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COMPARATIVE STATICS UNDER UNCERTAINTY WITH THE MONOTONE PROBABILITY RATIO ORDER REVISITED*

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This paper proposes a 'left-side monotone likelihood ratio' (L-MLR) order which is a more restrictive concept than the 'monotone probability ratio' (MPR) order and generalizes the comparative static result for the MPR change to the non-linear case in the standard one-argument decision model. The L-MLR defines an order on random distribution that lies between 'monotone likelihood ratio' (MLR) and MPR. Transitivity, however, does not hold for L-MLR and we show that an important relationship exists between the orders, L-MLR and MPR. This paper also shows that an MPR change can always be decomposed into a sequence of L-MLR shifts.

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I. INTRODUCTION

Landsberger and Meilijson (1990), and Eeckhoudt and Gollier (1995) have shown the importance, for comparative static analysis, of the ratio between an initial and a final probability density function (pdf) and the ratio between an initial and a final cumulative distribution function (CDF). The 'monotone likelihood ratio' (MLR) order from Landsberger and Meilijson is defined by requiring the pdf ratio to be monotone, and the 'monotone probability ratio' (MPR) order from Eeckhoudt and Gollier is given by imposing the monotonicity restriction on the CDF ratio. As special cases of the first-degree stochastic dominance (FSD) order, each of the orders specifies a particular subset of the set of all FSD changes in a random variable.

In a simple two asset portfolio problem with one risky asset and one safe asset, Landsberger and Meilijson showed that an MLR change in the random return of the risky asset leads to an increase in the demand for the asset by all investors with non-decreasing utility. Eeckhoudt and Gollier extended this result by showing that the MPR order, which specifies a more general class of changes in randomness than the MLR condition, is sufficient for the comparative static statement for all risk-averse investors. This extension exhibits the trade-off between the size of the admissible set of changes in randomness (MLR changes $\rightarrow$ MPR changes) and the size of the set of decision-makers (non-decreasing utility functions $\rightarrow$ non-decreasing and concave utility functions). However these comparative static results are restricted to decision models in which the payoff function is linear in both the choice variable and the random variable.

Given the set of decision-makers, the traditional approaches for the comparative static analysis separately restrict the structure of the economic decision model and the set of changes in distribution. However, Gollier (1995) provided the necessary and sufficient condition for all risk-averse individuals, which is a joint condition on both the model and the change in the cumulative distribution function. That is, he obtained the least constraining condition on changes in risk that yielded the general comparative static statements for a given economic model and for the class of risk-averse agents. His condition using the location-weighted probability mass functions is $T(x,\alpha;G,z) \le \gamma T(x,\alpha;F,z)$, $\forall x \in [a,b]$

where $\gamma > 0$, $T(x,\alpha;F,z) \equiv \int_a^x z_\alpha dF(s)$ and $T(x,\alpha;G,z) \equiv \int_a^x z_\alpha dG(s)$. He showed that a strong increase in risk defined by Meyer and Ormiston (1985) corresponds to the case where $\gamma = 1$, but did not deal with other values of γ. Therefore, it is not easy to find such cases directly from the Gollier condition.

This paper generalizes Eeckhoudt and Gollier's comparative static result to the non-linear decision model by further characterizing the MPR condition.[1] For any CDF order on distribution functions, two properties, 'transitivity and decomposability', have important implications with regard to comparative static analysis. If a CDF order is sufficient for making a comparative static statement, then the comparative static statement also follows for any change in CDF that can be decomposed into a sequence of changes that satisfy the given order because comparative static implications are transitive. When the given order is itself transitive, this extension results in no gain. Such is the case for the MLR and MPR orders which are transitive. However, if the order is not transitive, then the extension given by the method of decomposition gives a more general order. This is the process of Rothschild and Stiglitz (1970) used to extend the concept of a 'mean-preserving spread' (MPS) for an increase in risk, and is the method used in deriving the main comparative statics result in this paper.

To do this, we introduce a new concept, called the 'left-side monotone likelihood ratio' (L-MLR) criterion. The L-MLR defines an order on random distributions that lies between MLR and MPR. Transitivity, however, does not hold for L-MLR and we show that an important relationship exists between the orders, L-MLR and MPR. We show that an MPR change can always be decomposed into a sequence of L-MLR shifts. This implies that any comparative static statement made for the L-MLR changes can directly be generalized to the MPR changes, without any additional cost of assumptions. This is the method of proof of the main comparative static theorem. Moreover, we present some relationships among the subsets of FSD shifts in Appendix.

In the next section, we give three definitions of ordering CDF's (MLR,

[1] See also Kim and Ryu (2004) for the generalization of Landsberger and Meilijson's comparative static result (for the case of MLR with the set of non-decreasing utilities) to the non-linear case.

L-MLR, and MPR) and examine the basic relationships among the orders. Two important relationships, 'transitivity and decomposability', are discussed in section III. The main comparative static theorems are developed in section IV, and some concluding remarks are provided in the final section.

II. DEFINITIONS AND BASIC RELATIONSHIPS AMONG MLR, L-MLR, AND MPR

Let x be a random variable characterized by an initial and a final CDF, G and F, with their corresponding pdf's g and f, respectively. Both G and F are assumed to have their points of increase in bounded intervals. For notational convenience, we assume that the support of $G(x)$ is a finite interval $[x_1, x_3]$ and the support of $F(x)$ is another finite interval $[x_2, x_4]$ where $x_1 \leq x_2$ and $x_3 \leq x_4$.[2]

In this section, three different stochastic orderings are presented: two previously used ones, MLR and MPR, and a newly defined one, L-MLR. They are all defined by imposing monotonicity restrictions on the likelihood ratio between the pdf's f and g or on the probability ratio between the CDF's G and F. First, MLR and MPR orders are defined as:[3]

Definition 1. $F(x)$ represents a monotone likelihood ratio FSD shift from $G(x)$ (denoted by F MLR G) if there exists a non-decreasing function $h: [x_2, x_3] \to [0, \infty)$ such that $f(x) = h(x)g(x)$ for all $x \in [x_2, x_3]$.

