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# THE DISTRIBUTIONAL CHANGE IN SECOND-DEGREE STOCHASTIC DOMINANCE AND ITS DECOMPOSITION*

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*This paper introduces a new concept, a left-side strong second-degree stochastic dominance (L-SSSD) shift which includes a left-side strong increase in risk (L-SIR) shift and proves that an L-SSSD can be decomposed into an L-SIR and one subset of first-degree stochastic dominance (FSD) which is a subset of a monotone probability ratio (MPR). We show that one can obtain an intuitively appealing comparative statics result for L-SSSD shifts by using the risk preferences of risk-averse individuals with prudence.*

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Keywords: Left-side strong second-degree stochastic dominance (L-SSSD), Monotone probability ratio (MPR), Left-side strong increases in risk (L-SIR), Prudence

## Ⅰ. INTRODUCTION

An important question in the study of economic decisions under uncertainty is to find predictable directions of the choice variable selected by an economic agent when a given random parameter changes. In order to generate interesting comparative statics results, the common restrictions to impose on the changes in probability distribution function (PDF) or cumulative distribution function (CDF) are applied to general stochastic dominance orders which are the first-degree stochastic dominance (FSD), the second-degree stochastic dominance (SSD), and the third-degree stochastic dominance (TSD). It implies that these stochastic

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dominance (SD) rules play an important role in the comparative statics analysis. Stochastic dominance can be applied to portfolio choice, investment and production decision problems, and others under uncertainty.

In case of FSD shifts, first of all, Fishburn and Porter (1976) demonstrated that an FSD order does not allow a determinate general comparative statics statement for all risk-averse agents, even in the simplest case of portfolio problem. To obtain interesting comparative statics results for the FSD case, it is needed to restrict the changes in PDF or CDF of the random parameter. Imposing monotonocity restriction on the likelihood ratio between a pair of PDFs, Landsberger and Meilijson (1990) defined a monotone likelihood ratio (MLR) order which is widely used in the statistical literature. The MLR shift is a subset of general FSD shift. Eeckhoudt and Gollier (1995) considered a monotone probability ratio (MPR) order which is obtained by imposing monotonicity restriction on the ratio of a pair of CDFs. The MPR order also specifies a subset of FSD changes and is more general than the MLR order. Recently, Ryu and Kim (2003) introduced a left-side relatively weak FSD (L-RWFSD) order which is less restrictive than the MLR order.

As the subset of SSD shifts, Rothschild and Stiglitz (1971) defined the concept of an increase in risk. Meyer and Ormiston (1983) showed that an arbitrary Rothschild-Stiglitz (R-S) increase in risk simply does not allow general comparative statics statements to be made concerning the effect of an increase in risk on the choice made by a risk-averse agent. Since then, some researchers have focused on finding the constraints on the set of R-S increases in risk to obtain interesting comparative statics results for the risk-averse decision-maker: a strong increase in risk (SIR) in Meyer and Ormiston (1985), a relatively strong increase in risk (RSIR) in Black and Bulkley (1989), and a relatively weak increase in risk (RWIR) in Dionne, Eeckhoudt and Gollier (1993a, 1993b).

Following these lines, we propose a new concept of a left-side strong second-degree stochastic dominance (L-SSSD) shift that extends the subsets of R-S increases risk and prove that an L-SSSD can be always decomposed into an L-SIR and one subset of first-degree stochastic dominance (FSD) which is a subset of a monotone probability ratio (MPR). We also investigate the subset of SSD shifts that causes risk-verse decision-makers with non-negative prudence ($u'''\geq 0$) to adjust their choice variable in the same direction in a general decision model.

In this paper we impose somewhat stronger restrictions on the risk preference of decision-makers by considering relatively weak restriction on the type of CDF changes. We deal with the set of risk-averse individuals with a non-negative third derivative of their utility function. This class of utility functions also includes the concept of 'prudence' ($\eta=-u'''/u''$) introduced by Kimball (1990), which denotes a precautionary saving motive. Note that the term 'prudence' is meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty.

