The Korean Economic Review
Volume 42, Number 1, Winter 2026, 141-183.
DOI : 10.22841/kerdoi.2026.42.1.006

R&D Investments under Bertrand Competition*

Myeonghwan Cho**

We study firms' incentives to invest in R&D when they produce homogeneous goods and engage in Bertrand competition. In the model, the firms can reduce their marginal costs through R&D before competing in the market by setting the prices of their goods. We find that if the firms make non-randomized choices on their R&D investments in equilibrium, only one firm invests in R&D and supplies goods to the entire market. If the firms are allowed to randomize their R&D investment choices, all firms invest in R&D with probability one in any (symmetric) equilibrium. In addition, if firms' R&D investment costs are sufficiently low, there is a positive probability that all firms make the maximum R&D investment and share the market. The paper also analyzes how R&D spillovers and the number of competing firms affect firms' R&D investment incentives, and compares the R&D behavior of firms under Bertrand and Cournot competition.

JEL Classification: C72, D43, L13
Keywords: Bertrand Competition, R&D Investment, Mixed Strategy Equilibrium, Symmetric Equilibrium.

I. Introduction

It is generally perceived that R&D is important for firms competing in the market. It is frequently observed that, in many industries, firms make careful decisions about R&D investments and these decisions sometimes determine whether they survive or leave the market. For example, it is often said that the main reason for the decline of Kodak, which had a dominant position in the photographic film market, or the

Received: May 2, 2025. Revised: Aug. 4, 2025. Accepted: Nov. 26, 2025.
* This work was supported by the 2025 Research Fund of the University of Seoul. I would like to thank Seung-Yeop Lee for his hospitality during my visit to the University of South Florida. I also thank Gayeon Kim, Hyeonjin Sung, and the anonymous referees for their helpful comments.
** Professor, Department of Economics, University of Seoul, 163 Seoulsiripdaero, Dongdaemun-gu, Seoul, Republic of Korea, Tel: +82-2-6490-2067, e-mail: chomhmh@uos.ac.kr

decline of Nokia, which first created a cellular network in the mobile phone market, was the failure of R&D or innovation. It is also well known that Samsung and TSMC, which are leading companies in the semiconductor foundry market, invest heavily in R&D every year.

In a market, firms decide on R&D investments considering the benefits of gaining a superior position in the market through R&D and the cost of R&D investment. Their decisions on R&D investments are also influenced by their rival firms' decisions on R&D investments. For instance, the more aggressively rival firms invest in R&D, the more likely a firm is to increase its own R&D investment to avoid being left out of the market. Alternatively, if a firm finds it difficult to beat its rival firm that invests heavily in R&D competition, the firm may want to save on R&D investment costs rather than invest huge amounts in R&D. Firms' decisions on R&D investments can vary depending on the market structure and competitive situation, where they strategically interact with each other.[1]

With the recognition of the important role of R&D in the market, many studies have been conducted on the behavior of firms in their R&D investments. Moldovanu and Sela (2001), Kvasov (2007), and Siegel (2009) consider the R&D competition of the firms as an example of all-pay contests, where players bid irretrievable amounts to win a prize, and obtain many results that improve the understanding of all-pay contests.[2] Each firm's R&D investment and the profit earned by the firm with the higher investment correspond to an irrecoverable bid and the prize, respectively, in all-pay contests. Although these studies on the all-pay contest contribute to understanding R&D behavior, they do not reflect the features of R&D competition in that the value of the prize in the contests is exogenously given. In the R&D competition of firms, the prize of the competition is the profit that the winning firm earns in the market, and its value depends on the firms' R&D investments through reducing their costs or differentiating their products.

There have been many studies on the R&D competition where the profit of the firm winning the R&D competition is endogenously determined by their R&D investments. Symeonidis (2003) and Cellini and Lambertini (2004, 2011) study R&D competition where firms in an oligopoly differentiate their products through R&D, and d'Aspremont and Jacquemin (1988), Bester and Petrakis (1993), Qiu (1997), Hinloopen (2000), and Chen and Lee (2023) study R&D competition where firms reduce their costs of producing goods through R&D investments before engaging in

---

[1] The two representative views on how a firm's R&D investment is affected by a competitive situation are Schumpeter's view and Arrow's view. Schumpeter's view suggests that firms with a large surplus in a less competitive situation can and do invest more aggressively in R&D. On the other hand, Arrow's view suggests that the desire to survive in a competitive market is one of the main motives for R&D and so firms facing severe competition are more active in R&D.

[2] Other examples of all-pay contests are political lobbying, waiting in line, and advertising to attract loyal customers.

oligopoly competition. Lin and Saggi (2002) also consider the R&D competition in an oligopoly, where the firms can differentiate their products and reduce their costs of producing goods through R&D. Most of these studies focus on the R&D behavior of firms under Cournot or Bertrand competition and find that the R&D investments to reduce the cost appear greater when firms are involved in Cournot competition than when they are involved in Bertrand competition.

A common assumption in the previous studies on R&D competition is that the firms under Cournot or Bertrand competition produce differentiated goods. This assumption allows the profits of firms under Bertrand competition to continuously change in their marginal costs that are determined by the R&D investments, which induces the firms to make non-randomized choices on R&D investments as a pure strategy in an equilibrium. On the other hand, if the goods produced by firms are completely homogeneous, only the firm that wins the R&D competition supplies the goods to the entire market and earns a positive profit, while the other firms earn a profit of zero. This makes the firm's profit not continuous in their choices of R&D investments. In this situation, there may not exist an equilibrium where the firms make a non-randomized choice on R&D investments as in the previous studies.[3]

In this paper, we explore the R&D behavior of firms that produce homogeneous goods and are engaged in Bertrand competition. Particularly, we are interested in what choices they make for R&D investment and how the market price and quantity are determined in equilibrium. To this end, we first consider the market where two firms compete in prices and there is no spillover in their R&D. Given the market demand and the firm's R&D investment costs in general forms, there may or may not be an equilibrium where the firms make non-randomized choices on R&D investment, depending on the forms of the market demand and the R&D investment costs of the firms.[4] In addition, if there is an equilibrium where the firms make non-randomized choices on R&D, one firm invests in R&D and the other firm does not invest in R&D at all in the equilibrium. This results in a market where the firm that invests in R&D supplies goods to the entire market at a price set at the other firm's marginal cost.

Regardless of the existence of an equilibrium in which firms make non-

[3] Delbono and Denicoló (1990) investigates an R&D competition of firms that produce completely homogeneous goods. In his model, the R&D investments of firms continuously determine the probability of winning, which makes the profit of firms continuous in their choices on R&D investments.

[4] Our result on the non-existence of equilibrium where the firms make non-randomized choices on R&D investment may be related with the literature that discusses the existence of pure and mixed strategy equilibrium. Examples from general game-theoretic settings include Glicksberg (1952), Dasgupta and Maskin (1986), and Reny (1999). In the context of Bertrand competition, Kaplan and Wettstein (2000) and Blume (2003) study equilibria with homogeneous goods and cost asymmetries, showing the non-existence of pure strategy equilibria and the emergence of mixed strategy equilibria when costs differ. Most studies on firms' R&D behavior construct models in which firms make non-randomized decisions regarding R&D investment, and pay little attention to the existence of equilibrium itself.

randomized choices about R&D, there always exists an equilibrium where the firms make randomized choices on R&D investments. Assuming that the market demand and the R&D investment costs have specific forms, we can find an equilibrium where the firms make randomized choices on R&D investments and figure out how the firms behave with respect to R&D. In the equilibrium, all firms always invest in R&D to some extent. In addition, how much the firms make R&D investments and which firm supplies goods to the market depend on their R&D investment costs. That is, if the R&D investment costs are high, the firms make random choices on the R&D investments and the winner of the R&D competition is randomly determined to supply goods to the entire market. On the other hand, if the R&D investment costs are low, an equilibrium where all firms make the maximum investment in R&D to have the same marginal cost and supply goods to the market can appear with a positive probability. To the best of our knowledge, this kind of equilibrium is not suggested in previous studies on R&D competition.

We also extend the model to see how the spillover effect of R&D and the number of firms competing in the market affect the incentive for firms to invest in R&D. It is generally believed that a large spillover effect of R&D has a negative effect on the incentives for the firms to invest in R&D. A result in this study shows that this belief holds only when the spillover effect of R&D is sufficiently large. When the spillover effect of R&D is small, an increase in the spillover effect of R&D has a positive effect on the incentive for the firms to invest in R&D. Our extension also shows that as the number of firms competing in the market increases, the incentives for the firms to invest in R&D decreases. This can be understood because, under price competition, as the number of competing firms increases, the probability of individual firms winning the R&D competition and earning a positive profit decreases, which induces the firms to hesitate to invest in R&D.

Finally, we also compare the incentives for the firms to invest in R&D and the market equilibria when they are engaged in Bertrand competition versus when they are engaged in Cournot competition. We find that firms tend to invest more in R&D under Bertrand competition than under Cournot competition as R&D investment costs increase. In addition, even when the firms under Cournot competition make more R&D investments to have lower marginal costs, the market price can be higher under Cournot competition than under Bertrand competition.

II. Model

We consider a homogeneous goods market where the market demand is represented as a function $D(p)$ of the price p of goods. In this market, there are two firms (labeled $i = 1, 2$), which compete with each other by setting their prices.

Each firm i has a technology of constant return to scale and can choose its marginal cost c_i through R&D. That is, for given $c > 0$, firm i can reduce its marginal cost by $x_i \in [0, c]$ so that its marginal cost becomes $c_i = c - x_i$. If firm i reduces its marginal cost by x_i, it has to pay an R&D investment cost $R(x_i)$. We assume that $R(\cdot)$ is differentiable, strictly increasing, strictly convex, and satisfies $R(0) = 0$.

Let p_i be the price that firm i sets. Since the firms produce homogeneous goods, the consumers buy the goods from the firm that offers a lower price. If two firms set the same price, each firm can sell its goods to half of the consumers in the market. Thus, for the prices (p_i, p_j) of the firms, firm i's profit is determined as[5]

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c_i)D(p_i) & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c_i)D(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j. \end{cases} \tag{1}$$

To ensure that the equilibrium is well defined in subsequent analysis, we assume that the monopoly profit for firm i, $\pi_i = (p_i - c_i)D(p_i)$ is strictly concave in p_i for any c_i and has a unique maximizer at $p_i = \bar{p}_i > c$.[6] These assumptions ensure that $(p_i - c_i)D(p_i)$ is strictly decreasing as p_i is further away from $\bar{p}_i$ and $\bar{p}_i$ is strictly increasing in c_i.

If firm i reduces its marginal cost by x_i through R&D and the firms set the prices at (p_i, p_j), firm i's *payoff* is

$$U_i(p_i, p_j; x_i) = \pi_i(p_i, p_j) - R(x_i), \tag{2}$$

where $\pi_i(p_i, p_j)$ is firm i's profit in (1)

In this environment, the firms make decisions in two stages as follows. In the first stage, called the *R&D stage*, each firm i simultaneously chooses its marginal cost reduction $x_i \in [0, c]$ through R&D. In the second stage, called the *Bertrand stage*, each firm i observes their marginal costs $(c_i, c_j) = (c - x_i, c - x_j)$ and simultaneously chooses a price p_i for the goods it produces.

This situation can be interpreted as an extensive form game, where firm i's strategy (x_i, σ_i) consists of a choice $x_i \in [0, c]$ for R&D in the R&D stage and a choice $\sigma_i(x_i, x_j) \in [0, \infty)$ for the price of goods in Bertrand stage. Here, x_i means that firm i reduces its marginal cost by x_i through R&D. In addition, $\sigma_i(x_i, x_j)$

---

[5] In general, the profit of a firm is defined as its revenue minus its costs and the firm's costs include not only production cost but also R&D investment cost. However, for convenience of description, we refer to a firm's profit as its revenue minus its production cost, not the R&D investment cost, and a firm's payoff as its revenue minus its production cost and R&D investment cost.

[6] If the market demand is given linearly as $D(p) = 1 - p$ as in Section 4, these assumptions are satisfied.

means that, when firm i observes (x_i, x_j) in the R&D stage, it sets its price of goods at $p_i = \sigma_i(x_i, x_j)$. We allow the firms to play behavioral strategies so that they can choose randomized actions in the R&D and Bertrand stages. In other words, each firm i can choose a random variable x_i on $[0, c]$ in the R&D stage and a random variable $\sigma_i(x_i, x_j)$ on $[0, \infty)$ in Bertrand stage after observing (x_i, x_j).

III. R&D investment in equilibrium

In this section, we investigate how the firms competing with prices make decisions on R&D in subgame perfect equilibrium. A subgame perfect equilibrium can be found using backward induction. Consider Bertrand stage where the firms observe (x_i, x_j) chosen in R&D stage and are supposed to make decisions on the prices for goods. Given that (x_i, x_j) is chosen in the R&D stage, each firm i's marginal cost is fixed at $c_i = c - x_i$, and its R&D investment cost is also fixed at $R(x_i)$. Thus, the equilibrium strategy of the firms in Bertrand stage can be found by considering the firms' behavior to maximize their profits in (1). This is a typical model of price competition between two firms in a duopoly market. The firms' choices in equilibrium can be summarized as Proposition 1.

Proposition 1. *Consider a subgame of Bertrand stage that follows after the firms choose* (x_i, x_j) *in the R&D stage. If* $x_i = x_j$ *(i.e.,* $c_i = c_j$*), then the unique equilibrium in Bertrand stage is* $(p_i^*, p_j^*) = (c - x_i, c - x_j)$. *If* $x_i > x_j$ *(i.e.,* $c_i < c_j$ *), then Bertrand stage does not have a pure strategy Nash equilibrium. In addition, there exists a mixed strategy Nash equilibrium* (p_i^*, p_j^*) *such that* $p_i^* = c_j$ *and, for sufficiently small* $\eta > 0$, p_j^* *is a randomized choice of prices following a uniform distribution on* $[c_j, c_j + \eta]$.[7]

Proof. See Kaplan and Wettstein (2000) and Blume (2003). ■

Proposition 1 directly follows from Kaplan and Wettstein (2000) and Blume (2003), who study an equilibrium for the price competition between the firms with fixed marginal costs. If the firms make the same decision on the R&D investment, they have the same marginal costs in Bertrand stage. In this case, it is well known that the unique pure strategy equilibrium is for the firms to set their prices at their marginal costs (i.e., $p_i^* = c_i$ for each i). In addition, the non-existence of mixed strategy

[7] A subgame of Bertrand stage can be interpreted as a strategic form game where the marginal costs of the firms are fixed by their choices (x_i, x_j) in the R&D stage. Here, the pure strategy and the mixed strategy means a strategy in such a strategic form game. These strategies constitute a behavioral strategy in the entire game consisting of two stages.

equilibrium is ensured by Kaplan and Wettstein (2000).[8] If the firms make different R&D investments, they will have different marginal costs in Bertrand stage. Note that each firm j does not set a price lower than its marginal cost c_j and a price higher than $\bar{p}_j$ that maximizes $\pi_j = (p_j - c_j)D(p_j)$ because these prices are (weakly) dominated by the price set at its marginal cost and the price set at $\bar{p}_j$, respectively. If firm j with a higher marginal cost c_j sets a price p_j satisfying $c_j \leq p_j \leq \bar{p}_i$, its rival firm i with a lower marginal cost c_i can always improve its profit as long as it keeps a price lower than firm j's price p_j and so it cannot maximize its profit by setting a price lower than its rival firm.[9] If firm j with a higher marginal cost c_j sets a price p_j with $\bar{p}_i < p_j \leq \bar{p}_j$, its rival firm i with a lower marginal cost c_i will choose $\bar{p}_i$ to maximize its profit, which provides an incentive for firm j to deviate from p_j with $\bar{p}_i < p_j \leq \bar{p}_j$ by choosing a price slightly lower than $\bar{p}_i$. This explains why the firms with different marginal costs do not have a pure strategy equilibrium.