Definition 2. $F(x)$ represents a monotone probability ratio FSD shift from $G(x)$ (denoted by F MPR G) if there exists a non-decreasing function $H: [x_2, x_3] \to [0, 1]$ such that $F(x) = H(x)G(x)$ for all $x \in [x_2, x_3]$.

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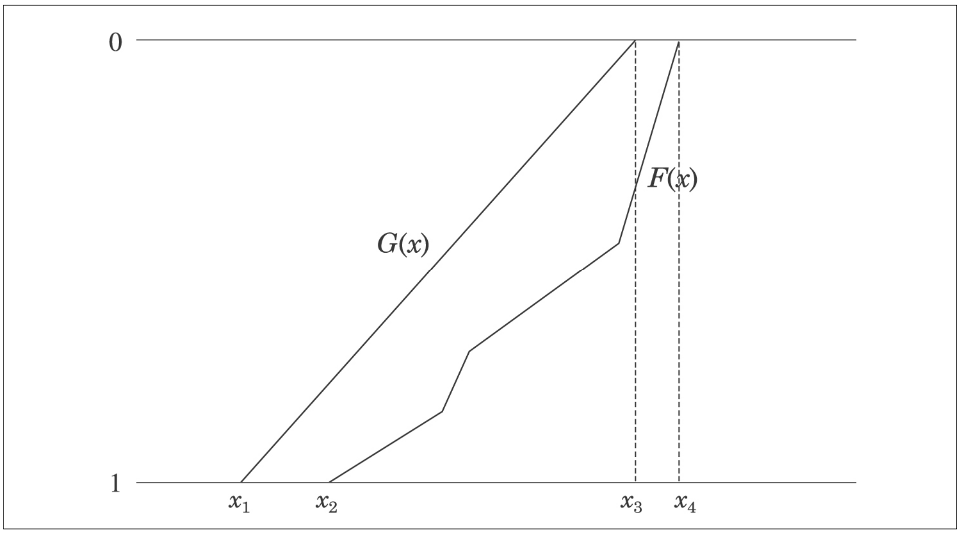
[2] In this paper, we consider only subsets of FSD shifts and assume that the final distribution F always dominates the initial distribution G in the sense of FSD (denoted by F FSD G).

[3] Following our notation about the support of CDF's F and G, Definitions 1 and 2 are slightly modified from the original ones used in Landsberger and Meilijson, and Eeckhoudt and Gollier, respectively.

[Figure 1] CDF representation of MPR



It is easy to confirm that both an MLR[4] and an MPR[5] are types of FSD changes. According to Definition 1, $g(x) \geq f(x)$, when $h(x) \leq 1$ and $g(x) \leq f(x)$, when $h(x) \geq 1$ and the function h non-decreasing implies that the pdf's f and g cross only once. Thus, an MLR satisfies the FSD condition that $G(x) \geq F(x)$ for all $x \in [x_1, x_4]$. For an MPR case in Definition 2, the condition that the ratio function H is less than one implies that the shift is an FSD change. In addition to these orders, we introduce a new ordering on random distributions that lies between MLR and MPR.

Definition 3. $F(x)$ represents a left-side monotone likelihood ratio FSD shift from $G(x)$ (denoted by F L-MLR G) if there exists a point $k \in [x_2, x_3]$ and a non-decreasing function h: $[x_2, k) \to [0,1]$ such that $f(x) = h(x)g(x)$ for all $x \in [x_2, k)$ and $g(x) \leq f(x)$ for all $x \in [k, x_3]$.[6]

[4] See Karlin and Rubin (1956), Landsberger and Meilijson (1990) and Ormistron and Schlee (1993), for special examples satisfying the MLR.

[5] See Bagnoli and Bergstrom (2005), and Eeckhoudt and Gollier (1995), for special examples satisfying the MPR.

[6] Only one crossing at a point $k \in [x_2, x_3]$ implies that $f(x) \leq g(x)$ for all $x \in [x_2, k)$ and $f(x) \geq g(x)$ for all $x \in (k, x_3]$. At the point k, it is usual that $f(k) = g(k)$, but if the pdf g or f is discontinuous at k, both the cases of $f(k) < g(k)$ and $f(k) > g(k)$ are possible, and thus

While an MLR requires the ratio of *f* to *g* to be non-decreasing to the right and to the left of the crossing point, the L-MLR order is defined by relaxing the monotonicity requirement for points to the right side of the crossing point.

It is obvious that an MLR change is also an L-MLR, but not vice versa. As an example of this case, consider a random variable which has only two outcomes. If the lower outcome becomes larger than before, then it is the case of an L-MLR shift but not an MLR shift. More generally, given any initial pdf $g(x)$ with support in $[x_1, x_3]$, define a conditional pdf g^{ℓ} by $g^{\ell}(t) = g(t)/G(T)$ with its support $[x_1, T]$ where $T \leq x_3$. Then any MLR change in g^{ℓ} is also an L-MLR change in the initial $g(x)$.

Property 1 below shows that the monotonicity restriction on likelihood ratio between a pair of pdf's for the MLR or the L-MLR change implies the monotone probability ratio between the corresponding CDF's.

Property 1. F MLR G $\Rightarrow$ F L-MLR G $\Rightarrow$ F MPR G.

