Credit Constraints, Utilitarian Market Failure and Social Enterprises*

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July 3, 2017

Abstract

Credit constraints create a gap between marginal utilities concerning primary needs satisfaction between consumers with and without credit constraints. Larger marginal utilities of consumers who spend their entire incomes for primary needs satisfaction cannot be realized through market exchange. Thus markets fail to maximize utilitarian social welfare, which is referred to as utilitarian market failure in this paper. The purpose of this paper is to bring out utilitarian market failure and to analyze the roles of social enterprises and government support for their production of social values in the face of this market failure. Social enterprises have objective functions given by both profits and the social values they produce. The social value is created from the employment of marginalized or disadvantaged workers. We consider the Cournot oligopoly model and compares the social welfare of two cases, one with only for-profit enterprises and the other with for-profit and social enterprises. Comparing the two cases of the markets, we show that the market with social enterprises can attain greater utilitarian social welfare than the market with only for-profit enterprises and, to attain the greatest utilitarian social welfare, social enterprises should assign an optimal weight on social values in their decision making. We show that when the weight on social values is suboptimal, government subsidization for the production of social values can improve social welfare by increasing the suboptimal weight on social values toward an optimal level. In other words, the government subsidization provides higher incentive for social enterprises (and for-profit enterprises as well) to put greater weights on social values in making their market decisions.

JEL classification: A13, P31

Keywords: Social enterprises, Social welfare, Cournot competition, Credit constraints

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1. Introduction

2. Low-Income Client as Market model

2.1. Low Income, Credit Constraint, and Market Failure

x is a good for basic human needs(e.g. primary goods or services provided in the market) and M is money. All consumers have an identical utility function which is given by $u(x, M) = v(x) + M$ where $v(x)$ is a primary needs satisfaction, $v'(x) > 0, v''(x) < 0$,. We assume the basic human needs implies that when there is too much dissatisfaction of basic human needs, satisfying them improves well-being quite significantly, which is captured by $lim_{x\to 0} = \infty$. The demand function of x, $p(x)$ is twice differentiable and $p'(x) < 0$. If the price(p) and the income(I) are given, the consumer's utility maximization problem is given by

$$max_{x,M} u(x, M) = v(x) + M \ s.t. \ px + M \leq I, M \geq 0.$$

Let $h(\cdot)$ be an inverse function of the marginal benefit function $v'(x)$. For any price $p > 0$, there is a threshold income $I^* > 0$ such that for a consumer whose income is above I^* her marginal benefit of x equals p, $x = h(p)$ and for a consumer whose income is less than I^* her marginal benefit of x is larger than p. Then $I^* = ph(p)$. Let a consumer whose income is higher than I^* be a high-income consumer and let a consumer whose income is less than I^* be a low-income consumer. A low-income consumers spends all income on buying x, i.e., $x = \frac{I}{p}$ since her marginal benefit of x is larger than the price, $v'(\frac{I}{p}) > p$. She wants to realize her greater marginal benefit but she cannot do due to the credit constraint which means that borrowing money cannot be allowed.

Let I_H, I_L, $p_H(x)$, and $p_L(x)$ be a high income, a low income, the demand curve of a high-income consumer, and the demand curve of a low-income consumer, respectively. Let x_H^* and x_L^* be the consumption of a high-income consumer and the consumption of a low-income consumer at the market equilibrium. The demand curve of a high-income consumer is given by $p_H(x) = v'(x)$ since $p^* = v'(x^*)$. However the demand curve of a low-income consumer is given not by $p_L(x) = v'(x)$ but by $p_L(x) = \frac{I_L}{x}$ since $p^* < v'(x_L^*)$.

Lemma 1 *The demand curve of a low-income consumer under the credit constraint under-estimates her marginal benefit.*

In the figure 1, I_M is the threshold income. A high-income consumer chooses C where her willingness-to-pay at x_H^* coincides with p^*. A low-income consumer chooses A where her willingness-to-pay at x_L^* is larger than p^*.

In the figure 2, a low-income consumer under the credit constraint consumes x_L^* at p^*. Her willingness-to-pay at x_L^*, namely, the price on her demand curve p^* is below her marginal benefit. The marginal benefit of the low-income consumer coincides with that of a high-income consumer, namely, the price on the demand curve of the high-income consumer p^{**}. It is appropriate to use the demand curve of a high-income consumer for measuring the surplus of a low-income consumer.

Assuming that there are consumers whose income is sufficiently close to zero, any competitive market equilibrium admits some credit-constrained consumers and fails to maximize

