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COOPERATION IN HETEROGENEOUS POPULATION*

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This paper analyzes a heterogeneous population in which selfish players and fair players are spatially distributed, and they are randomly and repeatedly matched to play a prisoner's dilemma. Players are assumed to behave in a myopic manner. By introducing the random experimentation of fair players, we show that the system described by a Markov process converges to a best possible equilibrium in the long run. Simulation results show that the structure of interactions plays an important role in determining the long-run cooperation rate.

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I. INTRODUCTION

The prisoner's dilemma is a well-known game in which players' mutual cooperation is desirable but difficult to enforce due to the rational behavior of selfish players. Selfishness and rationality are the two cornerstones on which conventional economic models have been established. However, it has been pointed out in the recent literature that many phenomena cannot be easily formulated as the outcome of rational choices by selfish agents. For example, in many laboratory experiments concerning the contribution to a public good, which can be characterized as a one-shot prisoner's dilemma, players contribute their resources on average nearly 40-60% of the socially optimal level.[1] Clearly, this

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[1] Dawes and Thaler (1988) provides a short survey of these public good experiments. For a

result is inconsistent with the prediction of standard game theory that prescribes zero contribution as the dominant strategy.

Since these experimental studies seem to imply that some players make contributions while others free ride, one natural theoretical extension is to take into account the fact that in many situations which are often modeled as a prisoner's dilemma, there exist a number of players who are not as "greedy" as typical selfish players. From this cause, we would like to examine a heterogeneous population in which selfish players and fair players interact. Fair players are characterized as suffering from guilty feeling when they free ride, whereas selfish players enjoy the material gain from exploiting other players.[2] Thus, fair players seek to coordinate with other players, while selfish players always defect.

Furthermore, observe that in a very large economy, agents tend to interact with only a relatively small subset of the whole population, whom we call neighbors. Generally, the set of neighbors of a particular agent partially overlaps with one another so that all agents are directly or indirectly linked with each other. The size of the neighborhood depends upon the characteristics of the game being played and the level of existing technology such as transportation or communication. For example, advanced means of transportation allow people to interact with a larger number of other people.

The object of this paper is to study how the cooperation can evolve in a heterogeneous population in which selfish players and fair players locally interact, and how the structure of interactions can affect the efficiency of the long-run equilibrium outcome.[3] For this purpose, we consider an economy in which two types of players are spatially distributed, and they are repeatedly and randomly matched to play a prisoner's dilemma. The stage game is complicated with incomplete information so that the payoff matrix depends on the types of matched pair. It is assumed that a player can remember his neighbors past actions but cannot distinguish their type.

In principle, when a player makes a decision in each period, he has to take into account the effect of today's decision on the future payoff through the influence on the future actions of his neighbors. For simplicity, however, we assume that the complexity of the problem forces players to optimize myopically, and that they naively expect to see the same profile of neighbors' actions as one in the previous period. Then players will choose one-shot optimal action in response to their neighbors' actions in the previous period. When players employ the myopic best response such as this, there exist multiple Nash equilibria, and thus a wide range of cooperation rates can be realized as

comprehensive survey, see Ledyard (1995).

[2] Fairness has been modeled in various ways. For example, see Hirshleifer (1985, 1993) and Rabin (1993).

[3] By using Ising model, Ellickson (1990) and Blume (1993) study local interactions in a homogeneous population.

outcomes. For any mix of two types, for example, the worst outcome in which everyone defects is a Nash equilibrium. There also exist other equilibrium outcomes in which some of fair players cooperate. We can call the best of the set of outcomes *best possible equilibrium*, so that in a best possible equilibrium the maximum amount of cooperation among fair players is achieved.

In order to single out the most plausible outcome in the long run, we will focus on one desirable characteristic of the fair players. Recognizing that current bad outcome is mainly due to coordination failure among themselves, some fair players attempt to act as leaders in solving the coordination problem. The intentional efforts of leaders can be modeled by assuming that in each period all fair players make random experiments, in which they cooperate with a small probability even when doing so is not myopically optimal. This leading cooperation is allowed to be withdrawn at any time if the leaders are not satisfied with a persistent non-cooperative neighborhood. Then some of fair players attempt to cooperate even in a completely defective environment. Clusters of these leading cooperators may induce other fair players to cooperate through their learning and myopic optimization, so that they can locally succeed in fostering cooperative environment. Therefore, we can expect that in the long run the economy would eventually evolve to a best possible equilibrium.

In this setting, the dynamics of the game can be described by a Markov process. We analyze the limit behavior of the system as the probability of experimentation goes to zero. We can show that in the long run the economy converges to the best possible equilibrium. Since the distribution of players certainly affects the cooperation rate in the best possible equilibrium, we need to look at the average cooperation rate for a given set of other parameter values. The average cooperation rate in the long-run equilibrium varies according to the proportion of selfish players, the size of neighborhoods, the size of population and the degree of overlap in the interaction structure.

For completeness, we carried out a number of computer simulations. In these simulations, we imposed a restriction on the payoff matrix, so that a fair player's myopic best response is to follow the majority action of his neighbors in the previous period. Two interesting results were found. First, if the population is quite selfish, or the matching rule is quite local, then the neighborhood size has a negative effect on the long-run cooperation rate. Second, the larger the population size is, the lower the cooperation rate. These two effects can be combined to explain why people living in a large and crowded city tend to be less cooperative.

The paper is organized as follows. In Section II, we discuss how our model is related to other literature. Section III describes the formal model, in which payoff matrices, matching rules, myopic best response, and random experimentation are detailed. In Section IV, we derive the main theorem concerning the existence of a unique long-run equilibrium. Section V investigates the implication of various matching rules for the long-run equilibrium by using simulation. Finally, Section Ⅵ summarizes our results and proposes several applications.

Ⅱ. THE RELATED LITERATURE

Axelrod (1984) addressed the question of how the cooperation can evolve in a population consisting only of selfish players. He set up the problem as an indefinitely repeated prisoner's dilemma, and showed that if players are sufficiently patient, then cooperation can evolve from small clusters of players who use TIT FOR TAT strategy. TFT strategy cooperates on the first move and then does whatever the opponent did on the preceding move. In his computer tournaments, Axelrod found that TFT was the best strategy of all strategies submitted for a repeated prisoner's dilemma.[4] Axelrod focused on how the long-run consideration of patient selfish players could lead them to cooperate on the basis of reciprocity. In contrast, we focus on the process by which myopic fair players get to cooperate among themselves in the face of selfish players who never cooperate. In Axelrod's framework, each player is assumed to recognize the other player in his interactions and to remember how the two of them have interacted. Thus players can base their decision on the history of the particular interaction. In our analysis, however, a player cannot recognize his opponent except knowing that he belongs to his neighbors. Hence, the average behavior of his neighbors in the past is taken into account by a player's strategy.