Proof: Note that the first relationship is obvious. Consider the second relationship "if F L-MLR G, then F MPR G." Since $G > 0$ for all $x \in (x_1, \infty)$, we can define a function H for the interval $[x_2, x_3]$ as,

$$H(x) = \begin{cases} 0, \text{ when } x = x_2 (F = 0) \\ F(x)/G(x), \text{ when } x \in (x_2, x_3](F \neq 0). \end{cases}$$

To prove this relationship, since $H \geq 0$, it is sufficient to show that the condition F L-MLR G implies that H is non-decreasing and less than or equal to one for the interval. It is clear that $H \leq 1$ for all $x \in [x_2, x_3]$ because an L-MLR shift is an FSD shift. The condition F/G non-decreasing for the interval $[x_2, x_3]$ is equivalent to:

$$f(x)G(x) - g(x)F(x) \geq 0, \text{ for all } x \in [x_2, x_3]. \tag{1}$$

Definition 3 includes the case where $f(x) \leq g(x)$ for all $x \in [x_2, k]$ and $f(x) \geq g(x)$ for all $x \in (k, x_3]$.

With the condition F L-MLR G in which the pdf's g and f cross only once, divide the interval $[x_2, x_3]$ into two sub-intervals, one such that $g \geq f$ and the other such that $g \leq f$. When $g \leq f$, the condition (1) is satisfied because $G \geq F$. When $g \geq f$, the L-MLR condition implies that, (i) if $g(x) = 0$, then $f(x) = 0$ and (ii) if $g(x) \neq 0$, then $h(x) = f(x)/g(x)$, where h is the ratio function used in Definition 3. Thus, for all values of x such that $g(x) = 0$, condition (1) is satisfied. For case (ii), condition (1) can be written as,

$$h(x)G(x) - F(x) \geq 0, \text{ for all } x \in [x_2, x_3] \text{ such that } g(x) \neq 0. \tag{2}$$

Since $h(x_2)G(x_2) - F(x_2) \geq 0$ and the condition h non-decreasing in x implies that the left side of (2) is non-decreasing, i.e., $\partial[h(x)G(x) - F(x)]/\partial x = h'G + hg - f = h'G \geq 0,$ condition (2) is satisfied. Hence, condition (1) is satisfied for all x such that $g \geq f$. This completes the proof. Q.E.D.

Now we have formal relationships among the above three CDF orders. Let's define the following four subsets of FSD shifts, each representing a set of pairs of CDF's (F, G) such as: Ω_{MLR} = {(F, G)| F MLR G}, Ω_{L-MLR} = {(F, G)| F L-MLR G}, Ω_{MPR} = {(F, G)|F MPR G}, and Ω_{FSD}={(F, G)|F FSD G}. Then Property 1 implies, $\Omega_{MLR} \subset \Omega_{L-MLR} \subset \Omega_{MPR} \subset \Omega_{FSD}$.

III. TRANSITIVITY AND DECOMPOSABILITY

This section examines the properties, transitivity and decomposability among the three CDF orders given in section II. First, we show that each of the orders, MLR and MPR, is transitive.

Property 2. If F_3 MLR F_2 and F_2 MLR F_1, then F_3 MLR F_1.

Proof: Let the support of F_i (and its corresponding pdf f_i) be a finite interval $[x_\ell^i, x_h^i]$, i = 1, 2, 3, where $x_\ell^1 \leq x_\ell^2 \leq x_\ell^3$ and $x_h^1 \leq x_h^2 \leq x_h^3$. Then the given two MLR shifts imply that there exist non-decreasing functions

$h_1:[x_\ell^2, x_h^1] \to [0,\infty)$ and $h_2:[x_\ell^3, x_h^2] \to [0,\infty)$ such that $f_2 = h_1 f_1$ for $x \in [x_\ell^2, x_h^1]$ and $f_3 = h_2 f_2$ for $x \in [x_\ell^3, x_h^2]$, respectively. This implies that, for the interval $[x_\ell^3, x_h^1]$, $f_2 = h_1 f_1$ and $f_3 = h_2 f_2$, and thus $f_3 = h_1 h_2 f_1$. Since $h_1 \cdot h_2$ is also non-decreasing and non-negative for the interval $[x_\ell^3, x_h^1]$, and by Definition 1, F_3 MLR F_1. Q.E.D.

Property 3. If F_3 MPR F_2 and F_2 MPR F_1, then F_3 MPR F_1.

Proof: With the same supports defined in the proof of Property 2, the given two MPR shifts imply that there exist non-decreasing functions $H_1:[x_\ell^2, x_h^1] \to [0,1]$ and $H_2:[x_\ell^3, x_h^2] \to [0,1]$ such that $F_2 = H_1 F_1$ for $x \in [x_\ell^2, x_h^1]$ and $F_3 = H_2 F_2$ for $x \in [x_\ell^3, x_h^2]$, respectively. This implies that, for the interval $[x_\ell^3, x_h^1]$, $F_2 = H_1 F_1$ and $F_3 = H_2 F_2$, and thus $F_3 = H_1 H_2 F_1$. Since $H_1 \cdot H_2$ is also non-decreasing and between 0 and 1 for all $x \in [x_\ell^3, x_h^1]$, and by Definition 2, F_3 MPR F_1. Q.E.D.

From Property 2 and 3, we have a general statement that a series of MLR (MPR) shifts results in an MLR (MPR) shift. That is, a series of $n+1\,(n \geq 1)$ MLR shifts such that F MLR G_n MLR G_{n-1} MLR $\cdots$ G_1 MLR G, implies that *F* MLR *G*, and the same relationship is applied for the MPR. In other words, any shift that can be decomposed into a series of MLR (MPR) shifts is also an MLR (MPR) shift. Thus, any shift that does not satisfy the MLR (MPR) order cannot be decomposed into a series of MLR (MPR) shifts.

