On-the-job Training: Investment or Compensation?

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Abstract

I analyze the firm’s decision of how much general but informal training to provide in the presence of on-the-job search. In particular, due to lack of formal measure and commitment technology on the training intensity, I assume that firms flexibly adjust their training intensity at any time, but hardly charge the cost of training to their workers. Even without any contractual agreement on training, firms still provide training not only to improve productivity (investment motive) but also to keep their workers from defecting to other firms (compensation motive). Furthermore, better paying firms are encouraged to provide more intensive training by investment motive, while they are discouraged by compensation motive. The implied wage-training pattern among ex-ante identical firms is reverse U-shaped. That is, training intensity rises with wage payment at low wages, but declines at high wages. Also, as the search friction is mitigated, the investment motive is deteriorated, but the compensation motive is intensified. I also show that the training intensity per trainee declines with the accelerated offer arrival rates in the numerical experiment with reasonable parameter values.

JEL Classification: D83, E24, J24, J33, J64

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1 Introduction

Most economists agree with that human capital accumulation on the job is the major contributor to individual wage growth as well as the economic growth.\textsuperscript{1} In fact, due to fast technological progress and knowledge spill-over, workers are required to continuously upgrade their skills even after schooling, and in many workplaces, firms pay for the cost of the human capital investment in the name of ‘on-the-job training’.\textsuperscript{2} The primary goal of this paper is to improve our understanding of firms’ investment decision on workers’ general human capital through both theoretical and empirical analysis. Furthermore, this paper attempts to examine how the training decision interacts with wage payment and search friction.

Barron et al. (1997) report that there is a great deal of measurement error in on-the-job training variables survey by survey, and respondent by respondent.\textsuperscript{3} Loewenstein and Spletzer (1998) find that if it is measured, the informal training provided by supervisors or colleagues is much more prevalent than training done in formal classes or workshops.\textsuperscript{4} Those empirical findings on the informal training tell us that the training intensity can be freely adjusted ex post and the cost of training is not explicitly charged to the workers. Then, it becomes non-trivial to understand why and how intensively the firm train their workers after the labor contract is signed. Furthermore, to make things harder, workers also engage in on-the-job search behavior. Falick and Fleischman (2004) find that 60% of vacancies in US labor market are taken by employed searchers and only 40% of vacancies are given to unemployed searchers and Topel and Ward (1992) find that a typical worker holds seven jobs during the first 10 years. In addition, Sim (2012) reports that the average job duration of white male high school graduates in U.S. is slightly longer than 2 years. It is important to point out that the on-the-job search behavior might encourage on-the-job training as a means of an optimal retention strategy or discourage it due to free rider problem by subsequent firms. Motivated by the above empirical findings, this paper analyzes why and how intensively the self interested firm train their workers without contractual agreement in the presence of on-the-job search.

This paper extends the wage posting framework proposed by Burdett and Mortensen (1998) to an equilibrium where firms optimally adjust their training intensity at any time. The flexible training decision embodies both investment motive to improve

\textsuperscript{1}For example, Altonji and Shakotko (1987) argue that general human capital accumulation takes lion’s share of wage growth.

\textsuperscript{2}Lynch and Black (1998) find that over 50% (but not all) of U.S. firms provide and pay for general training such as computer skills training and teamwork training.

\textsuperscript{3}Barron et al. (1997) show that the correlations between worker and establishment measures are less than 0.5, which are much lower than correlations for other variables that have been used in wage equations. Also, establishments report 25% more hours of training on average than do workers in the same questions.

\textsuperscript{4}For instance, in the 1994 wave of the NLSY79, just 1% of workers in the first year of a job attended formal classes or seminars, but 57% received instruction from supervisors for an average of 52 total hours and 46% received instruction from co-workers for an average of 69 hours.
productivity given retention probability and \textit{compensation motive} to enlarge retention probability by keeping workers from defecting to other poaching firms. In the Nash-revision subgame perfect equilibrium of my interest, all workers play the grim-trigger strategies; Initially, each worker expects that firms provide the ‘cooperative’ levels of training. If his employer does, the worker keeps his original ‘cooperative’ belief and makes job turnover decision based on the ‘cooperative’ belief. Otherwise, the worker updates his belief that the firm is not ‘cooperative’ and revises his job turnover decision. The sequential rationality requires that in the off-equilibrium path as well as the equilibrium path, the firm and the worker play their optimal strategies. Consequently, in the Nash-revision subgame perfect equilibrium, all firms decide their training intensity by considering both investment motive and compensation motive on equilibrium path, while they play sequentially rational but ‘non-cooperative’ Nash equilibrium strategies only with investment motive off equilibrium paths.

It is useful to review the debate among \textit{Becker (1964), Acemoglu and Pischke (1999)} and others. \textit{Becker (1964)} argues that under perfect competition, firms must provide the efficient level of general training and the cost of general training is paid for by workers through lower wages during training. In contrast, \textit{Acemoglu and Pischke (1999)} argue that the result is vulnerable to a wide range of deviations from perfect information and perfect competition in the labor market. For example, \textit{Acemoglu (1997)} shows that firms provide inefficient levels of training and share the cost of it in a labor market with search friction. He points out that the efficiency loss occurs because the training firm cannot raise a claim on the expected benefit of next employers.

Deviating from their seminal debate on the efficiency and incidence issue, this paper focuses on where the current labor market is located between the two different hypothetical worlds. In the perfectly competitive equilibrium as in \textit{Becker (1964)}, firms train their workers because if not, workers immediately switch to another poaching firms. Firms get zero profit regardless of their training decision. Only over-amplified compensation motive to prolong the expected match duration works in the equilibrium. In contrast, in the frictional labor market proposed in \textit{Acemoglu (1997)}, firms train their workers to enlarge their rent of the match. But their training decision does not affect workers’ job turnover behavior.\footnote{\textit{Acemoglu (1997)} assumes an exogenous reallocation shock rather than on-the-job search behavior} Only the investment motive determines the level of training, while the compensation motive never works. Given the underlying discrepancy, the model proposed by \textit{Acemoglu (1997)} cannot converge to the Beckerian world (with efficient level of training), even when search friction becomes mitigated. Then, the question how sensitively those underlying motives are working is important as much as the questions on what are the relevant sources of inefficiency.

