The Dynamics of Parliamentary Bargaining and the Vote of Confidence

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Abstract

We develop a dynamic model of parliamentary policymaking in which three parties bargain over two-dimensional policies and transferable benefits. The model captures an important aspect of parliamentary systems: a failure of critical legislation leads to government dissolution. Policies are continuing, so the policy outcome in a period becomes the status quo for the next. We fully characterize a Markov perfect equilibrium for sufficiently patient political parties. In the equilibrium, once a government forms, it is never dissolved. The policy dynamics under the consensus coalition and minimal winning coalitions exhibit strong persistency and Pareto-efficiency among governmental parties. By contrast, under minority governments, the policy outcome oscillates between two points that do not belong to the Pareto set. In the government formation processes, only minimal winning coalitions are formed with positive probability, and a party that is disadvantaged by the status quo policy is likely to be included in the government.

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1 Introduction

In parliamentary democracies, survival of governments relies on the confidence of a majority in parliament. This defining feature of parliamentarism distinguishes strategic situation of policymaking in parliamentary democracies from that in presidential-congressional democracies. Specifically, a government in the former systems can fall prior to a required election either through the vote of confidence or through the vote of no confidence. This study investigates how this institutional feature shapes policymaking by analyzing a dynamic model that takes the possibility of government dissolution into account.

The most prominent observation about parliamentary democracies is that elections rarely produce a party that holds a majority of seats in parliament. Political parties then need to make a coalition in order to adopt any policy other than the current status quo. Thus, bargaining games with more than two agents (e.g. Baron and Ferejohn, 1989; Banks and Duggan, 2000, 2006) are needed to model policymaking in parliamentary democracies. However, the confidence vote procedure provides political agents with distinct incentives in bargaining, which is not reflected in most of the existing models. In congressional systems, the consequence of a failure in important legislation is merely that the current status quo policy will remain in effect at least by the next legislative session. By contrast, rejecting a critical bill in parliamentary systems may result in a more serious situation. Provided that the vote of confidence is attached to the bill, a current government must resign if it fails to pass, which opens an opportunity to form a new government.

Then, strategic incentives of political parties in parliamentary systems differ from those in congressional systems. Most significantly, bargaining over policies would be a function of parties' expectation about which government would be formed if the current one were dissolved. If government positions are valuable for parties, members of the incumbent government may be willing to concede its policy preference in order to maintain their status as governmental parties. On the contrary, an opposition party may not easily accept policy proposals from the government to take an opportunity to be a member of the future government.

On the other hand, once a government falls, making a new government also relies on the coalition bargaining game among political parties. In parliamentary democracies, the executive mostly controls the legislative agenda. Thus, being a member of the government is instrumentally valuable even for political parties that are motivated solely by policy concerns. If parties cannot commit policy programs at the beginning of a new government, what parties do in government formation processes depends on their expectation about policymaking under alternative governments. Thus, the values of different governments are endogenous in the sense that they are derived from the policies that would be made once a government is invested. In sum, bargaining over government positions and bargaining over policies are necessarily interactive.

Hence, to study dynamics of political bargaining in parliamentary systems, it is essential to construct a model that includes both bargaining over policies and bargaining over governments. Such an effort is made by Baron (1998) and Diermeier and Feddersen (1998). This study takes approach of these papers, developing a game-theoretic model in which the game alternates back and forth between a policymaking stage and a government formation stage depending on previous bargaining outcomes.

This article seeks to deepen our understanding of the relationship between the two types of bargaining further than the existing studies by considering an important aspect of politics: endogeneity of status quo policies. We assume once a policy is adopted by parliament, it is remain in effect until another replaces it, which is in fact true in most policy areas. Under such an environment, policymaking in the current period affects not only the immediate payoffs for parties but also the strategic situation in the future and so the future payoffs. Moreover, since parties' policy choices may vary with the status quo, the value of each government also may vary with the status quo. Then, the optimal play in the government formation stage depends upon the status quo, which, in turn, will enter strategic consideration of policymakers. These complex strategic incentives of political parties are not captured by the existing studies with exogenous status quo policies.

The specific features of our model are summarized as follows. In each period when a ruling government is present, three political parties bargain to choose a policy in a two-dimensional Euclidean space and a distribution of private goods. Each party in the government is recognized to propose an alternative with equal probability. Then the proposal is subject to approval by the parliament. For a proposal to pass, it must be supported by a majority of parliament and by all members of the government. If the proposal passes, the same bargaining procedure is repeated with a new status quo policy. If the proposal fails to pass, then the status quo policy remains and the government is dissolved. Once a government is dissolved, parties bargain to form a new government. Each party is randomly selected as the formateur with probability one third, and the formateur party proposes a government (i.e., a coalition of parties). Then the proposed government is subject to a parliamentary vote. If it is accepted by a majority of parliament and all members of the proposed government, then the game moves onto policymaking stage, and otherwise, the same procedure of government formation is repeated.

We fully characterize a Markov perfect equilibrium for the game with patient players. In the equilibrium, every government is stable; once formed, no government is dissolved. In the government formation stage, no delay occurs, and only minimal winning governments are formed. Which minimal winning coalition is more likely to be formed depends on the status quo policy. If one party is disadvantaged by the status quo policy relative to another, then the former is more likely to be a member of the government than the latter. Thus, the minimal winning coalition that consists of the two most disadvantaged parties is most likely to be formed.

The dynamics of policy outcomes vary across different types of governments, but each government exhibits a simple dynamic of policy choices in the long run. The consensus government always implements a single policy, the average of the ideal policies of all parliamentary parties. The sequence of policy outcomes under any minimal winning coalition converges, within one period, to a single point, the midpoint of the contract curve for the governmental paries. On the other hand, the policy dynamics under minority governments show inefficiency due to inter-temporal tradeoffs in the governmental party's strategic consideration. Overall, the policy dynamics are more stable and efficient than those found by Fong (2006), who study the same setting without the vote of confidence. This suggests that the confidence vote institution enables parliamentary parties to bargain efficiently.

As mentioned earlier, Baron (1998) and Diermeier and Feddersen (1998) analyze formal models similar to our model. Diermeier and Feddersen (1998) develop a finite-period game where bargaining alternates between government formation and policymaking. Their model deals with a distributive setting, where parties bargain over allocations of a 'dollar.' They find that the vote of confidence procedure creates cohesive voting among the government members and allows the government collectively to capture more of the distributive benefits from the bargaining process. Baron (1998) extends this framework to an infinite horizon, and finds similar results. In contrast to ours, a status quo is randomly drawn in each period in these models. Thus, these models investigate how a random event affects stability of coalition governments to expense the dynamic property of policy-making, while we take the opposite approach in this study.

The bargaining protocol we use in this study is first developed by Baron and Ferejohn (1989) and extensively studied by Banks and Duggan (2000, 2006) in a general setting. In these models, interaction among players ceases once an agreement is reached, and the reversion point is exogenously fixed over time. Hence, their models are more applicable to study bargaining in parliament within one session while our model is applied to multiple legislative sessions in the time between elections by simplifying the bargaining procedure in each session.

Government formation in parliamentary systems has been paid much scholarly attention to. Most theoretical studies approach this topic by analyzing one stage models with the assumptions of either purely office motivated parties (Riker, 1962), purely policy motivated parties (Schofield, 1993*a*,*b*, 1995), or mixed motivated parties (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001; Sened, 1996). Beyond the static approach, Baron (1991) and Kalandrakis (2010*b*) study government formation by developing infinite-horizon games where bargaining over governments is repeated if the agreement is not reached in a given period. In all of these models, a government is interpreted as either an agreed upon policy, or a distribution of a fixed prize, or both. By contrast, in this study a government is a distribution of policymaking power, and policies and distributions of benefits are decided after a government is formed. Thus, in contrast to the above studies, we consider an environment where policy commitment is impossible at the time of investiture of governments.

This approach is also taken by Austen-Smith and Banks (1990) and Laver and Shepsle (1990, 1996). In these studies, a government is an allocation of different cabinet positions to political parties. Policy commitment is assumed to be impossible, and a party that holds a particular cabinet position is regarded as a dictator of policymaking in a corresponding policy area. As a result, they find that policy outcomes generically are not in the Pareto set for the governmental parties. By contrast, we abstract away from the qualitative differences between government positions in the model, and we require an enacted policy to be subject to an agreement in each period. Thus, unlike Austen-Smith and Banks (1990) and Laver and Shepsle (1990, 1996), we implicitly assume that the collective government or parliament is able to monitor cabinet ministers so that legislation in all jurisdictions can be negotiated at the same time.

Our study also contributes to the literature of dynamic legislative bargaining with endogenous status quo policies, which has been growing remarkably. Assuming that policies continue, different aspects of dynamic bargaining are investigated by various studies (Baron, 1996; Battaglini and Coate, 2008; Cho, 2012*b*,*a*; Diermeier and Fong, 2011; Duggan and Kalandrakis, 2012; Fong, 2006; Jeon, 2012; Kalandrakis, 2004, 2010*a*; Nunnari, 2011; Penn, 2009). Our model departs from these models by incorporating the vote of confidence procedure. Under this institutional feature, a vote for a policy is at the same time a vote for a current government. Thus, while bargainers compare a currently proposed policy to continuing the same game with a status quo in the existing models, they compare a policy proposal under a current government to dissolving a government in our model. This institutional difference is reflected by the difference between findings in this study and those in Fong's (2006) paper, which will be discussed in detail later.

The rest of this paper is organized as follows. Section 2 develops the model. Section 3 presents the findings from the analysis of the model followed by a concluding section. Formal proofs are contained in the Appendix.

2 Model

2.1 The Game

We consider an infinite period bargaining game where the players are three political parties in a parliament, $P = \{1, 2, 3\}$. The collection of all subsets of P is denoted by 2^P, and the collection of all parliamentary coalitions is $\Omega = 2^P \setminus \{\emptyset\}$.

