Absence of safe assets, large sovereign debt, and low bond yields[♦]

Masaya Sakuragawa[*] and Yukie Sakuragawa[**]

Preliminary, not quoted

Abstract

Why governments in advanced countries can sustain large debt has received increasing concern. We investigate a country's fiscal sustainability by estimating the fiscal space following Ghosh *et al.* (2013). Using a panel data of 23 advanced countries over the period 1985-2014 that includes the period of European fiscal crises, we estimated the non-linear reaction function. All countries have fiscal spaces except for Japan, Portugal, and Greece. The low real interest rate relative to the real GDP growth rate contributes to fiscal sustainability. When adding the effect of "absence of safe assets", the concept of which relates the incomplete international risk diversification of portfolio, we find the fiscal space more likely. Japan has the fiscal space if the extent of the absence of safe assets is high and if the growth-adjusted interest rate is negative.

♦ This research was supported by Keio Gijuku Academic Development Funds.
* Professor, Keio University, 2-15-45, Mita, Minato-ku, Tokyo, 108-8345, (Tel)81-03-5427-1832, masaya822@gmail.com
** Associate Professor, Atomi University, 1-9-6, Nakano, Niiza-shi, Saitama, 352-8501, (Tel)81-48-478-4110, sakuraga@atomi.ac.jp.

1.Introduction

High levels of outstanding government debt in advanced countries are one of important issues. In 1990s, the economy would fall into recessions if the debt-GDP ratio is over 100 percent.[1] The European Union imposed the requirement of joining the membership for a country's debt-GDP ratio to be less than 60 percent. However, in the 2000s, the debt-GDP ratio has increased among advanced countries. Stimulating fiscal spending after the global financial crisis led to the higher ratios of the public debt to GDP, which in turn triggered the fiscal crises in the Euro area. After the crises ended, the ratios of the public debt to GDP remain the highest for many of them, but the nominal bond yields fell to the low levels except for Greece. Figure 1 illustrates the public debt to GDP and the nominal bond yield for five large-indebted countries, Japan, Italy, Portugal, Greece, and Ireland.

The primary concern is to understand why the governments can sustain large debt. Ghosh *et al*. (2013) compute the fiscal space, defined as the difference between projected debt limits and actual debt in advanced countries using the data until 2007, and find that none of Japan, Italy, Portugal, Iceland, and Greece has fiscal space. According to theirs, those five countries should have defaulted on the government debt. The growing literature also presents pessimistic scenarios for fiscal sustainability.

We highlight two elements that are supposed to have contributed to sustaining the large debt for advanced countries. The first is the recent tendency for the low real interest rate relative to the real GDP growth rate. Table 1 shows that the averages of the real GDP growth rate and the real long-term bond yield over the 23 countries for

[1] Reinhalt and Rogoff (2010) stated in his book that once the debt-GDP ratio becomes over 100 percent, the economy falls into stagnation.

different sample periods. The long-term bond yield is intended as the proxy of the safe interest rate. The bond yields at least until 2008, just before the global financial crisis, could be close to the real interest rate. The GDP growth rate declined from 2.9 percent for the period 1990-1999 to 2.1 percent for 2000-2009. Likewise, the real bond yields of 10-years maturity declined from 4.7 percent to 2.1 percent. The gap between the two rates declined over time. The growth-adjusted interest rate is almost zero for the latter period. For 2010-2015, the growth-adjusted interest rate is close to zero. Remarkably, if several European countries that faced fiscal troubles are excluded, the growth-adjusted interest rate is minus one percent! The zero or negative growth-adjusted interest rate provides a room for rolling over the debt without imposing additional fiscal costs.

The second relates incomplete international risk diversification of portfolio. Coeurdacier and Rey (2013) present evidence showing that there is the strong home bias with respect to cross-border bond holdings. Investors will find it difficult to find safe alternatives to domestic government bonds if the country-specific risk is not independent each other.[3] This tendency is remarkable in countries with huge net foreign assets, such as Japan and Switzerland. The Japanese yen is used as a flight to quality, and tends to appreciate when Japan is hit by the adverse global shock. It appreciated substantially relative to both the US dollar and the euro for years following the global financial crisis, and Japanese investors who took the large international position got losses. Sakuragawa and Sakuragawa (2016) describe this situation as the "absence of safe assets", and show that when there are no safe

[3] In the standard theory of international finance, with the independently and identically distributed shock across countries, rational investors should hold internationally diversified portfolio.

alternatives, the bond yield does not react to the worsening fiscal conditions and the government can sustain the large debt.

We investigate a country's fiscal sustainability by estimating the fiscal space following Ghosh *et al.* (2013). Using a data of 23 advanced countries over the period 1985-2014 that includes European fiscal crises, we estimated the cubic government reaction function. The primary balance-GDP ratio reacts positively to the debt-GDP ratio from 60 percent, peaks at 200 percent and around, and starts to react negatively beyond it. We used the estimated reaction function to find the fiscal space. We can safely conclude that the low real interest rate since 2000 permitted the high debt-GDP ratio to be sustained. We estimated the more disciplined reaction function when the data for Japan is excluded. Investigating the sustainability country by country, we find fiscal spaces for all countries except for Portugal, Japan, and Greece.

When considering the absence of safe assets, we find the fiscal space more likely. Japan has the fiscal space if the government behaves following the disciplined reaction function, if the extent of the absence of safe assets is high, and if the growth-adjusted interest rate is negative.

Literature review

A huge literature provides quantitative analysis of fiscal sustainability. By limiting the research applied to Japan, the literature includes Doi and Ihori (2003), Dekle (2005), Broda and Weinstein (2005), Ihori *et al*. (2006), Sakuragawa and Hosono (2010, 2011), Doi, Hoshi, and Okimoto (2011), Hoshi and Ito (2013), Hansen and Imrohoroglu (2016), Matsuoka (2015), Miyazawa and Yamada (2015), and others.

The literature on sovereign default is related. Most of these papers consider the government's strategic default by developing countries (e.g., Calvo 1988, Cole and Kehoe 2000, Arellano 2008, and others). In contrast, we model fiscal default as a problem of the inability to pay, triggered by the stochastic shock to the primary balance. We believe that the inability to pay rather than strategic default is more likely to be relevant for analyzing public debt in countries where domestic residents hold almost all the debt.

This paper is related to the literature on the home bias. An ever-growing number of studies investigate the determinants of home bias from both rational and behavioral perspectives. The determinants proposed by those studies include transaction costs (Glassman and Riddick, 2001), real exchange rate risks (Fidora, Fratzscher and Thimann, 2007), information barriers (Ahearne, Griever and Warnock, 2004), corporate governance issues (Dahlquist et al., 2003), and lack of familiarity (Portes and Rey, 2005). Coeurdacier and Rey (2013) provide an extensive survey on home bias. In their seminal contribution, Cole and Obstfeld (1990) shows that gains from international risk sharing is small as the change in the terms of trade helps to share risk internationally. Martin and Rey (2004) show that even small transaction cost may lead to sizable home bias when home and foreign stocks are close substitutes.

Several papers examine the pricing of sovereign risk. Gruber and Kamin (2012), Beirne and Fratzscher (2013), Aizenman, Hutchisonb, and Jinjarakc (2013) estimate the sovereign risk.

This paper is organized as follows. In section 2, we set up a model of fiscal crisis. In section 3, we analyze model's features. In section 4, we estimate the fiscal reaction function for advanced countries and calculate the fiscal space for each. In section 5, we conclude.

2. Model

Consider an economy that lasts for infinity and consists of a large number of investors and the government. We first state the behavior of the government. Letting S_t denote the primary surplus. We assume that the government expenditure is zero so that S_t equals the tax. The government's budget constraint is written by

(1) $$D_{t+1} - D_t = R_{t+1} D_t - S_{t+1},$$

so long as the government rolls over the debt. We use a simple default rule; the government makes the promised payments so long as the government can roll over the debt, but otherwise pays only the fraction $(1-h)$ of the debt, where h is the haircut rate.

The government is committed to follow a fiscal reaction function;

(2) $$\frac{S_{t+1}}{Y_t} = \mu + f\left(\frac{D_t}{Y_t}\right) + \varepsilon_{t+1},$$

The parameter μ captures systematic determinants of the primary balance other than the past debt, $f(.)$ is the response of the primary surplus to the debt, which is a continuously differentiable function. Hereafter we denote $d_t \equiv D_t / Y_t$. The stochastic variable ε_t follows the distribution function $G(\varepsilon_t)$ that is continuously differentiable defined over the interval $[\varepsilon^{\min}, \varepsilon^{\max}]$. The stochastic primary surplus reflects the uncertainty for the tax revenues.

We obtain the debt dynamics from (1) and (2) by

$$(3) \qquad \mu + f(d_t) + \varepsilon_{t+1} = \frac{1+R_{t+1}}{1+g} d_t - d_{t+1},$$

where we assume that the output Y_t grows at the constant rate g. The LHS is the fiscal surplus, and the RHS is the "net" growth-adjusted interest payment. To consider the possibility of fiscal default, the function has the property that there exists a debt d^m such that, for any $d_t > d^m$,

$$(A) \qquad \mu + f(d^m) + \varepsilon^{\min} < \frac{1+R_{t+1}}{1+g} d^m - d^m \quad \text{and} \quad f'(d^m) < \frac{1+R_{t+1}}{1+g}$$

given R_{t+1}. At $d_t = d^m$ and with the worst shock, the primary surplus cannot cover the interest payment, and once the economy falls into this fiscal difficulty, the response of primary balance is so weak that the government cannot escape from this situation. We rewrite (3) by

$$\mu + f(d_t) + \varepsilon_{t+1} = \left(\frac{1+R_{t+1}}{1+g} - 1\right) d_t - \Delta d_t$$

Figures 2A and 2B plot the non-stochastic images of debt dynamics. The upper graph shows the case for the unique intersection. This intersection corresponds to the debt limit, denoted $\bar{d}$, as defined below; if $d_t < \bar{d}$, the primary surplus always cover the interest payment so that the debt decreases over time, ceasing finally to issue the debt. However, if $d_t > \bar{d}$, the primary surplus is insufficient to cover the interest payment, which makes it impossible for the government to roll over the debt. Thus in the deterministic case, the default probability is zero up to the debt limit, jumping to unity thereafter. The lower graph shows the case for two intersections, in which he reaction of the primary surplus to the debt is strong around $\tilde{d}$. If $d_t < \bar{d}$, the debt decreases monotonically, converging to some positive level $\tilde{d}$.

There are a large number of investors who are risk neutral. They face two assets, the government bonds of home country, and the other asset which we call "capital". As in Sakuragawa and Sakuragawa (2016), the fiscal default produces a negative externality on the return on capital. The gross return of capital is $(1+A)$ in the nondefault state and falls to $(1-\theta)(1+A)$ in the default state, with $0 \le \theta \le 1$.[5] The positive value of θ implies the correlation of risk across the two assets. We can think of θ as a no-risk hedge measure because θ captures the degree to which capital does not hedge fiscal risk.

We provide three stories that explain this situation. In Japan, domestic investors hold more than 95 percent of the JGBs on issue. Many of the large bondholders are domestic financial intermediaries who are supposed to be unsophisticated in that they have weak access to foreign markets.[6] They find it difficult to hedge the fiscal risk by holding foreign safe assets but rather have to hold domestic assets that are vulnerable to fiscal risk. Investors find no safe assets in the event of a fiscal crisis.

This closed-economy story is applied to the United States that is a country of the largest GDP in the world. If the US's government defaults on Treasuries, its adverse effects are anticipated to hit the world economy as well as home. Consequently, investors have no safe alternatives than Treasuries. The global financial crisis spread

[5] Cohen and Sachs (1986), Cole and Kehoe (2000), and Arellano (2008) model fiscal default costs as having negative implications on output.

[6] The financial intermediaries are owned or regulated by the government, including the central bank, private depository institutions, private insurance companies, publicly financial institutions, and social security funds.

out from the United States to the world, The GDP growth rate of the United States fell down by 3.2 percent in 2009, which is smaller than the decline in the global growth rate, higher than 4 percent, and US dollar appreciated relative to other currencies except for the Japanese yen.