Recently, \textit{Fu (2011)} extends Burdett and Mortensen (1998) framework by incorporating formal training together with piece-rate sharing rule. In her model, firms post the lifetime value of the jobs for workers, and they choose profit-maximizing combination of the training intensity and piece-rate wage scheme in order to deliver the committed values. Firms offering higher values to their workers expect longer job tenures so that
they provide more intensive training. But, firms do not perceive training as a means of affecting the retention probability.\textsuperscript{6} As a consequence, the investment motive solely works in her model, which arises a potential risk of underestimating the actual training intensity. It also predicts that the training intensity increases with both wage payment and search friction, which isolates her model from Beckerian world.

In contrast to Fu (2011), I incorporate both motives. My numerical experiments show that the equilibrium wage-training shows a reverse U-shaped curve. Intuitively, better paying jobs have more stronger investment motive than others because they have already committed high wages. But they have weaker compensation motive at the same time due to the same reason. The former reinforces the training intensity with wage payments, while the latter lowers it. By putting them together, I got a hump shaped wage-training pattern. Given the non-monotone pattern, I also conduct several experiments by changing job turnover rate and training cost parameters. Interestingly, as the offer arrival rates are accelerated, the reinforced compensation motive enforces firms to train their workers more intensively, but the discouraged investment motive reduces training intensity. Furthermore, a higher job offer arrival rate suppresses the wage offer distribution toward the marginal product of labor, in which the level of training declines following the reversed U shaped curve. Although the analytic prediction is somewhat ambiguous, all numerical experiments with reasonable parameter values report that the training intensity per trainee increases with search friction.

The paper proceeds as follows. In section 2, I build up the theoretical model and characterize the steady state equilibrium that I am interested in. In section 3, I present the result of numerical experiments. Then, section 4 concludes. ‘On-the-job Search and On-the-job Training’ is also an empirically relevant issue and it has some significant policy implications. The empirical study on this topic will be added later.

2 Model

2.1 Primitives

There are unit measures of risk neutral workers and firms. A worker, either employed or unemployed, is a lifetime income maximizer without saving nor borrowing.\textsuperscript{7} A newly born worker starts his career as an unemployed worker with $y_t$ units of human capital. An unemployed worker collects unemployment benefit $b$ per instant and finds another job offer at rate $\lambda_u$. If he finds a job offer, he accepts it and immediately starts working. It is natural to think that in equilibrium no job offer fails in attracting unemployed workers. An employed worker receives wage $w$ and finds a job offer at rate $\lambda$, where $\lambda_u > \lambda$. If he finds a seemingly better job, he immediately switches to the new job. He independently and stochastically accumulates additional $\Delta$ units of human capital

\textsuperscript{6}Her model solves for the optimal training decision given the probabilities.

\textsuperscript{7}Note that it is assumed to prevent ‘selling the firm scheme’.
through on-the-job training during his career. It is assumed that there are \( n \) discrete levels of human capital. By construction,

\[
y_i \in \{y_1, y_2, \ldots, y_n\}, \quad \text{where} \quad y_{i+1} - y_i = \Delta > 0.
\]

An employed worker is laid off and becomes unemployed at rate \( \delta \) due to an exogenous separation shock. Both employed and unemployed workers retire at rate \( \rho \) and the retirees are replaced by another born workers.

Each firm keeps one vacant job per instant. The firm with a vacancy posts a constant piece rate to recruit a worker. Let \( F : [\phi, \overline{\phi}] \to [0, 1] \) be the distribution function of the piece rates offered on the steady state equilibrium, where \( [\phi, \overline{\phi}] \) represents the equilibrium support. The usual equal profit condition implies the expected value of the vacant jobs are same regardless of the piece rate on equilibrium. The job with a \( y_i \)-type worker and the piece rate \( \phi \) accrues profit flow \((1 - \phi)y_i\) to the firm and pays \( \phi y_i \) per instant to the worker. The firm with an operating job decides how intensively it trains the worker. If the firm with a \( y_i \)-type worker chooses a particular level of training intensity \( x \in [0, \infty) \), it incurs cost \( c(x) \) and the worker improves his (general) productivity at rate \( \mu_i x \). It is assumed that \( c'(\cdot) > 0, \ c''(\cdot) > 0, \ c(0) = 0, \) and \( \mu_1 > \mu_2 > \cdots > \mu_n = 0 \). The job is destroyed when the worker leaves either voluntarily or involuntarily.

2.2 Repetition of Training Games

In this subsection, I evenly discretize the time horizon with infinitely many intervals of length \( dt \) and construct a repeated training game between the operating firm and the employed worker. In the next subsection, I will send \( dt \) to zero to focus on the limiting equilibrium as in Coles (2001).

At the beginning of period \( t \), the operating firm with a \( y_i \)-type worker chooses training intensity \( x_i \). Once it chooses \( x_i \), the firm cannot change the training intensity within the period. The employed worker produces output \( y_i \) and keeps searching for another job. At the end of each period, the worker may accumulate \( \Delta \) units of human capital with probability \( \mu_i dt \), be separated from the job with probability \( \delta dt \), retire with probability \( \rho dt \), or find another job offer with probability \( \lambda dt \). For expositional convenience, I simply assume that all probabilistic events are exclusive. One can easily see that this is innocuous when \( dt \to 0 \). If the new offer gives a higher expected value than the former job does, he switches to the new job from the next period. Let \( \{x_{is}^{e}(\phi)\}_{s=t}^{\infty} \) and \( \{x_{is}(\phi)\}_{s=t}^{\infty} \) be the expected training schedule and the equilibrium training schedule of the firm with piece rate \( \phi \), respectively. Separately from \( \{x_{is}^{e}\}_{s=t}^{\infty} \), I also define \( \{\bar{x}_{is}^{e}\}_{s=t}^{\infty} \) to be the employed worker’s expectation on the training schedule by his current employer. Shortly, I use \((x_{is}^{e}(\phi), \bar{x}_{is}^{e}(\phi), x_{is}(\phi))\) instead of \((\{x_{is}^{e}(\phi)\}_{s=t}^{\infty}, \{\bar{x}_{is}^{e}(\phi)\}_{s=t}^{\infty}, \{x_{is}(\phi)\}_{s=t}^{\infty})\) when it is innocuous.