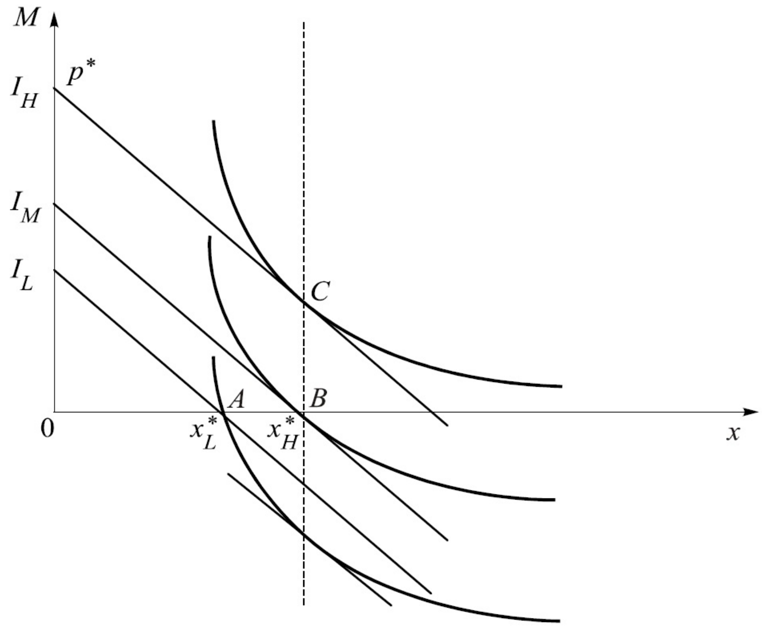

Figure 1: Credit constraint and optimal consumption

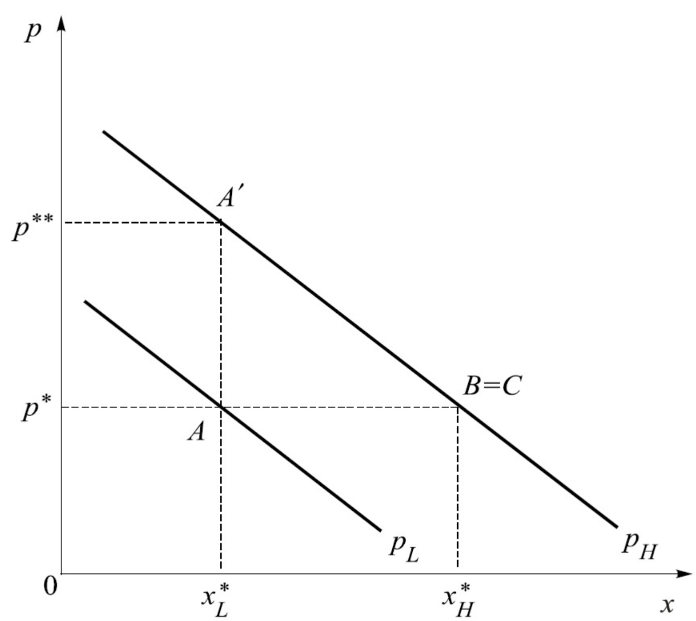

Figure 2: The demand curves of each type consumers

utilitarian social welfare(social surplus). Transferring the goods for basic human needs from a rich consumer(with lower marginal benefit) to a poor consumer under the credit constraint(with higher marginal benefit) improves utilitarian social welfare. The same applies to the markets with imperfect competition as well.

2.2. The model

There are two types of consumers, the one is a high-income consumer that is not under the credit constraint and the other is a low-income consumer that is under the credit constraint. The types of consumers are complete informations. And the market is divided into a high-income group market and a low-income group market.

There are n for-profit enterprises and m social enterprises. All enterprises have an identical cost function $C(x)$ which is strictly increasing($C'(x) > 0$, $C''(x) > 0$). Let $x_{HP,i}$, $x_{HS,j}$, and $x_{LS,j}$ be the supply of for-profit enterprise $i(i = 1, 2, \cdots, n)$ in a high-income group market, the supply of social enterprise $j(j = 1, 2, \cdots, m)$ in a high-income group market, and the supply of social enterprise j in the low-income group market, respectively. We suppose that $X_H = \sum_{i=1}^{n} x_{HP,i} + \sum_{j=1}^{m} x_{HP,j}$, $X_L = \sum_{j=1}^{m} x_{LS,j}$, $X_{-i} = X_H - x_{HP,i}$, and $X_{-j} = X_H - x_{HS,j}$.

While for-profit enterprise i maximizes its profit, social enterprise j pursues both profit and social value. Social enterprises produce social value by providing x for low-income consumers under the credit constraint. From lemma 1, social value can be measured using the surplus of the low-income group and is given by

$$SV(X_L) = (1 + \eta) \int_0^{X_L} p_H(x)dx$$

, where η is a coefficient of positive social externality of improving the quality of life of the low-income group. We assume that the payoff of social enterprise j is the weighted average of profit and social value and that α is the social entreprenuer's weight on social value. Then the payoff of social enterprise j is given by

$$U_{S,j} = (1 - \alpha)\pi_{S,j} + \alpha_j SV(X_L).$$

For simplicity, we assume that all social entrepreneurs are identical and have a same weight on social value, $\alpha \in (0, 1)$ and that social enterprises provide x for free[1].

The social welfare is the sum of the total consumer surplus(of both high-income and low-income consumers), the total producer surplus, and the positive externality of social value, and is given by

$$SW = \int_0^{X_H} p_H(x)dx + (1 + \eta) \int_0^{X_L} - \sum_i C(x_{HP,i}) - \sum_j C(X_{HS,j} + X_{LS,j}).$$

We assume that all enterprises produce perfectly substitutable goods, which means that each enterprise's reaction curve has a negative slope: $p''_H x_{HP,i} + p'_H < 0$, $p''_H x_{HS,j} + p'_H < 0$.

[1]The assumption implies that for-profit enterprises do not provide x in the low-income group market. Without this assumption we are also able to derive the same results.

2.3. The results

To investigate the effect of social enterprises on social welfare, We compare Cournot equilibria under the oligopoly with only for-profit enterprises and the oligopoly with both for-profit and social enterprises.

Proposition 1 *As social enterprises put more weight on the social value(α increases), the market share of the social enterprises(resp. for-profit enterprises) decreases(increases) in the high-income group market.*

Proof. See Appendix.

If social enterprises supply more quantities in the low-income group market, the social value produced increases. Social welfare improves initially but declines eventually. There is an optimal weight level α^* where the social welfare is maximized.

Proposition 2 *Comparing the oligopoly with only for-profit enterprises and the oligopoly with the same total number of both for-profit and social enterprises, the latter market(under some α) attains a higher level of social welfare regardless of η.*

Proof. See Appendix.

In the oligopoly with only for-profit enterprises, each low-income consumer's marginal benefit is higher than the price. But she cannot consume more of x due to credit constraint. Since the marginal benefit of low-income consumers is higher than the marginal benefit of high-income consumers, there is a possibility of improving utilitarian social welfare. Social enterprises help increasing x consumption of low-income consumers by providing social services for free, which increases low-income consumer benefits. On the other hand, additional production for low-income consumers may increase production cost too much. If the former effect of increasing consumer benefits dominates the latter effect of increasing production cost, the overall social welfare improves. In addition, social enterprises also help resolving inefficiency due to under-production in the standard Cournot competition.