The rest of the paper is organized as follows. In section Ⅱ, we give three definitions of ordering CDFs (MPR, L-SIR, and L-SSSD orders) and numerical and graphical examples, and present a general economic model in which a decision-maker maximizes his expected utility of the payoff variable depending on a choice variable and a random variable. Section Ⅲ provides comparative statics result for the L-SSSD order and indicates how our general result applies to specific economic models. Finally, section Ⅳ contains concluding remarks.

## Ⅱ. DEFINITIONS AND THE MODEL

In this section we describe the definitions of MPR, L-SIR, and L-SSSD orders and numerical and graphical examples, and present a model of a decision-maker maximizing the expected utility. We assume that the supports of $x$ under $G(x)$ are a finite interval $[x_1, x_4]$, under $G_1(x)$ are another finite interval $[l, x_3]$, and under $F(x)$ are other finite interval $[x_2, x_3]$, where $x_1 \leq l \leq x_2 \leq x_3 \leq x_4$. First, Eeckhoudt and Gollier (1995) introduced the concept of a monotone probability ratio (MPR) order that is defined by imposing monotonicity restriction on the ratio of a pair of CDFs. This restriction replaces the restriction on the ratio of a pair of PDFs used by Landsberger and Meilijson (1990) who define an MLR order.

**Definition 1.** $F(x)$ represents a monotone probability ratio FSD shift from $G(x)$ (denoted by $F$ MPR $G$) if there exists a non-decreasing function $h:[x_2, x_3] \to [0,1]$ such that $F(x) = h(x)G(x)$ for all $x \in [x_2, x_3]$.

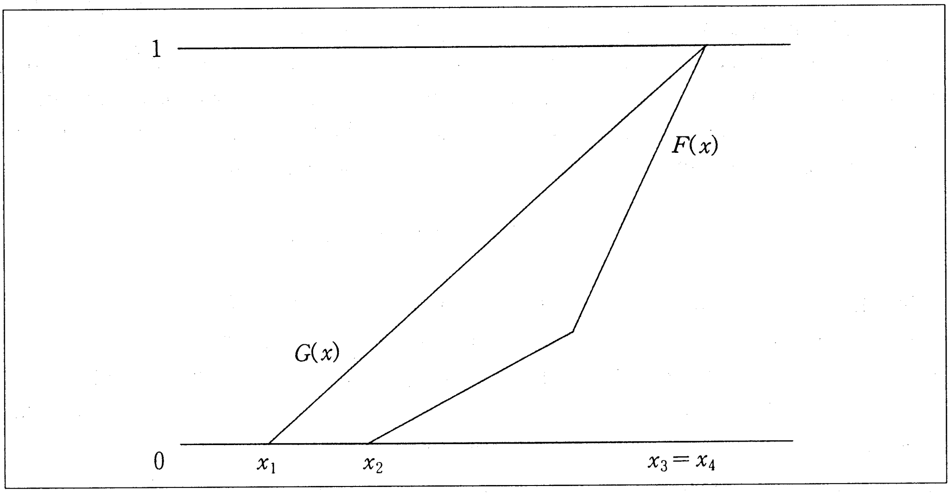
***Numerical example***: Consider the following two random variables with cumulative distribution functions $F(x)$ and $G(x)$, respectively; $F(x) = (1/2)x - (1/2)$ for $1 \leq x \leq 3/2$, $(3/2)x - 2$ for $3/2 \leq x \leq 2$, and $G(x) = (1/4)x^2$ for $0 \leq x \leq 2$. Note that $F(x) = 0$ for $0 \leq x \leq 1$ and $G(0) = 0$, and $F(2) = 1 = G(2)$. It is easy to show that $F(x)$ and $G(x)$ satisfy the condition of Definition 1:

$$d\left[\frac{F(x)}{G(x)}\right]/dx = \frac{-2x+4}{x^3} \geq 0 \text{ for } 1 \leq x \leq 3/2,$$

$$d\left[\frac{F(x)}{G(x)}\right]/dx = \frac{-6x+16}{x^3} \geq 0 \text{ for } 3/2 \leq x \leq 2.$$

The fact that $h \in [0,1]$ specifies $F(x) \leq G(x)$ for all $x \in [x_1, x_4]$, and thus an MPR shift is an FSD shift. An MPR order is less restrictive than an MLR order since the former does not restrict the number of times of crossing between the PDFs $f$ and $g$. Note that the MLR ranking implies the MPR one. Figure 1 illustrates an example of MPR.

