t$ securities market:

$$\begin{aligned} m_t + b_t + q_t a_t = \tilde{m}_t + (1 + i_{t-1})b_{t-1} + a_{t-1}\eta_t + (2z^*_{t-1} + kz^{*2}_{t-1})p_{t-1}\zeta_{t-1} \\ - (2z^*_{t-1} + kz^{*2}_{t-1})p_{t-1}c_{t-1}. \end{aligned} \tag{7}$$

In the goods markets, output is sold for cash or trade credit or devoted to transaction costs. When $\zeta_t$ is sold on credit for all shoppers from less than a distance $z^*_t$ away, a total of $(2z^*_t + kz^{*2}_t)\zeta_t$ units of output is exhausted on trade credit sales and associated costs. The remaining output is sold for cash, so the worker receives $[n_t - (2z^*_t + kz^{*2}_t)\zeta_t]p_t$ units of currency during the goods trading session. The shopper makes a fraction $(1 - 2z^*_t)$ of purchases using cash. Hence, the quantity of currency held until the next period is determined by:

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[9] Any consumption bundle $c_t(z)$ maximizing utility satisfies $c_t(z) = c_t$ almost everywhere and $c_t(z) > c_t$ otherwise. Because the set of $z$ for which this inequality is satisfied is of measure zero, $c_t(z)$ is assumed, without loss of generality, to equal $c_t$ for all $z \in [0, 1]$.

[10] In general, $\zeta_{t-1}$ is not a constant but a function of $z_{t-1}$. To focus on the substitution between the use of cash and credit in exchange, however, we have assumed that $\zeta_{t-1}(z) = \zeta_{t-1}$, for all $z \in [0, 1]$.

$$\tilde{m}_{t+1} = m_t + [n_t - (2z_t^* + kz_t^{*2})\xi_t]p_t - (1 - 2z_t^*)p_t c_t. \tag{8}$$

Currency acquired from goods market cash sales during period $t$ cannot be used for purchases at date $t$ because these occur simultaneously at spatially separated locations. The household thus faces an *endogenous* cash-in-advance constraint:

$$(1 - 2z_t^*)p_t c_t = m_t. \tag{9}$$

## 2.7. Equilibrium

In a stationary symmetric monetary equilibrium, (i) household maximizes preferences, (5′), subject to three constraints, (7), (8) and (9); (ii) the government budget constraint, (1), is satisfied; and (iii) the markets for money, bond, claim and goods clear at each date:

$$\begin{aligned} m_t &= M_t = (1 + g_{t-1})M_{t-1} \\ b_t &= B_t, \\ a_t &= 0, \\ n_t &= (1 + kz_t^{*2})c_t, \ \forall t. \end{aligned} \tag{10}$$

The first-order-conditions are given by:

$$c_t : U_1^t - \beta\lambda_{1t+1}(2z_t^* + kz_t^{*2})p_t - (\lambda_{2t} + \lambda_{3t})(1 - 2z_t^*)p_t = 0, \tag{11}$$

$$n_t : -U_2^t + \lambda_{2t}p_t = 0, \tag{12}$$

$$b_t : \beta\lambda_{1t+1}(1 + i_t) - \lambda_{1t} = 0, \tag{13}$$

$$a_t : \beta\lambda_{1t+1}\eta_{t+1} - \lambda_{1t}q_t = 0, \tag{14}$$

$$m_t : -\lambda_{1t} + \lambda_{2t} + \lambda_{3t} = 0, \tag{15}$$

$$\tilde{m}_{t+1} : \beta\lambda_{1t+1} - \lambda_{2t} = 0, \tag{16}$$

where $\lambda_{1t}$, $\lambda_{2t}$ and $\lambda_{3t}$ are Lagrange multipliers associated with the constraints (7), (8) and (9), respectively, and $U_j^t$ is the partial derivative of $U(c_t, 1 - n_t)$ with respect to its $j$th argument, $j = 1, 2$.

## Ⅲ. QUALITATIVE PROPERTIES OF EQUILIBRIUM

### 3.1. Economy with Multiple Means of Payment

Under an environment in which both cash and trade credit provide transactions services, in equilibrium, $z_t^* = i_t/k < 1/2$, as implied by (4).