Blume (2003) also shows that the mixed strategy in Proposition 1 is an equilibrium when the firms with different marginal costs are involved in a price competition. This can be interpreted as follows. Under the mixed strategy (p_i^*, p_j^*) in Proposition 1, firm i's (expected) profit is $\pi_i^* = (c_j - c_i)D(c_j) > 0$ and firm j's (expected) profit is $\pi_j^* = 0$. Given that firm i chooses $p_i^* = c_j$, firm j cannot earn a positive profit no matter what it chooses as its price p_j. This means that firm j will not deviate from p_j^* given that firm i chooses $p_i^* = c_j$. Next, suppose that firm j randomly chooses its price p_j^* as in Proposition 1. If firm i chooses $p_i < c_j$, its profit is $\pi_i = (p_i - c_i)D(p_i)$, which is smaller than π_i^*. If firm i chooses $p_i > c_j + \eta$, it receives a profit of zero. If firm i chooses p_i with $c_j \leq p_i < c_j + \eta$, its expected profit is

$$\pi_i = (p_i - c_i)D(p_i)\left(1 - \frac{p_i - c_j}{\eta}\right). \tag{3}$$

Differentiating this equation with respect to p_i, we have

$$\frac{d\pi_i}{dp_i} = (D(p_i) + (p_i - c_i)D'(p_i))\left(1 - \frac{p_i - c_j}{\eta}\right) - (p_i - c_i)D(p_i)\frac{1}{\eta}. \tag{4}$$

For sufficiently small η, $\frac{d\pi_i}{dp_i}$ in (4) has a negative value for any $p_i \in [c_j, c_j + \eta]$.

[8] Kaplan and Wettstein (2000) show that the firms with the same marginal cost have an equilibrium other than marginal-cost pricing if and only if the revenue increases to infinity as the price increases. That is, $\lim_{p\to\infty} pD(p) = \infty$, which conflicts with our assumption that the monopoly profit $\pi_i = (p_i - c_i)D(p_i)$ has the unique maximizer.

[9] Note that the assumptions for $\pi_i(p_i, p_j)$ in (1) ensure that $\bar{p}_i < \bar{p}_j$ and $\pi_i = (p_i - c_i)D(p_i)$ is strictly increasing in $p_i < \bar{p}_i$.

This means that, if firm i raises its price p_i from $p_i^* = c_j$, firm i suffers a decrease in profit. Thus, when firm j chooses a randomized price p_j^* as in Proposition 1, $p_i^* = c_j$ is optimal for firm i.

We note that the mixed strategy equilibrium (p_i^*, p_j^*) in Proposition 1 is not the only equilibrium in Bertrand stage. If the firms make different R&D investments in the R&D stage, Bertrand stage has numerous equilibria that are similar to the mixed strategy equilibrium (p_i^*, p_j^*) in Proposition 1. Indeed, for any price p_i with $c_i < p_i \leq c_j$, we can construct an equilibrium where firm i with a low marginal cost chooses p_i as its price. An example of such equilibria is a strategy profile that firm i chooses a price p_i with $c_i < p_i \leq c_j$ and firm j chooses a randomized price p_j following a uniform distribution over $[p_i, p_i + \eta]$. Under this equilibrium, the probability that firm j chooses a price p_j lower than its marginal cost c_j is greater than zero. This means that firm j chooses a weakly dominated strategy in the equilibrium. In an equilibrium where the firms choose only the undominated strategies in Bertrand stage, a firm i with a lower marginal cost sets a price p_i at the other firm j's marginal cost c_j and firm j randomly chooses prices that are not lower than its marginal cost c_j. The mixed strategy (p_i^*, p_j^*) in Proposition 1 is an equilibrium in which the firms do not choose dominated strategies with a positive probability.[10] In what follows, we assume that the firms play (p_i^*, p_j^*) in Proposition 1 in Bertrand stage.

Given that the firms choose the prices (p_i^*, p_j^*) as in Proposition 1 in Bertrand stage, if the firms chooses (x_i, x_j) satisfying $x_i \geq x_j$ (i.e., $c_i \leq c_j$), the market price p^* is determined at the price $p_i^* = c - x_j$ that firm i chooses. In addition, each firm i's (expected) profit $\pi_i(x_i, x_j)$ is

$$\pi_i(x_i, x_j) = \begin{cases} (x_i - x_j)D(c - x_j) > 0 & \text{if } x_i > x_j \\ 0 & \text{if } x_i \leq x_j. \end{cases} \tag{5}$$

Firm i gets a profit of zero if it invests in R&D less than or equal to its rival firm j. If firm i invests in R&D more than its rival firm j, it earns a positive profit in Bertrand stage. In addition, this positive profit of firm i increases as firm i increases its R&D investment, given that its rival firm j's R&D investment is fixed.

We next determine the firm's decision in the R&D stage given that the firms play as in Proposition 1 in Bertrand stage. The firm's decision (x_i, x_j) on R&D can be found by solving a reduced game, where each firm i simultaneously chooses $x_i \in [0, c]$ and its payoff is given by

[10] See Kartik (2011) for details.

$$U_i(x_i, x_j) = \begin{cases} (x_i - x_j)D(c - x_j) - R(x_i) & \text{if } x_i > x_j \\ -R(x_i) & \text{if } x_i \leq x_j. \end{cases} \tag{6}$$

The equilibrium strategy (x_i^*, x_j^*) for the firms in the R&D stage is a Nash equilibrium in this reduced game.

To find a Nash equilibrium in the R&D stage, we define $\bar{x}$ as $\bar{x} = x_i$ that maximizes firm i's payoff $U_i(x_i, 0)$ on $[0, c]$ given that its rival firm j chooses $x_j = 0$. We also define $\bar{\bar{x}}$ as $\bar{\bar{x}} = x_i$ that maximizes firm i's payoff $U_i(x_i, \bar{x})$ on $[\bar{x}, c]$ given that firm j chooses $x_j = \bar{x}$. That is,

$$\bar{x} = \text{argmax}_{x_i} U_i(x_i, 0) \quad \text{subject to} \quad x_i \in [0, c] \tag{7}$$

$$\bar{\bar{x}} = \text{argmax}_{x_i} U_i(x_i, \bar{x}) \quad \text{subject to} \quad x_i \in [\bar{x}, c]. \tag{8}$$

Since $R(\cdot)$ is continuous and strictly convex, $\bar{x}$ and $\bar{\bar{x}}$ are uniquely well defined.

In Proposition 2, we construct a pure strategy equilibrium (x_i^*, x_j^*) in the R&D stage and provide a condition for such an equilibrium to exist.

Proposition 2 *If* $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) > 0$ *, there is no pure strategy equilibrium in the R&D stage. If* $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) \leq 0$*, there is a pure strategy equilibrium* (x_i^*, x_j^*) *in the R&D stage and* (x_i^*, x_j^*) *satisfies that* $x_i^* = \bar{x}$ *for a firm* i *and* $x_j^* = 0$ *for the other firm* j*.*

Proof. First, suppose that (x_i, x_j) satisfies $0 < x_j \leq x_i \leq c$. Firm j's payoff at (x_i, x_j) is $U_j(x_i, x_j) = -R(x_j) < 0$. Then, firm j has an incentive to deviate from x_j by choosing $x'_j = 0$ to get the payoff $U_j(x_i, x'_j) = -R(0) = 0$. This means that (x_i, x_j) satisfying $0 < x_j \leq x_i \leq c$ cannot be an equilibrium strategy in the R&D stage.

Suppose that (x_i, x_j) satisfies $0 = x_j \leq x_i \leq c$. Then, firm i's payoff at (x_i, x_j) is $U_i(x_i, 0) = x_i D(c) - R(x_i)$, which has a unique maximizer at $x_i = \bar{x}$. Thus, any (x_i, x_j) satisfying $0 = x_j \leq x_i \leq c$ and $x_i \neq \bar{x}$ cannot be an equilibrium in the R&D stage.

Finally, we will find a condition under which $(x_i, x_j) = (\bar{x}, 0)$ is an equilibrium in the R&D stage. By the definition of $\bar{x}$, $x_i = \bar{x}$ is firm i's best response to firm j's choice of $x_j = 0$. If $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) > 0$, firm j can make its payoff positive by choosing $x'_j = \bar{\bar{x}}$ instead of $x_j = 0$. This means that $x_j = 0$ cannot be firm j's best response to $x_i = \bar{x}$. Thus, $(x_i, x_j) = (\bar{x}, 0)$ cannot be an equilibrium in the R&D stage. If $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) \leq 0$, firm j's payoff at $(\bar{x}, x_j)$ is $U_j(\bar{x}, x_j) \leq U_j(\bar{x}, \bar{\bar{x}}) < 0$ for x_j satisfying $\bar{x} \leq x_j \leq c$ and $U_j(\bar{x}, x_j) = -R(x_j) < 0$ for x_j satisfying $0 < x_j < \bar{x}$. Since firm j can earn a payoff $U_j(\bar{x}, 0) = -R(0) = 0$ by choosing $x_j = 0$, $x_j = 0$ is firm j's best response to firm i's choice of $x_i = \bar{x}$. Thus, $(x_i, x_j) = (\bar{x}, 0)$ is an equilibrium in the R&D stage. ■

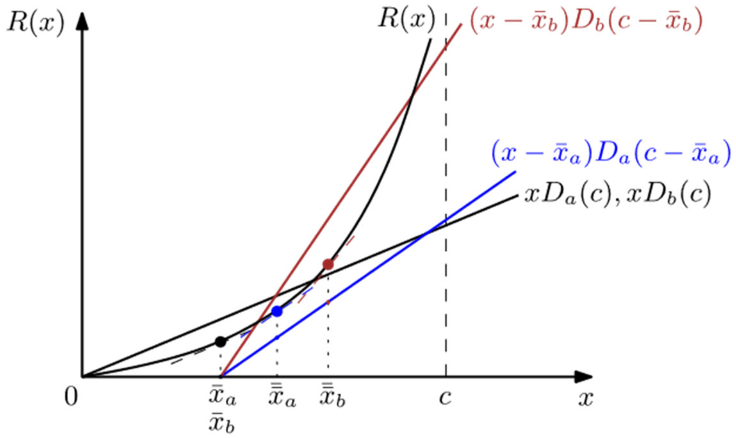
Proposition 2 says that if the firms' R&D investment appears as a pure strategy, one firm does not invest in R&D at all and the other firm invests in R&D to maximize its payoff. This implies that, even when the firms are completely homogeneous in advance, it is possible for them to make different choices on R&D and generate a difference in their costs of producing goods. This difference in their costs induces a firm making a larger R&D investment to take the upper hand in price competition and supply goods to the entire market. For intuition, consider a situation where a firm j does not make an R&D investment at all and its rival firm i makes an R&D investment optimal for itself. If firm j has to spend more on R&D investment than the profit it can earn in price competition, firm j will choose not to invest in R&D and will exit the market in the Bertrand stage. In other words, if firm j's optimal R&D investment level $\bar{\bar{x}}$ in response to its rival firm's R&D investment $\bar{x}$ is not beneficial, firm j will not change the choice of no R&D investment. This may explain why $(x_i, x_j) = (\bar{x}, 0)$ is a pure strategy equilibrium in the R&D stage when $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) \leq 0$ holds. On the other hand, if firm j can take a cost advantage in price competition and make a profit greater than its R&D investment, firm j will not maintain its choice of no R&D investment but will make an appropriate level of R&D investment to earn a positive profit. However, if firm j invests in R&D in this way, it may induces firm i to make no R&D investment. If this happens, then firm j will change its R&D investment to the level that firm i originally chooses. Continuing this reasoning for each firm's reaction to the other firm's choice on R&D, we may understand why the firms do not have a pure strategy equilibrium in the R&D stage when $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) > 0$ holds.

Proposition 2 may lead us to the conjecture that, given the market demand $D(\cdot)$, a pure strategy equilibrium in the R&D stage exists only when the R&D investment cost $R(x)$ is sufficiently large. However, this is not always true. To see this, suppose that the investment cost $R(x)$ is sufficiently low. Then, firm i's best response $x_i = \bar{x}$ to $x_j = 0$ is sufficiently high and very close to or equal to c. If firm i chooses $x_i = c$, then firm j cannot win the R&D competition, and so its best response to $x_i = c$ is to choose $x_j = 0$ and leave the market in the Bertrand competition. If firm i chooses $x_i = \bar{x}$ very close to c, then firm j could win the R&D competition by choosing $x_j = c$ as an R&D investment and could earn a positive profit in the Bertrand stage. However, such a profit for firm j in the Bertrand stage is very small because the difference in marginal costs is very small. If this happens, firm j would not make an R&D investment and would leave the market in the Bertrand competition. This explains how a pure strategy equilibrium can exist in the

R&D stage even when $R(x)$ is sufficiently low.[11]

In addition, Figure 1 illustrates how the market demand $D(\cdot)$ affects the existence of pure strategy equilibrium in the R&D stage, given the R&D investment cost $R(\cdot)$. Here, we consider two kinds of market demands: one is represented as $D_a(\cdot)$ and the other is represented as $D_b(\cdot)$. Since the increase in the demand quantity under $D_b(\cdot)$ is greater than that under $D_a(\cdot)$ when the price decreases from $p = c$ to $p' = c - \bar{x}$, we can interpret $D_b(\cdot)$ as having greater elasticity than $D_a(\cdot)$. For $k = a, b$, $xD_k(c)$ and $(x - \bar{x}_k)D_k(c - \bar{x}_k)$ are functions of x. In addition, $\bar{x}_k$ is the maximizer of $xD_k(c) - R(x)$ in $x \in [0, c]$, and $\bar{\bar{x}}_k$ is the maximizer of $(x - \bar{x}_k)D_k(\bar{x}_k) - R(x)$ in $x \in [\bar{x}_k, c]$. For the market demand $D_a(\cdot)$, $(\bar{\bar{x}}_a - \bar{x}_a)D_a(c - \bar{x}_a) - R(\bar{\bar{x}}_a) < 0$ holds and so $(x_i^*, x_j^*) = (\bar{x}, 0)$ can be an equilibrium strategy in the R&D stage. For the market demand $D_b(\cdot)$, $(\bar{\bar{x}}_b - \bar{x}_b)D_b(c - \bar{x}_b) - R(\bar{\bar{x}}_b) > 0$ holds and so the firms do not have a pure strategy equilibrium in the R&D stage.[12]

[Figure 1] Effects of $D(\cdot)$ on the pure strategy equilibrium



If the firms play a pure strategy $(x_i^*, x_j^*) = (\bar{x}, 0)$ as in Proposition 2 in the R&D stage and choose their prices (p_i^*, p_j^*) as in Proposition 1 in Bertrand stage, then the market price is determined at $p_i^* = c$ that firm i chooses in the price competition and only firm i supplies the goods in the market. In this equilibrium, firm i's profit is $\pi_i^* = \bar{x}D(c)$ and firm j's profit is $\pi_j^* = 0$. Their payoffs in the equilibrium are

[11] In Section 4, we provide an example where the market demand is given and the existence of pure strategy equilibrium depends on the R&D investment cost $R(\cdot)$.

