Currency Devaluation and Unemployment in an Open Economy

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May 2017

Abstract

China has been criticized for adopting the low yuan policy to take unfair advantage of its trading partners. This paper considers the optimal exchange rate policy of a Keynesian open economy with unemployed resources. Under certain conditions, yuan devaluation lowers the dollar price of China's exportable good. In the case of Cobb-Douglas utility and production functions, indirect utility is monotone increasing and concave in the exchange rate. Yuan devaluation is shown to reduce unemployment. However, currency devaluation is shown to have no effect on foreign debt. Moreover, the optimal exchange rate is one which guarantees full employment. Thus, China may be pursuing the low yuan policy to achieve full employment. A numerical example illustrates the main propositions.

Highlights
Yuan devaluation decreases the dollar price of China's exportable good.
Yuan devaluation is shown to reduce domestic unemployment.
The exchange rate that guarantees full employment is optimal.

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JEL classification code: F3
Keywords: unemployment, yuan devaluation, optimal exchange rate

1. Introduction

Due to its mounting currency reserves since the 1990s, China's exchange rate policy has been under intense scrutiny. According to the State Administration of Foreign Exchange of People's Bank of China (PBC), China's foreign exchange reserve was $22 billion in 1993. China's foreign exchange reserve has since increased steadily, to $3.1 trillion in October 2016. Such a dramatic rise in China's cumulative trade surplus has provoked much debate concerning China's currency valuation and misalignment.

Most major currencies are freely floating vis-à-vis other currencies, except the renminbi. It is argued that China may be deliberately depressing the yuan in the hope of stimulating domestic production. In the celebrated Mundell-Fleming model, currency devaluation influences a country's balance of payments, thereby affecting production and unemployment. Gylfason and Schmid (1983) show that devaluation has positive output effects in a study of ten countries.

In an open economy, the government may be more interested in the output effects of currency devaluation. Helpman (1976) considered a single-period framework with a nontraded good and showed devaluation increases employment, while Cuddington (1981) investigated the contemporaneous effect of devaluation. More recently, Batra and Beladi (2013) suggest that both China and Japan kept their currency values low relative to those of other nations such as the United States and Europe in order to maintain unemployment below a target rate.[1] Jin and Choi (2013) noted that while some profits might be generated in the short run by slightly deviating from the equilibrium exchange rates, in the long run excessive hoarding of reserve assets can only result in losses to PBC's balance of payment account. Jin et al (2016) showed that in a two-period model nonintervention is the optimal exchange rate policy. However, the prevailing view is that China has intentionally depressed the value of the yuan to gain unfair advantages in the global market. (Cheung et al., 2009, Cheung, 2012)

In a developing country like China, the goal of keeping the unemployment rate low might be more important than other economic issues. Thus, Chinese government may have adopted the low yuan policy, not to take advantage of its trading partners, but to stimulate its domestic economy.

This paper investigates optimal exchange rate for a Keynesian open economy.[2] Open economy macroeconomic models are predominantly based on an economy producing a single homogeneous good.[3] In order to consider currency devaluation and output effects, we consider an open economy which produces two tradable goods. Using the Cobb-Douglas utility and production functions, we show that under certain conditions, the exchange rate which guarantees full employment is the optimal policy.

Section 2 introduces the basic two-sector, two-country model with unemployment. Section 3 investigates the effect of yuan devaluation on exchange rate pass-through into the yuan price of China's exportable good. Section 4 examines the effect of yuan devaluation on trade imbalance. Section 5 considers the effect of yuan devaluation on income and welfare, while Section 6 explores the effect on unemployment. Section 7 shows a numerical example which illustrates the main propositions. Section 8 offers concluding remarks.

2. The Two-Sector Keynesian Model with Unemployment

In this section we describe a Keynesian open economy model with two goods to consider China's optimal exchange rate policy. Let China's importable good Z be the numéraire, i.e., its dollar price $P^* = 1$, and let δ denote the yuan price of the dollar. Exchange rate pass-through into the import price is perfect,[4] and the yuan price of the importable is P. An increase in δ represents an increase in the yuan price of the dollar, and hence a yuan depreciation. We assume that the dollar price of the importable good P^* is fixed in the importing country, and its yuan price is $P = P^*\delta$, where δ is the yuan price of the dollar.

Each country is assumed to fix the price of its exportable in terms of its own currency. That is, the yuan price of good *C*, which China exports, is *b*, regardless of yuan depreciation. Likewise, the dollar price P^* of good *Z* is fixed, equal to unity. The relative price of good *Z* in China can be written as: $\frac{P}{b} = \frac{P^*\delta}{b} = \frac{P^*}{b/\delta} = \frac{P^*}{b^*}$. That is, the relative price of good *Z* is the same in both countries, regardless of the exchange rate. However, a change in the exchange rate may affect the relative price of good *Z*.

Assumptions

We now consider a two-sector Keynesian model of two countries producing two goods, *C* and *Z*. Unemployment exists in both the capital and labor markets. The wage rate *w* and capital rental *r* are assumed to be fixed in the short run. To lay the basis for analyzing the effects of yuan devaluation, we employ the following assumptions:

(i) Two factors, capital *K* and labor *L*, are used to produce two goods, *C* and *Z*. China is assumed to export *C* and import *Z*.

(ii) The dollar price of good *Z* is normalized, i.e., $P^* = 1$.

(iii) The Chinese government pegs the yuan to the dollar, and the yuan price of the importable is $P = P^*\delta = \delta$.

(iv) Cobb-Douglas production functions are used in both industries.

(v) Unemployment exists in both capital and labor markets.

(vi) Consumer preferences are given by a Cobb-Douglas utility function in both countries.

China

China produces two goods, using two factors, capital (*K*) and labor (*L*) inputs. Domestic outputs of the traded goods are given by $C = F(L_C, K_C) = A_C L_C^{\alpha_1} K_C^{\beta_1}$, and $Z = G(L_Z, K_Z) = A_Z L_Z^{\alpha_2} K_Z^{\beta_2}$, where L_j and K_j denote the amounts of labor and capital inputs employed in sector j = *C*, *Z*. Both production functions are assumed to exhibit decreasing returns to scale (DRS), i.e., $\alpha_1 + \beta_1 < 1,$ and $\alpha_2 + \beta_2 < 1.$ DRS implies that the production functions $F(.)$ and $G(.)$ are monotone increasing and concave.[5]

China's Supplies of Tradable Goods

Since labor and capital inputs are unemployed and are immobile internationally, $w \neq w*$ and $r \neq r*.$ Total profit of the Chinese economy in yuan is

$$\Pi = \Pi_C + \Pi_Z = bA_C L_C^{\alpha_1} K_C^{\beta_1} + PA_Z L_Z^{\alpha_2} K_Z^{\beta_2} - wL_C - rK_C - wL_Z - rK_Z, \tag{1}$$

where L_i, and K_i are input demands of labor and capital in sector *i* = *C*, *Z*. The central planner's problem is to choose L_C, L_Z, K_C, and K_Z subject to $L_C + L_Z < \bar{L}$, $K_C + K_Z < \bar{K}$. Due to unemployment, a production mix of *C* and *Z* does not occur along a production possibility frontier (PPF). The first order conditions are

$$\begin{aligned} &b\alpha_1 A_C L_C^{\alpha_1 - 1} K_C^{\beta_1} = w, \ \ b\beta_1 A_C L_C^{\alpha_1} K_C^{\beta_1 - 1} = r, \\ &P\alpha_2 A_Z L_Z^{\alpha_2 - 1} K_Z^{\beta_2} = w, \ \ P\beta_2 A_Z L_Z^{\alpha_2} K_Z^{\beta_2 - 1} = r. \end{aligned} \tag{2}$$