From (13) and (15), (11) becomes:

$$U_1 = \Xi(i_t)p_t\lambda_{1t},$$

$$\text{where } \Xi(i_t) \equiv \frac{[2z_t^*(1+kz_t^*/2)+(1-2z_t^*)(1+i_t)]}{1+i_t} = 1-\frac{i_t^2}{(1+i_t)k}. \quad (17)$$

The term in the numerator of the definition of $\Xi(i_t)$ is the *effective* gross nominal interest rate paid on the purchase of one unit of consumption at date $t$. It reflects the fact that a fraction $2z_t^*$ of each unit of consumption is bought with trade credit at an average gross interest rate of $1+kz_t^*/2$, and a fraction $1-2z_t^*$ is bought with cash that instead could have been lent in the securities market at gross interest rate $1+i_t$. Thus, $\Xi(i_t)$ is the average cost (sum of resource and opportunity costs) of monetary and trade credit exchange. Note that $\Xi(i_t)$ is less than one and is decreasing in $i_t$.

Equations (13) and (17) imply:

$$\frac{1}{\Xi(i_t)p_t} = \frac{\beta(1+i_t)U_1^{t+1}}{\Xi(i_{t+1})p_{t+1}U_1^t}. \quad (18)$$

(18) relates the real rate of return on currency to the marginal rate of intertemporal substitution. Under the assumption of quasi-linear preferences (5′), eliminating $P_t$ from (18) by the use of cash-in-advance constraint (9) and the money market clearing condition in (10) yields:

$$\frac{1-2i_t/k}{(1+i_t)\Xi(i_t)} = \frac{\beta}{1+g_t}\frac{1-2i_{t+1}/k}{\Xi i_{t+1}}. \quad (19)$$

Equation (19) becomes a functional equation in nominal interest rate; the left-hand-side of (19) depends only on $i_t$, while the right-hand-side depends only on $i_{t+1}$ and current rate of money growth, $g_t$.

Notice that, given the equilibrium $i_t$ from (19), the behavior of all other endogenous variables can be derived. This feature of the model implies that a monetary policy which alters the nominal interest rate has the novel real effects, since

$\partial i_t / \partial g_t > 0$ from (19).

Combining the first-order-conditions (11), (12), (13), (15) and (16) gives:

$$\frac{U_1^t}{U_2^t} \equiv \frac{1}{\alpha c_t} = 1 + i_t - \frac{i_t^2}{k}. \tag{20}$$

Coupled with the feasibility condition in (10), $n_t = (1 + kz_t^{*2})c_t$, (20) can be solved for $c_t$ and $n_t$ to get:

$$c_t = \frac{1}{\alpha} \frac{k}{k + ki_t - i_t^2}, \tag{21}$$

$$n_t = \frac{1}{\alpha} \frac{k + i_t^2}{k + ki_t - i_t^2}, \tag{22}$$

and, since $\zeta_t = 2z_t^* c_t$, real trade credit balances are given by:

$$\zeta_t = \frac{2i_t c_t}{k}. \tag{23}$$

Equation (21) implies that an increase in the rate of money growth (thus, inflation rate) which raises the nominal interest rate causes consumption to decrease. However, the effect on employment is ambiguous, as (22) shows. When the inflation rate rises, the representative worker tends to substitute out of market activity into leisure, as in Cooley and Hansen (1989). With historically relevant inflation rate, in the model economy, this substitution effect dominates so that inflation causes people to work less. On the other hand, there is a negative wealth effect associated with an increase in the inflation tax, as in Cole and Stockman (1992). In our economy as the inflation rate goes beyond a certain level, people work more due to a dominating negative wealth effect.[11]

Equation (23) suggests that the response of trade credit balances to an expansionary monetary policy can also be ambiguous: the representative shopper economizes on her cash balances in the face of a positive inflation tax by purchasing a wider range of goods without money (i.e., by increasing real trade credit balances), but the decline in consumption has an offsetting effect. With quasi-linear

[11] As the following section shows, moderate inflation rates yield $i_t(4 + i_t) < k$ which guarantees that $\partial n_t / \partial i_t < 0$. With higher inflation rates (beyond 15% annually), however, the relationship between inflation and leisure time becomes negative in our economy.

preferences (5′), however, real trade credit balances become $\zeta_t = \frac{2}{\alpha} \frac{i_t}{\alpha k + k i_t - i_t^2}$ which always goes up as the nominal interest rate increases.