3. Employment Model

3.1. Labor Productivity, Credit Constraint, and Market Failure

In the employment model, we assume the economy with three goods, a good z that is consumed to satisfy basic human needs, money M, and labor l. All workers have an identical utility function which is given by $U(z, l, M) = v(z) - e(l) + M$ where $e(\cdot)$ is the effort cost of labor and $v(\cdot)$ is the benefit from consuming the basic human needs good. For standard monotonicity, we assume that $e'(l) > 0$, $e''(l) > 0$, $v'(z) > 0$, and $v''(z) < 0$. As the Low-income Client as Market Model, we also assume that $lim_{z \to 0} v'(z) = \infty$.

We consider two types of workers, one is a regular worker and the other is a disadvantaged worker. Each worker earns wage which equals her marginal productivity in the competitive labor market. Regular workers earn wage in the rate of $w(> 0)$ per unit labor. Each disadvantaged worker has lower productivity than regular workers owing to various reasons such as lack of skills and physical defects. Let $\lambda \in [0,1)$ be a productivity parameter which reflects how the marginal productivity of disadvantaged workers is lower than that of regular workers. Then the disadvantaged workers earn λw in the competitive labor market. Given the price of z and the regular wage w, the utility maximization problem of a disadvantaged worker is given by,

$$max_{z,l,M} U(z,l,M) = v(z) - e(l) + M \ s.t. \ pz + M \leq \lambda wl < I_m, \ z \geq 0, \ M \geq 0, \ 0 \leq l \leq \bar{l}$$

,where I_m is the threshold income needed to satisfy the basic human needs fully and $\bar{l}$ is the maximum labor supply.

Whether a disadvantaged worker is under the credit constraint or not depends on her marginal productivity. If λ is too low, $v'(\lambda w\bar{l}) > p$ which means that the disadvantaged worker cannot satisfy her human basic needs fully although she chooses the maximum labor hour. If λ is so high that $v'(\lambda w\bar{l}) \leq p$, the credit constraint is dependent on the level of labor supply. Let $H(\cdot)$ be an inverse function of the marginal benefit function $v'(z)$. The threshold income I_m is an income such that $I_m = pH(p)$. Let l_m be a threshold labor hour such that $l_m = \frac{I_m}{\lambda w} = \frac{pH(p)}{\lambda w}$. The marginal effort cost at l_m is less than the wage of the disadvantaged worker if and only if the disadvantaged worker obtains an income higher than the threshold income. That is, $e'(l_m) \leq \lambda w \leftrightarrow I > I_m$. So if $e'(l_m) > \lambda w$, a disadvantaged worker uses all income for satisfying basic human needs. There are two cases in which a disadvantaged worker is under the credit constraint. We deal with only the former case in which λ is too low.

In the figure 3, the disadvantaged workers use all income I_d for consuming z_d since $v'(z_d) > p$. Let assume that social enterprises provide the disadvantaged with wage $w_d \in (\lambda w, \overline{w})$ where $\overline{w}$ is less than w. The disadvantaged workers' income will increase from $I_d (= \lambda w\bar{l})$ to $I'_d (= w_d\bar{l})$. The consumption of z will increase from $z_d (= \frac{I_d}{p})$ to $z'_d (= \frac{I'_d}{p})$ and their welfare will increase from U_d to U'_d. We obtain the following equation[2].

$$\triangle U_d = U'_d - U_d = v(z'_d) - v(z_d) = \int_{z_d}^{z'_d} v'(z)dz > \int_{z_d}^{z'_d} pdz = I'_d - I_d$$

If we do not take the credit constraint into consideration, the surplus of the disadvantaged workers will be under-estimated. So we consider the credit constraint to measure the surplus of them appropriately. The change of the surplus is given by

$$\triangle U_d = v(\frac{w_d\bar{l}}{p}) - v(\frac{\lambda w\bar{l}}{p}) \equiv (1 + \theta(\lambda))(w_d - \lambda w)\bar{l}$$

, where $\theta(\lambda)$ is a coefficient that reflects the level of the credit constraint. If there is the credit constraint, $\theta(\lambda) > 1$. If not, $\theta(\lambda) = 0$.

[2] We can show the same thing in the latter case where $e'(l_m) > \lambda w$.

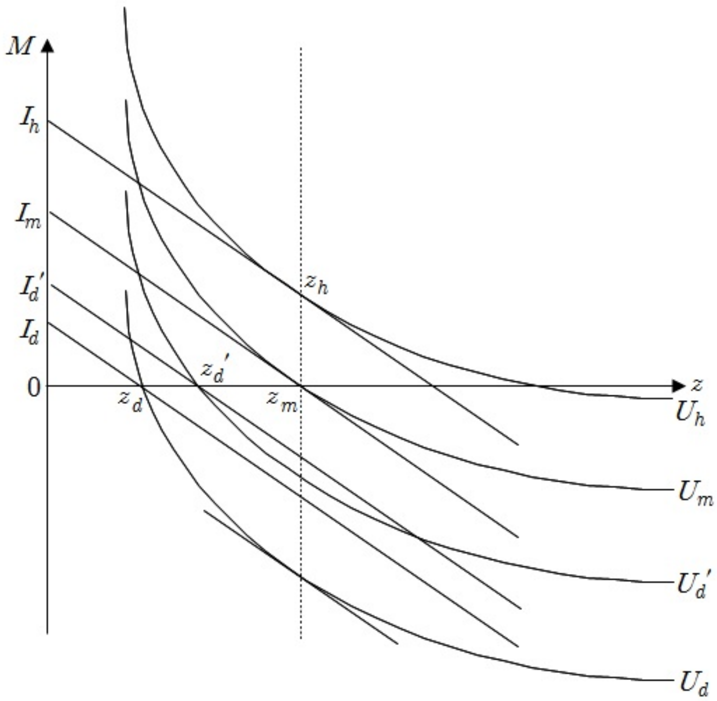

Figure 3: Labor productivity and credit constraint

3.2. Social value

In the employment model the social value is composed of two parts, the one is the increase of a disadvantaged worker's surplus produced by the social enterprise's employment and the other is the positive effect of the enhancing the quality of life of the disadvantaged worker.[3] We assume that the positive effect exists in proportion to the increase of the disadvantaged's surplus and that the proportional rate is $\eta(\geq 0)$. Let γ be the coefficient that reflects the credit constraint and the positive effect, i.e., $1+\gamma = (1+\theta(\lambda))(1+\eta)$.