**[Figure 1]** $F$ MPR $G$




Ryu and Kim (2004) considered a type of risk increases that is a subset of R-S increase in risk, called a left-side strong increase in risk (L-SIR). The L-SIR order is a less stringent type of R-S increase in risk than the SIR order proposed by Meyer and Ormiston (1985), who impose the restriction on the difference between the two CDFs.

**Definition 2.** $G(x)$represents a left-side strong increase in risk from $F(x)$ (denoted by $G$ L-SIR $F$) if

(a) $\int_{x_1}^{y}[G(x)-F(x)]dx \geq 0$ for all $y \in [x_1, x_4]$,

(b) $\int_{x_1}^{x_4}[G(x)-F(x)]dx = 0$,

(c) There exists a point $m \in [x_2, x_3]$ such that $G(x)-F(x)$ is non-increasing on $x \in (x_2, m)$ and $F(x) \geq G(x)$ for all $x \in [m, x_4]$.

***Numerical example***: Consider the following two random variables with cumulative distribution functions $F(x)$ and $G(x)$, respectively; $F(x)=(1/2)x+1/2$ for $-1 \leq x \leq 0$, $(3/4)x+1/2$ for $0 \leq x \leq 1/2$, $(1/12)x+5/6$ for $1/2 \leq x \leq 2$ and $G(x)=(1/4)x+1/2$ for $-2 \leq x \leq 2$. Note that $F(x)$ and $G(x)$ cross at the point $m(x=0)$ and $F(x)=0$ for $-2 \leq x \leq -1$ and $G(-2)=0$, and $F(2)=1$ $=G(2)$. After simple calculations, $F(x)$ and $G(x)$ satisfy the following conditions of Definition 2:

(a) $\int_{-2}^{y}[G(x)-F(x)]dx \geq 0$ for all $y \in [-2, 2]$,

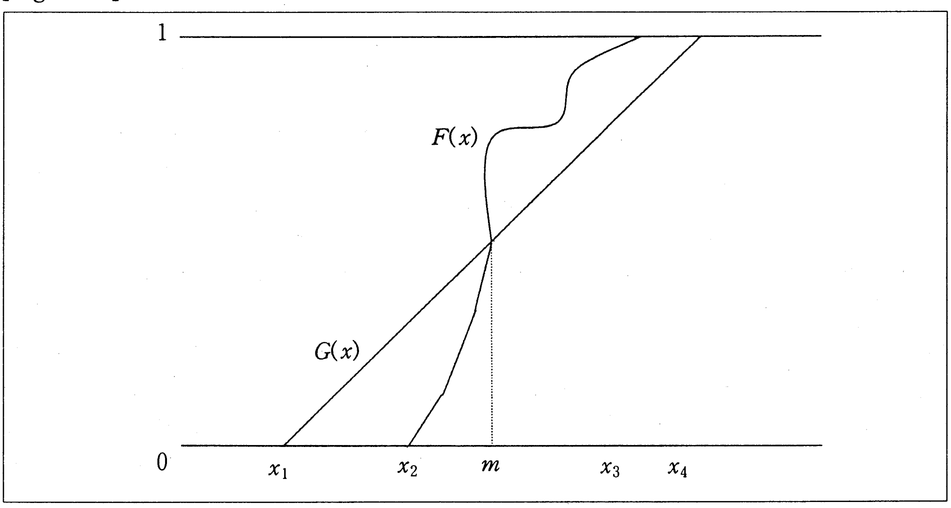
(b) $\int_{-2}^{2}[G(x)-F(x)]dx = 0$,

(c) There exists a point $m(x=0) \in [-1, 1/2]$ such that $d[G(x)-F(x)]/dx = -1/4 < 0$ for all $x \in [-1, 0]$ and $F(x) \geq G(x)$ for all $x \in [0, 2]$.