Kandori, Mailath and Rob (1993) analyzed the long-run behavior in a large population in which players are repeatedly and randomly matched to play a coordination game. They showed that evolutionary forces created by mutations and myopic behavior by players lead them to coordinate to a risk-dominant equilibrium in the long run. In our model the stage game played depends on the types of two matched players. Since fair players in our model are analogous to players in a coordination game in the sense that they want to coordinate with other players, we can employ the modeling strategy of KMR. When players globally interact in a fairly large population as assumed in KMR, however, it will take too long for the evolutionary forces to be felt. Then the analysis of long-run equilibrium is not relevant for the prediction of a real outcome. Ellison (1993) shows that if the matching process is local rather than global, the system can quickly converge to the long-run equilibrium. In our heterogeneous population game, the structure of interactions affects not only the rate of convergence to the limit, but also the limit itself.

In evolutionary game theory, mutations play a crucial role in making the system converge to the long-run equilibrium. In our model, random experimentations replace mutations. But random experimentations differ from the

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[4] For a short report of this tournament result, see Axelrod and Hamilton (1981).

mutations in the previous literature on two aspects. First, we introduce random experimentation to model fair players' intentional efforts to solve coordination failure amongst fair players, while mutations in KMR and Ellison are modeling mistakes by players. Second, experimentation in our model is only one way of moving from defection to cooperation, while KMR and Ellison's mutations allow two-way randomization between two actions.

Ⅲ. THE MODEL

Consider a large population consisting of N players, letting N also represent the set of players. Each player is one of two types: selfish (S) or fair (F). S and F will also be used to denote the number of people in each type. Notice that $N \equiv S \cup F$ and $S \cap F = \phi$. The percentage of selfish players is denoted by m. Players are randomly and repeatedly matched for play in a two-person Prisoner's Dilemma. Time is discrete, indexed by $t = 1, 2, 3, \cdots$. In each period, player i chooses one of two possible actions $a_{it} \in \{C, D\}$, where C and D denote "cooperate" and "defect" respectively. Depending on the types of the matched pair, one of three payoff matrices becomes relevant. In Table 1, these payoffs are tabulated, where we have assumed that $x > 0$, $l > 0$, and $g > y > 0$. Here l measures the loss from being exploited and y measures the pecuniary gain from free riding; however, fair players also suffer from "guilty" feeling of g when they cheat other people.

[Table 1] Three Possible Games

(a) Selfish vs. Fair

	C	D
C	$x,\ x$	$-l,\ x+y-g$
D	$x+y,\ -l$	$0,\ 0$

(b) Selfish vs. Selfish

	C	D
C	$x,\ x$	$-l,\ x+y$
D	$x+y,\ -l$	$0,\ 0$

(c) Fair vs. Fair

	C	D
C	$x,\ x$	$-l,\ x+y-g$
D	$x+y-g,\ -l$	$0,\ 0$

Players interact only with a subset of the population. To be concrete, suppose that players are uniformly distributed on a circle or torus of N sites and are not allowed to move.[5] Interactions on a circle (torus) can be thought of as one (two)-dimensional.[6] Let us symbolize local interaction on a circle by $C\#$, where # denotes the number of neighbors; for example, $C4$ stands for the local interaction on a circle with 4 neighbors. Likewise, $T4$ denotes a local matching on a torus with 4 neighbors. Figure 1 illustrates some examples of various interaction structures. Even when the numbers of neighbors are equal, the degree of overlap can be very different depending on the dimension of the interaction structure. Some interactions have a highly overlapping structure in the sense that a neighbor of an agent's neighbor is likely to be also a neighbor. In other structures, the neighborhoods of two distinct agents may only slightly overlap or in fact are completely disjoint. For example, the probability of a neighbor of an agent's neighbor also being his neighbor is 1/2 in $C4$ but 0 in $T4$.

[Figure 1] Examples of Interaction Structuresa

○	○	*	●	*	○	○

(a) C2

○	*	*	●	*	*	○

(a) C2

○	○	○	○	○
○	○	*	○	○
○	*	●	*	○
○	○	*	○	○
○	○	○	○	○

(c) T4

○	○	○	○	○
○	*	*	*	○
○	*	●	*	○
○	*	*	*	○
○	○	○	○	○

(d) T8

○	○	*	○	○
○	*	*	*	○
*	*	●	*	○
○	*	*	*	○
○	○	*	○	○

(e) T12

[a] The symbol * represents a player who is one of the neighbors of the player ●.

In every period, each player is randomly matched with one of his closest $2k$ neighbors with equal probability, where k is a positive integer smaller than $N/2$. Then the probability that player i meets j in a given period, π_{ij}, is defined by:[7]

$$\pi_{ij}=\begin{cases} \dfrac{1}{2k} & \text{if } i-j=\pm1,\pm2,\cdots,\pm k\,(nod\ N) \quad \text{mod} \\ 0 & otherwise \end{cases}$$

[5] Schelling (1971) analyzes the segregation phenomena when the locally interacting agents are allowed to move. And Ely (1995) extends the model of KMR (1993) and Ellison (1993) to the case when players can choose the neighborhoods to which they belong.

[6] A circle can be thought of as simply a line with both ends connected. Similarly, if unfolded, a torus becomes a square.

[7] When we consider two-dimensional matching, we might have to use a coordinate (i, j) rather than i to locate a player. For notational simplicity, however, we focus on one-dimensional matching. All of our results still apply to the two-dimensional matching cases.

