The Korean Economic Review
Volume 41, Number 2, Summer 2025, 349-374.
DOI : 10.22841/kerdoi.2025.41.2.005

Minimum Contribution Requirement in an Infinitely Repeated Public Goods Game*

Doruk İriş**

This research studies the impact of minimum contribution requirements on public goods provision in infinitely repeated games. While in static setups such requirements enhance welfare by surpassing non-cooperative Nash equilibria, their effectiveness in dynamic settings where cooperation relies on Nash reversion strategies remains an open question. This study reveals that the introduction of a minimum contribution requirement, positioned between the Nash equilibrium and the agreed cooperative level, hinders long term cooperation among symmetric players, under a quasi-linear and the constant elasticity of substitution (CES) utilities.

JEL Classification: H41
Keywords: Cooperation, Public Goods, Repeated Game, Minimum Contribution

I. Introduction

The pervasive concern of insufficient public goods provision and the response of introducing a minimum contribution requirement resonate across diverse domains. Examples span from mandatory membership fees for social clubs or organizations and minimum participation requirements for certain events, to government mandates for additional taxes or aid to constituents, and partners agreeing to contribute a minimum level of effort for household chores or financial support in marriages.

More specifically, for instance, the North Atlantic Treaty Organization (NATO) currently man-dates its members to contribute relatively small amounts for daily

Received: July 23, 2024. Revised: Jan. 31, 2025. Accepted: May 12, 2025.
* I thank Attila Ambrus, Euncheol Shin, and Pellumb Reshidi for insightful discussions that significantly enhanced this work. I also extend my gratitude to Huseyin Yildirim for his unwavering support, which played a pivotal role in bringing this article to fruition. Lastly, I would like to express my thanks to the Duke University Department of Economics, where I initiated this project during my visit.
** Associate Professor, School of Economics, Sogang University, GN618, Sinsu-dong, Mapo-gu Seoul, S. Korea, Phone: +82-2-705-8562, e-mail: dorukiris@gmail.com

operations. The main operations and missions rely on members' voluntary contributions and the contributions aim to reach 2% of members' GDP's. However, these voluntary contributions often fall below this target for many member states. As a result, NATO Secretary General Jens Stoltenberg stated the need to make support "less dependent on short term voluntary offers and more dependent on long term NATO commitments" and ensuring "long term, predictable, robust support" in NATO Brussels meeting in 2024.[1] Secondly, European Union (EU) mandates the contribution of member states to the budget and in return provide many EU-wide public goods.[2] Among many public goods, Stability and Growth Pact (SGP), which aims to coordinate the fiscal policy of the different member states and ensure the sustainability of public finances, enforces many fiscal rules, e.g., maintaining 3% deficit and the debt-to-GDP ratio to be below 60%, to the member states. Before these rules were adopted in 2023, there were intense discussions, arguing them to be over-restrictive and not having theoretical justifications.[3] Thirdly, Homeowner Associations (HOAs) are wide spread in the US, which mandate homeowners within their community to contribute and in return set community standards and provide amenities.[4] However, how much these fees should be and what standards and amenities should be provided within a community is highly debatable. Finally, in all these examples, determining how much contribution should be enforced is a challenge to the decision-makers and some still decide to voluntarily contribute above the enforced levels.

Contributions to public goods are subject to the free-rider problem. Thus, to overcome this prob-lem, policies and agreements enforcing a joint minimum contribution surpassing the non-cooperative Nash equilibrium levels would be indisputably welfare improving in static settings. However, would such minimum contribution requirements for the provision of public goods continue to improve the welfare in the long-run? Surprisingly, the long-term effects of minimum contribution requirements on the provision of public goods have not been examined.

This study centers on a public goods game akin to Bergstrom et al. (1986) and Park (1987), but distinguished by its infinite repetition with per-period discounting and players adopting grim-trigger (Nash reversion) strategies. In this context, it is widely acknowledged that a threshold discount factor exists, above which players can sustain cooperation, and below which coopera-tion fails (Friedman, 1971; Fudenberg and Maskin, 1996). Our setup bears resemblance to that of Pecorino

[1] See Funding NATO and Secretary General's speech ahead of the meetings of NATO Ministers of Foreign Affairs in Brussels.

[2] Such as Common Agricultural Policy providing subsidies to the farmers, European Defence Fund, EU's research and innovation program Horizon Europe, European Stability Mechanism (ESM) providing financial assistance to member states in economic distress.

[3] See The New European Fiscal Rules.

[4] See HOA details and Statistics.

(1999) but diverges in its incorporation of a player-enforced minimum contribution requirement.[5]

In our setup, players agree on both a minimum contribution requirement and a cooperative contribution level. While both are agreements, minimum contribution level is enforced by a contract or formal agreement, whereas the cooperative contribution level, which must be at least as high as the minimum contribution requirement, is voluntary. Sustaining cooperation at this level require self-enforcement.[6]

To comprehend the impact of a minimum contribution requirement on cooperation levels, we scrutinize its influence on the threshold discount factor. If the introduction of such a requirement decreases (increases) the threshold discount factor, it serves to facilitate (hinder) cooperation. In such infinitely repeated games with per period discounting and players adopting Nash reversion strategies, how players reach an agreement to play a cooperative contribution is not a concern. Therefore, the main question of the paper is as follows: let g^C be a cooperative individual contribution level that is sustainable for a given discount factor. What would happen if a well-intended institution increases the minimum contribution requirement in an attempt to improve the welfare? My analysis encompasses symmetric players with both a quasi-linear and CES utilities, thereby capturing a wide spectrum of decision-making behaviors. This distinction also allows us to assess whether the presence or absence of income effects influences the impact of minimum contribution requirements on cooperation.

Remarkably, our findings reveal that imposing a minimum contribution requirement, when set between the non-cooperative Nash equilibrium and the agreed cooperative level, obstructs cooperation among symmetric players. This result shows that the interplay of infinite repetition and enforced restrictions alters the incentives for sustained cooperation and a minimum contribution requirement has opposing effects in static and dynamic setups. The underlying rationale for this outcome hinges on two key effects: the introduction of a minimum contribution requirement con-strains both non-cooperative Nash contributions and a deviating player's contribution while other players continue to contribute. On the one hand, this diminishes the incentive to deviate because the deviating player cannot reduce

[5] Pecorino (1999) studies the role of the group size in such infinitely repeated public goods games. Hadjiyiannis et al. (2012a, 2012b) are other related papers studying the role of reciprocal preferences and players' expectations in an infinitely repeated public goods game.

[6] To clarify this distinction with an example: consider a scenario where firms sign a joint venture agreement that outlines objectives, initial contributions, rights to profits, and responsibilities for losses. Such agreements represent what I refer to as the minimum contribution requirement, as they are enforceable. However, joint venture agreements are inherently incomplete; they fail to specify numerous details, requiring firms to cooperate beyond what is explicitly stated in the agreement to fulfill their tasks. I refer to the total contribution of each firm, which may exceed the minimum contribution requirement, as their cooperative contribution level.

its contribution below the enforceable minimum contribution requirement. On the other hand, it strengthens the incentive to deviate because reverting to Nash contributions, now equivalent to the minimum required contribution, represents a milder punishment for the deviating player. However, our analysis shows that for a minimum contribution requirement positioned between the non-cooperative Nash equilibrium and the agreed cooperative level, the increase in the incentive to deviate always dominates, resulting in an increase in the threshold discount factor. In other words, a cooperative individual contribution g^C that is initially sustainable for a discount factor may not be sustainable anymore upon an increase in the minimum contribution requirement. Therefore, we conclude that such minimum contribution requirements hinder cooperation in the long-run.

The rest of the paper proceeds as follows. Section 2 sets up the model. Section 3 presents the results, first under a fairly general utility function, then under a quasi-linear utility function and, finally, under the CES utility function. Section 4 discusses several related aspects and assumptions. Section 5 concludes the paper. All the proofs and calculations are left to the appendix. Finally, for transparency, I provide a supporting Mathematica file and its PDF, which contains the details of the numerical analyses.

II. Model

A group of n players voluntarily and repeatedly contribute to a public good in each period $t=1,2,\ldots$. Let x_i, g_i, and w_i be player i's private consumption, contribution, and income. Normalizing prices to one, i's budget constraint is

$$x_i + g_i = w_i . \tag{1}$$

As in Bergstrom et al. (1986), the total supply of public good is $G=\sum_i g_i$. Moreover, player i's utility in a given period depends on his private consumption and the total supply of public good $u_i(x_i, G)$. Let $\delta_i \in [0,1)$ be i's per period discount factor. The difference from Pecorino (1999) is that players agreed on an exogenously determined enforceable minimum contribution level that is higher than the non-cooperative Nash contribution $\underline{g} > g^N$ and in any given period, player i cannot contribute below this minimum, $g_i \geq \underline{g}$.