Conditions (a) and (b) imply that the L-SIR order is an R-S increase in risk. That is, $F$ dominates $G$ in the second-degree and the mean of the random variable is kept constant. Condition (c) imposes the restriction that the two CDFs cross only once at a point $m$. This condition implies that, to the left of the point $m$, the L-SIR order requires the same restriction used by Meyer and Ormiston to define the SIR order. Note that, to the right of the point $m$, restrictions $(F \geq G)$ imposed on L-SIR shifts are less stringent than those on SIR shifts. Therefore, the set of SIR shifts is a subset of the set of L-SIR shifts.

Figure 2 illustrates an example of a left-side strong increase in risk and a case where restrictions on the difference between the two CDFs on the interval $x \in [m, x_3)$ to obtain a strong increase in risk are not met. Note that the L-SIR order can be obtained from the SIR one by relaxing the restrictions imposed to the right of the point $m$.

**[Figure 2]** $G$ L-SIR $F$




Now, we introduce a left-side strong second-degree stochastic dominance (L-SSSD) shift which includes an L-SIR shift as a special case.

**Definition 3**. $F(x)$ represents a left-side strong second-degree stochastic dominance shift from $G(x)$ (denoted by $F$ L-SSSD $G$) if

(a) $\int_{x_1}^{y}[G(x)-F(x)]dx \geq 0$ for all $y \in [x_1, x_4]$,

(b) There exists a point $m \in [x_2, x_3]$ such that $F(x) \leq G(x)$ for all $x \in [x_1, m)$ and $F(x) \geq G(x)$ for all $x \in [m, x_4]$,

(c) $G(x)-F(x)$ is non-increasing on for all $x \in (x_2, m]$.

***Numerical example***: Consider the following two random variables with cumulative distribution functions $F(x)$ and $G(x)$, respectively; $F(x)=x+1/2$ for $-1/2 \leq x \leq 0$, $(3/4)x+1/2$ for $0 \leq x \leq 1/2$, $(1/12)x+5/6$ for $1/2 \leq x \leq 2$ and $G(x)=(1/4)x+1/2$ for $-2 \leq x \leq 2$. Note that $F(x)$ and $G(x)$ cross at the point $m(x=0)$ and $F(x)=0$ for $-2 \leq x \leq -1/2$ and $G(-2)=0$, and $F(2)=1=G(2)$. After simple calculations, $F(x)$ and $G(x)$ satisfy the following conditions of Definition 3:

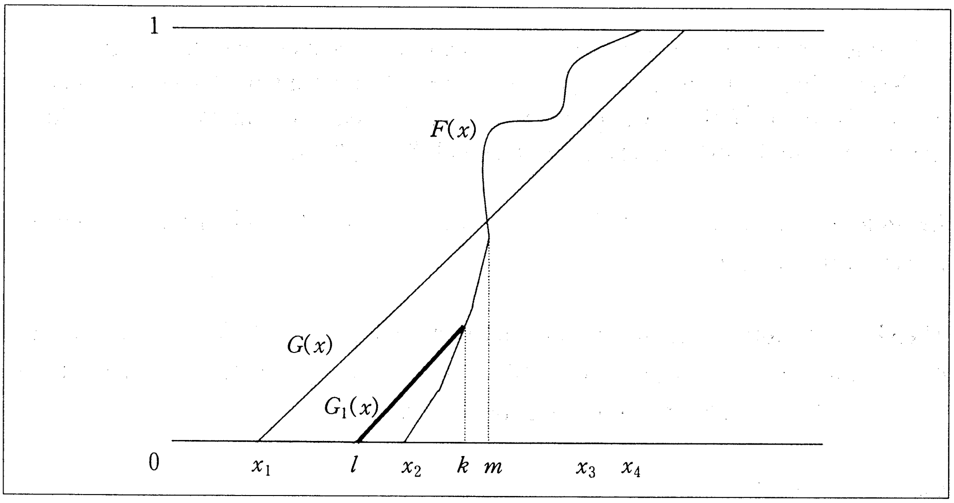
(a) $\int_{-2}^{y}[G(x)-F(x)]dx \geq 0$ for all $y \in [-2, 2]$,

(b) There exists a point $m(x=0) \in [-1/2, 1/2]$ such that $F(x) \leq G(x)$ for all $x \in [-2, 0)$ and $F(x) \geq G(x)$ for all $x \in [0, 2]$,

(c) $d[G(x)-F(x)]/dx = -\frac{3}{4} < 0$ for all $x \in (-1/2, 0]$.