The input demands for labor and capital in the production of the two goods are as follows:

$$L_C = \left(\frac{b\alpha_1^{1-\beta_1}\beta_1^{\beta_1}A_C}{w^{1-\beta_1}r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}, K_C = \left(\frac{b\alpha_1^{\alpha_1}\beta_1^{1-\alpha_1}A_C}{w^{\alpha_1}r^{1-\alpha_1}} \right)^{1/(1-\alpha_1-\beta_1)},$$
$$L_Z = \left(\frac{P\alpha_2^{1-\beta_2}\beta_2^{\beta_2}A_Z}{w^{1-\beta_2}r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}, K_Z = \left(\frac{P\alpha_2^{\alpha_2}\beta_2^{1-\alpha_2}A_Z}{w^{\alpha_2}r^{1-\alpha_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \tag{3}$$

The supplies of goods, *C* and good *Z*, are written as:

$$C(b,P) = A_C L_C^{\alpha_1} K_C^{\beta_1} = \left(\frac{b^{\alpha_1+\beta_1}\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)},$$
$$\mathrm{Z}(b,P) = A_Z L_Z^{\alpha_2} K_Z^{\beta_2} = \left(\frac{P^{\alpha_2+\beta_2}\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \tag{4}$$

China's Demands for Tradable Goods

Preferences of Chinese consumers are represented by a Cobb-Douglas utility function,

$$U(c,z) = c^{\gamma} z^{1-\gamma}, \tag{5}$$

where c and z are China's consumption of the exportable and importable, respectively. The equilibrium condition for optimal consumption is:

$$\frac{U_c}{U_z} = \frac{b}{P}, \tag{6}$$

where *b* and *P* are the yuan prices of exportable good *C* and importable good *Z*, respectively.

Let *S* denote the dollar amount of money that the Chinese government lends to the United States. When $S > 0$, consumers are required to spend only $E = I - \delta S$ yuan to purchase consumption goods. Thus, consumer demands for the two goods are written as: $c = \frac{\gamma E}{b}$ and $z = \frac{(1-\gamma)E}{P}$, where *I* and $E = I - \delta S$ are China's income and expenditure in yuan. Since both factors are unemployed and the total profit is distributed to consumers, the total income of China is $I = w(L_C + L_Z) + r(K_C + K_Z) + \Pi = bC + PZ.$ The budget constraint in yuan is given by:

$$bc + Pz = E = I - \delta S. \tag{7}$$

China's national income is

$$I = bC + PZ = \left(\frac{b\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \tag{8}$$

Suppose China lends S dollars to the United States. Then China's expenditure in yuan is

$$E = I - \delta S = \left(\frac{b\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} - \delta S. \tag{9}$$

China's consumer demands for two tradable goods are written as:

$$\begin{aligned} c &= \gamma \frac{\delta}{b} \left(\frac{b^{\frac{1}{1-\alpha_1-\beta_1}}}{\delta} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} - S \right), \\ z &= (1-\gamma) \left(\frac{b^{\frac{1}{1-\alpha_1-\beta_1}}}{\delta} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} - S \right). \end{aligned} \tag{10}$$

United States

The United States is also assumed to produce the two goods, using two factors, capital (K) and labor (L) inputs. Since both inputs are unemployed, production does not occur on the PPF. Recall that the United States and China have the same technologies in the production of two goods, C and Z. U.S. outputs of the tradable goods are given by

$C^* = F(L_C^*, K_C^*) = A_C L_C^{*\alpha_1} K_C^{*\beta_1}$, and $Z^* = G(L_Z^*, K_Z^*) = A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2}$, where L_j^* and K_j^* denote the labor and capital inputs employed in sector $j = C^*, Z^*$.

U.S. Supplies of Tradable Goods

The total profit of the United States in dollar is

$$\Pi^* = \Pi_{C^*}^* + \Pi_{Z^*}^* = b^* A_C L_C^{*\alpha_1} K_C^{*\beta_1} + P^* A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2} - w^*(L_{C^*}^* + L_{Z^*}^*) - r^*(K_{C^*}^* + K_{Z^*}^*). \quad (11)$$

where b^* and $P^* = 1$ are dollar prices of good C and Z, and L_i^* and K_i^* are input demands of labor and capital in sector $i = C^*, Z^*$. The central planner's problem is to choose L_C^*, L_Z^*, K_C^*, and K_Z^* to maximize the total profit in (11) subject to $L_C^* + L_Z^* < \bar{L}^*$, $K_C^* + K_Z^* < \bar{K}^*$.

The first order conditions are

$$\begin{aligned} &b^* \alpha_1 A_C L_C^{*\alpha_1 - 1} K_C^{*\beta_1} = w^*,\ b^* \beta_1 A_C L_C^{*\alpha_1} K_C^{*\beta_1 - 1} = r^*, \\ &P^* \alpha_2 A_Z L_Z^{*\alpha_2 - 1} K_Z^{*\beta_2} = w^*,\ P^* \beta_2 A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2 - 1} = r^*. \end{aligned} \quad (12)$$

U.S. input demands are given by

$$\begin{aligned} L_C^* &= \left(\frac{b^* \alpha_1^{1-\beta_1} \beta_1^{\beta_1} A_C}{w^{*1-\beta_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}, K_C^* = \left(\frac{b^* \alpha_1^{\alpha_1} \beta_1^{1-\alpha_1} A_C}{w^{*\alpha_1}\ r^{*1-\alpha_1}} \right)^{1/(1-\alpha_1-\beta_1)}, \\ L_Z^* &= \left(\frac{P^* \alpha_2^{1-\beta_2} \beta_2^{\beta_2} A_Z}{w^{*1-\beta_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}, K_Z^* = \left(\frac{P^* \alpha_2^{\alpha_2} \beta_2^{1-\alpha_2} A_Z}{w^{*\alpha_2}\ r^{*1-\alpha_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \quad (13)$$

The optimal supplies are:

$$\begin{aligned} C^*(b^*, P^*) &= A_C L_C^{*\alpha_1} K_C^{*\beta_1} = \left(\frac{b^{*\alpha_1+\beta_1}\ \alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{*\alpha_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}, \\ Z^*(b^*, P^*) &= A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2} = \left(\frac{P^{*\alpha_2+\beta_2}\ \alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{*\alpha_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \quad (14)$$

U.S. Demands for Tradable Goods

Preferences of American consumers are represented by a utility function, $U(c^*, z^*) = (c^*)^{\gamma}(z^*)^{1-\gamma}$, where c^* and z^* are the U.S. demands for *C* and *Z*, respectively. National income of the United States is $I^* = wr^*(L_C^* + L_Z^*) + r^*(K_C^* + K_Z^*) + \Pi^* = b^*C^* + P^*Z^*$. U.S. expenditure in dollar is given by:

$$b^*c^* + P^*z^* = I^* + S = b^*C^* + P^*Z^* + S, \qquad (15)$$

where *b** and *P** are the dollar prices of *C** and *Z**, respectively.