From (9), real money balances are given by:

$$\frac{M_t}{p_t} = \left(1 - \frac{2i_t}{k}\right) c_t . \tag{24}$$

As implied by (24), with quasi-linear preferences (5′), real money balances decrease in response to a positive inflation tax. Thus, a positive inflation tax causes shoppers to use trade credit for a larger fraction of their purchases and money for a smaller fraction. This causes an increase in the share of output devoted to the real resource costs of trade credit, driving a larger wedge between output and consumption in the feasibility condition in (10), $n_t = (1 + kz_t^{*2})c_t$. Notice that (24) is an aggregate money demand function that accounts for the use of trade credit and its associated transactions cost on the use of currency and that this interest-elastic demand for money gives rise to the Bailey-type welfare costs of inflation.

When we define the velocity of money demand as the ratio of nominal output to the stock of money, velocity is equal to, from (24),

$$v_t \equiv \frac{p_t n_t}{M_t} = \frac{k + i_t^2}{k - 2i_t}, \tag{25}$$

which varies positively with the nominal interest rate.

Now, we obtain an expression for the real rate of return in the securities market from the first-order-condition for $a_t$, (14). Together with (17), this gives:

$$q_t = \frac{\beta \Xi(i_t) p_t \eta_{t+1} U_1^{t+1}}{\Xi(i_{t+1}) p_{t+1} U_1^t} . \tag{26}$$

In particular, (26) implies that the nominal rate affects the risk-free real rate of return in the securities market. The real rate of return, $r_t$, can be obtained from a claim that yields $\eta_{t+1} = p_{t+1}/p_t$ in period $t+1$:

$$\frac{1}{1 + r_t} = \frac{\beta \Xi(i_t) U_1^{t+1}}{\Xi(i_{t+1}) U_1^t} . \tag{27}$$

Note that (27) states that the current real rate is increasing in the current nominal rate, since $\Xi'(i_t) < 0$. The higher the current nominal interest rate is relative to the next period nominal rate, the more shoppers substitute trade credit for cash in transactions. This substitution results in a greater waste of resources on transactions costs in the current period. Thus, relatively fewer resources are available for current consumption raising the current real interest rate. This comovement between nominal and real rates implies that smoothing nominal interest rate reduces the distortion in intertemporal choice of consumption due to the impediments to exchange.

### 3.2. Economy without Credit

In cash-only economy, the equilibrium conditions are simpler, because $k = \infty$, $z_t^* = 0$, and $\Xi(i_t) = 1$, $\forall t$. The first-order-conditions are identical to those for trade credit economy, except that (11) now becomes:

$$U_1^t - \lambda_{1t} p_t = 0. \tag{11'}$$

Equations (11') and (13) yield an equation analogous to (19):

$$\frac{1}{1+i_t} = \frac{\beta}{1+g_t}. \tag{19'}$$

Comparison of (19') to (19) indicates that the current nominal interest rate in the cash-only economy depends only on current money growth rate, while next period nominal rate also affects $i_t$ in the economy with multiple means of payment.

The equilibrium nominal interest rate can be obtained directly from (19'):

$$i_t = \frac{1+g_t}{\beta} - 1. \tag{19''}$$

Note that (20) becomes:

$$\frac{U_1^t}{U_2^t} \equiv \frac{1}{\alpha c_t} = 1 + i_t. \tag{20'}$$

As before, given $i_t$, equilibrium $c_t$ and $n_t$ can be derived from (20') and the feasibility condition, $n_t = c_t$:

$$c_t = n_t = \frac{1}{\alpha}\,\frac{1}{1+i_t}. \tag{21'}$$

The cash-in-advance constraint, (9), now takes its usual form, $p_t c_t = m_t$, which yields the following real money balances:

$$\frac{M_t}{P_t} = c_t . \tag{24'}$$

Note that in the cash-only economy the demand for money is perfectly interest-inelastic and that velocity is always equal to one.