Given the overall symmetry in our framework, we focus on symmetric cooperative subgame-perfect equilibria in which (i) along the equilibrium path, players play a common agreed cooperative contribution level $q^C \in (\underline{g}, g^O)$ in each period, where g^O is the socially optimal contribution level; and (ii) at any point of the game if a player defects, all players revert to the non-cooperative Nash contribution of the stage game from the following period onwards.[7] In other words, players employ Nash reversion strategies to sustain cooperation.

Let u^C, u^D, and u^N be respectively a player's cooperative, defection, and noncooperative payoffs. Then, under Nash reversion strategies, a player cooperates if onetime gain from defection plus the discounted Nash payoff from the next period onward do not exceed the cooperative payoff:

$$\frac{1}{1-\delta}u^C \geq u^D + \frac{\delta}{1-\delta}u^N \Leftrightarrow \delta \geq \delta^* = \frac{u^D - u^C}{u^D - u^N}, \tag{2}$$

Thus, players can sustain cooperation if they are sufficiently patient, $\delta_i \geq \delta_i^*$.

Finally, suppose all players are homogeneous in all respects, and preferences represented by the following fairly general utility function:

$$u_i = (\alpha f(x_i)^\rho + (1-\alpha)h(G)^\rho)^{\frac{1}{\rho}} = (\alpha f(w_i - g_i)^\rho + (1-\alpha)h(G)^\rho)^{\frac{1}{\rho}} \tag{3}$$

The parameter α represents the preference weight on the private and public goods respectively, and $\frac{1}{1-\rho}$ denotes the elasticity of substitution between the two goods. Since players enjoy consuming both private and public goods, the functions f and h are increasing.

For the utility function (3), given the others' contribution with each $g \in [\underline{g}, g^O]$, the marginal per capita return (MPCR), which measures the individual's private benefit from increasing their contribution to the public good, relative to their cost, is equal to:

$$MPCR = \frac{(1-\alpha)h(G)^{\rho-1}h'(G)}{\alpha f(x)^{\rho-1}f'(x)}. \tag{4}$$

We have the following assumptions to ensure we have a public goods game.

Assumption 1

1. $MPCR < 1$*: Each player has an incentive to free-ride because their private gain from con-tributing is insufficient to justify the personal cost.*
2. $n \cdot MPCR > 1$*: Each player increasing contributions by one unit would generate a public good benefit exceeding the total private costs.*

---

[7] One can rightfully ask why not simply set $\underline{g} = q^O$? While this is possible in our stylized model, in reality agreeing on an optimal contribution level is not easy, e.g., see the examples in introduction, which I discuss the reasons more in Section 4. For now, the only importance of q^O is, it serves as the upper bound for g^C.

III. Results

I want to understand how does the enforceable minimum contribution requirement $\underline{g}$ affect the group's cooperation. In other words, how does it affect the threshold level of discount factors δ_i^* ?

I find the individual contributions at the social planner's optimal g^O, Nash equilibrium g^N, and when a player deviates while $n-1$ others cooperate for the general utility function (3). First, in the optimal case, the social planner maximizes the sum of utilities of all individuals, which simplifies as follows assuming symmetry in players:

$$\sum_{i=1}^{n} u_i = n(\alpha f(w-g)^{\rho} + (1-\alpha)h(ng)^{\rho})^{\frac{1}{\rho}}. \tag{5}$$

Maximizing the utility above with respect to g yields the following first order condition (FOC) after some simplifications and rearrangements:

$$\frac{f'(w-g)}{h'(ng)} = n\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{f(w-g)}{h(ng)}\right)^{1-\rho}. \tag{6}$$

This equation defines the optimal contribution g^O in implicit form, depending on the functions f and h. Specifically, on the left hand side (LHS), there is the individual's willingness to trade private consumption for the public good, i.e., marginal rate of substitution (MRS) between the private and public good. On the right hand side (RHS), the relative importance of the public good (in a collective sense) is reflected, scaled by the number of contributors n, the relative weights α and $1-\alpha$ in the utility function, and the substitutability parameter ρ.

In the Nash equilibrium, each individual maximizes their own utility u_i, taking the contributions of others as given. The individual's utility is:

$$u_i = (\alpha f(w_i - g_i)^{\rho} + (1-\alpha)h(G)^{\rho})^{\frac{1}{\rho}}, \tag{7}$$

where $G = g_i + (n-1)g$. Maximizing the utility function above with respect to g_i yields the following FOC after some simplifications and rearrangements:

$$\frac{f'(w_i - g_i)}{h'(G)} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{f(w_i - g_i)}{h(G)}\right)^{1-\rho}. \tag{8}$$

which defines the Nash contribution g^N in implicit form, depending on the functions f and h. Specifically, the LHS is again the MRS between the private and public good. The RHS is how much weight a player places on the public good relative to the private one and how the trade-off between private and public goods is influenced by the current levels of consumption, the relative weights α and $1-\alpha$ in the utility function, and the substitutability parameter ρ.

Finally, when player i deviates and the other $n-1$ players contribute g^C, the total public good is $G = g_i + (n-1)g^C$. The utility of the deviating player is:

$$u_i = (\alpha f(w_i - g_i)^\rho + (1-\alpha)h(g_i + (n-1)g^C)^\rho)^{\frac{1}{\rho}}. \tag{9}$$

Maximizing the utility above with respect to g_i yields the following FOC after some simplifica-tions and rearrangements:

$$\frac{f'(w_i - g_i)}{h'(g_i + (n-1)g^C)} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{f(w_i - g_i)}{h(g_i + (n-1)g^C)}\right)^{1-\rho}, \tag{10}$$

which defines the deviation contribution g^D in implicit form, depending on the functions f and h. The intuition from the LHS and RHS is similar to te FOC of the Nash contributions case. However, when player i deviates while $n-1$ others play g^C, the free-riding incentives (e.g., assuming h to be concave and hence satisfying diminishing marginal return) yields $g^D < g^N$. Given that $g^N < g^C$, the utility levels for a player will be $u^D > u^C > u^N$.

Suppose the group agrees at the outset to a minimum contribution $\underline{g} \in (g^N, g^C)$ and this is the only amount that can be explicitly enforced. Suppose it will bind for both the Nash equilibrium and the deviation cases, meaning that players set $\underline{g}$ as their Nash and deviation contributions, since it is the lowest possible contribution level. As a result, $u^D > u^C > u^N$. Specifically, the three utility functions are:

$$\begin{aligned} u^C &= (\alpha f(x^C)^\rho + (1-\alpha)h(G^C)^\rho)^{\frac{1}{\rho}}, \text{ where } x^C = w - g^C \text{ and } G^C = ng^C \\ u^N &= (\alpha f(\underline{x})^\rho + (1-\alpha)h(\underline{G})^\rho)^{\frac{1}{\rho}}, \text{ where } \underline{x} = w - \underline{g} \text{ and } \underline{G} = n\underline{g} \\ u^D &= (\alpha f(\underline{x})^\rho + (1-\alpha)h(G^D)^\rho)^{\frac{1}{\rho}}, \text{ where } G^D = \underline{g} + (n-1)g^C \end{aligned} \tag{11}$$

Proposition 1 summarizes how the minimum contribution requirement affects the utility functions and the degree of cooperation in the long-run. To simplify notation, I use the subscripts to refer to the respective partial derivatives.