An outcome of the game at $t = 1, 2, \ldots$ is a pair (x^t, g^t), where x^t is a public policy and $g^t = (g_1^t, g_2^t, g_3^t)$ is a distribution of non-policy benefits. The set of public policies is a two-dimensional disk $X = \{x \in \mathbb{R}^2 | \, ||x|| \leq d\}$. Let $G \geq 0$ be the size of total non-policy benefits. Let $\mathcal{G} = \{g \in \mathbb{R}^3 \mid \sum_{i \in P} g_i = G\}$ be the set of all distributions of G. Note that g_i may be a negative number. Each $g \in G$ may be interpreted as (re)allocation of patronage positions or side payment using resources that are considered as private goods from the parties' perspective.

Each party i is endowed with an ideal policy $\tilde{x}^i \in X$ and a quadratic loss function $u_i : X \to \mathbb{R}$ given by $u_i(x) = -||x - \tilde{x}^i||^2$. We focus on symmetric preferences assuming that the ideal points of the parties form a unit equilateral triangle; for all $i, j \in P$, $||\tilde{x}^i - \tilde{x}^j|| = 1$. The locations of ideal points in the policy space are normalized so that the centroid of the triangle is the origin; $\frac{1}{3}\sum_{i \in P} \tilde{x}^i = (0, 0)$. We assume $d \geq \sqrt{3}$; the policy space is large relative to the distances between parties.[1] Each party i's stage utility from a outcome $(x^t, y^t) \in X \times \mathcal{G}$ is $u_i(x^t) + g_i^t$.

The sequence of bargaining in a given period t depends on the *state* in the period, denoted by s^t. A state consists of two components: a current status quo policy and a current government (including a state with no government). Let $S = X \times 2^P$ be the set of all possible states. A period t is called a *policy period with status quo x and government C* if $s^t = (x, C)$ for a nonempty C, and the bargaining in period t takes place as follows. First, each party in the current government ($i \in C$) is selected as a proposer with probability $\frac{1}{|C|}$. That is, proposal power is monopolized by the government and is equally distributed among the members of the government. Second, the selected party proposes a policy and a distribution of G, say $(y, g) \in X \times \mathcal{G}$. Third, all parties simultaneously vote to either accept or reject the proposal. For a proposal to pass, the following conditions must be satisfied; (1) it must obtain a majority of votes in parliament (votes by at least two parties); (2) all parties in C must accept the proposal; (3) if $g_i < 0$, then i must accept the proposal. If proposal (y, g) passes, then the outcome at t is (y, g) and the next period becomes a policy period with status quo y and government C, i.e., $s^{t+1} = (y, C)$. Otherwise, the outcome at t is $(x, (0, 0, 0))$ and the government falls, and hence the next period becomes an *organization*

[1] Our findings will be unchanged if we assume the policy space X is an arbitrary convex, compact set in $\mathbb{R}^2$ that contains the disk $\{x \in \mathbb{R}^2 | \, ||x|| \geq \sqrt{3}\}$. However, letting X be a disk substantially shorten the presentation and the proofs of the results.

period with status quo x; that is, $s^{t+1} = (x, \emptyset)$.

It is worthwhile to elaborate on the three conditions for a proposal to pass. With the assumption that no party holds a majority of seats in parliament, the first condition follows from the majority voting rule that is prevalent in almost all parliamentary democracies. The rationale for the second condition is that a governmental party can always voluntarily resign. If there is a serious disagreement on policy programs among members of the government and, as a result, a governmental party prefers dissolving the current government to keeping it, the government will fall by resignation of the discontented member. Since all proposals in the model are interpreted as critical bills with which the confidence motion is attached, we consider rejection by a government party as resignation that leads to a dissolution of the government. The third condition is reasonable because a negative transfer to a party is possible only by extracting some private goods from the party, which must be agreed by the party.

In any organization period t, parties bargain over compositions of the government, i.e., proposal power in the next policy period. The bargaining is described as follows. First, each party is selected as the *formateur* with probability $\frac{1}{3}$. Second, the selected formateur proposes a government $C \in \Omega$. Third, all parties simultaneously vote to either accept or reject the proposed government. If at least two parties in the parliament and all parties in the proposed government accept the proposal, then the government is formed and the next period becomes a policy period, i.e., $s^{t+1} = (x, C)$. Otherwise, the next period remains an organization period with the same status quo, i.e., $s^{t+1} = (x, \emptyset)$. No policy-making activity occurs in an organization period, and therefore, $(x^t, g^t) = (x, (0, 0, 0))$. The initial state $s^1 \in S$ is exogenously determined before the game begins.

The payoff for party i from the sequence of outcomes (x^t, g^t) is

$$(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} [u_i(x^t) + g_i^t],$$

where $\delta \in (0, 1)$ is a common discount factor.

2.2 Strategies and Equilibrium Concept

We analyze this game for *Markov perfect equilibria.* A Markov strategy is a strategy that prescribes the same action in any pair of periods t and t' if the states of both periods are identical and previous actions within the periods are identical. In other words, each player's action in every period t depends only on s^t and the event that occurred at t; and moreover, if $s^t = s^{t'}$, then actions at t and t' are identical. In the context of our game, proposals depend only on the current state and votes depend only on the current state and the current proposal. This type of equilibria is relatively tractable and may have a focal quality due to its simplicity.

A Markov strategy for party i is a pair (π_i, A_i) where π_i is a proposal strategy and A_i is a voting strategy. Let $\Theta = (X \times \mathcal{G}) \cup \Omega$, and let $\mathcal{P}(\Theta)$ be the set of probability measures on Θ. Generally, a proposal strategy for i is a mapping $\pi_i : S \to \mathcal{P}(\Theta)$. Without delving into measurability issues, it is sufficient for the purpose of our analysis to assume that π_i has finite support for every state. Then, for each $s \in S$ and each $\theta \in \Theta$, let $\pi_i(\theta|s)$ denote the probability that party i proposes θ conditional on the party is selected at state s. For any subset $\Theta' \subseteq \Theta$, let $\pi_i(\Theta'|s) = \sum_{\theta \in \Theta'} \pi_i(\theta|s)$ be the probability that i's proposal at state s belongs to Θ'. We need the restriction that, for all $x \in X$, $\pi_i(\Omega|x, \emptyset) = 1$ and if $C \neq \emptyset$, then $\pi_i(X \times \mathcal{G}|x, C) = 1$. That is, parties propose governments in organization periods and policies in policy periods. A voting strategy for party i is an acceptance correspondence $A_i : S \twoheadrightarrow \Theta$. For each $s \in S$, $A_i(s)$ consists of proposals party i would accept if proposed at state s. Any acceptance correspondence must satisfy that, for all $x \in X$, $A_i(x, \emptyset) \subseteq \Omega$, and if $C \neq \emptyset$, then $A_i(x, C) \subseteq X \times \mathcal{G}$. A profile of Markov strategies of the game is denoted by $\sigma = (\pi_i, A_i)_{i \in P}$.

For each state $s \in S$, a strategy profile σ generates a probability distribution on the sequences of outcomes from any period given that the current state is s. With this, each party has the *ex ante* expected payoff conditional on the current state being s. We refer to it as the *continuation value* for party i at s in σ and denote it by $v_i^\sigma(s)$. The set of passable proposals at s in σ, denoted by $A^\sigma(s)$, is well defined from the voting strategies in σ. From the voting rule we specified, A^σ is described as follows: (1) For all $x \in X$, for all $C \in \Omega$, and for all $(y, g) \in X \times \mathcal{G}$, $(y, g) \in A^\sigma(x, C)$ if and only if $|\{i \in P|(y, g) \in A_i(x, C)\}| \geq 2$ and $C \cup \{i \in P|g_i < 0\} \subseteq \{i \in P|(y, g) \in A_i(x, C)\}$. (2) For all $x \in X$, $C \in A^\sigma(x, \emptyset)$ if and only if $|\{i \in P|C \in A_i(x, \emptyset)\}| \geq 2$ and $C \subseteq \{i \in P|C \in A_i(x, \emptyset)\}$.

Given that proposal strategies have finite support, the continuation values at policymaking states satisfy the following. For every $i \in P$, every $x \in X$ and every $C \in \Omega$,

$$
\begin{aligned}
v_i^\sigma(x, C) &= \sum_{j \in P} \sum_{(y,g) \in A^\sigma(x,C)} \pi_j(y, g|x, C)\Big[(1-\delta)(u_i(y) + g_i) + \delta v_i^\sigma(y, C)\Big] \qquad (1)\\
&\quad + \Big[1 - \sum_{j \in P} \pi_j(A^\sigma(x, C)|x, C)\Big]\Big[(1-\delta)u_i(x) + \delta v_i^\sigma(x, \emptyset)\Big]. \qquad (2)
\end{aligned}
$$

The expression (1) represents the case in which a proposal passes in parliament, and, thus, party i receives payoff from the proposal in the current period and the continuation payoff from the same government with the proposal being the status quo in the next period. The second part represents the case of rejected proposals in which party i receives payoff from the status quo x in the current period, and the next period becomes an organization period.