If we denote the dimension of the interaction structure by d, then $M \equiv (2k, d)$ defines a matching rule. For convenience, let us redefine the set of possible actions as $A \equiv \{1, 0\}$, where C has been replaced by 1 and D by 0. Clearly then, the expected payoff to player i in period t, u_{it}, can be written as a function of his own action $(a_{it} \in A)$ and his neighbors' actions $(a_{-it} \in A^{N-1})$,

$$u_{it} \equiv u_i(a_{it},\ a_{-it}):$$
$$\pi_{ij} = \begin{cases} (1-a_{it})(\frac{x+y}{2k}\sum_{j=1}^{k} a_{i\pm jt}) + a_{it}(\frac{x+l}{2k}\sum_{j=1}^{k} a_{i\pm jt} - l) & \text{if } i \in S \\ (1-a_{it})(\frac{x+y-g}{2k}\sum_{j=1}^{k} a_{i\pm jt}) + a_{it}(\frac{x+l}{2k}\sum_{j=1}^{k} a_{i\pm jt} - l) & \text{if } i \in F. \end{cases}$$

Let us describe the spatial distribution of players' types by an $N \times 1$ vector, $e^0 \in \{0, 1\}^N$, where 0 denotes a selfish type and 1 a fair type. Now, we can define a heterogeneous population game G as follows.

DEFINITION 3.1 *$G \equiv (E, M, \phi)$ is a repeated game played by a heterogeneous population $E \in (N, m, e^0)$, with the matching rule $M \equiv (2k, d)$ and the payoffs $\phi \equiv (x, y, g, l)$.*

We assume that a player can observe his neighbors' past actions but cannot distinguish their types. In principle, players wish to maximize the lifetime payoff, $V(t) = \sum_{s=t}^{\infty} \beta^{s-t} u_{is}$, where β is a discount factor. In other words, when a player makes a decision in each period, he should take into account the effect of today's decision on the future payoff through the influence on the future actions of his neighbors. Since the future actions of his neighbors depend on their other neighbors' actions as well, however, it is difficult for a player to form rational belief as to how his neighbor will respond to his today's action. Therefore, we assume that players believe that their current actions do not affect their neighbors' future actions, i.e., $E_i[a_{-is} \mid a_{it} = 0] = E_i[a_{-is} \mid a_{it} = 1]$ for all $s > t$. Under this belief, players behave myopically. Then a myopic player does not care about the future and simply chooses an action to maximize his current period expected payoff, which depends on his neighbors' current period actions.[8] We also assume that players have static belief that their neighbors will stick to the same actions as in the previous period, i.e., $E_i[a_{-it} \mid a_{-it-1}] = a_{-it-1}$. Then players' optimal actions can be described by the following best response function, $BR_i: a_{t-1} \equiv (a_{1t-1}, a_{2t-1}, \cdots, a_{Nt-1}) \rightarrow a_{it}$:[9]

[8] Ellison (1997) explores the question of when the assumption of myopia can be justified in a large population. In contrast to our model, he assumes that each player knows only about the matches in which he has been involved.

[9] Here, we assume that when they are indifferent, fair players cooperate.

$$BR_i(a_{t-1}) = \begin{cases} 0 & \text{if } i \in S, \\ 0 & \text{if } i \in F \text{ and } \frac{1}{2k} \Sigma_{j=1}^{k} a_{i \pm jt-1} < \frac{l}{g-y+l}, \\ 1 & \text{if } i \in F \text{ and } \frac{1}{2k} \Sigma_{j=1}^{k} a_{i \pm jt-1} \geq \frac{l}{g-y+l}. \end{cases}$$

In other words, selfish players always choose to defect no matter how other people behave. The optimal action of a fair player, however, depends on what proportion of his neighbors cooperated in the previous period, i.e., he will cooperate if and only if the proportion of cooperative neighbors is not less than a critical level. Now, using this best response function, we can define a Nash equilibrium of the game.

DEFINITION 3.2 *For a given game* G, *an action profile* $a \equiv (a_1, a_2, \cdots, a_N) \in A^N$ *is a Nash equilibrium if and only if* $BR_i(a) = a_i$ *for all* $i \in N$.

Since fair players' best responses depend on how other people behave, there exist multiple Nash equilibria from the worst one of complete defection, *ALL D,* to a *best possible equilibrium*, in which fair players achieve the maximum level of cooperation among themselves. Of course, the level of cooperation in the best possible equilibrium varies with the parameters of the model, such as the configuration of players in the population and the matching rule.[10]

We would like to know whether the economy settles at the best possible equilibrium in the long run. Suppose that initially the economy is at the worst equilibrium in which both selfish players and fair players defect. Selfish players, being greedy, can be said to deserve the low level of utility from *ALL D*. For fair players, however, the Pareto inferior outcome should be attributed to the lack of coordination among themselves as well as to the fear of being exploited by selfish players. Once coordination failure is recognized as the main reason for a bad outcome, some fair players are motivated to play a role in mobilizing incentives to improve the current bad state. Then they will attempt to cooperate even in a non-cooperative environment. Clusters of these leaders may induce other fair players to cooperate through their myopic best response and learning about the changing environment. In some neighborhoods, these coordination efforts may be discouraged due to a large fraction of selfish players relative to fair leaders. In the long run, however, the economy may eventually move from a non-cooperative state to the best possible equilibrium.

To capture this idea, let us assume that in each period fair players, in the hope of minimizing the inefficiency of possibly being locked in a bad equilibrium due to their myopic behavior, choose to cooperate with probability ε, even when their best response calls for defection.[11] On the other hand,

[10] From now on, by the configuration of players we mean the spatial distribution of players, e^0.

selfish players are not allowed to make any experimentation because here we introduce random experimentation to model intentional efforts rather than mistakes. Then the best response function should be replaced by a *behavior rule*, $B_i: a_{t-1} \rightarrow a_{it}$, which is given by:

$$B_i(a_{t-1})=\begin{cases} 0 \text{ if } i \in S, \\ 0 \text{ with probability } (1-\varepsilon) \text{ if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i \pm jt-1} < \frac{1}{g-y-l}, \\ 1 \text{ with probability } \varepsilon \text{ if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i \pm jt-1} < \frac{1}{g-y-l}, \\ 1 \text{ if } i \in F \text{ and } \frac{1}{2k}\sum_{j=1}^{k} a_{i \pm jt-1} \geq \frac{1}{g-y-l}. \end{cases}$$

Although all fair players make random efforts to improve the outcome, not all of them will be successful. Intuitively, we expect that those who interact with a relatively fair neighborhood will be able to succeed in fostering a cooperative neighborhood, while those who interact with a relatively selfish neighborhood are unable to succeed. In order to make a clear distinction between these two types of fair players according to the quality of their neighborhoods, we introduce the notion of a discouragement operator D.