Proposition 1 *For the general utility function (3),*

1. The utility functions u^N increases with $\underline{g}$, u^D decreases with $\underline{g}$, and u^C is *unaffected.*

2. *The impact of* $\underline{g}$ *on* δ^*. *determined by the following condition:*

$$\frac{\partial \delta^*}{\partial \underline{g}} \propto \underbrace{u^D_{\underline{g}}}_{(-)} \underbrace{(u^C - u^N)}_{(+)} + \underbrace{u^N_{\underline{g}}}_{(+)} \underbrace{(u^D - u^C)}_{(+)}. \tag{12}$$

3. *The cross-partial derivative:*

$$\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C} \propto (\underbrace{u^D_{\underline{g}g^C}}_{(-/?)} (u^C - u^N) + \underbrace{u^D_{\underline{g}}}_{(-)} \underbrace{u^C_{g^C}}_{(+)} + \underbrace{u^N_{\underline{g}}}_{(+)} \underbrace{(u^D_{g^C} - u^C_{g^C})}_{(?)})(u^D - u^N) - 2\underbrace{u^D_{g^C}}_{(+)} \underbrace{N_{\underline{g}}}_{(?)}, \tag{13}$$

where $N_{\underline{g}} = u^D_{\underline{g}}(u^C - u^N) - u^N_{\underline{g}}(u^D - u^C)$ *is the numerator of* $\frac{\partial \delta^*}{\partial \underline{g}}$.[8]

The results in Point 1 follow Assumption 1. Intuitively, an increase in $\underline{g}$ causes all players to increase their contributions in non-cooperative outcome, resulting in higher benefits from public good that exceed the total decrease in private goods, given that $n \cdot MPCR > 1$. Thus, this increase makes the non-cooperative outcome more cooperative. However, an increase $\underline{g}$ affects only the deviating player's contribution in the deviating scenario. Since this player cannot lower its contribution as much as before, their utility u^D decreases, as $MPCR > 1$.

The result in Point 2 addresses the main question of the paper.[9] The first term in (12) reflects the positive impact of the minimum contribution rule on the degree of cooperation: With a more binding minimum contribution rule, deviation becomes less appealing and players can achieve a higher utility through cooperation compared to the Nash utility. Conversely, the second term in (12) represents the negative impact of the minimum contribution rule on the degree of cooperation: as the Nash utility increases with the minimum contribution requirement, reverting to the Nash outcome becomes less undesirable than before.

In the proof, I investigate meaningful sufficiency conditions under which one effect dominates the other. However, it becomes evident that determining which of these two effects prevails depends on the exact functional forms of f and h. The result in Point 3 is part of this investigation and concludes that the overall effect of the level of cooperative contributions g^C on $\partial \delta^* / \partial \underline{g}$ remains ambiguous without specifying the functional forms of f and h.

In the following subsections, I study this problem for a quasi-linear utility and the general CES utility functions by assuming specific functional forms for the f and h functions. These represent special cases of the utility function (3).

[8] The term $u^D_{\underline{g}g^C}$ becomes negative for linear or concave h, but ambiguous otherwise.

[9] The $\propto$ sign is read as "directly proportional."

3.1. A Quasi-Linear Utility with Natural Logarithm Function

Suppose preferences represented by the following quasi-linear utility function:[10]

$$u_i = x_i + a\ln G = w - g_i + a\ln G . \tag{14}$$

Proposition 2 *Assume identical players with quasi-linear utilities (14), the individual contribution levels at the optimal, non-cooperative, and if one player deviates while other $n-1$ players cooperate, are as follows:*

$$g^O = a\,,\quad g^N = \frac{a}{n},\quad g^D = \begin{cases} 0, & if\ \ g^C \geq \frac{a}{n-1} \\ a-(n-1)g^C, & if\ \ g^C < \frac{a}{n-1} \end{cases}$$

The utility functions under the three possible cases are:

$$u^C = x^C + a\ln(G^C)\,,\quad u^N = \underline{x} + a\ln(\underline{G})\,,\quad u^D = \underline{x} + a\ln(G^D)$$

The minimum contribution requirement hinder cooperation, i.e., $\frac{\partial \delta^*}{\partial \underline{g}} > 0$ *if*

$$(a/\underline{g}-1)\ln(G^D/G^C) - (1-a/G^D)\ln(g^C/\underline{g}) + \left(\frac{g^C}{\underline{g}} - 1\right) + \left(\frac{g^C - \underline{g}}{G^D}\right) > 0 \tag{15}$$

Can the minimum contribution requirement ever facilitate cooperation under the assumptions of the model, i.e., the reverse of the inequality (15) holds? Unfortunately, it is not possible to derive closed-form solutions where the inequality holds with equality. This is because the function on the LHS of the inequality is a transcendental function, which cannot be solved analytically. Therefore, the inequality can only be analyzed numerically.

<u>Numerical Analysis:</u> Based on the utility function (14), the findings in Proposition 2, and the setting of the model $(a > g^C > \underline{g} > \frac{a}{n})$, I evaluate the LHS of the inequality (15) using the following values: $a \in \{0.1,\ldots,10\}$ with increments of 0.1, $g^C \in \{\frac{a}{n}+0.001,\ldots,a\}$ with increment of 0.01, $\underline{g} \in \{\frac{a}{n},\ldots,a-0.1\}$ with increments of 0.1, and $n \in \{2,4,10,20,100,10000,1000000\}$. I find the minimum value of the function a positive number. Therefore, the condition (15) "always" holds, indicating that the minimum contribution requirement hinders cooperation.

[10] As in (3), the results are inconclusive for a generic non-linear function in quasi-linear preferences.

Secondly, for the same parameter values, I evaluate the cross-partial derivative $\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C} > 0$, which is also a transcendental function. To analyze this, I again find the minimum value of this function, which consistently yields a positive number. This result implies that the greater the cooperative contributions the group can agree upon g^C, the more the minimum contribution requirement hinders cooperation.

The inequality (15) and the results from the numerical analysis demonstrate that the minimum contribution requirement does not facilitate the long-term contribution. But what is the underlying mechanism driving this result? It is important to notice that if players cannot agree on a cooperation level g^C higher than the minimum contribution requirement $\underline{g}$, i.e., $g^C = \underline{g}$, then the three utility levels in (11) become equivalent as $x^C = \underline{x}$ and $G^C = \underline{G}$. As a result, the impact of the minimum contribution requirement on long-term cooperation becomes negligible, i.e., $\frac{\partial \delta^*}{\partial \underline{g}} = 0$. This suggests that the essential assumption of the model, which leads to the negative impact of the minimum contribution requirement on long-term cooperation, is the possibility of achieving a higher degree of cooperation.

3.2. CES Utility

Are the results we obtained so far unique to the quasi-linear preferences studied above, or do they hold under a broader class of preferences? To investigate this, I repeat the above analysis with the following CES utility, while again assuming identical players:

$$u_i = (\alpha x_i^{\rho} + (1-\alpha) G^{\rho})^{\frac{1}{\rho}} = (\alpha (w_i - g_i)^{\rho} + (1-\alpha) G^{\rho})^{\frac{1}{\rho}}, \tag{16}$$

where $\alpha \in (0,1)$ and $\rho \in (-\infty, 1)$. Here, the substitution between private and public good con-sumptions grows with ρ, where $\rho \to \{0, -\infty, 1\}$ respectively represent Cobb-Douglas, Leontief, and Linear preferences. Furthermore, MPCR for CES utility is:

$$MPCR = \frac{(1-\alpha) x^{1-\rho}}{\alpha G^{1-\rho}}$$

Proposition 3 *Let* $\kappa := (\frac{1-\alpha}{\alpha})^{\frac{1}{1-\rho}} n^{\frac{\rho}{1-\rho}}$ *and assume identical players with CES utilities (16), the individual contribution levels at the optimal, non-cooperative, and if player* i *deviates while other* $n-1$ *players cooperate, are as follows:*

$$g^O = \frac{\kappa}{\kappa + 1} w, \quad g^N = \frac{\kappa}{\kappa + n^{\frac{1}{1-\rho}}} w, \quad g^D = \frac{w \kappa n^{\frac{-\rho}{1-\rho}} - (n-1) g^C}{1 + \kappa n^{\frac{-\rho}{1-\rho}}}$$

The utility functions with the minimum contribution requirement:

$$u^C = (\alpha (x^C)^\rho + (1-\alpha)(G^C)^\rho)^{\frac{1}{\rho}}, \quad u^N = (\alpha (\underline{x})^\rho + (1-\alpha)(\underline{G})^\rho)^{\frac{1}{\rho}},$$
$$u^D = (\alpha (\underline{x})^\rho + (1-\alpha)(G^D)^\rho)^{\frac{1}{\rho}}$$

The minimum contribution requirement hinder cooperation, i.e., $\frac{\partial \delta^*}{\partial \underline{g}} > 0$ *if*

$$\left(\frac{u^N}{u^D}\right)^{\frac{1-\rho}{\rho}} \left(\frac{-\alpha \underline{x}^{\rho-1} + (1-\alpha) n \underline{G}^{\rho-1}}{\alpha \underline{x}^{\rho-1} - (1-\alpha) G^{D^{\rho-1}}}\right)\left(\frac{u^D - u^C}{u^C - u^N}\right) > 1 \tag{17}$$