Euro zone countries seem to have safe alternatives, but are not necessarily exceptional. In the recent fiscal crisis of the Euro zone, many countries of that region increased the exposure to the domestic bonds relative to foreign bonds (Battistini, Pagano, and Simonelli 2013). The pessimistic belief that the fiscal risk of one country spills over to that of other (see Antonakakis and Vergos 2013, Altera and Beyerb 2014, and Grauwea and Ji 2014), and triggers the collapse of the Euro system made investor of the Euro zone choose holding the bond denominated in the domestic currency more than the internationally diversified portfolio.

What is common to these three stories is that in case of fiscal default, investors find it difficult to find safe alternatives to domestic government bonds. In the model, "capital" is the domestic stocks or loans to Japan, foreign assets general to the US, and the Germany bonds to the euro zone countries.

Gourinchas and Jeanne (2012) and Caballero and Farhi (2014) propose the notion of a shortage of safe assets, arguing that only some developed countries have an ability to supply safe assets. This also implies that the number of safe assets available to investors is too limited to diversify risk.

Finally, we assume that once the default has occurred, the government cannot issue the government bond at least during this period.

3. The Analysis

First of all, we characterize the fiscal default. In doing do, we consider the "debt limit", denoted $\bar{d}_{t+1}$, as the maximum level of debt under which investors are willing to buy the bonds. For the present, we start the analysis by assuming that there exists the debt limit. The government can roll over the debt until the limit. If the shock ε_{t+1} is so small that the debt that is necessary to cover the interest payment exceeds the debt limit; $\bar{d}_{t+1} < d_{t+1}$, the government cannot roll over the debt and has to default. Given the debt limit $\bar{d}_{t+1}$, there exists a cutoff shock $\bar{\varepsilon}_{t+1}$, below which the debt that the government has to issue to avoid default exceeds the debt limit;

(4) $$\bar{\varepsilon}_{t+1} = \frac{1+R_{t+1}}{1+g} d_t - \mu - f(d_t) - \bar{d}_{t+1} \equiv \bar{\varepsilon}(R_{t+1}, d_t, \bar{d}_{t+1}),$$

Accordingly the probability of fiscal default is defined by

(5) $$\pi_{t+1} = prob.[\varepsilon_{t+1} < \bar{\varepsilon}_{t+1}] = G(\bar{\varepsilon}(R_{t+1}, d_t, \bar{d}_{t+1})).$$

The function $G(.)$ has the following property:

(i) $$\partial \pi_{t+1} / \partial R_{t+1} = G'(\bar{\varepsilon}_{t+1}) \frac{d_t}{1+g} > 0,$$

(ii) $$\partial \pi_{t+1} / \partial d_t = G'(\bar{\varepsilon}_{t+1}) \{ \frac{1+R_{t+1}}{1+g} - f'(d_t) \} > 0 \quad \text{if} \quad \frac{1+R_{t+1}}{1+g} > f'(d_t), \text{ and}$$

(iii) $$\partial \pi_{t+1} / \partial \bar{d}_{t+1} = -G'(\bar{\varepsilon}_{t+1}) < 0.$$

The higher is either the bond yield or the current debt (if $\frac{1+R_{t+1}}{1+g} > f'(d_t)$), the default probability is higher, but the larger is the debt limit, the default probability is smaller.

If investors hold the government bonds, they should be indifferent between capital and the bonds. The no-arbitrage condition is written as

(6) $$(1-\pi_{t+1})(1+R_{t+1})+\pi_{t+1}(1-h)(1+R_{t+1})=(1-\pi_{t+1})(1+A)+\pi_{t+1}(1-\theta)(1+A),$$

for $d_t > 0$. The LHS is the expected return on the government bonds; investors receive the full of the face value if the debt is rolled over, and only the fraction $(1-h)$ otherwise. The RHS is the expected return on capital; agents receive the full return $(1+A)$ if the debt is rolled over, but only the part $(1-\theta)(1+A)$ if the fiscal default occurs. It is useful to rearrange (6) to have meaningful implications by

(7) $$1+R_{t+1}=(1+A)\frac{1-\theta\pi_{t+1}}{1-h\pi_{t+1}},$$

This equation has several implications. First, taking the default probability as given, the bond yield is lower as either the haircut rate h declines or the no-risk-hedge measure θ rises. Table 2 shows some calculation by assuming $A=0.02$ and the default probability is 10 percent. The impact of the haircut on the yield is intuitive. The first column shows that the bond yield is higher as the haircut rate rises in the conventional case for $\theta=0$. The primary concern is the impact of θ. For example, the second row $(h=0.1)$ shows that the bond yield is lower as the no-risk-hedge measure θ rises. As θ rises, the bond yield becomes less likely to reflect the risk premium. This is because, when a fiscal default occurs, the return on the government bonds declines, and also does the opportunity cost of holding the bonds.

Second, there exists the threshold under which the risk premium disappears. When $h=\theta$, the bond yield equals the risk-free return on capital, as the triangular components of the matrix.

Third, if $h<\theta$, the bond yield is lower than the return on capital, as the upper components against the triangular show. Then the fiscal crisis damages more on the return on capital than on government bonds. This is not unrealistic. The government can favor the bondholders at the sacrifice of the shareholders. The government could compensate the bondholders for the losses in the event of fiscal crisis by exploiting the income of wealthy capital holders. We summarize this argument as follows.

Proposition 1: The bond yield R_{t+1} is higher than the normal return on capital A if and only if $\theta < h$.

The bond yield is higher than the normal return on capital if the fiscal crisis damages more on the return on the bonds than other assets.

We turn to the determinant of the bond yield. We use (7) to specify the net return function on the government bond. Formally, we define the net return function by

(8) $$\Omega(R_{t+1}, d_t, \bar{d}_{t+1},)$$

$$= (1+R_{t+1})\{1 - hG(\bar{\varepsilon}(R_{t+1}, d_t, \bar{d}_{t+1}))\} - (1+A)\{1 - \theta G(\bar{\varepsilon}(R_{t+1}, d_t, \bar{d}_{t+1})\},$$

for $0 \le \theta < 1$ and $0 \le h < 1$. The first term is the expected return from the government bonds and the second term is the expected return from capital.[9]

Differentiating this function in terms of $1+R_{t+1}$, we have

(9) $$\partial\Omega(.)/\partial R_{t+1} = \{1 - hG(\bar{\varepsilon}(R, d, \bar{d}))\} - \{(1+R)h - (1+A)\theta)\}\, G'(\bar{\varepsilon})\frac{d_t}{1+g}.$$

The first term in the right-hand side is the marginal expected gain from the interest receipt, and the second term is the marginal cost of increasing the default probability. Additionally, we have the following properties:

(i) $$\lim_{R\to A} \partial\Omega(.)/\partial R_{t+1} = 1 - (1+A)(h-\theta)\, G'(\varepsilon^{\min})\frac{d_t}{1+g},$$

[9] When $\theta = h$, $\Omega(.) \equiv (R-A)\{1 - hG(.)\}$ holds. Then $\Omega(.) = 0$ if and only if $R = A$.

(ii) $\lim_{R\to\infty} \partial\Omega(.)/\partial R_{t+1} = -\{(1+R)h-(1+A)\theta)\}\ G'(\varepsilon^{\max})\dfrac{d_t}{1+g}$,

(iii) $\partial\Omega(.)/\partial d_t = -\{(1+R)h-(1+A)\theta\}\ G'(\bar{\varepsilon}_{t+1})\{\dfrac{1+R_{t+1}}{1+g} - f'(d_t)\}$, and

(iv) $\partial\Omega(.)/\partial \bar{d}_{t+1} = \{(1+R)h-(1+A)\theta\}\ G'(\bar{\varepsilon}_{t+1})$.

Proposition 1 states that the term $\{(1+R)h-(1+A)\theta\}$ is positive only when $\theta < h$. We focus on this case and then this function is defined for $R_{t+1} \in (A,+\infty)$.

Proposition 2: Suppose that $\lim_{R\to A} \partial\Omega(.)/\partial R_{t+1} > 0$, and $\theta < h$. The net return function $\Omega(R_{t+1}, d_t, \bar{d}_{t+1},)$ is (i) first increasing, later reaches a peak, and then decreasing in the bond yield, (ii) decreasing in the current debt, and (iii) increasing in the debt limit.

The first property implies that the net return function has a maximum, implying that there may exist a maximum bond yield above which the bond market shuts down. The second and third properties are closely related to the presence of the debt limit.

If there exists the debt limit, there should exist no bond yield satisfying $\Omega(.) > 0$ for any debt beyond the debt limit. As Proposition 2 states, the net return function is decreasing in the current debt, but increasing in the debt limit. Once the current debt reaches the debt limit, further debt should make the function $\Omega(.)$ negative. This holds only if

$$\partial\Omega(.)/\partial d_t + \partial\Omega(.)/\partial \bar{d}_{t+1}\Big|_{d_t=\bar{d}_{t+1}} = \{(1+R)h-(1+A)\theta\}G'(\bar{\varepsilon}_{t+1})\{\frac{1+R_{t+1}}{1+g} - f'(d_t) - 1\} < 0,$$

requiring

(B) $$\frac{1+R_{t+1}}{1+g} > 1 + f'(d_t) .$$

When (B) holds, the function $\Omega(R_{t+1}, d_t, d_t)$ is decreasing in d_t. Condition (B) states that the growth-adjusted bond yield is sufficiently higher than the sensitivity of the primary surplus to the debt. Condition (A) guarantees that there is some large debt d^m beyond which (B) always holds, and that $\Omega(R_{t+1}, d_t, d_t)$ is decreasing in d_t. This does not guarantee that $\Omega(R_{t+1}, d_t, d_t)$ is decreasing in d_t globally. Recalling Figure 2, the function in decreasing around d^m, but increasing around $\tilde{d}$.

Therefore, when Condition (A) holds, there exists a debt limit above which agents are never willing to hold the government bonds at any bond yield, as is familiar from the literature on equilibrium credit rationing (e.g., Stiglitz and Weiss, 1981 and Williamson, 1986). We can define the two stationary limit variables $\{\bar{d}, \bar{R}\}$ that satisfy

(10) $$\Omega(\bar{R}, \bar{d}, \bar{d}) = 0 \text{, and}$$

(11) $$\partial\Omega(\bar{R}, \bar{d}, \bar{d}) / \partial R_{t+1} = 0 ,$$

and $\bar{d}_{t+1} > d^m$, where we call $\bar{R}$ the "bond yield limit", which is the bond yield that the agents are willing to hold the bonds if the actual debt would be $\bar{d}$. Accordingly, the limit default probability is written as

(12) $$\bar{\pi} = G\{\frac{1+\bar{R}}{1+g}\bar{d} - \mu - f(\bar{d}) - \bar{d}\} .$$

This default probability is in general less than unity so that if the debt were to increase beyond $\bar{d}$, the default probability suddenly jumps to unity.

Once the debt limit $\bar{d}$ has been determined, there is a bond yield $R_{t+1} = R(d_t, \bar{d})$ that satisfies

(13) $$\Omega(R_{t+1}, d_t, \bar{d}) = 0$$

so long as $d_t < \bar{d}$, while otherwise there is no finite bond yield.[10] Accordingly, the default probability is given by

(14) $$\pi_{t+1} = G\{\frac{1+R_{t+1}}{1+g} d_t - \mu - f(d_t) - \bar{d}\},$$

if $d_t < \bar{d}$, while otherwise, $\pi_{t+1} = 1$. We establish the following.

Proposition 3: Suppose that $\theta < h$ and Condition (A) holds. (i) There exists a debt limit $\bar{d}$, above which agents are never willing to hold the government bonds at any bond yield. (ii) There exists a finite bond yield at which the government can issue the debt for $d_t \in (0, \bar{d}]$. (iii) There exists no finite bond yield at which the government can issue the debt for $d_t \in (\bar{d}, +\infty)$.

The first property implies that there exists the debt limit under mild conditions. The sensitivity of the primary balance to the debt is too weak, and the fiscal crisis must damage more on the return on the bonds than the return on capital, that is $\theta < h$. The second property implies that if debt remains at or below the debt limit, there are

[10] Note that there are multiple bond yields that satisfy $\Omega(.) = 0$. Among them, the lowest is chosen as equilibrium. There are only two yields that satisfy $\Omega(.) = 0$ if this function is globally concave in terms of R if the functional form on $G(.)$ satisfies $2G(.) + G'(.)\{(1+R)h - (1+A)\theta\} > 0$. Strictly speaking, this restriction is not necessary for all the main results to hold. Without this assumption, the number of equilibria may be more than three, and the analysis will be a little complicated.

always agents who are willing to lend to the government at a positive finite bond yield. The third implies that if debt exceeds the debt limit, there is no finite bond yield that compensates for the default risk to agents. As a result, the government faces a complete loss of access to the market.