Transitivity does not hold for the L-MLR order. It is easy to find an example in which the sum of two single crossing L-MLR changes, from f_1 to f_2 and from f_2 to f_3, involves multiple crossings between f_1 and f_3. In general, the relationships F_3 L-MLR F_2 and F_2 L-MLR F_1 do not necessarily mean F_3 L-MLR F_1. In this case, since an L-MLR shift is also an MPR shift, Property 3 implies F_3 MPR F_1. In other words, a CDF change that can be decomposed into a series of L-MLR shift is not always an L-MLR shift but it is an MPR shift. Considering the reverse case, there exists an important relationship between the L-MLR order and the MPR order. The next property shows that an MPR shift can always be decomposed into a series of L-MLR shifts where the number of crossing between two pdf's is finite.

Property 4. Assuming that the number of crossing between two pdf's is finite, any MPR shift can be decomposed into a series of L-MLR shifts, that is, if F MPR G, then there exists a series of CDF's $G_1, \cdots, G_n$ such that F L-MLR G_n L-MLR G_{n-1} L-MLR $\cdots$L-MLR G_1 L-MLR G.

Proof: Given an arbitrary of pdf's f and g satisfying the condition F MPR G, let's define a function $h: R \to [0, \infty)$ as

$$h(x) = \begin{cases} f(x)/g(x), \text{when } g(x) \neq 0 \\ \text{the defined last value of } f(x)/g(x), \text{when } g(x) = f(x) = 0 \\ \infty, \text{when } g(x) = 0 \text{ and } f(x) \neq 0. \end{cases}$$

Under the MPR condition, the function h is generally not monotone and thus there are points at which h changes from non-increasing to non-decreasing. Let's denote all such points as c_i, i = 1, 2, $\cdots$, n where $n \geq 0$ [7] and $c_n < c_{n-1} < \cdots < c_1$, and for each c_i define a CDF G_i and its corresponding pdf g_i as,

$$G_i(x) = \begin{cases} \lambda_i G(x), \text{ when } x < c_i \\ F(x), \text{ when } x \geq \text{ c}_i \end{cases} \tag{3}$$

and

$$g_i(x) = \begin{cases} \lambda_i g(x), \text{when } x < c_i \\ f(x), \text{when } x \geq \text{c}_i \end{cases} \tag{4}$$

where $\lambda_i = F(c_i)/G(c_i)$, respectively. If we set $G = G_0$ and $F = G_{n+1}$ with $\lambda_0 = 1$, $\lambda_{n+1} = 0$, $c_0 = x_4$ and $c_{n+1} = x_2$, then from (4) we see that, for every i = 1, 2, $\cdots$, n+1, g_i is generally related to g_{i-1} as

$$g_i(x) = \begin{cases} (\lambda_i / \lambda_{i-1}) g_{i-1}(x), \text{ when } x < c_i \\ (h / \lambda_{i-1}) g_{i-1}(x) (= f(x)), \text{ when } c_i \leq x < c_{i-1} \\ g_{i-1}(x)(= f(x)), \text{ when } x \geq c_{i-1}. \end{cases} \tag{5}$$

[7] When $n = 0$, it just implies that the MPR shift is an MLR shift.

With the given condition F MPR G, $\lambda_i / \lambda_{i-1} < 1$. From (3), the fact that $G_i(c_i) = F(c_i)$ and $G_i(x) \le F(x)$ for all $x \in [x_1, c_i)$ implies that, at every point c_i, g_i meets f from below. This implies $\lambda_i \le h(c_i)$. For the interval $[c_i, c_{i-1})$, the function h is initially non-decreasing and changes to non-increasing through the end of the interval. Accordingly, (5) implies that there exist a point $k_i \in [c_i, c_{i-1})$ [8] and a non-decreasing function $h_i : [x_1, k_i) \to [0,1]$ such that $g_i(x) = h_i(x) g_{i-1}(x)$ for all $x \in [x_1, k_i)$ and $g_i(x) \ge g_{i-1}(x)$ for all $x \in [k_i, x_4]$, where $h_i(x)$ is given as,

$$h_i(x) = \begin{cases} \lambda_i / \lambda_{i-1}, \text{ for } x \in [x_1, c_i) \\ h(x) / \lambda_{i-1}, \text{ for } x \in [c_i, k_i). \end{cases}$$

Hence, by Definition 3, G_i L-MLR G_{i-1} for all $i = 1, 2, \cdots, n+1$. Q.E.D

To illustrate the result in Property 4, consider an example of an FSD shift given in Figure 2. As the figure shows, since the pdf's g and f cross seven times, the shift is neither an MLR nor an L-MLR shift. Following the method given in the proof of Property 4, four intermediate pdf's g_i 's, for i = 1, 2, 3, 4, are defined. The shift from g to f is divided into five shifts, from g to $g_1, \cdots$, and from g_4 to f. Applying the conditions in Definition 3, it is easy to show that each shift is an L-MLR shift. Thus the example shows that a shift from g to f, satisfying the MPR[9] condition, can be decomposed into five L-MLR shifts such that: F L-MLR G_4 L-MLR G_3 L-MLR G_2 L-MLR G_1 L-MLR G.