Revenue from production is

$$I^* = b^*C^* + P^*Z^* = \left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}. \qquad (16)$$

Since the U.S. borrows *S* dollars from China, the total expenditure of the United States is

$$E^* = I^* + S = \left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} + S. \qquad (17)$$

The equilibrium condition for optimal consumption is:

$$\frac{U_{c^*}}{U_{z^*}} = \frac{b^*}{P^*}. \qquad (18)$$

U.S. consumer demands satisfying the equilibrium condition in (18) and the budget constraint in (16) are written as:

$$\begin{aligned} c^* &= \frac{\gamma}{b^*}\left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \frac{\gamma}{b^*}\left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} + \frac{\gamma S}{b^*}, \\ z^* &= \frac{1-\gamma}{P^*}\left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \frac{1-\gamma}{P^*}\left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} + \frac{(1-\gamma)S}{P^*}, \end{aligned} \qquad (19)$$

where *w** and *r** are the wage rate and capital rent, respectively.

World Market Equilibrium and Trade Balance

Recall that the relative price of the exportable is $b/P = b/\delta$. A competitive equilibrium is attained when producers and consumers in both markets behave as price takers. Recall that when unemployment exists in the labor market, any increases in $\bar{K}$ or $\bar{L}$ have no effect on China's domestic outputs. Note that due to unemployment the supply of each good depends on its own price, but not on the price of the other good. Thus, the supply functions are written as: $C = C(b)$ and $Z = Z(P)$.

The market clearing condition for good *C* is written as:

$$c(b,\delta,I-\delta S)+c^*(b,\delta,I^*+S)=C(b,\delta)+C^*(b,\delta). \qquad (20)$$

Despite unemployment in the factor markets, a Keynesian equilibrium exists when aggregate demand is equal to aggregate supply in each sector. The labor market imperfection only causes labor unemployment, but does not preclude Walras' Law in the output markets. Thus, all outputs produced are sold at the equilibrium prices. Walras' Law suggests that if the market for good *C* is in equilibrium, the market for the other output, *Z*, is also in equilibrium, i.e.,

$$z(b,\delta,I-\delta S)+z^*(b,\delta,I^*+S)=Z(b,\delta)+Z^*(b,\delta). \qquad (21)$$

Thus, there exists a unique value of δ which clears the market for good *C* in (20). Note that when capital is not fully utilized, $\partial Z/\partial K = 0$. Likewise, when labor unemployment exists, $\partial Z/\partial L = 0.$ The same conditions hold if capital and labor are not fully employed in the United States, i.e., $\partial Z^*/\partial K^* = \partial Z^*/\partial L^* = 0.$

Exchange Rate Pass Through

Let (b,δ) be a pair of yuan prices at the competitive benchmark equilibrium satisfying (20).[6] Then by Walras' law, the price pair (b,δ) also satisfies (21). That is, if the market for good *C* clears for given prices (b,δ), the other product market (i.e., for good *Z*) also clears. Substituting (4), (10), (14) and (19) into (20), we obtain the world market clearing condition for good *C*:

$$\begin{aligned} & b^{1/(1-\alpha_1-\beta_1)}\left(\frac{b\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}+b^{1/(1-\alpha_1-\beta_1)}\left(\frac{(1/\delta)^{\alpha_1+\beta_1}\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w*^{\alpha_1}\ r*^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} \\ & =\delta\frac{\gamma}{(1-\gamma)}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w*^{\alpha_2}\ r*^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}+\delta^{\frac{1}{1-\alpha_2-\beta_2}}\frac{\gamma}{(1-\gamma)}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \tag{22}$$

Since good *Z* is the numéraire, $P^*=1,$ we have $P=\delta,$ and $b=b^*\delta.$ Recall that China and the United States have identical technologies, and in the absence of trade barriers, output prices are equalized. The equilibrium yuan price of good *C* is:[7]

$$b(\delta)=\frac{\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}\left(\delta^{\frac{1}{1-\alpha_2-\beta_2}}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}+\delta\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w*^{\alpha_2}\ r*^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}\right)^{(1-\alpha_1-\beta_1)}}{\left(\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}+\left(\frac{1}{\delta}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w*^{\alpha_1}\ r*^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\right)^{(1-\alpha_1-\beta_1)}}. \tag{23}$$

The equilibrium dollar price of good *C* is:[8]

$$b*(\delta)=\frac{\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}\left(\delta^{\frac{1}{1-\alpha_2-\beta_2}}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}+\delta\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w*^{\alpha_2}\ r*^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}\right)^{(1-\alpha_1-\beta_1)}}{\left(\delta^{\frac{1}{1-\alpha_1-\beta_1}}\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}+\delta\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w*^{\alpha_1}\ r*^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\right)^{(1-\alpha_1-\beta_1)}}. \tag{24}$$

Thus, both the yuan and dollar prices of good *C* depend on the exchange rate, δ.

3. Devaluation and Exchange Rate Pass-Through

We now consider the effect of yuan devaluation on the yuan and dollar prices of the exportable good *C*. Yuan devaluation is represented by an increase in δ, which raises the yuan price *P* of good *Z*. Differentiating (23) with respect to δ, we have

$$\frac{db(\delta)}{d\delta}=\frac{(1-\alpha_1-\beta_1)(A_3)\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}}{(A_1)^{(1-\alpha_1-\beta_1)}(A_2)^{(\alpha_1+\beta_1)}}+\frac{(\alpha_1+\beta_1)\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}\left(\frac{1}{\delta}\right)^{\frac{1}{1-\alpha_1-\beta_1}}(A_2)^{(1-\alpha_1-\beta_1)}}{\left(\frac{\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w*^{\alpha_1}\ r*^{\beta_1}}\right)^{1/(\alpha_1+\beta_1-1)}(A_1)^{(2-\alpha_1-\beta_1)}}.\quad(25)$$

where

$$A_1=\left(\frac{\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}+\left(\frac{1}{\delta}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\left(\frac{\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w*^{\alpha_1}\ r*^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)},$$

$$A_2=\delta^{\frac{1}{1-\alpha_2-\beta_2}}\left(\frac{\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}+\delta\left(\frac{\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w*^{\alpha_2}\ r*^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)},$$

$$A_3=\frac{\delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}}{1-\alpha_2-\beta_2}\left(\frac{\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}+\left(\frac{\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w*^{\alpha_2}\ r*^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}.$$

If both industries exhibit DRS ($\alpha_1+\beta_1<1$ and $\alpha_2+\beta_2<1$), then $db/d\delta>0$. If *b* rises proportionately, then $b/\delta=b*$ is constant, i.e., yuan devaluation has no effect on the dollar price of good *C*.