In an organization period with status quo x, the current payoff for party i is $u_i(x)$ since there is no policymaking. If a proposed government is formed, then party i receives $v_i^\sigma(x, C)$ in the next period. If a proposed government is rejected in the parliament, then it receives the continuation payoff $v_i^\sigma(x, \emptyset)$ at the same organization state in the next period. Hence, for every $i \in P$ and every $x \in X$,

$$\begin{aligned} v_i^\sigma(x,\emptyset) &= (1-\delta)u_i(x) + \delta \sum_{j\in P} \sum_{C\in A^\sigma(x,\emptyset)} \pi_j(C|x,\emptyset)v_i^\sigma(x,C) \\ &\quad +\delta[1-\pi(A^\sigma(x,\emptyset)|x,\emptyset)]v_i^\sigma(x,\phi). \end{aligned} \tag{3}$$

We now discuss the conditions for a strategy profile to be an equilibrium. First, let us consider voting strategies. For each state $s = (s_1, s_2) \in S$, let $R_i^\sigma(s) = (1-\delta)u_i(s_1) + \delta v_i^\sigma(s)$. Suppose, in any policy period t with the state (x, C), a proposer proposes a policy y and a distribution of the benefit g. If the proposal is accepted in parliament, then the outcome in the current period will be (y, g) and the state of the next period will be (y, C). Thus, party i expects to receive payoff

$$(1-\delta)[u_i(y) + g_i] + \delta v_i^\sigma(y,C) = (1-\delta)g_i + R_i^\sigma(y,C). \tag{4}$$

If the proposal fails to pass, then the status quo x remains in effect in the current period and the benefits will not be distributed. Moreover, the government must be dissolved and thus the next period will be an organization period. Then the expected payoff for party i is

$$(1-\delta)u_i(x) + \delta v_i^\sigma(x,\emptyset) = R_i^\sigma(x,\emptyset), \tag{5}$$

which is party i's *reservation value.* Then, the party weakly prefers passing (y, g) to rejecting it if and only if

$$g_i \geq \frac{R_i^\sigma(x,\emptyset) - R_i^\sigma(y,C)}{1-\delta}. \tag{6}$$

We say a voting strategy A_i is *undominated at* (x, C) *in* σ if

$$A_i(x, C) = \left\{ (y, g) \in X \times \mathcal{G} \,\middle|\, g_i \geq \frac{R_i^\sigma(x, \emptyset) - R_i^\sigma(y, C)}{1 - \delta} \right\}. \tag{7}$$

The condition (7) is equivalent to ruling out stage weakly dominated voting strategies with the additional assumption that indifferent parties accept proposals.

In any organization period with status quo x, if a proposed government C passes in parliament, party i expects to receive the payoff $(1-\delta)u_i(x) + \delta v_i^\sigma(x, C)$. If C is rejected by a parliamentary majority, the i's expected payoff is $(1-\delta)u_i(x) + \delta v_i^\sigma(x, \emptyset)$. We say a voting strategy A_i is *undominated at* $(x, \emptyset)$ *in* σ if

$$A_i(x, \emptyset) = \{C \in \Omega |\; v_i^\sigma(x, C) \geq v_i^\sigma(x, \emptyset)\}. \tag{8}$$

Now let us consider proposal strategies in any policy period with status quo x and government C. Suppose a member of C, say i, is selected as a proposer. If party i proposes any $(y, g) \in A^\sigma(x, C)$, the proposal will pass and the state of the next period will become (y, C). Hence, the expected payoff from proposing $(y, g) \in A^\sigma(x, C)$ is $(1-\delta)[u_i(y) + g_i] + \delta v_i^\sigma(y, C)$, which is equal to $(1-\delta)g_i + R_i^\sigma(y, C)$. If the party proposes any alternative that is not passable, the status quo policy is maintained and the next period will become an organization period. Then the expected payoff from proposing any $(y, g) \notin A^\sigma(x, C)$ is simply $R_i^\sigma(x, \emptyset)$. To maximize its expected payoff, party i would propose an alternative that maximizes $(1-\delta)g_i + R_i^\sigma(y, C)$ among the passable proposals if there is an alternative in $A^\sigma(x, C)$ that gives payoff greater than $R_i^\sigma(x, \emptyset)$. And if all alternatives in $A^\sigma(x, C)$ give payoff less than $R_i^\sigma(x, \emptyset)$, the party would propose any alternative that will not pass in order to dissolve the government. Formally, let $\overline{A}_i^\sigma(x, C) = \arg\max_{(y,g) \in A^\sigma(x,C)} [(1-\delta)g_i + R_i^\sigma(y, C)]$ for each $(x, C) \in X \times \Omega$ and each $i \in P$. We say a proposal strategy π_i is *sequentially rational at* (x, C) *in* σ if it satisfies the following:(1) if $\sup\{(1-\delta)g_i + R_i^\sigma(y, C) | (y, g) \in$

$A^\sigma(x,C)\} > R_i^\sigma(x,\emptyset)$, then $\pi_i(\overline{A}_i^\sigma(x,C)|x,C) = 1$; and (2) if the inequality is reversed, then $\pi_i(A^\sigma(x,C)|x,C) = 0$.

Lastly, we discuss proposal strategies in organization periods. Suppose the status quo is x and party i is designated as the formateur. Regardless of its government proposal, party i will receive utility from x in the current period. If party i proposes a government C that is acceptable by a parliamentary majority and every member of C, then i will receive payoff $v_i^\sigma(x,C)$ in the next period. And if the party proposes a government that is acceptable by no majority, then its expected payoff from the next period will be $v_i^\sigma(x,\emptyset)$. We say a proposal strategy π_i is *sequentially rational at* $(x,\emptyset)$ *in* σ if the following holds: (1) if $\max\{v_i^\sigma(x,C)|C \in \Omega\} > v_i^\sigma(x,\emptyset)$, then $\pi_i(\arg\max_{C\in A^\sigma(x,\emptyset)} v_i^\sigma(x,C)|x,C) = 1$; and (2) if the inequality is reversed, then $\pi_i(A^\sigma(x,\emptyset)|x,C) = 0$. We define our solution concept as follows.

Definition 1 A profile of strategies $\sigma = (\pi_i, A_i)_{i\in P}$ is a *Markov perfect equilibrium* (MPE) if, for every i and every $s \in S$, A_i is undominated at s in σ, and π_i is sequentially rational at s in σ.

2.3 Preliminary Analysis

In this section, we provide the essential logic of optimal policymaking strategies, which helps to understand the results in the next section. Suppose a party proposes a proposal that is accepted in parliament. When a policy proposal passes, there is a set of parties that accept the proposal, which we call a *policy coalition* following the terminology by Diermeier and Feddersen (1998). A policy coalition does not have to be identical to the current governing coalition. For example, a minority government has to find a majority coalition that will accept the government proposal. However, a proposer must find a policy coalition that contains the government coalition in order to prevent any coalition partner to resign from the government. For each government coalition $C \in \Omega$, define the set of all possible policy coalitions by

$$\Omega(C) = \{D \in \Omega | C \subseteq D \text{ and } |D| \geq 2\}.$$

Parties' choices of proposals in policy periods can be understood as a two-step decision making: choosing a policy coalition and choosing a policy. Consider the situation where the government coalition is C and the status quo policy is x. Suppose that proposal (y, g) by proposer i is accepted by policy coalition $D \in \Omega(C)$. Note that any party that is not in the policy coalition has a zero share of G. Thus, the proposer's share of G is $g_i = G - \sum_{j \in D \setminus \{i\}} g_j$. Then, the proposer's payoff is

$$(1-\delta)\left[G - \sum_{j \in D \setminus \{i\}} g_j\right] + R_i^\sigma(y, C). \tag{9}$$

Since party i maximizes its own payoff under the constraint that the members of the policy coalition accept the proposal, the condition in (6) must be binding. Thus, for every coalition partner $j \in D \setminus \{i\}$,

$$g_j = \frac{R_j^\sigma(x, \emptyset) - R_j^\sigma(y, C)}{1-\delta}. \tag{10}$$

That is, all members of the policy coalition except the proposer receives payoff $R_j^\sigma(x, \emptyset)$; their payoffs are equal to their reservation values. Substituting the right hand side of equation (10) for g_j in (9), we see that the proposer i's payoff is equal to

$$(1-\delta)G + \sum_{j \in D} R_j^\sigma(y, C) - \sum_{j \in D \setminus \{i\}} R_j^\sigma(x, \emptyset). \tag{11}$$

Thus, the policy proposal must maximize the sum of (far-sighted) policy payoffs of all parties in the policy coalition. That is,

$$y \in \arg\max_{z \in X} \sum_{j \in D} R_j^\sigma(z, C). \tag{12}$$

Thus, when $\sum_{j \in D} R_j^\sigma(z, C)$ has a unique maximizer for each $D \in \Omega(C)$, the policy proposal does not depend on who the proposer is if a same policy coalition is chosen.

Next, the proposer can choose any policy coalition D under the constraint that $D \in \Omega(C)$. Among the possible policy coalitions, the proposer chooses a coalition that maximizes the payoff specified in (11). Thus, in equilibrium, the policy coalition D must be such that

$$D \in \arg\max_{B \in \Omega(C)} \left[\max_{z \in X} \sum_{j \in B} R_j^\sigma(z, C) - \sum_{j \in B \setminus \{i\}} R_j^\sigma(x, \emptyset) \right]. \tag{13}$$

Thus, if D satisfies (13) and y satisfies (12), choosing policy coalition P and proposing y is at least as good as proposing any proposal in $A^\sigma(x, C)$. Then, the proposal strategy is sequentially rational at x in σ if the proposing party cannot increase its payoff by intentionally dissolving the government. That is,

$$(1 - \delta)G + \sum_{j \in D} R_j^\sigma(y, C) - \sum_{j \in D \setminus \{i\}} R_j^\sigma(x, \emptyset) \geq R_i^\sigma(x, \emptyset), \tag{14}$$

which is equivalent to

$$(1 - \delta)G + \sum_{j \in D} R_j^\sigma(y, C) \geq \sum_{j \in D} R_j^\sigma(x, \emptyset) \tag{15}$$

From (10) and (11), once we know i's chosen policy coalition and policy proposal, we can identify the distribution of transferable benefits. That is, given policy proposal y, status quo x, and policy coalition D, let $g^i(y, x|\sigma)$ is such that: for every $j \in D \setminus \{i\}$,

$g_j^i(y,x|\sigma) = \frac{R_j^\sigma(x,\emptyset)-R_j^\sigma(y,C)}{1-\delta}$; for every $k \in P \setminus D$, $g_k^i(y,x|\sigma) = 0$; and $g_k^i(y,x|\sigma) = G - \sum_{j\in D\setminus\{i\}} g_j^i(y,x|\sigma)$. Then, with an abuse of notation, we use the following abbreviation: $\pi_i(y,D|x,C) \equiv \pi_i(y,g^i(y,x|\sigma)|x,C)$. That is, $\pi_i(y,D|x,C)$ is the probability that i chooses policy coalition C and policy y conditional on the current state being (x,C). Our first lemma summarizes what we have discussed.