The real interest rate in cash-only economy also takes its standard form so that the Fisherian independence of real from nominal rates holds:

$$\frac{1}{1+r_t} = \frac{\beta U_1^{t+1}}{U_1^t} . \tag{27'}$$

In the economy without credit, an expansionary monetary policy which raises the nominal interest rate causes consumption, employment, and real money balances to decrease, as (21') and (24') imply.

## Ⅳ. QUANTITATIVE PROPERTIES OF EQUILIBRIUM

### 4.1. Parameter Values

In this subsection, we discuss how parameter values are assigned to the model. The parameters in the economy include the preference parameter, $\alpha$, the real resource cost parameter of trade credit, $k$, and the subjective discount factor, $\beta$, which is set to 0.9606. The preference parameter, $\alpha$, which identifies the consumer's relative liking for leisure can be determined from the fact that the average share of available time devoted to work, $n$, has been constant. Kapteyn and Kooreman (1987) report the value of 0.4, while King and Rebelo (1993) use the value of 0.2. Also, Christiano (1991) indicates that the empirical ratio of market-to-nonmarket activity averages 0.28 implying that $n = 0.22$. To obtain a value for $\alpha$, I set $n = 0.3058$ which is the annual average in U.S. over 1948-1988.[12] From the steady state version of (22) or (21'), we know that $\alpha = 1/n$ at the economy's optimum ($i = 0$). This expression implies an estimate of $\alpha = 3.2701$.

The transactions cost parameter, $k$, can be singled out from survey studies of how people actually make their transactions. Avery et al. (1986, 1987) indicate that in 1984 and again in 1986, when inflation was about 4%, U.S. households

[12] Data are from the U.S. Commerce Department, Bureau of Economic Analysis, *Survey of Current Business*, 1985-1989. Also, from the works of Den Haan (1990) and Gillman (1993), we can get $n = 0.3$ approximately.

made 82% of their transactions with M1. This fraction corresponds to the value of $1-2i/k$ under 4% inflation in the model.[13] Our model is parameterized so that with 4% annual inflation, this constant fraction is about 80%. This implies an estimate of $k = 0.827$.

Table 1 indicates that the shopper uses money in a smaller range of transactions when inflation is higher. Thus, the steady state velocity of money rises with the inflation rate, as (25) shows.[14] Only with interest-elastic money demand, we can capture Bailey-type real resource cost of inflation. Under our parameter values the interest elasticity can be computed as follows. From (24), the steady state interest elasticity of money demand, $\varepsilon_{M/P,i}$, is given by:

$$\varepsilon_{\frac{M}{P},i} = -\left[\frac{2i}{k-2i} + \frac{(k-2i)i}{k+ki-i^2}\right]. \tag{28}$$

With 10% annual inflation, the interest elasticity of money demand in the economy with multiple means of payment is $-0.6247$, as Table 1 shows.[15] In sharp contrast, the cash-only economy yields money demand that has zero interest elasticity, as (24′) implies. As the Bailey measure describes, for the costly credit and cash-only economies, this ranking of the interest elasticities bears a direct connection to the ranking of the magnitude of welfare costs of inflation.

### 4.2. The Welfare Costs of Inflation

Following Cooley and Hansen (1989), among others, I compute the welfare costs of monetary policies that call for constant rates of money growth. These policies give rise to steady state equilibria in which each variable grows at a constant rate.

Specifically, Table 1 describes steady state equilibria under the benchmark policy that adopts the Friedman (1969) rule, under which the money supply is contracted at the rate of time preferences so as to make the nominal interest rate equal to zero. With an annual rate of time preference of about 4%, the steady state real interest rate in this economy is approximately 4%. Thus, following the Friedman rule generates a 4% annual rate of price deflation. Table 1 compares

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[13] This follows from the fact that in steady state a fraction $2z^* = 2i/k$ of each unit of consumption is bought with trade credit while the other fraction $1-2z^*$ is purchased with cash which is defined as M1.

