Two Sectors and Two Shocks: How do Surprise and News Shocks Impact on Economy

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Abstract

I estimate the decomposition of total factor productivity (TFP) shocks by two sectors: (a) investment and (b) consumption. I also identify sectoral shocks by the timing of shocks realization, surprise shocks (unanticipated) and news shocks (anticipated), to investigate negative correlations among TFP in investment sector and macro variables. I find that surprise shocks to investment sector drive recession in short run. In contrast, positive comovements in response to TFP news shocks to investment sector immediately trigger economic boom. A two-sector DSGE (Dynamic Stochastic General Equilibrium) model confirms that price rigidity of investment goods is a key factor to generate the responses to the sector-specific TFP surprise shocks and news shocks as the empirical response: high rigidity for the surprise shocks and low rigidity for the new shocks. This result suggests that the model should adjust with the degree of the price rigidity depending on the surprise shocks or the news shocks.

Keywords: News shocks; Structural vector autoregressive model; Two-sector New Keynesian model; Sectoral total factor productivity;

JEL Classification: E32, O41, O47.

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1 Introduction

What drives economic growth and fluctuations? There are many candidates to answer this question; monetary and fiscal policies, preference, oil price shocks, etc. Among various alternatives, aggregate total factor productivity (hereafter, TFP) has been widely accepted as the main source of business cycles since Prescott (1986) argued that technology shocks account for more than half of fluctuations in the U.S postwar period. Aggregate TFP determines how efficiently an economy produces total output by transforming factors of production such as aggregate labor and capital. Meanwhile, Greenwood, Hercowitz, and Krusell (1997) propose investment-specific technology as the most important driving force of economic growth and fluctuations. It provokes the attention on the role of technology about investment goods.

The research question begins with Figure 1, 2 and Table 1. First, Figure 1 traces out the evolutions of utilization-adjusted TFPs (as proxies of pure technology) in two sectors of the U.S.: consumption and investment[1]. At low frequency, the level of TFP for producing investment goods grows faster than the one in consumption sector. Until mid-1960s, investment-goods-producing technology came along with consumption-goods-producing technology at the similar growth rates. Afterwards, the growth rate of technology in investment sector overwhelms the one in consumption sector. Second, Figure 2 shows the ratio of investment to consumption in real term. The fraction of investment relative to consumption has been gradually taking a large part over time before the great depression in 2007-2008. In light of the two graphs, technology shocks to investment sector and capital accumulation would play an important role in economic growth and business cycles.

However, Table 1 shows that, in terms of growth rate, there are negative comovements of macroeconomic variables with TFP to investment sector, whereas TFP to consumption sector is positively correlated with them except hours worked. This data description on correlation

[1]The procedure of utilization adjustment on TFP series follows Basu, Fernald, Kimball (2006) as updated Basu, Fernald, Fisher, Kimball (2013). It is available at http://www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xls.

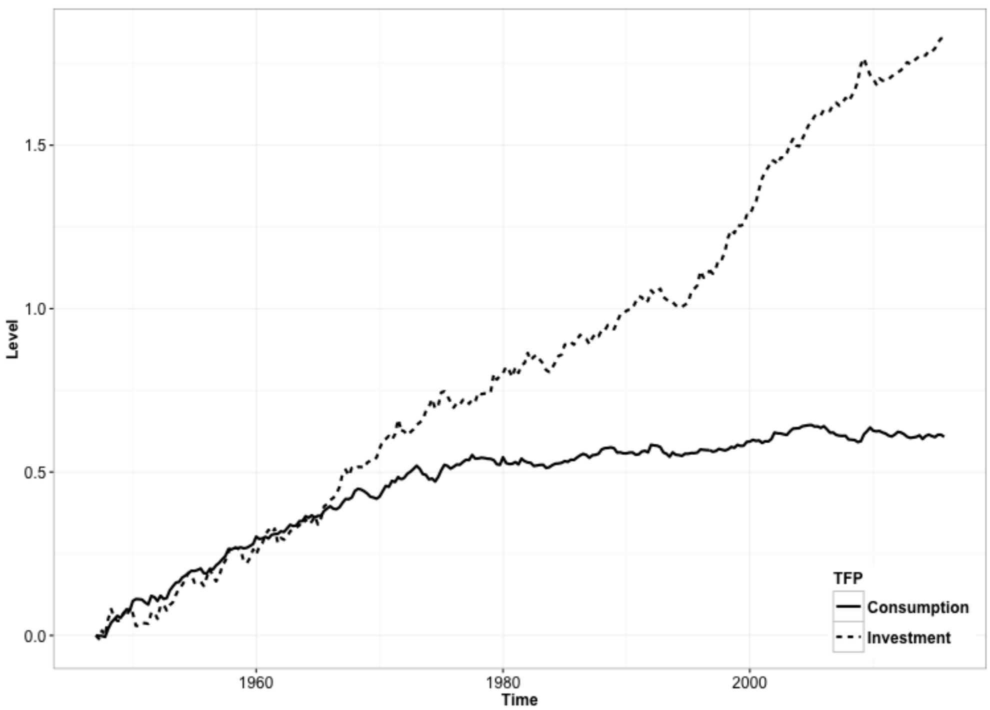

Figure 1: TFP series of consumption sector and investment sector

Note: TFP levels are normalized at zero in 1947:Q1.

coefficients motivates the first question on the role of structural shocks about technology by sectors. In other words, technology shocks would result expansionary or contractionary responses of macro variables in the short run from where the source of structural shocks comes. To explore the counterintuitive consequences on investment sector, I separate TFP shocks into two parts by a structural vector autoregressive model (hereafter, SVAR); (1) surprise shocks which are observed and materialized at the same time and (2) news shocks which are observed in advance before being effective in economy.[2] The economic intuition is that the surprise shocks are responsible for generating the negative correlation because firms are not allowed to choose the optimal level of labors immediately due to sticky price or other real frictions. If this assumption is true, this eventually leads to market inefficiency in short run. Meanwhile, I presume that news shocks possibly serve as an opposite way. News shocks let people change their behavior gradually based on their expectation, so that people

[2]Beaudry and Portier (2006) sheds light on the importance of news shocks which generates economic boom-burst cycle under expectation without changing in technology.

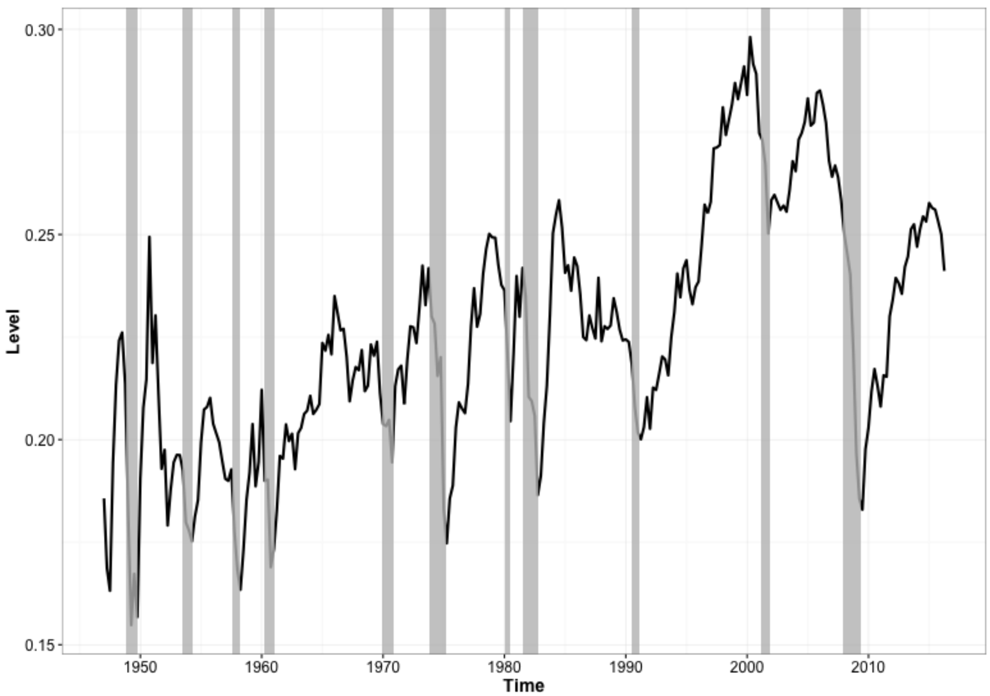

Figure 2: Ratio of investment to consumption in real term.
Note: The shaded areas in gray correspond to NBER recession dates.

increase demand for both consumption and investment goods based on their optimistic views in the future, driving into higher level of the macro variables. Moreover, I incorporate two final-goods sectors in the traditional one-sector New Keynesian model (hereafter, NK model) to compare with the inferences from the identified shocks by SVAR and discuss the potential and limitation of the models.

Table 1: Correlation Coefficients of Growth Rates

	GDP	Consumption	Investment	Hours worked	FFR	Inflation
TFP^I	-0.27	-0.15	-0.29	-0.40	0.05*	-0.03*
TFP^C	0.36	0.18	0.23	-0.14	-0.12*	-0.16

[a] ***Asterisk(*)*** *denotes insignificant for testing a hypothesis of no correlation at a 5% significance level.*

To identify news shocks about TFP progress on both sectors, I define news shocks to impose restrictions into SVAR featuring with an empirical measure of forward-looking variables as Barsky and Sims (2011): news shocks are identified as the shocks that best explain future movements in technology that has no immediate impact on the level of technology, but gradually proliferates its influence on technology. I impose the additional zero restriction on

SVAR which ensures to extract the uncorrelated news shocks on two sectors. Furthermore, I extract the surprise TFP shocks as the reduced-form innovation in the level of utilization-adjusted TFPs in a SVAR, which is implementable by imposing the assumption that surprise shock only determines the level of TFPs in short run. In two-sector NK model, I apply a Bayesian estimation methodology to infer the stochastic dynamics and the fraction explained by the structural shocks such as the surprise shocks and the news shocks to each sector.

The main finding in this paper is twofold. First, the surprise shocks to investment sector drive a recession in short run. It coincides negative correlations of TFP in investment sector with GDP, consumption, investment and hours worked. In contrast, the news shocks drive positive comovements on economy. I also identify surprise shocks and news shocks to consumption sector. In consumption sector, the surprise shocks are expansionary, but the news shocks are contractionary. Second, the estimated two-sector DSGE model is able to generate the same responses as the results of SVAR for the surprise shocks but not the news shocks to both sectors in the short-run. Upon the investigation of parameters, the model confirms that price rigidity of investment goods is a key factor to generate the responses to the sector-specific TFP shocks as the empirical responses: high rigidity for the surprise shocks and low rigidity for the news shocks. This result suggests that the model should adjust with the degree of the price rigidity depending on the surprise shocks or the news shocks.

The rest of the paper is organized as follows. Section 2 explores the VAR identification procedures for identifying sector-specific news shocks and surprise shocks. Section 3 presents the data set: sectoral TFP series and macro variables. Section 4 shows the identification results from SVAR. Section 5 and 6 build the two-sector NK model and investigate its estimation result. Section 7 concludes the research.

2 Identification Strategies

The identification procedure begins at an assumption that a level of technology about each sector follows a stochastic process primarily driven by two structural shocks: surprise and news shocks. The surprise shocks impact the level of sector-specific technology at the same period in which agents observe it. It is identical to a transitory technology shock conventionally used for standard real business cycle models. The next one is news shock to technology that agents can observe it in advance but influences technology later. The time lag from perception to materialization of shocks differentiates news shocks from surprise shocks. Suppose that A_t is the level of technology about a sector and ϵ_t^s and ϵ_t^n are uncorrelated surprise shock and news shock respectively at time t. By following the above definition, I can describe, for instance, the stochastic process of technology with a unit root as:

$$\ln A_t = \ln A_{t-1} + \epsilon_t^s + \epsilon_{t-j}^n \text{ for } j > 0 \tag{1}$$

where ϵ^s and ϵ^n denote surprise shock and news shock respectively. The determinants of technology at time t are the technology at the last period and the current structural shock ϵ_t^s, which is observed and materialized at the same time. Moreover, a term ϵ_{t-j}^n for the news shock also influences the level of current technology.

2.1 News Shocks

I first consider a VAR that includes empirical measures of utilization-adjusted TFP series for two-sector as proxies of pure technology and several macroeconomic aggregates. The news shocks to each sector are identified as the shock that best explains future movements of a target TFP over a horizon and that are orthogonal to current TFPs in both sectors.

Suppose that I have the vector moving average (Wald) representation of a reduced-form

VAR in level driven by structural shocks $\epsilon_t = (\epsilon_t^s \ \epsilon_t^n)'$:

$$Y_t = C(L)u_t = C(L)\tilde{A}Q\epsilon_t \tag{2}$$

where Y_t is a $n \times 1$ vector ($n > 2$) of observables of length T, $C(L)(= I + C_1L + C_2L^2 + \cdots)$ is a lag polynomial, u_t is a $n \times 1$ vector of innovations with variance-covariance matrix $\mathbb{E}[u_t u_t'] = \Sigma$. By assuming that there exist an $n \times n$ matrix $\tilde{A}$ and an $n \times 2$ orthonormal matrix Q for a linear mapping from a vector of structural shocks ϵ_t to innovations such as $u_t = \tilde{A}Q\epsilon_t$, identification problem of structural shocks is the same as identifying an impact matrix $\tilde{A}$ such that $\tilde{A}\tilde{A}' = \Sigma$. However, there are numerous alternatives for choosing $\tilde{A}$. Any impact matrix satisfying $A^*QQ'A^{*'} = \Sigma$ projects the innovations to irrelevant noise ϵ_t^*, not to the structural shocks ϵ_t (i.e. $\epsilon_t^* = Q'A^{*-1}u_t$). The structural shocks, therefore, cannot be uniquely determined by identifying an impact matrix A.

Instead, consider a column vector of the orthonormal matrix Q with a fixed matrix $\tilde{A}$ such as the lower triangle matrix of Cholesky decomposition of the variance-covariance matrix Σ. As Barsky and Sims (2011), the current objective is to choose the column vector of the orthonormal matrix linearly combined to exogenous news shocks ϵ_t^N that best explains the future fluctuations of a target variable (e.g. sectoral TFP) over a forecast horizon and that are orthogonal to the current TFP series of both sectors. For example, I want to show how to identify the news shocks to TFP in consumption sector ("TFP^C" henceforth). Identifying the news shocks to TFP in investment sector ("TFP^I" henceforth) is following the same way.[3] I put TFP^C and TFP^I in the first and second positions in the VAR system orderly and denote a vector of structural shocks as $\epsilon_t = (\epsilon_t^s \ \epsilon_t^n)'$. The h-step ahead forecast error of $\ln TFP_t^C$ in Y_t denoted by $y_{1,t}$ is

$$y_{1,t+h} - \mathbb{E}_t y_{1,t+h} = e_1' \left[\sum_{\tau=0}^{h-1} C_\tau \tilde{A}Q\epsilon_{t+h-\tau} \right] \tag{3}$$

[3] I sequentially identify the sectoral news shocks from different VAR specification in order to consider spillover externalities across two sectors.

where e_1 is a selection vector with 1 in the first position and zero elsewhere. The identification of news shocks to consumption sector requires finding the second column matrix q_2 of the orthonormal matrix Q which maximizes the sum of contribution to the forecast error variance of TFP^C over a range of horizons H subject to the restriction that these shocks have no contemporaneous effect on TFP^C and TFP^I. Formally, this identification strategy requires solving the following optimization problem given the Cholesky decomposition matrix $\tilde{A}$

$$q_2^* = \underset{q_2}{\operatorname{argmax}}\, e_1' \left[\sum_{h=1}^{H} \sum_{\tau=0}^{h-1} C_\tau \tilde{A} q_2 q_2' \tilde{A}' C_\tau' \right] e_1 \quad \text{s.t} \tag{4}$$

$$q_2(1,1) = 0 \tag{5}$$

$$q_2(2,1) = 0 \tag{6}$$

$$q_2' q_2 = 1. \tag{7}$$

The first two constraints assure that news shocks to TFP_C do not affect the current levels of technology in all sectors. The last holds the unit length for the orthonormal vector Q. This optimization problem can be reduced to the problem for finding the eigenvector corresponding to the largest eigenvalue of the matrix by a trace operator. It is called the maximum forecast error variance (hereafter, MFEV) approach. After obtaining the optimal column of the impact matrix q_2, it is straightforward to identify the vector of the sectoral news shocks ϵ^n combining with the matrix, $\tilde{A}$, and the error (residuals) u_t. The analysis on impulse responses and variance decompositions can be performed by Bayesian VAR with Minnesota priors.