DEFINITION 3.3 *Discouragement operator,* $D \equiv (D_1, D_2, \cdots, D_N)$ *is defined by:*

$$D_i(e_i, e_{-i}) = \begin{cases} 0 \quad \text{if } e_i = 0, \\ 0 \quad \text{if } e_i = 1 \text{ and } \frac{1}{2k}\sum_{j=1}^{k} e_{i \pm j} < \frac{l}{g-y+l}, \\ 1 \quad \text{if } e_i = 1 \text{ and } \frac{1}{2k}\sum_{j=1}^{k} e_{i \pm j} \geq \frac{l}{g-y+l}. \end{cases}$$

For expositional purpose, suppose that all fair players happen to experiment to cooperate simultaneously in a given period. In this case the action profile is equivalent to e^0. Then those surrounded by fairly many selfish players will be disappointed by their neighbors' defective behavior and with high probability they will switch to defection in the next period. But with some positive probability these players will continue to cooperate because it is assumed that all fair players make random experiments in each period. In the worst case all of these disappointed fair players will defect and the action profile becomes equivalent to $D(e^0)$. Then the neighbors of these disappointed players will be disappointed too so that with high probability they will also change to defect in the next period. The discouragement process continues like this.

Since we have a finite number of players, it will take only finite number of

[11] We assume that this experimentation is reversible in that players are allowed to go back to defection whenever their experimental cooperation is not immediately matched by sufficient number of neighbors.

rounds for this discouragement process to be completed. Let us denote the number of rounds it takes to complete the discouragement process by T. Then we have

$$
\begin{aligned}
&e^1 = D(e^0) \\
&e^2 = D(e^1) = D^2(e^0) \\
&\vdots \\
&e^T = D^T(e^0) = e^*(e^0) = D^{T+1}(e^0) < D^{T-1}(e^0).
\end{aligned}
$$

Obviously, there exists a unique limit

$$e^*(e^0) = \lim_{t\to\infty} D^t(e^0).$$

Using this limit, we can divide fair players into two categories.

DEFINITION 3.4 *A fair player* $i \in F$ *is called an active-fair player if* $e_i^* = 1$ *and a discouraged-fair player if* $e_i^* = 0$.

In other words, fair players become discouraged when they are in the middle of fairly many selfish players or other discouraged-fair players. Notice that whether a fair player is discouraged depends not only on the types of their neighbors but the types of their neighbors' neighbors and their neighbors and so on. In each period all fair players experiment. Some of these fair players will find their cooperation matched by other fair players' cooperative behavior. But if they are discouraged-fair players, we can expect they will soon find some of their neighbors stop cooperating. Then they will also stop cooperating except for the purpose of experimentation. Only the active-fair players will be able to eventually succeed in establishing cooperative clusters.

DEFINITION 3.5 *A discouraged-fair player is called a round-t-discouraged-fair player if he gets discouraged at round t: i.e., the set of round-t-discouraged-fair players can be represented by*

$$DF^t = \{i \in F \mid e_i^t \equiv D_i^t(e^0) = 0, \text{but } e_i^{t-1} \equiv D_i^{t-1}(e^0) = 1\} \text{ for } t = 1, 2, \cdots, T.$$

Ⅳ. CHARACTERIZATION OF THE LONG-RUN EQUILIBRIUM

In order to analyze the dynamics of the system, let us define a state of an economy as a profile of actions of players. Then the state space can be represented by $A^N \equiv \{0, 1\}^N$. Although the total number of possible states is 2^N, along the equilibrium path at most 2^F states will be observed because selfish players have a dominant strategy and are not allowed to make any mistakes or experimentation. In order to give a systematic order to these 2^F states, we introduce a *lexicographic ordering.*

DEFINITION 4.1 *For any two states* $a, b \in A^N$, *in which all selfish players defect, we define a lexicographic ordering as follows.* $a > b$ *if and only if one of the following conditions holds.*

(I-1) $a \cdot e^* > b \cdot e^*$

(II-1) $a \cdot e^* = b \cdot e^* < \| e^* \|$ and $\| a \| > \| b \|$

(II-2) $a \cdot e^* = b \cdot e^* < \| e^* \|$ and $\| a \| = \| b \|$ and $\sum_{i=1}^N i a_i > \sum_{i=1}^N i b_i$

(III-1) $a \cdot e^* = b \cdot e^* = \| e^* \|$ and $a \cdot (e^0 - e^1) > b \cdot (e^0 - e^1)$

(III-2) $a \cdot e^* = b \cdot e^* = \| e^* \|$, $a \cdot (e^0 - e^1) = b \cdot (e^0 - e^1)$ and $a \cdot (e^1 - e^2) > b \cdot (e^1 - e^2)$

$\vdots$

(III-T) $a \cdot e^* = b \cdot e^* = \| e^* \|$, $a \cdot (e^{t-1} - e^t) = b \cdot (e^{t-1} - e^t)$ for $t = 1, 2, \cdots, T-1$ and $a \cdot (e^{T-1} - e^T) > b \cdot (e^{T-1} - e^T)$

(IV) $a \cdot e^* = b \cdot e^* = \| e^* \|$, $a \cdot (e^{t-1} - e^t) = b \cdot (e^{t-1} - e^t)$ for $t = 1, 2, \cdots, T$ and $\sum_{i=1}^N i a_i > \sum_{i=1}^N i b_i$

For any two states, we compare the numbers of active-fair players who cooperate. If this test does not work and some active-fair players are defecting, we count the number of cooperators. And if even this criterion does not answer, we give a particular order by using the player-numbers. Now consider the states in which all active-fair players are cooperating and in addition to this some discouraged-fair players are cooperating. In this case, we look at when those cooperating fair players get discouraged. The more fair players who get discouraged in the early round cooperate, the bigger state it will be. And if even this complicated criterion cannot distinguish the states then we finally rely on the player-numbers.

Based on this ordering, we can assign a number to each state like 1 to the state $(0, 0, \cdots, 0)$ and $n \equiv 2^F$ to the state in which all fair players cooperate. Then the state space can be redefined by $Z = \{1, 2, \cdots, n\}$. In the theorem below, we will show that active-fair players cooperate and discouraged-fair players and selfish players defect in the long-run equilibrium. For convenience, let n^* be a number assigned by the lexicographic ordering to a state in which all active-fair players cooperate and all discouraged-fair and selfish players defect. Clearly, the value of n^* depends on the parameters of the game, as e^* does. Let z_t be the state of the system at time t. Then on the modified state space, Z, we can define transition probabilities as follows:

$$p_{ij}(\varepsilon)=prob[z_{t+1}=j \mid z_t=i] \text{ for any } i,j\in Z \text{ and } \varepsilon>0.$$

By using the lexicographic ordering in counting the states, we get some nice properties of $p_{ij}(\varepsilon)$.