Let L_r an L_d be regular workers and disadvantaged workers. Let $f(L_r + \lambda L_d, K)$ be the production function in which two types of workers are perfectly substitutable and λ is the marginal rate of technical substitution of disadvantaged workers for regular workers. Let $g(x, K)$ be the inverse function of f which is the conditional labor demand function, then $L_r(x, K) = g(x, K)$ and $L_d(x, K) = \frac{g(x,K)}{\lambda}$. The short-run labor cost function of for-profit enterprises is $c_p(x, w) = wL_r(x.K) = wg(x, K)$ and that of social enterprises is $c_s(x, w_d) = w_d L_d(x, K) = \frac{w_d g(x,K)}{\lambda}$. The social value by a social enterprise which hires L_d is given by

$$SV = (1+\theta(\lambda))(1+\eta)(w_d - \lambda w)L_d(x, K) = (1+\gamma)(w_d - \lambda w)L_d(x, K).$$

[3]Haugh(2006) classified the enterpreneurial outcomes into economic, social, and environmental outcomes. Improved quality of life was the direct social outcomes and the increased individual confidence, extent of personal networks, team-working skills were the indirect social outcomes. Our classification of the social value can be seen similar to Haugh's classification.

Hereinafter, K, w, and w_d will be hidden from the functions.

The social value produced by a social enterprise can also be approached from the viewpoint of the cost. A social enterprise hires not regular workers but disadvantaged workers. The social enterprise must endure the higher variable cost than a for-profit enterprise to make the same production. This may be because a social enterprise judges that social value produced offsets the cost disadvantage. Therefore, we can assume that the social value recognized by the social enterprise is the same as the increment of the variable cost generated by employing the disadvantaged workers. It is also possible to consider the case where the fixed costs are additionally required while employing the disadvantaged workers such as the disabled workers. In that case, a social enterprise needs to establish facilities for workers with disabilities and it must endures the higher fixed cost, too. However we will not consider an additional fixed cost when measuring social value in terms of cost since it is not constantly required. The relationship between the income increase of the disadvantaged workers and the variable cost increase of the social enterprise is as follows.

Lemma 2 $SV = (1+\gamma)(w_d - \lambda w)L_d(x) = (1+\gamma)(c_s(x) - c_p(x))$.

Proof. See Appendix.

As the previous model, the payoff of a social enterprise is the weighted average of profit and and social value and the weight is α. The payoff maximization problem of a social enterprise is given by

$$max_{x_s,L_r,L_d,w_d} U = (1-\alpha)(p(x_s+\overline{X})x_s - wL_r - w_d L_d) + \alpha(1+\gamma)(w_d - \lambda w)L_d$$

, where $\overline{X}$ is the total supply of other enterprises. We obtain lemma 3 by solving the payoff maximization problem.

Lemma 3 *If a social enterprise's weight on the social values is larger than $\frac{1}{2+\gamma}$, the social enterprise hires only the disadvantaged workers at wage $\overline{w}$.*

Proof. See Appendix.

$\alpha \geq \frac{1}{2+\gamma}$ means that social enterprises consider social value in which the credit constraint and the positive effect are reflected more important than profit. Since the social value is more important than the profit, the marginal benefit of the social value will be greater than the marginal cost of the social value. To realize the higher marginal benefit, social enterprises raise wages for disadvantaged workers and hire only them. In the real world, social enterprises employ both regular and disadvantaged workers. This phenomenon can be seen as a result of the adjustment of the employment level and the wage level of the disadvantaged workers since it is necessary to be sustainable. For simplicity, we assume that social enterprises employ only the disadvantaged workers as lemma 3.

3.3. The model and result

The demand function of x, $p(x)$ is twice differentiable and $p'(x) < 0$. There are n for-profit enterprises and m social enterprises. We assume that all for-profit enterprises are identical and that all social enterprises are also identical. The cost function of $i(i = 1, 2, \cdots, n)$ for-profit enterprise is given by $C_{P,i}(x) = c_{p,i}(x) + F_{P,i} = c_P(x) + F_P$ and the cost function of $j(j = 1, 2, \cdots, m)$ social enterprise is given by $C_{S,j}(x) = c_{S,j}(x) + F_{S,j} = c_S(x) + F_S$ where c is the variable cost and F is the fixed cost. We also assume that $c'_S(x) > c'_P(x) > 0$, $c''_S(x) > c''_P(x) > 0$, and $F'_S(x) \geq F'_P(x)0$. Let $x_{P,i}$ and $x_{S,j}$ be the supply of i for-profit enterprise and the supply of j social enterprise. We suppose that $X = \sum i = 1^n x_{P,i} + \sum j = 1^m x_{S,j}$, $X_{-i} = X - xP, i$, and $X_{-j} = X - x_{S,j}$. Enterprises' payoff functions are assumed to be the same as the previous model. Finally we assume perfectly substitutable goods and Cournot quantity competition . By lemma 2, we measure social value using the cost functions. Then the social welfare function is given by

$$SW = \int_0^X p(x)dx - \sum_{i=1}^{n} C_P(x_{P,i}) - \sum_{j=1}^{m} C_S(x_{S,j}) + \sum_{j=1}^{m} SV(x_{S,j}).$$

To investigate the effect of social enterprises on social welfare, we use the same method as the previous model.