Note that while Definition 2 includes the mean-preserving condition, Definition 3 does not always satisfy it. Figure 3 illustrates an example of a left-side strong SSD shift and a case where the mean-preserving condition to obtain a left-side strong increase in risk is not met. Observe that the bold line $lk$ in Figure 3 connecting a point $l$ to a point $k$ plays a decisive role in decomposing an L-SSSD into an L-SIR and one subset of FSD which is a subset of an MPR and making $\int_{x_1}^{x_4}[G(x)-G(x)]dx=0$ (both $G(x)$ and $G_1(x)$ have equal means) which implies an increase in risk in the R-S sense and satisfies condition (b) of Definition 2 for the L-SIR. For a numerical example of the L-SSSD, the line lk connecting a point $l(x=-3/2)$ to a point $k(x=-1/4)$ is represented by $G_1(x)=(1/5)x+3/10$ for $-3/2 \leq x \leq -1/4$ in Figure 3.

**[Figure 3]** $G$ L-SSSD $F$




In this paper, the payoff function is restricted to be linear in the random variable. We consider the simple form of the general decision model such as $z(x,\alpha)\equiv z_0+\alpha x$, where $z_0$ is an exogenous constant, $\alpha$ is a decision variable, and $x$ is a random variable. The economic decision problem can be written as

$$\alpha_F \in \arg\max{}_{\alpha} Eu(z_0+\alpha_F x)=\int_{x_2}^{x_3} u(z_0+\alpha_F x)dF(x). \tag{1}$$

We assume that utility function $u(z)$ is three times differentiable with $u'(z)\geq 0$, $u''(z)\leq 0$ and $u'''(z)\geq 0$; thus, the decision maker is a risk averter with non-negative prudence.

The necessary and sufficient condition for the choice of $\alpha_F$ to maximize the expected utility is

$$\int_{x_2}^{x_3} u'(z_0+\alpha_F x)xdF(x)=0. \tag{2}$$

It is well known that $\alpha_F$ has the same sign as $E_F(x)=\int_{x_2}^{x_3} xdF(x)$ [see Dionne, Eeckhoudt and Gollier (1993a), and Eeckhoudt and Gollier (1995)]. Therefore, we assume that $E_F(x)$ is positive. In order to prove $\alpha_F\geq\alpha_G$ for a specified change in the CDF from $F$ to $G$, it is sufficient to show that, for all $x\in[x_1, x_4]$,

$$Q(\alpha_F)=\int_{x_1}^{x_4} u'(z_0+\alpha_F x)xd[F(x)-F(x)]\geq 0. \tag{3}$$

## Ⅲ. COMPARATIVE STATICS ANALYSIS AND APPLICATIONS

In this section, we consider general comparative statics statement regarding the L-SSSD shifts and present several examples of applications which illustrate the use of Theorem. Before we prove the main theorem, we introduce the following Lemma for an L-SIR presented in Ryu and Kim (2004).

**Lemma**. If $G$ L-SIR $F$ and $z_{xx}=0$, then $\alpha_F \geq \alpha_G$ for all risk-averse decision makers with $u''' \geq 0$.

**Sketch of Proof**: Using the payoff function $z(x, \alpha)$ in (1), let $x^*$ be the value of $x$ satisfying $z_\alpha(x, \alpha_F)=0$, and assume that $x^*$ exists on the interval $[x_2, x_3]$. We consider the following two cases:

Case (i): $x_2 \leq x^* \leq m$.
Let's rewrite $Q(\alpha_F)$ in (3) as

$$Q(\alpha_F)=\int_{x_1}^{x^*} u'(z) z_\alpha(f-g) dx+\int_{x^*}^{m} u'(z) z_\alpha(f-g) dx+\int_{m}^{x_4} u'(z) z_\alpha(f-g) dx.$$