We assume that *b* rises with δ, but less than proportionately, and that the dollar price of good *C* decreases with δ. Differentiating (24) with respect to δ yields

$$\frac{\partial\left(b*(\delta)\right)}{\partial\delta}=\frac{(1-\alpha_1-\beta_1)\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}A_3}{(A_0)^{(1-\alpha_1-\beta_1)}(A_2)^{(\alpha_1+\beta_1)}}-\frac{(1-\alpha_1-\beta_1)\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}(A_2)^{(1-\alpha_1-\beta_1)}A_4}{(A_0)^{(2-\alpha_1-\beta_1)}},$$

where A_i's are functions of δ as follows:

$$A_0 = \delta^{\frac{1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1{}^{\alpha_1} \beta_1{}^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta \left(\frac{\alpha_1{}^{\alpha_1} \beta_1{}^{\beta_1} A_C}{w^{*\alpha_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)},$$

$$A_2 = \delta^{\frac{1}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2{}^{\alpha_2} \beta_2{}^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} + \delta \left(\frac{\alpha_2{}^{\alpha_2} \beta_2{}^{\beta_2} A_Z}{w^{*\alpha_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)},$$

$$A_3 = \frac{\delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}}{1-\alpha_2-\beta_2} \left(\frac{\alpha_2{}^{\alpha_2} \beta_2{}^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} + \left(\frac{\alpha_2{}^{\alpha_2} \beta_2{}^{\beta_2} A_Z}{w^{*\alpha_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)},$$

$$A_4 = \frac{\delta^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}}{1-\alpha_1-\beta_1} \left(\frac{\alpha_1{}^{\alpha_1} \beta_1{}^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{\alpha_1{}^{\alpha_1} \beta_1{}^{\beta_1} A_C}{w^{*\alpha_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}.$$

In general, the sign of $d(b/\delta)/d\delta$ depends on the values of $\alpha_1,\ \beta_1,\ \alpha_2,\ \beta_2,\ w,\ w^*,\ r,$ and r^*. If both industries exhibit the same DRS ($\alpha_2+\beta_2=\alpha_1+\beta_1$), then a yuan devaluation has no effect on the relative price of the good C, $\frac{d(b/\delta)}{d\delta}=0$. If $\delta>1$, industry C exhibits higher returns to scale than industry Z ($\alpha_1+\beta_1>\alpha_2+\beta_2$), and the U.S. wage is higher and capital rent lower than those in China ($w^*>w$, and $r^*<r$), then $\frac{d(b/\delta)}{d\delta}<0$. In this case, the U.S. import of good C increases. This implies that the dollar price of good C decreases with δ, i.e., $db^*/d\delta<0.$ Figure 1 illustrates the case where the dollar price of good C decreases with δ until the full employment rate δ^f is reached, and is constant thereafter because exchange rate pass-through into the export price is perfect.

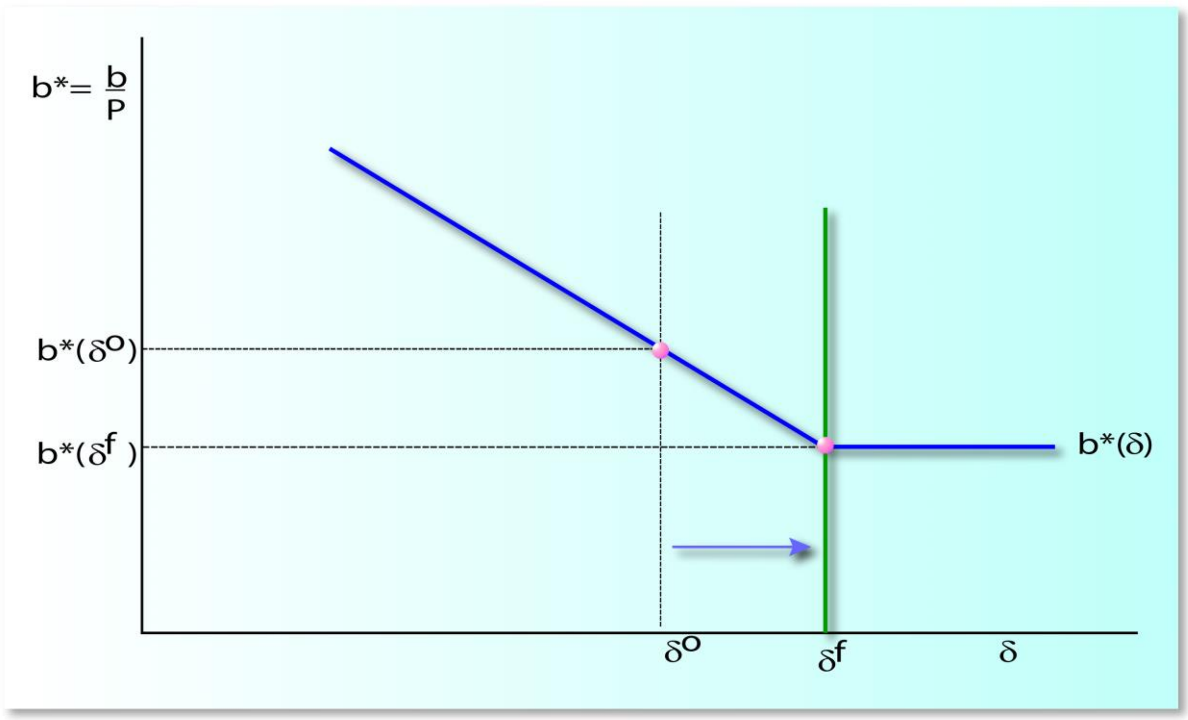


Figure 1. Yuan Devaluation and Terms of Trade

Let δ^o denote the benchmark equilibrium value of δ, at which the market for good *C* clears when producers and consumers in both countries behave as price takers. In the absence of government intervention, the benchmark equilibrium also means trade is balanced between the two countries. If China devalues the yuan from the benchmark equilibrium rate, then the yuan price of good *C* increases. We assume that the yuan price of good *C* increases at a decreasing rate, i.e., $b'(\delta) > 0,$ and $b''(\delta) < 0.$

Once full-employment is reached, it can be shown that the dollar price of good *C* is constant,

$$b^* = \left(\left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{\bar{L}+\bar{L}^*}{\bar{K}+\bar{K}^*}\right)\frac{(1-\gamma)}{\gamma}\frac{(1-\alpha_2)^{1+\alpha_2}}{(1-\alpha_1)^{1+\alpha_1}}\right)^{(\alpha_2-\alpha_1)}\left(1-\left(\frac{\alpha_2}{1-\alpha_2}\right)\frac{(\bar{K}+\bar{K}^*)}{(\bar{L}+\bar{L}^*)}\right)^{(\alpha_2-\alpha_1)}$$
$$+\left(\left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{\bar{L}+\bar{L}^*}{\bar{K}+\bar{K}^*}\right)\right)^{(\alpha_2-\alpha_1)}\left(\frac{A_Z(\alpha_2)^{\alpha_2}(1-\alpha_2)}{A_C(\alpha_1)^{\alpha_1}(1-\alpha_1)}\right),$$

where $\bar{K}$ and $\bar{L}$ are fixed endowments of China's capital and labor inputs, and $\bar{K}*$ and $\bar{L}*$ are U.S. endowments of the same inputs. Thus, once full employment is reached, a further increase in δ has no effect on the dollar price of good *C*, i.e., $db*/d\delta = d(b/\delta)/d\delta = 0.$

If $\alpha_1+\beta_1>\alpha_2+\beta_2,$ yuan devaluation lowers the dollar price of good *C*, which increases China's exports. If $\alpha_1+\beta_1=\alpha_2+\beta_2,$ yuan devaluation has no effect on the terms of trade. On the other hand, if $\alpha_1+\beta_1<\alpha_2+\beta_2,$ then yuan devaluation raises the relative price of good *C* and may defeat the purpose of reducing unemployment.