2.2 Surprise Shocks

I sequentially identify the surprise technology shocks to each industrial sector as the reduced-form innovation by imposing zero restrictions on all elements of the first column vector q_1 of the impact matrix Q but not in the first position: $q_1 = (1\, 0 \cdots 0)'$.[4] It can be used to extract the contemporaneous shocks after estimating the residual $\hat{u}_t$ in VAR system and then linearly combine it with the impact matrices $\tilde{A}$ and q_1 (i.e. $\epsilon_t^s = q_1' \tilde{A}^{-1} \hat{u}_t$). In addition, the partial identification of the matrix Q infer the dynamic analysis such as impulse responses and variance decompositions.

3 Data

The empirical analysis conducts by using U.S. data over the period 1954-Q4 to 2015-Q3. There are two key TFP series in the VAR exercise: utilization-adjusted TFP series of consumption and investment sectors, respectively.[5] The VAR systems also contain output, consumption, investment, and hours worked to measure the significance of news to macroeconomic variables. In addition, I include Federal Funds Rate (FFR henceforth) as an indicator of monetary policy and inflation in GDP deflator for changes in overall price level. For the Bayesian estimation of the two-sector NK model, I use the same vector of macroeconomic variables for the observation equation. The vector for the observation equation, however, does not include the TFP series because they are set as the exogenous disturbances.

All macroeconomic variables are obtained in quarterly frequency. The nominal series and price indexes for output, consumption, and investment are taken from Bureau of Economic Analysis (BEA). Data for total labor is from Bureau of Labor Statistics (BLS). The dataset is measured in log of per-capita real variables with all seasonally adjusted and based in 2009. In detail, output is Gross Domestic Product (GDP) and consumption is the real values of composite series of expenditures on nondurable goods and services. Investment consists of

[4] It shows with the column vector q_2 that the level of technology is solely determined by the current TFP shocks, realized news shocks, and the level of previous period as the example (1).

[5] The utilization-adjusted TFP series for the U.S. business sector are available from Fernald (2014), measured as percentage changes in quarterly and annually basis, updated on his webpage. They filter the Solow residuals in annual frequency by non-technological factors such as non-constant returns to scale and imperfect competition as well as variable factor utilization that could affect it. Although these high-quality data are only available in annual basis, quarterly utilization-only adjusted TFP series can be a close proxy for pure technology since much of gap between TFP and technology results from utilization

durable goods and fixed investment.[6] Labor is hours worked in the non-farm business sector. Converting real measure of individual components such as GDP is straightforward, but it is not as simple as output to construct real measures of the composite series like consumption and investment because investment consists of durable good consumptions besides fixed investment. Composite real measures can be obtained by weighing growth rates of real variables based on the share of total nominal costs and then convert the weighted growth rates to real variables in levels.Federal funds rate (FFR) is taken from the FRED II database of the St. Louis Fed and inflation is measured as the growth rate of GDP deflator.

4 Identification results

This section presents the main results of the paper based on impulse responses of macro variables to four identified structural shocks: surprise shocks and news shocks to consumption and investment sectors (hereafter, C-sector and I-sector respectively). According to the signs in the responses of output, consumption, and investment on impact, I group economic shocks by expansionary and contractionary technology shocks. I also discuss possible dynamics for my empirical results, point out some unresolved issues, and suggest avenues for future research at the end of each subsection.

4.1 Expansionary shocks

Both traditional real business cycle and New Keynesian models support the positive comovements among macro variables to the positive TFP shocks even though the response of labor (employment or hours worked) is controversial on the reaction to positive shocks about technology. By following the novel identification approach about the structural shocks, surprise shocks to C-sector and news shocks to I-sector about technology prompt positive responses of output, consumption, and investment on impact.

[6] The treatment of consumer durables as a form of investment is standard in the business cycle literature (see for example Cooley and Prescott, 1995; Christiano et al., 2005; Del Negro et al., 2007)

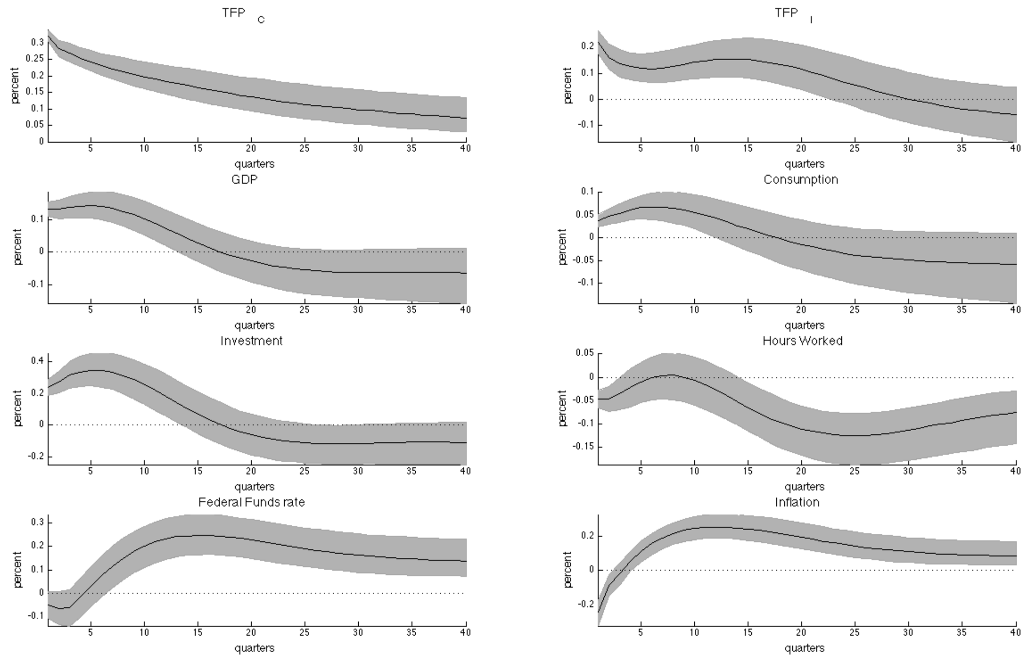

Figure 3: Impulse Responses to Surprise Shocks to Consumption Sector

Note: The shaded areas in gray correspond to 16 to 84 percent posterior coverage intervals.

Figure 3 displays the impulse responses to 1 percent innovation[7] in surprise shocks to C-sector in the baseline VAR model. The positive technology shocks to consumption sector directly lift up the TFP in C-sector and then it gradually decreases in long run like transitory aggregate TFP shocks in real business cycle models. As the pattern of the response of TFP in C-sector, the macro variables also steadily converge to the pre-shock levels after surging or falling on impact: surprise technology shocks to C-sector raise output (0.13%), consumption (0.04%), and investment (0.24%), whereas it decreases hours worked (-0.05%). Distinct from TFP, the responses of macro fundamentals reach at the maximum around 4 to 7 quarters with hump-shaped pattern, which means that the shocks are gradually transmitted to macro variables over time instead of jumping to the highest levels on impact. In response to the unanticipated rise in the level of TPF in C-sector, Federal funds rate decreases a little and inflation significantly declines on impact. Thus, real interest rates drop as a consequence of

[7] All impulse response functions, hereafter, are generated by 1 percent innovation to the corresponding shocks.

surprise consumption shocks.

There are two following interesting questions: what factor does cause the negative response of hours worked and why does investment positively respond to shocks to C-sector? The decline in hours worked is not in line with the property of standard real business cycle models which feature positive comovements of macro variables by positive technology shocks. It is consistent on the empirical results from Galí (1999) and Francis and Ramey (2005) although Figure 3 is derived from the sectoral technology approach instead of aggregate technology.[8] Nominal price rigidity would commonly provide as a source of negative response of labor input. Economic agents optimally react to the positive TFP shocks to C-sector as supply shocks in consumption-good market which leads to a decrease in price and increase in quantity of C-sector. However, the nominal price rigidity in C-sector hinders selecting the optimal choice of price and quantity. It induces the rigidity of demand on labor input for producing consumption goods that forces producing less consumption goods rather than the optimal level of choice under flexible price in response to positive TFP shocks to consumption section. Therefore, I can suggest that firms producing consumption goods do not require as much labor input as pre-shock level. Notwithstanding the drop in the labor input used to produce consumption goods by price rigidity, the amount of investment goods can be boosted if more labor inputs are injected into I-sector. In other words, the positive TFP shocks to C-sector trigger a spillover to I-sector by sectoral input reallocation mechanism.

Next, news shocks about technological progress in I-sector, which are orthogonally identified to the level of total factor productivity in C-sector, have zero effect on its sectoral TFP on impact, but later it consistently increases its level up to the persistent point over time. Figure 4 illustrates the joint dynamics to news shocks about TFP for producing investment goods. It also gives rise to the expansionary economy for all macro variables on impact, even significantly hours worked too. The impulse responses of output, consumption, investment,

[8] Although they conclude a negative response of hours worked (or employment) with only permanents shocks, I derive the same result with respect to surprise and permanent (news) shocks.

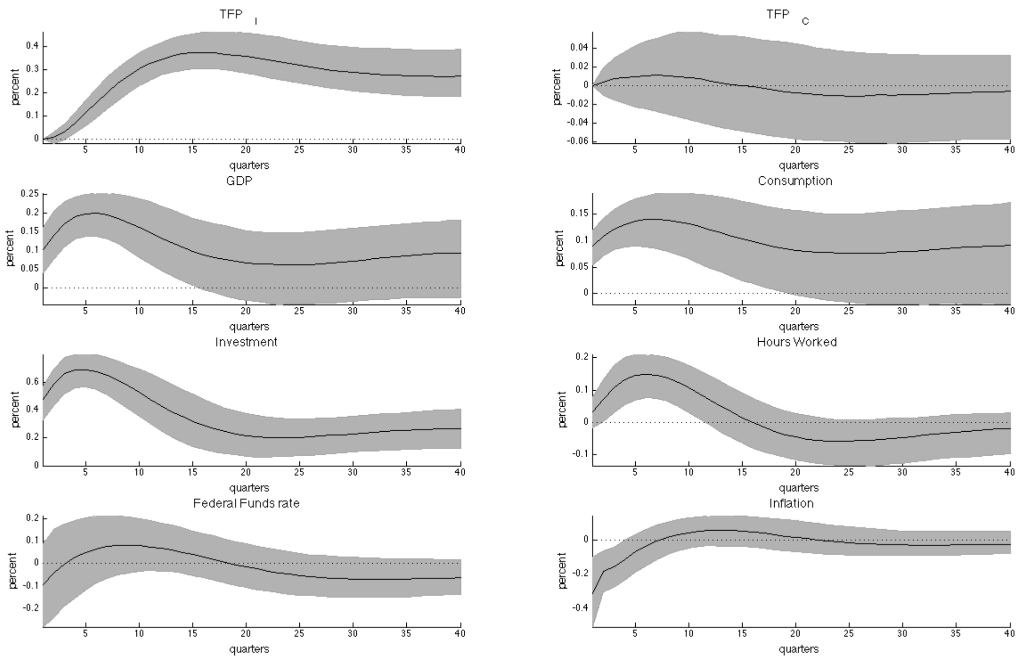

Figure 4: Impulse Responses to News Shocks to Investment Sector

Note: The shaded areas in gray correspond to 16 to 84 percent posterior coverage intervals.

and hours worked on impact are 0.10%, 0.09%, 0.47% and 0.03% respectively. Their maximum responses reach from 4 to 6 period later after arriving the news shocks. In long run, output, consumption, and investment converge to certain levels as TFP of I-sector whereas labor goes to zero.

The overall responses resemble to the responses from the framework of real business cycle models which the exogenous TFP shocks generate positive comovements to all macro fundamentals. Due to the increase in hours worked at economically significant level with the expectation on the surge of TFP level in I-sector, it causes the jumps of macro variables on impact and they ratchet up until the certain convergence levels. The increase in labor cannot be justified by NK models, especially in one-sector model. In detail, the wealth effect of news shocks about aggregate TFP motivates the decline in labor by increasing consumption and leisure. Therefore, the empirical responses to investment news shocks is in line with the research suggested by Jaimovich and Rebelo (2009) to generate positive comovements in the RBC framework. They propose preferences that allow to parameterize the strength of short run wealth effect on the labor supply that generate positive aggregate and sectoral comovements of macro variables by adding three elements into a neoclassical model: variable capital utilization, adjustment costs to investment, and a parameter in preferences. By imposing the elements, they successfully achieve positive responses of aggregate and also sectoral labors to the surprise and news shocks to the fundamentals (technology shocks). In addition, there would be the labor input reallocation between two sectors by the different magnitudes of responses of consumption and investment to investment news shocks. Without the materialization of news shocks on impact, the large proportion of the increase in labor input would be used to produce investment goods through the channel for sectoral input reallocation.

4.2 Contractionary shocks

The remaining two extracted shocks fall into the category of contractionary structural shocks which precipitate negative comovements of key variables on impact: surprise shocks to I-sector and news shocks to C-sector about each sectoral technology. The remarkable point is the large downswing in labor input transmitted via two shocks respectively because it can provide the first clue of recession by the positive shocks in short run. Therefore, it is worth to investigate it at the end of this section about the implication of contractionary shocks how they cause a sizable reduction on labor input.

The identified contemporaneous TFP shocks to I-sector, which I hereafter refer to as surprise investment shocks, enhance the level of TFP for the production of investment goods on impact. Positive TFP shocks would trigger positive comovements of macro variables as expected in light of real business cycle models. However, Figure 5 illustrates that the empirical result is against my expectation. Positive surprise investment shocks are contractionary to all important business-cycle variables, with statistically significant declines in output, consumption, hours worked, and even investment in short run. To be specific, output, con-

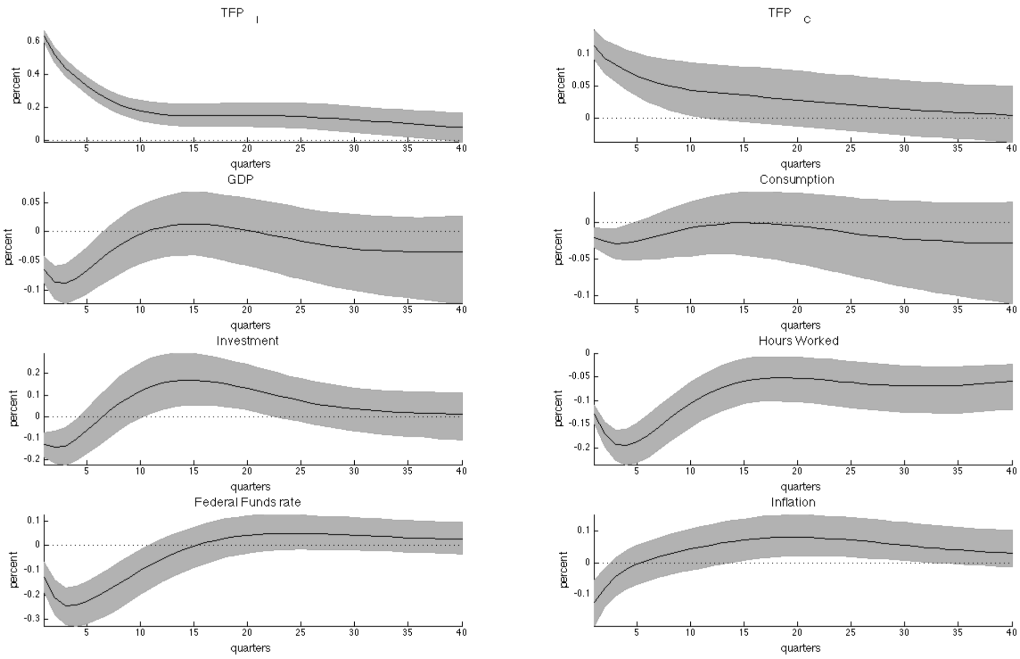

Figure 5: Impulse Responses to Surprise Shocks to Investment Sector

Note: The shaded areas in gray correspond to 16 to 84 percent posterior coverage intervals.

sumption, investment, and labor decline by -0.06%, -0.02%, -0.13%, and -0.13% respectively on impact. Macro variables reach at minimum troughs at about 3 quarters and then they gradually revert near pre-shock levels. Even if output and consumption attain the highest points at 14 quarters, there are no significant signs that surprise investment shocks positively influence these variables according to the confidence intervals from 16% to 84%. Contrary to two variables, investment significantly changes its response to the shocks to positive after 10 quarters and reaches at peak (0.17%) at 13 quarters. In long run, the consequence of surprise investment shocks to investment reverts to zero as other macro variables.