Using assumptions on preference and the L-SIR definition, and adding and subtracting $u'[z(m, \alpha_F)] \int_{x_1}^{x^*} z_\alpha(f-g) dx$, we have

$$\begin{aligned} Q(\alpha_F) \geq & \{u'[z(x_2, \alpha_F)]-u'[z(m, \alpha_F)]\} \int_{x_1}^{x^*} z_\alpha(f-g) dx \\ & +u'[z(m, \alpha_F)] \int_{x_1}^{m} z_\alpha(f-g) dx+\int_{m}^{x_4} u'(z) z_\alpha(f-g) dx. \end{aligned} \tag{4}$$

From the given assumptions and the L-SIR definition, (4) becomes

$$Q(\alpha_F) \geq\{u'[z(x_2, \alpha_F)]-u'[z(m, \alpha_F)]\} \int_{x_1}^{x^*} z_\alpha(f-g) dx.$$

Because $u'(z)$ is non-increasing in $x$, $Q(\alpha_F) \geq 0$ if $\int_{x_1}^{x} z_\alpha(f-g) dx \geq 0$ for all $x \in [x_1, x^*]$. Let $T(x, \alpha_F)=\int_{x_1}^{x} z_\alpha(f-g) dx=z_\alpha H(x)-\int_{x_1}^{x} z_{\alpha x} H(s) ds$, where $H(x) \equiv [F(x)-G(x)]$. Since $z_\alpha$ is negative and $H(x) \leq 0$ on $[x_1, x_2]$ and $H(x)$ is non-decreasing on $[x_2, m]$, we have that

$$T(x, \alpha_F) \geq z_\alpha H(x)-\int_{x_1}^{x} z_{\alpha x} H(s) ds$$

$$\geq z_\alpha H(x) - H(x)\int_{x_2}^{x} z_{\alpha x} ds = z_\alpha(x_2, \alpha_F)H(x)$$

for all $x \in [x_2, m]$. Since $z_0(x_2, \alpha_F)H(x)$ is non-negative for all $\alpha_F \in [\alpha_2, \alpha_3]$, $T(x, \alpha_F) \geq 0$ for all $x \in [x_2, m]$. Therefore, $Q(\alpha_F) \geq 0$.

Case (ii): $m \leq x^* \leq x_3$.
Integrating $Q(\alpha_F)$ by parts, $Q(\alpha_F)$ can be written as

$$Q(\alpha_F) = \int_{x_1}^{x_4} [u''(z) z_x z_\alpha + u'(z) z_{\alpha x}][G(x) - F(x)]dx.$$

Note that, when the assumption of $u''' \geq 0$ is used, $u''(z)z_x z_\alpha + u'(z) z_{\alpha x}$ is positive and non-increasing in $x$ in the interval $[x_1, x^*]$, and it has its maximum at $x = x^*$ in the interval $[x^*, x_4]$ because $u''(z) z_x z_\alpha$ is always non-positive and $u'(z) z_{\alpha x}$ is non-increasing in $x$. Since $m \leq x^*$, this implies that

$$[u''(z) z_x z_\alpha + u'(z) z_{\alpha x}]|_{x \leq m} \geq [u''(z) z_x z_\alpha + u'(z) z_{\alpha x}]|_{x=m}$$

$$\geq [u''(z) z_x z_\alpha + u'(z) z_{\alpha x}]|_{x \geq m}.$$

Therefore, we have the following inequality,

$$Q(\alpha_F) \geq [u''(z) z_x z_\alpha + u'(z) z_{\alpha x}]|_{x=m} \int_{x_1}^{x_4} [G(x) - F(x)]dx = 0.$$

See Ryu and Kim (2004) for more details of proof. Q.E.D.

The following comparative statics result for an L-SSSD indicates that, when $z_{xx} = 0$, one can explore the subset of SSD shifts for the risk-averse decision-makers with non-negative prudence.

**Theorem**. If $G$ L-SSSD $F$, $z_{\alpha x} \geq 0$ $(z_{\alpha x} \leq 0)$ and $z_{xx} = 0$, then $\alpha_F \geq (\leq) \alpha_G$ for all risk-averse decision-makers with $u''' \geq 0$.