<u>*Cobb-Douglas:*</u> *As* $\rho \to 0$, *this condition becomes:*

$$\left(\frac{G^D}{\underline{G}}\right)^\alpha \left(\frac{n(1-\alpha)\underline{x} - \alpha \underline{G}}{\alpha G^D - (1-\alpha)\underline{x}}\right)\left(\frac{u^D - u^C}{u^C - u^N}\right) > 1. \tag{18}$$

Analyzing whether these conditions always hold under the assumptions of the model proved to be cumbersome. As a result, I relied on numerical analysis once again.[11]

<u>*Numerical Analysis:*</u> Using the Cobb-Douglas utility function, the findings in Proposition 3 and its proof, and the assumptions of the model $(g^O(w,\alpha) \geq g^C > \underline{g} > g^N(w,\alpha,n) > 0)$, I evaluate $\frac{\partial \delta^*}{\partial \underline{g}}$ under the following parameter values: $w \in \{1,5,10\}$, $n \in (2,4,10,20,100\}$, $\alpha \in (0,0.1,0.2,\ldots,1\}$, $\underline{g} \in \{g^N(w,\alpha,n)+0.01,\ldots, g^C - 0.01\}$ with increments of 0.1, and $g^C \in \{\underline{g}+0.01,\ldots,g^O(w,\alpha)\}$ with increments of 0.01. Across all evaluated parameter combinations, $\frac{\partial \delta^*}{\partial \underline{g}} > 0$ consistently holds. Specifically, in Mathematica, I identified a term whose negative values are both necessary and sufficient for $\frac{\partial \delta^*}{\partial \underline{g}} > 0$ to hold. The maximum value of this term across all evaluations remains negative. Therefore, the condition ensuring that the minimum contribution requirement hinders cooperation holds under the Cobb-Douglas utility function as well.

Secondly, I evaluate $\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C}$ using the following slightly modified values for the parameters: $\alpha \in \{0.1,0.2,\ldots\}$, $\gamma \in \{0.1,0.2,\ldots,0.9\}$, and $\beta \in (0.01,0.02,\ldots,0.99\}$.[12]

11 The cross-partial derivative $\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C}$ proved to be complex and I only present it in the supporting file for both the general CES and Cobb-Douglas utility functions.

12 I excluded $\alpha = \{0,1\}$ because some terms in the cross-partial derivative approach infinity, which causes issues in the calculations.

For each combination of these parameter values, I set values for g^C and $\underline{g}$ as follows, which ensures satisfying the assumptions of the model:

$$g^C = \beta \cdot g^O(w,\alpha) + (1-\beta) \cdot g^N(w,\alpha,n)$$
$$\underline{g} = \gamma \cdot g^C + (1-\gamma) \cdot g^N(w,\alpha,n) .$$

I confirmed that the cross-partial derivative is always positive for the given parameter ranges. Therefore, as in the analysis of quasi-linear utility, the more cooperative contributions the group can agree upon, the stronger the hindering effect of the minimum contribution requirement on cooperation.

Finally, I repeat the same exercise for the CES function, varying the level substitutability and complementarity between the private and public goods using $\rho \in \{-100, -0.5, 0.5, 0.9\}$ and the same parameters values as above. The results again confirm that $\frac{\partial \delta^*}{\partial \underline{g}} > 0$. However, while I find $\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C} > 0$ for $\rho = \{-100, -20, -1, -0.5, -0.2, 0.2, 0.5, 0.6\}$ as in the previous cases, I find some instances where $\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C} < 0$ when the private and the public goods are highly substitutable, specifically for $\rho = \{0.7, 0.8, 0.9\}$. This indicates that in these cases, the hindering effect of the minimum contribution requirement on cooperation weakens as the groups agrees on more cooperative contributions. Nevertheless, for the majority of parameter combinations, $\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C} > 0$ continues to hold even when the private and public goods are highly substitutable.

IV. Discussion

In my highly stylized model, using both a quasi-linear and CES utilities, I could explicitly find the optimal contributions. This raises a reasonable question: why not simply set $\underline{g} = g^O$? However, in reality, decision makers often face a far more complex problem than what my stylized model captures. They may have limited cognitive abilities and time to make decisions, be heterogeneous in their incomes and valuations of public goods, face uncertainties and trade-offs between contributions to different public goods, or experience turnover due to re-elections or selling their properties. Additionally, they may hold differing view on fairness.[13] As a result, identifying and agreeing on an optimal contribution level may not

[13] For example, should each country contribute equally to tackling climate change, despite some being more respon-sible for the problem and benefiting more from activities that caused it? Even if everyone agrees that those more responsible should contribute more, determining the quantities is extraordinarily challenging.

feasible.[14] Nevertheless, it seems reasonable to argue that the agreed cooperative contribution level and the minimum contribution level are unlikely to exceed the optimal contribution level.

My main conclusion is any minimum contribution requirement above the Nash contribution would hinder cooperation. This conclusion relies on the assumption that players can agree on a cooperative contribution level. As discussed in introduction, this model is silent about how players could reach such an agreement. In fact, in many real life examples players often fail to reach such agreements. For instance, countries implement tax systems, a mandatory contribution rule, to create necessary funds to finance many public goods. Without such enforced contributions, it is highly unlikely that citizens could voluntarily contribute to finance sizable amounts of public goods. Therefore, such minimum enforced contribution requirements are beneficial if they can be set above the agreed upon sustainable contribution levels, which would be the non-cooperative Nash contribution level if they cannot reach an agreement at all. This would be a greater challenge if there is no central authority that could introduce such minimum contribution requirements.

While I focus on the symmetric players in the model for simplicity, real-world examples typically involve asymmetric players. Asymmetries in size or wealth levels among players could lead to differences in non-cooperative Nash contributions and optimal contributions. Similarly, the minimum contribution requirement and agreed cooperative contribution levels may vary, as seen in the examples like climate change negotiations or the NATO 2% of GDP rule. Alternatively, players might have different returns or weights from public good contributions. For instance, Cornes and Hartley (2007) introduce such asymmetries using a CES utility function, albeit within a static model.[15] Regardless, I conjecture that the hindering effect of the minimum contribution rule remains unchanged as long as, for each player, the assumption $g^N < \underline{g} < g^C \leq g^O$ holds, as this significantly align the incentive of all players.

An interesting extension of the current model is the case where players agree on a minimum contribution requirement that increases over time.[16] Initially, this requirement may be set below the cooperative contribution level g^C and hence hinder cooperation. However, if it eventually exceeds g^C, it could become beneficial for the cooperation. Speculatively, the greater the extent to which the minimum contribution requirement exceeds g^C, and the more patient (i.e., the higher discount factor) the players are, the easier it may become for them to sustain

[14] In practice, a negotiated minimum contribution requirement often serves as a compromise. It provides participants with some assurance while avoiding the complexities of determining and enforcing an optimal level of cooperation.

[15] See also Liu and Sandler (2024), who study different public good aggregation rules in addition to the summation, which is the focus of this paper.

[16] I thank Prof. Euncheol Shin for suggesting this idea.

cooperation under such a rule.

Finally, in this dynamic game, alternative strategies such as tit-for-tat or getting-even strategies could be analyzed instead of Nash reversion. The level of cooperation that players can sustain may vary depending on the strategies employed. Nonetheless, I posit that introducing a minimum contribution requirement would not qualitatively alter the incentives faced by players, thereby preserving the validity of my main conclusions.

V. Concluding Remarks

I study infinitely repeated public goods games with minimum contribution requirements, shedding light on their influence in sustaining cooperation among symmetric players. By analyzing the interplay between enforcement mechanisms, Nash reversion strategies, and dynamic settings, I aim to understand the impact of minimum contribution requirements on cooperation dynamics. Interestingly, unlike the static setup in which minimum contribution requirements help to overcome the free-riding problem, my findings reveal that the introduction of a minimum contribution requirement, when positioned between the non-cooperative Nash equilibrium and a sustainable cooperative level, obstructs cooperation in the long run.

These findings reverberate beyond theoretical constructs, finding real-world resonances. In the context of the EU, where member countries have a certain level of obligation and enforcement mechanisms, a pertinent example lies in the allocation of financial resources within the EU budget. While the EU sets goals for member countries to contribute a certain percentage of their GDP to the common budget, negotiations often result in compromises, and countries may agree to a minimum contribution requirement below the ideal level. This could stem from varying economic conditions, national priorities, or political considerations. The EU's ability to enforce compliance provides a structured framework, yet the negotiated minimum contribution requirement allows flexibility for member countries, acknowledging their diverse circumstances. However, this flexibility, when translated into commitments below the highest sustainable cooperative level, raises concerns about the long-term sustainability of EU cooperation. If member countries consistently opt for minimum contributions requirements that fall short of the highest sustainable cooperative level, it may lead to inadequate funding for crucial EU initiatives, hindering the organization's ability to address collective challenges and pursue strategic objectives effectively over an extended period. The negotiated minimum contribution requirement, while offering adaptability, could pose a risk to the overall cohesion and progress of the EU in the long run. This example underscores the com-plexities and potential challenges associated with strategically positioning minimum contribution requirements in cooperative frameworks.