Another contractionary shock is the news shock about TFP at C-sector. Once the notice of news shocks in C-sector, output and investment decrease by following the plummet of labor input at the same time. Despite the shrinkage of hours worked, households are unwilling to change in consumption activity. The consumption smoothing in response to this shock leads to vast drop in investment as well as output. By taking a close look at impulse response

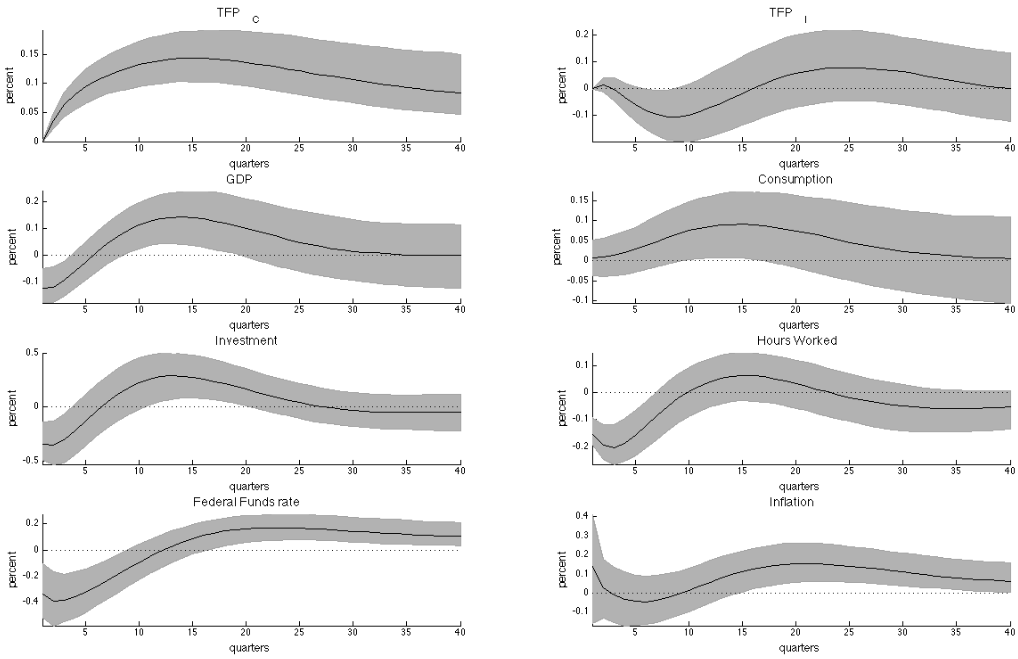

Figure 6: Impulse Responses to News Shocks to Consumption Sector

Note: The shaded areas in gray correspond to 16 to 84 percent posterior coverage intervals.

functions, all macro variables illustrate negative comovements except consumption: output (-0.12%), consumption (0.01%), investment (-0.34%), hours worked (-0.15%). The variables reach at the peak in 13 quarters as TFP in C-sector having hump-shaped behavior. They then return the pre-shock level as the effect of transitory shocks because news shocks to C-sector sluggishly decrease in long run in contrary to other persistent news shocks.

As aforementioned, householders commonly lose their willingness to work in response to those two contractionary structural shocks. In order to explore the impacts of the contractionary shocks, it is necessary to study about what factors cause the sharp drop in labor input. First, the small response of consumption accounts for the fact that there is little change in labor input in C-sector with respect to the surprise investment shock while the large proportion of decline in total labor affects the reduction on production of investment goods by reducing the labor input for producing investment goods. Since the dynamic path of the consumption seems neutral in response to the shocks over the time span, it bolsters the idea that the surprise investment shocks mostly affect I-sector but not C-sector. While the surprise investment shocks only influence its own sector, the news shocks to C-sector proliferate to both sectors. Second, due to the consumption smoothing in the response on the two contractionary shocks, a structural model requires the strong habit formation in consumption to fit the empirical results well. In addition to the habit formation, a structural model requires high adjustment costs of investment to avoid the production of investment goods instead of producing consumption goods. To be specific, wealth effect of positive contractionary shocks about technology induces the increase in the demand on consumption and leisure. However, labor input cannot be assigned to produce the consumption goods because of the existence of habit formation. And also, the labor input cannot flow into I-sector due to high adjustment costs of investment. Therefore, consumption and hours worked decrease and householders leisure goes up.

5 Model

The model is in line with the vintage of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) with wage and price rigidities and several real frictions. It builds a standard medium scale DSGE (Dynamic Stochastic General Equilibrium) model which has a part of production of the final goods in two different sectors: C-sector and I-sector. The aggregate consumption for household directly improves the level of utility and the total investment contributes the accumulation of physical capital as a production input for future.

5.1 Production

5.1.1 Final Goods Firms

The final output goods, $Y_{c,t}$ and $Y_{i,t}$, are constant elasticity of substitution (CES) aggregates of a continuum of intermediate goods indexed from zero to one. As the model splits the final good productions by a sector, the intermediate goods, $Y_{c,t}(m)$ and $Y_{i,t}(n)$, are separately produced in different sectors.

$$Y_{x,t} = \left(\int_0^1 Y_{x,t}(l)^{\frac{1}{\mu_{x,t}}} dl \right)^{\mu_{x,t}} \tag{8}$$

where $x = c,\ i$ and $l = m,\ n$ respectively. $\mu_{x,t}$ determines the elasticity of substitution between differentiated intermediate goods in the two sectors. Following Smets and Wouters (2007), the shock processes for the price markups $\mu_{x,t}$ are

$$\ln \mu_{x,t} = (1 - \rho_x) \ln \mu_x + \rho_x \ln \mu_{x,t-1} + \sigma_x \left(\epsilon_{x,t} - \phi_x \epsilon_{x,t-1} \right) \tag{9}$$

where, for $x = c,\ i$, ρ_x is the AR(1) coefficient, ϕ_x is the MA(1) coefficient, σ_x is the standard deviation, and $\epsilon_{x,t}$ is an i.i.d. standard Gaussian distribution.

Suppose that the final-good markets are perfectly competitive. Hence, the final good firms solve the profit maximization problem by choosing a bundle of intermediate goods given prices of final-goods in each sector. The optimal demands on intermediate goods are downward-sloping to the relative prices to aggregate price levels[9]:

$$Y_{x,t}(l) = \left(\frac{P_{x,t}(l)}{P_{x,t}} \right)^{\left(-\frac{\mu_{x,t}}{\mu_{x,t}-1} \right)} Y_{x,t}. \tag{10}$$

Under the assumption of zero-profit in perfectly competitive markets, a sectoral aggregate price index is:

$$P_{x,t} = \left(\int_0^1 P_{x,t}(l)^{\frac{1}{1-\mu_{x,t}}} dl \right)^{1-\mu_{x,t}} \tag{11}$$

where $x = c,\ i$.

[9]The final good producers solve the optimization problem to maximize their nominal profits by taking the cost as the aggregate of intermediate goods and their costs: $\max_{Y_{x,t}(l)} P_{x,t} Y_{x,t} - \int_0^1 P_{x,t}(l) Y_{x,t}(l) dl$ for $x = c,\ i$ and $l = m,\ n$ respectively.

5.1.2 Intermediate Goods Firms

There are a continuum of intermediate goods firms indexed by $l = m$, n producing differentiated intermediates for exclusive use of each sector in monopolistic competitive markets and the masses of these firms are normalized to one. The intermediate producer produces an output with constant return to scale technology in labor and capital service binding to a fixed cost of production[10]. The productive inputs are capital services $K_{x,t}$ and labor services (hours worked) $L_{x,t}$ paid a nominal wage W_t for $x = c$, i. The production functions are:

$$Y_{c,t} = A_t K_{c,t}(m)^{\alpha_c} L_{c,t}(m)^{1-\alpha_c} - A_t V_t^{\frac{\alpha_c}{1-\alpha_i}} \Phi_c \tag{12}$$

$$Y_{i,t} = V_t K_{i,t}(n)^{\alpha_i} L_{i,t}(n)^{1-\alpha_i} - V_t^{\frac{1}{1-\alpha_i}} \Phi_i \tag{13}$$

where α_c, $\alpha_i \in (0, 1)$ denote capital shares in production.

I model the sectoral TFP processes, A_t and V_t, which basically conforms Schmitt-Grohe and Uribe (2012) and the processes consist transitory shocks and stochastic trends. For example, I specify the TFP in C-sector A_t. The demeaned log of the levels of the sectoral TFP can be described as the sum of two components

$$\log A_t = z_t + x_{c,t} \tag{14}$$

where z_t is the transitory part which represents the surprise shocks process and x_t denotes the stochastic trend part corresponding to the news shocks;

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}^{surprise} \quad \text{and} \quad \Delta x_{c,t} = (1 - \rho_{z,n})\Delta x_c + \rho_{z,n} \Delta x_{c,t-1} + \sigma_{c,x} \epsilon_{z,t-1}^{news} \tag{15}$$

[11]. The structure of the shock process for the I-sector TFP, V_t, has the same process as the

[10] The fixed costs ensure that profits are zero along a stationary balanced growth path and allow us to provide the entry and exit of producers (Christiano et al. (2005)).

[11] Impulse responses to news shocks such as persistent shocks are created by the cumulative summation of impulse responses in Dynare.

C-sector TFP, A_t, with parameters: ρ_v and σ_v for the surprise shocks: $\rho_{v,n}$ and $\sigma_{i,x}$ for the news shocks.

The existence of price rigidity hinders the optimal control of prices as it is necessary to the change in the economy. Hence, instead of profit maximization by price, the firms will choose production inputs so as to minimize cost, given a price, subject to the constraint that it produces enough to meet demand. The cost minimization leads to the optimal input ratio which is the same as the ratio of input prices:

$$\frac{W_t/P_{c,t}}{Q_{i,t}r_{k,t}} = \frac{1-\alpha_x}{\alpha_x}\frac{K_{x,t}(l)}{L_{x,t}(l)} \quad \forall l = m,\, n \quad \text{and} \quad x = c,\, i. \tag{16}$$

$Q_{i,t}$ $(= \frac{P_{i,t}}{P_{c,t}})$ stands for the relative price of investment in consumption units. Since all firms hire labor and capital service at the same ratio and faces the same factor prices, the nominal marginal costs, $MC_{x,t}$, are identical over the producers by a sector[12].

As aforementioned, an intermediate firms are not freely able to adjust a price in every period. In particular, each period there is an exogenous probability that a firm can adjust its price. This is exactly Calvo-style of sticky prices with probability ξ_x. Consider the pricing problem of a firm given the opportunity to adjust its price in a given period. Since there is a chance that the firm will get stuck with its price for multiple periods, the pricing problem becomes dynamic. Firms will discount their nominal profit by the stochastic discount factor $D_{t,t+s}$.[13] I open the possibility that firms that cannot adjust their prices are able to index their prices to the rate between lagged inflation and inflation at steady state.Therefore, the dynamic optimization problem for choosing the prices can be written:

$$\max_{P_{x,t}(l)} \mathbb{E}_t \sum_{s=0}^{\infty} \xi_x^s D_{t,t+s}\left[P_{x,t}(l)\Pi_{t,s}^x - MC_{x,t+s}\right] Y_{x,t+s}(l) \text{ s.t. } Y_{x,t+s}(l) = \left(\frac{P_{x,t}(l)\Pi_{t,s}^x}{P_{x,t}}\right)^{-\frac{\mu_{x,t+s}}{\mu_{x,t+s}-1}} Y_{x,t+s} \tag{17}$$

[12]Hereafter, I refer x to both sectors c and i respectively.

[13]The optimization problem for households will define the stochastic discount factor as the ratio of marginal utility in consumption and nominal prices of consumption goods.

and then the optimal pricing rule for intermediate goods producers in each sector can be written:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \xi_x^s D_{t,t+s} \frac{1}{\mu_{x,t+s} - 1} \left[P_{x,t}(l) \Pi_{t,s}^x - \mu_{x,t+s} MC_{x,t+s} \right] Y_{x,t+s}(l) = 0 \tag{18}$$

5.2 Households

Households choose consumption, bond-holdings, wages, labor supply, capital accumulation, and capital utilization. Assume that each household is a price-taker in the final good markets and a monopolistic competitor in the labor market. Under the assumption on the labor market, households optimally decide their nominal wages for the supply of their differentiated labor inputs given the aggregate wage index and the downward-demand of labor packers, which aggregate the differentiated labor inputs that is sold to the intermediate-goods producers. The process of nominal wage-setting is also in the line with Calvo (1983). Households have to supply the labor to meet the market demand on labor input from the labor packers so long as the nominal wage is settled in each period.

I first consider the problem of the labor packer, which generates a downward-sloping demand for labor and implies the aggregate wage index. Then I consider the problem of the household given the labor packers' decision.

5.2.1 Labor Packer

Labor packers bundle the specialized labor inputs from households to provide the aggregate labor to firms in each sector. As the intermediate good producers, the way to aggregate the differentiated labors into total labor input is identical to CES technology:

$$L_t = \left(\int_0^1 L_t(h)^{\frac{1}{\mu_{w,t}}} dh \right)^{\mu_{w,t}} \tag{19}$$

where $\mu_{w,t}$ is the wage markup and h indexes the differentiated labor, which populates an unit interval.

The profit maximization given wages determines the downward-sloping demand on the differentiated labor input, which shows the negative relationship between relative wage and relative demand on labor:

$$L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\frac{\mu_{w,t}}{\mu_{w,t-1}}} L_t \tag{20}$$

In a way exactly analogous to intermediate goods, an aggregate wage index can be written by zero-profit condition of perfectly competitive market:

$$W_t = \left(\int_0^1 W_t(h)^{\frac{1}{1-\mu_{w,t}}} dh\right)^{1-\mu_{w,t}} \tag{21}$$

5.2.2 Household Problem

A heterogenous household derives utility from consumption and leisure. The assumption on the additively separable preference $U(C_t, L_t)$ in consumption and leisure and the existence of state-contingent claims builds upon the work of Erceg et al. (2000). Without the assumption, households will choose a different set of consumption and leisure because Calvo rigidity on the nominal wage makes households charge different wages. However, Erceg et al. (2000) argue that households would have an identical choice set of consumption, capital accumulation, capital utilization, and bond-holdings with the existence of state contingent claims that hedge against the wage risk caused by the wage friction.[14] For this reason, I omit the household index h except wages and labor input.

The additively separable preference for household $h \in [0, 1]$ is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t U(C_t - bC_{t-1}, L_t(h)) \tag{22}$$

[14] Wage risk share in Christiano et al. (2005)

where $\beta \in (0,1)$ is a subjective discount factor, C_t, denotes consumption, $L_t(h)$, denotes hours worked, and b measures the importance of habit formation. The shock process on the discount factor Λ_t follows

$$\ln \Lambda_t = \rho_\Lambda \ln \Lambda_{t-1} + \sigma_\Lambda \epsilon_{\Lambda,t} \tag{23}$$

where $\rho_\Lambda \in (-1,1)$ is the persistence parameter, σ_Λ is the standard deviation, and $\epsilon_{\Lambda,t}$ is an i.i.d standard normal process.

In each period t, the household faces the budget constraint

$$W_t(h)L_t(h) + P_{i,t}r_{k,t}u_tK_{t-1} + \Pi_t + B_t \geq P_{c,t}C_t + P_{i,t}\left(I_t + a(u_t)K_{t-1}\right) + \mathbb{E}_tD_{t,t+1}B_{t+1} + T_t \tag{24}$$

where I_t stands for investment, $r_{k,t}$ is the real rental rate of capital, B_{t+1} is the nominal bond-holding that is sold at the discount rate $D_{t,t+1}$ in period t and returns the nominal par value B_{t+1} in period $t+1$, $W_t(h)$ is the nominal wage for an individual household h's labor skill, K_{t-1} is the stock of physical capital at the beginning of period, u_t is the utilization rate of physical capital, Π_t is the profit share, and T_t is a lump-sum taxes for the government. The cost function of utilization $a(u_t)$in investment units is increasing and convex. The utilization of physical capital ($u = 1$) at steady state incurs zero cost of utilization, meaning that $a(1) = 0$.

Households accumulate the stock of physical capital up according to the law of motion which will be rent to intermediate producers;

$$K_t = (1-\delta)K_{t-1} + \chi_t\left(1 - S\left(\frac{I_t}{K_{t-1}}\right)\right)I_t \tag{25}$$

where δ denotes the rate of depreciation of physical capital by period. The increasing and convex function $S(\cdot)$ represents the adjustment cost of investment which is proportional to the amount of physical capital accumulated the last period. At steady state, the adjustment cost of investment is also zero as the cost charged in utilization. Justiniano et al. (2011) argue that the marginal efficiency of investment (MEI) χ_t, which affects the transformation of investment into the future physical capital, is the most important driving force of business fluctuation in the post-war period. The shocks evolve in the way of the stationary AR(1) process;

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \sigma_\chi \epsilon_{\chi,t} \tag{26}$$

where $\rho_{chi} \in (0,1)$ is the persistence parameter, σ_χ is the standard deviation, and $\epsilon_{\chi,t}$ is an i.i.d standard normal process.