[12] From these arguments, one may conjecture that the higher the elasticity of market demand, the more difficult it is for a pure strategy equilibrium to exist in the R&D stage. However, we were unable to verify whether this conjecture is true or not.

$$U_i^* = \bar{x}D(c) - R(\bar{x}) > 0 \text{ and } U_j^* = 0. \tag{9}$$

In an equilibrium where the firms play a pure strategy $(x_i^*, x_j^*) = (\bar{x}, 0)$ in the R&D stage, R&D is conducted by firm i that supplies the goods to the market, thereby reducing firm i's marginal cost and the social marginal cost of producing goods. This reduction of marginal cost may be interpreted as a social benefit from R&D. Although firm i pays a cost to R&D investment, it earns a positive profit and payoff in the equilibrium. Note that without R&D investment, the firms get a payoff of zero by setting the prices at their marginal cost in the price competition. Thus, firm i that receives a positive profit (or, payoff) in the equilibrium is one that gets the social benefit from R&D, while firm j that receives a payoff of zero with giving up producing goods does not get the social benefit from R&D. In addition, the market price is determined at $p^* = c$ regardless of whether R&D investment occurs. Thus, the consumers also do not get the social benefit from R&D to reduce marginal cost.

Proposition 2 provides the characteristics of a pure strategy equilibrium when it exists in the R&D stage, but does not suggest anything about the existence or characteristics of mixed strategy equilibrium. Indeed, the existence of mixed strategy equilibrium in the R&D stage can be established through the discussions of Glicksberg (1952).[13] However, to find the characteristics or the explicit form of mixed strategy equilibrium in the R&D stage, we may need more assumptions on the market demand $D(\cdot)$ and the R&D investment cost $R(\cdot)$.

IV. R&D investment under a specific environment

In this section, we consider the specific forms of $D(\cdot)$ and $R(\cdot)$ and characterize a mixed strategy equilibrium in the R&D stage. Let the market demand be given as[14]

$$Q = D(p) = 1 - p. \tag{10}$$

Let each firm i's R&D investment cost be given as, for $r > 0$,

$$R(x_i) = \frac{1}{2} r x_i^2. \tag{11}$$

Here, r represents the cost intensity of R&D investment, and we say that the R&D

[13] Glicksberg (1952) shows that a strategic form game in which the strategy sets are compact and the players' payoff functions are continuous has a mixed strategy equilibrium.

[14] Since this demand function can be obtained from a general linear demand function $D(p) = a - bp$ $(a > 0,\ b > 0)$ by taking appropriate units for quantity and price, the results in this section can be extended to general linear demand functions.

investment cost is high when r is high. The marginal cost c when a firm does not make any R&D investment is given to satisfy $0 < c < 1$.

We first apply the results in Section 3 to this setting. There are three cases depending on c and r.

Case 1: $c < \frac{1}{1+r}$

It is straightforward that $\bar{x} = c$ is the unique maximizer of $U_i(x_i, 0) = x_i(1 - c) - \frac{1}{2}rx_i^2$ with respect to $x_i \in [0, c]$. In addition, $\bar{\bar{x}} = c$ is the unique maximizer of $U_i(x_i, \bar{x}) = (x_i - c)(1 - c + c) - \frac{1}{2}rx_i^2$ with respect to $x_i \in [c, c]$. Since $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) = -\frac{1}{2}rc^2 < 0$ holds, Proposition 2 implies that $(x_i^*, x_j^*) = (c, 0)$ is the pure strategy equilibrium in the R&D stage.

Case 2: $\frac{1}{1+r} \leq c \leq \frac{1+r}{1+r+r^2}$

It is straightforward that $\bar{x} = \frac{1-c}{r}$ is the unique maximizer of $U_i(x_i, 0) = x_i(1 - c) - \frac{1}{2}rx_i^2$ with respect to $x_i \in [0, c]$. In addition, $\bar{\bar{x}} = c$ is the unique maximizer of $U_i(x_i, \bar{x}) = (x_i - \frac{1-c}{r})(1 - c + \frac{1-c}{r}) - \frac{1}{2}rx_i^2$ with respect to $x_i \in [\frac{1-c}{r}, c]$. Note that

$$(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) = \left(c - \frac{1-c}{r}\right)\left(1 - c + \frac{1-c}{r}\right) - \frac{1}{2}rc^2 \leq 0 \qquad (12)$$

if and only if either (i) $r > 1$ or (ii) $r \leq 1$ and

$$c \leq \frac{(2+r)(1+r) - r\sqrt{(1-r)(1+r)}}{r^3 + 2r^2 + 4r + 2} \qquad (13)$$

is satisfied.[15] This provides the conditions for $(x_i^*, x_j^*) = (\frac{1-c}{r}, 0)$ to be a pure strategy equilibrium in the R&D stage.

Case 3: $\frac{1+r}{1+r+r^2} < c$

It is straightforward that $\bar{x} = \frac{1-c}{r}$ is the unique maximizer of $U_i(x_i, 0) = x_i(1 -$

[15] For $r \leq 1$,

$$\frac{1}{1+r} < \frac{(2+r)(1+r) - r\sqrt{(1-r)(1+r)}}{r^3 + 2r^2 + 4r + 2} < \frac{1+r}{1+r+r^2}$$

is satisfied.

$c) - \frac{1}{2}rx_i^2$ with respect to $x_i \in [0, c]$. In addition, $\bar{\bar{x}} = \frac{(1+r)(1-c)}{r^2}$ is the unique maximizer of $U_i(x_i, \bar{x}) = (x_i - \frac{1-c}{r})(1 - c + \frac{1-c}{r}) - \frac{1}{2}rx_i^2$ with respect to $x_i \in [\frac{1-c}{r}, c]$. Note that

$$(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) \\ = \left(\frac{(1+r)(1-c)}{r^2} - \frac{1-c}{r}\right)\left(1 - c + \frac{1-c}{r}\right) - \frac{1}{2}r\left(\frac{(1+r)(1-c)}{r^2}\right)^2 \leq 0 \tag{14}$$

is satisfied if and only if $r \geq 1$. Thus, in this case, $(x_i^*, x_j^*) = (\frac{1-c}{r}, 0)$ is a pure strategy equilibrium in the R&D stage if and only if $r \geq 1$.

[Figure 2] Pure strategy equilibrium in R&D stage

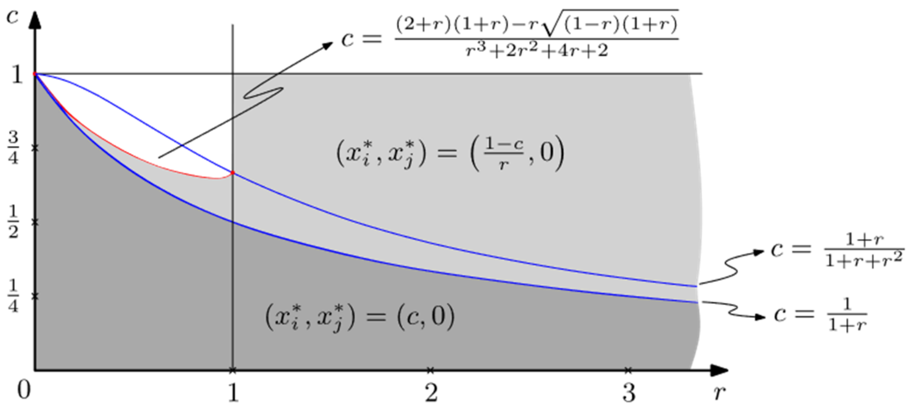


Figure 2 describes the conditions on c and r for the pure strategy equilibrium to exist in the R&D stage. If (r, c) lies in the dark gray area, the R&D stage game has a pure strategy equilibrium of $(x_i^*, x_j^*) = (c, 0)$ for some i and $j \neq i$. If (r, c) lies in the light gray area, the R&D stage game has a pure strategy equilibrium of $(x_i^*, x_j^*) = (\frac{1-c}{r}, 0)$ for some i and $j \neq i$. If (r, c) lies in the white area, the R&D stage game does not have a pure strategy equilibrium.

For intuition behind Figure 2, suppose that c is low. Then, it is less costly for firm i to choose the maximum level of R&D investment, $x_i = c$, and this can deter firm j from investing in R&D. Moreover, when firm j does not invest in R&D at all, it becomes optimal for firm i to choose the maximum investment $x_i = c$ at a low cost. As c increases, however, the cost for firm i to choose $x_i = c$ in order to deter firm j's R&D investment also increases. This implies that even when firm j does not invest in R&D at all, it may no longer be optimal for firm i to choose the maximum investment $x_i = c$ if c is high. In other words, when firm j makes no R&D investment, it may be optimal for firm i to choose $x_i = \bar{x}$ which is lower than

c. If firm i chooses such a reduced level of R&D investment, then firm j will have to invest in R&D at a level higher than x_i in order to win the R&D competition. In that case, the cost of R&D investment for firm j would exceed the profit it could obtain in the Bertrand competition, leading firm j to choose not to invest in R&D at all and leave the Bertrand competition.

We can also see that, in the equilibrium where the firms play the pure strategy $(x_i^*, x_j^*) = (c, 0)$ or $(x_i^*, x_j^*) = (\frac{1-c}{r}, 0)$ in the R&D stage, the market price is determined at $p^* = c$ and the market quantity is $Q^* = 1 - p^* = 1 - c$. In the equilibrium, firm i's and j's profits are

$$\pi_i^* = p^*Q^* - (c - x_i^*)Q^* \tag{15}$$
$$= \begin{cases} c(1-c) > 0 & \text{if } (x_i^*, x_j^*) = (c, 0) \\ \frac{(1-c)^2}{r} > 0 & \text{if } (x_i^*, x_j^*) = \left(\frac{1-c}{r}, 0\right) \end{cases}$$
$$\pi_j^* = 0, \tag{16}$$

and their payoffs are

$$U_i^* = \pi_i^* - \frac{1}{2}r(x_i^*)^2 \tag{17}$$
$$= \begin{cases} \frac{c(2-2c-cr)}{2} > 0 & \text{if } (x_i^*, x_j^*) = (c, 0) \\ \frac{(1-c)^2}{2r} > 0 & \text{if } (x_i^*, x_j^*) = \left(\frac{1-c}{r}, 0\right) \end{cases}$$
$$U_j^* = 0. \tag{18}$$

We next derive an equilibrium where the firms play mixed strategies in the R&D stage. In particular, we construct a symmetric mixed strategy equilibrium for the R&D stage, where the firms play the same strategy in the R&D stage. Suppose that the firms play a same mixed strategy F in the R&D stage, where F is a distribution of x_i on $[0, c]$. Let $\underline{s} = \inf\{x \in [0, c] : F(x) > 0\}$ and $\bar{s} = \sup\{x \in [0, c] : F(x) < 1\}$.[16] We assume that F is continuous in $[\underline{s}, \bar{s})$ and differentiable in $(\underline{s}, \bar{s})$. It will turn out that the firm's equilibrium strategy F we will construct later satisfies these assumptions. For $x \in [\underline{s}, \bar{s})$, let $f(x) = \frac{dF(x)}{dx}$.

Lemma 1 provides a necessary condition under which a mixed strategy F is an equilibrium strategy for the firms in the R&D stage.

Lemma 1 *If a mixed strategy F is an equilibrium strategy for the firms in the R&D stage, then $\underline{s} = 0$ holds.*

---

[16] That is, $\underline{s}_i$ and $\bar{s}_i$ are the infimum and the supremum of the support of F.

Proof. Suppose that $\underline{s} > 0$ holds. Since F is right continuous, there exists $\varepsilon > 0$ such that $\int_{\underline{s}}^{\underline{s}+\varepsilon} dF(x_i) > 0$ and for any $x_i \in [\underline{s}, \underline{s}+\varepsilon]$,

$$\int_{\underline{s}}^{x_i} (x_i - x_j)D(c - x_j)dF(x_j) < \frac{1}{2}r\underline{s}^2 \leq \frac{1}{2}rx_i^2. \tag{19}$$

This implies that, given that firm j plays F, firm i receives a payoff less than 0 when it chooses $x_i \in [\underline{s}, \underline{s}+\varepsilon]$. Note that, if firm i chooses $x'_i = 0$ when firm j plays F, firm i's expected payoff is 0. Thus, firm i can improve its payoff by shifting the probability that is assigned to $[\underline{s}, \underline{s}+\varepsilon]$ under F to $x'_i = 0$. This contradicts that the mixed strategy F is the best response of firm i to firm j's strategy F. ■

Given that firm j plays F in the R&D stage, if firm i chooses $x_i \in [0, \bar{s})$, firm i gets a profit of $\pi_i = (x_i - x_j)(1 - c + x_j)$ only when $x_j < x_i$ happens. In this case, firm i's expected payoff is

$$EU_i = \int_0^{x_i} (x_i - x_j)(1 - c + x_j)f(x_j)dx_j - \frac{1}{2}rx_i^2. \tag{20}$$

If $x_i \in (0, \bar{s})$ belongs to the best response to F, it has to maximize EU_i in (20) and so has to satisfy the first order necessary condition

$$\frac{dEU_i}{dx_i} = \int_0^{x_i} (1 - c + x_j)f(x_j)dx_j - rx_i = 0. \tag{21}$$

Applying the integration by parts and rearranging the equation in (21), we have

$$\begin{aligned}\int_0^{x_i} (1-c)f(x_j)dx_j + \int_0^{x_i} x_j f(x_j)dx_j - rx_i \\ = (1-c)F(x_i) + x_iF(x_i) - \int_0^{x_i} F(x_j)dx_j - rx_i = 0.\end{aligned} \tag{22}$$

Letting $\mu(x_i) = \int_0^{x_i} F(x_j)dx_j$, (22) can be rewritten as a differential equation

$$(1-c)\mu'(x_i) + x_i\mu'(x_i) - \mu(x_i) - rx_i = 0. \tag{23}$$

Solving the differential equation for μ_i, we have that for some constant K,

$$\begin{aligned}\mu(x_i) &= \int_0^{x_i} F(x_j)dx_j \\ &= r(1-c) + r(1-c+x_i)\ln(1-c+x_i) + K(1-c+x_i).\end{aligned} \tag{24}$$

Taking a differentiation on the equation (24), we have

$$\frac{d\mu(x_i)}{dx_i} = F(x_i) = r + r\ln(1 - c + x_i) + K \tag{25}$$

From Lemma 1, $\underline{s} = 0$ and $F(0) = 0$ must hold. Thus, K in (25) has to satisfy $K = -(r + r\ln(1 - c))$.