**Proof**: Before proving Theorem, observe that if two distributions $F$ and $G$ have equal mean, an L-SSSD is degenerate to an L-SIR. Note that $G$ L-SSSD $F$ can be decomposed into $G$ L-SIR $G_1$ and one subset of FSD (imposing monotonicity restriction on the difference of two CDFs, $G_1 - F$) which is a subset of MPR (imposing monotonicity restriction on the ratio of two CDFs, $F/G$ in Definition 1) in Figure 3.

(i) In order to explore the relationship between $G$ and $G_1$, let's define an intermediate CDF $G_1(x)$ such that

$$G_1(x)=\begin{cases} \delta G(x), & when\ x<k \\ F(x), & when\ x\geq k \end{cases}$$

where $\delta=F(k)/G(k)$ and $k$ is a selected point on the interval $[x_2, m]$ such that $G_1(x)$ has the same mean as $G(x)$. Therefore, $G$ L-SIR $G_1$ is established and implies Definition 2.

(ii) Now, consider the relationship between $F$ and $G_1$. Note that condition (c) of Definition 3 implies $\frac{f(x)}{g_1(x)}\geq 1$. From Definition 1, $F$ MPR $G$ means $\frac{f(x)}{g(x)}\geq\frac{F(x)}{G(x)}\geq 0$. Therefore, condition (c) of Definition 3 implies MPR.

Since L-SSSD shifts are decomposed into L-SIR shifts and one subset of MPR shifts, the comparative statics result in Theorem is held by Eeckhoudt and Gollier (1995) and Lemma. Q.E.D.

While the linearity assumption $(z_{xx}=0)$ restricts the set of decision problems to which our result is applicable, linear payoffs prevail in many economic environments such as those analyzed by Sandmo (1971), Rothschild and Stiglitz (1971), Fishburn and Porter (1976), Dionne, Eeckhoudt and Gollier (1993a), and Eeckhoudt and Gollier (1995). Applications include the standard portfolio model, the optimal behavior of a competitive firm with price uncertainty, the coinsurance problem, and others.

In particular, now we give some examples which provide appropriate application of Theorem. First, consider that a perfectly competitive firm chooses output $q$, and that the profit function is given by $\pi=p\cdot q-c(q)-F$, where $p$ is uncertain output price, $c(q)$ is the variable cost function with $c'(q)>0$ and $c''(q)\geq 0$, and $F$ is fixed cost. The decision problem is to maximize the expected utility of profits, $EU(\pi)$. This model is analyzed by Sandmo (1971) and later by Ishii (1977) and Eeckhoudt and Hansen (1980). For this model, the payoff function is given by $z(x,a)=\pi(p,q)$; hence, $z_{ax}=\pi_{qp}=1$ and $z_{axx}=\pi_{qpp}=0$. Therefore, applying Theorem, an L-SSSD shift leads to decrease in the output level of risk-averse firm with $u'''\geq 0$.

Paroush and Kahana (1980) examined the behavior of a cooperative firm that maximizes the expected utility of profits per unit of labor. Assume a price taking firm that produces output $q$ using a single input called labor $L$. Let $q=f(L)$ be the production function with $f'(L)>0$ and $f''(L)<0$. The cooperative firm is assumed to choose $L$ in order to maximize the expected utility from per capita profits $\pi/a=(p\cdot f(L)-w\cdot L-F)/L$, where $p$ is uncertain output price, $w$ is the wage rate, and $F$ is fixed cost. Applying

Theorem, the payoff function is given by $z(x,\alpha)=\pi(p.L)/L=\hat{\pi}(p,L)$. Hence, $z_{\alpha x}=\hat{\pi}_{Lp}=[Lf'(L)-f(L)]/L^2$ is less than zero from the first-order condition and $z_{\alpha xx}=\pi_{Lpp}=0$. Therefore, an L-SSSD shift causes risk-averse firms with $u'''\geq 0$ to increase employment and thus output.

Finally Feder (1977) investigated the decision problem of hiring workers whose productivity is uncertain but changes directly with the level of formal education. The profit function from employing a worker is given by $\pi=\theta F(s)-w(s)-T$, where $\theta$ is a random variable representing individual characteristics, $s$ is the level of education, $\theta F(s)$ is the productivity with $F'(s)>0$, $w(s)$ is the wage that depends on the education level, and $T$ is fixed training cost. In our notation, the payoff function can be represented by $z(x,\alpha)=\pi(\theta,s)$; thus $z_{\alpha x}=\pi_{s\theta}=F'(s)>0$ and $z_{\alpha xx}=\pi_{s\theta\theta}=0$. Therefore, for all risk-averse firms with $u'''\geq 0$, an L-SSSD in the distributional changes of worker's productivity decreases the education level at which hiring takes place.