4. Devaluation and Foreign Debt

The yuan value of China's export is written as

$$\begin{aligned}
&bX(\delta, I-S) = bC(b) - bc(b,\delta, I-\delta S)\\
&= bC(b) - b\frac{\lambda bC(b)+\lambda bZ(b,\delta)-\lambda\delta S}{b} = (1-\lambda)bC(b) - \lambda PZ(b,\delta) + \lambda\delta S \qquad (26)\\
&= (1-\gamma)b(b*\delta)^{\frac{(\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)}}\left(\frac{\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} - b\frac{\gamma\delta^{\frac{(\alpha_2+\beta_2)}{(1-\alpha_2-\beta_2)}}}{b*}\left(\frac{\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} + \lambda\delta S.
\end{aligned}$$

The yuan value of China's import is written as

$$\begin{aligned}
&PM(\delta, I-\delta S) = Pz(\delta, I-\delta S) - PZ(\delta)\\
&= P\frac{(1-\lambda)bC(b)+(1-\lambda)PZ(b,\delta)-(1-\lambda)\delta S}{P} - PZ(\delta)\\
&= (1-\lambda)bC(b) - \lambda PZ(b,\delta) - (1-\lambda)\delta S \qquad (27)\\
&= (1-\gamma)Pb*\delta^{\frac{(\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)}}\left(\frac{\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\\
&-\gamma P\delta^{\frac{(\alpha_2+\beta_2)}{(1-\alpha_2-\beta_2)}}\left(\frac{\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} - (1-\lambda)\delta S.
\end{aligned}$$

The yuan value of China's import, $PM(\delta, I-\delta S)$, and that of export, $bX(\delta, I-\delta S)$, are functions of the exchange rate. We assume that China's trade surplus is lent fully to the United States. Thus, China's trade surplus also depends on δ:

$$\begin{aligned} &B(\delta) = bX(\delta, I-\delta S) - PM(\delta, I-\delta S) \\ &= (1-\lambda)bC(b) - \lambda PZ(b,\delta) + \lambda\delta S \\ &-(1-\lambda)bC(b) + \lambda PZ(b,\delta) + (1-\lambda)\delta S = \delta S, \end{aligned} \tag{28}$$

The dollar value of China's trade surplus is:

$$\frac{B}{\delta} = S.$$

which implies *the dollar value of China's trade surplus is independent of* δ.

Proposition 1: If the income elasticity of demand for each good is unity, a yuan devaluation has no effect on the dollar value of China's trade surplus.

Note that China's trade surplus is a means of forced savings for China's consumers, and is a policy variable in China. Once the Chinese government chooses the lending amount *S*, Chinese consumers are forced to spend that much less. As a result, China incurs a trade surplus, equal to *S*, which is not affected by currency manipulations.

5. Yuan Devaluation, Income and welfare

Devaluation and GDP

We now consider the effect of yuan devaluation on national income. Let *I* denote the yuan value of China's outputs, i.e.,

$$\begin{aligned} I &= bC(b,P) + PZ(b,P) \\ &= b^{\frac{1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1{}^{\alpha_1} \beta_1{}^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta^{\frac{1}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2{}^{\alpha_2} \beta_2{}^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \quad (29)$$

Differentiating (29) with respect to δ yields

$$\frac{dI}{d\delta} = b^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1{}^{\alpha_1} \beta_1{}^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \frac{1}{1-\alpha_2-\beta_2} \delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2{}^{\alpha_2} \beta_2{}^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} > 0. \quad (30)$$

Thus, the yuan value of China's national income rises with δ.

Devaluation and Welfare

Next, consider the effect of a yuan devaluation on consumer welfare, using (8) and (23), and the Cobb-Douglas utility function in (5). Expenditure is written as a function of δ:

$$E(\delta) = I(\delta) - \delta S.$$

Given the demand functions in (10), the indirect utility of the Chinese consumers is given by

$$V(b, \delta, I(\delta) - \delta S) \equiv U\left(c(b, \delta, I(\delta) - \delta S), z(b, \delta, I(\delta) - \delta S)\right). \quad (31)$$

We now investigate whether China gains from yuan devaluation. Differentiating (31) with respect to δ, and using Roy's identities, $V_b = -V_I c$ and $V_P = -V_I z$, we obtain

$$\begin{aligned} \frac{dV}{d\delta} &= V_b (db/d\delta) + V_P + V_I \left(I'(\delta) - S \right) \\ &= V_b (db/d\delta) + V_P + V_I \left(b'(\delta) C + Z + bb' C'(b) + \delta Z'(\delta) - S \right) \\ &= V_I \left((C-c) b'(\delta) + (Z-z) + bb' C'(b) + \delta Z'(\delta) - S \right), \end{aligned} \quad (32)$$

where $\theta \equiv bb'C'(b)+\delta Z'(\delta)$ is the change in national income resulting from a reduction in unemployment through a change in the exchange rate. Once full employment is reached, $\theta = 0$. Thus, as δ approaches δ^f, $\theta(\delta)$ must converge to 0. Thus, we assume that $\theta'(\delta) < 0$. Note that $\delta S = bX - \delta Q$, and hence $Q = \frac{b}{\delta}X - S$, or $(z-Z) = \frac{b}{\delta}(C-c) - S$. Thus, (32) is written as

$$\frac{dV}{d\delta} = V_I\left((C-c)b'(\delta) - \frac{b(\delta)}{\delta}(C-c)+\theta\right) = V_I\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right). \qquad (33)$$

If unemployment exists in China, then $db(\delta)/d\delta = b' < \frac{b}{\delta} = b^*$. Assume that there is a unique solution δ^* in the interval $(\delta^o, \delta^f]$ to the utility maximization problem. From (33), δ^* satisfies the first order condition,

$$\frac{dV}{d\delta} = V_I\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right) = 0. \qquad (34)$$

Recall that once full employment is reached $\theta = 0$, and hence $b'(\delta) - \frac{b(\delta)}{\delta} = 0$. Thus, δ^f satisfies (34). Differentiating (34) with respect to δ, we obtain

$$\begin{aligned}\frac{d^2V}{d\delta^2} &= V_{IP}\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right) + V_{Ib}\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right)b'(\delta) \\ &+V_I\left(X'(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+X(\delta)b''(\delta)-\frac{X(\delta)}{\delta}\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta'(\delta)\right) \\ &+V_{II}\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right)I'\end{aligned}$$

When evaluated at δ^f, $\frac{d^2V}{d\delta^2}$ reduces to

$$\frac{d^2V}{d\delta^2} = V_I\left(X(\delta)b''(\delta) + \theta'(\delta)\right) < 0.$$

which implies that the indirect utility is concave in δ. Thus, δ^f is the optimal solution.