From the discussion so far, for $x_i \in (\underline{s}, \bar{s})$ to be the best response to F, F must have the form of

$$F(x_i) = r\ln\left(\frac{1-c+x_i}{1-c}\right). \tag{26}$$

In addition, since $F(\bar{s}) = 1$ and $\bar{s} \leq c$, it should be satisfied that

$$\bar{s} = \min\{(e^{1/r} - 1)(1 - c), c\}, \tag{27}$$

where $(e^{1/r} - 1)(1 - c)$ is the value of x_i satisfying $F(x_i) = 1$ for F in (26). We note that $\bar{s} = c$ if and only if

$$c \leq 1 - e^{-\frac{1}{r}}, \text{ or equivalently } r \leq -\frac{1}{\ln(1-c)}. \tag{28}$$

If R&D investment cost r is small enough to satisfy (28) with strict inequality, the distribution $F(x_i)$ in (26) has a point mass at $x_i = c$ with a probability $1 - F(c)$.

The mixed strategy F having the form in (26) is a necessary condition for F to be an equilibrium strategy for the firms in the R&D stage, and it does not imply that the mixed strategy F in (26) constitutes an equilibrium. In Proposition 3, we show that F in (26) is an equilibrium strategy in the R&D stage.[17]

Proposition 3 *A mixed strategy profile such that each firm i plays F in (26) is an equilibrium in the R&D stage.*

Proof. Suppose that $(e^{1/r} - 1)(1 - c) \leq c$ is satisfied. Then, $\bar{s} = (e^{1/r} - 1)(1 - c)$ holds. In addition, the distribution F is continuous on $[0, \bar{s}]$ and its derivative f is, for $x_i \in [0, \bar{s}]$

$$f(x_i) = \frac{dF(x_i)}{dx_i} = \frac{r}{1-c+x_i}. \tag{29}$$

[17] Note that the assumption of symmetry in r_i is crucial for both firms' playing F in (26) to constitute an equilibrium in the R&D stage. If we allow for heterogeneity in firms' R&D investment costs, a mixed strategy equilibrium in the R&D stage may still exist, but characterizing the equilibrium is not an easy task.

Plugging this into (20), we have that, for any $x_i \in [0, \bar{s}]$,

$$EU_i = \int_0^{x_i} (x_i - x_j) r dx_j - \frac{1}{2} r x_i^2 = 0, \tag{30}$$

which means that, if firm i chooses $x_i \in [0, \bar{s}]$ as an R&D investment when firm j plays F, firm i's expected payoff is $EU_i = 0$. If firm i chooses $x'_i > (e^{1/r} - 1)(1 - c)$ as an R&D investment when firm j plays F, firm i's expected payoff satisfies

$$\begin{aligned} EU_i &= \int_0^{(e^{1/r}-1)(1-c)} (x'_i - x_j) r dx_j - \frac{1}{2} r (x'_i)^2 \\ &= -\frac{1}{2} r (e^{1/r} - 1)^2 (1 - c)^2 + r (e^{1/r} - 1)(1 - c) x'_i - \frac{1}{2} r (x'_i)^2 \\ &= -\frac{1}{2} r \left((e^{1/r} - 1)(1 - c) - x'_i \right)^2 < 0, \end{aligned} \tag{31}$$

This means that when firm j plays F, $x_i \in [0, \bar{s}]$ is optimal for firm i to maximize its expected payoff.

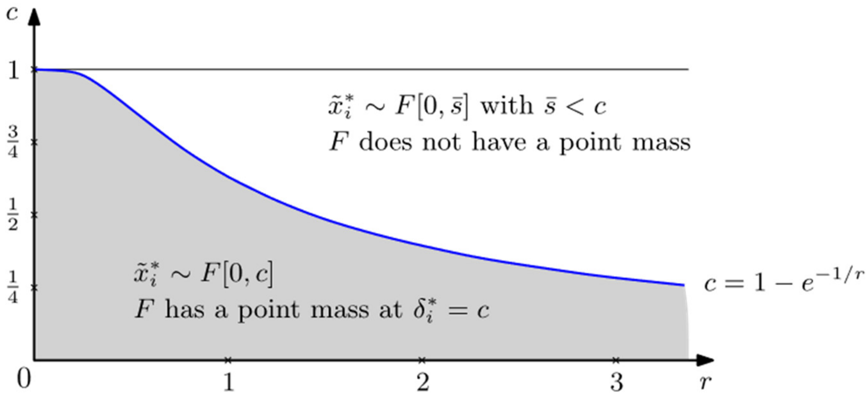
Suppose that $c < (e^{1/r} - 1)(1 - c)$ is satisfied. Then, $\bar{s} = c$ and the distribution F has a point mass at $x_i = c$ with probability $1 - F(c)$. Applying the previous arguments, we can see that if firm i chooses $x_i \in [0, c)$ as an R&D when firm j plays F, firm i's expected payoff is $EU_i = 0$. In addition, if firm i chooses $x_i = c$ when firm j chooses $x_j = c$ as an R&D investment, firm i's profit is $\pi_i(c, c) = 0$. Thus, given that firm j plays F, firm i's expected payoff is

$$EU_i = \int_0^{c} (c - x_j) r dx_j + (1 - F(c)) \pi_i(c, c) - \frac{1}{2} r c^2 = 0 \tag{32}$$

when it chooses $x_i = c$. Therefore, when firm j plays F in the R&D stage, any $x_i \in [0, c]$ is the optimal choice for firm i, which gives firm i an expected payoff of $EU_i = 0$. ■

Proposition 3 shows that in the R&D stage, there is an equilibrium in which the firms randomly choose their R&D investment in the same way. In the equilibrium, the firms choose their R&D investments along a distribution F that is continuous on $[0, c)$. This means that the probability that the firms make the same choice on R&D investment is 0, unless both of them choose $x_i = c$ for R&D. In addition, the probability that each firm i's marginal cost c_i is realized to be lower than c is 1, which implies that the market price $\tilde{p}^*$ is always determined to be lower than c in the equilibrium. In the equilibrium, it happens with a positive probability that the firms have different marginal costs to produce the goods, one firm i supplies goods to the entire market, and the market price $\tilde{p}^*$ is determined at the other firm j's marginal cost c_j. When the R&D investment cost is low enough (i.e., r is small to satisfy (28)), both firms make the maximum amount of R&D investment (i.e., $x_i = c$ for each i) with a positive probability in the equilibrium. If this happens, each firm i in the price competition has the marginal cost of $c_i = 0$ and the market price is determined at $\tilde{p}^* = 0$ in the equilibrium. When the R&D investment cost is large (i.e., r is large not to satisfy (28)), both the firms never make the maximum amount of R&D investment so that the market price $\tilde{p}^*$ is always determined to be greater than 0 in the equilibrium. Corollary 1 summarizes these observations (See also Figure 3).

[Figure 3] Mixed strategy equilibrium in R&D stage



Corollary 1 *Consider an equilibrium where each firm i plays σ_i^* that follows the distribution F in (26) in the R&D stage. Let $(\tilde{x}_i^*, \tilde{x}_j^*)$ be the realization of the R&D investment by the firms and $\tilde{p}^*$ be the realization of the market price in the equilibrium. For any $r > 0$, $\tilde{x}_i^* > 0$, $\tilde{x}_j^* > 0$ and $\tilde{p}^* < c$ happen with probability 1. If $c < 1 - e^{-1/r}$ (i.e., $r < -\frac{1}{ln(1-c)}$), then $\tilde{x}_i^* = \tilde{x}_j^* = c$ and $p^* = 0$ happen with a positive probability. If $c > 1 - e^{-1/r}$ (i.e., $r > -\frac{1}{ln(1-c)}$), then $\tilde{x}_i^* < c$ and $\tilde{x}_j^* < c$ with $\tilde{x}_i^* \neq \tilde{x}_j^*$ and $\tilde{p}^* > 0$ happen with probability 1.*

Proof. The results directly follow from Proposition 3. ■

In the equilibrium where the firms play the mixed strategy F in the R&D stage, each firm i can earn a positive (ex-post) profit in the price competition. Indeed, firm i can get a positive profit when it makes more R&D investment than its rival firm j (i.e., $x_i > x_j$ is realized). However, in terms of expectations, the positive profits earned by the firms are offset by their costs of R&D investment. Thus, in the equilibrium with the mixed strategy F in the R&D stage, each firm i's expected payoff is $EU_i = 0$.[18] In addition, since the firms reduce their marginal costs to less than c through R&D with probability 1, the market price is always determined to be lower than c. This can be interpreted as the consumers always receiving the benefit of cost reduction from the R&D investment. In this respect, an equilibrium where the firms play the mixed strategy F in the R&D stage has different features from the equilibrium where they play a pure strategy in the R&D stage.

[Figure 4] Market price p^* in the equilibrium

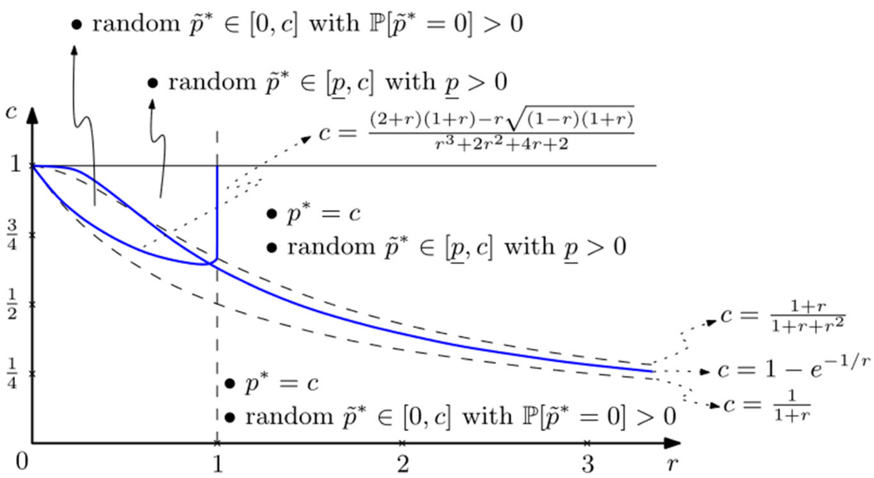


Figure 4 illustrates how the equilibrium market price p^* appears depending on c and r. Given that the initial cost of production (i.e., c) is high, if the R&D investment cost r is low enough, then there is an equilibrium where the firms play pure strategy in the R&D stage and the consumers pay the price $p^* = c$ for the goods and do not get the benefit from the R&D investment. In addition, if the R&D investment cost r is higher but less than 1, then such an equilibrium with the pure strategies in the R&D stage disappears and only the equilibrium where the firms use the mixed strategies in the R&D stage survives. In the equilibrium with the mixed strategies, both firms always make some R&D investment and the consumers pay the price $\tilde{p}^*$ lower than c and get the benefit from the R&D investment.

A conjecture may suggest that the lower the cost of R&D investment, the more the firms invest in R&D, which results in a lower market price. Accordingly, one might infer that the lower the cost of R&D investment, the greater the consumer surplus. However, our result implies that this conjecture is not always true and that how the market price is determined is unpredictable. Note that the equilibrium with the pure strategies exists when r is low enough and disappears when r is high (but less than

---

[18] This is shown in the proof of Proposition 3. See the equation (30).

1). Thus, when the cost of R&D investment is sufficiently low, the competition for R&D intensifies and induces a firm to give up on the R&D competition. As a result, a firm surviving in the R&D competition can set the highest price that can monopolize market demand. If the R&D investment cost r is high enough (greater than 1), the equilibrium in which the firms play the pure strategy in the R&D stage appears again. In this case, the market price can be set at $p^* = c$ or can be randomly determined over $[0, c]$ in equilibrium.

As long as the firms play mixed strategies in the R&D stage, the lower the cost of R&D investment, the more the firms invest in R&D. An increase in R&D investment reduces the production cost of the firms and the equilibrium market price of the goods on average. Corollary 2 represents these results.

Corollary 2 *In the equilibrium where the firms play the mixed strategy F in (26) in the R&D stage, the expectation of the R&D investment decreases in r and the expectation of market price $\tilde{p}^*$ increases in r.*

Proof. To see the effect of r on the equilibrium, we consider $F(x_i)$ in (26) as a function of r as well as x_i, and denote $F(x_i; r)$. If $r > \hat{r}$, then $F(x_i; r) \geq F(x_i; \hat{r})$ for any $x_i \in [0, c]$ with a strict inequality at some x_i. This means that $F(x_i; \hat{r})$ has first order stochastic dominance over $F(x_i; r)$ and so

$$\int_0^c x_i dF(x_i; r) < \int_0^c x_i dF(x_i; \hat{r}). \tag{33}$$

This proves that the expectation of the R&D investment decreases in r. Letting $z = \min\{x_i, x_j\}$, the (ex-post) market price $\tilde{p}^*$ in the equilibrium is determined at $\tilde{p}^* = c - z$. Given that the firms play $F(x_i; r)$ in (26) in the R&D stage, the distribution of z generated by F is

$$\begin{aligned} G(z; r) &= 2F(z; r)(1 - F(z; r)) + F(z; r)^2 \\ &= 2r\ln\left(\tfrac{1-c+z}{1-c}\right)\left(1 - r\ln\left(\tfrac{1-c+z}{1-c}\right)\right) + \left(r\ln\left(\tfrac{1-c+z}{1-c}\right)\right)^2. \end{aligned} \tag{34}$$

Note that, for any z satisfying $0 \leq z \leq (e^{1/r} - 1)(1 - c)$

$$\frac{dG(z;r)}{dr} = 2\ln\left(\tfrac{1-c+z}{1-c}\right)\left(1 - r\ln\left(\tfrac{1-c+z}{1-c}\right)\right) > 0. \tag{35}$$

This implies that for r and $\hat{r}$ satisfying $r > \hat{r}$, $G(z; \hat{r})$ has first order stochastic dominance over $G(z; r)$. Given r, in the equilibrium where each firm i plays $F(x_i, r)$ in the R&D stage, the expectation of market price $\tilde{p}^*$ is

$$\mathbb{E}[\tilde{p}^*] = \int (c-z)dG(z;r). \tag{36}$$

Since $\tilde{p}^* = c - z$ is decreasing in z, the first order stochastic dominance of $G(z;\hat{r})$ over $G(z;r)$ implies

$$\int (c-z)dG(z;r) > \int (c-z)dG(z;\hat{r}). \tag{37}$$

This proves that the expectation of market price $\tilde{p}^*$ increases in r. ■

V. Discussion

5.1 R&D spillover

One of the issues receiving a lot of attention in R&D is its spillover effect. That is, each firm i's benefits from the R&D are passed on to other firms without payments. We can reflect this feature of R&D in our model by letting each firm i's marginal cost c_i be determined by its own R&D x_i and its rival firm j's R&D x_j as

$$c_i = c - x_i - \beta x_j, \tag{38}$$

where $0 < \beta < 1$.[19] β can be interpreted as a measure of R&D spillover. To ensure that each firm i's marginal cost c_i is non-negative, we assume that for each i, $0 \le x_i \le \frac{c}{2}$ holds.