[14] Recall that in Cooley and Hansen's (1989) single-good cash-in-advance model, for example, the velocity of money is practically constant. Thus, money demand is highly interest-inelastic. Though our model describes a velocity more dependent on relative prices, it matches somewhat poorly the reasonable estimate of U.S. velocity which averaged 5.4 since 1959 with an average inflation rate of 5%.

[15] Lucas (1988), among others, suggests the interest elasticities of money demand from the upper half of the estimated ranges.

these equilibria with those obtaining under the policy that yields a constant zero inflation rate, 5% and 10% annual rates of inflation, and so forth. This choice makes our measures directly comparable with the figures reported by others.

To obtain a measure of welfare loss, consider the following standard criterion. A monetary regime-II with various annual inflation rates results in a welfare cost of $100\xi\%$ of output relative to a monetary regime-I adopting the Friedman rule, where $\xi$ uniquely satisfies,

$$\frac{1}{1-\beta}\, U(c^{I}, 1-n^{I}) = \frac{1}{1-\beta}\, U(c^{II} + \xi n^{II}, 1-n^{II}). \tag{29}$$

The left-hand-side of (29) is the steady state level of welfare under the benchmark policy that yields a constant $-4\%$ annual inflation rate. Our welfare measure is based on the increase in output that a representative consumer would require to be as well off as under the reference regime-I. The term $\xi n^{II}$, in the right-hand-side of (29), is a lump-sum equivalent variation made to households to obtain the equality in utility terms.

According to our measure of welfare loss, in the economy with multiple means of payment, a 5% annual inflation rate yields a welfare cost of 1.26% of output, and 10% inflation costs 2.89% of output relative the Friedman rule. The relatively larger welfare estimate stems from the following two distortions associated with the inflation tax. First, when inflation is higher, the amount of leisure time increases while consumption and hours worked (thus, output) fall. As in standard cash-in-advance models, inflation lowers welfare through its negative effect on the return to work. Second, inflation brings about additional welfare cost as the consumer dissipates real resources to avoid inflation as in Bailey (1956). Having multiple means of payment, the shopper inefficiently economizes on her cash balances in the face of a positive inflation tax by purchasing a wider range of goods with trade credit that uses up real resources.

The model is parameterized so that the representative worker devotes approximately 30% of his time endowment to labor. Table 1 shows that as the inflation rate rises, the representative household tends to substitute out of market activity, which requires either money or costly credit, and into leisure, which can be enjoyed without the use of a means of exchange.

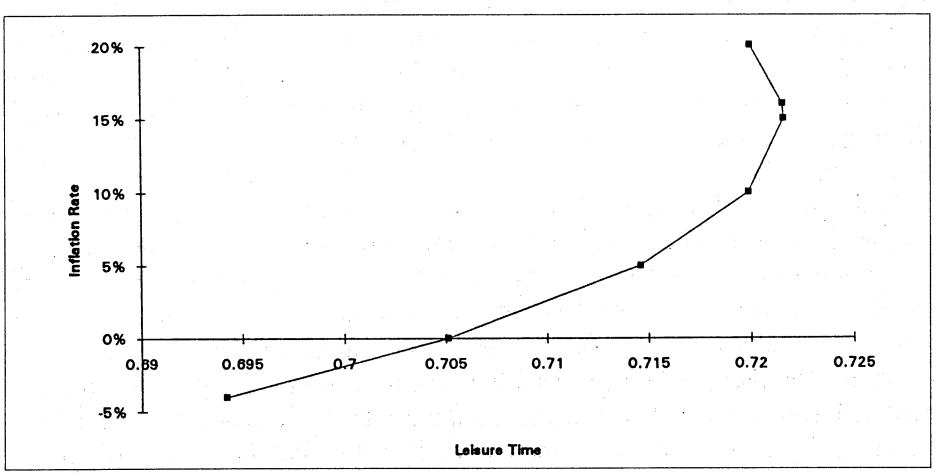
In addition to this substitution effect, however, there is a negative wealth effect associated with an increase in the inflation tax. While the substitution effect always dominates in the typical cash-in-advance models like Cooley and Hansen (1989), in our model of multiple means of payment, as in Cole and Stockman (1992), the wealth effect becomes dominant so that an increase from 15% inflation actually rises the household's labor supply. This interesting feature is presented in Figure 1. Even though higher inflation rates cause people to work more, the welfare costs of inflation rise monotonically with inflation rates. This reflects