In the context of climate change mitigation strategies, many experts expect the Paris Agreement to produce insufficient incentives for countries to lower their emissions in comparison to business as usual (see, e.g., Barrett and Dannenberg, 2016). The main criticism is that it adopts an assessment and review framework, which lacks an enforcement mechanism. [17] While an enforced minimum contribution requirement might seem an intuitive solution if countries could agree on such an agreement, this research cautions against overlooking potential long-term consequences. In the realm of climate agreements, my findings resonate as we observe that a minimum emission reduction target negotiated below the sustainable cooperative level may impede long-run cooperation. Countries committing to lower targets might face less pressure to invest in sustainable practices or technologies, leading to a scenario where overall global emissions remain high. The presence of a minimum contribution requirement below the sustainable cooperative level could undermine the effectiveness of the agreement, hindering the achievement of broader environmental goals over time.

Finally, I hope that the highly stylized framework developed here can be a starting point, incorporating more plausible assumptions such as re-election as in Buchholz et al. (2005), heterogeneous players, and fairness concerns.

[17] See Barrett (2005) for a review on the literature of international environmental agreements.

Appendix: Proofs and Calculations

A.1 Calculations

A.1.1 MPCR

Given the contributions of others, the marginal utility of a player from increasing x and his own contribution G are, respectively:

$$\frac{\partial u}{\partial x} = \alpha u^{1-\rho} f(x)^{\rho-1} f'(x)$$
$$\frac{\partial u}{\partial G} = (1-\alpha) u^{1-\rho} h(G)^{\rho-1} h'(G).$$

Thus, with some simplifications:

$$MPCR = \frac{\frac{\partial u}{\partial G}}{\frac{\partial u}{\partial x}} = \frac{(1-\alpha)h(G)^{\rho-1} h'(G)}{\alpha f(x)^{\rho-1} f'(x)}.$$

A.2 Proofs

Proof of Proposition 1. The partial derivatives of u^N and u^D with respect to $\underline{g}$ are:

$$u^N_{\underline{g}} = u^{N^{1-\rho}} \cdot (-\alpha f(w-\underline{g})^{\rho-1} f'(w-\underline{g}) + n(1-\alpha)h(\underline{G})^{\rho-1} h'(\underline{G})) \tag{A1}$$
$$u^D_{\underline{g}} = u^{D^{1-\rho}} \cdot (-\alpha f(w-\underline{g})^{\rho-1} f'(w-\underline{g}) + (1-\alpha)h(G^D)^{\rho-1} h'(G^D)) \tag{A2}$$

1. From (A1), $u^N_{\underline{g}} > 0$ if

$$\frac{n(1-\alpha)h(\underline{G})^{\rho-1} h'(\underline{G})}{\alpha f(w-\underline{g})^{\rho-1} f'(w-\underline{g})} > 1 \Leftrightarrow n \cdot MPCR > 1,$$

which holds by assumption.
From (A2), $u^D_{\underline{g}} < 0$ if

$$\frac{(1-\alpha)h(G^D)^{\rho-1} h'(G^D)}{\alpha f(w-\underline{g})^{\rho-1} f'(w-\underline{g})} < 1 \Leftrightarrow MPCR < 1,$$

which holds by assumption.

2. Taking derivative of the δ^* with respect to $\underline{g}$, ignoring the positive denominator and simplifying yields:

$$\frac{\partial \delta^*}{\partial \underline{g}} = \frac{u^D_{\underline{g}}(u^D - u^N) - (u^D - u^C)(u^D_{\underline{g}} - u^N_{\underline{g}})}{(u^D - u^N)^2}$$
$$\propto \underbrace{u^D_{\underline{g}}}_{(-)} \underbrace{(u^C - u^N)}_{(+)} + \underbrace{u^N_{\underline{g}}}_{(+)} \underbrace{(u^D - u^C)}_{(+)},$$

Search for a sufficiency condition: To understand the forces that could drive a positive relationship between δ^* and $\underline{g}$, I investigate under what conditions, we have i) $u^N_{\underline{g}} > | u^D_{\underline{g}} |$ and ii) $u^D - u^C > u^C - u^N \Leftrightarrow u^D + u^N > 2u^C$.

Condition i) can be written as:

$$u^{N^{1-\rho}} T^N > u^{D^{1-\rho}} | T^D |,$$

where I respectively call the second terms in (A1) and (A2) as T^N and T^D, and T^D is the only negative term. Thus,

$$| T^D | = \alpha f(w - \underline{g})^{\rho-1} f'(w - \underline{g}) - (1-\alpha) h(G^D)^{\rho-1} h'(G^D)$$

Since $u^D > u^N$ and $1-\rho \geq 0$, we have $u^{D^{1-\rho}} \geq u^{N^{1-\rho}}$, which holds with equality only at $\rho = 1$. On the other hand, dividing both T^N and $| T^D |$ by the private good term: $\alpha f(w-\underline{g}) u^{\rho-1} f'(w-\underline{g})$ does not change the order of magnitudes, thus $T^N \geq | T^D |$ if

$$-1 + \frac{n(1-\alpha)h(\underline{G})^{\rho-1} h'(\underline{G})}{\alpha f(w-\underline{g})^{\rho-1} f'(w-\underline{g})} \geq 1 - \frac{(1-\alpha)h(G^D)^{\rho-1} h'(G^D)}{\alpha f(w-\underline{g})^{\rho-1} f'(w-\underline{g})}$$
$$\Leftrightarrow \frac{n(1-\alpha)h(\underline{G})^{\rho-1} h'(\underline{G}) + (1-\alpha)h(G^D)^{\rho-1} h'(G^D)}{\alpha f(w-\underline{g})^{\rho-1} f'(w-\underline{g})} \geq 2$$
$$\Leftrightarrow \frac{(1-\alpha)}{\alpha} \cdot \frac{n \frac{h'(\underline{G})}{h(\underline{G})^{1-\rho}} + \frac{h'(G^D)}{h(G^D)^{1-\rho}}}{\frac{f'(w-\underline{g})}{f'(w-\underline{g})^{1-\rho}}} \geq 2$$

First, all terms on the LHS is positive. The last step facilitates the terms, since $1-\rho \geq 0$ for any ρ and holds with equality only at $\rho = 1$.

For any $\underline{g}, g^C, w, \rho$,

- The smaller is the α, the easier for $T^N \geq |T^D|$ to hold. However, from (11), the smaller the α, the larger is the u^D over u^N, since $G^D > \underline{G}$. Thus, the overall effect of α on condition i) is ambiguous.
- The impact of n depends on the function h since in addition to its direct effect, $h(\cdot)$ increase with n, among others. Also, its impact on h' depends on whether the function is concave, linear, or convex. Regardless, the impact is inconclusive without the exact form of h – more on this below.
- The functions f, h being (more) concave or convex (or linear) have inconclusive impacts on the relationship, since both numerator and denominator in h'/h and f'/f change in the same direction and one cannot know which of the two change faster without the exact form of the functions f, h. Specifically, since $g^C > \underline{g}$, $G^D > \underline{G}$. Because h is increasing and $1-\rho \geq 0$ for any $\rho \in (-\infty, 1]$,

  $$h(G^D)^{1-\rho} \geq h(\underline{G})^{1-\rho},$$

  which holds with equality only at $\rho = 1$. On the other hand, $h'(G^D) < h'(\underline{G})$ for concave, $h'(G^D) = h'(\underline{G})$ for linear, and $h'(G^D) > h'(\underline{G})$ for convex h.

A sufficient condition for $T^N \geq |T^D|$ can be for the function h to exponentially increasing. However, in this case we have again $u^{D^{1-\rho}} \geq u^{N^{1-\rho}}$ and, thus, the overall effect remains ambiguous.