\(^8\)The high order terms converge to zero at much faster rates.
When the $y_t$-type unemployed worker finds a job offer, he constructs his own belief on the lifetime value of the offer $E_i(\phi, x^e_i(\phi))$ as well as the training schedule $x^e_i(\phi)$ for each $\phi \in [\underline{\phi}, \overline{\phi}]$. The implied lifetime value of a $y_t$-type unemployed worker at period $t$, $U_i$, is described by

$$U_i = \int_t^{t+dt} e^{-r s} bs + e^{-r dt} \left[ \lambda_d dt \mathbb{E}_F[E_i|U_i] + (1 - \rho dt - \lambda_d dt) U_i \right],$$

where

$$\mathbb{E}_F[E_i|U_i] = \int_{\overline{\phi}}^{\overline{\phi}} \max \left[ E_i(\phi', x^e_i(\phi')) \right] dF(\phi').$$

Equation (2) represents the expected value of a job offer based on the expected piece rate $\phi'$ and the training schedule associated with it. At the beginning of each period, the employed worker having another outside offer makes the turnover decision. That is, if the outside offer gives a higher expected value, he switches to the new job. Otherwise, he stays. Given $(\phi, \{x^e_i\}_{s=t+1}^\infty)$ and $U_i$, $E_i(\phi, \{x^e_i\}_{s=t+1}^\infty)$ is described by

$$E_i(\phi, \{x^e_i(\phi)\}_{s=t+1}^{\infty}) = \int_t^{t+dt} e^{-r(s-t)} \phi y_i ds + e^{-r dt} \left[ \delta dt U_i + \mu_i x_i dt E_i(\phi, \{x^e_i(\phi)\}_{s=t+1}^{\infty}) + \lambda dt \mathbb{E}_F[E_i|E_i(\phi, \{x^e_i(\phi)\}_{s=t+1}^{\infty}) + (1 - \rho dt - \delta dt - \mu_i x_i dt - \lambda dt) E_i(\phi, \{x^e_i(\phi)\}_{s=t+1}^{\infty})] \right].$$

where

$$\mathbb{E}_F[E_i|E_i(\phi, \{x^e_i(\phi)\}_{s=t+1}^{\infty})] = \int_{\overline{\phi}}^{\overline{\phi}} \max \left[ E_i(\phi', x^e_i(\phi')) \right] dF(\phi').$$

Throughout the paper, I do not write down the expression for the case with $i = n$ separately. Note that by construction it is allowed to delete any terms having either $\mu_n$ or $x_n(\cdot)$ in any expression. On equilibrium, the consistency together with symmetry requires

$$x_i^e(\phi) = x_i^e(\phi) = \tilde{x}_i^e(\phi) \quad \text{and} \quad E_i(\phi, x_i^e(\phi)) = E_i(\phi, x_i^e(\phi)) = E_i(\phi, \tilde{x}_i^e(\phi)).$$

Suppose that $x_i^e(\phi)$ is uniquely defined so that $E_i(\phi, x_i^e(\phi))$ is also uniquely defined. Let $\underline{E}_i$ and $\overline{E}_i$ be the lower and upper bound of the expected lifetime value possibly given to $y_t$-type workers, respectively. Define $\phi_i^* : [\underline{E}_i, \overline{E}_i] \to [\underline{\phi}, \overline{\phi}]$ to be a mapping from his current lifetime value to an equilibrium threshold of piece rate such that for any $\phi \in [\underline{\phi}, \overline{\phi}]$,

$$\phi' \leq \phi^*_i(E_i(\phi, x_i^e(\phi))) \iff E_i(\phi', x^e(\phi')) \leq E_i(\phi, x_i^e(\phi)).$$

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8Strictly speaking, I have to use $U_i^e$. But for simplicity I slightly abuse the notation.
Given $E_i(\phi, \tilde{x}_i^e(\phi))$, the worker chooses threshold $\phi_i^*(E_i(\phi, \tilde{x}_i^e(\phi)))$. If the worker is offered a piece rate $\phi'$ higher than or equal to $\phi_i^*(E_i(\phi, \tilde{x}_i^e(\phi)))$, he accepts it. Otherwise, he rejects and stays with the current firm. On a symmetric equilibrium, $\phi_i^*(E_i(\phi, x_i^*(\phi)))$ is uniquely defined if and only if

$$\frac{dE_i(\phi, x_i^*(\phi))}{d\phi} > 0, \text{ for any } \phi \in [\underline{\phi}, \overline{\phi}].$$

(7)

It is useful to reemphasize that condition (7) does not require monotonicity on the equilibrium training decision $x_i^*(\phi)$. Instead, it requires monotonicity on the equilibrium life time value $E_i(\phi, x_i^*(\phi))$ with respect to $\phi$. It’s a weaker condition in the sense that $E_i(\phi, x_i^*(\phi))$ is possibly monotone increasing in $\phi$ even when $x_i^*(\phi)$ is not monotone. Finally, on a symmetric equilibrium,

$$\phi_i^*(E_i(\phi, x_i^*(\phi))) = \phi.$$ 

(8)

Denote by $J_i(\phi)$ the value of an operating job with $(y_i, \phi)$. The firm optimally determines the training intensity $x_t$ to maximize

$$J_i(\phi) = \max_{x_t \in [0, \overline{x}_i(\phi)]} \int_t^{t+dt} e^{-r(s-t)}[y_i(1 - \phi) - c(x_t)]ds + e^{-rdt}\left[\mu_i x_t dt J_{i+1}(\phi) + (1 - \rho dt - \delta dt - \lambda dt (1 - F(\phi_i^*(E_i(\phi, \tilde{x}_i^e(\phi))))) - \mu_i x_t dt J_i(\phi)\right],$$