Proposition 3 *When the fixed cost gap($F_S - F_P$) is small enough, social enterprises can improve social welfare.*

Proof. See Appendix.

If the fixed cost of for-profit enterprises are the same as that of social enterprises, social enterprises which put an proper weight on social value improve social welfare. If the fixed cost of social enterprises is larger than that of for-profit enterprises, social enterprises have negative effects on social value in the viewpoint of the cost efficiency. However, social enterprises produce social value and mitigate the under-production problem of the oligopoly market. To investigate whether social enterprises can improve social welfare or not, we must compare the opposing effects.

Assuming the 2-period model in which the fixed cost is required only in the first period, there is more room for evaluating the effect of social enterprises on the social welfare positively. Even if the cost inefficiency decreases the social welfare in the first period, the social welfare can increase due to the positive effects in the second period.

4. Social Progress Credit

In section 2 and 3, we admit a spectrum of social entrepreneurs depending on how much weight they put on social value as opposed to profit. Depending on the weight on social value, social enterprises may improve or disimprove social welfare.

Social value cannot be evaluated by monetary terms since it is not traded in the market. It is difficult for each social enterprises to know the relative importance of social value to profit.

Even if social enterprises get to know the socially optimal weight, they produce more or less social value than the optimal level, which optimizes the payoff of social enterprises. Thus it is important to induce proper incentive for the creation of social value.

There are many ways to adjust the decision of social enterprises. We introduce a subsidy policy, social progress credit(SPC). SPC is a subsidy which is paid in proportion to social value that a social enterprise creates. For example, if $SPC = \beta SV$ is introduced, the payoff function of social enterprises is changed as follows.

$$U_S = (1-\alpha)(\pi_S + SPC) + \alpha SV = (1-\alpha)\pi_S + ((1-\alpha)\beta + \alpha)SV.$$

SPC makes social entrepreneurs behave as if they put a higher weight on social value than they really do when $\beta > 0$.

In the low-income client as market model, social value is equal to the total consumer surplus of the low-income group market. There is a need for standards that determine the extent to which each social enterprise contributes to the social value produced by all of them. A supply of a social enterprise in the low-income group market can be an alternative, since the social value is produced in proportion to the supply. So we suppose that $SPC = \beta x_{LS}$. In the employment model, social value produced by a social enterprise is proportional to the product of the number of disadvantaged workers and the wage rate higher than the marginal productivity. We suppose that $SPC = \beta(w_d - w_L)L_d$. Generally speaking, β can be interpreted as the rate of credit for social value.

Measuring social value, SPC is a negative factor in the viewpoint of the government but SPC is a positive factor in the viewpoint of social enterprises. If there is no loss of welfare in the process of a subsidy, SPC have not a direct effect on social welfare. SPC will change decisions of social enterprises, which has an indirect effect on social welfare(White, 1996; Tomaru, 2006). The social welfare functions are assumed to be the same as those of section 2 and 3.

Proposition 4 *If the weight on social value is lower than the optimal level, social progress credit can improve social welfare.*

Proof. See Appendix.

5. Conclusion

References

[1] Alter, K., Social Enterprise Typology, Virtue Ventures, 2007.

[2] Chu, C. Y. Cyrus, Warren Buffet versus Muhammad Yunus, Journal of institutional and Theoretical Economics, Vol. 171, No. 4, 2015, pp.696-708.

[3] Davister, C., Defourny, J., Gregoire, O., Work Integration Social Enterprises in the European Union: an Overview of Existing Models, Working Paper Series, No. 04, 2004.

[4] Haugh, Helen, Social enterprise: beyond economic outcomes and individual returns. Social entrepreneurship. Palgrave Macmillan UK, 2006. 180-205.

[5] Kato, K., Tomaru, Y., Mixed oligopoly, privatization, subsidization, and the order of firms' moves: Several types of objectives, Economics Letters, Vol. 96, No. 2, 2007, pp.287-292.

[6] Mark D. White, Mixed oligopoly, privatization and subsidization, Economics Letters, Vol. 53, 1996, pp.189-195.

[7] Matsumura, T., Partial privatization in mixed duopoly, Journal of Public Economics, Vol. 70, 1998, pp.473-483.

[8] Matsumura, T., Kande, O., Mixed Oligopoly at Free Entry Markets, Journal of Economics, Vol. 84, 2005, pp.27-48.

[9] Tomaru Yoshihiro, Mixed Oligopoly, Partial Privatization and Subsidization, Economics Bulletin, Vol. 12, No. 5, 2006, pp.1-6.

[10] Yunus, M., Building Social Business: The New Kind of Capitalism that Serves Humanity's Most Pressing Needs, Public Affairs, 2010.

Appendix

Proof of proposition 1

The first order conditions of for-profit enterprises and social enterprises are as follows.

$$p'_H(X_H)'x_{HP,i} + p_H(X_H) - C'(x_{HP,i}) = 0 \tag{1}$$

$$p'_H(X_H)'x_{HS,j} + p_H(X_H) - C'(x_{HS,j} + x_{LS,j}) = 0 \tag{2}$$

$$-C'(x_{HS,j} + x_{LS,j}) + \frac{\alpha}{1-\alpha}SV'(X_L) = 0 \tag{3}$$

From the assumption that enterprises are identical, $x_{HP,1} = x_{HP,2} =, ..., = x_{HP,n} = x_{HP}$, $x_{HS,1} = x_{HS,2} =, ..., = x_{HS,m} = x_{HS}$, $x_{LS,1} = x_{LS,2} =, ..., = x_{LS,m} = x_{LS}$ in the equilibrium. Putting $X_H = nx_{HP} + mx_{HS}$ and $X_L = mx_{LS}$ into (1),(2), and (3), and differentiating them totally yields the followings.