Proposition 2: Assume that $b' < b/\delta$. Then China's optimal policy is yuan devaluation until full employment is reached.

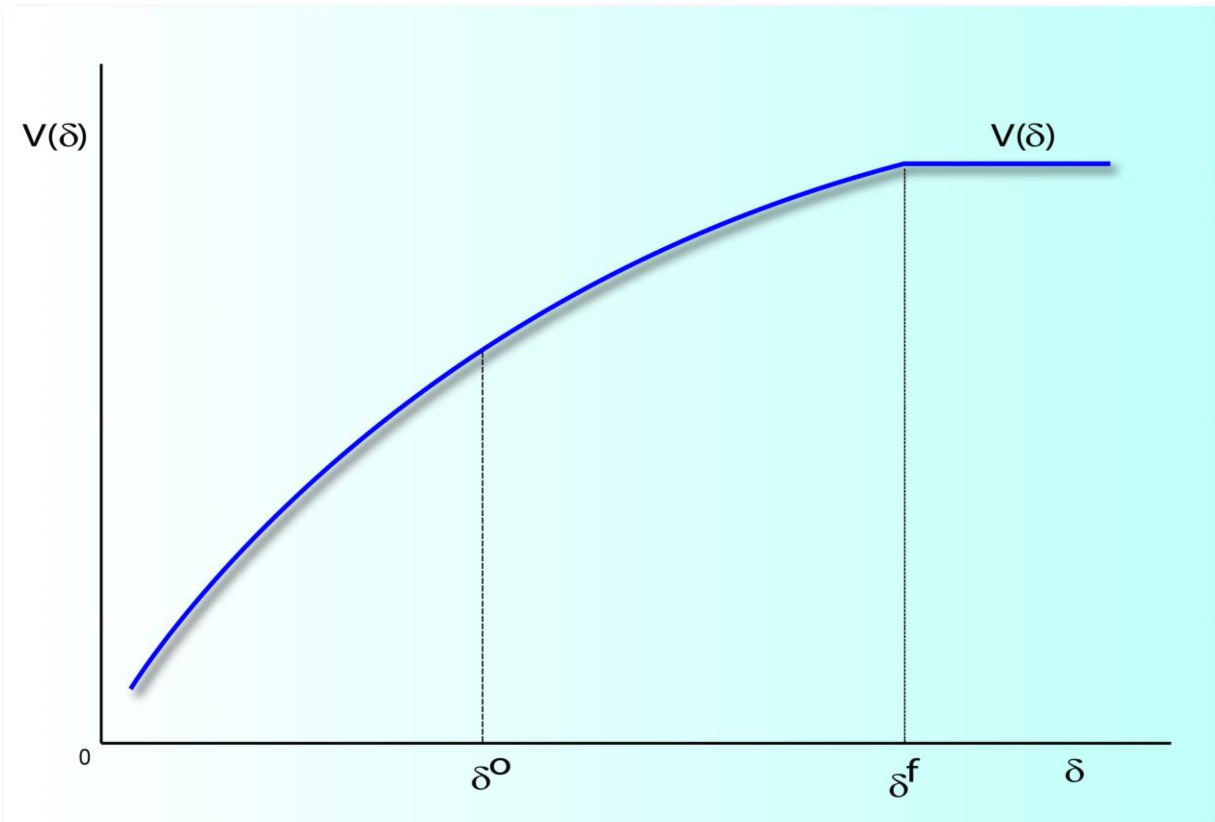


Figure 2. Yuan Devaluation and Indirect Utility

Figure 2 illustrates that China's optimal policy is yuan devaluation, raising δ above δ^o. Since the indirect utility is monotone increasing, δ can be raised to its upper limit, δ^f, thereby reaching a maximum utility. A further increase in δ does not affect the dollar price of good *C*, because exchange rate pass-through into the export price is perfect.

Chen (2014) showed that in the case of China, from 1995 to 2007, exchange rate pass-through into the (long-term) export price was 42.6 percent. This finding suggests that China has not pursued the optimal exchange rate policy, and China has devalued the yuan below the full employment rate, δ^f.

6. Yuan Devaluation and Unemployment

We now consider the effect of yuan devaluation on unemployment. The number of unemployed workers is

$$\begin{aligned} U &= L - L_C(\delta) - L_Z(\delta) \\ &= L - b^{1/(1-\alpha_1-\beta_1)} \left(\frac{\alpha_1^{1-\beta_1} \beta_1^{\beta_1} A_C}{w^{1-\beta_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} - \delta^{1/(1-\alpha_2-\beta_2)} \left(\frac{\alpha_2^{1-\beta_2} \beta_2^{\beta_2} A_Z}{w^{1-\beta_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \quad (35)$$

Once the full employment exchange rate δ^f is reached, the labor constraint is binding, i.e.,

$$L - (b(\delta^f))^{1/(1-\alpha_1-\beta_1)} \left(\frac{\alpha_1^{1-\beta_1} \beta_1^{\beta_1} A_C}{w^{1-\beta_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} - (\delta^f)^{1/(1-\alpha_2-\beta_2)} \left(\frac{\alpha_2^{1-\beta_2} \beta_2^{\beta_2} A_Z}{w^{1-\beta_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} = 0.$$

Differentiating (35) with respect to δ yields

$$\frac{\partial U}{\partial \delta} = -b' \frac{b^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}}{(1-\alpha_1-\beta_1)} \left(\frac{\alpha_1^{1-\beta_1} \beta_1^{\beta_1} A_C}{w^{1-\beta_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} - \frac{\delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}}{(1-\alpha_2-\beta_2)} \left(\frac{\alpha_2^{1-\beta_2} \beta_2^{\beta_2} A_Z}{w^{1-\beta_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} < 0.$$

This implies that a yuan devaluation reduces labor unemployment. Thus, there is some merit in the argument that China has is undervaluing renminbi to reduce its domestic unemployment.

Proposition 3: Assume the Cobb-Douglas production in Section 2, a yuan devaluation reduces China’s domestic unemployment.