(9)

where $\overline{x}_i(\phi)$ solves for $J_i(\phi) = 0$. That is,

$$(1 - \phi)y_i - c(\overline{x}_i(\phi)) + \mu_i \overline{x}_i(\phi) J_{i+1}(\phi) = 0.$$  

(10)

As a benchmark, consider the symmetric equilibrium in which the firm and the employed worker play their own stage-game Bayesian-Nash equilibrium strategies. Note that it is not necessary for the forward-looking worker to update his belief $\{\tilde{x}_{it}(\phi)\}_{s=t}^{\infty}$ after observing the training intensity $x_t$ at period $t$. That is,

$$\frac{\partial F(\phi_i^*(E_i(\phi, \{\tilde{x}_{it}(\phi)\}_{s=t+1}^{\infty})))}{\partial x_t} = 0$$

This constitutes the least efficient sub game perfect equilibrium. For any $(y_i, \phi)$, the least efficient sub game perfect equilibrium strategy, denoted by $\hat{x}_i(\phi)$, should solve for

$$c' \hat{x}_i(\phi) = \mu_i(J_{i+1}(\phi) - J_i(\phi)).$$

(11)

In the equilibrium, $x_i^e(\phi) = \hat{x}_i(\phi) = x_i^*(\phi)$. The left hand side in (11) represents the marginal cost of training. The right hand side represents the marginal benefit of training associated with the investment motive. In the least efficient sub game perfect
equilibrium, all firms train their workers in expecting higher profit in future. Without affecting workers’ belief on future training schedule, firms have no compensation motive.

Now, I enlarge my scope to the dynamic environment where the worker updates his belief on the training schedule \( \{ \bar{x}_{it}(\phi) \}_{t=1}^{\infty} \) after observing \( x_t \). In particular, I focus on the symmetric equilibrium in which the worker plays the Nash-revision trigger strategy as follows.

(i) If the \( y_i \)-type employed worker with piece rate \( \phi \) observes \( x_{i1} \) at the first period, he believes that \( \bar{x}_i(\phi) = x_{i1} \) forever and sets the threshold at \( \phi_1^*(E_i(\phi, \bar{x}_i(\phi) = x_{i1})) \).

(ii) If the firm keeps the same training intensity \( x_{i1} \) in subsequent periods, the worker keeps the same belief and threshold.

(iii) If the firm deviates from \( x_{i1} \) at any period, the worker believes \( x_i^*(\phi) = \hat{x}_i(\phi) \) forever and sets the threshold at \( \phi_i^*(E_i(\phi, \hat{x}_i(\phi))) \) afterwards. The off-equilibrium strategy \( \hat{x}_i(\phi) \) is prescribed in equation (11).

Given \( (y_i, \phi) \) and the trigger strategy by the worker, \( x_{i1} \) should solve for

\[
c'(x_{i1}) = \mu_i(J_{i1}(\phi) - J_i(\phi)) + \lambda \left[ \frac{dF(\phi_i^*(E_i(\phi, x_{i1})))}{d\phi_i^*} \frac{\partial \phi_i^*(E_i(\phi, x_{i1}))}{\partial E_i} \frac{\partial E_i}{\partial x_{i1}} \right] J_i(\phi). \tag{12}
\]

Equation (12) reveals an interesting point. The left-hand side represents marginal cost of training. The first term on the right-hand side shows the firm’s marginal benefit from improving the worker’s productivity (investment motive). The second term represents the marginal benefit from longer retention (compensation motive). In particular, the square bracket represents the marginal increase in retention probability with respect to training as a means of compensation. For convenience, define \( MC(x) \) and \( MB(x) \) to be the left hand side function and the right hand side function in (12), respectively.

It is important to point out that the solution for (12) achieves the unique global maximum within \( [0, \bar{x}_i(\phi)] \) if and only if there exists \( x_i^*(\phi) \in [0, \bar{x}_i(\phi)] \) such that

\[
MB(x) \begin{cases} 
> MC(x) & \text{for } x \in [0, x_i^*(\phi)) \\
= MC(x) & \text{for } x = x_i^*(\phi) \\
< MC(x) & \text{for } x \in (x_i^*(\phi), \bar{x}_i(\phi)] 
\end{cases} \tag{13}
\]

As an alternative sufficient condition instead of the second order sufficient condition, it tells us that the global maximum is achieved at an interior intersection point as long as the marginal benefit curve crosses the marginal cost curve just once from above. The alternative sufficient condition in (13) requires neither monotonicity of marginal benefit nor monotonicity of the difference in marginal benefit and marginal
cost. One can easily see that it is less restrictive than the usual second order sufficient condition. In the numerical simulation, I check whether the solution satisfies the alternative sufficient condition in (13). Figure 1 presents a numerical example in which the unique global maximum is achieved at the intersection point in spite of the non-monotone marginal benefit curve.