LEMMA 1. (a) $p_{ij}(\varepsilon)=0$ for $j<n^*\le i$,
(b) $\sum_{j=n^*}^{n} p_{ij}(\varepsilon)>0$ for $i<n^*$.

Proof
(a) In states $i\ge n^*$, all active-fair players cooperate, but in states $j<n^*$, some active-fair players defect. As long as all active-fair players cooperate, their behavior rule tells them to cooperate, regardless of the value of ε.
(b) For a given state $i<n^*$, we can always find a state $j(n^*\le j\le n)$ in which all active-fair players cooperate, and all discouraged-fair players choose their myopic best responses to the state i. Then for these i and j, we have $p_{ij}(\varepsilon)>0$ due to the fair players' random experiments. □

LEMMA 2. (a) $\lim_{\varepsilon\to 0} p_{ii}(\varepsilon)=1$ if $i=n^*$,
(b) $\lim_{\varepsilon\to 0} p_{ii}(\varepsilon)=1$ if $i>n^*$,
(c) $\lim_{\varepsilon\to 0} p_{ij}(\varepsilon)=1$ if $j>i>n^*$.

Proof
(a) The fact that n^* is the best response to itself shows part (a).
(b) In states $i>n^*$, some discouraged-fair players are cooperating. Then, some of them will change their actions to defection in the next period. Therefore, the same state cannot be repeated in the following period, which implies part (b).
(c) In states $i>n^*$, some discouraged-fair players are cooperating. Among these players, consider the players who get discouraged at the earliest round. And suppose that these players become discouraged at round t; in other words, they belong to DF^t. Then these players' best response is to choose D in the next period. And it may be possible that some other players become cooperators induced by those players' cooperation in the previous period. However, these new cooperators belong to DF^s, where $s>t$. According to our lexicographic ordering, hence, to this state a smaller number will be assigned. This establishes part (c). □

Due to the behavior rule derived from myopic best response and random experimentation, the dynamics of the game can be described by a Markov process. Now, we introduce the notion of stationary distribution on a simplex $\triangle\equiv\{\mu\in R_+^n \mid \sum_{i=1}^{n}\mu_i=1\}$.

DEFINITION 4.2 *$\mu(\varepsilon)$ is a stationary distribution if and only if $\mu(\varepsilon)=\mu(\varepsilon)$ $P(\varepsilon)$, where $P(\varepsilon)$ is a $n\times n$ transition probability matrix.*

As we have seen in Lemma 1(a), in our model, $P(\varepsilon)$ is not strictly positive. So we cannot guarantee the uniqueness of the stationary distribution. However, this cannot be a problem because we are interested in the long-run behavior of the game when the probability of random experimentation is small and $P(\varepsilon)$ has some nice properties in the limit as in Lemma 2. To be precise, let us define the limit distribution and the set of long run equilibria.

DEFINITION 4.3 *The limit distribution μ^* is defined by $\mu^*=\lim_{\varepsilon\to 0}\mu(\varepsilon)$.*

DEFINITION 4.4 *The set of long run equilibria is defined as $LRE(\mu^*)=\{i\in Z \mid {}^{*}_{i}>0\}$.*

Now, we can show the following theorem.

THEOREM 1. *For a given heterogeneous population game G, there exists a unique long-run equilibrium in which all active-fair players cooperate and all discouraged-fair and selfish players defect, i.e., $\mu^*_{n^*}=1$ or equivalently $LRE(\mu^*)=n^*$.*

Proof

Lemma 1(a) simplifies the transition probability matrix $P(\varepsilon)$ as follows.

$$P(\varepsilon)=\begin{pmatrix} p_{11}(\varepsilon) & p_{12}(\varepsilon) & \cdots & p_{1n^*-1}(\varepsilon) & p_{1n^*}(\varepsilon) & \cdots & p_{1n}(\varepsilon) \\ p_{21}(\varepsilon) & p_{22}(\varepsilon) & \cdots & p_{2n^*-1}(\varepsilon) & p_{2n^*}(\varepsilon) & \cdots & p_{2n}(\varepsilon) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n^*-11}(\varepsilon) & p_{n^*-12}(\varepsilon) & \cdots & p_{n^*-1n^*-1}(\varepsilon) & p_{n^*-1n^*}(\varepsilon) & \cdots & p_{n^*-1n}(\varepsilon) \\ 0 & 0 & \cdots & 0 & p_{n^*n^*}(\varepsilon) & \cdots & p_{n^*n}(\varepsilon) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & p_{nn^*}(\varepsilon) & \cdots & p_{nn}(\varepsilon) \end{pmatrix}$$

From $\mu(\varepsilon)P(\varepsilon)=\mu(\varepsilon)$, we have[12]

$$\begin{aligned} &\mu_1 p_{1n^*}+\mu_2 p_{2n^*}+\cdots+\mu_n p_{nn^*}=\mu_{n^*} \\ &\mu_1 p_{1n^*+1}+\mu_2 p_{2n^*+1}+\cdots+\mu_n p_{nn^*+1}=\mu_{n^*+1} \\ &\vdots \\ &\mu_1 p_{1n}+\mu_2 p_{2n}+\mu_n p_{nn}=\mu_n \end{aligned}$$

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[12] From now on, for notational simplicity, we replace $p_{ij}(\varepsilon)$ and $\mu_i(\varepsilon)$ by p_{ij} and μ_i, respectively.