The household optimally resets the nominal wages for their differentiated labor input to maximize their preference when they are able to freely adjust them given an exogenous probability. Due to the stickiness of wages, households also have to solve a dynamic problem because the possibility on being stuck at the wage determined in the past exists. The first order necessary condition for wage-setting becomes;

$$\mathbb{E}_t \sum_{s=0}^{\infty} \xi_w^s D_{t,t+s} \frac{1}{\mu_{w,t+s} - 1} \left[W_t(h) \Pi_{t,s}^w - \mu_{w,t+s} MRS_{t+s}(h) P_{c,t+s} \right] L_{t+s}(h) = 0 \tag{27}$$

where ξ_w denotes the fraction of households who are not free to optimize their wages, μ_w is a wage-markup which follows the same process as μ_x for intermediate producers, and $MRS_t(h)$ is the marginal rate of substitution between leisure and income for household h. Households who was not permitted to re-optimize the nominal wages apply the indexation rule $\Pi_{t,s}^w$ to partially match with inflation and trends of technological growth.

[14]The product of ratios of stochastic trend combined to the indexation rule for wages ensures the existence of steady state in stationary economy.

5.3 Central Bank and Market Clearing

The monetary policy rule is described by

$$R_t = \kappa R_{t-1}^{\rho_r} \left[r\pi_c \left(\frac{\pi_{c,t}}{\pi_c} \right)^{\phi_\pi} y_t^{\phi_y} \right]^{1-\rho_r} e^{\sigma_r \epsilon_{r,t}} \tag{28}$$

where $\kappa \equiv y^{-\phi_y(1-\rho_r)}$, y is the steady state value of detrended GDP, and r denotes the real interest rate at steady state. ρ_r, ϕ_π, and ϕ_y are the policy parameters about the sensitivity on economic conditions. $\epsilon_{r,t}$ represent a monetary policy shocks with an i.i.d. Gaussian process. σ_r is the standard deviation of the monetary policy shock.

Output (GDP) is set to be measured in consumption goods as the numeraire. Output is defined as

$$Y_t = C_t + Q_{i,t} I_t + \left(1 - \frac{1}{e_t} \right) Y_t \tag{29}$$

where e_t is GDP measurement error.[15]

Total outputs from two sectors are balanced according to

$$Y_{c,t} = C_t \text{ and } Y_{i,t} = I_t + a(u_t) K_{t-1} \tag{30}$$

respectively. The bond market is clearing at $B_t = 0$. The production factor markets holds the equilibrium which the sum of the all factors are the same as the aggregates.

$$L_t = \int_0^1 L_{c,t}(m)dm + \int_0^1 L_{i,t}(n)dn, \;\; u_t K_{t-1} = \int_0^1 K_{c,t}(m)dm + \int_0^1 K_{i,t}(n)dn \tag{31}$$

[15] The GDP measurement error evolves according to $\ln e_t = \rho_e \ln e_{t-1} + \sigma_e \epsilon_{e,t}$ where $\rho_e \in (0,1)$, $\epsilon_{e,t}$ follows i.i.d standard normal distribution, and σ_e is a standard deviation of shock of measurement error.

Lastly, the total demands on the intermediates are the same as what they are supplied.

$$G_{c,t}Y_{c,t} = A_t K_{c,t}^{\alpha_c} L_{c,t}^{1-\alpha_c} - A_t V_t^{\frac{\alpha_c}{1-\alpha_i}} \Phi_c, \text{ where } G_{c,t} \equiv \int_0^1 \left(\frac{P_{c,t}(m)}{P_{c,t}} \right)^{-\frac{\mu_{c,t}}{\mu_{c,t}-1}} dm \tag{32}$$

$$G_{i,t}Y_{i,t} = V_t K_{i,t}^{\alpha_i} L_{i,t}^{1-\alpha_i} - V_t^{\frac{1}{1-\alpha_i}} \Phi_i, \text{ where } G_{i,t} \equiv \int_0^1 \left(\frac{P_{i,t}(n)}{P_{i,t}} \right)^{-\frac{\mu_{i,t}}{\mu_{i,t}-1}} dn \tag{33}$$

6 Bayesian Inference

I solve the model by the mapping of the DSGE model from nonlinear first-order systems of expectational difference equations into linear representation. Bayesian estimation involves the derivation of likelihood from a filtering of data and priors. Based on Bayes Theorem, posterior densities are proportional to the product of the likelihood and the priors, which are obtained numerically by Markov Chain Monte Carlo simulator.[16]

6.1 Calibration and Priors

I adopt the calibration results reported in Smets and Wouters (2007) and Liu, Fernald, and Basu (2012) and calibrate the sample statistics from the data so that I hold a number of the calibrated parameters fixed during the estimation. The parameters are chosen for the calibration if they have already had a general economic consensus over literatures or they are not directly related to the inference on the effect of sector-specific shocks. On econometric purpose, the estimated model can relieve a computational burden by adding precise calibrated parameters.

I choose eighteen parameters fixing during estimation and I report them in Table 2: the subjective discount factor β; the depreciation rate of physical capital δ; sectoral capital share α_c and α_i; sectoral steady state growth rates of TFP g_c and g_i in percent; price and wage markups μ_c, μ_i, and μ_w; sectoral indexations η_c and η_i; persistency, sensitivity of inflation,

[16] All estimations are done by DYNARE (see Adjemian et al. (2011), http://www.dynare.org). I compute the mode under the Sim's method by minimization. I use a random-walk Metropolis-Hastings algorithm for the generation of the posteriors. I try 500,000 draws and them burn in 20% of them.

Table 2: Calibrated parameters

Parameter	Value	Description
β	0.9962	Subjective discount factor
δ	0.0330	Depreciation rate of physical capital
α_c	0.3300	Capital Share in C-sector
α_i	0.3300	Capital Share in I-sector
g_c	0.1410	Steady-state growth rate of TFP in C-sector
g_i	0.4340	Steady-state growth rate of TFP in I-sector
μ_c	1.1100	Steady-state price markup in C-sector
μ_i	1.1100	Steady-state price markup in I-sector
μ_w	1.2000	Steady-state wage markup
ρ_r	0.6000	Persistency of Monetary Policy
π_ϕ	2.5000	Sensitivity of Inflation of C-sector on Monetary Policy
π_y	0.5000	Sensitivity of Output Growth on Monetary Policy
η_c	0.2200	Indexation in C-sector
η_i	0.2200	Indexation in I-sector

and sensitivity of output of monetary policy are ρ_r, π_ϕ, and π_y respectively. The subscripts c and i stand for C-sector and I-sector respectively. They are used to primarily identify the endogenous values at a steady state and the model is linearized around the steady state for parameter estimation.

Table 3 reports the priors. Bayesian estimation of the DSGE model requires the priors which reflect the belief on the parameters before the estimation. The distribution of priors should be chosen by meeting standards from microeconomics evidences and data. For instance, the priors of price rigidities have beta distribution since its feasible values as exogenous probabilities are in $[0, 1]$. All priors for the specific parameters for two-sector model conforms Görtz and Tsoukalas (2016) and others are in line with the Smets and Wouter (2007).

6.2 Posteriors

The last three column in Table 3 shows the mean, 90% highest posterior density interval (HPDI) as a result of the posterior estimation. Most parameters are consistent with Smets and Wouters (2007) except the sector-specific parameters. The sector-specific parameters

Table 3: Prior and posteriro distributions

Parameter	Prior distribution			Posterior distribution		
	Distribution	Mean	SD	Mean	5%	95%
b	Beta	0.7500	0.0293	0.7155	0.7130	0.7200
S''	Gamma	5.0000	0.5000	5.1150	4.5696	5.2772
ξ_c	Beta	0.9000	0.0500	0.9324	0.9099	0.9546
ξ_i	Beta	0.8000	0.0600	0.7992	0.7935	0.8026
ξ_w	Beta	0.7500	0.1000	0.7715	0.7638	0.7870
σ_u	Normal	2.2600	0.3000	2.2415	2.1762	2.2985
η	Beta	0.5000	0.0100	0.4976	0.4064	0.4995
η_w	Beta	0.5900	0.0500	0.5790	0.5632	0.5862
Shock process						
ρ_z	Beta	0.9000	0.1000	0.9228	0.9084	0.9451
ρ_v	Beta	0.9000	0.1000	0.9614	0.9473	0.9721
$\rho_{z,n}$	Beta	0.9000	0.1000	0.9266	0.9132	0.9413
$\rho_{v,n}$	Beta	0.9000	0.1000	0.9632	0.9479	0.9725
ρ_χ	Beta	0.9000	0.1000	0.8019	0.7728	0.8226
σ_z	InvGamma	0.0100	0.1500	0.0142	0.0068	0.0221
σ_v	InvGamma	0.0100	0.1500	0.0727	0.0212	0.1342
$\sigma_{c,x}$	InvGamma	0.0100	0.1500	0.0083	0.0051	0.0135
$\sigma_{i,x}$	InvGamma	0.0100	0.1500	0.0242	0.0135	0.0424
σ_χ	InvGamma	0.0100	0.1500	0.0233	0.0069	0.0457

such as the price rigidities are also consistent with Görtz and Tsoukalas (2016). In contrary to others, the TFP shock process has two components to explore transitory and persistent disturbances together. It shows the shocks process more persistent than the results from other literatures. Regardless of the surprise shocks or the news shocks, the persistency are similar in the same industrial sector. In case of the standard deviation, the shocks in I-sector are more volatile than them in C-sector.

6.3 Variance Decompositions

In this section, I discuss the relative contribution and importance of the TFP stochastic disturbances which proliferate the influence on the economic actors. The forecast error variance decompositions show the fraction of fluctuations explained by a structural shock onto interesting variables. Due to the two separation of final-good sector, the variance

Table 4: Unconditional variance decompositions at posterior estimates

Innovation	C-Surprise	C-News	I-Surprise	I-News	MEI
Output	20	6	19	28	1
Consumption	19	6	45	28	1
Investment	31	9	41	19	0.2
Hours worked	19	9	45	19	1
C-Hours worked	18	5	46	39	1
I-Hours worked	28	9	41	22	0.5
Federal funds rate	18	5	47	29	1

decompositions analysis unveils the role of surprise shocks and news shocks by C-sector and I-sector. I adopt the unconditional variance decomposition to investigate not only the business frequency but also variations in long term, as obtained from the HP filter with smoothing parameter 1,600, because the identified news shocks in the SVAR are inclined to spread persistently to economy. This section answers on two questions. First, the result from the decompositions discovers the relative contribution on the shares of variance in all observables with respect to the sector -specific shocks to C-sector and I-sector respectively. Second, it reveals the significant amount of variation by surprise and news shocks on sectoral TFPs in the model.

The total amount of variations generated by the aggregate technology shocks to I-sector dominates the most of business fluctuations observed on the data.[17] To be specific, the shocks to I-sector account for 47%, 73%, 60%, 64% of the variance in output, consumption, investment, and hours worked respectively. Even if the aggregate technology shock to C-sector explains the business cycles less than its counterpart, it still drives a considerable variation of the observables approximately 20% to 40%. In terms of the relative importance of sectoral technology shocks explained by the two-sector model, TFP shocks to I-sector influence more than 1/3 of the fluctuations in comparison with the shocks to C-sector in general. The conclusion by Greenwood, Hercowitz, Krusell (1997) also emphasize the role of TFP shocks to I-sector as the source of business cycle and economic growth to the economy.

The result from the decompositions gives rise to the question about why the technology

[17] I refer aggregate shocks in this section to the surprise shock and the news shocks altogether.

shocks to the investment shock dominates the fluctuation in all observable, even the variation of consumption. In Chen and Wemy (2015), they explore that the technology shocks in the investment sector, or the capital-producing sector in their terminology, by the maximum forecast error variance approach provoke the spillover into the rest of economy and enhance counterpart technology in long-run as well. According to their research, the spillover effect gradually amplifies the variation of consumption over time from approximately 60% to 80% until 80 quarters through the spillover which improves the technology for producing consumption goods through the technology to the I-sector. The model shows the evidence that their empirical argument on the spillover originated from the investment shocks can be supported by the model in short-run fluctuations according to the variance decompositions.

Görtz and Tsoukalas (2016) argue that consumption specific technology news shocks[18] account for the majority of the shares in economics fluctuations which are 37%, 30%, 31%, 50% of variance in output, consumption, investment and hours worked respectively. In order for their model to amplify the real effect of the news shocks on economy, they augment a DSGE model with a financial channel in each sector. The fractions explained by the aggregate technology news shocks are not distinct too much. Both models appeal the news shocks as a significant driver of business cycles as the surprise shocks. Nevertheless, there is a still big difference in the models on relative importance of news shocks from where they are originated. Even if they claim the strong rise in capital prices by increase in demand on investment goods is key for strong propagation of the news shocks, they cannot explain why the news shocks to the C-sector surpass its counterpart. In contrast, my estimated model shows an opposite result; the news shocks to investment shocks explains at least twice the fluctuations in observables. Although they are successful to generate the significant fluctuations by news shocks, it is unsatisfactory on the crucial role of investment shocks relative to consumption shocks as the empirical researches through spillover effect, neutrality of consumption shocks, etc.

[18] The technology news shocks they implement is the additional term on the shocks process with 2-year ahead observable stochastic disturbances.

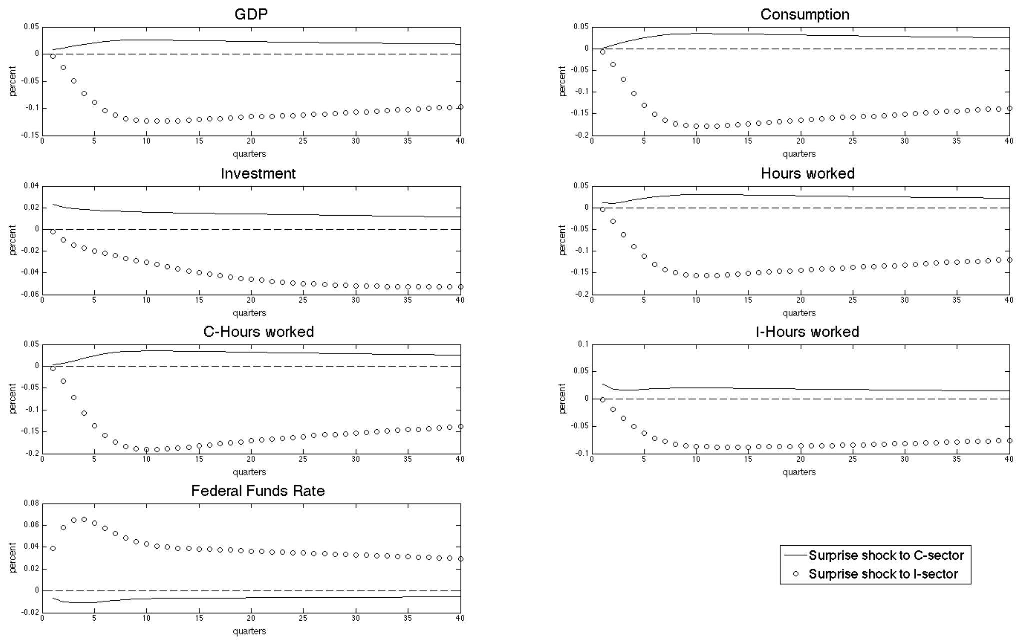

Figure 7: Impulse responses to sector-specific TFP surprise shocks from the NK model.

6.4 Impulse Responses

I explore the economic dynamics to the identified sector-specific shocks via the impulse responses from the SVAR models. It shows that shocks from different sector and different timing of realization cause significantly opposite responses on the U.S. economy. In this section, I would like to study the dynamic reactions to the technology shocks in the two-sector New Keynesian model and compare the responses generated by the model to the empirical results.

Figure 7 shows the estimated impulse responses to the surprise shocks to the sectoral TFP. On the outbreak of positive sectoral shocks, the economy has different adjustment processes to different shocks; the surprise shocks to C-sector triggers the expansions on real economy whereas the counterpart makes it fall into severe recession. The estimated impulse responses to surprise shocks to sectoral total factor productivity share important features with the ones from the SVAR models during short-run. However, the model still has a problem on the amplification of C-sector surprise shocks. In contrary to the empirical result, C-sector surprise shocks elicit only approximately a half amount of responses in the observables via the estimated model. And also, the I-sector surprise shocks persistently transmit its influence and it tend to vanish to zero sluggishly. Although there are minor differences in magnitude and persistency on the estimated model, the estimated DSGE model shows the potential on the replication of transitory sectoral shocks to the level of total factor productivity.