2.3 A Continuous Time Search Equilibrium

In this subsection, I send $dt$ to zero to incorporate the Nash-revision subgame perfect equilibrium on top of Burdett and Mortensen (1998) framework. Subtracting $e^{-rdt}U_i$ from both sides of (1), dividing by $dt$, sending $dt \to 0$, and reordering yields

$$rU_i = b - \rho U_i + \lambda u \int_0^\phi (E_i(\phi, x_i^*(\phi)) - U_i) d\phi.$$  \hspace{1cm} (14)

The Hamilton-Jacobi-Bellman equations for the $y_i$-type employed worker with $(\phi, x_i^*(\phi))$ are given by

$$rE_i(\phi, x_i^*) = \phi y_i - \rho E_i(\phi, x_i^*) + \delta(U_i - E_i(\phi, x_i^*)) + \mu_i x_i^*(E_i(\phi, x_i^*) - E_i(\phi, x_i^*))$$  
$$+ \lambda \int_0^\phi \max[E_i(\phi', x_i^*(\phi')) - E_i(\phi, x_i^*), 0] d\phi', \text{ and}$$  \hspace{1cm} (15)

$$rJ_i(\phi) = (1 - \phi)y_i - c(x_i^*) + \mu_i x_i^* J_{i+1}(\phi) - (\rho + \delta + \lambda(1 - F(\phi^*)) + \mu_i x_i^*) J_i(\phi).$$  \hspace{1cm} (16)

A recruiting firm optimally chooses a piece rate $\phi \in [0, 1]$ to post. Since they are ex ante identical, all recruiting firms must have the same expected profit in equilibrium, which implies

$$\sum_{i=1}^n \left[ (\lambda u_i + \lambda G_i(\phi)) J_i(\phi) \right] = \pi, \text{ if } \phi \in [\underline{\phi}, \bar{\phi}]$$  
$$< \pi \text{ otherwise,}$$  \hspace{1cm} (17)
where \( u_i \) is the mass of \( y_i \)-type unemployed workers and \( G_i(\phi) \) is the measure of \( y_i \)-type employed workers receiving a piece rate less than \( \phi \).\(^{10}\)

Now, I turn to the equilibrium support of piece rates. Since firms can adjust their level of investment,

\[
E_i(\phi, x_i^*(\phi)) = U_i \text{ for each } i \in \{1, 2, \cdots, n - 1\}. 
\]

When \( i = n \), I know that \( E_n(\phi, 0) \geq U_n \). It is not clear whether there exists another equilibrium with \( E_n(\phi, 0) > U_n \). But for simplicity, I just look for an equilibrium with \( E_n(\phi, 0) = U_n \).

Given the above equilibrium selection rule, I obtain

\[
\phi = \frac{1}{y_n} \left[ (r + \rho + \lambda)U_n - \frac{\lambda}{\lambda_u}[(r + \rho + \lambda_u)U_n - b] \right] 
\]

Once the lower bound \( \underline{\phi} \) is fixed, the upper bound \( \overline{\phi} \) is determined by

\[
\sum_{i=1}^{n} \lambda_u u_i J_i(\underline{\phi}) = \sum_{i=1}^{n} [\lambda_u u_i + \lambda G_i(\underline{\phi})] J_i(\overline{\phi}), \text{ where } F(\overline{\phi}) = 1. 
\]

Then, given the equilibrium support \([\underline{\phi}, \overline{\phi}]\) and the other equilibrium objects, \((17)\) returns the equilibrium piece rate offer distribution \( F \). This requires a fixed point problem of \( F \) in a functional space.

**Definition** The steady state equilibrium with the on-the-job training and on-the-job search consists of the value equations \( \{U_i, E_i, J_i\}_{i=1}^{n} \), workers’ beliefs \( \{x_i^c\}_{i=1}^{n} \), policy functions \( \{\phi_i^c(E_i, y_i), x_i^c(\phi)\}_{i=1}^{n} \), and steady state measures \( \{F, \{u_i\}_{i=1}^{n}, \{G_i\}_{i=1}^{n}\} \), which jointly satisfies the following conditions:

(i) Given \((y_i, \phi)\) and \( F \), a job searcher constructs his belief \( x_i^c(\phi) \). When the worker finds a job offer, he switches to the new job if it gives a higher value than the his reservation value, which generates \( \phi_i^c(E_i) \). Equation \((14)\) and \((15)\) jointly determine \( \{U_i, E_i\}_{i=1}^{n} \).

(ii) Given \((y_i, \phi)\) and \( F \), firms choose the optimal level of training \( x_i^c(\phi) \). The consistency requires \( x_i^c(\phi) = x_i^c(\phi) \). Equation \((16)\) determines \( \{J_i\}_{i=1}^{n} \).

(iii) Given \( \{J_i, u_i, G_i\}_{i=1}^{n} \), recruiting firms optimally post their own piece rate \( \phi \). Then, the equal profit condition \((17)\) holds.

(iv) \( \{F, \{u_i\}_{i=1}^{n}, \{G_i\}_{i=1}^{n}\} \) are stationary.

\(^{10}\) The usual arguments apply to ensure that any equilibrium offer distribution will be continuous and have no gaps.
Finally, I would like to finish this subsection by characterizing the steady state wage-earning distributions. A $y_1$-type unemployed worker finds a job at rate $\lambda_u$, while a $y_1$-type employed worker becomes unemployed at rate $\delta$. Also, the $y_1$-type employed worker retires at rate $\rho$ and the retiree is replaced by a new born $y_1$-type unemployed worker. In steady state,

$$\lambda_u u_1 = \delta G_1(\bar{\phi}) + \rho(1 - u_1).$$

A $y_1$-type unemployed worker finds a job with piece rate lower than or equal to $\phi$ at rate $\lambda_u F(\phi)$. A $y_1$-type employed worker with piece rate $\phi$ may retire, be laid off, find a better job, or become the $y_2$-type at rate $\rho + \delta + \lambda(1 - F(\phi)) + \mu_1x_1^*(\phi)$. Then, the measure $G_1(\phi)$ is characterized by

$$\lambda_u F(\phi) u_1 = (\rho + \delta + \lambda(1 - F(\phi))) G_1(\phi) + \int_{\phi}^{\phi} \mu_1x_1^*(\phi')dG_1(\phi').$$

Taking derivative of (22) with respect to $\phi$ yields

$$\frac{dG_1(\phi)}{d\phi} = \frac{\lambda_u f(\phi) u_1 + \lambda f(\phi) G_1(\phi)}{\rho + \delta + \lambda(1 - F(\phi)) + \mu_1x_1^*(\phi)}.$$  

Solving the differential equation (23) requires the integrating factor $I_1(\phi)$ and an initial condition. Let

$$I_i(\phi) = \exp \left[ \int_{\phi}^{\phi} \frac{-\lambda f(\phi')}{\rho + \delta + \lambda(1 - F(\phi')) + \mu_1x_1^*(\phi')} d\phi' \right], \text{ for } i = 1, 2, \cdots, n$$