Note that $c_i < c_j$ if and only if $x_i > x_j$. Applying the same arguments as in Proposition 1, firm i can supply the goods to the entire market at price $p_i^* = c_j$ only when it invests in R&D more than its rival firm j. If firm i invests in R&D less than or equal to its rival firm j, firm i earns a profit of zero. That is, given that the firms choose (x_i, x_j) as their R&D investments in the R&D stage, firm i's profit is

$$\pi_i(x_i,x_j) = \begin{cases} (1-\beta)(x_i - x_j)D(c - x_j - \beta x_i) > 0 & \text{if } x_i > x_j \\ 0 & \text{if } x_i \le x_j, \end{cases} \tag{39}$$

and its payoff is

$$U_i(x_i,x_j) = \begin{cases} (1-\beta)(x_i - x_j)D(c - x_j - \beta x_i) - R(x_i) & \text{if } x_i > x_j \\ -R(x_i) & \text{if } x_i \le x_j. \end{cases} \tag{40}$$

[19] This specification of R&D spillover is also found in d'Aspremont and Jacquemin (1988).

Similarly to Section 3, let $\bar{x}$ be the maximizer x_i of $U_i(x_i, 0)$ on $[0, \frac{c}{2}]$, and $\bar{\bar{x}}$ be the maximizer x_i of $U_i(x_i, \bar{x})$ on $[\bar{x}, \frac{c}{2}]$. Straightforward calibrations yield that

$$\bar{x} = \min\left\{\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2}, \frac{c}{2}\right\} \tag{41}$$
$$\bar{\bar{x}} = \min\left\{\frac{(1-\beta)(1-c)(r-4\beta+3\beta^2+1)}{(r-2\beta+2\beta^2)^2}, \frac{c}{2}\right\}. \tag{42}$$

The arguments analogous to Proposition 2 lead to the conclusion that an equilibrium where the firms play the pure strategy $(x_i^*, x_j^*) = (\bar{x}, 0)$ in the R&D stage exists if and only if $U_i(\bar{\bar{x}}, \bar{x}) \le 0$.

Let the market demand $D(\cdot)$ and the R&D investment cost $R(\cdot)$ be given as in Section 4. Under this specification of model, straightforward (but tedious and messy) calculations yield the conditions for the existence of an equilibrium where the firms play pure strategies in the R&D stage in an equilibrium. Indeed, one can show that there is an equilibrium where the firms play a pure strategy (x_i^*, x_j^*) in the R&D stage depending on the following conditions:

(i) if $r \le \rho_1(\beta)$, then $(x_i^*, x_j^*) = \left(\frac{c}{2}, 0\right)$,
(ii) if $\rho_1(\beta) < r < \rho_2(\beta)$ and $c \le \psi_1(r, \beta)$, then $(x_i^*, x_j^*) = \left(\frac{c}{2}, 0\right)$,
(iii) if $\rho_1(\beta) < r < \rho_2(\beta)$ and $\psi_1(r, \beta) < c \le \psi_2(r, \beta)$, then $(x_i^*, x_j^*) = \left(\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2}, 0\right)$,
(iv) if $\rho_1(\beta) < r < \rho_2(\beta)$ and $\psi_2(r, \beta) < c$, then any pure strategy in the R&D stage cannot survive in an equilibrium,
(v) if $\rho_2(\beta) \le r$ and $c \le \psi_1(r, \beta)$, then $(x_i^*, x_j^*) = \left(\frac{c}{2}, 0\right)$,
(vi) if $\rho_2(\beta) \le r$ and $\psi_1(r, \beta) < c$, then $(x_i^*, x_j^*) = \left(\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2}, 0\right)$,

where $\rho_1(\beta)$ and $\rho_2(\beta)$ are thresholds of r that depend on β and $\psi_1(r, \beta)$ and $\psi_2(r, \beta)$ are thresholds of c that depend on r and β. The explicit forms of $\rho_1(\beta)$, $\rho_2(\beta)$, $\psi_1(r, \beta)$, and $\psi_2(r, \beta)$ are found in the Appendix.

Figure 5 describes how the pure strategy (x_i^*, x_j^*) is determined in an equilibrium when $\beta = \frac{1}{5}$. If (r, c) is in the light gray area, $(x_i^*, x_j^*) = \left(\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2}, 0\right)$ can survive as an equilibrium. If (r, c) is in the dark gray area, $(x_i^*, x_j^*) = \left(\frac{c}{2}, 0\right)$ can survive as an equilibrium. The qualitative features explained for Figure 2 in Section 4 are preserved even when we allow the R&D spillover.

[Figure 5] Pure strategy equilibrium in the R&D stage with spillover ($\beta = \frac{1}{5}$)

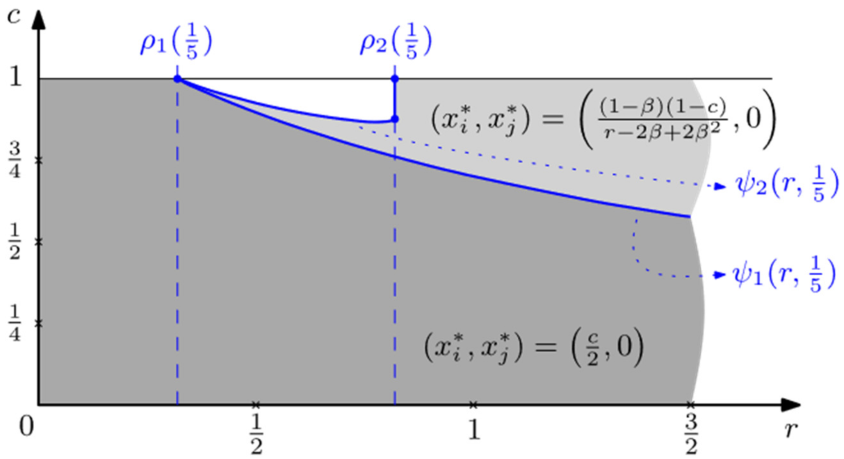


Figure 6 describes how a change in β affects the equilibrium strategy (x_i^*, x_j^*) in the R&D stage.[20] As β increases from 0 to 1, $\rho_1(\beta)$ initially increases and then decreases to 0 while $\rho_2(\delta)$ decreases to 0. In addition, as β increases from 0 to 1, $\psi_2(r,\beta)$ which is defined for $r \leq \rho_2(\beta)$ gets close to 1. Thus, when the R&D spillover effect is small (i.e., β is small), there is a tendency that the set of (r, c) ensuring the existence of equilibrium with the pure strategy in the R&D stage expands as β increases. However, this is not always true when the R&D spillover effect is great (i.e., β is large).[21] In addition, we can show that the set of (r, c) for which no pure strategy in the R&D stage can survive as an equilibrium converges to the point $(r, c) = (0,1)$ as β increases to 1.[22] In other words, if the R&D spillover effect is sufficiently large so that a firm can benefit from its rival firm's R&D achievement almost entirely, there is always an equilibrium where one firm i makes an R&D investment and the other firm j benefits from firm i's R&D achievement. In this equilibrium, for almost every (r, c), firm i's R&D investment is $x_i^* = \frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2} < \frac{c}{2}$, which is negligibly small when β is almost closed to 1. This observation coincides with the general perception that the positive externality of the R&D spillover has a negative effect on the firm's incentives to invest in R&D. However, this perception may not be true when the R&D spillover effect β is low. For example,

---

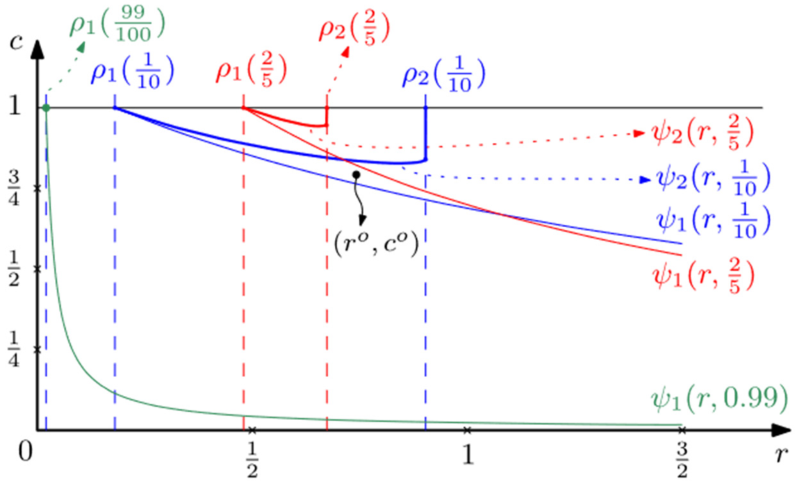
[20] $\rho_1(\frac{99}{100})$ and $\rho_2(\frac{99}{100})$ are distinct but too close to be distinguished in the figure. In addition, $\psi_2(r, \frac{99}{100})$ is defined for $r \leq \rho_2(\beta)$ and is so close to $r = 1$ that it cannot be represented in the figure.

[21] To speak more formally, given β, let $B(\beta)$ be the set of (r, c) for which there exists an equilibrium such that the firms play a pure strategy in the R&D stage. If $\beta < \beta'$ for small β and β', then $B(\beta) \subset B(\beta')$. In addition, for large β and β' with $\beta < \beta'$, $B(\beta) \not\subset B(\beta')$ is possible.

[22] More rigorously, as β converges to 1, $\rho_1(\beta)$ and $\rho_2(\beta)$ converge to 0 and $\psi(r,\beta)$, which is defined for $r \leq \rho_2(\beta)$, converges to 1 for any r with $0 < r \leq \rho_2(\beta)$.

if (r, c) is given at (r^o, c^o) as in Figure 6, firm i's R&D investment is $x_i^* = \frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2} < \frac{c}{2}$ when $\beta = \frac{1}{10}$ and $x_i^* = \frac{c}{2}$ when $\beta = \frac{2}{5}$. This means that firm i makes more R&D investment when the R&D spillover effect is high than when the R&D spillover effect is low.

[Figure 6] Effect of β on the R&D investment



When the R&D investment has spillover effects, we can also find an equilibrium where the firms play a mixed strategy in the R&D stage. The same methods used in Section 4 for the firm's payoff in (40) lead us to the conclusion that the mixed strategy F having the form

$$F(x;\beta) = \frac{r}{2\beta(1-\beta)}\left(1 - \left(\frac{1-c}{(1-c)+(1+\beta)x}\right)^{\frac{2\beta}{1+\beta}}\right) \tag{43}$$

for each firm i constitutes an equilibrium. The support of $F(x;\beta)$ is $[0, \bar{s}]$, where $\bar{s} = \min\{\hat{x}, \frac{c}{2}\}$ for $\hat{x}$ satisfying $F(\hat{x};\beta) = 1$. We can see that the qualitative results (i.e., Corollaries 1 and 2) in Section 4 are also preserved in the presence of R&D spillover. For example, if c is sufficiently low, there is a positive probability that the firms make a maximum amount of R&D investment (i.e., $\tilde{x}_i^* = \tilde{x}_j^* = \frac{c}{2}$) in the equilibrium. In addition, the higher the R&D investment cost r, the lower the R&D investment of firms and the higher the market price on average.

[Figure 7] $F(x;\beta)$ with $r=\frac{1}{2}$ and $c=\frac{4}{5}$

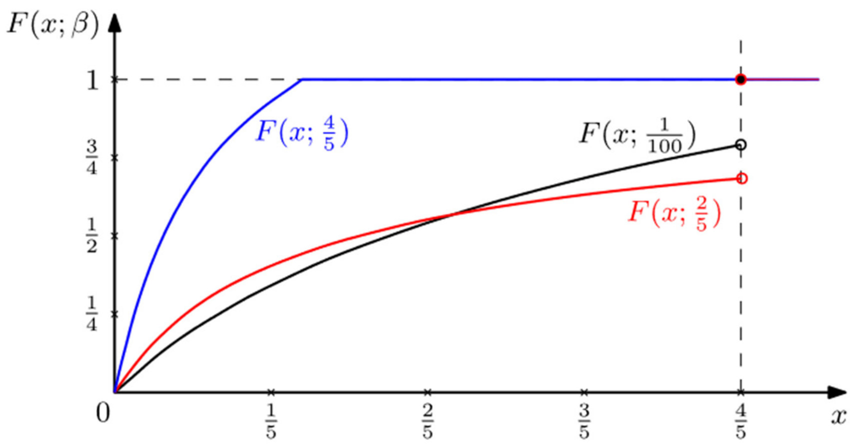


Figure 7 illustrates how the mixed strategy $F(x;\beta)$ varies depending on β, given that $r=\frac{1}{2}$ and $c=\frac{4}{5}$. Here, we can see that $F(x;\beta)$ does not change monotonically as β increases. In particular, when β is low, $F(x,\beta)$ may increase in an interval of x and decrease in the other interval of x as β increases. This also implies that the general belief that the R&D spillover effect negatively affects the incentives of firms to invest in R&D is not always true.[23] However, notice that, for any $x \in (0,c)$, $F(x,\beta)$ in (43) goes to infinity as β converges to 1 (i.e., $\lim_{\beta\to 1} F(x,\beta)=\infty$). This implies that, when the R&D spillover effect is large enough (i.e., β is sufficiently high), the belief holds true. In addition, when the R&D spillover effect is extremely large (i.e., β is close to 1), the firms make negligibly small R&D investments, which induces the market price $\tilde{p}^*$ to be approximately equal to c.

5.2 Number of firms

The relationship between market competitiveness and R&D investment is also an issue that people are interested in. One view, suggested by Schumpeter (1943), is that technological innovation, such as R&D, can be better motivated in the firms that earn high surplus under weak competitive pressure. On the other hand, others such as Arrow (1972) suggest that the firms under high competitive pressure are more actively pursuing innovation such as R&D to survive in the competition.

[23] Numerically, we can also see that for $F(x,\beta)$ in (43) with $r=\frac{1}{2}$ and $c=\frac{4}{5}$, $\int xdF(x,\frac{1}{100}) \approx 0.39768$, $\int xdF(x,\frac{2}{5}) \approx 0.39899$, and $\int xdF(x,\frac{4}{5}) \approx 0.21895$. This means that the expectation of firm i's R&D investment increases when β increases from $\frac{1}{100}$ to $\frac{2}{5}$.

To find a partial answer to this issue, we consider a market where more than two firms make a decision on R&D investment and compete in prices. Let $N = \{1, \dots, n\}$ with $n \geq 2$ be the set of firms. Except for the number of firms, other settings are the same as in Section 4.

To characterize the equilibria in this setting, we first consider Bertrand stage where the firms choose their prices given that each firm i's marginal cost is c_i. For convenience, let $c_{min} = \min\{c_1, \dots, c_n\}$ be the minimum of the c_i's, and let $N_{min} = \{i \in N : c_i = c_{min}\}$ be the set of firms that have the smallest marginal cost. There are two cases: (i) the number of firms in N_{min} is greater than 1 (i.e., $|N_{min}| > 1$), and (ii) the number of firms in N_{min} is 1 (i.e., $|N_{min}| = 1$).