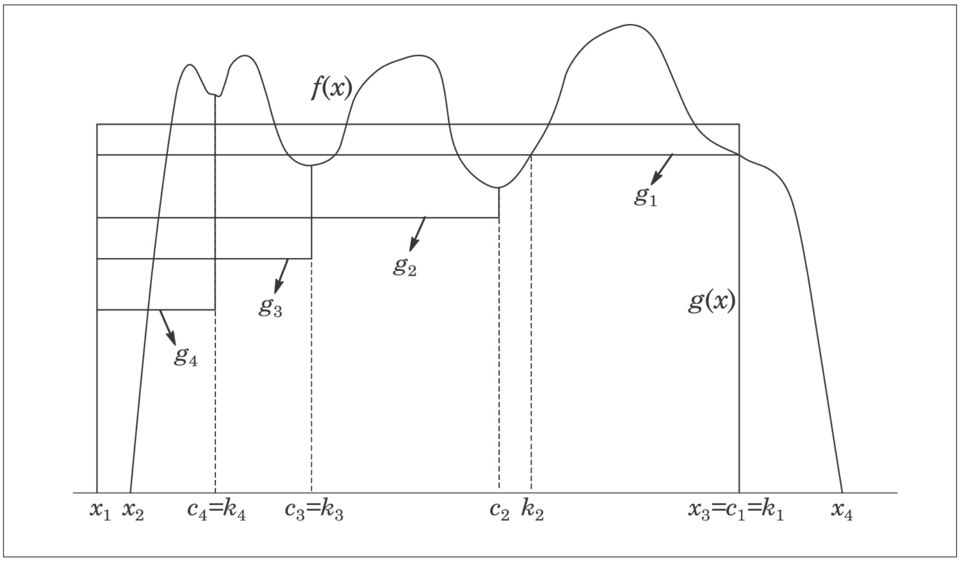
[8] If $g_i(c_i) \ge g_{i-1}(c_i)$, then c_i is the point k_i and from (3),

$$g_i(x) = (\lambda_i / \lambda_{i-1}) g_{i-1}(x), \text{ for } x \in [x_1, c_i)$$
$$g_i(x) \ge g_{i-1}(x), \text{ for } x \in [c_i, x_4]$$

which imply G_i L-MLR G_{i-1}. This case, as will be shown in Figure 1, leads to also G_{i+1} L-MLR G_{i-1}, and more generally, if $g_{i+j}(c_{i+j}) \ge g_{i-1}(c_{i+j})$ for every $j = 0, 1, \cdots, m$, then G_{i+m+1} L-MLR G_{i-1}.

[9] Another implication from the example in Figure 1 is that the shift from g to f is guaranteed to be an MPR shift, without examining its CDF representation. It is because the five L-MLR shifts in the figure are also understood as the five MPR shifts (by Property 1) which are transitive (by Property 3).

[Figure 2] *F* MPR G that is the sum of five L-MLR shifts



IV. COMPARATIVE STATICS ANALYSIS

Following the general one-argument decision model in Kraus (1979), Katz (1981), and Meyer and Ormiston (1985), the relatively standard notation for this general decision model is

$$\max_{b} E[u(z(x, b))], \tag{6}$$

where b is a choice variable, x is an exogenous random variable, and z is the payoff function which depends on both the choice variable and the random variable. It is also assumed that the utility function u is non-decreasing and weakly-concave. This is a weaker restriction than the one used in Eeckhoudt and Gollier's analysis in which strict concavity is required to guarantee an interior unique solution for the decision problem with a linear payoff. Our analysis does not exclude the risk-neutral agents because the sufficient second-order condition for the maximization problem (6) is not used in obtaining our comparative static results.

Given the general one-argument model (6), let b_G and b_F be optimal choices under the CDF's G and F, respectively. First, we examine the sign of $b_F - b_G$ when an L-MLR change from an initial CDF *G* to a final CDF *F* occurs.

Theorem 1. For all risk-averse decision makers, $b_F \geq b_G$ if

(a) *F* L-MLR *G*

(b) $z_x \geq 0$ and $z_{bx} \geq 0$.

Proof: With the CDF's *F* and *G* given, each expected utility function can be expressed as a function of the choice variable *b*,

$$EU_F(b) = \int_{x_2}^{x_4} u[z(x,b)]f(x)dx \quad \text{and} \quad EU_G(b) = \int_{x_1}^{x_3} u[z(x,b)]g(x)dx,$$

respectively. To prove theorem, it suffices[10] to show that, for a pair of points b_1 and b_2,

$$\begin{aligned} &\text{if } b_1 \leq b_2 \text{ and } EU_G(b_1) \leq EU_G(b_2), \\ &\text{then } EU_F(b_1) \leq EU_F(b_2). \end{aligned} \tag{7}$$

This is because (7) implies that $\Delta_F = EU_F(b_G) - EU_F(b) \geq 0$ for every $b \leq b_G$, which in turn implies $b_F \geq b_G$.[11] Thus, assuming that $\Delta_G = EU_G(b_2) - EU_G(b_1) \geq 0$ where $b_1 < b_2$, we show that the following is non-negative,

$$\Delta_F = \int_{x_2}^{x_4} A(x)f(x)dx \tag{8}$$

where $A(x) = u[z(x, b_2)] - u[z(x, b_1)]$. Since $z_{bx} \geq 0$, the difference $z(x, b_2) - z(x, b_1)$ is non-decreasing in *x*. This implies that the assumption $\Delta_G = \int_{x_1}^{x_3} A(x)g(x)dx \geq 0$ excludes the case where $z(x, b_2) - z(x, b_1) \leq 0$ for all $x \in [x_2, x_3]$ because it contradicts the assumption. If $z(x,b_2) - z(x,b_1) \geq 0$ for all $x \in [x_2, x_3]$, then $A \geq 0$ for

[10] We follow the technique used in Landsberger and Meilijson (1990).