Proposition 4: Suppose that $\theta < h$ and Condition (A) holds. (i) The bond yield R_{t+1} is higher if the current debt is large, the no-risk-hedge measure θ is low, and the haircut rate is high. (ii) The debt limit $\bar{d}$ is larger if the no-risk-hedge measure θ is high, and the hair cur rate is low.

The overall features of Propositions 3 and 4 need the assumption of $\theta < h$. The debt limit is not defined for $\theta = h$ because then $\Omega(R_{t+1}, d_t, d_t)$ is independent of the debt level d_t. The following is straightforward.

Proposition 5: Suppose that $\theta < h$ and Condition (A) holds. When θ approaches h from below, the bond yield approaches A, and the debt limit approaches infinity.

This implies a very strong implication on the debt sustainability. By choosing the appropriate combination (θ, h), any large debt is sustainable.

Finally in this section, we state the sequence of the determination of the equilibrium. The equilibrium of this stochastic economy with debt is a sequence of endogenous variables $\{d_t, R_t, \pi_t, \bar{d}_t, \bar{R}_t\}_{t=0}^{\infty}$, and the stochastic variable $\{\varepsilon_t\}_{t=0}^{\infty}$, satisfying (3), (10), (11), (13), (14), given the initial debt d_{-1}.

At the beginning of period t, d_{t-1}, is predetermined, and the shock ε_t is revealed. If $\varepsilon_t < \bar{\varepsilon}_t$, the default occurs, and solving the model is simple, while otherwise the default does not occur, and (3) determines the new debt d_t.

In the next step we solve other four variables. First, equations (10) and (11) determine the debt limit $\bar{d}$ and the bond yield limit $\bar{R}$. Second, given the current debt d_t and the debt limit $\bar{d}$, equations (13) and (14) determine the current bond yield R_{t+1}, and the current default probability π_{t+1}.

We should be careful to the characterization of equilibria. Strictly speaking, there are multiple equilibria when the debt level is within the debt limit. The equilibrium in which no finite interest rate compensates for the default risk to investors is also supported as a stable equilibrium irrespective of the debt level. As Romer (2001) shows, each of the multiple equilibria is self-fulfilling equilibrium. If investors believe that the probability of default should be low, the low interest rate is realized. On the other hand, if investors believe that the probability of default must be very high, they do not hold the bonds at any interest rate, which forces the government to face the complete loss of access to the market. In this paper, we focus on the equilibrium in which investors purchase the bonds so long as the debt level is within the debt limit.

4. Empirical Investigation

This section applies the theoretical framework developed above to a sample of 23 advanced countries using data for the period 1985-2014. We estimate the fiscal reaction function (2), and calculate the fiscal space for each country.

A. Government fiscal reaction function

A-1, Specification and estimation method

We specify a non-linear fiscal reaction function. A non-linear response of fiscal behavior is found by Bohn(1998), Ostry and Abiad (2005) , Mendoza and Ostry (2008), and Ghosh *et al.* (2013). We consider a cubic fiscal reaction function in terms of the ratio of debt to GDP, following Mendoza and Ostry (2008), Ghosh *et al.* (2013) and others. Once we allow for the possibility that the fiscal discipline eventually weakens at higher debt levels, like Ghosh *et al.* (2013), a finite debt limit exists in general.

A number of variables apart from debt are likely to affect a country's surplus-generating capacity. Following the literature by Mendoza and Ostry(2008) and Ghosh *et al.* (2013), we include conditioning variables into the estimation; the GDP gap, the government expenditure gap, inflation rate, and the trade openness. In addition, we also consider the growth rate of the trend of GDP and the ratio of current account to GDP. The GDP gap controls for the effect of business cycles. The government expenditure gap measures the effect of temporary fluctuations in government outlays. The inflation rate captures the effect on the fiscal balance because of the bracket-creep effects and/or because that government needs greater effort to counter the effects of higher interest rates accompanying higher inflation.

The growth rate of GDP trend captures the effect of natural increase in tax revenue. The effect on the ratio of current account to GDP is basically positive. Because the country with higher current account surplus is in good economic condition, government can secure more tax revenue.[11] It has a positive effect on the fiscal surplus. However, if the government thinks that the domestic investors have the strong home bias and the government does not need to borrow from abroad at higher interest rate because of the higher rate of current account, that it, θ is high, the positive effect of large current account on the surplus is weakened because of this weak discipline. Data source and data description are shown in Appendix 2.

We estimate a cross-country panel data that varies across countries (indexed by i) and over time (indexed by t). We allow for country-specific fixed effects. The estimation equation is written as;

$$s_{it} = a_1 d_{it-1} + a_2 d_{it-1}^2 + a_3 d_{it-1}^3 + c_i + bX + u_{it}, \quad t = 1985, \ldots, 2014,$$

where s_{it}, d_{it}, c_i, and u_{it} note primary surplus to GDP ratio, debt to GDP ratio, country-specific effect, and error term, respectively and X is a vector of control variables. In addition, we allow for the possibility that an unobserved shock this period will affect the fiscal reaction by the government for at least the next several periods. When serial correlation of disturbances across periods is present but we estimate equation ignoring it, estimation results we obtain are consistent but inefficient estimates of the regression coefficients and biased standard errors. To

[11] For example, an oil export country has large current account surplus when oil price rises up.

address this problem, we allow for serial correlation in the error term and model u_{it} as an AR(1) process; $u_{it} = \rho u_{it-1} + \omega_{it}$, $|\rho| < 1$ and $\omega_{it} \sim \mathrm{IID}(0, \sigma_{\omega}^2)$.

A-2, Estimation results

Table 3 shows estimation results for various specifications of the cubic fiscal reaction function. Estimation 1 shows the result of the estimation in which the GDP gap and the government expenditure gap are used as the control variables. Estimation 2 shows the result of the estimation equation in which all six variables we explained above are used as the control variables.

The coefficients of the cubic functional form, capturing the increasing but slowing response of the primary balance to lagged debt, are statistically significant for results of estimations 1 and 2. The estimated coefficients of other determinants included in the fiscal reaction function are also plausible. The primary balance responds positively to the output gap. Temporary increases in government outlays as captured by the government expenditure gap variable, affect the primary balance negatively. Inflation rate has a positive effect on the primary balance in the benchmark estimation. Trade openness has no effect on the primary balance, which is different result from Ghosh *et al.* (2013). Growth rate of GDP trend affect the primary balance positively. Current account to GDP has a positive effect on the primary balance, which means that government can secure tax revenue much more and the surplus-generating capacity becomes large when positive current account becomes large. We examine the robustness of estimated nonlinear fiscal reaction function to different estimation methods for estimation 2 in Table 3. This examination is shown in Appendix 3A.

Figure 3 shows the scatter diagram with the fitted value ($\hat{s}_{it}$) calculated from the result of estimation 2 in Table 3 and lagged debt to GDP ratio (d_{it-1}).The reaction function exhibits a non-monotone movement as the debt-GDP ratio rises. The primary balance-GDP ratio first decreases as the debt-GDP ratio rises; the government tends to have a weak concern on the fiscal position when the debt outstanding is small. As the debt-GDP ratio increases, the primary balance-GDP ratio rises but the responsiveness eventually starts to weaken. Finally at very high level of the debt-GDP ratio, the primary balance- GDP ratio decreases.

The bold red line in Figure 4 shows the fiscal reaction function obtained by estimation 4 in Table 3.The marginal response of primary balance-GDP to lagged debt-GDP starts to decline at the debt-GDP ratio around 130 percent and becomes negative at the debt-GDP ratio around 200 percent. These numbers are larger than the ones Ghosh *et al.*(2013) estimates.[12] One possible reason is the difference in the sample period; their sample period is 1985-2007 but ours is 1985-2014, which includes the period after the global financial crisis in 2009.

We estimate the estimation equation with six control variables for 1985-2007 instead of 1985-2014. Estimation 3 in Table 3 shows the estimation result. We do not obtain the statically significant relation about the fiscal surplus and the debt-GDP ratio, though Ghosh *et al.*(2013) obtain statistically significance of the debt-GDP ratio to the fiscal surplus. The fact that the fiscal surplus responds to the debt-GDP

[12] By Ghosh *et al*. (2013), the marginal response of primary balance to lagged debt starts to decline at the debt-GDP ratio around 90-100 percent and becomes negative at the debt-GDP ratio around 150 percent.

ratio less in the period 1985-2007 but responds strongly it in the period 1985-2014 may suggest government cares about the fiscal reaction in the fiscally severe period.

Because in general large shocks such as wars, fiscal fallout of financial crisis or fiscal crisis may cause temporary deviations from usual primary balance response and the slope coefficients of the primary, we control the impact of the European sovereign fiscal crisis starting from 2010 using the coefficient dummy with debt-GDP ratio for 4 countries (Greece, Portugal, Ireland, and Italy) and for 3 years (2010-2012). Estimation 4 in Table 3 shows the result of the estimation equation. These four countries (Greece, Portugal, Ireland, and Italy) have a statistically different fiscal response during the fiscal crisis.

Next, we estimate the fiscal reaction function using the sample that excludes Japan's data. Japan's government may decide the fiscal surplus responding to debt-GDP ratio differently from other countries. Ghosh *et al.* (2013) note that the response of primary balance to lagged debt in Japan is different from other countries for their sample period. Beirne and Fratzscher (2013) analyze the pricing of sovereign risk for 31 advanced and emerging economies during the European sovereign debt crisis, but Japan is not included in their analysis. In Appendix 3B, we examine the slope homogeneity for each country. For Greece, Ireland and Japan, the null hypothesis of slope homogeneity is rejected.

Estimation 5 in Table 3 shows the estimation result using the sample that excludes Japan's data. Even then, the non-linearity of fiscal reaction function is supported. We examine the robustness of estimated nonlinear fiscal reaction function to different estimation methods for estimation 5 in Table 3. This examination is shown in Appendix 3C.

Figure 4 compare two shapes between fiscal reaction functions obtained by estimations using all countries (result of estimation 2 in Table 3, drawn by the red and bold line) and the countries excluding Japan (result of estimation 5 in Table 3, drawn by the black and dash line). In the case for estimation using countries excluding Japan, the marginal response of primary balance to lagged debt starts to decline at the debt-GDP ratio around 130 percent and becomes negative at the debt-GDP ratio around 210 percent, which numbers are not so different with ones in the case for estimation using all countries. However, compared with these two fiscal reaction functions, the results using the countries excluding Japan show that the responsiveness is stronger in average. When the debt ratio is 160 percent, the primary balance is about 5.6 percent for the estimation using the countries excluding Japan but it is about 3.0 percent for the estimation using all countries. When the debt ratio is 180 percent, the primary balance is about 6.6 percent for the estimation excluding Japan, but it is about 3.4 percent for the estimation using all countries.

Finally, we estimate the reaction function controlling the impact of the fiscal crisis in Europe for the sample excluding Japan's data. Sovereign dummy we use is same as the one when we use in the estimation 4. Estimation 6 in Table 3 shows the estimation result. Greece, Portugal, Ireland and Italy have a statistically different fiscal response during the fiscal crisis.

B. Fiscal Space for the "average" country

We choose parameters. We choose the benchmark pair (g , A) from the data for the period of 2000-2009 in Table 1. We set the GDP growth rate at g =0.021, and A = 0.021. The data of the bond yield until 2008 is supposed not to reflect the fiscal risk, and regarded as the return on the asset with no fiscal risk. We examine the effect of the rise in the real interest rate by varying from 0.021 to 0.03, and 0.04.

Benjamin and Wright (2008) estimate the recovery rate for upper-middle income countries to be about 60-100 percent; hence, for advanced economies we assume that a recovery rate is 90 percent, that is, the hair cut rate h=0.1. We choose θ by θ=0, 0.05, 0.08 to keep $\theta < h$. Reinhalt and Rogoff (2010), in their sample of 224 domestic crises, document that the declines in output around the period of a domestic debt crisis amount to eight percent on average.

We specify the $G(.)$ function by the triangle density function for the support $[-0.5\varepsilon^m, 0.5\varepsilon^m]$. The mean is zero and the standard deviation is $\varepsilon^m / 2\sqrt{3}$. As ε^m gets larger, the tail risk increases, and the default probability increases.