In the case of $|N_{min}| > 1$, any equilibrium $(p_1^*, \dots, p_n^*)$ of Bertrand stage has to satisfy that for each firm k, $p_k^* \geq c_{min}$ (if p_k^* is a mixed strategy, p_k^* assigns the probability 1 for a set of prices $p_k \geq c_{min}$), and that there are at least two firms i and j in N_{min} such that $p_i^* = p_j^* = c_{min}$. For intuition, if $p_i^* < c_{min}$ for some i, there is a firm k that wins Bertrand competition with $p_k^* < c_k$ but gets a negative profit. Then, firm k has an incentive to choose $p_k = c_k$ to make its payoff 0. In addition, if $p_i^* = c_{min}$ and $p_j^* > c_{min}$ for all firm j except firm i, then firm i can improve its profit by slightly increasing its price.[24] We also note that the strategy profile $(p_1^*, \dots, p_n^*)$ with $p_i^* = c_i^*$ for each i is an equilibrium in Bertrand stage. If $n \geq 3$, this is not the unique equilibrium and there are infinitely many equilibria in Bertrand stage. However, in any equilibrium, each firm i gets a profit of zero.

For the case of $|N_{min}| = 1$, let $c_{2nd} = \min\{c_i : i \notin N_{min}\}$ be the second lowest marginal cost of the firms. In this case, we also restrict our attention to the equilibria where the firms do not play the dominated strategies in Bertrand stage as discussed in Section 3. This restriction implies that each firm i chooses prices $p_j \geq c_j$ with probability 1 in any equilibrium $(p_1^*, \dots, p_n^*)$. Then, we can see that any equilibrium $(p_1^*, \dots, p_n^*)$ satisfies that any firm j with $c_j = c_{2nd}$ does not choose $p_j = c_j$ with a positive probability. For intuition, note that if there is a firm j with $c_j = c_{2nd}$ that chooses $p_j = c_j$ with a positive probability, firm i with $c_i = c_{min}$ cannot maximize its profit although it can earn a positive (expected) profit by choosing a price p_i satisfying $c_i < p_i < c_{2nd}$. By a similar argument, switching the roles of firms i and j, we can also see that firm i never chooses prices $p_i > c_{2nd}$ with a positive probability in any equilibrium. In addition, given that each firm j with $c_j > c_{min}$ chooses prices $p_j \geq c_j$ with a probability 1, firm i with $c_i = c_{\min}$ will not choose prices $p_i < c_{min}$ with a positive probability because it can improve its profit by slightly raising its price from p_i. This, in any equilibrium

[24] Formal characterization of equilibria is straightforward but contains tedious arguments for a variety of situations with allowing mixed strategies. Thus, we provide here the intuitions for the characteristics of equilibria and leave the rigorous characterization of equilibria to the readers.

$(p_1^*, \dots, p_n^*)$, firm i with $c_i = c_{min}$ has to choose $p_i = c_{2nd}$. Note that a strategy profile $(p_1^*, \dots, p_n^*)$ such that firm i with $c_i = c_{min}$ chooses $p_i^* = c_{2nd}$, the firms j with $c_j \neq c_{min}$ randomly choose their prices p_j^* following a uniform distribution on $[c_j, c_j + \eta]$ for sufficiently small η is such an equilibrium for Bertrand stage.[25] In this equilibrium, firm i earns a profit of $(c_{min} - c_i)D(c_{min})$ and the other firms receive a profit of zero. Of course, given that the firms do not play dominated strategies in Bertrand stage, this is not the unique equilibrium strategy in Bertrand stage. However, if the firms do not play dominated strategies in Bertrand stage, the firms receive the same profits in any equilibrium.

Since $c_i < c_j$ if and only if $x_i > x_j$ happens in the R&D stage, the above observations imply that when the firms choose $(x_1, \dots, x_n)$ as their R&D investments in the R&D stage, each firm i's profit is

$$\pi_i(x_1, \dots, x_n) = \begin{cases} (x_i - x_{2nd})D(c - x_{2nd}) > 0 & \text{if } x_i > x_j \text{ for all } j \neq i \\ 0 & \text{otherwise} \end{cases} \tag{44}$$

and its payoff is

$$U_i(x_1, \dots, x_n) = \begin{cases} (x_i - x_{2nd})D(c - x_{2nd}) - R(x_i) & \text{if } x_i > x_j \text{for all } j \neq i \\ -R(x_i) & \text{otherwise,} \end{cases} \tag{45}$$

where $x_{2nd} = \max\{x_j : j \neq i\}$.

We next discuss how the firms make a decision on their R&D investments given that the firms' payoffs are determined as in (45). We define $\bar{x}$ and $\bar{\bar{x}}$ as in Section 3. The condition for a pure strategy in the R&D stage to constitute an equilibrium, as stated in Proposition 2, also applies to markets with more than two firms. To understand this, consider two firms, i and j, that choose x_i^* and x_j^* with $0 < x_i^* \leq x_j^*$. Firm i, receiving a payoff of 0, has an incentive to deviate from x_i^* by choosing $x_i = 0$. Thus, there should be at most one firm i that chooses $x_i^* > 0$ in any equilibrium. Given that all other firms j choose $x_j^* = 0$, firm i's optimal choice is $x_i^* = \bar{x}$. A slight modification of the proof of Proposition 2 establishes that if $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) > 0$, there is no pure strategy equilibrium in the R&D

[25] This can be verified through a small modification of the arguments in Section 3. For example, when firm i with $c_i = c_{min}$ chooses $p_i \in [c_{2nd}, c_{2nd} + \eta]$, its expected payoff is

$$\pi_i = (p_i - c_i)D(p_i)\left(1 - \left(\frac{p_i - c_{2nd}}{\eta}\right)^{|N_{2nd}|}\right),$$

where $N_{2nd} = \{j : c_j = c_{2nd}\}$ is the set of firms j with $c_j = c_{2nd}$. Differentiating π_i with respect to p_i, we can confirm that $\frac{d\pi}{dp_i} < 0$ for sufficiently small η. This implies that firm i will not deviate from $p_i^* = c_{min}$ by choosing a price p_i with $c_{min} < p_i \leq c_{2nd}$.

stage. On the other hand, if $(\bar{\bar{x}} - \bar{x})D(c - \bar{x}) - R(\bar{\bar{x}}) \leq 0$, there is a pure strategy equilibrium $(x_1^*, \ldots, x_n^*)$ in the R&D stage and $(x_1^*, \ldots, x_n^*)$ satisfies that $x_i^* = \bar{x}$ for some i and $x_j^* = 0$ for all $j \neq i$. Note that in an equilibrium where the firms play a pure strategy in the R&D stage, the market price is determined at $p^* = c$. This implies that, as long as the firms play a pure strategy in the R&D stage, the number of firms does not affect the market equilibrium and the consumer surplus.

For the mixed strategy equilibrium in the R&D stage, let the market demand $D(\cdot)$ and the R&D investment cost $R(\cdot)$ be given as in Section 4. Suppose that a distribution F_n on $[0, c]$ represents a mixed strategy in the R&D stage that constitutes an equilibrium. For firm i, let $z_{-i} = \max\{x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n\}$. That is, z_{-i} is the largest R&D investment among the firms $j \neq i$. If the firms $j \neq i$ play F_n in the R&D stage, then z_{-i} follows the distribution

$$H(z_{-i}) = F_n(z_{-i})^{n-1}. \tag{46}$$

Putting appropriate assumptions on F_n, we can obtain that F_n and H have the support $[0, \bar{s}]$ for some $\bar{s} \in [0, c]$, and F_n and H have the derivatives f_n and h, respectively, on $(0, \bar{s})$.[26] Given that the other firms play F_n in the R&D stage, firm i earns a profit $\pi_i = (x_i - z_{-i})(1 - c + z_{-i})$ only when $z_{-i} < x_i$ happens. Thus, firm i's expected payoff when it chooses $x_i \in [0, \bar{s})$ is

$$EU_i = \int_0^{x_i} (x_i - z)(1 - c + z)h(z)dz - \frac{1}{2}rx_i^2, \tag{47}$$

where $h(z) = (n-1)F_n(z)^{n-2}f_n(z)$. Then, the same arguments applied to (20) imply that H has the form of

$$H(x_i) = F_n(x_i)^{n-1} = r\ln\left(\frac{1-c+x_i}{1-c}\right), \tag{48}$$

or equivalently, F_n has the form of

$$F_n(x_i) = \left(r\ln\left(\frac{1-c+x_i}{1-c}\right)\right)^{\frac{1}{n-1}} \tag{49}$$

for $x_i \in [0, \bar{s}]$, where $\bar{s} \leq c$ is taken appropriately. Analogously to Proposition 3, we can verify that $F_n(x_i)$ in (49) is an equilibrium strategy for the firms in the R&D stage.

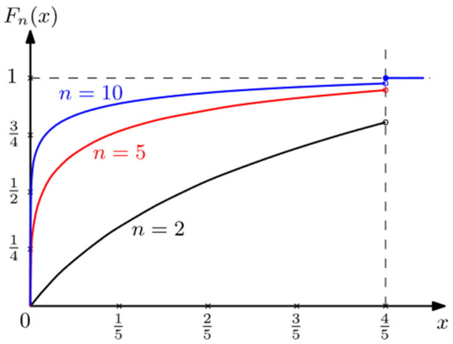
It is easy to verify that the qualitative results we obtain for the 2-firm market still hold even in markets with more than two firms. For example, under the equilibrium

[26] Similar arguments as in Section 4 can be applied to obtain this.

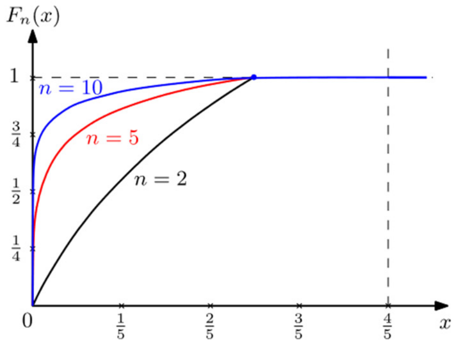
with a symmetric mixed strategy in the R&D stage, if the initial production cost c is low enough (or, the R&D investment cost r is low enough), $\tilde{x}_i^* = c$ for all i happens with a positive probability, and if c is high enough (or r is high enough), there is a positive probability that $\tilde{x}_i^* < c$ for all i and $\tilde{x}_i^* \neq \tilde{x}_j^*$ for any firms i and j (Corollary 1). In addition, given that the firms play the mixed strategy F_n in the R&D stage, the expectation of market price $\tilde{p}^*$ increases in r (Corollary 2).

To see how the number of firms affects the equilibrium, let F_n and $\hat{F}_n$ be the distributions defined as in (49) for n and $\hat{n}$ with $\hat{n} > n$. Note that for any $x_i \in [0, \bar{s})$, $F_n(x_i)$ in (49) increases and converges to 1 (i.e., $\lim_{n\to\infty} F_n(x_i) = 1$). This implies that F_n has first order stochastic dominance over $\hat{F}_n$ and so $\int x_i dF_n > \int x_i d\hat{F}_n$. In other words, each firm i invests more in R&D when competing with a small number of firms than when competing with a large number of firms. Figure 8 illustrates $F_n(x)$ for a variety of the number of firms given $r = \frac{1}{2}$ or $r = \frac{4}{5}$ and $c = \frac{4}{5}$.

[Figure 8] Effects of n on $F_n(x)$



(a) $r = \frac{1}{2}$ and $c = \frac{4}{5}$



(b) $r = \frac{4}{5}$ and $c = \frac{4}{5}$

The number of competing firms also affects the market price in the equilibrium. Note that the equilibrium market price $\tilde{p}^*$ is determined at the second highest marginal cost $c_i = c - x_i$, where x_i is the second highest R&D investment of the firms. Let $\hat{G}(z; n)$ be distribution of the second highest R&D investment z when n firms compete in the market and they play F_n in (49) in the R&D stage. That is,

$$\begin{aligned} \hat{G}(z; n) &= nF_n(z)^{n-1}(1 - F_n(z)) + F_n(z)^n \\ &= nF(z)\left(1 - F(z)^{\frac{1}{n-1}}\right) + F(z)^{\frac{n}{n-1}}, \end{aligned} \tag{50}$$

where $F(z) = r\ln\left(\frac{1-c+z}{1-c}\right)$. Note that, for any $z \in [0, \bar{s})$,

$$\frac{d\hat{G}(z;n)}{dn} = \left(F(z) - F(z)^{\frac{n}{n-1}}\right) + \frac{1}{n-1}F(z)^{\frac{n}{n-1}}\ln(F(z)) \tag{51}$$

satisfies

$$\lim_{n\to\infty} \frac{d\hat{G}(z;n)}{dn} = 0 \tag{52}$$

and

$$\frac{d^2\hat{G}(z;n)}{dn^2} = -\frac{1}{(n-1)^3}F(z)^{\frac{n}{n-1}}\ln^2(F(z)) < 0. \tag{53}$$

(52) and (53) together imply that, for any $z \in [0, \bar{s})$, $\frac{d\hat{G}(z;n)}{dn} > 0$ for all n. This implies that, for n and $\hat{n}$ with $\hat{n} > n$, $\hat{G}(z;n)$ has first order stochastic dominance over $\hat{G}(z;\hat{n})$, which implies

$$\int (c-z)d\hat{G}(z;n) < \int (c-z)d\hat{G}(z;\hat{n}), \tag{54}$$

where $\int (c-z)d\hat{G}(z;n)$ is the expectation of equilibrium market price $\tilde{p}^* = c - z$ when n firms compete in the market and play the mixed strategy F_n in the R&D stage.

To sum up, given that the firms randomly decide on their R&D investments, the more firms compete in the market, the less the firms invest in R&D and the higher the market price on average. Intuitively, given that the firms decide in the same way on the R&D investments, it becomes less likely that a firm wins in the R&D race when it competes with more firms in the market. This implies that the R&D investment is less attractive to the firms that compete with more firms in the market so they are more passive in investing in R&D. Thus, the firms competing with more firms have higher marginal costs than those competing with fewer firms in the market. In addition, the market price under Bertrand competition is determined at the second lowest marginal cost of the firms. Due to the firms' strategic behavior in investing in R&D, the more firms compete in the market, the higher the second lower marginal cost of the firms and the higher the market price.[27] Thus, if the firms can choose their marginal costs through R&D before competing in prices, consumers can be worse off in a market where firms face more competitive pressure.

The discussion in this section is in line with Schumpeter (1943)'s view that highly competitive markets are not necessary to promote innovation. Schumpeter (1943)

[27] It is worth mentioning that if the marginal costs of the firms are independently drawn from a distribution rather than determined by their strategic behavior, the second lowest marginal cost and the equilibrium market price may increase as the number of firms increases.

points out that the surplus a firm can get in a less competitive market can be a driving force for innovation. On the other hand, we suggest that firms are more active in R&D in the less competitive market because they perceive that they are more likely to be a winner in the R&D race.[28]

5.3 Cournot vs Bertrand

Another way for the firms to compete in the market is Cournot competition, where the firms simultaneously choose their quantities. It might be interesting to compare the firms' R&D investments between Bertrand and Cournot competitions. To this end, we consider the specification of the model in Section 4, where the market demand $D(\cdot)$ and the R&D investment cost $R(\cdot)$ are given as in (10) and (11), respectively. The decision procedure consists of two stages. In the first stage (the *R&D stage*), each firm i simultaneously chooses its R&D investment $x_i \in [0, c]$ to make its marginal cost $c_i = c - x_i$. In the second stage (the *Cournot stage*), each firm i simultaneously chooses its output q_i after observing their R&D investments (x_i, x_j) in the R&D stage. If the firms choose their outputs (q_i, q_j), the market price p is determined on the demand. That is, $p = 1 - (q_i + q_j)$. The equilibrium can be found through the backward induction.