The remaining question is whether the model generates the dynamic interactions to the sectoral news shocks observed in the SVAR models.[19] The answer is no. I obtain the similar result as Görtz and Tsoukalas (2016) which shows economic boom triggered by the C-sector news shocks but severe recession to the I-sector news shocks. The results come from the model of shocks processes which describes the news shocks as only lagged error terms. Hence, the economic conditions brought by the news shocks to the same industry almost resemble the one to the surprise shocks with slight response on impact of the shocks.

Due to the complexity of New Keynesian two-sector model, it is hard to directly figure out the mechanism of the model through precise inspections. Responses of variables influenced by productive factor markets, labor and investment-good, shows how the model generates opposite economic conditions to the sectoral technology shocks at a glance. Lets first look at the responses of labor market and capital market, respectively. C-sector surprise shocks induce decrease in real wage and increase in hours worked in the labor market. Larger increase in labor supply relative to the increase in labor demand can drive this reaction in the labor market. In capital market, increase in demand on (physical) capital pushes the economy, which can be determined by the evidence on increases in rental rate of capital and capital. With respect to the I-sector surprise shocks, every prices and productive factors responses in opposite ways. Consequently, supply on labor and demand on capital plummet in the response to the I-sector surprise shocks. As a result, the two surprise shocks portray totally distinct futures of the economy as I identified in the SVAR models. Whereas, the way to add a news shock to the model in the form of lagged disturbances does not make a

[19] Due to the persistent feature on the news shocks, the impulse responses on the news shocks are the same as the cumulative sum of responses to the lagged shocks in the two-sector model.

big difference to the transitory shocks. Because of the structural resemblance, the responses to the news shocks take the almost same effects on the productive factors markets as the surprise shocks. The non-replicability on the responses to the news shocks is the limitation of the current two-sector models.

To find out the cause of the limitation on the model, I first pick an interesting parameter and then I scrutinize the changes of impulse responses generated by different values within the convergence range while holding all other parameters at estimated values. The parameter controlling the degree of the nominal price rigidity of investment goods[20] is the only parameter which dramatically changes from positive responses to negative responses or vice versa to the shocks to both sectors while others show minor changes in terms of amplitude and persistency. To be specific, the high degree of stickiness of investment goods makes the model replicate the responses to the surprise shocks to both sectors: expansions to C-sector surprise shocks and recessions to I-sector surprise shocks. However, high price stickiness on investment goods does not satisfy the model for the news shocks. In case of the news shocks, low degree of price rigidity of investment goods enables the model to generate the empirical responses.

Why does the model require different degrees of nominal price rigidity of investment goods to generate the empirical responses to surprise and news shocks? A possible explanation is that the nominal price rigidity should be endogenously determined by the timing of realization of the shocks. In New Keynesian models, the intermediate firms set up their prices in advance based on what they expect on the aggregate price level in order to match the optimal relative prices. In response to a surprise shocks, some fraction of firms does not have an enough time to adjust their prices to the realized aggregate price level. However, if the news shocks arrive at the economy, the firms are able to prepare on the change of their prices because the lagged realization of news shocks allows a sufficient time. This factor makes the model requires higher price rigidity for the surprise shocks than the one for the

[20]The degree of price rigidity of investment goods varies from 0.6 to 0.95.

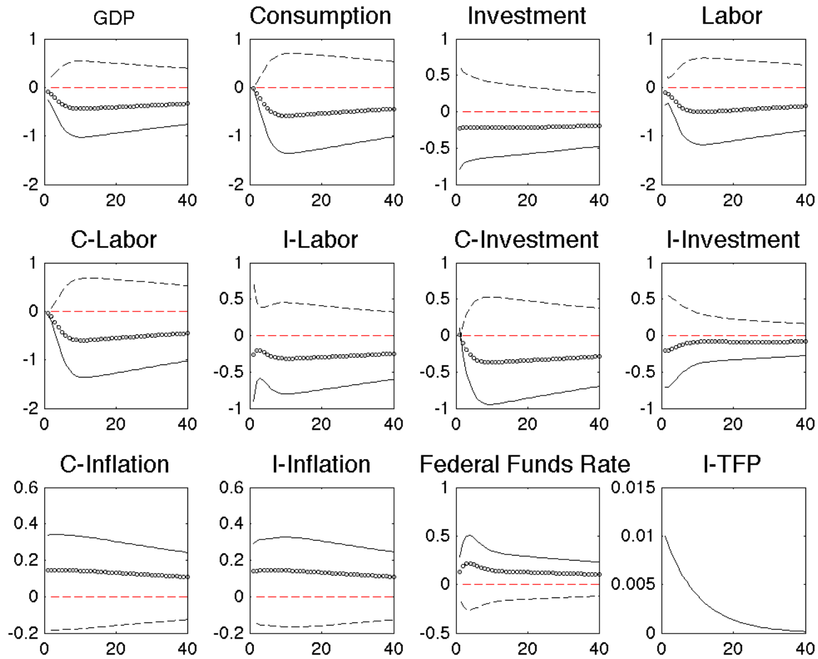

Figure 8: Impulse responses to TFP Shocks to I-sector from low degree (dotted lines) to high degree (solid lines) on price rigidity of investment goods from DSGE model

news shocks.

7 Conclusion

In this paper, I question why TFP in investment sector is negatively correlated with macroeconomic variables: output, consumption, investment, and hours worked. My economic intuition behind it is that the sudden arrival and realization of shocks (e.g., surprise shocks) will cause an inefficiency in I-sector because producers of investment goods cannot adjust their production factors or nominal price on time, but the producers are able to prepare on the advance of TFP with enough time. This insight motivates me to identify the news shocks as well as surprise shocks to two different sectors. By SVAR models, I discover that the news shocks to I-sector trigger economic boom in short run while the surprise shocks to I-sector plunge into recession. In case of C-sector, the surprise shocks are expansionary, but the news shocks are contractionary.

The estimated two-sector DSGE model accounts for the transmission mechanism of the shocks and indicates which parameters are important to replicate the empirical responses from SVAR. The model confirms that a large fraction of the U.S. economy is driven by expansionary shocks: the surprise shocks to C-sector and the news shocks to I-shocks. On the sectoral level, TFP shocks to I-sector dominates the shocks to C-sector through the spillover effects that are supported by Chen and Wemy (2015). Moreover, the model can generate the surprise TFP shocks to the C-sector and the I-sector respectively as the results from SVAR, but it fails to generate the responses to the TFP news shocks. To accomplish it, the nominal price rigidity of investment goods should be adjusted to generate the empirical responses according to the surprise shocks or the new shocks. This result implies the necessity of the endogenous nominal rigidity by the different shock processes.

In conclusion, this paper shows the fruitful opportunity for future research to study the different roles of sectoral TFP shocks on the business cycles of the U.S. economy. Although the vintage of the NK frameworks does not explain all empirical inferences from SVAR, it provides the evidence that the model is still valid for the surprise shocks by sectors, and also it suggests the way to overcome the limitation of the model to satisfy the surprise shocks and the news shocks together.

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Appendix A Utilization-Adjusted TFP series

I adopt quarterly utilization-adjusted series on total factor productivity (hereafter, TFP) which are originally proposed by Basu et al. (2006). Although they focused only on the aggregate total factor productivity in the initial stage, they recently apply the utilization adjustment on the series of sectoral TFP to allow an analysis on the role of investment-specific technical progress. (Basu et al. (2013)) In this section, I would like to discuss about how they define on the aggregate TFP and apply utilization adjustment to introduce a proxy of pure-technology. Furthermore, it presents their method for decomposition of aggregate TFP into investment and consumption components.

First, aggregate TFP is derived from Cobb-Douglas production function in capital and labor. By differentiating the production function, it gives:

$$\Delta \ln Y = \alpha \Delta \ln K + (1-\alpha)\Delta \ln L + \Delta \ln U + \Delta \ln A, \tag{A.1}$$

where $\Delta \ln U = \alpha \Delta \ln Z + (1-\alpha)\Delta \ln E$. Y is a total output in business sector. K and L is aggregate capital input and labor input respectively. Z is capital utilization and E is effort per unit of labor. Given an estimate of the contribution of utilization, $\Delta \ln U$, utilization-adjusted TFP growth is:

$$\Delta \ln TFP_{util} = \Delta \ln Y - \alpha \Delta \ln K - (1-\alpha)\Delta \ln L - \Delta \ln U \tag{A.2}$$

As $\Delta \ln TFP_{util}$ equals technology growth $\Delta \ln A$, it is a proxy of pure technology through the procedure of utilization adjustment on TFP growth. At this moment, the question emerges how to estimate unobservable non-technological factor. To be specific, the challenge is to find an appropriate proxy for unobserved output utilization variation $\Delta \ln U$. Under the idea that a cost-minimizing firm operates on all margins, they adopt hours worked per employee as a good proxy for utilization in industry level and then aggregate them with respect to the industry weight. By implementing above procedures, they are able to build the times series of utilization-adjusted aggregate TFP as technology.

In order to explore the role of sectoral technology, they use relative prices to decompose aggregate TFP into TFPs for consumption sector, C, and investment sector, I. They define investment as the sum of private business equipment, intellectual property investment and durable consumptions. Everything other than investment is regarded the final goods in consumption sector. They express aggregate TFP growth as the weighted average of sectoral TFP growths for both sectors:

$$\Delta \ln TFP = w^I \Delta \ln TFP^I + (1 - w^I) \Delta \ln TFP^c, \tag{A.3}$$

where w^i is the share of sector $i \in (C, I)$. By a cost minimization problem in each sector with strong assumptions, changes in relative TFP is the same as changes in relative prices:

$$\Delta \ln TFP^I - \Delta \ln TFP^C = \Delta \ln P^C - \Delta \ln P^I. \tag{A.4}$$

$\Delta \ln P^I$ is the prices of equipment and software combined with durable consumptions; $\Delta \ln P^C$ is the price of business output except for the price of investment. They impose Equantion A.3 and A.4 quarter by quarter. After deriving sectoral TFP, they use input-output data from Basu et al. (2013) in order to find the measure of utilization for each sector. Finally, they calculate quarterly utilization-adjusted sector-specific TFPs as they did in Equation A.2.

Appendix B Relevance to U.S Recessions

One of drawbacks of neoclassical and real business cycle models is that they do not provide the decent elucidation about recessions. Pigou (1926) argues that an aggregate optimistic expectation about future state of economy can generate boom-recession cycles without the materialization of expectations. In this section, I would like to show the empirical relationship between identified (surprise and news) shocks and recession within sample periods. In order to find out the relevance to recessions with shocks, the shock possibly related to recessions should pass the first test which checks how many recession periods fit to the negative structural shocks. If it passes, I measure the cross-autocorrelations to confirm the shock closely related to recession.

Figure 9, 10 , 11 and 12 illustrate the time series of identified shocks and plot each of them with recession dates as defined by the National Bureau of Economic Research (NBER). To enhance the readability of the time series, I plot one-year moving average on the identified shocks instead of actually identified shocks.[21] I count a number of recession periods associated with each of negative extracted shocks. As a result of the first test, news shocks to investment shocks overwhelm all other shocks in terms of explanation of recessions. It matches with nearly 7 recessions out of 8 recession-period the exception being the early 2000 recession while other shocks explain a little (0, 1, or 2 recessions). In detail, surprise shocks to consumption sector and investment sector overlap 2 and 0 recessions respectively. News shocks to consumption sector only correspond with 1 recession period.

The next stage is to check autocorrelations of the identified shocks with cyclical factors of GDP.[22] I compute the autocorrelation functions of lag k:

$$C_{y^c,\epsilon}(k) = \frac{cov\left(\epsilon_t, y^c_{t+k}\right)}{\sqrt{var(y^c_t)}\sqrt{var(\epsilon_t)}} \text{ for } k = 0, 1, 2, \cdots \tag{B.1}$$

where y^c and ϵ are a cyclical factor of GDP and an identified shock respectively. To figure out the link of shocks on business fluctuations, I measure cross-autocorrelation function (XCF) between cyclical factor of output varying time from 0 to 20 and the shocks holding time 0. The last graph of Figure 13 shows XCF for news shocks to investment that pass the first

[21]Specifically, it is calculated as $\epsilon^A_t = (\epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3})/4$ where ϵ^A_t is the one-year moving average of identified shocks and ϵ_t is the extracted structural shocks.

[22]I apply the HP(Hodrick-Prescott) filter on time series of real GDP with the parameter $\lambda = 1,600$ to separate trend and cycle.

test. At a glance, the each of lagged GDP for 4-period are significantly correlated to the news shocks to investment sector with the exception of zero-lagged GDP on the account of the feature of news shocks. Overall, the news shocks about technology in investment sector are able to explain the most of recession patterns and play a prominent role to drive business cycles within sample periods.

Appendix C Model Details and Derivations

This section provides the model details and derivations required for solution and estimation of the model. I begin with the pricing and wage decisions of firms and households, respectively, followed by the normalization of the model to render it stationary model from nonstationary model, the description of the steady state and the log-linearized model equations.

C.1 Nonstationary Economy

C.1.1 Intermediate Good Producers

There are a continuum of intermediates, so these producers behave as monopolistically competitive. They are not freely able to control prices each period as in Calvo-style sticky prices. Since their optimization problem inherently involves the different prices among the firms, we apply cost minimization on the firms' problem instead of profit maximization by duality. The cost minimization problems on both sectors determines the intermediate good producers optimal choices on prices, labor, and capital.

$$\min_{K_{c,t},L_{c,t}} W_t L_{c,t}(m) + P_{i,t} r_{k,t} K_{c,t}(m) \text{ s.t. } Y_{c,t}(m) = A_t K_{c,t}(m)^{\alpha_c} L_{c,t}(m)^{1-\alpha_c} - A_t V_t^{\frac{\alpha_c}{1-\alpha_i}} \Phi_c \tag{C.1}$$

A Lagrangian is:

$$\mathcal{L} = W_t L_{c,t}(m) + P_{i,t} r_{k,t} K_{c,t}(m) + \phi(m) \left(Y_{c,t}(m) - A_t K_{c,t}(m)^{\alpha_c} L_{c,t}(m)^{1-\alpha_c} + A_t V_t^{\frac{\alpha_c}{1-\alpha_i}} \Phi_c \right) \tag{C.2}$$

The first order conditions show that:

$$\frac{W_t}{P_{i,t} r_{k,t}} = \frac{1-\alpha_c}{\alpha_c} \frac{K_{c,t}(m)}{L_{c,t}(m)} \quad \forall m \tag{C.3}$$

Because the firms have the same factor prices, the optimal factor ratios are the same for all firms and a nominal marginal cost $\phi(m)$ (hereafter $MC_{c,t}$) as well.

$$\frac{W_t / P_{i,t}}{Q_{i,t} r_{k,t}} = \frac{1-\alpha_c}{\alpha_c} \frac{K_{c,t}}{L_{c,t}} \tag{C.4}$$

$$MC_{c,t} = \alpha_c^{-\alpha_c} (1-\alpha_c)^{-(1-\alpha_c)} (P_{i,t} r_{k,t})^{\alpha_c} W_t^{1-\alpha_c} A_t^{-1} \tag{C.5}$$

The firms cannot optimally adjust their prices with the probability ξ_c at each period. However, according to the indexation rule, the firms unable to freely adjust the prices can balance their prices catching up with inflation:

$$P_{c,t}(m) = \pi_{c,t-1}^{\eta_c} \pi_c^{1-\eta_c} P_{c,t-1}(m) \text{ where } \pi_{c,t} = \frac{P_{c,t}}{P_{c,t-1}}. \tag{C.6}$$

The firms choose their prices given the probability of price rigidities which makes their profit maximization problems being dynamic problems with respect to prices:

$$\max_{P_{c,t}(m)} \mathbb{E}_t \sum_{s=0}^{\infty} \xi_c^s D_{t,t+s} \left[P_{c,t}(m) \Pi_{t,s}^c - MC_{c,t+s} \right] Y_{c,t+s}(m) \text{ s.t. } Y_{c,t+s}(m) = \left(\frac{P_{c,t}(m) \Pi_{t,s}^c}{P_{c,t}} \right)^{-\frac{\mu_{c,t+s}}{\mu_{c,t+s}-1}} Y_{c,t+s} \tag{C.7}$$

In this case, the firms also contemplate the indexation rules.

$$\Pi^c_{t,s} = \begin{cases} 1, & \text{if s=0.} \\ \prod_{l=1}^{s} \pi^{\eta_c}_{c,t+l-1}\pi_c^{1-\eta_c}, & \text{otherwise} \end{cases} \tag{C.8}$$