$$\frac{dx_{HP}}{d\alpha} = \frac{1}{(1-\alpha)^2}\frac{SV'(mx_{LS})C''(x_{HS}+x_{LS})G}{J(GH-FI) - [C''(x_{HS}+x_{LS})]^2 F} \tag{4}$$

$$\frac{dx_{HS}}{d\alpha} = -\frac{F}{G}\frac{dx_{HP}}{dx_\alpha} \tag{5}$$

$$\frac{dx_{LS}}{d\alpha} = \frac{GH-FI}{C''(x_{HS}+x_{LS})G}\frac{dx_{HP}}{d\alpha} \tag{6}$$

where $F = np''_{HP}x_{HP} + (n+1)p'_H - C''(x_{HP})$, $G = m(p''_H x_{HP} + p'_H)$, $H = n(p''_H x_{HS} + p'_H)$, $I = mp''_H x_{HS} + (m+1)p'_H - C''(x_{HS}+x_{LS})$, $J = C''(x_{HS}+x_{LS}) - \frac{\alpha}{1-\alpha}mSV''(x_{LS})$. From

$p''_H x_{HP} + p'_H < 0$ and $p''_H x_{HS} + p'_H < 0$, $F < 0$, $G < 0$, $H < 0$, and $I < 0$. From $SV''(mx_{LS}) = (1+\gamma)p'_L(mx_{LS}) < 0$, $J > 0$. From $|GH| < |FI|$, $GH - FI < 0$. And since $|J| > |C''(x_{HS} + x_{LS})|$ and $|GH - FI| > |C''(x_{HS} + x_{LS})F|$, $J(GH - FI) - [C''(x_{HS} + x_{LS})]^2 F < 0$. Therefore $\frac{dx_{HP}}{d\alpha} > 0, \frac{dx_{HS}}{d\alpha} < 0$, and $\frac{dx_{LS}}{d\alpha} > 0$. Q.E.D.

Proof of proposition 2
Assume the oligopoly with $(n+m)$ for-profit enterprises. Let x_E and SW_E denote the output of each for-profit enterprise and the social welfare in the equilibrium, respectively.

$$p'_H x_E + p_H - C'(x_E) = 0 \tag{7}$$

$$SW_E = \int_0^{(n+m)x_E} p_H(x)dx - (n+m)C(x_E) \tag{8}$$

The first order condition is (7) and social welfare is given by (8). Next assume the oligopoly with n for-profit enterprises and m social enterprises. Let x_{HP}, x_{HS}, x_{LS}, and $SW_{P,S(\alpha)}$ denote the output of a for-profit enterprise, the output of a social enterprise in H-type market, the output of a social enterprise in L-type market, and the social welfare in the equilibrium, respectively.

$$p'_H x_{HP} + p_H - C'(x_{HP}) = 0 \tag{9}$$

$$p'_H x_{HS} + p_H - C'(x_{HS} + x_{LS}) = 0 \tag{10}$$

$$\frac{\alpha(1+\eta)}{1-\alpha} p_H(mx_{LS}) = C'(x_{HS} + x_{LS}) \tag{11}$$

$$SW_{P,S(\alpha)} = \int_0^{nx_{HP}+mx_{HS}} p_H(x)dx + (1+\eta)\int_0^{mx_{LS}} p_H(x)dx - nC(x_{HP}) - mC(x_{HP} + x_{LS}) \tag{12}$$

The first order conditions are (9) - (11) and social welfare is given by (12). If $\alpha = \alpha_1 = \frac{C'(x_E)}{(1+\eta)p_H(0)+C'(x_E)}$, then $x_E = x_{HP} = x_{HS}$ and $x_{LS} = 0$. So we have that $SW_E = SW_{P,S(\alpha_1)}$ and that $\frac{dSW_{P,S(\alpha)}}{d\alpha}|_{\alpha=\alpha_1} = n\frac{dx_{HP}}{d\alpha}[p_H - C'(x_E)] + m\frac{dx_{HS}}{d\alpha}[p_H - C'(x_E)] + m\frac{dx_{LS}}{d\alpha}[(1+\eta)p_H(0) - C'(x_E)] > 0$. Q.E.D.

Proof of lemma 2
From $c_p(x) = wL_r(x) = wg(x)$ and $c_s(x) = w_d L_d(x) = \frac{w_d g(x}{\lambda}$, $c_s(x) - c_p(x) = \frac{w_d g(x)}{\lambda} - wg(x) = (w_d - \lambda w)\frac{g(x)}{\lambda} = (w_d - \lambda w)L_d(x)$. Q.E.D.

Proof of lemma 3
If x_s is given, the payoff maximization problem is equal to the following minimization problem.

$$min_{L_r,L_d,w_d}(1-\alpha)(wL_r + w_d L_d) - \alpha(1+\gamma)(w_d - \lambda w)L_d \; s.t. \; L_r + \lambda L_d = g(x_s).$$

$(1-\alpha)(wL_r + w_d L_d) - \alpha(1+\gamma)(w_d - \lambda w)L_d = (1-\alpha)(w(g(x_s) - \lambda L_d) + w_d L_d) - \alpha(1+\gamma)(w_d - \lambda w)L_d = (1-\alpha)wg(x_s) - ((1-\alpha)(\lambda w - w_d) + \alpha(1+\gamma)(w_d - \lambda w))L_d = (1-\alpha)wg(x_s) - (\lambda w - w_d)((2+\gamma)\alpha - 1)L_d$. If $\alpha \geq \frac{1}{2+\gamma}$, $w_d = \overline{w}$, $L_r = 0$, and $L_d = \frac{g(x_s)}{\lambda}$. Q.E.D.