We use the parameters of the reaction function from the estimation 5 in Table 3. We set ε^m by using the standard deviation of the error term of the regression. We obtain the lower bound from the standard deviation in the estimation 5 in Table 3, in which the error term includes the autocorrelation. We obtain ε^m=0.051. We obtain the higher bound from the standard deviation in the estimation 5 in Table A4, in which the error term has no autocorrelation. We obtain ε^m=0.083. The term ε^m ranges from 0.05 to 0.08.

Table 4A shows debt limits for the "average" country. We pick up several features. First, the debt limit is over 200 percent for all the cases except for $(\theta, \varepsilon^m, A)$=(0, 0.08, 0.04), since the average debt-GDP ratio is 88.4 percent in 2014, the average country tends to have the fiscal space. Second, for given θ and A, as ε^m increases, the debt limit declines. Third, for given θ and ε^m, as A increases, the debt limit declines. For ε^m=0.08 and θ=0, as A ranges from low to high, the debt limit decline from 244 to 228, and then the fiscal space disappears. Forth, for given A and ε^m, as θ increases, the debt limit gets larger. There is the positive effect of the absence of safe assets on the fiscal sustainability. Fifth, the quantitative effect of the change in θ on the debt limit is larger as A goes up.

Table 4B shows the details of the effects the change in θ. As the debt limit gets larger, the yield limit and the default probability get higher.

C: Fiscal Space for 23 individual countries

We calculate the fiscal space for each of 23 countries. The government reaction functions vary across countries. The sensitivity of the primary balance to the debt is common to all countries, but the five explanatory variables and the constant term are specific to individual countries. We use the coefficient of the country dummy, the output gap, the government expenditure gap, the ratio of the current account to GDP, the trend GDP growth, and the inflation rate in the regressions to deal with the possible country's difference in the government reaction function. We use the coefficients of the estimation 5 in Table 3 that excludes Japan. In examining the Japan's fiscal space, we use the coefficients of the estimation 2 in Table 3 that includes Japan. We set ε^m=0.08 in the following analysis.

Column 2 in Table 5 depicts the baseline result when the riskless interest rate is A=0.021, and the economy access the safe asset freely (θ=0). We define the fiscal space by the difference in the predicted debt limit and the debt in 2014 so long as the former exceeds the latter. The debt limits range from 220 to 270 percent except for Japan and Portugal, and tend to be higher than Ghosh *et al*. (2013). Greece has the fiscal space, implying that Greece fell into the fiscal crisis although there was the fiscal space. Note that the fiscal space is defined by the assumption that the equilibrium is in a good equilibrium. One interpretation is that the Greek fiscal crisis was a panic equilibrium in which the pessimistic belief could make the default equilibrium self-fulfilling (e.g., Calvo 1988).

As shown in Table 1, after the global financial crisis, the real interest rate declined. Column 3 in Table 5 shows the effect of the decline in the real interest rate. Portugal has fiscal space, but Japan has not yet. Additionally, Table 1 shows that the GDP growth rates also fell down and did particularly for several European countries that had fiscal troubles. We recalculate fiscal space for each country by replacing the GDP growth rate by the average for the period 2010-15. Column 4 shows that Greece as well as Japan and Portugal has no fiscal space. We can conclude that all the countries have fiscal spaces except for Greece, Japan, and Portugal, given that there is no absence of safe assets. This finding is different from Ghosh *et al*. (2013), in which Italy and Ireland as well as these three countries find no fiscal spaces.

The primary concern is to explain why Japan sustains the high outstanding debt. As Table 1 shows, the Japanese bond yields are lower than the averages of all countries but the debt-GDP ratios are higher for all the intervals. As we have studied in the theoretical part, the absence of safe assets can explain this situation.

Column 5 in Table 5 show the case when θ=0.05. The debt limit is higher, but still short of the outstanding debt. Column 6 shows that when θ=0.08, the debt limit almost reaches the outstanding debt. Japan has fiscal space if the government behaves following the disciplined reaction function, if the extent of the absence of safe assets is high, and if the growth-adjusted interest rate is negative. Interestingly, when the haircut rate (h) is up to 0.2, we obtain almost the same debt limit when θ

is doubled to 0.16 (although the result is not listed). The ratio θ / h is more important than θ.

5. Conclusion

We investigate a country's fiscal sustainability by estimating the fiscal space following Ghosh *et al*. (2013). Using a panel data of 23 advanced countries over the period 1985-2014 that includes the period of European fiscal crises, we estimated the cubic reaction function that is non-linear and statistically significant. All countries have robust fiscal spaces except for Japan, Portugal, and Greece. The low real interest rate relative to the real GDP growth rate contributes to fiscal sustainability. When adding the effect of "absence of safe assets", the concept of which relates the incomplete international risk diversification of portfolio, we find the fiscal space more likely. Japan has the fiscal space if the extent of the absence of safe assets is high and if the growth-adjusted interest rate is negative.

Literature

Ahearne, A. G., Griever, W. L., Warnock, F. E., 2004. Information costs and home bias: an analysis of U.S. holdings of foreign equities. Journal of International Economics 62(2), 313-336

Aizenmana, Joshua., Michael Hutchisonb, and Yothin Jinjarakc, 2013, What is the risk of European sovereign debt defaults? Fiscal space, CDS spreads and market pricing of risk, *Journal of International Money and Finance*, 34, April, 37–59.

Altera, Adrian., and Andreas Beyerb, 2014, The dynamics of spillover effects during the European sovereign debt turmoil, *Journal of Banking & Finance*, 42, May, 134–153.

Antonakakis, Nikolaos., and Konstantinos Vergos, 2013, Sovereign bond yield spillovers in the Euro zone during the financial and debt crisis, *Journal of International Financial Markets, Institutions and Money*, 26, October, 258–272.

Arellano, C., 2008, Default risk and income fluctuations in emerging economies, *The American Economic Review*, 98(3), 690-712.

Broda, C., and Weinstein, D.E., 2005. Happy News from the Dismal Science: Reassessing the Japanese Fiscal Policy and Sustainability. In: Takatoshi, I., Patrick, H., Weinstein, D.E., (Eds.). *Reviving Japanese Economy*: MIT Press, 39-78.

Barro, Robert J., 2006, Rare Disasters and Asset Markets in the Twentieth Century, *The Quarter Journal of Economics*, 121(3), 823-866.

Battistini, Niccolò, Marco Pagano, and Saverio Simonelli, 2013, Systemic Risk and Home Bias in the Euro Area, *European Economy Economic Papers* 494.

Benguia, Julien, Enrique G. Mendoza, and Vincenzo Quadrinid, 2013, Capital mobility and international sharing of cyclical risk, *Journal of Monetary Economics*, 60(1), 42–62.

Benjamin, D., and M. Wright, 2009, Recovery Before Redemption? A Theory of Delays in Sovereign Debt Renegotiations, *Center for Applied Macroeconomic Analysis Working Paper* 15/2009.

Beirne, John., and Marcel Fratzscher, 2013, The pricing of sovereign risk and contagion during the European sovereign debt crisis, *Journal of International Money and Finance*, 34, April, 60–82.

Bohn, Henning, 1998, The behavior of US public debt and deficits, *the Quarterly Journal of economics*, 113(3), 949-963.

Caballero, R.J., and Farhi, E., 2014, On the role of safe asset shortages in secular stagnation. In: Coen, T., Baldwin, R., (Eds.). *Secular Stagnation: Facts, Causes, and Cures*: CEPR Press, 111-122.

Calvo, A.G., 1988. Servicing the Public Debt: The Role of Expectations. *The American Economic Review*, 78(4), 647-661.

Coeurdacier, Nicolas, and Rey, Hélène, 2013, Home Bias in Open Economy Financial Macroeconomics, *Journal of Economic Literature*, 51(1), March, 63-115.

Cohen, Daniel, and Jeffrey Sachs, 1986, Growth and external debt under risk of debt repudiation, *European Economic Review,* 30.3, 529-560.

Cole, Harold L. and Kehoe, T.J., 2000, Self-Fulfilling Debt Crises, *The Review of Economic Studies,* 67(1), 91-116.

Cole, Harold L., and Maurice Obstfeld, 1991, Commodity trade and international risk sharing: How much do financial markets matter? *Journal of Monetary Economics*, 28(1), August, 3-24.

Dahlquist, M., Pinkowitz, L., Stulz, R.M., Williamson, R., 2003. Corporate governance and home bias. *Journal of Financial and Quantitative Analysis,* 38(01), 87-110.

Dan Su and Xiang Li, 2016, Equity Home Bias Puzzle Revisited

Dekle, R., 2005. Understanding Japanese Saving: Does Population Aging Matter? : Routledge Curzon.

Doi, T., and Ihori, T., 2003, Sustainability of Government Deficits in Japan: Including Trends in Local Government Finance. In: Yoshino, N., Inukai, S., Tamaki, N., (Eds.). *Enhancing Market Functions in Japan*. : Keio University Press, 3-29

Doi, T., Hoshi, T., and Okimoto, T., 2011. Japanese Government Debt and sustainability of Fiscal Policy. *Journal of the Japanese and International Economies,* 25(4), 414-433

Fidora, M., Fratzscher, M., Thimann, C., 2007. Home bias in global bond and equity markets: The role of real exchange rate volatility. *Journal of International Money and Finance,* 26(4), 631-655.

Glassman, D.A., Riddick, L.A., 2001. What causes home asset bias and how should it be measured? *Journal of Empirical Finance,* 8(1), 35-54.

Gruber, Joseph, W., and Steven B. Kamin, 2012, Fiscal Positions and Government Bond Yields in OECD Countries, *Journal of Money, Credit and Banking,* 44(8), December, 1563–1587.

Grauwea, Paul D., and Yuemei Ji, 2013, Self-fulfilling crises in the Eurozone: An empirical test, *Journal of International Money and Finance*, 34, April, 15–36.

Ghosh, A.R., Kim, J.I., Mendoza, E.G., Ostry, J. D., Qureshi, M.S., 2013. Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *The Economic Journal,* 123(566), F4-F30.

Gourinchas, P.O., and Jeanne, O., 2012. Global safe assets. *BIS Working Paper,* No. 399.

Hansen, Gary D., and Selahattin İmrohoroğlu, 2016, Fiscal reform and government debt in Japan: A neoclassical perspective, *Review of Economic Dynamics,* 21, 201-224.

Hoshi, T., Ito, T., 2013. Is the Sky the Limit? Can Japanese Government Bonds Continue to Defy Gravity? *Asian Economic Policy Review,* 108(2), 218-247.

Ihori, T., Kato, R., Kawade, M., and Bessho, S., 2006. Public Debt Policy and Economic Growth in An Aging Japan. In: Kaizuka, K., Krueger, A., (Eds.). *Tackling Japan's Fiscal Challenges: Strategies to Cope with High Public Debt and Population Aging*. IMF: Palgrave Macmillan Ltd., 30-68.

Matsuoka, H., 2015, Fiscal limits and sovereign default risk in Japan. *Journal of the Japanese and International Economies,* 38, 13-30.

Mendoza, Enrique G., and Jonathan D. Ostry, 2008, International evidence on fiscal solvency: Is fiscal policy “responsible”? *Journal of Monetary Economics,* 55(6), 1081-1093.

Miyazawa, K., Yamada, J., 2015, The growth strategy of Abenomics and fiscal consolidation. *Journal of the Japanese and International Economies.* 37, 82-99.