Lemma 1 *Let $\sigma = (\pi_i, A_i)_{i\in L}$ and let $C \in \Omega$ be an arbitrary government. Assume that*

$$\max_{B\in\Omega(C)} \left(\max_{z\in X} \sum_{j\in B} R_j^\sigma(z,C) - \sum_{j\in B} R_j^\sigma(x,\emptyset) \right) \geq 0 \tag{16}$$

and that A_i is undominated at (x,C) in σ. If, for every $(y,D) \in X\times\Omega(C)$ with $\pi_i(y,D|x,C) > 0$,

$$y \in \arg\max_{z\in X} \sum_{j\in D} R_j^\sigma(z,C). \tag{17}$$

and

$$D \in \arg\max_{B\in\Omega(C)} \left[\max_{z\in X} \sum_{j\in B} R_j^\sigma(z,C) - \sum_{j\in B\setminus\{i\}} R_j^\sigma(x,\emptyset) \right], \tag{18}$$

then π_i is sequentially rational at (x,C) in σ.

3 Results

In this section, we fully characterize a MPE of the game for large enough discount factor δ. In the next proposition, we present the equilibrium in terms of the long-run outcomes in it. Before presenting the result, we need to define the concepts of transition probability and absorbing set. Given a strategy profile σ and a pair of states $s, s' \in S$, let $\zeta^\sigma(s'|s)$ denote the *transition probability from s to s' in σ*. For any subset $S' \subseteq S$, let $\zeta^\sigma(S'|s) = \sum_{s'\in S'} \zeta^\sigma(s'|s)$ be the probability that the state in the next period is in S' given that the current state is

s. We say a nonempty set $S' \subseteq S$ is an *absorbing set of* σ if, for all $s \in S'$, $\zeta^\sigma(S'|s) = 1$. In words, once the game reaches any state in an absorbing set S', it never leaves S'. Obviously, there is at least one absorbing set for every strategy profile since S itself is an absorbing set. We say S' is an *irreducible absorbing set of* σ if it is an absorbing set of σ and there is no proper subset of S' that is an absorbing set of σ.

For any coalition $C \in \Omega$, let $x^C = \frac{\sum_{i \in C} \tilde{x}^i}{|C|}$ be the average of all coalition members' ideal points. For any two-party coalition C, x^C is the midpoint of the two parties' ideal points, and x^P is the origin, the centroid of the triangle constituted by the three ideal points. Given the stage utility functions, for every coalition C, x^C is the unique maximizer of $\sum_{i \in C} u_i(x)$, i.e., the maximizer of the total (stage) policy utilities of the coalition. Given that we allow side payments in the bargaining, x^C is the unique efficient policy among the parties in C. Also, for each distinct pair of parties $i, j \in P$, let $y^{ij}(\delta) = \frac{2(\tilde{x}^i + \tilde{x}^j)}{4 - \delta(1+\delta)}$. The next proposition is our main result.

Proposition 1 *For some $\bar{\delta} \in (0,1)$, if $\delta \geq \bar{\delta}$, then there exists a MPE $\sigma = (\pi_i, A_i)_{i \in P}$ that satisfies the following:*

1. *For every $x \in X$, $\zeta^\sigma(x^P, P|x, P) = 1$.*
2. *For every $C \in \Omega$ with $|C| = 2$ and every $x \in X$, $\zeta^\sigma(x^C, C|x, C) = 1$.*
3. *For every $i \in P$, $\{j, k\} = P \setminus \{i\}$, and every $x \in X$, either $\zeta^\sigma(y^{ij}(\delta), \{i\}|x, \{i\}) = \zeta^\sigma(y^{ik}(\delta), \{i\}|x, \{i\}) = \frac{1}{2}$ or $\zeta^\sigma(x^P, \{i\}|x, \{i\}) = 1$.*
4. *For every $x \in X$, $\zeta^\sigma(x, \{1,2\}|x, \emptyset) + \zeta^\sigma(x, \{1,3\}|x, \emptyset) + \zeta^\sigma(x, \{2,3\}|x, \emptyset) = 1$.*
5. *For every $i \in P$ and every $x \in X$, $R_i^\sigma(x, \emptyset) = \frac{1}{3}[(1-\delta^2)\sum_{h \in P} u_h(x) + \delta^2(G - \frac{5}{4})]$.*
6. *There are seven irreducible absorbing sets of σ:*
   $\{(x^P, P)\}$, $\{(x^{\{1,2\}}, \{1,2\})\}$, $\{(x^{\{1,3\}}, \{1,3\})\}$, $\{(x^{\{2,3\}}, \{2,3\})\}$, $\{(y^{12}(\delta), \{1\}), (y^{13}(\delta), \{1\})\}$, $\{(y^{12}(\delta), \{2\}), (y^{23}(\delta), \{2\})\}$, *and* $\{(y^{13}(\delta), \{1\}), (y^{23}(\delta), \{3\})\}$.

The general characteristics of the equilibrium is as follows. First, every government is stable; once a government forms, it never fails to pass the vote of confidence. Second, policymaking under every government exhibits a certain degree of policy persistency. Once a game begins in a policy period, the game reaches a finite absorbing set. The equilibrium policymaking under the consensus government and each minimal winning coalition is strongly persistent: in the long run, each of the governments represents one policy. On the other hand, under each single party minority government, there are two distinct policies that alternate over time. Third, any organization state is transient. Once an organization period reaches, the parties instantly agree on the future government. No delay occurs in the government formation processes. Lastly, only minimal winning governments are formed in any organization periods.

In what follows, we discuss the properties of the MPE σ of Proposition 1 in detail, specifying strategies of the parties at different states.

3.1 The Consensus Government

First, we consider policymaking under the consensus government P. The equilibrium proposal strategy is such that: for every $i \in P$ and every $x \in X$, $\pi_i(x^P, P|x, P) = 1$. That is, every party proposes x^P, the efficient policy among the parties and chooses P as the policy coalition. Also, proposals are unanimously accepted.

Let x be an arbitrary status quo and consider the state $s = (x, P)$. Given the strategies, when party i is selected as the proposer, its payoff is

$$(1-\delta)G + \sum_{h \in P} R_h^{\sigma}(x^p, P) - \sum_{h \in P \setminus \{i\}} R_h(x, \emptyset) \tag{19}$$

from (11). Since the two other parties receive the reservation payoff $R_h(x, \emptyset)$, the sum of the continuation payoffs under the consensus government is

$$\sum_{h\in P} v_h^\sigma(x,P) = (1-\delta)G + \sum_{h\in P} R_h^\sigma(x^P,P). \tag{20}$$

Since this does not depend on x, $\sum_{h\in P} R_h^\sigma(x,P)$ is maximized at the point that maximizes the sum of all parties' stage utilities. Hence, x^P satisfies the condition (17) for the policy coalition P. Also, since P is the only available policy coalition for the consensus government, the condition (18) is satisfied.

Thus, by Lemma 1, the strategies are optimal if proposing x^P is preferred to dissolving the government. Substituting x^P for x in (20) and using the definition of R_h^σ, we obtain

$$\sum_{h\in P} R_h(x^P,P) = \sum_{h\in P} u_h(x^P) + \delta\left[(1-\delta)G + \sum_{h\in P} R_h^\sigma(x^P,P)\right], \tag{21}$$

and, thus,

$$\sum_{h\in P} R_h(x^P,P) = \delta G + \sum_{j\in P} u_h(x^P) = \delta G - 1. \tag{22}$$

Let $\bar{R}(x) = \frac{1}{3}[(1-\delta^2)\sum_{h\in P} u_h(x) + \delta^2(G-\frac{5}{4})]$. The fourth part of Proposition 1 states that, for every $h\in P$ and every $x\in X$, $R_h^\sigma(x,\emptyset) = \bar{R}(x)$. Then, since $\max_{x\in X}\sum_{h\in P} u_h(x) = -1$,

$$\max_{x\in X}\sum_{h\in P} R_h^\sigma(x,\emptyset) = \delta^2 G - (1-\delta^2) - \frac{5}{4}\delta^2 < \delta G - 1.$$

Hence, the condition (16) in Lemma 1 holds. Hence, the strategies are optimal.

We now derive the continuation values under the consensus government. Since all reservation payoffs are equal to $\bar{R}(x)$, when party i proposes, its payoff is

$$G - 1 - 2\bar{R}(x)$$

from (19) and (22). Party i receives this payoff with probability $\frac{1}{3}$ and it receives $R_i(x, \emptyset) = \bar{R}(x)$ when it does not propose. Thus,

$$v_i^\sigma(x, P) = \frac{1}{3}[G - 1]. \tag{23}$$

In sum, the consensus government always implements the efficient policy among all parties with respect to stage utilities. This finding extends the result of Baron and Diermeier (2001), who analyze a single period game with a bargaining protocol similar to the model in this paper. Notice that, once the game reaches state (x^P, P), it stays there forever. Thus, the singleton set $\{(x^P, P)\}$ is an irreducible absorbing set. Moreover, once the consensus government is formed, convergence to the absorbing state is obtained within one period. From then, only the distribution of G may vary across periods depending on the random recognition of the proposer. Finally, the sequence of outcomes generated by the consensus government is Pareto optimal. Since we allow side payments, the unique Pareto optimal policy sequence is the one in which x^P is chosen in every period. Hence, we may conclude that the consensus government is in fact consensual.

3.2 Minimal Winning Coalitions

Next, we discuss policymaking under any two party governments. Let $C = \{i, j\}$ and $k \in P \setminus C$. The equilibrium proposal strategies are such that, for every $x \in X$, $\pi_i(x^C, C|x, C) = \pi_j(x^C, C|x, C) = 1$. Thus, every government party in C chooses the governing coalition as the policy coalition and implements the efficient policy x^C among the governing parties for every status quo.