Consider Cournot stage where the firms observe (x_i, x_j) chosen in the R&D stage. If the firms choose their output (q_i, q_j), firm i's profit is

$$\pi_i(q_i, q_j) = \big(1 - (q_i + q_j)\big)q_i - (c - x_i)q_i. \tag{55}$$

A Nash equilibrium (q_i, q_j) can be obtained through a standard method as

$$(q_i, q_j) = \begin{cases} \left(0, \frac{1-c+x_j}{2}\right) & \text{if } x_i \leq \frac{x_j-(1-c)}{2} \\ \left(\frac{1-c+2x_i-x_j}{3}, \frac{1-c+2x_j-x_i}{3}\right) & \text{if } \frac{x_j-(1-c)}{2} < x_i < 1-c+2x_j \\ \left(\frac{1-c+x_i}{2}, 0\right) & \text{if } x_i \geq 1-c+2x_j, \end{cases} \tag{56}$$

which constitutes a subgame perfect equilibrium of the entire game.

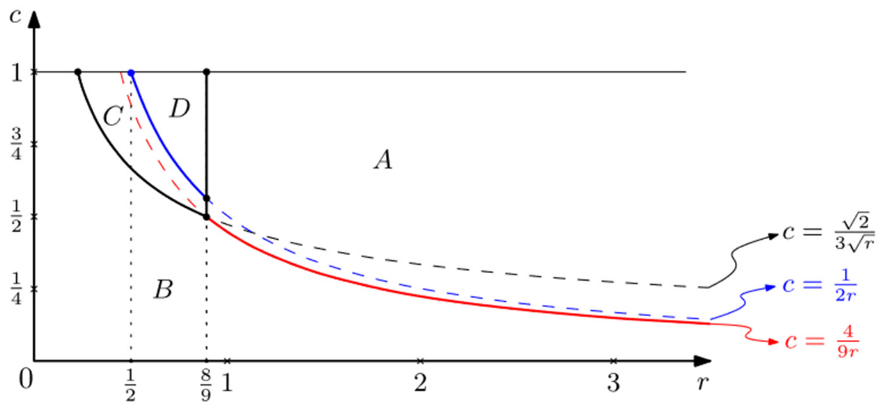
Given that the firms choose their outputs (q_i, q_j) as in (56), when the firms choose their R&D investments (x_i, x_j) in the R&D stage, firm i's payoff is determined as

[28] Cellini and Lambertini (2011) analyze R&D investment of firms that produce differentiated goods and are engaged in Bertrand competition, and also show that individual firms' R&D investment decreases as the number of firms increases.

$$U_i(x_i, x_j) = \begin{cases} -\frac{1}{2}rx_i^2 & \text{if } x_i \le \frac{x_j-(1-c)}{2} \\ \frac{(1-c+2x_i-x_j)^2}{9} - \frac{1}{2}rx_i^2 & \text{if } \frac{x_j-(1-c)}{2} < x_i < 1-c+2x_j \\ \frac{(1-c+x_i)^2}{4} - \frac{1}{2}rx_i^2 & \text{if } 1-c+2x_j \le x_i. \end{cases} \quad (57)$$

Note that, given x_j, if $r < \frac{8}{9}$, $U_i(x_i, x_j)$ is not concave at x_i satisfying $\frac{x_j-(1-c)}{2} < x_i < 1-c+2x_j$, and if $r < \frac{1}{2}$, $U_i(x_i, x_j)$ is not concave at x_i satisfying $1-c+2x_j \le x_i$. The non-concavity of the firm's payoff and the possibility of corner solutions cause complexity in finding the equilibrium for all $r > 0$ and c with $0 < c < 1$.

[Figure 9] Equilibrium under Cournot competition



Area	(x_i^C, x_j^C)	(q_i^C, q_j^C)	p^C
A	$\left(\frac{4(1-c)}{9r-4}, \frac{4(1-c)}{9r-4}\right)$	$\left(\frac{3r(1-c)}{9r-4}, \frac{3r(1-c)}{9r-4}\right)$	$\frac{6cr-4+3r}{9r-4}$
B	(c, c)	$\left(\frac{1}{3}, \frac{1}{3}\right)$	$\frac{1}{3}$
C	$(c, 0)$	$\left(\frac{1}{2}, 0\right)$	$\frac{1}{2}$
D	$\left(\frac{1-c}{2r-1}, 0\right)$	$\left(\frac{r(1-c)}{2r-1}, 0\right)$	$\frac{cr-1+r}{2r-1}$

Through straightforward but tedious calibrations, we can derive the equilibrium outcomes depending on (r, c) as shown in Figure 9. If (r, c) belongs to area A (i.e., $r \ge \frac{8}{9}$ and $c \ge \frac{4}{9r}$), each firm i chooses $x_i^C = \frac{4(1-c)}{9r-4}$ in the R&D stage and $q_i^C = \frac{3r(1-c)}{9r-4}$ in Cournot stage, which are obtained as interior solutions. In this equilibrium, the market price is $p^C = \frac{6cr-4+3r}{9r-4}$. If (r, c) belongs to area B (i.e.,

$c \leq \frac{\sqrt{2}}{3\sqrt{r}}$ and $c \leq \frac{4}{9r}$), each firm i's choice $x_i^C = c$ in the R&D stage is obtained as a corner solution, its choice in Cournot stage is $q_i^C = \frac{1}{3}$, and the market price is determined at $p^C = \frac{1}{3}$. If (r, c) belongs to area C (i.e., $r \leq \frac{8}{9}$ and $\frac{\sqrt{2}}{3\sqrt{r}} < c < \frac{1}{2r}$), a firm i makes an R&D investment by choosing $x_i^C = c$ and supplies goods in the market with $q_i^C = \frac{1}{2}$ at price $p^C = \frac{1}{2}$, and the other firm j does not invest in R&D and gives up supply of goods to the market. If (r, c) belongs to area D (i.e., $r \leq \frac{8}{9}$ and $c \geq \frac{1}{2r}$), a firm i makes an R&D investment by choosing $x_i^C = \frac{1-c}{2r-1}$ and supplies goods in the market with $q_i^C = \frac{r(1-c)}{2r-1}$ at price $p^C = \frac{cr-1+r}{2r-1}$, and the other firm j does not invest in R&D and gives up supply of goods to the market.

We numerically compare the equilibria under Cournot and Bertrand competitions, where the firms behave as in Section 4. Figure 10 illustrates the comparison of the firms' R&D investments under Cournot and Bertrand competition, where the firms play pure strategies in the R&D stage.[29] This comparison of R&D investment directly follows from Figure 2 and Figure 9. The gray area in Figure 10 represents (r, c) for which the Bertrand competition does not have a pure strategy equilibrium in the R&D stage. Figure 11 illustrates the comparison of the firms' R&D investments under Cournot and Bertrand competition, where the firms play mixed strategies as in Section 4. Here, $\mathbb{E}[\tilde{x}_i^B]$ is the expectation of each firm i's R&D investment under Bertrand competition (i.e., $\mathbb{E}[\tilde{x}_i^B] = \int x_i dF(x_i)$), given that the firms play the mixed strategy F in (26) in the R&D stage.[30] In these figures, we observe a tendency that the firms invest in R&D more under Bertrand competition than under Cournot competition when the R&D investment cost r is high.[31] It is intuitively clear that an increase in the R&D investment cost r has a negative effect on the incentive for the firms to invest in R&D. Under Bertrand competition, a decrease in the rival firm j's R&D investment x_j caused by a higher r induces a higher price of the goods in the market and so a higher profit for firm i when it wins the R&D race. This may induce firm i to invest more in R&D to win in the R&D race, which could partially offset the negative effect of an increase in r on firm i's incentive to invest in R&D.

---

[29] In this section, the superscripts 'B' and 'C' represent Bertrand and Cournot, respectively.

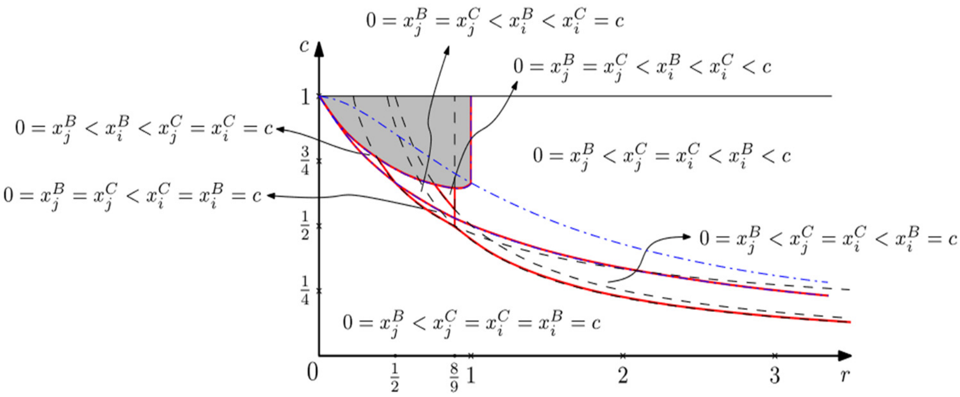
[30] We could not obtain the closed forms of the thresholds of $\mathbb{E}[\tilde{x}_i^B] = x_i^C$. The thresholds of $\mathbb{E}[\tilde{x}_i^B] = x_i^C$ consist of trajectories of (r, c) that satisfies the following:

$$\int_0^{(e^{1/r}-1)(1-c)} \left(\frac{xr}{(1-c)+x}\right) dx = \frac{4(1-c)}{9r-4} \left(\text{orequivalently}, \mathrm{e}^{1/\mathrm{r}} = \frac{9\mathrm{r}+5}{9\mathrm{r}-4}\right)$$

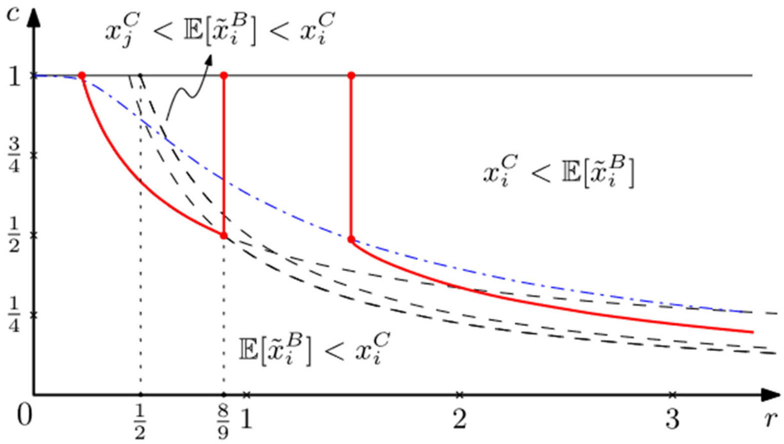
$$\int_0^c \left(\frac{xr}{1-c+x}\right) dx + c\left(1 - r\ln\left(\frac{1}{1-c}\right)\right) = \frac{4(1-c)}{9r-4}$$

[31] We also observe a tendency for the firms to invest in R&D more under Bertrand competition than under Cournot competition when the initial marginal cost c of production is high. Remind that, since the firms' profits depend on their reduced marginal cost $c_i = c - x_i$ by R&D investment, a higher initial marginal cost c can be interpreted as a higher R&D investment cost to achieve, given r in our model.

[Figure 10] R&D Investment: Cournot vs Bertrand(Non-random)

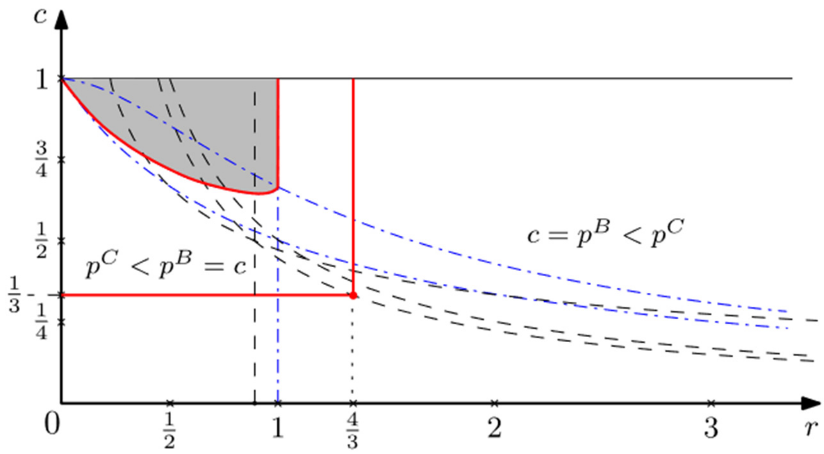


[Figure 11] R&D Investment: Cournot vs Bertrand(Random)

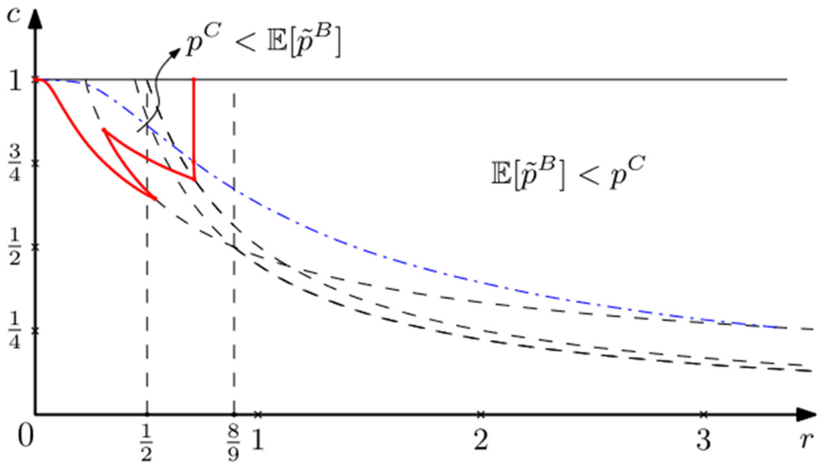


There are previous studies that compare firms' R&D investments for cost reduction under Cournot and Bertrand competition. For example, Qiu (1997) finds that firms invest more in R&D under Cournot competition, while Cellini and Lambertini (2011) argue that firms invest more under Bertrand competition. These studies assume that firms produce differentiated goods, leading to deterministic decisions on R&D investments. In contrast, we assume that firms produce homogeneous goods and randomly choose their R&D investments in Bertrand competition. Our result suggests that whether firms invest more in R&D under Cournot or Bertrand competition depends on their production costs and R&D investment costs.