Martin, Philippe., and Hélène Rey, 2004, Financial super-markets: size matters for

asset trade, *Journal of International Economics*, 64(2), December, 335–361.
Ostry, Jonathan David, and Abdul Abiad, 2005, *Primary surpluses and sustainable debt levels in emerging market countries*. No. 5-6. International Monetary Fund.
Portes, R., Rey, H., 2005. The determinants of cross-border equity flows. *Journal of International Economics* 65(2), 269-296
Ravn, M and H. Uhlig, 2002, On adjusting the Hodrick-Prescott filter for the frequency of observations, *The Review of Economics and Statistics*, 84, 371-375.
Romer, D., 2001. *Advanced Macro Economics*, Second Edition. McGraw-Hill Higher Education.
Reinhalt, C.M., Rogoff, K.S., 2010. *This Time is Different: Eight Centuries of Financial Folly*. Princeton University Press.
Sakuragawa, Masaya. and Yukie Sakuragawa, 2016, Absence of safe assets and fiscal crisis, *Journal of the Japanese and International Economies,* 40, 59–76.
Sakuragawa, M., Hosono, K., 2010. Fiscal Sustainability of Japan: A Dynamic Stochastic General Equilibrium Approach. *The Japanese Economic Review,* 61(4), 517-537.
Sakuragawa, M., Hosono, K., 2011. Fiscal Sustainability in Japan. *Journal of the Japanese and International Economies*, 25(4), 434-446.
Stiglitz, Joseph E., and Andrew Weiss, 1981, Credit rationing in markets with imperfect information, *The American economic review*, 71(3),393-410.
Williamson, Stephen D. 1986, Costly monitoring, financial intermediation, and equilibrium credit rationing, *Journal of Monetary Economics,* 18(2), 159-179.

Appendix 1: Numerical solution

We specify the $G(\varepsilon)$ function and the density as

$$G(\varepsilon_t) = \begin{cases} 0 & \text{for} \quad \varepsilon_t < -0.5\varepsilon^m \\ \dfrac{2}{(\varepsilon^m)^2}(\varepsilon_t + \dfrac{1}{2}\varepsilon^m)^2 & \text{for} \quad -0.5\varepsilon^m \le \varepsilon_t \le 0 \\ -\dfrac{2}{(\varepsilon^m)^2}(\varepsilon_t - \dfrac{1}{2}\varepsilon^m)^2 + 1 & \text{for} \quad 0 \le \varepsilon_t \le 0.5\varepsilon^m \\ 1 & \text{for} \quad \varepsilon_t \ge 0.5\varepsilon^m \end{cases} \quad \text{(A1)}$$

$$G'(\varepsilon_t) = \begin{cases} 0 & \text{for} \quad \varepsilon_t \le -0.5\varepsilon^m \\ \dfrac{4}{(\varepsilon^m)^2}(\varepsilon_t + \dfrac{1}{2}\varepsilon^m) & \text{for} \quad -0.5\varepsilon^m \le \varepsilon_t \le 0 \\ \dfrac{-4}{(\varepsilon^m)^2}(\varepsilon_t - \dfrac{1}{2}\varepsilon^m) & \text{for} \quad 0 \le \varepsilon_t \le 0.5\varepsilon^m \\ 0 & \text{for} \quad \varepsilon_t \ge 0.5\varepsilon^m \end{cases} \quad \text{(A2)}$$

Using (5), we define the cutoff

$$\bar{\varepsilon}_{t+1} = \frac{1+R_{t+1}}{1+g}d_t - \mu - f(d_t) - \bar{d} \equiv \frac{1+R_{t+1}}{1+g}d_t - B(d_t, \bar{d}) \equiv \bar{\varepsilon}(R_{t+1}, d_t, \bar{d}) \quad \text{(A3)},$$

where $B(d_t, \bar{d}) \equiv \mu + d_t + f(d_t) + (\bar{d} - d_t) \equiv B$. The default probability is

$$\pi_{t+1} = prob.[\varepsilon_{t+1} < \bar{\varepsilon}_{t+1}] \equiv G(\bar{\varepsilon}(R_{t+1}, d_t, \bar{d}))$$

$$= \begin{cases} 0 & \text{for} \quad \dfrac{1+R_{t+1}}{1+g}d_t \le B - \dfrac{1}{2}\varepsilon^m \\ \dfrac{2}{(\varepsilon^m)^2}\{\dfrac{1+R_{t+1}}{1+g}d_t - B + \dfrac{1}{2}\varepsilon^m\}^2 & \text{for} \quad B - \dfrac{1}{2}\varepsilon^m \le \dfrac{1+R_{t+1}}{1+g}d_t \le B \\ -\dfrac{2}{(\varepsilon^m)^2}(\dfrac{1+R_{t+1}}{1+g}d_t - B - \dfrac{1}{2}\varepsilon^m)^2 + 1 & \text{for} \quad B \le \dfrac{1+R_{t+1}}{1+g}d_t \le B + \dfrac{1}{2}\varepsilon^m \\ 1 & \text{for} \quad \dfrac{1+R_{t+1}}{1+g}d_t \ge B + \dfrac{1}{2}\varepsilon^m \end{cases}$$

(A4)

When $B-\frac{1}{2}\varepsilon^m \le \frac{1+R_{t+1}}{1+g}d_t \le B$, we write the net return function:

$$\Omega(R_{t+1},d_t,\bar{d})$$

$$=(1+R_{t+1})\{1-hG(\bar{\varepsilon}(R_{t+1},d_t,\bar{d}))\}-(1+A)\{1-\theta G(\bar{\varepsilon}(R_{t+1},d_t,\bar{d})\}$$

$$=(1+R_{t+1})[1-h\frac{2}{(\varepsilon^m)^2}\{\frac{1+R_{t+1}}{1+g}d_t-(B-\frac{1}{2}\varepsilon^m)\}^2]$$

$$-(1+A)[1-\frac{2\theta}{(\varepsilon^m)^2}\{\frac{1+R_{t+1}}{1+g}d_t-(B-\frac{1}{2}\varepsilon^m)\}^2]$$

$$=-\frac{2h}{(\varepsilon^m)^2}(\frac{d_t}{1+g})^2(1+R_{t+1})^3+\frac{2}{(\varepsilon^m)^2}\frac{d_t}{1+g}\{2h(B-\frac{1}{2}\varepsilon^m)+\theta(1+\widetilde{A})d_t\}(1+R_{t+1})^2$$

$$+\frac{1}{(\varepsilon^m)^2}\{(\varepsilon^m)^2-2h(B-\frac{1}{2}\varepsilon^m)^2-4\theta(1+\widetilde{A})(B-\frac{1}{2}\varepsilon^m)d_t\}(1+R_{t+1})$$

$$-\frac{1+A}{(\varepsilon^m)^2}\{(\varepsilon^m)^2-2\theta(B-\frac{1}{2}\varepsilon^m)^2\},$$

(A5)

where $1+\widetilde{A}\equiv(1+A)/(1+g)$.

The bond yield is determined to satisfy $\Omega(R_{t+1},d_t,\bar{d})=0$, and written as

$$R_{t+1}=R(d_t,\bar{d}), \qquad \text{(A6).}$$

We can calculate the debt limit $\bar{d}$ jointly with the "bond yield limit" $\bar{R}$ by $\Omega(\bar{R},\bar{d},\bar{d})=0$ and $\partial\Omega(\bar{R},\bar{d},\bar{d})/\partial(1+R_{t+1})=0$. From the latter equation, we obtain

$$1+\bar{R}=\frac{(1+g)^2}{6h\bar{d}^2}[\frac{2\bar{d}_{t+1}}{1+g}\{2h(\bar{B}-\frac{1}{2}\varepsilon^m)+\theta(1+\widetilde{A})\bar{d}\}+\sqrt{D_{\bar{R}}}]\equiv 1+\bar{R}(\bar{d}) \qquad \text{(A7),}$$

where $D_{\bar{R}}\equiv[\frac{2\bar{d}}{1+g}\{2h(\bar{B}-\frac{1}{2}\varepsilon^m)+\theta(1+\widetilde{A})\bar{d}\}]^2$

$$+\frac{6h\bar{d}^2}{(1+g)^2}\{(\varepsilon^m)^2-2h(\bar{B}-\frac{1}{2}\varepsilon^m)^2-4\theta(1+\widetilde{A})\bar{d}(\bar{B}-\frac{1}{2}\varepsilon^m)\}$$

and $\bar{B}\equiv\bar{B}(\bar{d},\bar{d})=\mu+\bar{d}+f(\bar{d})+(\bar{d}-\bar{d})=\mu+\bar{d}+f(\bar{d})$.

By incorporating (A7) into (A5) and rearranging it, we obtain

$\Omega(\bar{R}, \bar{d}, \bar{d})$

$$= -\frac{2h}{(\varepsilon^m)^2}(\frac{\bar{d}}{1+g})^2(1+\bar{R})^3 + \frac{2\bar{d}}{(\varepsilon^m)^2(1+g)}\{2h(\bar{B}-\frac{1}{2}\varepsilon^m) + (1+\tilde{A})\theta\bar{d}\}(1+\bar{R})^2$$

$$+\frac{1}{(\varepsilon^m)^2}\{(\varepsilon^m)^2 - 2h(\bar{B}-\frac{1}{2}\varepsilon^m)^2 - 4(1+\tilde{A})\theta\bar{d}(\bar{B}-\frac{1}{2}\varepsilon^m)\}(1+\bar{R})$$

$$-\frac{1+A}{(\varepsilon^m)^2}\{(\varepsilon^m)^2 - 2\theta(\bar{B}-\frac{1}{2}\varepsilon^m)^2\} \quad \text{(A8).}$$

The debt limit $\bar{d}$ has to satisfy $\Omega(\bar{R}, \bar{d}, \bar{d}) = 0$. Using (A7), this equation can be denoted as $\Omega(\bar{R}(\bar{d}), \bar{d}, \bar{d}) = 0$.

The probability of fiscal default is

$$\bar{\pi} = \frac{2}{(\varepsilon^m)^2}(\frac{1+\bar{R}}{1+g}\bar{d} - \bar{B} + \frac{1}{2}\varepsilon^m)^2 \quad \text{(A9)}$$

Given that the debt limit $\bar{d}$ has been determined, the current bond yield R_{t+1} is determined to satisfy $\Omega(R_{t+1}, d_t, \bar{d}) = 0$ using (A5). Similarly, the current default probability π_{t+1} is written, using (A4), by

$$\pi_{t+1} = \frac{1}{\varepsilon^m}\{\frac{1+R_{t+1}}{1+g}d_t - B(d_t, \bar{d}) + 0.5\varepsilon^m\},$$

With the realization of ε_{t+1}, we obtain the next debt by

$$d_{t+1} = \frac{1+R_{t+1}}{1+g_t}d_t - \{\mu + f(d_t) + \varepsilon_{t+1}\}$$

Appendix 2: Data description

This appendix explains variable definitions and data sources for estimating fiscal reaction function. Table A1 presents the basic statistics of our data.

Primary surplus to GDP ratio

We use the general government primary net lending/borrowing as shares of GDP from the IMF's World Economic Outlook Database (2016 April).

Gross government debt to GDP ratio

We use the general government gross debt as shares of GDP from the IMF's World Economic Outlook Database (2016 April).

GDP gap

This variable is calculated by difference rate between actual GDP and potential one.

Actual GDP is the gross domestic product evaluated by constant prices from the IMF's World Economic Outlook Database (2016 April) and potential GDP is calculated using the Hodric-Prescott filter with the smoothing parameter set 6.25, which value is recommended for annual data by Ravn and Uhlig (2002).

The formulation of GDP gap is the following; $GDP\ gap = \frac{GDP - GDP^{trend}}{GDP}$.

Government expenditure gap

This variable is calculated by difference rate between actual government expenditure and potential one. Actual government expenditure is the general government total expenditure from the IMF's World Economic Outlook Database (2016 April), which consists of total expense and the net acquisition of nonfinancial assets. Potential government expenditure is calculated using the Hodric-Prescott filter with the smoothing parameter set 6.25. The formulation of GDP gap is the following; $GOVE\ gap = \frac{GOVE - GOVE^{trend}}{GOVE}$.

Inflation rate

We use inflation rate calculated as annual change of average consumer prices from the IMF's World Economic Outlook Database (2016 April). We take the average in the previous three years of inflation rate.

Trade openness

Trade openness is calculated by sum of export and import to GDP and its data source is World Bank's World Development Indicators. We take the average in the previous three years of the trade openness.

Trend GDP growth

We make this variable as the growth rate of the potential GDP calculated above.

Current account balance

We use the current account balance as shares of GDP from the IMF's World Economic Outlook Database (2016 April). Current account is all transactions other than those in financial and capital items. The major classifications are goods and services, income and current transfers. We take the average in the previous three years of current account balance.