We first show that the strategies are optimal and then discuss the implications. Let $x \in X$ be an arbitrary status quo. Suppose i is selected as the proposer. Given the strategy, party i's payoff is, from (11),

$$(1-\delta)G + R_i^\sigma(x^C, C) + R_j^\sigma(x^C, C) - R_j^\sigma(x, \emptyset). \tag{24}$$

Since party j receives its reservation payoff $R_j^\sigma(x, \emptyset)$, the sum of the government parties' payoffs is $(1-\delta)G + R_i^\sigma(x^C, C) + R_j^\sigma(x^C, C)$. This will hold true when party j proposes. Hence,

$$v_i^\sigma(x, C) + v_j^\sigma(x, C) = (1-\delta)G + R_i^\sigma(x^C, C) + R_j^\sigma(x^C, C). \tag{25}$$

Since this does not depend on x, $R_i^\sigma(x, C) + R_j^\sigma(x, C)$ is maximized at the point that maximizes the sum of the two parties' stage payoffs. Therefore, x^C satisfies the condition (17) for policy coalition C.

We next derive the continuation values for the parties at (x, C). The equation (25) is true when $x = x^C$. Substituting x^C for x in (25) and using the definition of R functions, we obtain

$$R_i^\sigma(x^C, C) + R_j^\sigma(x^C, C) = (1-\delta)[u_i(x^C) + u_j(x^C)] + \delta[(1-\delta)G + R_i^\sigma(x^C, C) + R_j^\sigma(x^C, C)].$$

Then,

$$R_i^\sigma(x^C, C) + R_j^\sigma(x^C, C) = \delta G + u_i(x^C) + u_j(x^C) = \delta G - \frac{1}{2}.$$

From (24), when party i proposes, its payoff is

$$G - \frac{1}{2} - R_j^\sigma(x, \emptyset).$$

Since party i receives $R_i^\sigma(x, \emptyset)$ when it does not propose and party i is selected with probability $\frac{1}{2}$,

$$v_i^\sigma(x, C) = \frac{1}{2}[G - \frac{1}{2} + R_i^\sigma(x, \emptyset) - R_j^\sigma(x, \emptyset)].$$

From the fourth part of Proposition 1, $R_i^\sigma(x,\emptyset) = R_j^\sigma(x,\emptyset) = \bar{R}(x)$. Hence,

$$v_i^\sigma(x,C) = \frac{1}{2}[G - \frac{1}{2}] = v_j^\sigma(x,C), \tag{26}$$

where the last equality holds due to the symmetry.

Now consider the out party k. Whoever proposes, the policy x^C is implemented and party k receives no side payments, and the government C continues in power with the status quo x^C in the next period. Thus, for every $x \in X$,

$$v_k^\sigma(x,C) = (1-\delta)u_k(x^C) + \delta v_k^\sigma(x^C,C), \tag{27}$$

which does not depend on x. Since this is true when $x = x^C$, substituting x^C for x in the above equation and solving, we have that, for every $x \in X$,

$$v_k^\sigma(x) = u_k(x^C) = -\frac{3}{4}. \tag{28}$$

Since each party's continuation value under government C is constant across different status quo policies, $\sum_{h\in P} v_h^\sigma(x,C) = G - \frac{5}{4}$ does not depend on x. Hence, x^P is the unique maximizer of $\sum_{h\in P} R_h^\sigma(y,C)$. Since the available policy coalitions for government C are P and C, we have to show that each government party prefers the policy coalition C to P. If any proposer proposes x^C with policy coalition C, the payoff is

$$(1-\delta)G + R_i^\sigma(x^C,C) + R_j^\sigma(x^C,C) - \bar{R}(x). \tag{29}$$

If the proposer proposes x^P with policy coalition P, then the payoff is

$$(1-\delta)G + \sum_{h\in P} R_h^\sigma(x^P,C) - 2\bar{R}(x). \tag{30}$$

Then, the strategy is sequentially rational if and only if

$$\bar{R}(x) \geq \sum_{h \in P} R_h^\sigma(x^P, C) - R_i^\sigma(x^C, C) - R_j^\sigma(x^C, C). \tag{31}$$

From (26) and (28),

$$\sum_{h \in P} v_h^\sigma(x^P, C) - v_i^\sigma(x^C, C) - v_j^\sigma(x^C, C) = -\frac{3}{4}.$$

Also, $\sum_{h \in P} u_h(x^P) - u_i(x^C) - u_j(x^C) = -\frac{1}{2}$. Then, the RHS of (31) is equal to $-\frac{1}{2} - \frac{1}{4}\delta$. Then, the inequality (31) is equivalent to

$$\frac{1}{3}\left((1-\delta^2)\sum_{h \in P} u_h(x) + \delta^2[G - \frac{5}{4}]\right) \geq -\frac{1}{2} - \frac{1}{4}\delta \tag{32}$$

Since X is compact $\sum_{h \in P} u_h(x)$ is bounded. As δ approaches one, the LHS of (32) converges to $\frac{1}{3}G - \frac{5}{12}$, and the RHS converges to $-\frac{1}{2}$. Hence, for sufficiently high δ, the inequality (32) is true for every $x \in X$.

Lastly, $R_i^\sigma(x^C, C) + R_j^\sigma(x^C, C) = \delta G - \frac{1}{2}$, which is greater than $R_i^\sigma(x, \emptyset) + R_j^\sigma(x, \emptyset) = 2\bar{R}(x)$ for every $x \in X$. Thus, no government party wants to dissolve the government C. Therefore, the proposal strategies are sequentially rational.

Every minimal winning coalition government implements the midpoint of the contract curve for the government coalition regardless of the status quo policy when the discount factor is sufficiently high. Thus, with patient parties, the dynamics of policy outcomes is extremely simple under minimal winning coalitions as is the case under the consensus government. Once a minimal winning coalition is formed, the policy outcome moves to the midpoint of the governmental parties in one period, and stays there forever. The singleton set $\{(x^C, C)\}$ is an irreducible absorbing set. Although the consensus policy coalition is available for a minimal winning government, no proposer includes an out party as a member of the policy coalition. When the parties are sufficiently patient, the policy coalition is identical to the governing coalition. Moreover, the sequence of outcomes under any minimal winning coalition is Pareto optimal between the members of the government coalition.

This result is in contrast to the findings of previous models in which the vote of confidence procedure is not present. Baron and Diermeier (2001) study a single-period model of an election and parliamentary bargaining where the set of alternatives and the preferences of parties are the same as those in the current paper. In their result, when the status quo is extreme (far from the center of party preferences), the centroid x^P is adopted. In our model, this is not the case if the governing coalition is minimal winning and parties are very patient.

Fong (2006) studies an infinite-period version of Baron and Diermeier's model excluding elections but including the feature of endogenous status quo policies. In his model, each of the three parties is selected as a proposer with probability one third and makes a proposal that is subject to majority voting. Fong (2006) interprets interactions in the stage game as a government formation process in parliamentary democracies. However, we may interpret his model as a model of legislative policymaking in congressional systems, which is directly comparable to my model of parliamentary bargaining. Specifically, in his model the proposal right is equally distributed among all members of the legislature while only the members of the government have the proposal power in my model. This is one stylized difference between congressional and parliamentary systems. Moreover, in Fong's model, once a policy proposal does not pass, then the same policy bargaining is repeated. On the other hand, in my model, if a policy is not accepted in parliament, the government falls and a new government must form. This is another important difference between the two systems. Thus, we regard Fong's model as a model of a congressional legislature.[2]

Fong's finding is strikingly different from ours. In his equilibrium, when parties are

[2]Fong's own interpretation is of course plausible. In his interpretation, a period in the model is the time between two elections and he assumes election outcomes are relatively constant over time. When I regard his model as the one of congressional system, a period is interpreted as one legislative session.

patient, the policy coalition oscillates between minimal winning coalitions and the consensus coalition. That is, if a policy is adopted by a minimal winning coalition in one period, then the policy in the next period is passed by the consensus coalition. Once a policy is adopted by the consensus coalition, then the policy coalition in the next period is one of the minimal winning coalitions. Moreover, the policy dynamics is inefficient in Fong's equilibrium. While the consensus coalition implements the centrist policy, the policy choice by each minimal winning coalition is not on the contract curve of its members. The policy outcome recurrently goes out of the triangle constituted by the ideal points, the set of (stage game) Pareto optimal policies. This result shows that, when policies are continuing programs and thus farsighted parties are concerned with the current policy-making on the future bargaining, congressional bargaining may lead to inefficient outcomes. Baron, Diermeier and Fong (2012) also find a similar result in their two period model of elections and bargaining.

Such inefficiency does not occur under minimal winning governments in our model with the vote of confidence. The intuition of the difference is as follows. Consider a proposer i's strategic decision. The proposer needs at least one member, say j, as a policy coalition partner since support of a majority is needed. If the other party k's reservation value is sufficiently low, then proposer i wants to include the party as well so that it can extract some private goods from k. In Fong's model, if the status quo is far from the center, then k's reservation value is low, so party i forms the consensus policy coalition and implements the centrist policy. After that, the status quo is moderate, and so i wants to form a two-party coalition, say $\{i, j\}$. Considering only the current period payoff, party i may prefer to implement the midpoint of the ideal points of i and j, which maximizes the sum of the two parties' stage utilities. However, for the future payoffs, it is beneficial for i and j to make their future coalition partner k's reservation value as low as possible. Thus there is an incentive to agree to implement a policy that is even farther from k's ideal point than the efficient policy between i and j is. The actual equilibrium policy is determined by this tradeoff between the current payoff and the future payoff. As the future is more important for the parties, the degree of inefficiency becomes higher.

The critical distinction of our model from Fong's lies in the reservation values of the parties in policy bargaining. The reservation values depend on what would happen in the future once a proposal is rejected. In our model with the vote of confidence, rejecting a proposal leads the game to an organization period. Thus, if party k does not belong to the current governing coalition, the party likes to dissolve the current government hoping itself to be included in a new government. If the party is sufficiently concerned with the future, then such consideration outweighs the myopic policy payoff. Thus, the out party's reservation value is not as low as it needs to be for the consensus coalition to be formed. Moreover, as we will discuss later, if the status quo is relatively bad for party k, the party has a high chance to belong to the future government if the current one is dissolved. Thus, the capacity of the proposer to lower party k's reservation value in the next period by implementing an extreme policy is limited when there is a vote of confidence. Thus, our result implies the parliamentary structure of policymaking in which government dissolution is possible by a vote of confidence can prohibit inefficiency of policy dynamics that may occur otherwise.