Summing both sides of these equations gives rise to

$$\sum_{i=1}^{n} \mu_i \sum_{j=n^*}^{n} p_{ij} = \sum_{i=n^*}^{n} \mu_i.$$

This equation can be written as follows.

$$\sum_{i=1}^{n^*-1} \mu_i \sum_{j=n^*}^{n} p_{ij} = \sum_{i=n^*}^{n} \mu_i \sum_{j=1}^{n^*-1} (1-p_{ij}) = \sum_{i=n^*}^{n} \mu_i \tag{1}$$

Since $p_{ij}=0$ for $j<n^*\leq i$, the second term of the left-hand side of the equation (1) is reduced to $\sum_{i=n^*}^{n} \mu_i$. Then from the equation (1), we get $\sum_{i=1}^{n^*-1} \mu_i \sum_{j=n^*}^{n} p_{ij}=0$. Now notice that $\sum_{j=n^*}^{n} p_{ij}>0$ for $i<n^*$ from Lemma 1(b). From this it follows that $\mu_1=\mu_2=\cdots=\mu_{n^*-1}=0$, and thus we have $\mu_1^*=\mu_2^*=\cdots=\mu^*_{n^*-1}=0$. Now that we are interested in the set of long-run equilibria, we can focus on the reduced state space, $\overline{Z}=\{n^*, n^*+1, \cdots, n\}$. For this state space, from Lemma 2, we can write transition probability matrix in the limit as follows.

$$P(\varepsilon)=\begin{pmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\ p^*_{n^*+1n^*} & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\ p^*_{n^*+2n^*} & p^*_{n^*+2n^*+1} & 0 & & \cdots & 0 & 0 & 0 \\ \vdots & & & & & & & \\ p^*_{n-1n^*} & p^*_{n-1n^*+1} & & \cdots & & p^*_{n-1n-2} & 0 & 0 \\ p^*_{nn^*} & p^*_{nn^*+1} & & & \cdots & & p^*_{nn-1} & 0 \end{pmatrix}$$

Since P^* is a triangular matrix we can easily solve the equation $\mu^* P^* = \mu^*$. It can be shown that $\mu_n^*=\mu_{n-1}^*=\cdots=\mu^*_{n^*+1}=0$. Now it follows that $\mu^*_{n^*}=1$ from $\sum_{i=1}^{n}\mu_i^*=1$. □

V. SIMULATION RESULTS

In the long-run equilibrium, as shown in the previous section, active-fair players cooperate while selfish and discouraged-fair players defect. However, how many fair players are discouraged depends on the initial configuration of players and the matching rule. If the same population is spatially distributed in a different way, the long-run cooperation rate will also be different. For example, for a highly selfish population, the cooperation rate will be smaller if two types of players are mixed rather than segregated. We want to see how the "average" cooperation rate in the long-run equilibrium changes as we vary matching rules. For this purpose, we performed the following simulation analysis.[13]

The behavior rule specified in Section III can be used to generate various rules for fair players depending on the values of g, y, and l. For example, if $1 \leq \frac{g-y}{l} < \frac{k+1}{k-1}$, fair players will employ *follow-the-majority* rule in choosing their actions, which is described by[14]

$$B_i(a_{t-1}) = \begin{cases} 0 & \text{if } i \in S, \\ 0 & \text{with probability } (1-\varepsilon) \text{ if } i \in F \text{ and } \sum_{j=1}^{k} a_{i \pm jt-1} < k, \\ 1 & \text{with probability } \varepsilon \text{ if } i \in F \text{ and } \sum_{j=1}^{k} a_{i \pm jt-1} < k, \\ 1 & \text{if } i \in F \text{ and } \sum_{j=1}^{k} a_{i \pm jt-1} \geq k. \end{cases}$$

According to follow-the-majority rule, fair players cooperate as long as at least half of their neighbors cooperate. Follow-the-majority rule is useful in that it is a rule of thumb that can be easily adopted, and that it also simplifies the simulation analysis.

5.1. Long-Run Cooperation Rate

For a given fraction of selfish players, the long-run cooperation rate is determined by the initial configuration of each type, the size of neighborhood, the size of population, and the degree of overlap or dimension effect. In our simulation, we compute the cooperation rate among fair players in the long run equilibrium for 1,000 different games G, where only the spatial distribution of players' types (e^0) differs and all other parameters of the game such as N and m keep the same. And then we take the average of these cooperation rates. In the remaining part of this paper, the long-run cooperation rate means the average cooperation rate among fair players in the long run equilibrium. We maintain $N=100$ in all simulations except when we look at the population size effect.

Number of Neighbors Effect. Figure 2 shows how the long-run cooperation rate changes as the number of neighbor increases. As we have expected, the long-run cooperation rate becomes smaller as more people in the population are selfish. We know that in the case of global interactions, all fair players defect when $m>50\%$ and cooperate when $m \leq 50\%$. Thus as the number of neighbor approaches to the population size, the cooperation rate among fair players will converge to 100% or 0% depending on whether the fraction of selfish players is less than 50% or not. This effect can be called a globalization effect. The globalization effect is verified in Figure 2; when $m \leq 50\%$, as the number of neighbor becomes larger, the long-run cooperation rate is eventually rising up to

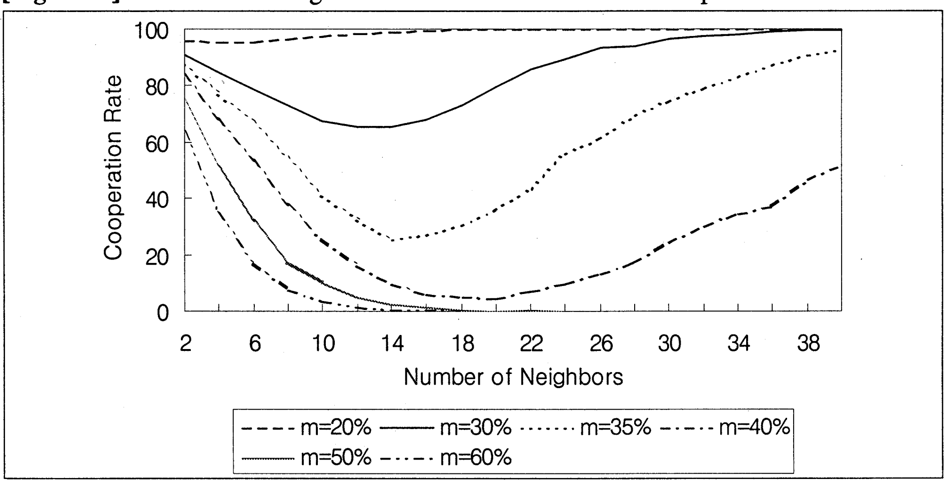
[13] There are many simulation analyses on the evolution of cooperation in various settings. For example, see Nowak and May (1992), Glance and Huberman (1993).