the fact that the wedge between output and consumption which should be devoted to transactions cost widens as inflation rate rises. The dominating negative wealth effect, therefore, is internally related with the Bailey-type welfare cost.

Under a constant rate of inflation, the representative shopper makes a constant fraction of her purchases with cash. The representative household makes all of its purchases with cash under the Friedman rule, since the zero nominal interest rate eliminates the opportunity cost of holding real balances. Our model is paramerized so that with 4% annual inflation, this constant fraction is about 80%. Table 1 indicates that as inflation rate increases from a −4% to 5%, the fraction of transactions using money declines from 100% to 77%: accordingly, real money balances fall from 0.31 to 0.22 while real trade credit balances rise from 0 to 0.06. As inflation rate increases further to 10%, real money balances fall to 0.18 while real trade credit balances rise to 0.1. Thus, the steady state velocity of money rises with inflation rates.

Inflation-induced substitution out of cash into trade credit drives a larger wedge between hour worked and consumption and results in a greater welfare cost of inflation. Recall that the current real interest rate is increasing in the current nominal rate. The higher the current nominal interest rate is, the more shoppers substitute trade credit for cash in transactions. This is so because, under our method of paramerization, money demand is reasonably interest-elastic: as Table 1 indicates, the interest elasticity is −0.6247 at a 10% annual inflation rate. This substitution results in a greater waste of resources on transactions costs in the current period. Accordingly, relatively fewer resources are available for current consumption raising the current real interest rate. This mechanism results in an additional welfare cost of inflation by distorting the intertemporal choice of consumption, as implied by (27).

**[Figure 1] Steady State Relationship between Inflation Rate and Leisure Time in the Economy with Multiple Means of Payment**



**[Table 1]** Steady State Welfare Costs of Inflation in the Economy with Multiple Means of Payment

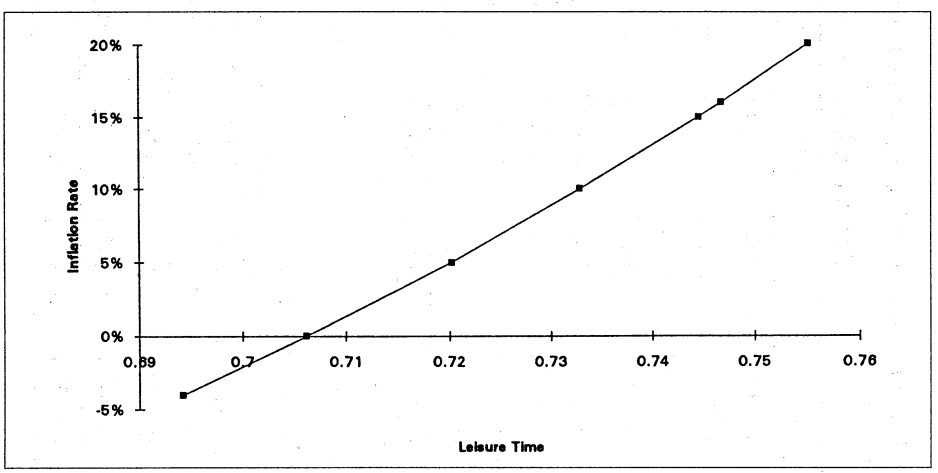
| | Annual Inflation Rate | | | | | | |
|---|---|---|---|---|---|---|---|
| | −4% | 0% | 5% | 10% | 15% | 16% | 20% |
| Output | 0.3058 | 0.2949 | 0.2854 | 0.2801 | 0.27838 | 0.27844 | 0.28 |
| Consumption | 0.3058 | 0.2943 | 0.2825 | 0.2731 | 0.2659 | 0.2646 | 0.2605 |
| Leisure | 0.6942 | 0.7051 | 0.7146 | 0.7199 | 0.72162 | 0.72156 | 0.72 |
| Real Money Balanes | 0.3058 | 0.2651 | 0.2189 | 0.1773 | 0.1391 | 0.1317 | 0.1035 |
| Real Trade Credit Balances | 0 | 0.0292 | 0.0636 | 0.0958 | 0.1268 | 0.1329 | 0.157 |
| Velocity of Money (M1) | 1 | 1.1123 | 1.3042 | 1.5799 | 2.0016 | 2.1129 | 2.7057 |
| Interest Elasticity of Money Demand | 0 | −0.1457 | −0.3572 | −0.6247 | −1.0014 | −1.0978 | −1.601 |
| Fraction of Transactions Using Cash | 1 | 0.9008 | 0.7748 | 0.6491 | 0.5231 | 0.4979 | 0.3973 |
| Welfare Cost (Percentage of Output) | 0 | 0.2713 | 1.2614 | 2.8918 | 4.9213 | 5.3872 | 7.3571 |