The first order condition simplifies their optimal behavior as

$$\mathbb{E}_t \sum_{s=0}^{\infty} \xi_c^s D_{t,t+s} \frac{1}{\mu_{c,t+s}-1} \left[P_{c,t}(m)\Pi^c_{t,s} - \mu_{c,t+s} MC_{c,t+s}\right] Y_{c,t+s}(m) = 0. \tag{C.9}$$

The intermediate good producers for investment goods follows similar procedure. I write down the equations for the investment sector in detail below for the replicability.

$$\min_{K_{i,t},L_{i,t}} W_t L_{i,t}(n) + P_{i,t} r_{k,t} K_{i,t}(n) \text{ s.t. } Y_{i,t}(n) = V_t K_{i,t}(n)^{\alpha_i} L_{i,t}(n)^{1-\alpha_i} - V_t^{\frac{1}{1-\alpha_i}} \Phi_i \tag{C.10}$$

The first order conditions shows:

$$\frac{W_t}{P_{i,t} r_{k,t}} = \frac{1-\alpha_i}{\alpha_i} \frac{K_{i,t}(n)}{L_{i,t}(n)} \quad \forall n \tag{C.11}$$

$$\frac{W_t/P_{i,t}}{Q_{i,t} r_{k,t}} = \frac{1-\alpha_i}{\alpha_i} \frac{K_{i,t}}{L_{i,t}} \tag{C.12}$$

$$MC_{i,t} = \alpha_i^{-\alpha_i}(1-\alpha_i)^{-(1-\alpha_i)}(P_{i,t} r_{k,t})^{\alpha_i} W_t^{1-\alpha_i} V_t^{-1} \tag{C.13}$$

Firms in investment sector cannot freely adjust their prices increases their prices by matching with inflation:

$$P_{i,t}(n) = \pi^{\eta_i}_{i,t-1}\pi_i^{1-\eta_i} P_{i,t-1}(n) \text{ where } \pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}} \tag{C.14}$$

Their price decision follows the same logic as the consumption sector.

$$\max_{P_{i,t}(n)} \mathbb{E}_t \sum_{s=0}^{\infty} \xi_i^s D_{t,t+s} \left[P_{i,t}(n)\Pi_{t,s}^i - MC_{i,t+s}\right] Y_{i,t+s}(n) \text{ s.t. } Y_{i,t+s}(n) = \left(\frac{P_{i,t}(n)\Pi_{t,s}^i}{P_{i,t}}\right)^{-\frac{\mu_{i,t+s}}{\mu_{i,t+s}-1}} Y_{i,t+s} \tag{C.15}$$

$$, \text{ where } \Pi_{t,s}^i = \begin{cases} 1, & \text{if s=0.} \\ \prod_{l=1}^{s} \pi_{i,t+l-1}^{\eta_i} \pi_i^{1-\eta_i}, & \text{otherwise} \end{cases} \tag{C.16}$$

And finally, the first order condition is simplified to

$$\mathbb{E}_t \sum_{s=0}^{\infty} \xi_i^s D_{t,t+s} \frac{1}{\mu_{i,t+s}-1} \left[P_{i,t}(n)\Pi_{t,s}^i - \mu_{i,t+s} MC_{i,t+s}\right] Y_{i,t+s}(n) = 0. \tag{C.17}$$

C.1.2 Final Good Producers

The final output good is a constant elasticity substitute (CES) aggregate of a continuum of intermediate goods by each sector:

$$Y_{x,t} = \left(\int_0^1 Y_{x,t}(l)^{\frac{1}{\mu_{x,t}}} dl\right)^{\mu_{x,t}} \tag{C.18}$$

, , where $x = C, I$ and $l = m, n$ respectively for intermediate good producers. They maximizes their profits given the final good price and intermediate good prices.

$$\max_{Y_{x,t}(l)} P_{x,t} Y_{x,t} - \int_0^1 P_{x,t}(l) Y_{x,t}(l) dl \tag{C.19}$$

The demand functions for intermediate goods and price index come from the first order conditions.

$$Y_{x,t}(l) = \left(\frac{P_{x,t}(l)}{P_{x,t}} \right)^{\left(-\frac{\mu_{x,t}}{\mu_{x,t-1}}\right)} Y_{x,t} \tag{C.20}$$

$$P_{x,t} = \left(\int_0^1 P_{x,t}(l)^{\frac{1}{1-\mu_{x,t}}} dl \right)^{1-\mu_{x,t}} \tag{C.21}$$

C.1.3 Households

Households are heterogenous with $h \in (0,1)$ and provide differentiated labor input to the intermediate goods firms. The preference with habit formation increases in current and previous consumptions and also labor generates disutility. Households choose their own consumption, bond-holding, wages, labor supply, capital accumulation, investment, and capital utilization. By Erceg (2000), there are only differences in wage and labor supply in this preference. The households' preference is constant relative risk averse (CRRA) utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t U \left(C_t(h) - bC_{t-1}, L_t(h) \right). \tag{C.22}$$

The budget flow is:

$$W_t(h)L_t(h) + P_{i,t}r_{k,t}u_tK_{t-1} + \Pi_t + B_t \geq P_{c,t}C_t + P_{i,t}\left(I_t + a(u_t)K_{t-1}\right) + D_{t,t+1}B_{t+1} + T_t \tag{C.23}$$

They accumulate the physical capital with an adjustment cost.

$$K_t = (1-\delta)K_{t-1} + \chi_t \left(1 - S\left(\frac{I_t}{K_{t-1}} \right) \right) I_t \tag{C.24}$$

The Lagrangian is:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\Lambda_t U\left(C_t - bC_{t-1}, L_t(h)\right)$$

$$+\mu_t \left\{ W_t(h)L_t(h) + P_{i,t} r_{k,t} u_t K_{t-1} + \Pi_t + B_t - P_{c,t}C_t - P_{i,t}\left(I_t + a(u_t)K_{t-1}\right) - D_{t,t+1}B_{t+1} - T_t \right\}$$

$$+ \mu_{k,t} \left\{ (1-\delta)K_{t-1} + \chi_t \left(1 - S\left(\frac{I_t}{K_{t-1}}\right)\right) I_t - K_t \right\} \Bigg] \quad \text{(C.25)}$$

The first order conditions are:

$$\partial C_t(h)\colon\ \Lambda_t U_{c,t} = \mu_t P_{c,t} + \beta b U_{c,t+1}\Lambda_{t+1} \quad \text{(C.26)}$$

$$\partial B_{t+1}\colon\ D_{t,t+1} = \beta \mathbb{E}_t \frac{\mu_{t+1}}{\mu_t} \quad \text{(C.27)}$$

$$\partial I_t\colon\ \mu_t P_{i,t} = \mu_{k,t}\chi_t \left\{ 1 - S\left(\frac{I_t}{K_{t-1}}\right) - S'\left(\frac{I_t}{K_{t-1}}\right)\frac{I_t}{K_{t-1}} \right\} \quad \text{(C.28)}$$

$$\partial K_t\colon\ \mu_{k,t} = \beta \mathbb{E}_t \left[\mu_{t+1} P_{i,t+1}\left(r_{k,t+1}u_{t+1} - a(u_{t+1})\right) + \mu_{k,t+1}\left\{ (1-\delta) + \chi_{t+1} S'\left(\frac{I_{t+1}}{K_t}\right)\left(\frac{I_{t+1}}{K_t}\right)^2 \right\} \right] \quad \text{(C.29)}$$

$$\partial u_t\colon\ r_{k,t} = a'(u_t) \quad \text{(C.30)}$$

Define $Q_{k,t} \equiv \frac{\mu_{k,t}}{\mu_t P_{c,t}}$, Tobin's Q in consumption good unit.

Divide C.28 by $\mu_t P_{c,t}$,

$$Q_{k,t} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}U_{c,t+1}}{\Lambda_t U_{c,t}} \left[Q_{i,t}(r_{k,t+1}u_{t+1} - a(t_{t+1})) + Q_{k,t+1}\left\{ (1-\delta) + \chi_{t+1} S'\left(\frac{I_{t+1}}{K_t}\right)\left(\frac{I_{t+1}}{K_t}\right)^2 \right\} \right] \quad \text{(C.31)}$$

From C.29,

$$
\begin{aligned}
D_{t,t+1} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{U_{c,t+1}}{U_{c,t}} \frac{P_{c,t}}{P_{c,t+1}} \right] \\
\frac{1}{R_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{U_{c,t+1}}{U_{c,t}} \frac{P_{c,t}}{P_{c,t+1}} \right]
\end{aligned} \tag{C.32}
$$

This is the details in the preference:

$$
U(C_t - bC_{t-1}) = \frac{(C_t - bC_{t-1})^{1-\sigma}}{1-\sigma} e^{(\sigma-1)V(L_t)} \text{ where } V(L_t) = \psi \frac{L_t^{1+\eta}}{1+\eta} \tag{C.33}
$$

$$
U_{c,t} = (C_t - bC_{t-1})^{-\sigma} e^{(\sigma-1)V(L_t)} V'(L_t) - \beta b \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (C_{t+1} - bC_t)^{-\sigma} e^{(\sigma-1)V(L_{t+1})} \tag{C.34}
$$

$$
-U_{l,t} = (C_t - bC_{t-1})^{1-\sigma} e^{(\sigma-1)V(L_t)} V'(L_t). \tag{C.35}
$$

As the intermediate good producers, households also cannot freely set their nominal wages each period. Each period, they can adjust their wages with the probability $1 - \xi_w$. Each household determines the wage based on the utility-maximization problem subject to budget constraint and labor demand. As a result, the first order condition with respecto to wage is:

$$
\mathbb{E}_t \sum_{s=0}^{\infty} \xi_w^s D_{t,t+s} \frac{1}{\mu_{w,t+s} - 1} \left[W_t(h) \Pi_{t,s}^w - \mu_{w,t+s} MRS_{t+s}(h) P_{c,t+s} \right] L_{t+s}(h) = 0 \tag{C.36}
$$

$$
\text{where } MRS_t(h) = -\frac{U_{l,t}}{U_{c,t}} \text{ and}
$$

$$
\Pi_{t,s}^w = \begin{cases} 1, & \text{if s=0.} \\ \prod_{l=1}^{s} \pi_{c,t+l-1}^{\eta_w} \pi_c^{1-\eta_w} \left(\frac{A_{t+l}}{A_{t+l-1}} \right) \left(\frac{V_{t+l}}{V_{t+l-1}} \right)^{\frac{\alpha_c}{1-\alpha_i}}, & \text{otherwise} \end{cases} \tag{C.37}
$$

C.1.4 GDP and Monetary Policy

GDP is defined as the sum of consumption, investment, and GDP measurement error. The relative price of investment goods in terms of consumption goods is $Q_{i,t}$. At steady state, there is no GDP measurement error that is $e_t = 1$.

$$Y_t = C_t + Q_{i,t}I_t + \left(1 - \frac{1}{e_t}\right) Y_t \tag{C.38}$$

The nominal interest rate follows the Taylor Rule determined by price level in consumption goods and growth rate in real GDP.

$$R_t = \kappa R_{t-1}^{\rho_r} \left[r\pi_c \left(\frac{\pi_{c,t}}{\pi_c}\right)^{\phi_\pi} y_t^{\phi_y} \right]^{1-\rho_r} e^{\sigma_r \epsilon_{r,t}} \tag{C.39}$$

,where $\kappa \equiv y^{-\phi_y(1-\rho_r)}$, y is the steady state value of detrended GDP.

C.1.5 Market Clearing Conditions and Equilibrium

This section presents the market clearing conditions and the optimal allocations in equilibrium. In steady state, there are no excess and shortage of inputs and outputs. The equations below in this section describes the perfect balance in supply and demand in economy.

$$B_t = 0, \quad \forall t \tag{C.40}$$

$$\int_0^1 L_{c,t}(m)dm + \int_0^1 L_{i,t}(n)dn = L_t \tag{C.41}$$

$$\int_0^1 K_{c,t}(m)dm + \int_0^1 K_{i,t}(n)dn = u_t K_{t-1} \tag{C.42}$$

$$C_t = Y_{c,t} \tag{C.43}$$

$$I_t + a(u_t)K_{t-1} = Y_{i,t} \tag{C.44}$$

$$G_{c,t}Y_{c,t} = A_t K_{c,t}^{\alpha_c} L_{c,t}^{1-\alpha_c} - A_t V_t^{\frac{\alpha_c}{1-\alpha_i}} \Phi_c, \text{ where } G_{c,t} \equiv \int_0^1 \left(\frac{P_{c,t}(m)}{P_{c,t}}\right)^{-\frac{\mu_{c,t}}{\mu_{c,t}-1}} dm \tag{C.45}$$

$$G_{i,t}Y_{i,t} = V_t K_{i,t}^{\alpha_i} L_{i,t}^{1-\alpha_i} - V_t^{\frac{1}{1-\alpha_i}} \Phi_i, \text{ where } G_{i,t} \equiv \int_0^1 \left(\frac{P_{i,t}(n)}{P_{i,t}}\right)^{-\frac{\mu_{i,t}}{\mu_{i,t}-1}} dn \tag{C.46}$$

C.2 Stationary Equilibrium

Since the model contains two nonstationary technology processes, V_t and A_t, I transform all nonstationary components in the previous sector into stationary equilibrium by dividing the technology processes. Hence, the variables are normalized as follows

$$y_t = \frac{Y_t}{A_t V_t^{\frac{\alpha_c}{1-\alpha_i}}}, \quad y_{c,t} = \frac{Y_{c,t}}{A_t V_t^{\frac{\alpha_c}{1-\alpha_i}}}, \quad c_t = \frac{C_t}{A_t V_t^{\frac{\alpha_c}{1-\alpha_i}}}, \quad w_t = \frac{W_t/P_{c,t}}{A_t V_t^{\frac{\alpha_c}{1-\alpha_i}}} \tag{C.47}$$

$$y_{i,t} = \frac{Y_{i,t}}{V^{\frac{1}{1-\alpha_i}}}, \quad i_t = \frac{I_t}{V^{\frac{1}{1-\alpha_i}}}, \quad k_t = \frac{K_t}{V^{\frac{1}{1-\alpha_i}}}, \quad k_{c,t} = \frac{K_{c,t}}{V^{\frac{1}{1-\alpha_i}}}, \quad k_{i,t} = \frac{K_{i,t}}{V^{\frac{1}{1-\alpha_i}}} \tag{C.48}$$

$$q_{i,t} = \frac{Q_{i,t}}{A_t V_t^{\frac{\alpha_c}{1-\alpha_i}}} V^{\frac{1}{1-\alpha_i}}, \quad q_{k,t} = \frac{Q_{k,t}}{A_t V_t^{\frac{\alpha_c}{1-\alpha_i}}} V^{\frac{1}{1-\alpha_i}} \tag{C.49}$$

$$\tilde{u}_{c,t} = U_{c,t} \left(A_t V_t^{\frac{\alpha_c}{1-\alpha_i}} \right)^{\sigma}, \quad \tilde{u}_{l,t} = U_{l,t} \left(A_t V_t^{\frac{\alpha_c}{1-\alpha_i}} \right)^{\sigma-1}, \quad mrs_t = \frac{MRS_t}{A_t V_t^{\frac{\alpha_c}{1-\alpha_i}}} \tag{C.50}$$

In steady steady, the capital utilization and adjustment costs are one and zero respectively:

$$u = 1, \quad a(1) = 0, \quad S\left(\frac{I_t}{K_{t-1}}\right)\Big|_{SS} = 0 \tag{C.51}$$

The stationary variables of in consumption sector is:

$$\frac{w_t}{q_{i,t} r_{k,t}} = \frac{1-\alpha_c}{\alpha_c} \frac{k_{c,t}}{L_{c,t}} \tag{C.52}$$

$$mc_{c,t} = \alpha_c^{-\alpha_c} (1-\alpha_c)^{-(1-\alpha_c)} (q_{i,t} r_{k,t})^{\alpha_c} w_t^{1-\alpha_c}, \text{where } mc_{c,t} = \frac{MC_{c,t}}{P_{c,t}} \tag{C.53}$$

From the price setting of intermediate goods producers in consumption sector,

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\xi_c \beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} \frac{\tilde{u}_{c,t+s}}{\tilde{u}_{c,t}} \frac{1}{\mu_{c,t+s}-1} \left[p_{c,t}(m) \left(\frac{\Pi^c_{t,s}}{\prod_{l=1}^{s} \pi_{c,t+l}} \right) - \mu_{c,t+s} mc_{c,t+s} \right]$$

$$y_{c,t+s}(m) \left[\left(\frac{A_{t+s}}{A_t} \right) \left(\frac{V_{t+s}}{V_t} \right)^{\frac{\alpha_c}{1-\alpha_i}} \right]^{1-\sigma} = 0, \text{ where } p_{c,t}(m) = \frac{P_{c,t}(m)}{P_{c,t}}. \quad \text{(C.54)}$$