The integrating factor $I_1(\phi)$ together with the initial condition $G_1(\phi) = 0$ yields

$$G_1(\phi) = I_1^{-1}(\phi) \int_{\phi}^{\phi} \frac{I_1(\phi')\lambda_u f(\phi') u_1}{\rho + \delta + \lambda(1 - F(\phi')) + \mu_1x_1^*(\phi')} d\phi'.$$

Then, plugging (25) into (21) and reordering yields

$$u_1 = \rho \left[ \lambda_u + \rho - \delta I_1^{-1}(\bar{\phi}) \int_{\phi}^{\phi} \frac{I_1(\phi')\lambda_u f(\phi') u_1}{\rho + \delta + \lambda(1 - F(\phi')) + \mu_1x_1^*(\phi')} d\phi' \right]^{-1}.$$ 

By the same reasoning as in (22) and (23), I obtain

$$\frac{dG_i(\phi)}{d\phi} = \frac{(\lambda_u u_i + \lambda G_i(\phi)) f(\phi) + \mu_{i-1}x_{i-1}^*(g_{i-1}(\phi))}{\rho + \delta + \lambda(1 - F(\phi)) + \mu_1x_1^*(\phi)}, \text{ for any } i \in \{2, 3, \cdots, n\}.$$

Then, for each $i \in \{2, 3, \cdots, n\}$,

$$G_i(\phi) = I_i^{-1}(\phi) \int_{\phi}^{\phi} \frac{I_i(\phi') [\lambda_u f(\phi') u_i + \mu_{i-1}x_{i-1}^*(g_{i-1}(\phi))]}{\rho + \delta + \lambda(1 - F(\phi')) + \mu_1x_1^*(\phi')} d\phi',$$

and

$$u_i = \int_{\phi}^{\phi} \frac{I_i(\phi') \mu_{i-1}x_{i-1}^*(g_{i-1}(\phi))}{\rho + \delta + \lambda(1 - F(\phi')) + \mu_1x_1^*(\phi')} \phi d\phi' \times \left[ \frac{\lambda_u + \rho}{\delta} I_i(\phi) - \int_{\phi}^{\phi} \frac{I_i(\phi') \lambda_u f(\phi')}{\rho + \delta + \lambda(1 - F(\phi')) + \mu_1x_1^*(\phi')} d\phi' \right]^{-1}.$$
2.4 The Solution Algorithm

In this subsection, I describe an alternative way of solving the model by converting the fixed point problem in the functional space into a fixed point problem into a finite dimensional vector space.

Taking derivative of (15) and (16) with respect to $\phi$ yields

$$[r + \delta + \rho + \lambda[1 - F(\phi)] + \mu_i x_i^*] \frac{dE_i(\phi, x^*(\phi))}{d\phi} = y_i$$

$$+ \mu_i x_i^* \frac{dE_{i+1}(\phi, x_{i+1}^*)(\phi)}{d\phi} + \mu_i (E_{i+1}(\phi, x_{i+1}^*) - E_i(\phi, x_i^*)) \frac{dx_i^*}{d\phi}, \text{ and}$$

$$[r + \rho + \lambda[1 - F(\phi)] + \mu_i x_i^*] \frac{dJ_i(\phi)}{d\phi} = -y_i - c'(x_i^*) \frac{dx_i^*}{d\phi}$$

$$+ \mu_i \frac{dx_i^*}{d\phi} (J_{i+1}(\phi) - J_i(\phi)) + \mu_i x_i^* \frac{dJ_{i+1}(\phi)}{d\phi} + \lambda \frac{dF(\phi)}{d\phi} J_i(\phi).$$

Plugging (30) into (12) yields

$$c'(x_i^*(\phi)) = \mu_i (J_{i+1}(\phi) - J_i(\phi))$$

$$+ \lambda \frac{dF(\phi)}{d\phi} \left[ \frac{y_i + \mu_i x_i^*(\phi) \frac{dE_{i+1}(\phi, x_{i+1}^*)(\phi)}{d\phi}}{\mu_i (E_{i+1}(\phi, x_{i+1}^*)(\phi)) - E_i(\phi, x_i^*)} + \frac{dx_i^*}{d\phi} \right]^{-1} J_i(\phi).$$

Equation (32) shows how $x_i^*(\cdot)$ proceeds to achieve the first order condition. By taking derivative of (17) with respect to $\phi$, I obtain

$$\sum_{i=1}^{n} \left[ (\lambda_i u_i + \lambda G_i(\phi)) \frac{dJ_i(\phi)}{d\phi} + \lambda \frac{dG_i(\phi)}{d\phi} J_i(\phi) \right] = 0.$$  

(33)

Plugging (25), (27), and (31) into (33) yields

$$\sum_{i=1}^{n} \left[ (\lambda_i u_i + \lambda G_i(\phi)) \frac{-y_i - c'(x_i^*) \frac{dx_i^*}{d\phi} + \mu_i \frac{dx_i^*}{d\phi} (J_{i+1}(\phi) - J_i(\phi)) + \mu_i x_i^* \frac{dJ_{i+1}(\phi)}{d\phi}}{r + \delta + \rho + \lambda (1 - F(\phi)) + \mu_i x_i^*} \right]$$

$$+ \lambda \frac{\mu_i x_i^* g_{i-1}(\phi)}{\rho + \delta + \lambda (1 - F(\phi)) + \mu_i x_i^*} J_i(\phi)$$

$$+ \sum_{i=1}^{n} \left[ \frac{(\lambda_i u_i + \lambda G_i(\phi)) \lambda J_i(\phi)}{r + \rho + \lambda (1 - F(\phi)) + \mu_i x_i^*} + \frac{(\lambda_i u_i + \lambda G_i(\phi)) \lambda J_i(\phi)}{\rho + \delta + \lambda (1 - F(\phi)) + \mu_i x_i^*} \right] \frac{dF(\phi)}{d\phi} = 0.$$  

(34)

Equation (34) shows how $F$ evolves with $\phi$ to maintain the equal profit. Consequently, the steady state equilibrium is described as follows.