Condition ii) Let me first rewrite (11), which can help reader to follow the next part:

$$u^C = (\alpha f(w - g^C)^\rho + (1-\alpha)h(G^C)^\rho)^{\frac{1}{\rho}}, \text{ where } G^C = ng^C$$
$$u^N = (\alpha f(w - \underline{g})^\rho + (1-\alpha)h(\underline{G})^\rho)^{\frac{1}{\rho}}, \text{ where } \underline{G} = n\underline{g}$$
$$u^D = (\alpha f(w - \underline{g})^\rho + (1-\alpha)h(G^D)^\rho)^{\frac{1}{\rho}}, \text{ where } G^D = \underline{g} + (n-1)g^C$$

Note that $u^D + u^N > 2u^C$ becomes, for:

$$\rho = 1: 2\alpha f(\underline{x}) + (1-\alpha)(h(\underline{G}) + h(G^D)) > 2\alpha f(x^C) + 2(1-\alpha)h(G^C)$$
$$\rho \in (-\infty, 1): (\alpha f(\underline{x})^\rho + (1-\alpha)h(G^D)^\rho)^{\frac{1}{\rho}} + (\alpha f(\underline{x})^\rho + (1-\alpha)h(\underline{G})^\rho)^{\frac{1}{\rho}} >$$
$$2(\alpha f(x^C)^\rho + (1-\alpha)h(G^C)^\rho)^{\frac{1}{\rho}},$$

where $x^C = w - g^C$ and $\underline{x} = w - \underline{g}$ with $\underline{x} > x^C$. Note that we have $f(\underline{x})^\rho > f(x^C)^\rho$ for $\rho \geq 0$ and $f(\underline{x})^\rho < f(x^C)^\rho$ for $\rho < 0$. On the other hand, the comparison of the public goods terms depends both on ρ and whether h is concave, linear, or convex. Regardless, since all these impacts are ambiguous on condition i), the overall effects remain ambiguous without specifying the functions f and h.

3. The cross-partial derivative:

$$\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C} = \frac{(u^D_{\underline{g}g^C}(u^C - u^N) + u^D_{\underline{g}} u^C_{g^C} + u^N_{\underline{g}}(u^D_{g^C} - u^C_{g^C}))(u^D - u^N)^2 - 2(u^D - u^N)u^D_{g^C} N_{\underline{g}}}{(u^D - u^N)^4}$$

where $N_{\underline{g}} = u^D_{\underline{g}}(u^C - u^N) + u^N_{\underline{g}}(u^D - u^C)$ is the numerator of $\frac{\partial \delta^*}{\partial \underline{g}}$. Thus,

$$\frac{\partial^2 \delta^*}{\partial \underline{g} \partial g^C} \propto (\underbrace{u^D_{\underline{g}g^C}}_{(-/?)} \underbrace{(u^C - u^N)}_{(+)} + \underbrace{u^D_{\underline{g}}}_{(-)} \underbrace{u^C_{g^C}}_{(+)} + \underbrace{u^N_{\underline{g}}}_{(+)} \underbrace{(u^D_{g^C} - u^C_{g^C})}_{(?)}) \underbrace{(u^D - u^N)}_{(+)} - 2\underbrace{u^D_{g^C}}_{(+)} \underbrace{N_{\underline{g}}}_{(?)}, \tag{A3}$$

where

$$u^D_{\underline{g}g^C} = \underbrace{(1-\rho)u^{D^{-\rho}}}_{+} \underbrace{u^D_{g^C}}_{+} \underbrace{Z^D}_{-} + \underbrace{u^{D^{1-\rho}}}_{+} \underbrace{Z^D_{g^C}}_{-/?},$$

$$Z^D_{g^C} = (1-\alpha)(n-1)(\underbrace{(\rho-1)}_{-} \underbrace{h(G^D)^{\rho-2} h'(G^D)^2}_{+} + \underbrace{h(G^D)^{\rho-1}}_{+} \underbrace{h''(G^D)}_{+/-})$$

Note that $Z^D_{g^C}$ becomes negative for concave and linear h, and ambiguous for convex h. As a result, $u^D_{\underline{g}g^C}$ is either negative or ambiguous. Furthermore, both the following terms are positive. However, which one is larger than the other is ambiguous for general f and h.

$$u^C_{g^C} = u^{C^{1-\rho}} \cdot (\underbrace{-\alpha f(x^C)^{\rho-1} f'(x^C)}_{-} + n(1-\alpha) \underbrace{h(G^C)^{\rho-1} h' f(G^C)}_{+})$$

$$u^D_{g^C} = u^{D^{1-\rho}} \cdot (n-1)(1-\alpha) \underbrace{h(G^D)^{\rho-1} h'(G^D)}_{+}$$

Thus, the sign of the cross-partial derivative is ambiguous. ■

Proof of Proposition 2.

- **Optimal:** $\max_G \sum u_i = nw - G + na \ln G \Rightarrow G^O = na$ and $g^O = a$.
- **Noncooperation:** Given G_{-i} and using (14), $\max_{g_i} u_i \Rightarrow \text{FOC}: \frac{\partial u_i}{\partial g_i} = -1 +$

$\frac{a}{G}=0 \Rightarrow G^N=a$ and $g^N=\frac{a}{n}$.

- **Deviation:** Given that everyone else contributes the cooperative share g^C,

$$\max_{g_i} u_i^D = w - g_i + a\ln((n-1)g^C + g_i)$$

$$\text{FOC}: \frac{\partial u_i}{\partial g_i} = -1 + \frac{a}{(n-1)g^C + g_i} = 0$$

$$\Rightarrow g_i = a-(n-1)g^C$$

Hence,

$$g_i^D = \begin{cases} 0, & \text{if } g^C \geq \frac{a}{n-1} \\ a-(n-1)g^C, & \text{if } g^C < \frac{a}{n-1} \end{cases}$$

Substituting the respective individual contributions found above into the utility function (14) yields the utility level under cooperation and Nash, u^C and u^N.

For deviation, if $g^C \geq a/(n-1)$, then $g^D = 0$ and thus $\underline{g} > a/n > g^D$ binds. On the other hand, if $g^C < a/(n-1)$, then $g^D = a-(n-1)g^C$. Let $g^C = \frac{a}{n-1} - \varepsilon \Rightarrow$ $g^D = a-(n-1)(\frac{a}{n-1}-\varepsilon) = \varepsilon(n-1)$, where $\varepsilon > 0$ is small enough such that $g^C = \frac{a}{n-1} - \varepsilon > a/n \Leftrightarrow \varepsilon(n-1) < a/n$. Since this also yields $\underline{g} > g^D$, the utility under deviation is:

$$u^D = w - \underline{g} + a\ln((n-1)g^C + \underline{g})$$

Clearly,

$$\frac{\partial u^N}{\partial \underline{g}} = -1 + \frac{a}{\underline{g}} > -1 + \frac{a}{a} = 0,$$

$$\frac{\partial u^D}{\partial \underline{g}} = -1 + \frac{a}{(n-1)g^C + \underline{g}} < -1 + \frac{a}{(n-1)g^C} \leq 0.$$

Condition (12) for quasi-linear utility, $\frac{\partial \delta^*}{\partial \underline{g}} > 0 \Leftrightarrow u_{\underline{g}}^N (u^D - u^C) > |u_{\underline{g}}^D| (u^C - u^N) \Leftrightarrow$

$$(a/\underline{g}-1)(g^C - \underline{g} + a\ln(G^D/G^C)) > (1-a/G^D)(\underline{g} - g^C + a\ln(g^C/\underline{g})) \Leftrightarrow$$

$$\left(\frac{g^C}{\underline{g}} - 1\right) + (a/\underline{g}-1)\ln(G^D/G^C) > \left(\frac{\underline{g} - g^C}{G^D}\right) + (1-a/G^D)\ln(g^C/\underline{g}) \Leftrightarrow$$

$$(a/\underline{g}-1)\ln(G^D/G^C)-(1-a/G^D)\ln(g^C/\underline{g})+\left(\frac{g^C}{\underline{g}}-1\right)+\left(\frac{g^C-\underline{g}}{G^D}\right)>0$$

Notice also that while we assume $g^C > \underline{g}$, if $g^C = \underline{g}$, the condition above holds with equality as all the terms in natural logarithm would be equal to 1 and the rest are equal to 0. ■

Proof of Proposition 3.