Appendix 3: Sensitivity analysis of the estimated nonlinear fiscal reaction function

This appendix examines the sensitivity analysis of the estimated nonlinear fiscal reaction function

A: Robustness to different estimation methods in the case using the six control variables and the sample for all countries

We examine the robustness of estimated nonlinear fiscal reaction function to different estimation method for estimation 2 in Table 3. Table A2 shows the results. Estimation 1 shows the same result as one in estimation 2 in Table 3, of which method is the fixed effects equation with assuming AR(1) error structure. Estimation 2 shows the result of the random effect model with assuming AR(1) error structure. Estimation 3 shows the result of the fixed effects equation with year effects included and with assuming AR(1) error structure. Estimation 4 shows the result of the estimation equation through pooled ordinary least squares method. Estimation 5 shows the results of the fixed effects equation without assuming AR(1) error structure. Estimation 6 shows the result of the fixed effects equation with year effects included and without assuming AR(1) error structure. We find that the non-linearity is supported in each estimation methods. The Hausman test for fixed effects - random effect is $\chi^2(9)$ equals 32.44 with p-value of 0.0002, which reject the null-hypothesis that the preferred model is the random effect at the 1percent significant level.

B: Slope Homogeneity

We examine the slope homogeneity of each country. We add the country-specific debt terms into the estimation equation for each country one by one. Estimation equation is written as

$$s_{it} = a_1 d_{it-1} + a_2 d_{it-1}^2 + a_3 d_{it-1}^3$$

$$+ a_1^j d_{it-1} \times dum_j + a_2^j d_{i,t-1}^2 \times dum_j + a_3^j d_{it-1}^3 \times dum_j + c_i + bX + u_{it},$$

$$t = 1985,..,2011,$$

where indices i and j denote each country.

Table A3 shows the results. For the countries excluding Greece, Ireland and Japan, the null hypothesis of slope homogeneity cannot be rejected. For Greece and Ireland, the joint significance test of parameter estimates indicates that those governments do a statistically different fiscal response, but they have a cubic fiscal reaction behavior. However, even if we exclude Japan data from our sample, we obtain the cubic fiscal reaction function.

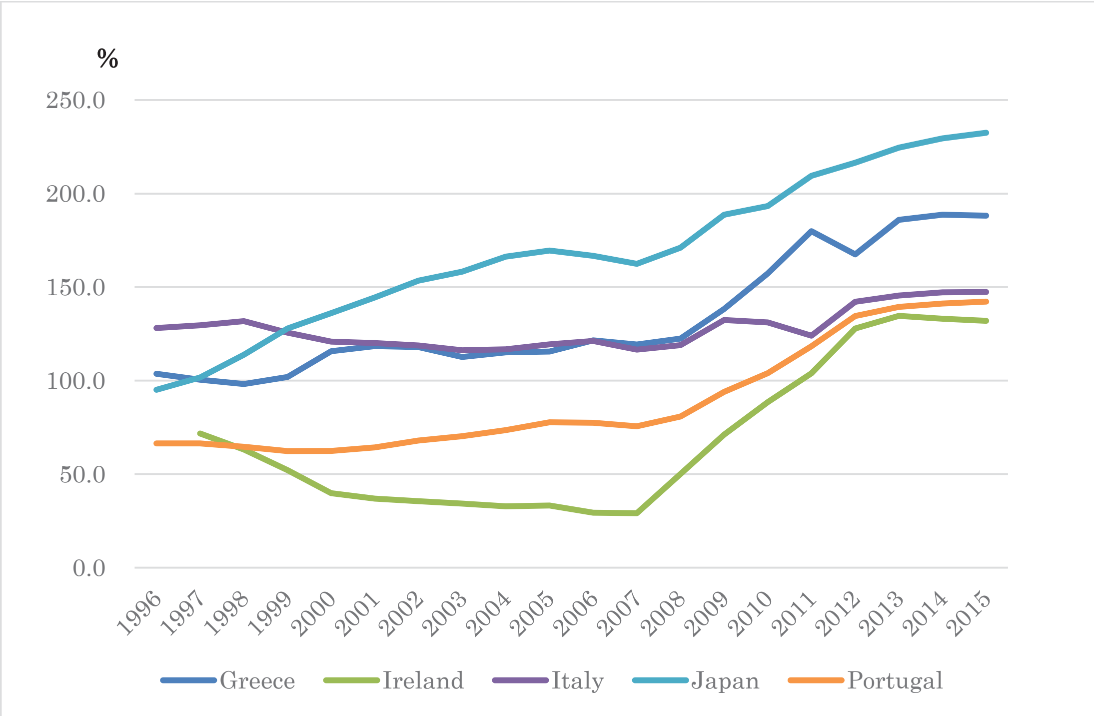
C: Robustness to different estimation methods in the case using the six control variables and the sample for the countries excluding Japan

We examine the robustness of estimated nonlinear fiscal reaction function to different estimation method for estimation 5 in Table 2. Table A4 shows the results. Estimations we do are same as ones we do in Appendix 3A.

We find that the non-linearity is supported in the four estimation methods, but we cannot find the relation of the non-linearity in the pooled ordinary least square method and fixed effect model without assuming AR(1) error structure. The Hausman test for fixed effects - random effect is $\chi^2(9)$ equals 18.85 with p-value of 0.027, which reject the null-hypothesis that the preferred model is the random effect at the 5percent significant level.

Figure 1 Public debt and the nominal bond yield

A. Debt/GDP



B. Nominal Bond Yield

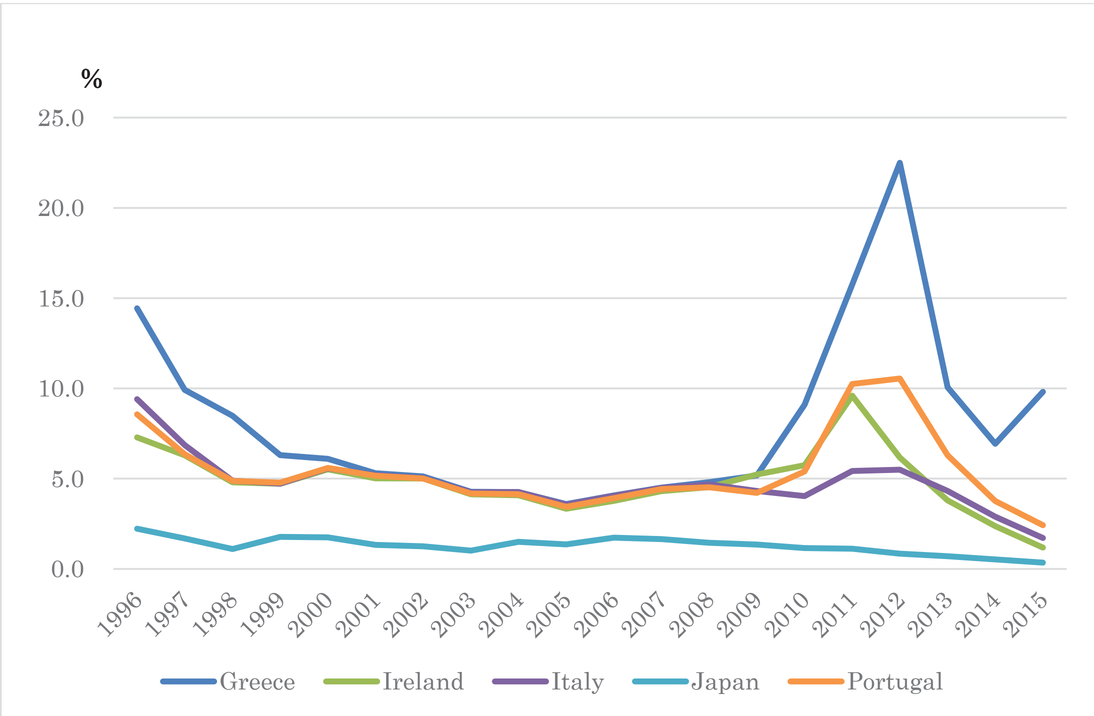


Figure 2 Non-stochastic images of debt dynamics

Figure 2A

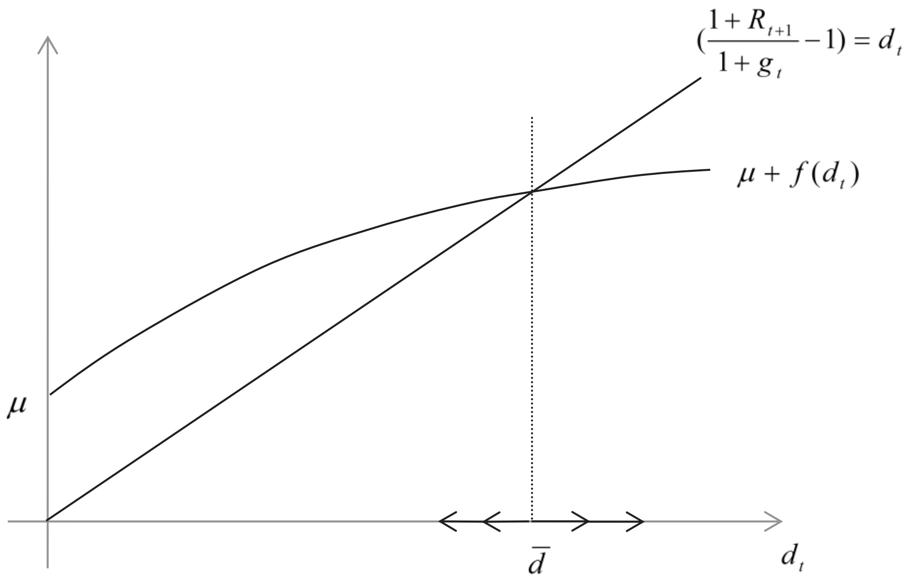


Figure 2B

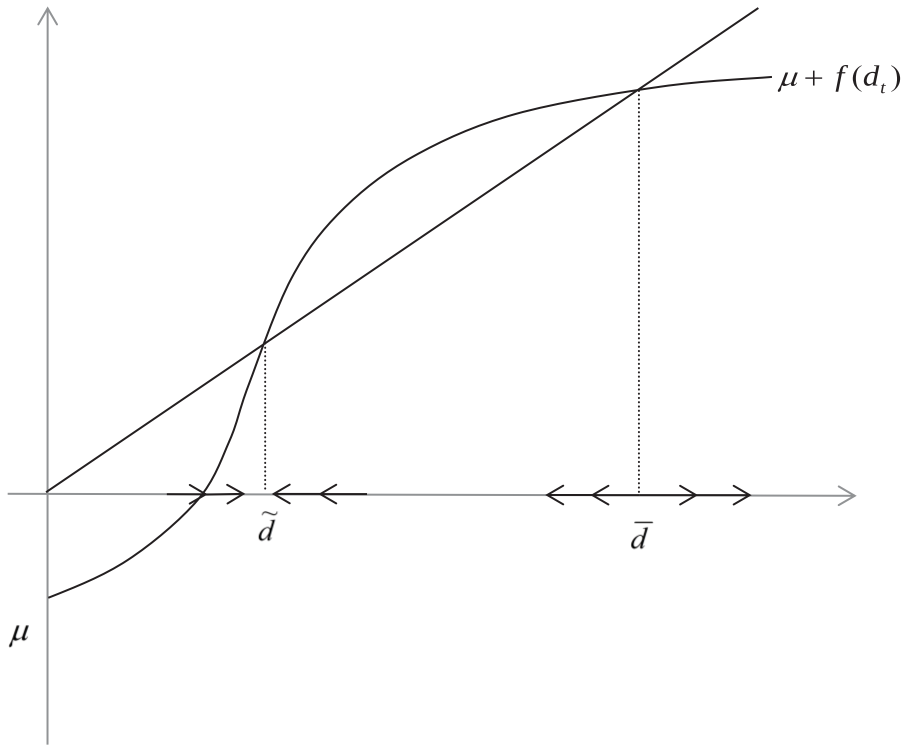
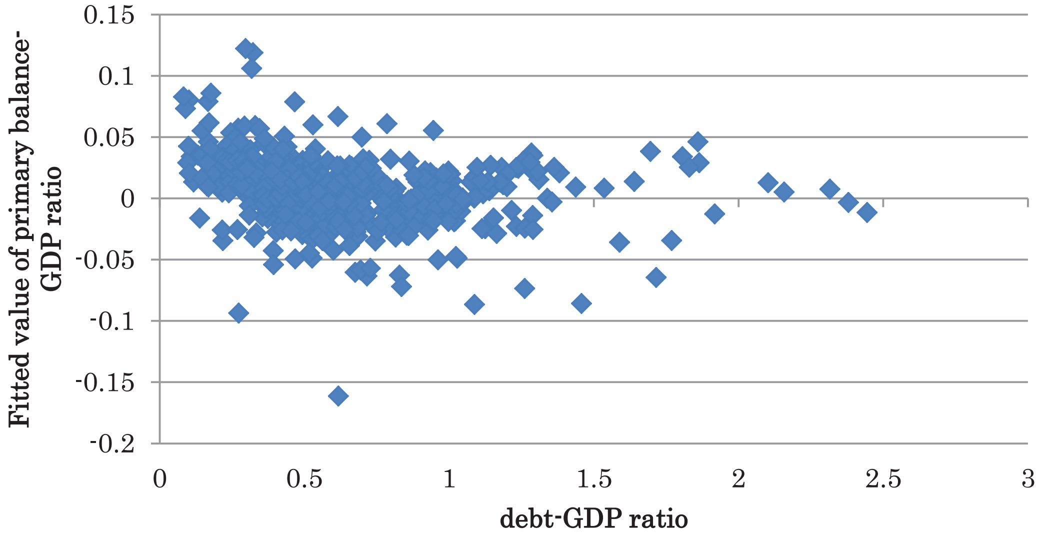


Figure 3 The scatter diagram with

the fitted value of primary balance and debt



Note) the fitted value of primary balance is calculated from the result of estimation 2 in Table 3.