Proof of proposition 3

Assume the oligopoly of $(n+m)$ for-profit enterprises. Let x_E and SW_E denote the output of each for-profit enterprises and the social welfare in the equilibrium, respectively. Then we have the followings.

$$p'x_E + p - c_P'(x_E) = 0 \tag{13}$$

$$SW_E = \int_0^{(n+m)x_E} p(x)dx - (n+m)C_P(x_E) \tag{14}$$

Next assume that there are n for-profit enterprises and m social enterprises. Let x_P, x_S, and $SW_{P,S(\alpha)}$ denote the output of a for-profit enterprise, the output of a social enterprise, and the social welfare in the equilibrium, respectively. Using the first order conditions, we get the followings.

$$p'x_P + p - c_P'(x_P) = 0 \tag{15}$$

$$(1-\alpha)[p'x_S + p - c_S'(x_S)] + \alpha(1+\gamma)[c_S'(x_S) - c_p'(x_S)] = 0 \tag{16}$$

$$SW_{P,S(\alpha)} = \int_0^{nx_P+mx_S} p(x)dx - nC_P(x_P) - mC_S(x_S) + m(1+\gamma)[c_S(x_S) - c_P(x_P)] \tag{17}$$

Total differntiation of (15) and (16) gives

$$\frac{dx_P}{d\alpha} = -\frac{L}{LM-KN}\frac{SV'(x_S)}{(1-\alpha)^2}, \tag{18}$$

$$\frac{dx_S}{d\alpha} = -\frac{K}{LM-KN}\frac{SV'(x_S)}{(1-\alpha)^2}, \tag{19}$$

where $K = np''x_P + (n+1)p' - c_P''(x_P)$, $L = m(p''x_P + p')$, $M = n(p''x_S + p')$, $N = mp''x_S + (m+1)p' - c_S''(x_S) + \frac{\alpha(1+\gamma)}{1-\alpha}[c_S''(x_S) - c_P''(x_S)]$. From the assumption of strategic substitutes and the second order conditions, $K < 0$, $L < 0$, $M < 0$, and $N < 0$. If $\alpha_1 = \frac{1}{2+\gamma}$, then $x_E = x_P = x_S$, $LM|_{\alpha=\alpha_1} = mn(p''x_E + p')^2$, and $KN|_{\alpha=\alpha_1} = (n(p''x_E + p') + p' - c_P''(x_E))(m(p''x_E + p') + p' - c_S''(x_E))$. $LM - KN|_{\alpha=\alpha_1} < 0$ gives the followings.

$$\frac{dx_P}{d\alpha}|_{\alpha=\alpha_1} < 0, \tag{20}$$

$$\frac{dx_S}{d\alpha}|_{\alpha=\alpha_1} > 0. \tag{21}$$

We know that $SW_{P,S(\alpha_1)} - SW_E = m\gamma[c_S(x_E) - c_P(x_E)] - m(F_S - F_P)$ and investigate two cases.

(Case i)$F_P = F_S$.

First, if $\gamma > 0$, $SW_{P,S(\alpha_1)} - SW_E > 0$.

Second, if $\gamma = 0$, $SW_{P,S(\alpha_1)} - SW_E = 0$.

$\frac{dSW_{P,S(\alpha_1)}}{d\alpha} = (n\frac{dx_P}{d\alpha}|_{\alpha=\alpha_1} + m\frac{dx_S}{d\alpha}|_{\alpha=\alpha_1})[p - c_P'(x_E)] + m\frac{dx_S}{d\alpha}|_{\alpha=\alpha_1}\gamma[c_S'(x_E) - c_P'(x_E)]$.

From (18) - (21), $n\frac{dx_P}{d\alpha}|_{\alpha=\alpha_1} + m\frac{dx_S}{d\alpha}|_{\alpha=\alpha_1} > 0$. Therefore $\frac{dSW_{P,S(\alpha_1)}}{d\alpha} > 0$. There is a $\alpha > \alpha_1$ which can improve the social welfare, even if $\gamma = 0$.

(Case ii)$F_P < F_S$.

First, if $\gamma = 0$, $SW_{P,S(\alpha_1)} - SW_E < 0$.
Second, if $\gamma > 0$, $SW_{P,S(\alpha_1)} - SW_E = \gamma[c_S(x_E) - c_P(x_E)] - (F_S - F_P)$, which is positive if $(F_S - F_P)$ is small enough. Q.E.D.

Proof of proposition 4
First, we prove the low-income client as market model. If $SPC(= \beta x_{LS})$ is introduced and $\alpha = \alpha_1$, the payoff function of social enterprise j is given by

$$U_{S,j} = (1-\alpha)[p_H(X_H)x_{HS,j} - C(x_{HS,j} + X_{LS,j}) + \beta x_{LS,j}] + \alpha SV(X_L),$$

while the payoff function of for-profit enterprise i is not changed. The first order conditions (1) and (2) are not changed. But the first order condition (3) is changed to

$$-C'(x_{HS,j} + x_{LS,j}) + \beta + \frac{\alpha}{1-\alpha}SV'(X_L) = 0. \tag{22}$$

From the assumption that enterprises are identical, $x_{HP,1} = x_{HP,2} =, ..., = x_{HP,n} = x_{HP}$, $x_{HS,1} = x_{HS,2} =, ..., = x_{HS,m} = x_{HS}$, $x_{LS,1} = x_{LS,2} =, ..., = x_{LS,m} = x_{LS}$ in the equilibrium. Putting $X_H = nx_{HP} + mx_{HS}$ and $X_L = mx_{LS}$ into (1),(2), and (22), and differentiating them totally yields the followings.

$$\frac{dx_{HP}}{d\beta} = \frac{C''(x_{HS} + x_{LS})I}{J(GH - FI) - [C''(x_{HS} + x_{LS})]^2 F} \tag{23}$$

$$\frac{dx_{HS}}{d\beta} = -\frac{F}{G}\frac{dx_{HP}}{d\beta} \tag{24}$$

$$\frac{dx_{LS}}{d\beta} = \frac{GH - FI}{C''(x_{HS} + x_{LS})G}\frac{dx_{HP}}{d\beta} \tag{25}$$

$$\begin{aligned}\frac{dSW_{P,S(\alpha_1,\beta)}}{d\beta} &= n\frac{dx_{HP}}{d\beta}[p_H(nx_{HP} + mx_{HS}) - C'(x_{HP})] + m\frac{dx_{HS}}{d\beta}[p_H(nx_{HP} + mx_{HS}) - C'(x_{HS} + x_{LS})] \\ &+ m\frac{dx_{LS}}{d\beta}[(1+\eta)p_H(mx_{LS}) - C'(x_{HS} + x_{LS})]\end{aligned} \tag{26}$$