[11] If there is no such pair of points b_1 and b_2 satisfying $b_1 \leq b_2$ and $EU_G(b_1) \leq EU_F(b_2)$, this is the case of corner solution and the optimal choice is the lowest among the feasible set. In this case, $b_F \geq b_G$ for any FSD shift from *G* to *F*.

all $x \in [x_2, x_4]$ and thus $\Delta_F \geq 0$. These cases are true for any FSD shift from G with its support $[x_1, x_3]$ to F with its support $[x_2, x_4]$ where $x_1 \leq x_2$ and $x_3 \leq x_4$.[12]

Now consider the case that, with b_1, b_2 and the payoff function z given, there exists a point $x^*(b_1, b_2, z) \in [x_2, x_3]$ such that $z(x, b_2) - z(x, b_1)$ is non-positive for all $x \leq x^*$ and non-negative for all $x \geq x^*$. This implies that $A \leq 0$ for all $x \leq x^*$ and $A \geq 0$ for all $x \geq x^*$. According to Definition 3, the condition F L-MLR G implies that there exists a point $k \in [x_2, x_3]$ such that $f(x) \leq g(x)$ for all $x \in [x_1, k)$ and $f(x) \geq g(x)$ for all $x \in [k, x_4]$.[13] Hence $f(x)$ can be expressed as,

$$f(x) = \begin{cases} [1-\delta(x)]g(x), \text{for } x \in [x_1, k) \\ [1+\eta(x)]g(x), \text{for } x \in [k, x_3) \\ f(x), \text{for } x \in [x_3, x_4] \end{cases}$$

where δ is non-increasing in x, $\delta = 1$ for $x \in [x_1, x_2)$, $0 \leq \delta \leq 1$ for $x \in [x_2, k)$, and η is a non-negative function.

Case (i): when $x^* \leq k$. Since $f(x) = 0$ for $x \in [x_1, x_2)$, (8) can be rewritten as,

$$\Delta_F = \int_{x_1}^{k} A(x)[1-\delta(x)]g(x)dx + \int_{k}^{x_3} A(x)[1+\eta(x)]g(x)dx \\ + \int_{x_3}^{x_4} A(x)f(x)dx. \tag{9}$$

Rearranging (9) gives,

$$\Delta_F = \Delta_G + \int_{x_1}^{k} A(x)[-\delta(x)]g(x)dx + \int_{k}^{x_3} A(x)\eta(x)g(x)dx$$

[12] We do not exclude the case $x_3 \leq x_2$ where all the possible outcome values of x under F are higher than under G. For any shift of this type, $\Delta_F \geq 0$.

[13] Since the proof for the case that $f(k) \leq g(k)$ and $f(x) \leq g(x)$ for all $x \in [x_1, k]$ and $f(x) \geq g(x)$ for all $x \in (k, x_4]$ follows the same procedure, we omit the case.

$$+\int_{x_3}^{x_4} A(x)f(x)dx.$$

Since $A(x)$ changes its sign from negative to positive at $x = x^*$ and $\delta(x)$ is non-increasing,

$$\Delta_F \geq \Delta_G - \delta(x^*)\int_{x_1}^{k} A(x)g(x)dx + \int_{k}^{x_3} A(x)\eta(x)g(x)dx$$
$$+\int_{x_3}^{x_4} A(x)f(x)dx.$$

Since $\Delta_G \geq 0$ by assumption, $0 \leq \delta(x^*) \leq 1$ and $\Delta_G \geq \int_{x_1}^{k} A(x)g(x)dx$, we have $\Delta_F \geq 0$.

Case (ii): when $k \leq x^*$. Let's rewrite (8) as,

$$\Delta_F = \int_{x_1}^{x^*} A(x)f(x)dx + \int_{x^*}^{x_3} A(x)f(x)dx + \int_{x_3}^{x_4} A(x)f(x)dx. \tag{10}$$

Integrating the first term in right side of (10) by parts, and by adding and subtracting,

$$\Delta_F = A(x)F(x)|_{x_1}^{x^*} - \int_{x_1}^{x^*} B(x)F(x)dx + \int_{x^*}^{x_3} A(x)f(x)dx$$
$$+\int_{x_3}^{x_4} A(x)f(x)dx$$
$$= A(x)[F(x)-G(x)]|_{x_1}^{x^*} + A(x)G(x)|_{x_1}^{x^*} - \int_{x_1}^{x^*} B(x)[F(x)-G(x)]dx$$
$$-\int_{x_1}^{x^*} B(x)G(x)dx + \int_{x^*}^{x_3} A(x)[f(x)-g(x)]dx + \int_{x^*}^{x_3} A(x)g(x)dx$$
$$+\int_{x_3}^{x_4} A(x)f(x)dx$$

where $B = dA/dx = u'(z(x,b_2))z_x(x,b_2) - u'(z(x,b_1))z_x(x,b_1)$. By rearranging the above equation,

$$\Delta_F = \Delta_G + A(x)[F(x)-G(x)]|_{x_1}^{x^*} - \int_{x_1}^{x^*} B(x)[F(x) - G(x)]dx$$

$$+\int_{x^*}^{x_3} A(x)[f(x)-g(x)]dx+\int_{x_3}^{x_4} A(x)f(x)dx\,.$$

Note that $z(x,b_2)\le z(x,b_1)$ when $x\le x^*$ and $z_x(x,b_2)\ge z_x(x,b_1)$ when $x\ge x_1$ by the assumption $z_{bx}\ge 0$. The assumptions $u'\ge 0$, $u''\le 0$ and $z_x\ge 0$ imply that $B(x)\ge 0$ for $x\in[x_1,x^*]$. Since $A\le 0$ for all $x\le x^*$ and $A\ge 0$ for all $x\ge x^*$, and the L-MLR condition implies that $F(x)\le G(x)$ for all $x\in[x_1,x_4]$ and $f(x)\ge g(x)$ for all $x\in[k,x_3]$, the assumption $\Delta_G\ge 0$ implies that $\Delta_F\ge 0$. Q.E.D.

While Theorem 1 itself is meaningful as a general comparative static statement, a further generalization is given in the next theorem. From Property 4 developed in section III, it is known that an MPR shift can be decomposed into a series of L-MLR shifts. This allows a direct generalization of the comparative static result in Theorem 1, without any additional cost of assumptions.