(i) Given $U_n$, our equilibrium selection rule, $U_n = E_n(\phi, 0)$, yields $\phi$ presented in (19).
(ii) Given \( \{U_i\}_{i=1}^{n-1} \) and \( \phi_i \), the initial condition \( U_i = E_i(\phi_i, x^*_i(\phi)) \) together with (15) yields \( x^*_i(\phi) \), which in turn determines \( J_i(\phi) \) for each \( i \) using (16).

(iii) Given \( \{U_i = E_i(\phi, x^*_i(\phi)), u_i\}_{i=1}^{n} \), the system of the differential equations (23), (27), (30), (31), (32) and (34) with \( F(\phi) = 0 \), \( \{G_i(\phi) = 0\}_{i=1}^{n} \), and the initial conditions in (i) and (ii) yields \( \{E_i, J_i, x^*_i, G_i\}_{i=1}^{n} \) and \( F \).

(iv) The terminal condition \( F(\bar{\phi}) = 1 \) determines \( \bar{\phi} \).

(v) Given \( \{x^*_i, G_i\}_{i=1}^{n} \) and \( [\phi, \bar{\phi}] \), (26) and (29) jointly restore \( \{u_i\}_{i=1}^{n} \).

(vi) Given \( \{E_i, J_i, x^*_i, G_i\}_{i=1}^{n} \) and \( [\phi, \bar{\phi}] \), (14) restores \( \{U_i\}_{i=1}^{n} \).

(vii) The finalized solution satisfies the conditions in (7) and (13).

One advantage of this alternative prescription is that it pins down the fixed point problem in a functional space into the fixed point problem in a finite dimensional vector space. In particular, if I assume \( n \) number of different types, I need to solve for the system of \( 4 \times n \) differential equations in (iii). Note that \( x^*_n = 0 \) reduces one equation, but \( F \) requires one more equation. From (i), (ii) and (iii), I also have \( 4 \times n \) initial conditions. Thus, I just need to solve for the fixed point of \( \{U_i\}_{i=1}^{n} \) in \( n \) dimensional vector space.

Note that I have to check (vii) ex post. Solving the system of differential equations gives me an equilibrium candidate under the implicitly assumption that workers’ threshold mapping is well defined and firms’ first order condition yields a global maximum. But these implicit assumptions may or may not be true. The conditions in (7) and (13) should be checked to ensure the existence of the optimal decision. The existence of the equilibrium fixed point of \( \{U_i\}_{i=1}^{n} \) is not proved analytically. Instead, I prove numerically by presenting numerical examples in the next section.

3 Numerical Experiment

3.1 Parameterization

In this section, I present the result of my numerical experiments. For simplicity, I proceed with two types of workers, unskilled workers and skilled workers. Table 1 presents the parameter values used in this section.

The quarterly interest rate is set to 0.012 which roughly targets the annual interest rate to 0.05. The separation rate and the job finding rate are set to 0.1 and 1.35 respectively following Shimer (2005). However, due to lack of precise knowledge, I put some easy numbers on other parameters in this numerical experiment. Later it will be carefully estimated or calibrated. For example, \( \lambda \) can be estimated through the average job duration and employment duration, and \( \rho \) can be estimated from the
average life span. $\mu$ can be inferred together with cost function parameters through the distribution of wages and wage growth rates and the correlation of them. I assume an exponential cost function.

$$c(x) = \alpha(e^{\gamma x} - 1), \text{ where } \alpha = 0.05, \text{ and } \gamma = 2.0$$

It satisfies $c(0) = 0$, $c' > 0$ and $c'' > 0$.

3.2 An Experiment with $n = 2$

[Figure 2] shows the relationship of piece rate (wage) and training. The graph on the top shows us that the equilibrium training intensity $x_1$ satisfies the condition in (13) and it is the unique global maximum within $[0, \pi_i(\phi)]$ for each $\phi$. All firms face the same marginal cost function regardless of piece rate. But the marginal benefit is affected by the piece rate that firms offer. The downward sloping solid line represents the marginal benefit from training associated with a low piece rate. The dotted line on the top is the one with a middle piece rate and the dashed line on the bottom is the one with a high piece rate. Although those three lines are downward sloping in [Figure 2], those are not monotone in an extended interval. However, in the entire closed interval, the marginal cost function crosses the marginal benefit curve once from the below, which is consistent with the curves in [Figure 1]. By connecting it to the bottom left one, one can see that the global maximum is unique at each piece rate, but it is not monotone with piece rate. The bottom left graph demonstrates the reverse U-shaped wage-training pattern when it is rotated anti-clockwise. I put training intensity on the horizontal axis and piece rate on the vertical axis. The graph on the bottom right shows that workers’ lifetime value strictly increases with piece rate, which is consistent with the condition in (7). The dotted line represents the value of unskilled workers and the dashed line shows the value of skilled workers.

[Figure 3] rotates the graph on the bottom left in [Figure 2] by putting piece rate on the horizontal axis and training intensity on the vertical axis. It shows firms’ training
decision at each piece rate. When the piece rate is small, the training intensity increases with piece rate. But when the piece rate is sufficiently large, the training intensity decreases with piece rate. In all experiments with different parameter values, the reversed U shaped curve is quite robust. In [Figure 3], the training intensity at the highest piece rate is lower than the training intensity at the lowest piece rate, which is also robust. It implies that firms with highest wage payment are not able to get enough gains from training because their values are already small enough so that the marginal benefit from extending the expected duration is not large. They do not have strong compensation motive.