Aggregate price index in the consumption sector:

$$\left[(1-\xi_c) p_{c,t}(m)^{\frac{1}{1-\mu_{c,t}}} + \xi_c \left\{ \left(\frac{\pi_{c,t-1}}{\pi_c} \right)^{\eta_c} \left(\frac{\pi_{c,t}}{\pi_c} \right)^{-1} \right\}^{\frac{1}{1-\mu_{c,t}}} \right]^{1-\mu_{c,t}} = 1. \quad \text{(C.55)}$$

From now on, all equations are about investment sector. Therefore, I omit the detailed explanation.

$$\frac{w_t}{q_{i,t} r_{k,t}} = \frac{1-\alpha_i}{\alpha_i} \frac{k_{i,t}}{L_{i,t}} \quad \text{(C.56)}$$

$$mc_{i,t} = \alpha_i^{-\alpha_i} (1-\alpha_i)^{-(1-\alpha_i)} (q_{i,t} r_{k,t})^{\alpha_i} w_t^{1-\alpha_i} q_{i,t}^{-1}, \text{where } mc_{i,t} = \frac{MC_{i,t}}{P_{i,t}} \quad \text{(C.57)}$$

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\xi_c \beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} \frac{\tilde{u}_{c,t+s}}{\tilde{u}_{c,t}} \frac{q_{i,t+s}}{q_{i,t}} \frac{1}{\mu_{i,t+s}-1} \left[p_{i,t}(n) \left(\frac{\Pi^i_{t,s}}{\prod_{l=1}^{s} \pi_{i,t+l}} \right) - \mu_{i,t+s} mc_{i,t+s} \right]$$

$$y_{i,t+s}(n) \left[\left(\frac{A_{t+s}}{A_t} \right) \left(\frac{V_{t+s}}{V_t} \right)^{\frac{\alpha_c}{1-\alpha_i}} \right]^{1-\sigma} = 0 \text{ ,where } p_{i,t}(n) = \frac{P_{i,t}(n)}{P_{i,t}}. \quad \text{(C.58)}$$

$$\left[(1-\xi_i) p_{i,t}(n)^{\frac{1}{1-\mu_{i,t}}} + \xi_i \left\{ \left(\frac{\pi_{i,t-1}}{\pi_i} \right)^{\eta_i} \left(\frac{\pi_{i,t}}{\pi_i} \right)^{-1} \right\}^{\frac{1}{1-\mu_{i,t}}} \right]^{1-\mu_{i,t}} = 1 \quad \text{(C.59)}$$

Stationary equilibrium for households decisions as follows:

$$q_{i,t} = q_{k,t}\chi_t \left[1 - S\left(\frac{i_t}{k_{t-1}}\left(\frac{V_t}{V_{t-1}}\right)^{\frac{1}{1-\alpha_i}}\right) - S'\left(\frac{i_t}{k_{t-1}}\left(\frac{V_t}{V_{t-1}}\right)^{\frac{1}{1-\alpha_i}}\right)\frac{i_t}{k_{t-1}}\left(\frac{V_t}{V_{t-1}}\right)^{\frac{1}{1-\alpha i}}\right] \tag{C.60}$$

$$\frac{1}{R_t} = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\tilde{u}_{c,t+1}}{\tilde{u}_{c,t}}\pi_{c,t+1}^{-1}\left\{\left(\frac{A_{t+1}}{A_t}\right)\left(\frac{V_{t+1}}{V_t}\right)^{\frac{\alpha_c}{1-\alpha_i}}\right\}^{-\sigma}\right] \tag{C.61}$$

$$q_{k,t} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\frac{\tilde{u}_{c,t+1}}{\tilde{u}_{c,t}}\left\{\left(\frac{A_{t+1}}{A_t}\right)\left(\frac{V_{t+1}}{V_t}\right)^{\frac{\alpha_c}{1-\alpha_i}}\right\}^{1-\sigma}\left(\frac{V_{t+1}}{V_t}\right)^{-\frac{1}{1-\alpha_i}} \times$$
$$\left[q_{i,t+1}\left(r_{k,t+1}u_{t+1} - a(u_{t+1})\right) + q_{k,t+1}\left\{(1-\delta) + \chi_{t+1}S'\left(\frac{i_{t+1}}{k_t}\left(\frac{V_{t+1}}{V_t}\right)^{\frac{1}{1-\alpha_i}}\right)\left(\frac{i_t}{k_{t-1}}\left(\frac{V_t}{V_{t-1}}\right)^{\frac{1}{1-\alpha i}}\right)^2\right\}\right] \tag{C.62}$$

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \frac{\Lambda_{t+s}}{\Lambda_t}\frac{\tilde{u}_{c,t+s}}{\tilde{u}_{c,t}}\left\{\left(\frac{A_{t+s}}{A_t}\right)\left(\frac{V_{t+s}}{V_t}\right)^{\frac{\alpha_c}{1-\alpha_i}}\right\}^{1-\sigma} \times$$
$$\left[\mu_{w,t+s}mrs_{t+s}(h) - w_t(h)\frac{\Pi_{t,s}^w}{\prod_{l=1}^s \pi_{c,t+l}\left(\frac{A_{t+l}}{A_{t+l-1}}\right)\left(\frac{V_{t+l}}{V_{t+l-1}}\right)^{\frac{\alpha_c}{1-\alpha_i}}}\frac{1}{\mu_{w,t+s}-1}L_{t+s}(h)\right] = 0 \tag{C.63}$$

$$w_t = \left[(1-\xi_w)w_t(h)^{\frac{1}{1-\mu_{w,t}}} + \xi_w\left\{w_{t-1}\left(\frac{\pi_{c,t-1}}{\pi_c}\right)^{\eta_w}\left(\frac{\pi_{c,t}}{\pi_c}\right)^{1-\eta_w}\right\}^{\frac{1}{1-\mu_{w,t}}}\right]^{1-\mu_{w,t}} \tag{C.64}$$

$$\tilde{u}_{c,t} = \left(c_t - bc_{t-1}\left(\frac{A_{t-1}}{A_t}\right)\left(\frac{V_{t-1}}{V_t}\right)^{\frac{\alpha_c}{1-\alpha_i}}\right)^{-\sigma} e^{(\sigma-1)V(L_t)}$$
$$- \beta b \mathbb{E}_t\left[\left(c_{t+1}\left(\frac{A_{t+1}}{A_t}\right)\left(\frac{V_{t+1}}{V_t}\right)^{\frac{\alpha_c}{1-\alpha_i}} - bc_t\right)^{-\sigma} e^{(\sigma-1)V(L_{t+1})}\right] \quad (C.65)$$

$$-\tilde{u}_{l,t} = \left(c_t - bc_{t-1}\left(\frac{A_{t-1}}{A_t}\right)\left(\frac{V_{t-1}}{V_t}\right)^{\frac{\alpha_c}{1-\alpha_i}}\right)^{1-\sigma} e^{(\sigma-1)V'(L_t)} \quad (C.66)$$

GDP identify presents as:

$$y_t = c_t + q_{i,t}i_t + \left(1 - \frac{1}{e_t}\right) y_t. \quad (C.67)$$

Monetary Policy follows as:

$$\ln R_t = \ln \kappa + \rho \ln R_{t-1} + (1-\rho_r)\left\{\ln R + \ln \pi_c + \phi_\pi \ln\left(\frac{\pi_{c,t}}{\pi_c}\right) + \phi_y \ln y_t\right\}. \quad (C.68)$$

Market Clearing Conditions and its Equilibriums follow as:

$$L_{c,t} + L_{i,t} = L_t \quad (C.69)$$

$$k_{c,t} + k_{i,t} = u_t k_{t-1}\left(\frac{V_{t-1}}{V_t}\right)^{\frac{1}{1-\alpha_i}} \quad (C.70)$$

$$y_{c,t} = c_t \quad (C.71)$$

$$y_{i,t} = i_t + a(u_t)k_{t-1}\left(\frac{V_{t-1}}{V_t}\right)^{\frac{1}{1-\alpha_i}} \quad (C.72)$$

$$k_t = (1-\delta)k_{t-1}\left(\frac{V_{t-1}}{V_t}\right)^{\frac{1}{1-\alpha_i}} + \chi_t\left\{1 - S\left(\frac{i_t}{k_{t-1}}\left(\frac{V_t}{V_{t-1}}\right)^{\frac{1}{1-\alpha_i}}\right)\right\} i_t \quad (C.73)$$

$$\frac{q_{i,t}}{q_{i,t-1}} = \frac{\pi_{c,t}}{\pi_{i,t}} \left(\frac{A_{t-1}}{A_t}\right) \left(\frac{V_{t-1}}{V_t}\right)^{\frac{\alpha_c - 1}{1-\alpha_i}} \tag{C.74}$$

$$G_{c,t} y_{c,t} = k_{c,t}^{\alpha_c} L_{c,t}^{1-\alpha_c} - \Phi_c \tag{C.75}$$

$$G_{i,t} y_{i,t} = k_{i,t}^{\alpha_i} L_{i,t}^{1-\alpha_i} - \Phi_i. \tag{C.76}$$

The shock processes in steady state are normalized as follows:

$$z_t = (1-\rho_z) g_a + \rho_z z_{t-1} + \sigma_z \epsilon_{z,t} \tag{C.77}$$

$$v_t = (1-\rho_v) g_v + \rho_v v_{t-1} + \sigma_v \epsilon_{v,t} \tag{C.78}$$

$$\ln \mu_{x,t} = (1-\rho_x) \ln \mu_x + \rho_x \ln \mu_{x,t-1} + \sigma_x \left(\epsilon_{z,t} - \phi_x \epsilon_{z,t-1}\right) \text{ where } x = c, i, w \tag{C.79}$$

$$\ln \Lambda_t = \rho_\Lambda \ln \Lambda_{t-1} + \sigma_\Lambda \epsilon_{\Lambda,t} \tag{C.80}$$

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \sigma_\chi \epsilon_{\chi,t} \tag{C.81}$$

$$\ln e_t = \rho_e \ln e_{t-1} + \sigma_e \epsilon_{e,t}. \tag{C.82}$$

C.3 Steady State

This section describes the steady state of the model. In steady state, all variables are not affected by the stochastic processes such as the technology shocks. They are all constants and have an unconditional mean.

$$mc_c = \frac{1}{\mu_c} \tag{C.83}$$

$$mc_i = \frac{1}{\mu_i} \tag{C.84}$$

$$r_k = \frac{e^{\frac{1}{1-\alpha_i} g_v}}{\beta e^{G(1-\sigma)}} - (1-\delta) \tag{C.85}$$

$$q_i = \left[\alpha_i^{-\alpha_i} (1-\alpha_i)^{1(1-\alpha_i)} r_k^{\alpha_i} \mu_i\right]^{\frac{1-\alpha_c}{1-\alpha_i}} \left[\alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c} r_k^{-\alpha_c} \mu_c^{-1}\right] \tag{C.86}$$

$$w = \left[\alpha_c^{\alpha_c} (1-\alpha_c)^{1-\alpha_c} r_k^{-\alpha_c} \mu_c^{-1}\right]^{\frac{1}{1-\alpha_c}} q_i^{-\frac{\alpha_c}{1-\alpha_i}} \tag{C.87}$$

$$\frac{k_c}{L_c} = \frac{\alpha_c}{1-\alpha_c} \frac{w}{q_i r_k} \tag{C.88}$$

$$\frac{k_i}{L_i} = \frac{\alpha_i}{1-\alpha_i}\frac{w}{q_i r_k} \tag{C.89}$$

$$\frac{k_i}{k} = \mu_i \left(\frac{k_i}{L_i}\right)^{1-\alpha_i} \frac{i}{k} \tag{C.90}$$

$$\frac{k_c}{k} = e^{-\frac{1}{1-\alpha_i} g_v} - \frac{k_i}{k} \tag{C.91}$$

$$\frac{L_c}{L_i} = \left(\frac{1-\alpha_c}{\alpha_c}\right)\left(\frac{1-\alpha_i}{\alpha_i}\right)^{-1}\left(\frac{k_c}{k_i}\right) \tag{C.92}$$

$$\frac{L}{L_i} = \frac{L_c}{L_i} + 1 \tag{C.93}$$

$$\frac{L}{L_c} = \frac{L}{L_i}\left(\frac{L_c}{L_i}\right)^{-1} \tag{C.94}$$

$$\frac{y_c}{L} = \mu_c^{-1}\left(\frac{k_c}{L_c}\right)^{\alpha_c}\left(\frac{L_c}{L}\right) \tag{C.95}$$

$$\frac{c}{L} = \frac{y_c}{L} \tag{C.96}$$

$$w = \mu_w mrs = \mu_w \frac{e^{G(\sigma-1)}}{e^{\sigma G} - \beta b} c V'(L) \tag{C.97}$$

$$V'(L)L = \mu_w^{-1}\frac{e^{\sigma G} - \beta b}{e^{G(\sigma-1)}} w \left(\frac{c}{L}\right)^{-1} \tag{C.98}$$

$$V(L) = \psi \frac{L_t^{1+\eta}}{1+\eta} \tag{C.99}$$

$$q_i = q_k \tag{C.100}$$

$$u = 1 \tag{C.101}$$

$$y = c + q_i \tag{C.102}$$

$$\pi_c = \beta R e^{\sigma G} \tag{C.103}$$

$$\pi_i = \pi_c e^{-G + \frac{1}{1-\alpha_i} g_v} \tag{C.104}$$

$$L = \left[\left(\frac{e^G - \beta b}{\psi \mu_w}\right) w \left(\frac{c}{L}\right)^{-1}\right]^{\frac{1}{1+\eta}} \tag{C.105}$$

C.4 Log-Linearized Equilibrium Conditions

The model is linearized by taking log on both sides of the equations and expanding them with Taylor expansion in stationary economy. The linearized solutions and the shock processes result in the solvable solutions for the model. I omit the expectation operator $\mathbb{E}_t$ for simplicity.

$$\hat{w}_t - \hat{q}_{i,t} - \hat{r}_{k,t} = \hat{k}_{c,t} - \hat{L}_{c,t} \tag{C.106}$$

$$\hat{w}_t - \hat{q}_{i,t} - \hat{r}_{k,t} = \hat{k}_{i,t} - \hat{L}_{i,t} \tag{C.107}$$

$$\hat{mc}_{c,t} = \alpha_c(\hat{q}_{i,t} + \hat{r}_{k,t}) + (1-\alpha_c)\hat{w}_t \tag{C.108}$$

$$\hat{mc}_{i,t} = \alpha_i(\hat{q}_{i,t} + \hat{r}_{k,t}) + (1-\alpha_i)\hat{w}_t \tag{C.109}$$

$$\hat{q}_{i,t} = \hat{q}_{k,t} + \hat{\chi}_t - S'' e^{\frac{2}{1-\alpha_i}g_v}\left(\frac{i}{k}\right)^2\left(\hat{i}_t - \hat{k}_{t-1} + \frac{1}{1-\alpha i}\hat{v}_t\right) \text{ where } S'' \equiv S''(\cdot)|_{ss} \tag{C.110}$$

$$\hat{R}_t + \hat{\Lambda}_t - \hat{\Lambda}_{t-1} + \hat{\tilde{u}}_{c,t+1} - \hat{\tilde{u}}_{c,t} - \hat{\pi}_{c,t+1} - \sigma\left(\hat{z}_{t+1} + \frac{\alpha_c}{1-\alpha_i}\hat{v}_{t+1}\right) = 0 \tag{C.111}$$

$$\begin{aligned}\hat{q}_{k,t} = {} & \hat{\Lambda}_{t+1} - \hat{\Lambda}_t + \hat{\tilde{u}}_{c,t+1} - \hat{\tilde{u}}_{c,t} + \hat{z}_{t+1} + \frac{\alpha_c(1-\sigma)-1}{1-\alpha_i}\hat{v}_{t+1} + \beta e^{G(1-\sigma)-\frac{1}{1-\alpha_i}g_v} \times \\ & \left[r_k\left(\hat{q}_{i,t+1} + r_k\hat{r}_{k,t+1}\right) + (1-\delta)\hat{q}_{k,t+1}S''\left(\frac{i}{k}e^{\frac{1}{1-\alpha_i}g_v}\right)^3\left(\hat{i}_{t+1} - \hat{k}_t + \frac{1}{1-\alpha_i}\hat{v}_{t+1}\right)\right]\end{aligned} \tag{C.112}$$

$$\hat{mrs}_t = -\hat{\tilde{u}}_{l,t} - \hat{\tilde{u}}_{c,t} \tag{C.113}$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + q_i\frac{i}{y}(\hat{q}_{i,t} + \hat{i}_t) + \hat{e}_t \tag{C.114}$$