## Ⅳ. CONCLUSIONS

This paper introduces a new concept of a left-side strong second-degree stochastic dominance (L-SSSD) order that represents a net improvement over a left-side strong increase in risk (L-SIR) one without any cost of additional assumptions and explores the trade-offs among the CDF change, the structure of the decision model, and the set of decision-makers for a subset of SSD shifts. This paper also shows that an L-SSSD includes an L-SIR as a special case, and that an L-SSSD can be always decomposed into an L-SIR and one subset of FSD which is a subset of MPR. Our result reveals that the effect of this shift can be determined for all risk-averse decision-makers with non-negative third derivative of utility functions. This implies that our result contains a larger set of changes in distribution and a smaller set of the risk preference of decision-makers. It can be also compared to the previous results derived from the distributional changes in FSD shifts: it includes a larger set of changes in distribution and a smaller set of the decision model and the risk preference of decision-makers.

## REFERENCES

Black, J.M. and Bulkley, G. (1989), "A Ratio Criterion for Signing the Effect of an Increase in Uncertainty," *International Economic Review*, 30, 119-130.

Dionne, G., Eeckhoudt, L. and Gollier, C. (1993a), "Increases in Risk and Linear Payoffs," *International Economic Review*, 34, 309-319.

Dionne, G., Eeckhoudt, L. and Gollier, C. (1993b), "Relatively Weak Increases in Risk and Their Comparative Statics," *Economics Letters*, 41, 269-272.

Eeckhoudt, L. and Gollier, C. (1995), "Demand for Risky Assets and the Monotone Probability Ratio Order," *Journal of Risk and Uncertainty*, 11, 113-122.

Eeckhoudt, L. and Hansen, P. (1980), "Mean-Preserving Changes in Risk with Tail-Dominance," *Theory and Decision*, 33, 23-39.

Fishburn, P.C. and Porter, B. (1976), "Optimal Portfolio with one Safe and one Risky Asset: Effects of Changes in Rate of Return and Risk," *Management Science*, 22, 1064-1073.

Feder, G. (1977), "The Impact of Uncertainty in a Class of Objective Functions," *Journal of Economic Theory*, 16, 504-512.

Ishii, Y. (1977), "On the Theory of the Competitive Firm under Price Uncertainty: Note," *American Economic Review*, 67, 768-769.

Kimball, M. (1990), "Precautionary Saving in the Small and the Large," *Econometrica*, 58, 53-73.

Landsberger, M. and Meilijson, I. (1990), "Demand for Risky Financial Assets: A Portfolio Analysis," *Journal of Economic Theory*, 50, 204-213.

Meyer, J. and Ormiston, M.B. (1983), "The Comparative Statics of Cumulative Distribution Function Changes for the Class of Risk Averse Agents," *Journal of Economic Theory*, 31, 153-169.

Meyer, J. and Ormiston, M.B. (1985), "Strong Increases in Risk and Their Comparative Statics," *International Economic Review*, 26, 425-437.

Paroush, J. and Kahana, N. (1980), "Price Uncertainty and the Cooperative Firm," *American Economic Review*, 70, 212-216.

Rothschild, M. and Stiglitz, J. (1971), "Increasing Risk: II. Its Economic Consequences," *Journal of Economic Theory*, 3, 66-84.

Ryu, Suyeol and Kim, Iltae (2003), "Comparative Statics on the Left-Side Relatively Weak First-Degree Stochastic Dominance Order and Its Application," *Seoul Journal of Economics*, 16, 59-69.

Ryu, Suyeol and Kim, Iltae (2004), "Left-Side Strong Increases in Risk and Their Comparative Statics," *Theory and Decision*, 57, forthcoming.

Sandmo, A. (1971), "On the Theory of the Competitive Firm under Price Uncertainty," *American Economic Review*, 61, 65-73.