Theorem 2. For all risk-averse decision makers, $b_F\ge b_G$ if

(a) F MPR G

(b) $z_x\ge 0$ and $z_{bx}\ge 0$.

Proof: Property 4 and Theorem 1 completes the proof. Q.E.D.

Theorem 2 is an easy and direct generalization of Theorem 1, which is given by the decomposability of the two orders, L-MLR and MPR. Accordingly, using the general one-argument decision model (6), this paper shows that the comparative statics result in Eeckhoudt and Gollier's analysis holds for the cases of non-linear payoffs without any additional cost of assumptions. The nonlinear example which provides appropriate application of Theorems 1 and 2 can be found in a competitive risk-averse firm that maximizes the expected utility of profits, $Eu(\pi)$ in which the profit function is given by $\pi(x,q)=p\cdot q-c(x,q)$, where p is the output price, q is the output level, x is a random variable, and $c(x,q)$ is the random cost function. It is assumed that the partial derivatives of the total cost function and marginal cost function with respect to x are positive

($c_x > 0$ and $c_{qx} > 0$). If we further assume that the partial derivative of the marginal function cost with respect to x is non-increasing and convex ($z_{\alpha xx} = \pi_{qxx} = -c_{qxx} \geq 0$ and $z_{\alpha xxx} = \pi_{qxxx} = -c_{qxxx} \leq 0$), Theorems 1 and 2 hold for a risk-averse firm.

Compared with Eeckhoudt and Gollier's result, the result in Theorem 2 has two improvements. The first is that our result uses a weaker restriction for the optimal solution of the decision problem. That is, the case of a corner or an unbounded solution and the risk neutral decision-makers are not excluded from our result.

Another improvement is that Theorem 2, using the general decision model (6), completes the Eeckhoudt and Gollier's discussion about the lack of symmetry in the effects of the minimum and the maximum price regulations in Eeckhoudt and Hansen (1980) who consider a competitive firm under price uncertainty. A minimum price regulation gives a change from one price distribution G to another F such that

$$F(x) = 0 \quad \text{for all} \quad x < x_{\min} \quad \text{and} \quad G(x) = F(x) \quad \text{for all} \quad x \geq x_{\min} \qquad (11)$$

and a maximum price regulation gives a change such that

$$F(x) = 1 \quad \text{for all} \quad x \geq x_{\max} \quad \text{and} \quad G(x) = F(x) \quad \text{for all} \quad x < x_{\max}, \qquad (12)$$

where $x_{\min}$ and $x_{\max}$ denote the minimum and the maximum price given for each price regulation, respectively. Eeckhoudt and Hansen found that, while the minimum price regulation increases the output level of a risk-averse competitive firm, the effect of the maximum price regulation on the output level of the firm is ambiguous. This lack of symmetry between two regulations can be explained by Theorem 2. The minimum price regulation (11) implies F MPR G but the maximum price regulation (12) does not imply G MPR F.

V. CONCLUDING REMARKS

Regarding the MPR order, Eeckhoudt and Gollier's comparative static result is restricted to decision models with linear payoffs. This paper has generalized their results into the standard one-argument decision model which includes the decision problems with non-linear payoffs. In the process, we introduce, a new concept of ranking distributions – an L-MLR order – which lies between the MLR and the MPR order, and we use a special technique for making general comparative static statements. More precisely, we show that a CDF change satisfying the MPR order can always be decomposed into a series of CDF changes that satisfy the L-MLR order. Then the comparative static statement made for an L-MLR change can also be applied for any MPR change, without requiring any additional cost of assumptions.

APPENDIX

Before we present some relationships among the subsets of FSD shifts, we define 'the right-side monotone likelihood ratio' (R-MLR) order in Kim and Ryu (2004, p.297) and 'the left-side relatively weak first-degree stochastic dominance' (L-RWFSD) order in Ryu and Kim (2003, p.62).

Definition 4. Given a point $p \in [x_2,\ x_3]$, $F(x)$ represents a 'right-side monotone likelihood ratio shift with respect to a point p' from $G(x)$ (denoted by F R-MLR(p) G) if there exists a point $k \in [x_2,\ p]$ and a non-decreasing function $h\colon [k,\ x_3] \to [1,\ \infty)$ such that $f(x) = h(x)g(x)$ for all $x \in [k,\ x_3]$ and $g(x) \geq f(x)$ for all $x \in [x_2,\ k)$.

Definition 5. $F(x)$ represents 'a left-side relatively weak FSD shift' from $G(x)$ (denoted by F L-RWFSD G) if

(a) There exists a point $m \in [x_2, x_3]$ such that $f(x) \leq g(x)$ for all $x \in [x_2, m)$ and

$f(x) \geq g(x)$ for all $x \in [m, x_3]$

(b) When $x^* \in [x_2, m)$, one needs the following condition:

$$\frac{f(x)}{g(x)} \leq \frac{f(x^*)}{g(x^*)}\ , \qquad x_2 \leq x \leq x^*$$

$$\frac{f(x)}{g(x)} \geq \frac{f(x^*)}{g(x^*)}\ , \qquad x^* \leq x \leq m$$

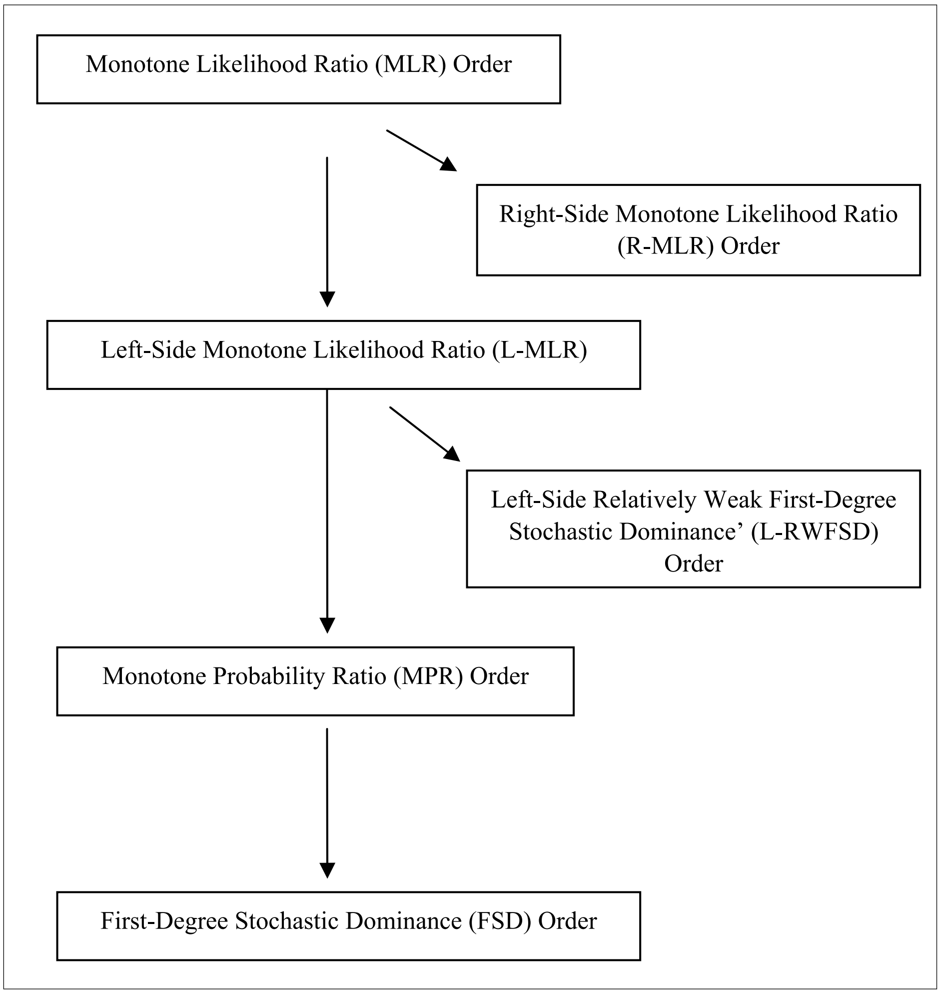
where x^* denotes the value of x satisfying $[z(x,\alpha_2) - z(x,\alpha_1)] = 0$.

Basic relationships among the subsets of FSD shifts are shown in Figure. 3 and summarized as:

1) MLR $\Rightarrow$ L-MLR $\Rightarrow$ L-RWFSD
2) MLR $\Rightarrow$ L-MLR $\Rightarrow$ MPR
3) MLR $\Rightarrow$ R-MLR $\Rightarrow$ MPR

The above orders yield the same comparative statics results for the risk averse decision makers. Therefore, the MPR order represents a net improvement to the MLR, L-MLR and R-MLR ones, and the L-RWFSD order shows a net improvement to the MLR and L-MLR ones.

[Figure 3] Relationships among the subsets of FSD shifts



Some properties of transitivity and decomposition are summarized as:

1) The MLR order is transitive.
2) The MPR order is transitive.
3) The MPR shift can be decomposed into a series of L-MLR shifts.

References

Bagnoli, M. and T. Bergstrom (2005), "Log-concave Probability and Its Applications," *Economic Theory*, Vol. 26, pp. 445-469.

Eeckhoudt, L. and C. Gollier (1995), "Demand for Risky Assets and the Monotone Probability Ratio Order," *Journal of Risk and Uncertainty*, Vol. 11, pp. 113-122.

Eeckhoudt, L. and P. Hansen (1980), "Minimum and Maximum Prices, Uncertainty, and the Theory of the Competitive Firm," *American Economic Review*, Vol. 70, pp. 1064-1068.

Gollier, C. (1995), "The Comparative Statics of Changes in Risk Revisited," *Journal of Economic Theory*, Vol. 66, pp. 522-535.

Karlin, S. and H. Rubin (1956), "Distributions Possessing a Monotone Likelihood Ratio," *Journal of the American Statistical Association*, Vol. 51, pp. 637-643.

Katz, E. (1981), "A Note on a Comparative Statics Theorem for Choice under Risk," *Journal of Economic Theory*, Vol. 25, pp. 318-319.

Kim, S. and S. Ryu (2004), "Comparative Statics under Uncertainty with the Monotone Likelihood Ratio Order," *Korean Economic Review*, Vol. 20, pp. 293-304.

Kraus, M. (1979), "A Comparative Statics Theorem for Choice under Risk," *Journal of Economic Theory*, Vol. 21, pp. 510-517.

Landsberger, M. and I. Meilijson (1990), "Demand for Risky Financial Assets: A Portfolio Analysis," *Journal of Economic Theory*, Vol. 50, pp. 204-213.

Meyer, J. and M.B. Ormiston (1985), "Strong Increases in Risk and their Comparative Statics, *International Economic Review*, Vol. 26, pp. 425-437.

Ormiston, M.B. and E. Schlee (1993), "Comparative Statics under Uncertainty for a Class of Economic Agents," *Journal of Economic Theory*, Vol. 61, pp. 412-422.

Rothschild, M. and J.E. Stiglitz (1970), "Increasing Risk: I. A Definition," *Journal of Economic Theory*, Vol. 2, pp. 225-243.

Ryu, S. and I. Kim (2003), "Comparative Statics on the Left-Side Relatively Weak First-Degree Stochastic Dominance Orders and Its Applications," *Seoul Journal of Economics*, Vol. 16, pp. 59-69.