Figure 4 Fiscal reaction functions for samples

for all countries and for the countries excluding Japan

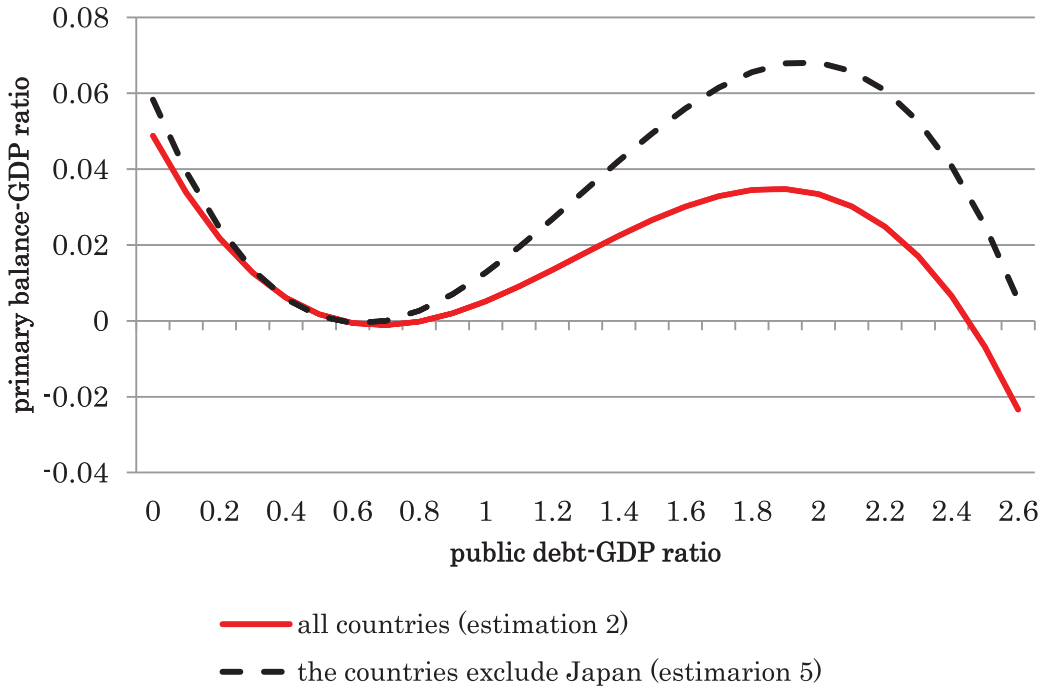


Table 1 GDP growth rate and the bond yield

A. 23 countries average

	1990-1999	2000-2009	2010-2015	2010-2015 (excluding GIPS*)
GDP growth	2.9	2.1	1.5	1.8
bond yield	4.7	2.1	1.7	0.8
Growth-adjusted real interest rate	1.8	0	0.2	-1.0
Debt/GDP	69.4	67.0	90.2	82.3

*) "excluding GIPS" implies that 5 countries that had fiscal crises (Greece, Ireland, Italy, Portugal, and Spain) are excluded.

B. Japan

	1990-1999	2000-2009	2010-2015
GDP growth	1.5	0.6	1.3
bond yield	2.3	1.7	0.6
Growth-adjusted real interest rate	0.8	1.1	-0.7
Debt/GDP	88.1	161.7	217.7

C. Average of 5 countries (Greece, Ireland, Italy, Portugal, and Spain)

	1990-1999	2000-2009	2010-2015
GDP growth	3.2	2.2	0.2
bond yield	5.3	1.7	5.2
Growth-adjusted real interest rate	2.1	-0.5	5.0
Debt/GDP	82.2	81.7	132.3

Table 2 Bond Yields for Different h and θ

h \ θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	2.0	1.0	0.0	-1.1	-2.1	-3.1	-4.1	-5.1	-6.2	-7.2	-8.2
0.1	3.0	2.0	1.0	-0.1	-1.1	-2.1	-3.2	-4.2	-5.2	-6.2	-7.3
0.2	4.1	3.0	2.0	1.0	-0.1	-1.1	-2.2	-3.2	-4.2	-5.3	-6.3
0.3	5.2	4.1	3.1	2.0	0.9	-0.1	-1.2	-2.2	-3.3	-4.3	-5.4
0.4	6.3	5.2	4.1	3.1	2.0	0.9	-0.1	-1.2	-2.3	-3.3	-4.4
0.5	7.4	6.3	5.2	4.1	3.1	2.0	0.9	-0.1	-1.2	-2.3	-3.4
0.6	8.5	7.4	6.3	5.3	4.2	3.1	2.0	0.9	-0.2	-1.3	-2.3
0.7	9.7	8.6	7.5	6.4	5.3	4.2	3.1	2.0	0.9	-0.2	-1.3
0.8	10.9	9.8	8.7	7.5	6.4	5.3	4.2	3.1	2.0	0.9	-0.2
0.9	12.1	11.0	9.8	8.7	7.6	6.5	5.4	4.2	3.1	2.0	0.9
1	13.3	12.2	11.1	9.9	8.8	7.7	6.5	5.4	4.3	3.1	2.0

Table 3 Estimation results of the Fiscal Reaction Funtion

estimation	1	2	3	4	5	6
					ex.Japan	ex. Japan
sample period	1985-2014	1985-2014	1985-2007	1985-2014	1985-2014	1985-2014
lagged debt	-0.116 **	-0.166 ***	-0.086	-0.123 ***	-0.212 ***	-0.169 ***
	(0.046)	(0.043)	(0.070)	(0.043)	(0.057)	(0.059)
lagged debt_square	0.121 **	0.166 ***	0.075	0.126 ***	0.224 ***	0.182 **
	(0.048)	(0.045)	(0.090)	(0.045)	(0.071)	(0.075)
lagged debt_cubic	-0.032 **	-0.043 ***	-0.023	-0.034 ***	-0.058 **	-0.047 *
	(0.014)	(0.013)	(0.032)	(0.013)	(0.025)	(0.028)
lagged debt				-0.327 ***		-0.325 ***
* sovereign dummy				(0.068)		(0.070)
lagged debt_square				0.472 ***		0.475 ***
* sovereign dummy				(0.103)		(0.108)
lagged debt_cubic				-0.159 ***		-0.163 ***
* sovereign dummy				(0.038)		(0.041)
output gap	0.481 ***	0.446 ***	0.366 ***	0.467 ***	0.438 ***	0.456 ***
	(0.043)	(0.043)	(0.056)	(0.043)	(0.045)	(0.045)
govt. expenditure gap	-0.430 ***	-0.415 ***	-0.239 ***	-0.369 ***	-0.410 ***	-0.364 ***
	(0.020)	(0.020)	(0.031)	(0.021)	(0.020)	(0.022)
inflation		0.162 **	0.043	0.138 *	0.155 **	0.131 *
		(0.072)	(0.076)	(0.072)	(0.073)	(0.073)
trade openness		-0.001	-0.006	-0.002	-0.002	-0.004
		(0.022)	(0.029)	(0.021)	(0.021)	(0.021)
trend GDP		0.865 ***	0.674 ***	0.881 ***	0.909 ***	0.922 ***
		(0.146)	(0.184)	(0.143)	(0.146)	(0.146)
curent account		0.187 ***	0.111	0.179 ***	0.193 ***	0.183 ***
		(0.061)	(0.068)	(0.060)	(0.061)	(0.060)
observations	570	570	392	570	541	541
number of countries	23	23	23	23	22	22
R-squared	0.156	0.343	0.208	0.386	0.404	0.435
AR(1) coefficient	0.866	0.812	0.781	0.807	0.791	0.789
sigma_e	0.016	0.015	0.014	0.015	0.015	0.015

Notes) The dependent variable is the primary balance to GDP ratio. Estimation method is fixed effects model with assuming AR(1) error structre. Standard errors are reported in parentheses; ***, ** and * denote statistically significance at 1, 5, and 10 percent level, respectively. Soveriegn dummy takes the value one for four countries (Greece, Portugal, Ireland and Italy) and for three years (2010-2012) and zero otherwise.

Table 4 Debt limit, Yield limit and Default Probability

Table 4A Debt limit

ε^m	θ	A=2.1%	A=3%	A=4%
$\varepsilon^m = 0.05$	θ=0	252	238	218
	θ=0.05	252	240	220
	θ=0.08	254	242	224
$\varepsilon^m = 0.06$	θ=0	218	236	212
	θ=0.05	221	236	214
	θ=0.08	228	240	222
$\varepsilon^m = 0.07$	θ=0	246	230	202
	θ=0.05	248	234	208
	θ=0.08	252	240	220
$\varepsilon^m = 0.08$	θ=0	244	228	40
	θ=0.05	246	230	200
	θ=0.08	251	238	218

Table 4B Debt limit, Yield limit, Default Probability

	θ	debt limit	yield limit	default prob.
A=2.1%	θ=0	244	2.26%	1.3%
$\varepsilon^m = 0.08$	θ=0.05	246	2.40%	5.2%
	θ=0.08	251	2.82%	28.2%

Table 5 Debt limits for 23 countries

	Actual debt	Projected debt limit				
	(1)	(2)	(3)	(4)	(5)	(6)
	Debt 2014	A=2.1%	A=0.8%	A=0.8% g (10-15)	A=0.8% $\theta=0.05$	A=0.8% $\theta=0.08$
Australia	34.1	240		250		
Austria	84.2	230		240		
Belgium	106.7	250		250		
Canada	86.2	250		270		
Denmark	44.6	240		250		
Finland	59.3	240		240		
France	95.6	230		240		
Germany	74.9	240		260		
Greece	178.4	220		40		
Iceland	82.5	260		250		
Ireland	107.5	260		290		
Israel	67.1	260		280		
Italy	132.5	230		220		
Japan	249.1	40	240	230	243	249
Korea	35.1	270		280		
Netherlands	68.2	250		250		
New Zealand	30.8	260		270		
Norway	27.9	270		270		
Portugal	130.2	40	230	40		
Spain	99.3	240		210		
Sweden	44.9	250		270		
United Kingdom	88.2	220		250		
United States	105.0	240		260		

Table A1 Summary of statistics

	Mean	Median	Minimum	Maximum	Std. Dev.
Primary surplus/ GDP	0.00061	-0.000045	-0.297	0.159	0.040
Government debt/GDP	0.634	0.577	0.082	2.445	0.354
GDP gap	0.000025	-0.00046	-0.067	0.054	0.014
Government expenditure gap	-0.00056	-0.0015	-0.107	0.277	0.028
Inflation rate	0.052	0.026	-0.0079	2.747	0.172
Trade openness	0.667	0.618	0.162	1.920	0.310
Trend GDP growth	0.024	0.024	-0.059	0.101	0.019
Current account/GDP	-0.0036	-0.0057	-0.200	0.149	0.044