3.3 Minority Governments

We discuss policymaking under single party governments. Let $\{i\}$ be an arbitrary minority government and let $P = \{i, j, k\}$. The equilibrium proposal strategies satisfies the following. There exists $\underline{d} \in (||y^{ij}(\delta)||, d]$ such that:

1. If $||x|| \leq \underline{d}$, then $\pi_i(y^{ij}(\delta), \{i, j\}|x, \{i\}) = \pi_i(y^{ik}(\delta), \{i, k\}|x, \{i\}) = \frac{1}{2}$.

2. If $||x|| > \underline{d}$, then $\pi_i(x^P, P|x, \{i\}) = 1$.

That is, the policy proposals vary across different status quo policies. On the one hand, when the status quo is relatively centrist (i.e., close to the centroid of the stage Pareto set), the government forms each minimal winning policy coalition with equal probability. On the other hand, when the status quo is relatively extreme, the government chooses P as the policy coalition. When the government chooses the consensus policy coalition, the centroid x^P is implemented. However, for centrist status quo policies, the policy outcome does not lie in the stage Pareto set. Notice that $y^{ij}(\delta) = \frac{2(\tilde{x}^i+\tilde{x}^j)}{4-\delta(1+\delta)}$ is the midpoint of the ideal points of i and j only when $\delta = 0$. As δ increases, $y^{ij}(\delta)$ moves farther from the ideal point of k keeping its distances from $\tilde{x}^i$ and $\tilde{x}^j$ equal to each other.

The sequential rationality of the proposal strategy is proved in the Appendix. Here, we examine the dynamics of policy outcomes generated by the strategy. Note that $\underline{d} > ||y^{ij}(\delta)|| = ||y^{ik}(\delta)||$. Hence, for each $y \in \{y^{ij}(\delta), y^{ik}(\delta)\}$, $\zeta^\sigma(y^{ij}(\delta), \{i\}|y, \{i\}) = \zeta^\sigma(y^{ik}(\delta), \{i\}|y, \{i\}) = \frac{1}{2}$, which implies that $\{y^{ij}(\delta), y^{ik}(\delta)\}$ is an irreducible absorbing set. If we begin with the status quo x with $||x|| \leq \underline{d}$, the absorbing state is reached within one period. After that, the two policies are implemented with equal probability. If the initial status quo x is far from the center of the policy space, the outcome in the first period is x^P. But then, the absorbing state is reached in the next period. In either case, the long-run distribution of the policy outcome places probability $\frac{1}{2}$ on each of $y^{ij}(\delta)$ and $y^{ik}(\delta)$.

Once a single minority government is formed, the governmental party does not have to worry about the other party's resignation. As such, a minority government has more discretionary power in choosing the policy coalition. That is, government i can choose party j or party k or both as the coalition partner. Instead of forming a stable coalition over time, minority governments rely on ad hoc policy coalitions in our equilibrium. This leads to an oscillation of policy outcomes and the overall inefficiency of policymaking. Hence, although minority governments are stable in the sense that they are never dissolved in equilibrium, policy choices under minority governments are not persistent in contrast to those under minimal winning and oversized governments.

3.4 Formation of Governments

We proceed to discuss government making strategies in organization periods. In the equilibrium, each formateur i proposes only minimal winning governments that contains party i itself with positive probability. That is, for every $x \in X$, $\pi_i(\{i,j\}|x,\emptyset) + \pi_i(\{i,k\}|x,\emptyset) = 1$. Thus, only minimal winning coalitions are formed if the game reaches an organization period.

For large enough δ, there is a continuum of government making strategies that support MPE with the characteristics in Proposition 1. We identify one in this section. Consider an organization period with an arbitrary status quo x, i.e., $s = (x, \emptyset)$. Let $P = \{i,j,k\}$ and without loss of generality assume $u_i(x) \geq u_j(x) \geq u_k(x)$. Let $\bar{u}(x) = \frac{\sum_{h \in P} u_h(x)}{3}$ denote the average of the parties' stage utilities from policy x. For each $h \in P$, let

$$\Delta_h(x) = \frac{6(1-\delta^2)[\bar{u}(x) - u_h(x)]}{\delta^2(G+1)}. \tag{33}$$

Since $\bar{u}(x)$ is the average stage utilities, $\sum_{h \in P} \Delta_h(x) = 0$. Also, $\Delta_k(x) \geq 0$, $\Delta_i(x) \leq 0$, and $\Delta_k(x) \geq \Delta_j(x) \geq \Delta_i(x)$. The magnitude of $\Delta_h(x)$, $|\Delta_h(x)|$, is a continuously decreasing function in δ. Since X is compact, $|\bar{u}(x) - u_h(x)|$ is bounded. Thus, as δ approach to one, $|\Delta_h(x)|$ converges to zero. Therefore, for sufficiently large δ, it must be the case that, for every $x \in X$, $\Delta_k(x) \leq 1$ and $\Delta_i(x) \geq -1$. From now on, we only consider such a large discount factor.

The government proposal strategy is the following:

1. $\pi_i(\{i,j\}|x,\emptyset) = \Delta_j(x)$ and $\pi_i(\{i,k\}|x,\emptyset) = 1 - \Delta_j(x)$.

2. $\pi_j(\{i,j\}|x,\emptyset) = 1 + \Delta_i(x)$ and $\pi_i(\{j,k\}|x,\emptyset) = -\Delta_i(x)$.

3. $\pi_k(\{j,k\}|x,\emptyset) = 1$ and $\pi_i(\{i,k\}|x,\emptyset) = 0$.

Given the ranges of $\Delta_i(x)$ and $\Delta_j(x)$, the strategies are well defined probability distributions.

We first assume that all government proposals are accepted and show that the strategies satisfy the fourth property of Proposition (1), i.e., for every $h \in P$, $R_h(x, \emptyset) = \bar{R}(x)$. For each $h \in P$, let $\mu(h|x)$ be the probability that party h belongs to the future government after the interaction at state $(x, \emptyset)$ in σ. We claim that for every $h \in P$,

$$\mu(h|x) = \frac{2}{3} + \frac{1}{3}\Delta_h(x). \tag{34}$$

Note that $\mu_h(x)$ is the probability that party h is selected as the formateur, $\frac{1}{3}$, plus the sum of the probabilities that the other parties include h in the proposed government coalition. Then,

$$\mu(i|x) = \frac{1}{3} + \frac{1}{3}(1 + \Delta_i(x)) = \frac{2}{3} + \frac{1}{3}\Delta_i(x),$$
$$\mu(j|x) = \frac{1}{3} + \frac{1}{3}(\Delta_j(x) + 1) = \frac{2}{3} + \frac{1}{3}\Delta_j(x),$$

and

$$\mu(k|x) = \frac{1}{3} + \frac{1}{3}(1 - \Delta_j(x) - \Delta_i(x)) = \frac{2}{3} - \frac{1}{3}(\Delta_j(x) + \Delta_i(x)) + = \frac{2}{3} + \frac{1}{3}\Delta_k(x),$$

as claimed.

From (26) and (28), a party's value of the minimal winning governments to which it belongs is $\frac{1}{2}(G - \frac{1}{2})$, and its value of the minimal winning government that excludes it is $-\frac{3}{4}$. For each $h \in P$, let $\bar{v}_h^\sigma(x)$ denote party i's expected payoff from the future government when the status quo policy is x. Then,

$$\bar{v}_h^\sigma(x) = \mu(h|x)\frac{1}{2}(G - \frac{1}{2}) + [1 - \mu(h|x)](-\frac{3}{4}). \tag{35}$$

By definition, $v_h^\sigma(x,\emptyset) = (1-\delta)u_h(x) + \delta\bar{v}_h^\sigma(x)$, and then

$$\begin{aligned} R_h^\sigma(x,\emptyset) &= (1-\delta^2)u_h(x) + \delta^2\bar{v}_i^\sigma(x) \\ &= (1-\delta^2)u_h(x) + \delta^2\Big(\mu(h|x)\frac{1}{2}(G-\frac{1}{2}) + [1-\mu(h|x)](-\frac{3}{4})\Big). \end{aligned} \tag{36}$$

Recall that

$$\bar{R}(x) = (1-\delta^2)\bar{u}(x) + \frac{1}{3}\delta^2(G-\frac{5}{4}). \tag{37}$$

Letting (36) equal to (37) and solving for $\mu(h|x)$, we obtain

$$\mu(h|x) = \frac{2}{3} + \frac{2(1-\delta^2)[\bar{u}(x)-u_i(x)]}{\delta^2(G+1)} = \frac{2}{3} + \frac{1}{3}\Delta_h(x), \tag{38}$$

which shows that, for every $h \in P$, $R_h^\sigma(x) = \bar{R}(x)$.

Next, we show that all government proposals are accepted. Party h accepts government C if and only if $v_h^\sigma(x,C) \geq v_h^\sigma(x,\emptyset)$, which is equivalent to $R_h^\sigma(x,C) \geq R_h^\sigma(x,\emptyset)$. Then, party h accepts any minimal winning government that includes h if and only if

$$(1-\delta)u_h(x) + \delta\left[\frac{1}{2}G - \frac{1}{4}\right] \geq (1-\delta^2)\bar{u}(x) + \delta^2\left[\frac{1}{3}G - \frac{5}{12}\right]. \tag{39}$$

Since $\frac{1}{2}G - \frac{1}{4} \geq \frac{1}{3}G - \frac{5}{12}$, the inequality holds for large enough δ. Hence, for patient parties, each minimal winning government is accepted by all members of the proposed government, which is a majority in parliament.

Lastly, we need to show that each formateur prefers minimal winning governments that includes the formateur to any other governments that can be formed. The value of the consensus government is $\frac{1}{3}(G-1)$, which is clearly less than the value of minimal winning coalitions $\frac{1}{2}(G-\frac{1}{2})$. A minority government may be accepted in parliament for some status quo policies. However, for large enough δ, each formateur prefers minimal winning governments to the minority government of its own. The proof is provided in the Appendix, in which we derive the continuation values under minority governments. The main intuition is that, although a party monopolizes proposal power under a minority government, the inefficiency of policy dynamics is costly enough for the value of the government to be lower than those of minimal winning governments. Hence, the government proposal strategies are sequentially rational.