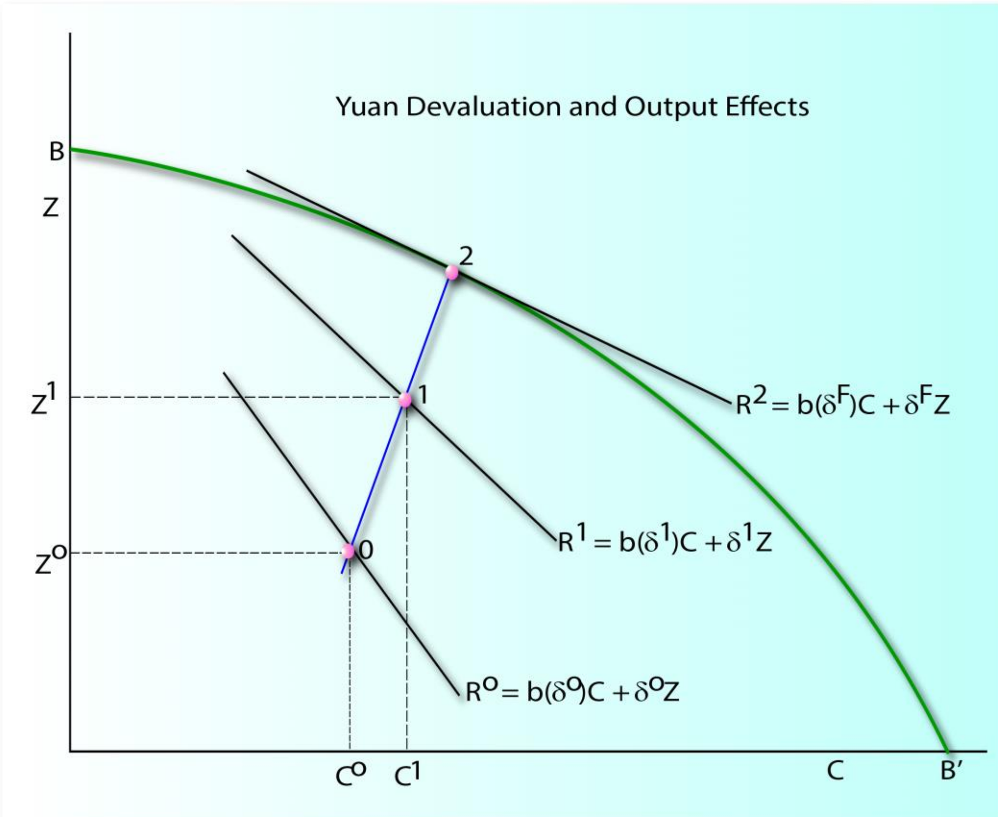


Figure 3. Yuan Devaluation and Output Effect

Figure 3 illustrates the effect of yuan devaluation in the presence of unemployment in China. The initial benchmark equilibrium occurs at point 0, where both *K* and *L* are unemployed and which is inside the PPF, labeled BB'. Since yuan devaluation affects the relative yuan price of good *C*. In this case, the supplies of *C* and *Z* rise, which causes a movement from point 0 to 1 inside the PPF. Yuan devaluation always raises domestic production of both goods in so far as unemployment exists. Thus, China's optimal policy is to choose δ to eliminate unemployment until production occurs at point 2 along the PPF, i.e., $\frac{\delta}{b} = \frac{\delta^f}{b(\delta^f)}$.

Consider an alternative scenario in which capital is fully utilized at point 0, but unemployment exists in the labor market. Note that the supply of good *Z* increases with *δ*. An increase in *δ* not only raises the yuan price of good *C*, but also raises the shadow price of capital. The latter effect partly offsets the former effect. Thus, the supply response of good *C* is likely to be less pronounced than when unemployment exists in both factor markets.[9]

7. Numerical Example

Since there is no tariff, the yuan price of the importable is $P = P^*\delta = \delta$. The price of the exportable good *C* is *b* and its dollar price of good *C* is b^*.

China and the United States

The United States and China are assumed to have identical production functions for the traded goods, *C* and *Z*: $C = F(L,K) = 1.3L^{.45}K^{.15}$ and $Z = G(L,K) = L^{.3}K^{.1}$. Assume that factor prices in China are: $w = .7$ and $r = 1$, and in the Unites States: $w^* = 1$ and $r^* = 0.7$. Labor and capital endowments are $\bar{L} = 4$ and $\bar{K} = 1$ in China and $\bar{L}^* = 1$ and $\bar{K}^* = 3$ in the United States. China's lending to the U.S. is *S* = 0.001. Direct utility functions of the Chinese and U.S. consumers are given by $u(c,z) = (cz)^{.4}$ and $u(c^*, z^*) = (c^* z^*)^{.4}$.

From the world market clearing condition in equation (24), we obtain the optimal yuan and dollar prices of good *C*:

$$b(\delta) = \left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44\left(1/\delta\right)^{1.5}} \right)^{0.4},$$

$$b^*(\delta) = \frac{b(\delta)}{P} = \frac{1}{\delta}\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44\left(1/\delta\right)^{1.5}} \right)^{0.4}.$$

Thus, as *δ* increases, the dollar price of good *C*, $b^*(\delta)$, decreases.

China's consumer demands for the two tradable goods are written as:

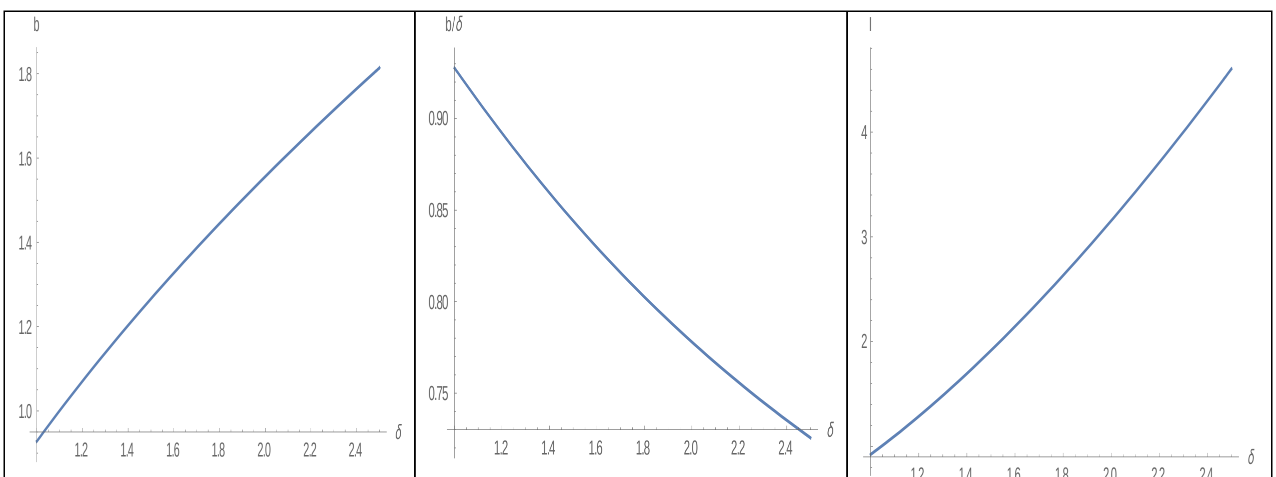
$$c = \frac{0.4}{b}\left(0.446\delta^{1.67} + 0.575\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44(1/\delta)^{1.5}}\right)^{0.4} - 0.001\right),$$

$$z = \frac{b}{\delta}c = \frac{0.4}{\delta}\left(0.446\delta^{1.67} + 0.575\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44(1/\delta)^{1.5}}\right)^{0.4} - 0.001\right).$$