[14] This condition says that the loss from cheating the other player should be slightly greater than that from being fooled. As the number of neighbors increases, the set of parameter values satisfying this condition becomes smaller.

100%. Before the globalization effect starts being felt, however, the neighborhood size has a negative effect on the long-run cooperation rate no matter how much selfish the population is. Therefore, the neighborhood size effect can be summarized as follows. First, if $m > 50\%$, then as the number of neighbors increases, the long-run cooperation rate monotonically decreases. Second, if $m \leq 50\%$, then the long-run cooperation rate initially falls and, beyond some point, rises again.

Follow-the-majority-rule involves a kind of bias toward cooperation. As the number of neighbor gets smaller, follow-the-majority rule of fair players makes them more likely to cooperate because it favors cooperation when there are equal number of cooperators and defectors, and the likelihood of this tie becomes higher when the number of neighbor is small. And this bias effect will be important when the number of neighbor is small and the fraction of selfish players is not too small. Figure 3 shows the simulation result under the assumption that fair players are biased toward defection in the sense that when they face the same number of cooperators and defectors they choose defection.[15] In this case the pattern of cooperation rate change is slightly different for the range of small size of neighborhood; when the population is neither too fair nor too selfish, the long-run cooperation rate is initially rising up to some point and beyond that point falls and then rises again. Before the globalization effect starts being felt, if m is not too low, initially the neighborhood size has a positive effect on the long-run cooperation rate because of the bias effect and then the neighborhood size effect becomes negative.

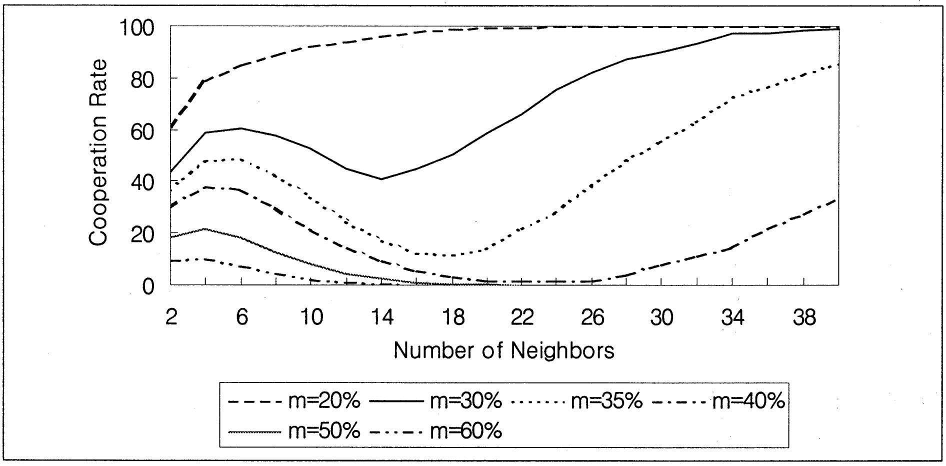
[Figure 2] Number of Neighbors Effect 1: Bias toward Cooperation[a]



[a] Here and in the following simulation results we set N=100, r=1,000; r denotes the number of different configurations with the same matching rule and the fraction of selfish people.

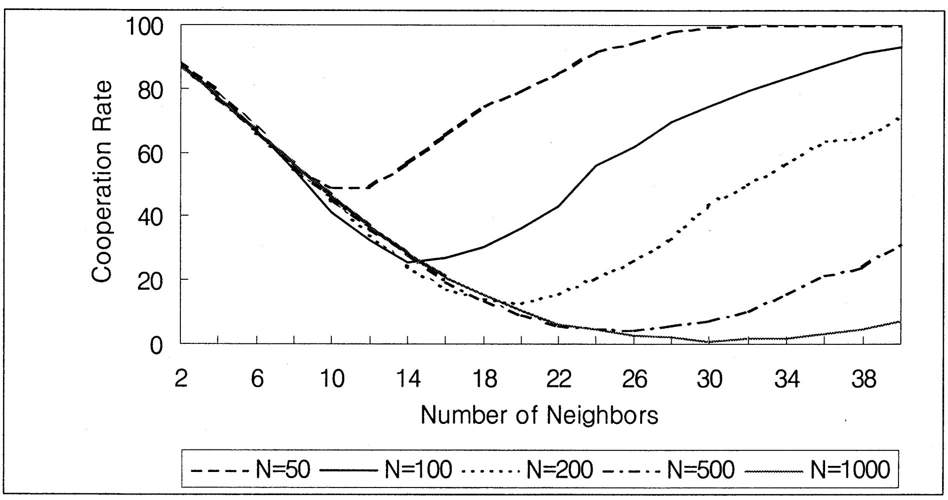
[15] This behavior rule will be obtained when $\frac{k-1}{k+1} < \frac{g-y}{l} < 1$.

[Figure 3] Number of Neighbors Effect 2: Bias toward Defection



Population-Size Effect. Figure 4 shows how the long-run cooperation rate is affected by the size of total population. Striking is the fact that the larger the population, the lower the minimum cooperation rate. This implies that large society is more likely to suffer from extremely low cooperation even when the population is not seriously selfish. The combination of the number of neighbors effect and the population size effect can explain the commonly observed fact that as people interact with more people, they become less cooperative. For example, people living in a large and crowded city like New York or Seoul are relatively less cooperative. As social interaction becomes broader with the spread of modern civilization, social cooperation becomes harder to realize.

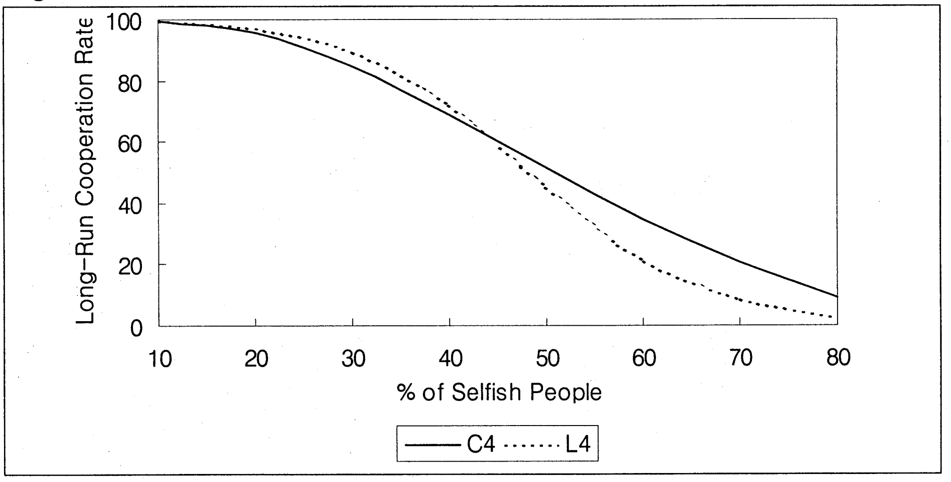
[Figure 4] Population Size Effect[a]