Although there are multiple equilibrium government proposal strategies that generate the same payoffs, the condition (34) must hold in all such equilibria. Thus, the probability that a party h belongs to the government is strictly increasing in $\Delta_h(x)$. Therefore, a party that is disadvantaged by the status quo is more likely to be included in the government than another party that is relatively advantaged by the status quo. The minimal winning government formed by the two most disadvantaged parties are the most likely outcome. However, as δ converges to zero, $\Delta_h(x)$ converges to zero. Hence, with almost perfectly patient parties, all minimal winning coalitions are almost equally likely to be formed.

4 Conclusion

The model in this paper reflects two important institutional characteristics in bargaining under parliamentarism: the possibility of dissolution of governments and the effects of current policymaking on bargaining in the future. We have characterized a Markov perfect equilibrium for sufficiently high discount factors. We find that all types of governments are stable in equilibrium and that, moreover, in the government formation game, delay can never occur. Different governments incur different streams of policy outcomes. The consensus government always chooses the same policy, which is the efficient policy among all members of parliament. Minimal winning coalitions adopt the midpoint of the contract curve for the governmental parties invariably over time. Under minority government, the policy outcome oscillates and inefficiency occurs.

Our results about policy dynamics depart from those of Fong (2006). In his model without the vote of confidence procedure, the policy outcome recurrently reaches outside the triangle formed by the ideal points of parties. This difference is mainly from the incentive of political parties to remain in the government coalition that is present in our model but not in Fong's (2006). We only observe inefficient dynamics under minority governments which will not be formed on the equilibrium path if the game begins from a government formation stage. This result of the efficient legislative outcomes with the vote of confidence is consonant with the findings by Baron (1998), and Diermeier and Feddersen (1998), and Huber (1996). That is, we extend their results, showing that the vote of confidence enhances the stability in parliamentary bargaining when the bargaining mainly concerns continuing policy programs.

Also, the results about government making in our model differ from those of Baron (1998) and Diermeier and Feddersen (1998). In both of the studies, each minimal winning coalition are formed with probability one third. This is the case in our model only when the status quo is the centroid of the Pareto set. In their models, the current status quo only affects the immediate payoffs of political parties. And since a new status quo will be drawn in the next period, what only matters is the expectation of the future status quo, which is fixed over time. Thus, every organization period is identical to each other, resulting in the invariant prediction of the government outcome. By contrast, we show that which government will be formed depends on the status quo policy by studying a fully dynamic model. Specifically, the minimal winning coalition government composed by the two most disadvantaged parties is most likely to be the government outcome.

Our study cannot predict government dissolution, formation of minority or oversized government, which we sometimes observe empirically. The model here is stylized in many ways and thus is not intended to make predictions directly testable to empirical data. The main limitation of the model is that it does not capture the electoral incentives of political parties which may be important factors determining parties' strategic consideration whether to dissolve the current government. However, this study highlights the essential difference between congressional bargaining and parliamentary bargaining in a dynamic environment and thus takes a step further to a comprehensive study of parliamentary democracies.

A Appendix

Proof of Proposition 1

Let $\sigma = (\pi_i, A_i)_{i \in L}$ be the strategy profile defined in the text. We already have shown that the optimality of policymaking strategies under the consensus government and minimal winning governments in Sections 3.1 and 3.2. We will show the optimality of policymaking strategies under minority governments.

We consider policymaking at state $(x, \{i\})$. Given the strategies in Section 3.3., for every x with $||x|| \leq \underline{d}$,

$$v_i^\sigma(x, \{i\}) = (1-\delta)G + R_i^\sigma(y^{ij}(\delta), \{i\}) + R_i^\sigma(y^{ij}(\delta), \{i\}) - \bar{R}(x), \tag{40}$$

$$v_j^\sigma(x, \{i\}) = \frac{1}{2}[R_j^\sigma(y^{ik}(\delta), \{i\}) + \bar{R}(x)], \tag{41}$$

and

$$v_k^\sigma(x, \{i\}) = \frac{1}{2}[R_k^\sigma(y^{ij}(\delta), \{i\}) + \bar{R}(x)]. \tag{42}$$

Thus,

$$\sum_{h \in P} v_h^\sigma(x, \{i\}) = (1-\delta)G + \sum_{h \in P} R_h^\sigma(y^{ij}(\delta), \{i\}). \tag{43}$$

Note that $||y^{ij}(\delta)|| \leq d$. Substituting $y^{ij}(\delta)$ for x in (43) and using the fact that $R_h^\sigma(y^{ij}(\delta), \{i\}) = (1-\delta)u_h(y^{ij}(\delta)) + \delta v_h^\sigma(y^{ij}(\delta), \{i\})$ for every h, we obtain

$$\sum_{h \in P} R_h^\sigma(y^{ij}(\delta), \{i\}) = \delta G + \sum_{h \in P} u_h(y^{ij}(\delta)). \tag{44}$$

Substituting this into (43), we obtain that, for every x with $||x|| \leq \underline{d}$,

$$\sum_{h \in P} v_h^\sigma(x, \{i\}) = G + \sum_{h \in P} u_h(y^{ij}(\delta)). \tag{45}$$

For every x with $||x|| > \underline{d}$,

$$v_i^\sigma(x, \{i\}) = (1-\delta)G + \sum_{h \in P} R_h^\sigma(x^P, \{i\}) - 2\bar{R}(x) \tag{46}$$

and

$$v_j^\sigma(x, \{i\}) = v_k^\sigma(x, \{i\}) = \bar{R}(x). \tag{47}$$

Then,

$$\sum_{h \in P} v_h^\sigma(x, \{i\}) = (1-\delta)G + \sum_{h \in H} R_h^\sigma(x^P, \{i\}). \tag{48}$$

Since $||x^P|| = 0 < \underline{d}$, we obtain from (45)

$$\sum_{h \in P} R_h^\sigma(x^P, \{i\}) = (1-\delta) \sum_{h \in H} u_h(x^P) + \delta \left[G + \sum_{h \in P} u_h(y^{ij}(\delta)) \right]. \tag{49}$$

Then, from (48),

$$\sum_{h \in P} v_h^\sigma(x, \{i\}) = G + (1-\delta) \sum_{h \in P} u_h(x^P) + \delta \sum_{h \in P} u_h(y^{ij}(\delta)). \tag{50}$$

We will show that $x^P \in \arg\max_{x \in X} \sum_{h \in P} R_h^\sigma(x, \{i\})$. Note that $\sum_{h \in P} u_h(x) = -3||x||^2 - 1$. Thus, on any subset $Y \in X$, $\sum_{h \in P} u_h(x)$ is maximized when $||x||$ is minimized. This implies that $x^P \in \arg\max_{||x|| \leq \underline{d}} \sum_{h \in P} R_h^\sigma(x, \{i\})$. Take any x with $||x|| > \underline{d}$. From (45) and (50),

$$\sum_{h \in P} v_h^\sigma(x^P, \{i\}) - \sum_{h \in P} v_h^\sigma(x, \{i\}) = (1-\delta) \left[\sum_{h \in P} u_h(y^{ij}(\delta)) - \sum_{h \in P} u_h(x^P) \right]$$

Then,

$$\begin{aligned}&\sum_{h\in P} R_h^\sigma(x^P,\{i\}) - \sum_{h\in P} R_h^\sigma(x,\{i\})\\ &= (1-\delta)\left[\sum_{h\in P} u_h(x^P) - \sum_{h\in P} u_h(x) - \delta\Bigg(\sum_{h\in P} u_h(x^P) - \sum_{h\in P} u_h(y^{ij}(\delta))\Bigg)\right]\\ &> 0,\end{aligned}$$

where the last inequality use $||x|| > d \geq ||y^{ij}(\delta)||$. Hence, $x^P \in \arg\max_{x\in X} \sum_{h\in P} R_h^\sigma(x,\{i\})$.

We next show that $y^{ij}(\delta) \in \arg\max_{x\in X}(R_i^\sigma(x,\{i\})+R_j^\sigma(x,\{i\})$. Take any x with $||x|| \leq \underline{d}$. Subtracting (42) from (45), we have

$$v_i^\sigma(x,\{i\}) + v_j^\sigma(x,\{i\}) = G + \sum_{h\in P} u_h(y^{ij}(\delta)) - \frac{1}{2}[R_k^\sigma(y^{ij}(\delta),\{i\}) + \bar{R}(x)]. \quad (51)$$

Then,

$$\begin{aligned}&R_i^\sigma(x,\{i\}) + R_j^\sigma(x,\{i\})\\ &= (1-\delta)[u_i(x)+u_j(x)] + \delta\Bigg(G + \sum_{h\in P} u_h(y^{ij}(\delta)) - \frac{1}{2}[R_k^\sigma(y^{ij}(\delta),\{i\}) + \bar{R}(x)]\Bigg). \quad (52)\end{aligned}$$

Note that, in the RHS of (52), only $[u_i(x)+u_j(x)]$ and $\bar{R}(x)$ depend on x. Also, recall that $\bar{R}(x) = \frac{1}{3}[(1-\delta^2)\sum_{h\in P} u_h(x) + \delta^2(G-\frac{5}{4})$. Then, maximizing $R_i^\sigma(x,\{i\}) + R_j^\sigma(x,\{i\})$ is equivalent to solving the problem

$$\max_{x\in X}\left[u_i(x)+u_j(x) - \frac{1}{6}\delta(1+\delta)\sum_{h\in P} u_h(x)\right] \quad \text{subject to } ||x|| \leq \underline{d} \quad (53)$$

The objective function in the problem (53) is written as

$$-||x-\tilde{x}^i||^2 - ||x-\tilde{x}^j||^2 + \frac{1}{6}\delta(1+\delta)||x||^2,$$

which is strictly concave. Applying the first order condition, we can easily see that the function is maximized at $y^{ij}(\delta) = \frac{2(\tilde{x}^i + \tilde{x}^j)}{4-\delta(1+\delta)}$.