Table A2　Robustness to difference estimation method for estimarion 2 in Table 3

estimaion	1	2	3	4	5	6
est. method	FE, AR(1)	RE, AR(1)	FE, AR(1), yea	pooled	FE	FE, year
lagged debt	-0.166 ***	-0.158 ***	-0.083 *	-0.146 ***	-0.121 ***	-0.089 ***
	(0.043)	(0.038)	(0.043)	(0.025)	(0.028)	(0.028)
lagged debt_square	0.166 ***	0.165 ***	0.098 **	0.165 ***	0.124 ***	0.088 ***
	(0.045)	(0.040)	(0.043)	(0.027)	(0.029)	(0.029)
lagged debt_cubic	-0.043 ***	-0.046 ***	-0.026 **	-0.054 ***	-0.037 ***	-0.027 ***
	(0.013)	(0.012)	(0.012)	(0.008)	(0.009)	(0.008)
output gap	0.446 ***	0.441 ***	0.231 ***	0.572 ***	0.529 ***	0.370 ***
	(0.043)	(0.043)	(0.054)	(0.087)	(0.071)	(0.091)
govt. expenditure gap	-0.415 ***	-0.414 ***	-0.405 ***	-0.433 ***	-0.421 ***	-0.395 ***
	(0.020)	(0.020)	(0.019)	(0.045)	(0.037)	(0.038)
inflation	0.162 **	-0.028	0.173 **	-0.039	-0.157 ***	-0.118 ***
	(0.072)	(0.040)	(0.069)	(0.030)	(0.029)	(0.033)
trade openness	-0.001	0.009	0.062 **	0.000	0.001	0.027 *
	(0.022)	(0.009)	(0.028)	(0.004)	(0.012)	(0.016)
trend GDP	0.865 ***	0.615 ***	0.339 *	0.520 ***	0.695 ***	0.298 ***
	(0.146)	(0.122)	(0.184)	(0.077)	(0.081)	(0.101)
curent account	0.187 ***	0.220 ***	0.150 **	0.350 ***	0.434 ***	0.431 ***
	(0.061)	(0.051)	(0.059)	(0.029)	(0.037)	(0.036)
observations	570	593	570	593	593	593
number of countries	23	23	23	23	23	23
R-squared	0.343	0.431	0.332	0.476	0.440	0.480
AR(1) coefficient	0.812	0.812	0.834	--	--	--
sigma_e	0.015	0.015	0.014	--	0.024	0.022
country fixed effects	yes	--	yes	no	yes	yes
time effects	no	no	yes	no	no	yes

Notes）The dependent variable is the primary balance to GDP ratio. Sample period is 1985-20014. Standard errors are reported in parentheses; ***, ** and * denote statistically significance at 1, 5, and 10 percent level, respectively. The Mehod of estimaion 1 is the same one of estimarion 2 inTable 2, which is the fixed effect model with assuming AR(1) error structure. The method of estimaiton 2 is the random effect model with assuming AR(1) error structure. The method of estimarion 3 is the fixed effect model with year effects and with assuming AR(1) error strucutre. The method of estimarion 4 is pooled ordinary least squares. The method of estimation 5 is the fixed effect model without assuming AR(1) error structure. The method of estimarion 6 is the fixed effect model with year efffect and without assuming AR(1) error

Table A3 Estimation results of the Fiscal Reaction Funtion: Slope homogenitity

country j	AUS	AUT	BEL	CAN	DNK	FIN	FRA	DEU
lagged debt	-0.167 ***	-0.166 ***	-0.171 ***	-0.167 ***	-0.166 ***	-0.146 ***	-0.168 ***	-0.167 ***
	(0.045)	(0.043)	(0.044)	(0.044)	(0.044)	(0.046)	(0.044)	(0.044)
lagged debt_square	0.167 ***	0.165 ***	0.172 ***	0.169 ***	0.168 ***	0.149 ***	0.167 ***	0.168 ***
	(0.046)	(0.045)	(0.046)	(0.045)	(0.045)	(0.046)	(0.045)	(0.045)
lagged debt_cubic	-0.044 ***	-0.043 ***	-0.045 ***	-0.044 ***	-0.044 ***	-0.039 ***	-0.044 ***	-0.044 ***
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
lagged debt	-0.279	-7.432	-3.895	4.423	-0.031	-1.075	0.449	4.291
* dummy country j	(1.823)	(14.31)	(4.897)	(5.552)	(1.160)	(0.721)	(1.604)	(5.020)
lagged debt_square	1.064	11.105	3.524	-5.746	0.130	2.789	-0.618	-6.610
* dummy country j	(9.009)	(20.94)	(4.428)	(6.801)	(2.355)	(2.116)	(2.594)	(7.926)
lagged debt_cubic	-1.119	-5.442	-1.058	2.433	-0.169	-2.398	0.282	3.300
* dummy country j	(14.14)	(10.15)	(1.318)	(2.757)	(1.462)	(1.951)	(1.350)	(4.108)
observations	570	570	570	570	570	570	570	570
number of countries	23	23	23	23	23	23	23	23
R-squared	0.354	0.031	0.012	0.006	0.333	0.134	0.164	0.012
AR(1) coefficient	0.812	0.809	0.813	0.810	0.812	0.810	0.812	0.812
sigma_e	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
Joint significance test (p-value)	0.988	0.898	0.776	0.696	0.807	0.300	0.976	0.799

country j	GRC	ISL	IRL	ISR	ITA	JPN	KOR	NLD
lagged debt	-0.131 ***	-0.186 ***	-0.122 ***	-0.166 ***	-0.176 ***	-0.211 ***	-0.159 ***	-0.164 ***
	(0.044)	(0.047)	(0.045)	(0.043)	(0.044)	(0.056)	(0.044)	(0.044)
lagged debt_square	0.127 ***	0.180 ***	0.117 ***	0.164 ***	0.178 ***	0.222 ***	0.158 ***	0.164 ***
	(0.046)	(0.047)	(0.047)	(0.045)	(0.046)	(0.070)	(0.046)	(0.045)
lagged debt_cubic	-0.034 ***	-0.046 ***	-0.030 ***	-0.043 ***	-0.047 ***	-0.057 ***	-0.041 ***	-0.043 ***
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.025)	(0.013)	(0.013)
lagged debt	-1.829 ***	-0.538	-0.471	-24.075	14.117	0.050	0.893	-3.649
* dummy country j	(0.566)	(0.503)	(0.272)	(22.26)	(11.75)	(0.361)	(1.630)	(5.856)
lagged debt_square	1.557 ***	1.115	0.476	31.365	-12.528	-0.172	-5.161	6.332
* dummy country j	(0.469)	(0.902)	(0.421)	(27.66)	(10.53)	(0.250)	(8.678)	(10.51)
lagged debt_cubic	-0.405 ***	-0.653	-0.103	-13.459	3.670	0.053	7.472	-3.644
* dummy country j	(0.125)	(0.496)	(0.198)	(11.41)	(3.143)	(0.056)	(14.01)	(6.199)
observations	570	570	570	570	570	570	570	570
number of countries	23	23	23	23	23	23	23	23
R-squared	0.043	0.271	0.267	0.000	0.012	0.425	0.326	0.011
AR(1) coefficient	0.796	0.813	0.817	0.811	0.812	0.790	0.812	0.813
sigma_e	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
Joint significance test (p-value)	0.002	0.380	0.003	0.293	0.330	0.003	0.636	0.853

country j	NZL	NOR	PRT	ESP	SWE	GBR	USA
lagged debt	-0.177 ***	-0.178 ***	-0.171 ***	-0.167 ***	-0.170 ***	-0.163 ***	-0.166 ***
	(0.044)	(0.046)	(0.044)	(0.044)	(0.044)	(0.044)	(0.043)
lagged debt_square	0.174 ***	0.177 ***	0.165 ***	0.166 ***	0.167 ***	0.164 ***	0.169 ***
	(0.046)	(0.047)	(0.046)	(0.045)	(0.045)	(0.045)	(0.045)
lagged debt_cubic	-0.045 ***	-0.046 ***	-0.042 ***	-0.043 ***	-0.043 ***	-0.043 ***	-0.044 ***
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
lagged debt	-0.702	-0.387	1.108	0.223	-2.576	-0.648	4.209 **
* dummy country j	(0.800)	(1.941)	(0.940)	(1.038)	(3.798)	(1.086)	(2.097)
lagged debt_square	2.307	1.744	-1.041	-0.364	5.325	1.141	-5.850 **
* dummy country j	(2.244)	(5.059)	(1.104)	(1.705)	(7.492)	(2.010)	(2.788)
lagged debt_cubic	-1.930	-2.021	0.318	0.197	-3.391	-0.650	2.601 **
* dummy country j	(1.921)	(4.308)	(0.413)	(0.889)	(4.798)	(1.181)	(1.215)
observations	570	570	570	570	570	570	570
number of countries	23	23	23	23	23	23	23
R-squared	0.261	0.310	0.014	0.274	0.026	0.228	0.000
AR(1) coefficient	0.807	0.813	0.816	0.812	0.811	0.811	0.807
sigma_e	0.015	0.015	0.015	0.015	0.015	0.015	0.015
Joint significance test (p-value)	0.399	0.474	0.188	0.986	0.567	0.918	0.145

Notes) The dependent variable is the primary balance to GDP ratio. Estimation method is fixed effects model with assuming AR(1) error structre using data of 23 countries. Standard errors are reported in parentheses; ***, ** and * denote statistically significance at 1, 5, and 10 percent level, respectively. The variable of dummy country j takes the value one for country j and zero otherwise.

Table A4 Robustness to difference estimation method for estimation 5 in Table 3 (excluding Japan's data)

estimaion	1		2		3		4		5		6	
est. method	⁼E, AR(1)		RE, AR(1)		FE, AR(1), yea		pooled		FE		FE, year	
lagged debt	-0.212	***	-0.215	***	-0.137	**	-0.165	***	-0.148	***	-0.155	***
	(0.057)		(0.049)		(0.054)		(0.036)		(0.041)		(0.039)	
lagged debt_square	0.224	***	0.237	***	0.173	***	0.163	***	0.135	**	0.169	***
	(0.071)		(0.063)		(0.067)		(0.049)		(0.054)		(0.051)	
lagged debt_cubic	-0.058	**	-0.065	***	-0.049	**	-0.031		-0.020		-0.037	*
	(0.025)		(0.023)		(0.024)		(0.021)		(0.021)		(0.020)	
output gap	0.438	***	0.429	***	0.240	***	0.573	***	0.536	***	0.411	***
	(0.045)		(0.045)		(0.056)		(0.082)		(0.070)		(0.086)	
govt. expenditure gap	-0.410	***	-0.408	***	-0.398	***	-0.399	***	-0.389	***	-0.363	***
	(0.020)		(0.020)		(0.019)		(0.042)		(0.036)		(0.036)	
inflation	0.155	**	-0.035		0.177	**	-0.055	**	-0.137	***	-0.048	
	(0.073)		(0.039)		(0.071)		(0.028)		(0.028)		(0.032)	
trade openness	-0.002		-0.002		0.049	*	-0.015	***	0.006		0.019	
	(0.021)		(0.009)		(0.027)		(0.004)		(0.011)		(0.014)	
trend GDP	0.909	***	0.628	***	0.317	*	0.631	***	0.815	***	0.360	***
	(0.146)		(0.121)		(0.189)		(0.075)		(0.083)		(0.106)	
curent account	0.193	***	0.264	***	0.185	***	0.423	***	0.451	***	0.464	***
	(0.061)		(0.051)		(0.057)		(0.028)		(0.037)		(0.034)	
observations	541		563		541		563		563		563	
number of countries	22		22		22		22		22		22	
R-squared	0.404		0.503		0.398		0.535		0.501		0.550	
AR(1) coefficient	0.791		0.791		0.796		--		--		--	
sigma_e	0.015		0.015		0.014		--		0.023		0.021	
country fixed effects	yes		--		yes		no		yes		yes	
time effects	no		no		yes		no		no		yes	

Notes) The dependent variable is the primary balance to GDP ratio. Sample period is 1985-20014. Standard errors are reported in parentheses; ***, ** and * denote statistically significance at 1, 5, and 10 percent level, respectively. The Mehod of estimaion 1 is the same one of estimarion 2 inTable 2, which is the fixed effect model with assuming AR(1) error structure. The method of estimaiton 2 is the random effect model with assuming AR(1) error structure. The method of estimarion 3 is the fixed effect model with year effects and with assuming AR(1) error strucutre. The method of estimarion 4 is pooled ordinary least squares. The method of estimation 5 is the fixed effect model without assuming AR(1) error structure. The method of estimarion 6 is the fixed effect model with year efffect and without assuming AR(1) error