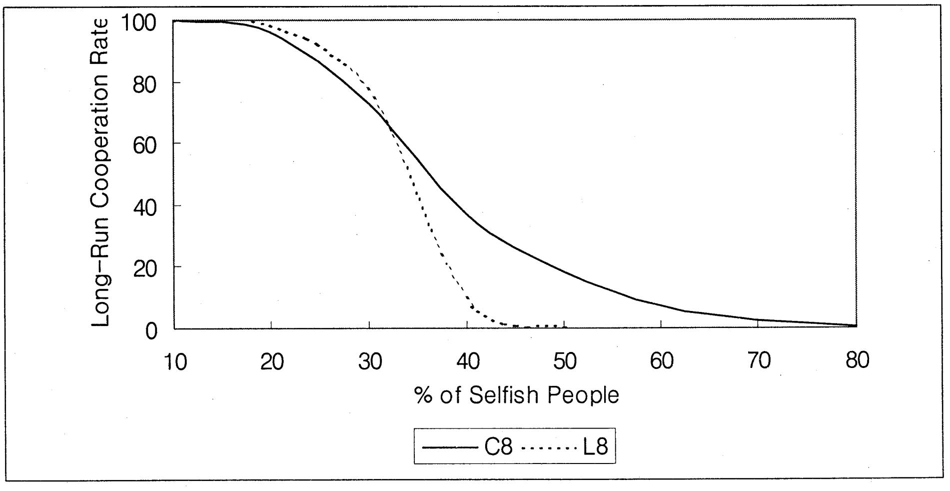
[a] m=35%.

Dimension Effect. Figures 5 and 6 depict the effect of the degree of overlap on the long-run cooperation rate. Beyond some level of m, the long-run cooperation rate is lower in two-dimensional matching. There are two factors behind this dimension effect. First, a high degree of overlap implies that discouraged-fair players have many neighbors who have discouraged them, and with whom they can discourage other neighbors. Second, a high degree of overlap also implies that discouraged-fair players have few neighbors whom they can discourage since many of their neighbors are already discouraged or selfish. As m increases, the second factor becomes more important. Therefore, when the fraction of selfish people is relatively high, one-dimensional matching yields more cooperative outcome.

[Figure 5] Dimension Effect 1



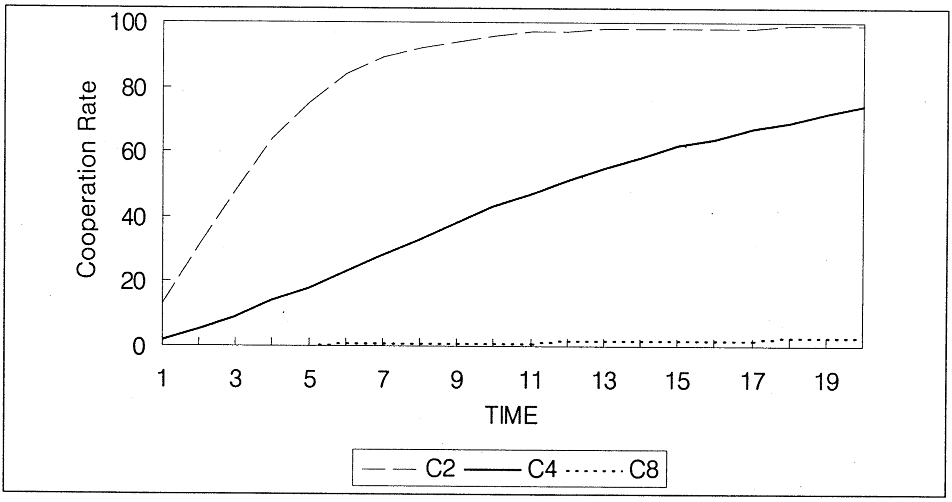
[Figure 6] Dimension Effect 2



5.2. The Rate of Convergence to Long-Run Equilibrium

For low m, the number of neighbors effect is negligible. However, it takes more time with larger number of neighbors case to converge to the long-run equilibrium. Therefore, the number of neighbors effect does matter even for low m during the transition period. This point is clear in Figure 7.[16]

[Figure 7] Cooperation Rate over Time[a]



[a] m=10%, ε=0.05.

Ⅵ. CONCLUSION

In this article we have tried to explain how the people will be able to cooperate in a heterogeneous society. For this, we examined a variant of prisoners' dilemma in a heterogeneous population where two types of players locally interact. Selfish players have standard preferences and hence always defect. Fair players experience a utility loss if they defect when their opponent cooperates. If this utility loss is sufficiently high, then the interaction among fair players becomes a coordination game. Players adjust their choices over time by playing a myopic best response to the actions taken by their neighbors in the previous period. In addition, fair players are allowed to experiment by choosing cooperation with some probability even when it is not a best response. This defines a Markov process. We have shown that in the limit, as the probability of experimentation approaches zero, the ergodic distribution puts probability one on the stage-game Nash equilibrium in which the smallest number of players defect. Another main finding is that the structure of interactions plays an important role in determining the level of cooperation in a heterogeneous

[16] Our computation is based on the assumption that the system starts from *ALL D.*

population.

Our model can be applied to explain the collusion in monopolistically competitive markets. In theses markets, firms tend to compete with a small number of rival firms, where the number of rival firms for each firm is determined by substitutability among products. If the interaction is global, collusion among fair firms will be either perfect success or complete failure, depending only on the composition of two types in this industry. However, with the local interaction, the degree of collusion will be in between these two extremes, where the average price level will depend on the number of rival firms as well as the proportion of each type in the industry.

Moreover, our model can be used to address sociological issues such as the effect of the interaction structure on the rate of drug usage among the youth, the rate of firearms possession, the diffusion of fashion, and student behavior in school.

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