If $\alpha_1 < \alpha^*$ and $\beta = 0$, the increase of α improves social welfare.

$$\begin{aligned}\frac{dSW_{P,S(\alpha_1,\beta=0)}}{d\alpha} &= n\frac{dx_{HP}}{d\alpha}[p_H(nx_{HP} + mx_{HS}) - C'(x_{HP})] \\ &+ m\frac{dx_{HS}}{d\alpha}[p_H(nx_{HP} + mx_{HS}) - C'(nx_{HP} + mx_{HS})] \\ &+ m\frac{dx_{LS}}{d\alpha}[(1+\eta)p_H(mx_{LS}) - C'((nx_{HP} + mx_{HS}))] \\ &= n\frac{dx_{HP}}{d\alpha}[p_H(nx_{HP} + mx_{HS}) - C'(x_{HP})] \\ &- m\frac{F}{G}\frac{dx_{HP}}{d\alpha}[p_H(nx_{HP} + mx_{HS}) - C'(nx_{HP} + mx_{HS})] \\ &+ m\frac{GH - FI}{C''(x_{HS} + x_{LS})G}\frac{dx_{HP}}{d\alpha}[(1+\eta)p_H(mx_{LS}) - C'((nx_{HP} + mx_{HS}))] > 0\end{aligned} \tag{27}$$

From proof of proposition 1,

$$\begin{aligned}&[p_H(nx_{HP}+mx_{HS})-C'(x_{HP})]\\&-m\frac{F}{G}[p_H(nx_{HP}+mx_{HS})-C'(nx_{HP}+mx_{HS})]\\&+m\frac{GH-FI}{C''(x_{HS}+x_{LS})G}[(1+\eta)p_H(mx_{LS})-C'((nx_{HP}+mx_{HS}))]>0.\end{aligned} \tag{28}$$

From (26) and (28), $\frac{dSW_{P,S(\alpha_1,\beta=0)}}{d\beta}>0$.

Second, we prove the employment model. If $SPC(=\beta SV_{S,j})$ is introduced and $\alpha=\alpha_2$, the payoff function of social enterprise j is given by

$$U_{S,j}=(1-\alpha)[p(X)x_{S,j}-C(x_{S,j}]+[(1-\alpha)\beta+\alpha SV_{S,j}),$$

while the payoff function of for-profit enterprise i is not changed. The first order condition (15) is not changed. But (16) is changed to

$$(1-\alpha)[p'x_{S,j}+p-c'_S(x_{S,j})]+[(1-\alpha)\beta+\alpha](1+\gamma)[c'_S(x_{S,j})-c'_p(x_{S,j})]=0. \tag{29}$$

Using the assumption that enterprises are identical, putting $X_=nx_P+mx_S$ into (15) and (29), and differentiating them totally yields the followings where $O=mp''x_S+(m+1)p'-c''_S(x_S)+\frac{(1-\alpha)\beta+\alpha}{1-\alpha}(1+\gamma)[c''_S(x_S)-c''_P(x_S)]<0$.

$$\frac{dx_P}{d\beta}=-\frac{L}{LM-KO}SV'(x_S) \tag{30}$$

$$\frac{dx_S}{d\beta}=\frac{K}{LM-KO}SV'(x_S) \tag{31}$$

$$\frac{dSW_{P,S(\alpha_2,\beta)}}{d\beta}=n\frac{dx_P}{d\beta}[p-c'_P(x_P)]+m\frac{dx_S}{d\beta}[p(X)-c'_P(x_S)+\gamma(c'_S(x_S)-c'_P(x_S))] \tag{32}$$

If $\alpha_2<\alpha^*$ and $\beta=0$, the increase of α improves social welfare.

$$\begin{aligned}\frac{dSW_{P,S(\alpha_2,\beta=0)}}{d\alpha}&=n\frac{dx_P}{d\alpha}[p-c'_P(x_P)]+m\frac{dx_S}{d\alpha}[p-c'_P(x_S)+\gamma(c'_S(x_S)-c'_P(x_S))]\\&=-\frac{SV'(x_S)}{(1-\alpha_2)^2}n\frac{L}{LM-KN}[p-c'_P(x_P)]\\&+\frac{SV'(x_S)}{(1-\alpha_2)^2}m\frac{K}{LM-KN}[p-c'_P(x_S)+\gamma(c'_S(x_S)-c'_P(x_S))]>0.\end{aligned} \tag{33}$$

From $\frac{SV'(x_S)}{(1-\alpha_2)^2}>0$,

$$n\frac{L}{LM-KN}[p-c'_P(x_P)]+m\frac{K}{LM-KN}[p-c'_P(x_S)+\gamma(c'_S(x_S)-c'_P(x_S))]>0. \tag{34}$$

At $\beta = 0$, $N = O$. Substituting O with N in (32) yields

$$\begin{aligned} \frac{dSW_{P,S(\alpha_2,\beta=0)}}{d\beta} &= -n\frac{L}{LM-KN}SV'(x_S)[p - c'_P(x_P)] \\ &+ m\frac{K}{LM-KN}SV'(x_S)[p - c'_P(x_S) + \gamma(c'_S(x_S) - c'_P(x_S))]. \end{aligned} \tag{35}$$

From (34) and (35), $\frac{dSW_{P,S(\alpha_2,\beta=0)}}{d\beta} > 0$. Q.E.D.