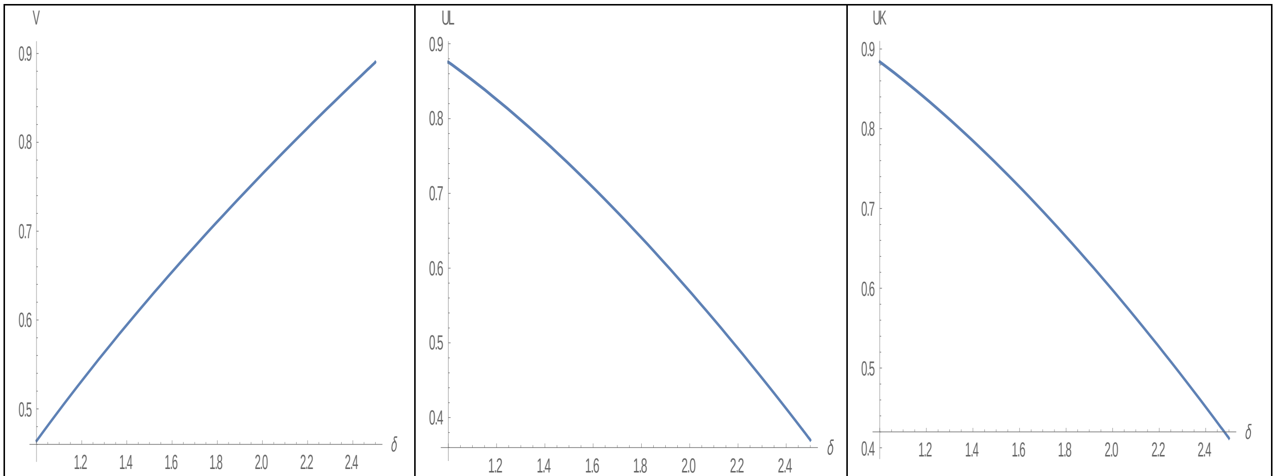
China's consumer welfare is measured by the indirect utility,

$$\begin{aligned} &V(b,\delta, I(\delta)-\delta S) = (cz)^{.4} \\ &= \frac{0.48}{b^{0.4}\delta^{0.4}}\left(0.446\delta^{1.67} + 0.575\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44(1/\delta)^{1.5}}\right)^{0.4} - 0.001\right)^{0.8}. \end{aligned} \tag{36}$$

We now investigate whether China gains from a yuan devaluation. Mathematica was used to generate Figure 4, which shows the impacts of yuan devaluation on the yuan and dollar prices of good *C*, national income, indirect utility, and labor and capital unemployment. When $\delta \approx 2.44$, both capital and labor inputs are fully utilized and a maximum utility is attained. Until δ^f is reached, exchange rate pass-through into the yuan price is imperfect, i.e., $b' < b/\delta$. Once δ^f is reached, a further increase in δ does not affect the dollar price of good C, b^*.



Yuan Price of Good *C*	Dollar Price of Good *C*	GDP



Indirect Utility	Unemployment of Labor	Unemployment of Capital

Figure 4. Effects on Yuan Devaluation

8. Concluding Remarks

This paper investigates optimal exchange rate policy for a two-sector, Keynesian open economy. A Cobb-Douglas utility function is used to investigate China’s optimal exchange rate policy. Under certain conditions, the indirect utility function is monotone increasing and concave in δ, which suggests the existence of an optimal exchange rate.

China has been widely criticized for keeping the yuan low in order to take advantage of its trading partners. However, this paper suggests that China may have pursued the low yuan policy to reduce domestic unemployment. For the case of Cobb-Douglas utility function, China’s optimal exchange rate is that rate which guarantees full employment. Such a policy cannot be criticized as deliberately devaluing the yuan to take unfair advantage of China’s trading partners. The current U.S.-China trade imbalance may be caused by other reasons than the undervalued yuan.

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Yue, Kan and Kevin Honglin Zhang, “How much does China’s exchange rate affect the U.S. trade deficit?” *The Chinese Economy* 46 (2013), 80-93.

[1] Currency devaluation of China may not be the only source of US trade deficits. For instance, Beladi and Oladi (2014) suggest that outsourcing may widen US trade deficits. Also, Yue and Zhang (2013) emphasize that US trade deficit would not be reduced very much by a change in the Chinese exchange rate.

[2] Of course, the first best policy is to remove wage and rent rigidity in the factor markets. Given this rigidity, Chinese government may be using yuan devaluation as a second best policy.

[3] In the same vein, Bruno (1976) considered a two-sector model, but defines the exchange rate as the ratio of the price of the tradable goods to that of the nontradable good.

[4] Devereux (2000) analyzed the impact of devaluation on the trade balance when exchange rate pass-through is imperfect.

[5] For instance, $F_{LL} = \alpha_1(\alpha_1 - 1)A_C L_C^{\alpha_1 - 2} K_C^{\beta_1} < 0,$ $F_{KK} = \beta_1(\beta_1 - 1)A_C L_C^{\alpha_1} K_C^{\beta_1 - 2} < 0,$ $F_{LK} = \alpha_1\beta_1 A_C L_C^{\alpha_1 - 1} K_C^{\beta_1 - 1} > 0,$ and $F_{LL}F_{KK} - \left(F_{LK}\right)^2 = \alpha_1\beta_1(1 - \alpha_1 - \beta_1)\left(A_C L_C^{\alpha_1 - 1} K_C^{\beta_1 - 1}\right)^2 > 0.$

[6] If the benchmark equilibrium value of \$1 is 6 RMB, new RMB notes can be issued at the rate of 6 RMB for 1 new RMB.

[7] $$\gamma\left(\frac{\delta\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}=(1-\gamma)\left(\frac{b\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}$$

[8] $$\gamma\left(\frac{\delta\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}=(1-\gamma)\left(\frac{b\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}$$

[9] Under assumption $\partial r/\partial\delta>0,$ the effect of exchange rate on the output C and Z are

$$\frac{\partial C(b,P)}{\partial\delta}=\frac{1}{1-\alpha_1-\beta_1}\left(\frac{b^{\alpha_1+\beta_1}\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\left(\frac{(\alpha_1+\beta_1)\delta}{b}\frac{\partial b}{\partial\delta}-\frac{\beta_1\delta}{r}\frac{\partial r}{\partial\delta}\right)\frac{1}{\delta}$$

$$=\frac{1}{1-\alpha_1-\beta_1}\left(\frac{b^{\alpha_1+\beta_1}\alpha_1{}^{\alpha_1}\beta_1{}^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\left(-(\alpha_1+\beta_1)\varepsilon_{b\delta}-\beta_1\varepsilon_{r\delta}\right)\frac{1}{\delta},$$

$$\mathrm{Z}(b,P)=\frac{1}{1-\alpha_2-\beta_2}\left(\frac{P^{\alpha_2+\beta_2}\alpha_2{}^{\alpha_2}\beta_2{}^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}\left((\alpha_2+\beta_2)\varepsilon_{P\delta}-\beta_2\varepsilon_{r\delta}\right)\frac{1}{\delta}>0.$$