Take any x with $||x|| > \underline{d}$. From (47) and (50),

$$v_i^\sigma(x, \{i\}) + v_j^\sigma(x, \{i\}) = G + (1-\delta)\sum_{h \in P} u_h(x^P) + \delta \sum_{h \in P} u_h(y^{ij}(\delta)) - \bar{R}(x). \qquad (54)$$

In the RHS, only $\bar{R}(x)$ depends on x. Subtracting (54) from (51), we obtain

$$\begin{aligned}
&[v_i^\sigma(y^{ij}(\delta), \{i\}) + v_j^\sigma(y^{ij}(\delta), \{i\})] - [v_i^\sigma(x, \{i\}) + v_j^\sigma(x, \{i\})] \\
&= (1-\delta)\left[\sum_{h \in P} u_h(y^{ij}(\delta)) - \sum_{h \in P} u_h(x^P)\right] + \frac{1}{2}[\bar{R}(x) - R_k^\sigma(x^{ij}, \{i\})]. \qquad (55)
\end{aligned}$$

From (42), $v_k^\sigma(y^{ij}(\delta), \{i\}) = \frac{1}{2}\bar{R}(y^{ij}(\delta)) + \frac{1}{2}R_k^\sigma(y^{ij}(\delta), \{i\})$. By the definition of $R_k^\sigma(y^{ij}(\delta), \{i\})$,

$$R_k^\sigma(y^{ij}(\delta), \{i\}) = (1-\delta)u_k(y^{ij}(\delta)) + \frac{1}{2}\delta \bar{R}(y^{ij}(\delta)) + \frac{1}{2}\delta R_k^\sigma(y^{ij}(\delta), \{i\}). \qquad (56)$$

Then

$$\bar{R}(x) - R_k^\sigma(y^{ij}(\delta), \{i\}) = \frac{2(1-\delta)[\bar{R}(x) - u_k(y^{ij}(\delta))] + \delta[\bar{R}(x) - \bar{R}(y^{ij}(\delta))]}{2-\delta}. \qquad (57)$$

Let $H(x,\delta) = \frac{2-\delta}{1-\delta}[\bar{R}(x) - R_k^\sigma(y^{ij}(\delta), \{i\})]$. Then,

$$H(x,\delta) = \bar{R}(x) - u_k(y^{ij}(\delta)) + \frac{1}{3}\delta(1+\delta)\left[\sum_{h \in P} u_h(x) - \sum_{h \in P} u_h(y^{ij}(\delta))\right]. \qquad (58)$$

Let

$$L(x,\delta) = \frac{2-\delta}{1-\delta}\Big([R_i^\sigma(y^{ij}(\delta), \{i\}) + R_j^\sigma(y^{ij}(\delta), \{i\})] - [R_i^\sigma(x, \{i\}) + R_j^\sigma(x, \{i\})]\Big).$$

From (55),

$$
\begin{aligned}
L(x,\delta) &= (2-\delta)[u_i(y^{ij}(\delta)) + u_j(y^{ij}(\delta)) - u_i(x) - u_j(x)] && (59)\\
&\quad +(2-\delta)\delta\left[\sum_{h\in P} u_h(y^{ij}(\delta)) - \sum_{h\in P} u_h(x^P)\right] + \frac{1}{2}\delta H(x). && (60)
\end{aligned}
$$

For sufficiently high δ, $L(x,\delta)$ is minimized at the boundary point of X that maximizes the distance from $\tilde{x}^k$. For convenience, let x in the above equation denote the minimizer. As δ converges to one, $y^{ij}(\delta)$ converges to $\tilde{x}^i + \tilde{x}^j$. Then, as δ approaches one, (59) converges to $3d^2 - \frac{5}{9} - (d + \frac{\sqrt{3}}{3})^2$. Also, the first term in (60) converges to -1, and $H(x)$ converges to $\frac{1}{3}G - \frac{5}{12} + (d + \frac{\sqrt{3}}{3})^2 - \frac{2}{3}(3d^2 - 1)$. Then,

$$
\lim_{\delta\to 1} L(x,\delta) = 2d^2 - \frac{1}{2}(d + \frac{\sqrt{3}}{3})^2 - \frac{103}{72} + \frac{1}{6}G \geq \frac{137}{72} + \frac{1}{6}G, \tag{61}
$$

where the inequality uses that $d \geq \sqrt{3}$. Hence, for sufficiently large δ, $L(x,\delta) > 0$. Hence, $y^{ij}(\delta) \in \arg\max_{x\in X}(R_i^\sigma(x,\{i\}) + R_j^\sigma(x,\{i\})$.

We have shown that x^P satisfies (17) for policy coalition P and that $y^{ij}(\delta)$ satisfies (17) for policy coalition $\{i,j\}$. Due to symmetry, it is clear that $y^{ik}(\delta)$ satisfies the condition for policy coalition $\{i,k\}$. We also have to show that, for each x, the choice of the policy coalition satisfies (18). Take any $x \in X$. By choosing $\{i,j\}$, party i receives payoff (40), and by choosing P, i receives payoff (46). Thus, choosing $\{i,j\}$ is optimal only if

$$
\begin{aligned}
\bar{R}(x) &\geq \sum_{h\in P} R_h^\sigma(x^P,\{i\}) - R_i^\sigma(y^{ij}(\delta),\{i\}) - R_j^\sigma(y^{ij}(\delta),\{i\}) && (62)\\
&= (1-\delta)\left[\sum_{h\in P} u_h(x^P) - \sum_{h\in P} u_h(y^{ij}(\delta))\right] + R_k^\sigma(y^{ij}(\delta),\{i\}) && (63)\\
&= (1-\delta)\left[\sum_{h\in P} u_h(x^P) - \sum_{h\in P} u_h(y^{ij}(\delta))\right] + \frac{2(1-\delta)u_k(y^{ij}(\delta)) + \delta\bar{R}(y^{ij}(\delta))}{2-\delta}, && (64)
\end{aligned}
$$

where we use that $\sum_{h\in P} v_h^\sigma(x^P,\{i\}) = \sum_{h\in P} v_h^\sigma(y^{ij}(\delta),\{i\})$ to derive (63) and use (56) to derive (64). Note that (64) does not depend on x and that $\bar{R}(x)$ is strictly decreasing in $||x||$. Set $\underline{d}$ so that $||x|| = \underline{d}$ makes (62) holds with equality. Then, for every $||x||$ with $||x|| \leq \underline{d}$, choosing $\{ij\}$ is optimal, and, for every $||x||$ with $||x|| > \underline{d}$, choosing P is optimal.

Finally, we have to verify that $||x^P|| \leq \underline{d}$ and $||y^{ij}(\delta)|| \leq \underline{d}$. The inequality (62) holds for $x = y^{ij}(\delta)$ if and only if

$$(2-\delta)\bar{R}(y^{ij}(\delta)) \geq (2-\delta)(1-\delta)\left[\sum_{h\in P} u_h(x^P) - \sum_{h\in P} u_h(y^{ij}(\delta))\right] + 2(1-\delta)u_k(y^{ij}(\delta)) + \delta\bar{R}(y^{ij}(\delta)).$$

This is equivalent to

$$2\bar{R}(y^{ij}(\delta)) \geq (2-\delta)\left[\sum_{h\in P} u_h(x^P) - \sum_{h\in P} u_h(y^{ij}(\delta))\right] + 2u_k(y^{ij}(\delta)). \tag{65}$$

As $\delta \to 1$, the LHS of (65) converges to $\frac{2}{3}G - \frac{5}{6}$ whereas the RHS converges to $-\frac{5}{3}$. Hence, for sufficiently large δ, $||y^{ij}(\delta)|| < \underline{d}$. Then, $||x^P|| = 0 < \underline{d}$. Hence, the proposal strategies under minority governments are sequentially rational.

Next, we prove that the proposal strategies at $(x,\emptyset)$ is sequentially rational. In Section 3.4., we have already shown that each formateur prefers minimal winning coalitions to the consensus government. Here we show that each formateur i prefers government $\{i,j\}$ to government $\{i\}$. Let x be an arbitrary status quo. First, suppose $||x|| > \underline{d}$. By (46) and (49),

$$\begin{aligned} v_i^\sigma(x,\{i\}) &= G + (1-\delta)\sum_{h\in P} u_h(x^P) + \delta\sum_{h\in P} u_h(y^{ij}(\delta),\{i\}) - 2\bar{R}(x) \\ &= (1-\frac{2}{3}\delta^2)G + (1-\delta)\left[\sum_{h\in P} u_h(x^P) - \frac{2}{3}(1+\delta)\sum_{h\in P} u_h(x)\right] \\ &\quad +\delta\left[\sum_{h\in P} u_h(y^{ij}(\delta)) + \frac{5}{6}\right]. \end{aligned} \tag{66}$$

As δ approaches to one, (66) converges to $\frac{1}{3}G - \frac{7}{6}$ which is less than $\frac{1}{2}(G - \frac{1}{2})$. Thus, $v_i^\sigma(x, \{i, j\}) > v_i^\sigma(x, \{i\})$.

Now suppose $x \leq \underline{d}$. By (40), (44), and (56),

$$\begin{aligned} v_i^\sigma(x, \{i\}) &= (1-\delta)G + \sum_{h \in P} R_h^\sigma(y^{ij}(\delta), \{i\}) - R_k^\sigma(y^{ij}(\delta), \{i\}) - \bar{R}(x) \\ &= G + \sum_{h \in P} u_h(y^{ij}(\delta)) - \frac{2(1-\delta)u_k(y^{ij}(\delta)) + \delta \bar{R}(y^{ij}(\delta))}{2-\delta} - \bar{R}(x). \qquad (67) \end{aligned}$$

As δ approaches one, (67) converges to $\frac{1}{3}\delta - 2$, which is less than $\frac{1}{2}(G - \frac{1}{2})$. Thus, $v_i^\sigma(x, \{i, j\}) > v_i^\sigma(x, \{i\})$, which completes the proof. ■

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