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)\left(\phi_\pi\hat{\pi}_{c,t} - \phi_y\hat{y}_t\right) + \sigma_r\epsilon_{r,t} \tag{C.115}$$

$$\hat{L}_t = \frac{L_c}{L}\hat{L}_{c,t} + \frac{L_i}{L}\hat{L}_{i,t} \tag{C.116}$$

$$e^{\frac{1}{1-\alpha_i}g_v}\left(\frac{k_c}{k}\hat{k}_{c,t} + \frac{k_i}{k}\hat{k}_{i,t}\right) = \hat{u}_t + \hat{k}_{t-1} - \frac{1}{1-\alpha_i}\hat{v}_t \tag{C.117}$$

$$\hat{c}_t = \hat{y}_{c,t} \tag{C.118}$$

$$\hat{y}_{i,t} = \hat{i}_t + r_k \frac{k}{i} e^{-\frac{1}{1-\alpha_i} g_v} \hat{u}_t \tag{C.119}$$

$$\hat{y}_{c,t} = \mu_c \left[\alpha_c \hat{k}_{c,t} + (1-\alpha_c) \hat{L}_{c,t} \right] \tag{C.120}$$

$$\hat{y}_{i,t} = \mu_i \left[\alpha_i \hat{k}_{i,t} + (1-\alpha_i) \hat{L}_{i,t} \right] \tag{C.121}$$

$$\hat{q}_{i,t} - \hat{q}_{i,t-1} = \hat{\pi}_{c,t} - \hat{\pi}_{i,t} - \hat{z}_t + \frac{1-\alpha_c}{1-\alpha_i} \hat{v}_t \tag{C.122}$$

$$\hat{k}_t = (1-\delta) e^{-\frac{1}{1-\alpha_i} g_v} \left(\hat{k}_{t-1} - \frac{1}{1-\alpha_i} \hat{v}_t \right) + \frac{i}{k} \left(\hat{\chi}_t + \hat{i}_t \right) \tag{C.123}$$

$$\hat{\pi}_{c,t} = \frac{(1-\xi_c\beta)(1-\xi_c)}{\xi_c(1+\beta\eta_c)} \left\{ \hat{mc}_{c,t} + \left(\frac{\mu_c - 1}{\mu_c} \right) \hat{\mu}_{c,t} \right\} + \frac{\eta_c}{1+\beta\eta_c} \hat{\pi}_{c,t-1} + \frac{\beta}{1+\beta\eta_c} \hat{\pi}_{c,t+1} \tag{C.124}$$

$$\hat{\pi}_{i,t} = \frac{(1-\xi_i\beta)(1-\xi_i)}{\xi_i(1+\beta\eta_i)} \left\{ \hat{mc}_{i,t} + \left(\frac{\mu_i - 1}{\mu_i} \right) \hat{\mu}_{i,t} \right\} + \frac{\eta_i}{1+\beta\eta_i} \hat{\pi}_{i,t-1} + \frac{\beta}{1+\beta\eta_i} \hat{\pi}_{i,t+1} \tag{C.125}$$

$$\begin{aligned}(1+\beta)\hat{w}_t - \hat{w}_{t-1} - \beta \hat{w}_{t+1} = {} & \frac{(1-\xi_w)(1-\xi_w\beta)}{\xi_w} \left[\left(\frac{\mu_w - 1}{\mu_w} \right) \hat{\mu}_{w,t} + \hat{mrs}_t - \hat{w}_t \right] \\ & - (1+\beta\eta_w)\,\hat{\pi}_{c,t} + \eta_w \hat{\pi}_{c,t-1} + \beta \hat{\pi}_{c,t+1}\end{aligned} \tag{C.126}$$

$$\hat{r}_{k,t} = \sigma_u \hat{u}_t \text{ where } \sigma_u = \frac{a''(1)}{a'(1)} \tag{C.127}$$

$$-\hat{\hat{u}}_{l,t} = \eta \hat{L}_t \tag{C.128}$$

$$\begin{aligned}(e^G - b)(e^G - b\beta)\hat{\hat{u}}_{c,t} = {} & -(e^{2G} - \beta b^2)\hat{c}_t + be^G \hat{c}_{t-1} + \beta b e^G \hat{c}_{t+1} \\ & - be^G \left(\hat{z}_t + \frac{\alpha_c}{1-\alpha_i} \hat{v}_t \right) + \beta b e^G \left(\hat{z}_{t+1} + \frac{\alpha_c}{1-\alpha_i} \hat{v}_t \right)\end{aligned} \tag{C.129}$$

Appendix D Convergence Diagnostics

I adopt the Monte Carlo Markov Chain (MCMC) univariate diagnostics (Brooks and Gelman, 1998).[23] Diagnostic tests: 1. sequence of draws should be from the invariant

[23] Dynare provide the univariate version instead of multivariate version because of computational speed problem.

distributions and 2. moments should not change within and between sequences. I compute 3 sets of MCMC statistics: mean, variance, and skewness. For each of these, I compute a statistic related to the within-sequence value of each of these and a sum of within-sequence statistic and a between-sequence variance. If these two statistics are closer, a parameter estimate converges in a specific level.

To be specific,

$$W = \frac{1}{J}\sum_{j=1}^{J}\frac{1}{I-1}\sum_{i=1}^{I}\left(\psi_{i,j} - \bar{\psi}_j\right)^2 \tag{D.1}$$

$$B = \frac{I}{J}\sum_{j=1}^{J}\left(\bar{\psi}_j - \bar{\psi}\right)^2 \tag{D.2}$$

$$V = \frac{I-1}{I}W + \frac{B}{I} \tag{D.3}$$

, where $\psi_{i,j}$ is i^{th} draw in j^{th} sequence, $\bar{\psi}_j$ is a mean of j^{th} sequence, $\bar{\psi}$ is a mean across all available data, W denotes a estimate of an average variance within sequences, B/I is a estimates of the variance of the mean across sequences, and V is the sum of within-sequence statistic and a between-sequence variance for $i = 1, \cdots, I$ and $j = 1, \cdots, J$.

Based on the above test, I conclude that all 18 parameter estimates satisfies the condition of convergences on all three moments. The statistics settle down with the values of moments for each estimates as the number of simulations increases.

8 Graphs and Tables

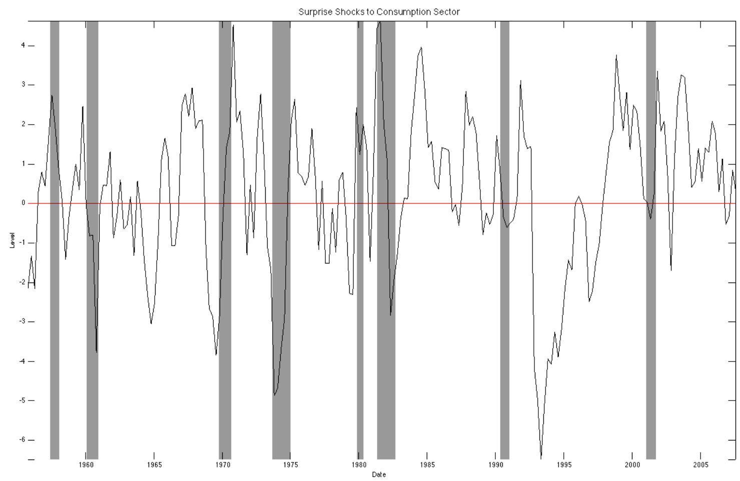

Figure 9: Surprise technology shocks to Consumption sector

Note: The shaded areas in gray correspond to NBER recession dates.

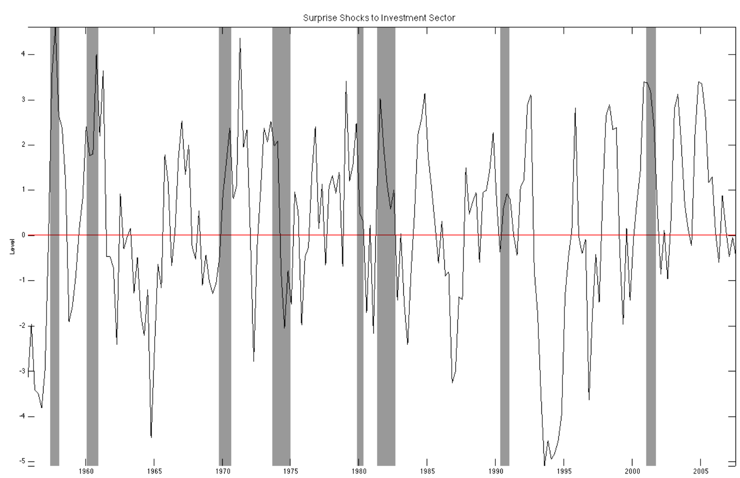

Figure 10: Surprise technology shocks to Investment sector

Note: The shaded areas in gray correspond to NBER recession dates.

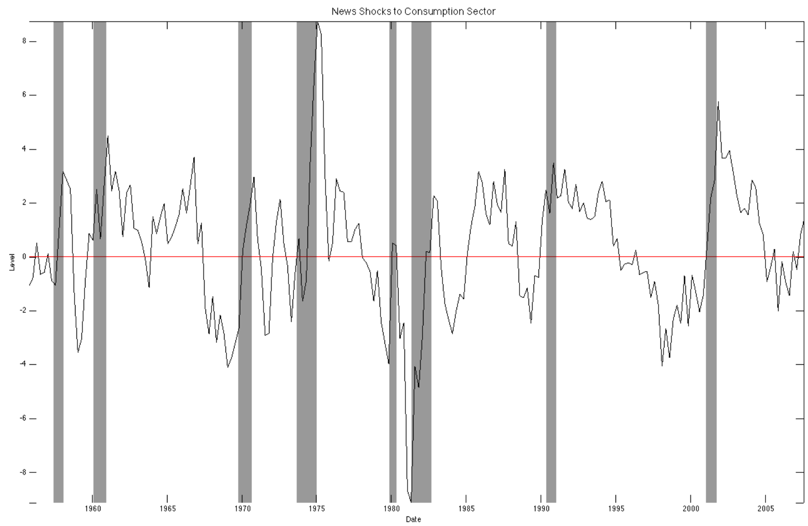

Figure 11: News technology shocks to Consumption sector
Note: The shaded areas in gray correspond to NBER recession dates.

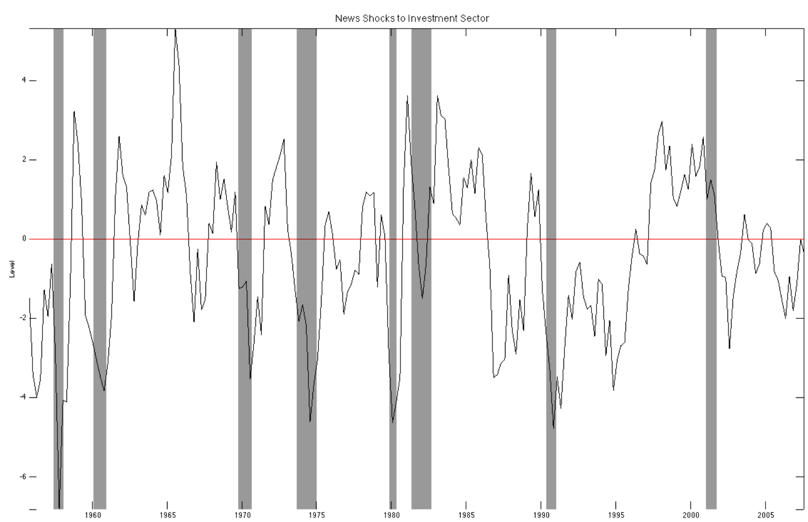

Figure 12: News technology shocks to Investment sector
Note: The shaded areas in gray correspond to NBER recession dates.

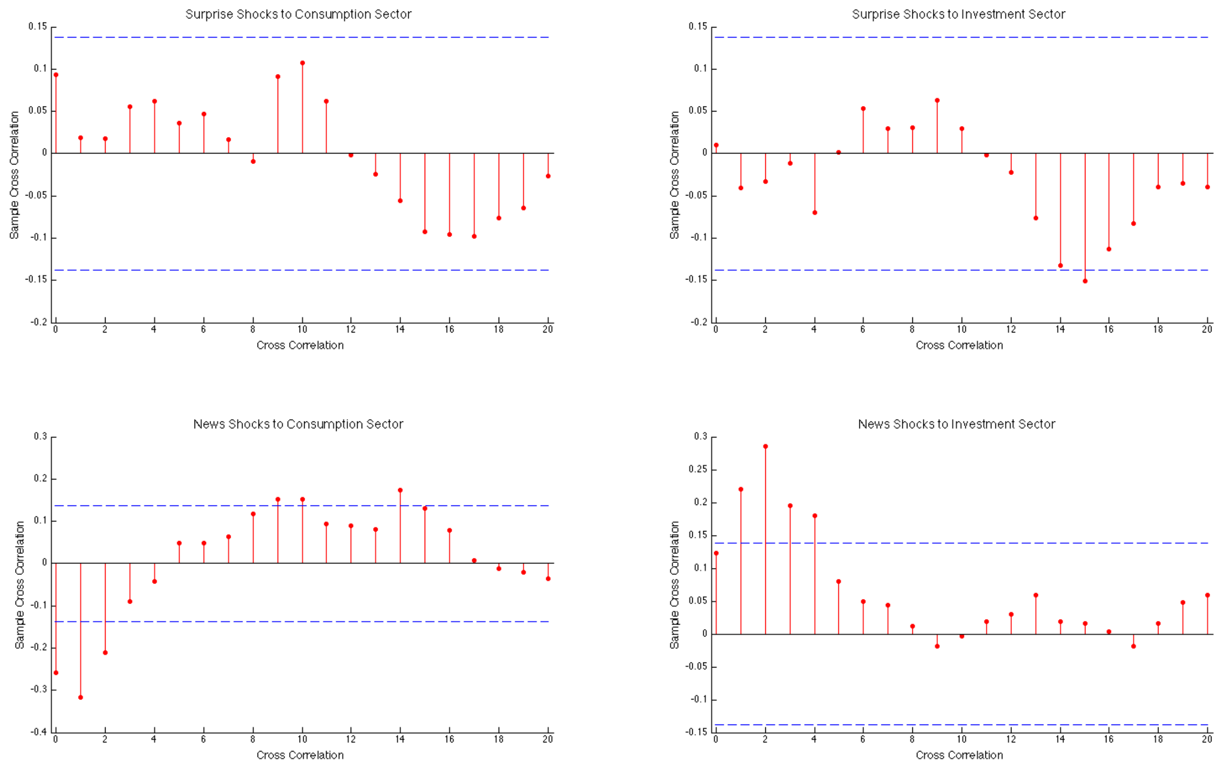

Figure 13: Cross Correlation between Identified Shocks and Output

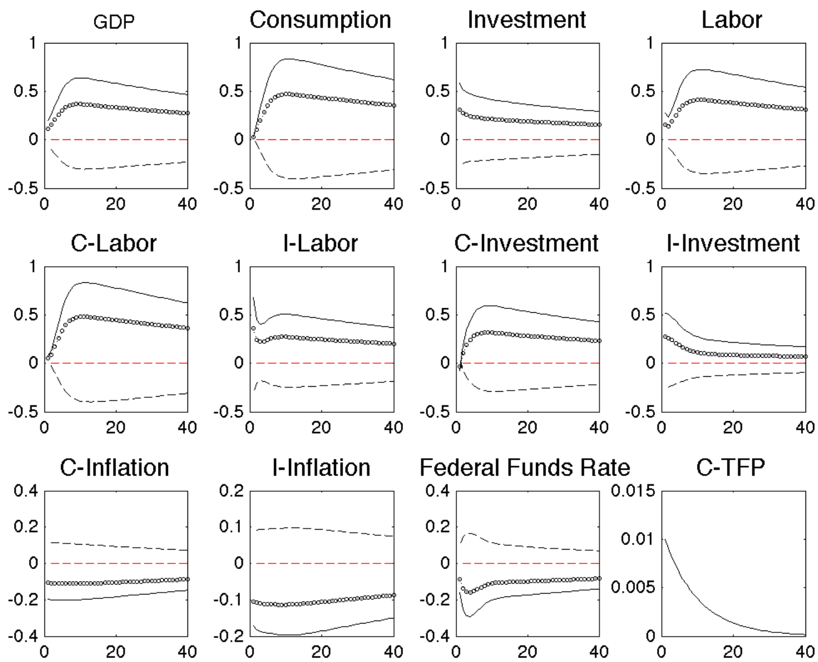

Figure 14: Impulse responses to TFP shocks to C-sector from low degree (dotted lines) to high degree (solid lines) on price rigidity of investment goods from DSGE model