[Figure 12] Market price: Cournot vs Bertrand(Non-random)



[Figure 13] Market price: Cournot vs Bertrand(Random)



We can also compare the market prices under Cournot and Bertrand competition. Figure 12 compares the market prices when the firms make non-random choices on the R&D investments under Bertrand competition. Under the Bertrand competition, the winner of the R&D competition always sets the price p^B at its rival firm's marginal cost c, while the market price under Cournot competition depends on the firms' marginal costs. Therefore, an increase in c raises the market price under both Cournot and Bertrand competition, whereas an increase in r lowers the market price only under Cournot competition by reducing the firms' marginal costs. Figure 13 compares the market prices given that the firms make random choices on the R&D investments under Bertrand competition. When the firms make extreme choices on the R&D investments under Cournot competition (i.e., $x_i^C = c$ or $x_i^C = 0$ for some i), the market price under Cournot competition can be lower than that under

Bertrand competition on average. However, if the firms do not make the extreme choices on the R&D investments under Cournot competition (i.e., $0 < x_i^C < c$ for all i), the market price under Cournot competition is higher than that under Bertrand competition on average, which is consistent with the previous literature comparing R&D investments between Cournot and Bertrand competition. This can be explained by the fact that the market price under Cournot competition is higher than the firm's marginal costs, whereas the market price under Bertrand competition is determined at the second lowest marginal cost among the firms.

VI. Conclusion

This paper examines the R&D behavior of firms that are engaged in price competition in the market. As a firm invests more in R&D, it can participate in the price competition with a lower marginal cost. In a market where firms compete on price, only the firm that has a lower marginal cost than its rival can supply goods to the entire market and earn a positive profit. And, if one firm invests enough in R&D, the other firm may choose to give up R&D investment and exit the market instead of investing heavily in R&D. In this paper, we provide the conditions for the existence of equilibrium in which the firms make such choices on the R&D investments and the production of goods. In addition, if we restrict the firms to make non-randomized choices on the R&D investments, an equilibrium in which both the firms make R&D investments does not appear.

If the firms can make randomized choices on their R&D investments, then there is an equilibrium in which both the firms invest in R&D and the average sizes of their R&D investments depend on their R&D investment costs. That is, as the firms' R&D investment costs decrease, the firms invest more in R&D on average. In addition, if the firms' R&D investment costs are high enough, then both the firms invest in R&D but only one firm winning the R&D race supplies goods to the entire market with probability 1 in an equilibrium. On the other hand, if their R&D investment costs are low enough, then it happens with a positive probability that both firms make the maximum R&D investments, and all of them supply goods to the market. These may explain why the firms make various choices on R&D investments in the real world. One firm that is relatively active in R&D investment survives to supply goods to the entire market. Alternatively, several firms survive in the market by investing in R&D as much as possible, and they all supply goods to the market.

We also investigate the effects of R&D spillover and the number of competing firms on the R&D behavior of firms. Given that the firms make randomized choices on R&D investments, when the R&D spillover effect is small, the firms may or may not invest more in R&D as the R&D spillover effect increases. However, when the R&D

spillover effect is large, an increase in the R&D spillover effect has a negative effect on the R&D investments of the firms. In addition, the more firms compete in the market, the less the firms invest in R&D. This is because an increase in the number of firms does not affect the prize value of winning the R&D race, but reduces the probability that each firm wins the R&D race through the R&D investments.

Finally, we compare the R&D incentives of firms in a situation where the firms compete in quantities and in a situation where they compete in prices. Given that the firms producing homogeneous goods are allowed to make randomized choices on R&D investments, we find that the firms may or may not invest more in R&D when they are engaged in a quantity competition than when they are engaged in a price competition, depending on the R&D investment costs. Many of the previous studies suggest that firms invest more in R&D when they are engaged in quantity competition than when they are engaged in price competition. Our finding implies that such a suggestion from the previous studies may not always be true.

Appendix

Appendix: Pure strategy equilibrium with R&D spillover

Here, we present the pure strategy (x_i^*, x_j^*) for the firms in the R&D stage that constitutes an equilibrium under the specification in Section 5.1. To this end, for $\beta \in (0,1)$, let

$$\rho_1(\beta) = 2\beta(1-\beta) \tag{58}$$
$$\rho_2(\beta) = (1-\beta)\left(\beta + \sqrt{2\beta^2 - 2\beta + 1}\right) \tag{59}$$

and for $r > 0$ and $\beta \in (0,1)$

$$\psi_1(r,\beta) = \frac{2(1-\beta)}{2+r-4\beta+2\beta^2} \tag{60}$$
$$\psi_2(r,\beta) = 2(1-\beta) \times \frac{\begin{pmatrix} 4+5r-18\beta-10r\beta+30\beta^2-22\beta^3+6\beta^4+5r\beta^2+r^2 \\ -(r-2\beta+2\beta^2)\sqrt{1-r^2-4\beta+2r\beta+6\beta^2-4\beta^3+\beta^4-2r\beta^2} \end{pmatrix}}{\begin{pmatrix} 2(1-\beta)(r-4\beta+2\beta^2+2) \\ \times\left((2-\beta)(r-2\beta+2\beta^2)+2(1-\beta)\right)+r(r-2\beta+2\beta^2)^2 \end{pmatrix}} \tag{61}$$
$$\psi_3(r,\beta) = \frac{2(1-\beta)(r-4\beta+3\beta^2+1)}{(r-2\beta+2\beta^2)^2+2(1-\beta)(r-4\beta+3\beta^2+1)} \tag{62}$$

where $\psi_2(r,\beta)$ is defined for (r,β) satisfying $1-r^2-4\beta+2r\beta+6\beta^2-4\beta^3+\beta^4-2r\beta^2 \geq 0$ or equivalently $0 < r \leq \rho_2(\beta)$. Then, for any $\beta \in (0,1)$, $\rho_1(\beta) < \rho_2(\beta)$ holds, and for any (r,β) (or, (r,β) satisfying $r < \rho_2(\beta)$), $\psi_1(r,\beta) < \psi_3(r,\beta)$ (or, $\psi_1(r,\beta) < \psi_2(r,\beta) < \psi_3(r,\beta)$) holds. We consider the following 8 cases.

Case A-1: $r \leq \rho_1(\beta)$
In this case, $\bar{x} = \frac{c}{2}$ is the maximizer of $U_i(x_i, 0)$ in (40) on $[0, \frac{c}{2}]$ and $\bar{\bar{x}} = \frac{c}{2}$ is the maximizer of $U_i(x_i, \bar{x})$ on $[\bar{x}, \frac{c}{2}]$. It is trivial that $U_i(\bar{\bar{x}}_i, \bar{x}) < 0$. Thus, $(x_i^*, x_j^*) = (\frac{c}{2}, 0)$ can be an equilibrium strategy for the firms in the R&D stage.

Case A-2: $\rho_1(\beta) < r < \rho_2(\beta)$ and $c \leq \psi_1(r,\beta)$
In this case, $\bar{x}$ and $\bar{\bar{x}}$ are the same as in Case 1. Thus, $(x_i^*, x_j^*) = (\frac{c}{2}, 0)$ can be an equilibrium strategy for the firms in the R&D stage.

Case A-3: $\rho_1(\beta) < r < \rho_2(\beta)$ and $\psi_1(r,\beta) < c \leq \psi_2(r,\beta)$
In this case, $\bar{x} = \frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2} < \frac{c}{2}$ is the maximizer of $U_i(x_i, 0)$ in (40) on

$[0,\frac{c}{2}]$ and $\bar{\bar{x}}=\frac{c}{2}$ is the maximizer of $U_i(x_i,\bar{x})$ on $[\bar{x},\frac{c}{2}]$. Since the condition of $c\le\psi_2(r,\beta)$ ensures that $U_i(\bar{\bar{x}},\bar{x})\le 0$, $(x_i^*,x_j^*)=\left(\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2},0\right)$ can be an equilibrium strategy for the firms in the R&D stage.

<u>Case A-4</u>: $\rho_1(\beta)<r<\rho_2(\beta)$ and $\psi_2(r,\beta)<c<\psi_3(r,\beta)$
In this case, $\bar{x}$ and $\bar{\bar{x}}$ are the same as in Case A-1, and the condition of $\psi_2(r,\beta)<c$ implies that $U_i(\bar{\bar{x}}_i,\bar{x})>0$. Thus, any pure strategy (x_i,x_j) in the R&D stage cannot constitute an equilibrium.

<u>Case A-5</u>: $\rho_1(\beta)<r<\rho_2(\beta)$ and $\psi_3(r,\beta)\le c$
In this case, $\bar{x}=\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2}<\frac{c}{2}$ is the maximizer of $U_i(x_i,0)$ in (40) on $[0,\frac{c}{2}]$ and $\bar{\bar{x}}=\frac{(1-\beta)(1-c)(r-4\beta+3\beta^2+1)}{(r-2\beta+2\beta^2)^2}\le\frac{c}{2}$ is the maximizer of $U_i(x_i,\bar{x})$ on $[\bar{x},\frac{c}{2}]$. Then, the condition of $r<\rho_2(\beta)$ implies that $U_i(\bar{\bar{x}}_i,\bar{x})>0$. Thus, any pure strategy (x_i,x_j) in the R&D stage cannot constitute an equilibrium.

<u>Case A-6</u>: $\rho_2(\beta)\le r$ and $c\le\psi_1(r,\beta)$
In this case, $\bar{x}$ and $\bar{\bar{x}}$ are the same as in Case A-1. Thus, $(x_i^*,x_j^*)=(\frac{c}{2},0)$ can be an equilibrium strategy for the firms in the R&D stage.

<u>Case A-7</u>: $\rho_2(\beta)\le r$ and $\psi_1(r,\beta)<c<\psi_3(r,\beta)$
In this case, $\bar{x}$ and $\bar{\bar{x}}$ are the same as in Case 3. The condition of $\rho_2(\beta)\le r$ implies that $U_i(\bar{\bar{x}}_i,\bar{x})\le 0$ for any $c\in(0,1)$, so $(x_i^*,x_j^*)=\left(\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2},0\right)$ can be an equilibrium strategy for the firms in the R&D stage.

<u>Case A-8</u>: $\rho_2(\beta)\le r$ and $\psi_3(r,\beta)\le c$
In this case, $\bar{x}$ and $\bar{\bar{x}}$ are the same as in Case 5. Then, the condition of $\rho_2(\beta)\le r$ implies that $U_i(\bar{\bar{x}}_i,\bar{x})\le 0$. Thus, $(x_i^*,x_j^*)=\left(\frac{(1-\beta)(1-c)}{r-2\beta+2\beta^2},0\right)$ can be an equilibrium strategy for the firms in the R&D stage.

References

Arrow, K. J. (1972), "Economic Welfare and the Allocation of Resources for Invention", in C. K. Rowley, ed., *Readings in Industrial Economics: Volume Two: Private Enterprise and State Intervention*, London: Macmillan Education UK, 219–236.

Bester, H. and Petrakis, E. (1993), "The incentives for cost reduction in a differentiated industry", *International Journal of Industrial Organization*, 11(4), 519–534.

Blume, A. (2003), " Bertrand without fudge", *Economics Letters*, 78(2), 167–168.

Cellini, R. and Lambertini, L. (2004), "Private and social incentives towards investment in product differentiation", *International Game Theory Review*, 06(4), 493–508.

________________________. (2011), "R&D incentives under Bertrand competition: A differential game", *Japanese Economic Review*, 62(3), 387–400.

Chen, J. and Lee, S.-H. (2023), "Cournot-Bertrand comparisons under R&D competition: Output versus R&D subsidies", *International Journal of Economic Theory*, 19(1), 77–100.

Dasgupta, P. and Maskin, E. (1986), "The existence of equilibrium in discontinuous economic games, I: Theory", *Review of Economic Studies*, 53(1), 1–26.

d'Aspremont, C. and Jacquemin, A. (1988), "Cooperative and noncooperative R&D in duopoly with spillovers", *American Economic Review*, 78(5), 1133–1137.

Delbono, F. and Denicoló, V. (1990), "R&D investment in a symmetric and homogeneous oligopoly: Bertrand vs Cournot", *International Journal of Industrial Organization*, 8(2), 297–313.

Glicksberg, I. L. (1952), "A further generalization of the Kakutani fixed point theorem, with application to Nash equilibrium points", *Proceedings of the American Mathematical Society*, 3(1), 170–174.

Hinloopen, J. (2000), "Strategic R&D co-operatives", *Research in Economics*, 54(2), 153–185.

Kaplan, T. R. and Wettstein, D. (2000), "The possibility of mixed-strategy equilibria with constant-returns-to-scale technology under Bertrand competition", *Spanish Economic Review*, 2, 65–71.

Kartik, N. (2011), "A note on undominated Bertrand equilibria", *Economics Letters*, 111(2), 125–126.

Kvasov, D. (2007), "Contests with limited resources", *Journal of Economic Theory*, 136(1), 738–748.

Lin, P. and Saggi, K. (2002), "Product differentiation, process R&D, and the nature of market competition", *European Economic Review*, 46(1), 201–211.

Moldovanu, B. and Sela, A. (2001), "The optimal allocation of prizes in contests", *American Economic Review*, 91(3), 542–558.

Qiu, L. D. (1997), "On the dynamic efficiency of Bertrand and Cournot equilibria", *Journal of Economic Theory*, 75(1), 213–229.

Reny, P. (1999), "On the existence of pure and mixed strategy Nash equilibria in discontinuous games", *Econometrica*, 67(5), 1029–1056.

Schumpeter, J. A. (1943), *Capitalism, Socialism and Democracy*, New York: Harper and Brothers.

Siegel, R. (2009), "All-pay contests", *Econometrica*, 77(1), 71–92.

Symeonidis, G. (2003), "Comparing Cournot and Bertrand equilibria in a differentiated duopoly with product R&D", *International Journal of Industrial Organization*, 21(1), 39–55.

베르트랑 경쟁하에서의 R&D 투자 행태 분석*

조 명 환**

초 록 이 논문은 동질적 재화를 생산하며 베르트랑 경쟁을 하는 기업들의 R&D 투자유인을 분석하였다. 모형에서 기업들은 시장에서 가격경쟁을 하기 전에 R&D를 통해 한계비용을 낮출 수 있다고 가정하였다. 분석 결과, 기업들이 R&D 투자에 대해 확률적이 아닌 확정적 선택을 하는 균형에서는 하나의 기업만 R&D에 투자하여 전체 시장에 공급하는 반면, 기업들이 R&D 투자수준을 확률적으로 선택하는 균형에서는 모든 기업이 확률 1로 어느 정도의 R&D 투자를 하는 것으로 나타났다. 그리고 R&D 투자비용이 충분히 낮으면, 모든 기업은 최고 수준의 R&D 투자를 선택하고 시장을 공유할 수 있음도 확인하였다. 또한, 이 논문은 R&D의 확산효과 정도와 경쟁기업의 수가 R&D 투자유인에 미치는 영향을 분석하고, 베르트랑 경쟁과 쿠르노 경쟁하에서 기업들의 R&D 행동을 비교하였다.

핵심 주제어: 베르트랑 경쟁, R&D 투자, 혼합전략균형, 대칭균형

경제학문헌목록 주제분류: C72, D43, L13

투고 일자: 2025. 5. 2. 심사 및 수정 일자: 2025. 8. 4. 게재 확정 일자: 2025. 11. 26.

* 본 연구는 2025년도 서울시립대학교 교내학술연구비에 의하여 지원되었음을 밝힌다. 그리고, 저자가 University of South Florida에 방문하였을 때 호의를 베풀어준 Prof. Seung-Yeop Lee에게 감사를 드리고, 여러 유익한 조언을 해준 김가연 학생, 성현진 학생과 익명의 심사자들께도 깊은 감사의 뜻을 전한다.

** 서울시립대학교 경제학부 교수, e-mail: chomhmh@uos.ac.kr