- **Optimal:** Drop the subscript i from the utility function and take derivative wrt g:

$$\max_g u = (\alpha(w-g)^\rho + (1-\alpha)(ng)^\rho)^{\frac{1}{\rho}}$$

$$\frac{\partial u}{\partial g} = \frac{1}{\rho}(\alpha(w-g)^\rho + (1-\alpha)(ng)^\rho)^{\frac{1-\rho}{\rho}}(-\alpha\rho(w-g)^{\rho-1} + (1-\alpha)\rho n^\rho g^{\rho-1}) = 0$$

$$\Rightarrow -\alpha\rho(w-g)^{\rho-1} + (1-\alpha)\rho n^\rho g^{\rho-1} = 0 \Leftrightarrow$$

$$\left(\frac{w-g}{g}\right)^{\rho-1} = \frac{(1-\alpha)}{\alpha}n^\rho \Rightarrow \frac{g}{w-g} = \left(\frac{1-\alpha}{\alpha}n^\rho\right)^{\frac{1}{1-\rho}}$$

Let

$$\kappa := \left(\frac{1-\alpha}{\alpha}n^\rho\right)^{\frac{1}{1-\rho}} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\rho}} n^{\frac{1}{1-\rho}}.$$

Then, $\frac{g}{w-g} = \kappa$ and thus,

$$g^O = \frac{\kappa}{\kappa+1}w \quad \text{and} \quad w - g^O = \frac{1}{\kappa+1}w.$$

- *Cobb-Douglas:* As $\rho \to 0$, $\kappa \to \frac{1-\alpha}{\alpha}$ and $g^O \to (1-\alpha)w$.
- *Leontief:* As $\rho \to -\infty$. $\kappa \to 1$ and $g^O \to \frac{w}{2}$ as $\rho \to -\infty$.
- *Linear:* As $\rho \to 1$,

$$\kappa \to \begin{cases} 0, & \text{if } \frac{1-\alpha}{\alpha}n < 1, \\ 1, & \text{if } \frac{1-\alpha}{\alpha}n = 1, \\ \infty, & \text{if } \frac{1-\alpha}{\alpha}n > 1. \end{cases} \qquad g^O \to \begin{cases} 0, & \text{if } \frac{1-\alpha}{\alpha}n < 1, \\ \frac{1}{2}w, & \text{if } \frac{1-\alpha}{\alpha}n = 1, \\ w, & \text{if } \frac{1-\alpha}{\alpha}n > 1. \end{cases}$$

- **Non-Cooperation:** Given G_{-i} and (16), $\max_{g_i} u_i$ yields:

$$\text{FOC}: \frac{\partial u_i}{\partial g_i} = \frac{1}{\rho}(\alpha(w_i - g_i)^\rho + (1-\alpha)(g_i + G_{-i})^\rho)^{\frac{1-\rho}{\rho}}(-\alpha\rho(w_i - g_i)^{\rho-1}$$
$$+(1-\alpha)\rho(g_i + G_{-i})^{\rho-1}) = 0$$
$$\Rightarrow -\alpha\rho(w_i - g_i)^{\rho-1} + (1-\alpha)\rho(g_i + G_{-i})^{\rho-1} = 0$$
$$\Leftrightarrow (1-\alpha)(ng)^{\rho-1} = \alpha(w-g)^{\rho-1}$$
$$\Leftrightarrow \left(\frac{w-g}{ng}\right)^{\rho-1} = \frac{1-\alpha}{\alpha} \Leftrightarrow \left(\frac{ng}{w-g}\right)^{1-\rho} = \frac{1-\alpha}{\alpha}$$
$$\Leftrightarrow \frac{g}{w-g} = n^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\rho}} = n^{\frac{-1}{1-\rho}} n^{\frac{\rho}{1-\rho}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\rho}} = \kappa n^{\frac{-1}{1-\rho}}$$
$$\Leftrightarrow g^N = \frac{\kappa n^{\frac{-1}{1-\rho}}}{\kappa n^{\frac{-1}{1-\rho}} + 1} w = \frac{\frac{\kappa}{n^{\frac{1}{1-\rho}}}}{\frac{\kappa}{n^{\frac{1}{1-\rho}}} + 1} w = \frac{\kappa}{\kappa + n^{\frac{1}{1-\rho}}} w$$

– *Cobb-Douglas:* As $\rho \to 0$, $\kappa \to \frac{1-\alpha}{\alpha}$ and $g^N \to \frac{1-\alpha}{1-\alpha+\alpha n} w$ (so, $g^N < g^O$)
– *Leontief:* As $\rho \to -\infty$, $\kappa \to 1$ and $g^N \to \frac{w}{2}$. So, $g^N = g^O$ in this case.
– *Linear:* As $\rho \to 1$,

$$\kappa \to \begin{cases} 0, & \text{if } \frac{1-\alpha}{\alpha} n < 1, \\ 1, & \text{if } \frac{1-\alpha}{\alpha} n = 1, \\ \infty, & \text{if } \frac{1-\alpha}{\alpha} n > 1. \end{cases} \quad\Bigg|\quad g^N \to \begin{cases} 0, & \text{if } \frac{1-\alpha}{\alpha} n < 1, \\ \frac{1}{2} w, & \text{if } \frac{1-\alpha}{\alpha} n = 1, \\ w, & \text{if } \frac{1-\alpha}{\alpha} n > 1. \end{cases}$$

So, $g^N = g^O$ in this case.

- **Deviation:** Given that everyone else contributes the cooperative share g^C,

$$\max_{g_i} u_i^D = (\alpha(w_i - g_i)^\rho + (1-\alpha)((n-1)g^C + g_i)^\rho)^{\frac{1}{\rho}}$$
$$\frac{\partial u_i}{\partial g_i} = \frac{1}{\rho}(\alpha(w - g_i)^\rho + (1-\alpha)((n-1)g^C + g_i)^\rho)^{\frac{1-\rho}{\rho}}$$
$$(-\alpha\rho(w - g_i)^{\rho-1} + (1-\alpha)\rho((n-1)g^C + g_i)^{\rho-1}) = 0$$
$$\Rightarrow -\alpha(w - g_i)^{\rho-1} + (1-\alpha)((n-1)g^C + g_i)^{\rho-1} = 0$$
$$\Leftrightarrow (1-\alpha)((n-1)g^C + g_i)^{\rho-1} = \alpha(w - g_i)^{\rho-1}$$
$$\Leftrightarrow \frac{1-\alpha}{\alpha} = \left(\frac{(n-1)g^C + g_i}{w - g_i}\right)^{1-\rho} \Leftrightarrow \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\rho}} = \frac{(n-1)g^C + g_i}{w - g_i}$$

$$\Leftrightarrow w\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\rho}} -(n-1)g^{C} = g_i\left(1+\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\rho}}\right)$$

$$\Leftrightarrow g^{D} = \frac{w(\frac{1-\alpha}{\alpha})^{\frac{1}{1-\rho}} -(n-1)g^{C}}{1+(\frac{1-\alpha}{\alpha})^{\frac{1}{1-\rho}}} = \frac{w\kappa n^{\frac{-\rho}{1-\rho}} -(n-1)g^{C}}{1+\kappa n^{\frac{-\rho}{1-\rho}}}$$

- *Cobb-Douglas:* As $\rho \to 0$, $\kappa \to \frac{1-\alpha}{\alpha}$ and $g^{D} \to w(1-\alpha)-\alpha(n-1)g^{C}$. For Cobb-Douglas, $g^{D} \le g^{N}$ always holds:

$$w(1-\alpha)-\alpha(n-1)g^{C} \le \frac{1-\alpha}{1-\alpha+\alpha n}w,$$

which binds the most at $g^{C} = g^{N}$ and holds with equality:

$$w(1-\alpha)-\alpha(n-1)\left(\frac{1-\alpha}{1-\alpha+\alpha n}w\right) = \frac{1-\alpha}{1-\alpha+\alpha n}w$$

$$1-\alpha(n-1)\left(\frac{1}{1-\alpha+\alpha n}\right) = \frac{1}{1-\alpha+\alpha n}$$

$$1-\alpha+\alpha n-\alpha(n-1) = 1.$$

All terms cancel each other out.