**[Table 2]** Steady State Welfare Costs of Inflation in the Economy with Credit

| | Annual Inflation Rate | | | | | | |
|---|---|---|---|---|---|---|---|
| | −4% | 0% | 5% | 10% | 15% | 16% | 20% |
| Output | 0.3058 | 0.2938 | 0.2978 | 0.2671 | 0.2554 | 0.2532 | 0.2448 |
| Leisure | 0.6942 | 0.7062 | 0.7202 | 0.7329 | 0.7446 | 0.7468 | 0.7552 |
| Welfare Cost (Percentage of Output) | 0 | 0.1021 | 0.3931 | 0.8611 | 1.527 | 1.6983 | 2.3284 |

In the (nested) cash-only economy, a 5% and 10% inflation results in a much smaller welfare cost of 0.39% and 0.86% of output, respectively. As noted, this is so because this economy captures only the inflation-induced inefficiency of substitution from goods to leisure and excludes any Bailey-type real resource cost of avoiding inflation. As opposed to the costly credit economy, as Table 2 and Figure 2 show, the amount of leisure time increases monotonically with the rate of inflation in the cash-only economy.

**[Figure 2] Steady State Relationship between Inflation Rate and Leisure Time in the Economy without Credit**




## V. CONCLUSION

In this paper we have tried to construct a monetary equilibrium model which can serve as a better basis for studying the welfare costs of inflation. In the model economy which extends the work of Lucas and Stokey (1983), both money and trade credit provide transactions services and the mix of two alternative means of payment is endogenous. The economy captures the Bailey's (1956) real resource cost of avoiding inflation tax in alternative means of exchange as well as the fact that inflation lowers welfare through its negative effect on the return to work.

With an explicit transactions technology parameterized to mimic the way U.S. households use cash and other means of payment in making their transactions, a welfare cost of 2.89% of output results from a 10% annual inflation rate relative to the Friedman (1969) rule. While a (nested) cash-only economy in which the transactions cost of trade credit becomes prohibitively high yields much smaller estimate of 0.86% of output.

The relatively larger welfare estimate in the economy with multiple means of payment stems from the following two distortions associated with the inflation tax. First, as in Cooley and Hansen (1989), agents substitute out of market activity by taking more leisure under moderate inflation. One interesting feature of our model is that the wealth effect which is dominated by this substitution effect at moderate inflation rates becomes dominant as the inflation rate goes beyond a certain level, as in Cole and Stockman (1992). Second, inflation-induced substitution out of cash into trade credit drives a larger wedge between output and consumption and results in a greater welfare cost of inflation by distorting the intertemporal choice of consumption, since real interest rate in the model is determined in part by the path of nominal rates.

The result strengthens the view that regards price stability as the most widely-cited objective for monetary policy. In particular, the comovement between nominal and real rates implies that smoothing nominal interest rate reduces the distortion in intertemporal choice of consumption due to the impediments to exchange.

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