- *Leontief:* As $\rho \to -\infty$, $\kappa \to 1$ and $g^{D} \to \frac{wn-(n-1)g^{C}}{1+n}$.
- *Linear:* As $\rho \to 1$,

$$\kappa \to \begin{cases} 0, & \text{if } \frac{1-\alpha}{\alpha}n<1, \\ 1, & \text{if } \frac{1-\alpha}{\alpha}n=1, \\ \infty, & \text{if } \frac{1-\alpha}{\alpha}n>1. \end{cases} \Bigg| \quad g^{D} \to \begin{cases} 0, & \text{if } \frac{1-\alpha}{\alpha}n<1, \\ \max\{0, \frac{w-(n-1)g^{C}}{2}, & \text{if } \frac{1-\alpha}{\alpha}n=1, \\ w, & \text{if } \frac{1-\alpha}{\alpha}n>1. \end{cases}$$

The utility functions and the relevant partial derivatives for (12) are:

$$u^{C} = (\alpha(w-g^{C})^{\rho} + (1-\alpha)(G^{C})^{\rho})^{\frac{1}{\rho}},$$

$$u^{N} = (\alpha(w-\underline{g})^{\rho} + (1-\alpha)(n\underline{g})^{\rho})^{\frac{1}{\rho}},$$

$$u^{D} = (\alpha(w-\underline{g})^{\rho} + (1-\alpha)((n-1)g^{C}+\underline{g}))^{\rho})^{\frac{1}{\rho}}$$

$$u^{N}_{\underline{g}} = \frac{1}{\rho}(u^{N})^{\frac{1-\rho}{\rho}}(-\alpha\rho(\underline{x})^{\rho-1} + (1-\alpha)\rho n(\underline{G})^{\rho-1})$$

$$u^{D}_{\underline{g}} = \frac{1}{\rho}(u^{D})^{\frac{1-\rho}{\rho}}(-\alpha\rho(\underline{x})^{\rho-1} + (1-\alpha)\rho(G^{D})^{\rho-1})$$

The condition (12) for the CES utility, i.e., $\frac{\partial\delta^*}{\partial\underline{g}} > 0 \Leftrightarrow u_{\underline{g}}^N(u^D - u^C) > |u_{\underline{g}}^D|(u^C - u^N) \Leftrightarrow$

$$u^{N\frac{1-\rho}{\rho}}(-\alpha(\underline{x})^{\rho-1} + (1-\alpha)n(\underline{G})^{\rho-1})(u^D - u^C) >$$
$$u^{D\frac{1-\rho}{\rho}}(\alpha(\underline{x})^{\rho-1} - (1-\alpha)(G^D)^{\rho-1})(u^C - u^N)$$

Notice that only some ρ's cancels out. Rewriting this condition:

$$\left(\frac{u^N}{u^D}\right)^{\frac{1-\rho}{\rho}}\left(\frac{-\alpha\underline{x}^{\rho-1} + (1-\alpha)n\underline{G}^{\rho-1}}{\alpha\underline{x}^{\rho-1} - (1-\alpha)G^{D^{\rho-1}}}\right)\left(\frac{u^D - u^C}{u^C - u^N}\right) > 1$$

Cobb-Douglas: The utility functions for Cobb-Douglas preferences:

$$u^C = x^{C^\alpha} G^{C^{1-\alpha}},$$
$$u^N = \underline{x}^\alpha \underline{G}^{1-\alpha},$$
$$u^D = \underline{x}^\alpha G^{D^{1-\alpha}}$$

Recall Proposition 1 for the signs.

$$u_{\underline{g}}^N = (w-\underline{g})^{\alpha-1}(n\underline{g})^{-\alpha}[-\alpha n\underline{g} + n(1-\alpha)(w-\underline{g})] > 0$$
$$= \underline{x}^{\alpha-1}(\underline{G})^{-\alpha} n\underbrace{[w(1-\alpha) - \underline{g}]}_{+} > 0$$
$$u_{\underline{g}}^D = (w-\underline{g})^{\alpha-1}((n-1)g^C + \underline{g})^{-\alpha}[-\alpha((n-1)g^C + \underline{g}) + (1-\alpha)(w-\underline{g})] < 0$$
$$= \underline{x}^{\alpha-1}(G^D)^{-\alpha}\underbrace{[-\alpha G^D + (1-\alpha)\underline{x}]}_{-} < 0$$

The condition (12) for the Cobb-Douglas utility, i.e., $\frac{\partial\delta^*}{\partial\underline{g}} > 0$ if

$$\underline{G}^{-\alpha}[-\alpha\underline{G} + n(1-\alpha)\underline{x}](\underline{x}^\alpha G^{D^{1-\alpha}} - x^{C^\alpha} G^{C^{1-\alpha}}) >$$
$$G^{D^{-\alpha}}[\alpha G^D - (1-\alpha)\underline{x}](x^{C^\alpha} G^{C^{1-\alpha}} - \underline{x}^\alpha \underline{G}^{1-\alpha}),$$

Notice that $\underline{x}^{\alpha-1}$ canceled from both sides and used $|u_{\underline{g}}^D|$, thus all terms are positive. The condition

$$\left(\frac{G^D}{\underline{G}}\right)^\alpha\left(\frac{n(1-\alpha)\underline{x} - \alpha\underline{G}}{\alpha G^D - (1-\alpha)\underline{x}}\right)\left(\frac{\underline{x}^\alpha G^{D^{1-\alpha}} - x^{C^\alpha} G^{C^{1-\alpha}}}{\underline{x}^{C^\alpha} G^{C^{1-\alpha}} - \underline{x}^\alpha \underline{G}^{1-\alpha}}\right) > 1.$$

References

Barrett, S. (2005), "The Theory of International Environmental Agreements," Handbook of Environmental Economics, edited by K.-G. M¨aler and J. R. Vincent, Elsevier, 3(28), 1457–1516. https://doi.org/10.1016/S1574-0099(05)03028-7.

Barrett, S. and A. Dannenberg (2016), "An Experimental Investigation into 'Pledge and Review' in Climate Negotiations," *Climatic Change*, 138, 339–351, https://doi.org/10.1007/s10584-016-1711-4.

Buchholz, W., A. Haupt, and W. Peters (2005), "International Environmental Agreements and Strategic Voting," *Scandinavian Journal of Economics*, 107, 175-195. https://doi.org/10.1111/j.1467-9442.2005.00401.x.

Bergstrom, T., L. Blume, and H. Varian (1986), "On the Private Provision of Public Goods," *Journal of Public Economics*, 29(1), 25–49. https://doi.org/10.1016/0047-2727(86)90024-1.

Cornes, R., and R. Hartley (2007), "Weak Links, Good Shots and Other Public Good Games: Building on BBV," *Journal of Public Economics*, 91(9), 1684–1707. https://doi.org/10.1016/j.jpubeco.2007.07.007.

Fudenberg, D., and E. Maskin (1986), "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information," *Econometrica*, 54(3), 533–554. https://doi.org/10.2307/1911307.

Hadjiyiannis, C., D. İriş, and C. Tabakis (2012a), "International Environmental Cooperation under Fairness and Reciprocity," *The B.E. Journal of Economic Analysis & Policy*, 12(1). https://doi.org/10.1515/1935-1682.2917.

______________________________ (2012b), "Multilateral Tariff Cooperation under Fairness and Reciprocity," *Canadian Journal of Economics*, 45(3), 925–941. https://doi.org/10.1111/j.1540-5982.2012.01721.x.

Friedman, J. W. (1971), "A Non-cooperative Equilibrium for Supergames," *The Review of Economic Studies*, 38(1), 1–12. https://doi.org/10.2307/2296617.

Liu, W. L., and T. Sandler (2024), "Public Goods, Group Size, and Provision Aggregation," *Journal of Economic Behavior & Organization*, 223, 146–167. https://doi.org/10.1016/j.jebo.2024.04.028.

Park, W. K. (1987), "A Theoretical Analysis of the Efficient Provision of a Public Good," *Korean Economic Review*, 2, 157–172.

Pecorino, P., (1999), "The Effect of Group Size on Public Good Provision in a Repeated Game Setting," *Journal of Public Economics*, 72(1), 121–134. https://doi.org/10.1016/S0047-2727(98)00091-7.

무한 반복 공공재 게임에서의 최소 기여 요건*

Doruk İriş**

초 록 본 연구는 무한 반복 게임에서 최소 기여 요건이 공공재 제공에 미치는 영향을 분석한다. 정적인 상황에서는 이러한 요건이 비협조적 내쉬 균형을 넘어 후생을 향상시키는 것으로 알려져 있으나, 협력이 내쉬 보복 전략에 의존하는 동적 환경에서는 그 효과가 불분명하다. 본 연구는 최소 기여 요건이 내쉬 균형과 합의된 협조 수준 사이에 설정될 경우, 대칭적 플레이어 간 장기 협력을 방해할 수 있음을 규명한다. 이 결과는 준선형 및 CES 효용 함수 하에서 도출되었다.

핵심 주제어: 협력, 공공재, 반복 게임, 최소 기여 요건
경제학문헌목록 주제분류: H41

투고 일자: 2024. 7. 23. 심사 및 수정 일자: 2025. 1. 31. 게재 확정 일자: 2025. 5. 12.
* 서강대학교 경제학과 부교수, e-mail: dorukiris@gmail.com