[Figure 4] provides a decomposition of the marginal benefit. I obtain the curves by plugging the equilibrium value $x^*_1(\phi)$ into (12). It explains why firms with different piece rates choose different training intensity on equilibrium. The marginal benefit is given by

$$
\mu_1(J_2(\phi) - J_1(\phi)) + \lambda \left[ \frac{dF(\phi^*(E_1(\phi, x)))}{d\phi^*_1} \frac{\partial \phi^*_1(E_1(\phi, x)) \partial E_1(\phi, x)}{\partial x} \right] J_1(\phi).
$$

The first term (investment motive) in (35) is represented by the bottom dotted curve.
The dotted line is upward sloping when

$$
\frac{d}{d\phi} \left( J_2(\phi) - J_1(\phi) \right) = \frac{-(y_2 - y_1)\phi + \lambda \frac{dF(\phi)}{d\phi} \left( J_2(\phi) - J_1(\phi) \right)}{r + \delta + \rho + \lambda(1 - F(\phi))}.
$$

A small increase in $\phi$ lowers the profit flow on the nominator, but raises the retention probability at the same time. Under reasonable parameter values, the increase in the second term dominates the decrease in the first term. Hence, as $\phi$ increases, the marginal gains from longer retention dominates the marginal loss in profit flow. Indeed, [Figure 5] shows that $J_2(\phi) - J_1(\phi)$ increases in $\phi$ in this experiment. Furthermore, one can expect that the dotted line is convexly increasing if $F$ is also convex. The convex offer distribution $F$ is inherited from Burdett and Mortensen (1998), which is confirmed in [Figure 6].

The second term (compensation motive) in (35) shows a hump shaped curve. Taking
the derivative of the square bracket in (35) with respect to $\phi$ yields

$$
\lambda J_1(\phi) \frac{d}{d\phi} \left[ \cdot \right] + \lambda \left[ \cdot \right] \frac{dJ_1(\phi)}{d\phi}.
$$

Intuitively, the first term captures the effect of the enlarged expected duration by a higher piece rate on the marginal benefit of training, while the second term captures the negative effect of the higher payment by the higher piece rate. If the piece rate is sufficiently small and $\frac{d}{d\phi} \left[ \cdot \right]$ is positive, the first term has a large positive value. If the positive effect dominates the negative effect from the second term, the training intensity rises with piece rate $\phi$. In contrast, if the piece rate is sufficiently large so that the first term has a small or negative value, it is dominated by the second term and the training intensity declines. In [Figure 4], the marginal benefit curve by compensation motive indeed rises a little and then declines with piece rate.

Summing up the effects of the investment and compensation motive, I get the total marginal benefit curve. At a smaller piece rate, both investment motive and compensation motive increase training intensity. But when the negative effect of discouraged compensation motive dominates the positive effect of encouraged investment motive, I get a reversed U shape marginal benefit curve. Since total marginal benefit equals to the marginal cost which has a monotone one-to-one relationship with $x_i^*$, I can infer that the shape of $x_i^*$ is inherited from the curve of total marginal benefit.

Finally, [Figure 5] shows that the value differential between the job with unskilled workers and and skilled workers. [Figure 6] provides the equilibrium distribution of piece rates offered by recruiting firms. As usual in wage posting models such as Burdett and Mortensen (1998) and Fu (2011), the cumulative distribution function is convex, which can be corrected by introducing heterogeneity as in Burdett and Mortensen (1998).

### 3.3 Further Experiment

In this subsection, I present the result of comparative statics analysis. First, I analyze the relationship between search friction and firms’ training decision. To this end, I accelerate the offer arrival rates, both $\lambda_u$ and $\lambda$, by 5% and 10% maintaining all other parameters same.

As before, from the first term (investment motive) on the right hand side of (12), I obtain

$$
\mu_1(J_2(\phi) - J_1(\phi)) = \frac{(y_2 - y_1)(1 - \phi) + c(x_i^*(\phi))}{r + \delta + \rho + \lambda(1 - F(\phi)) + \mu_1 x_i^*(\phi)},
$$

which clearly decreases with $\lambda$ at each $x_i^*(\phi)$ as in Acemoglu (1997). Moreover, faster offer arrival rates for both employed and unemployed workers also suppress the piece
rate offer distribution toward the marginal product of labor, which lowers $F(\phi)$ at each piece rate $\phi$. [Figure 7] shows this.

In [Figure 7], the solid line represents the cumulative piece rate offer distribution under $\lambda_u = 1.35$ and $\lambda = 0.3$. The dotted line represents the case with $\lambda_u = 1.4175$ and $\lambda = 0.315$, and the dashed line represents the case with $\lambda_u = 1.485$ and $\lambda = 0.33$. As the offer arrival rates are accelerated, the support shrinks toward the marginal product of labor and the cumulative distribution is more concentrated on high piece rates. Overall, as search friction is mitigated, the whole wage support shrinks toward the marginal product of labor and the investment motive for training is discouraged. This is contradictory to Becker (1964).

However, the marginal benefit from the second term (compensation motive) on the right hand side of (12) is ambiguous since it contains many equilibrium objects. If the compensation motive is also discouraged with higher offer arrival rates, the overall training intensity declines. But from the numerical experiments with different parameter values, it turns out that this is not the case. If higher offer arrival rate accelerates competition among firms, firms should train their worker more intensively in order to keep their workers. In this case, the overall training intensity may or may
not rise with the offer arrival rate. Interestingly, [Figure 8] reports that as the offer arrival rates are accelerated, the hump shaped training intensity curve makes a bigger swing with higher peak and deeper trough.

[Figure 9] compares the overall provision of training. I order all recruiting firms by their piece rate offers on the horizontal axis. The firm with the least piece rate offer is located at 0, while the firm with the highest piece rate offer is at 1. The vertical axis stands for training intensity. Roughly speaking, 70% of recruiting firms train their worker more intensively with higher offer arrival rates, while top 30% of recruiting firms provide less training.

However, it is not clear whether the increase in training level at each piece rate also implies an increase in training opportunity per trainee. [Figure 9] shows that as $\lambda$ increases more firms move toward the top and bottom ends of the support, at which actually firms invest less. Therefore, it is not clear whether each trainee in the economy with a higher $\lambda$ receives more training opportunity in average. Moreover, Table 2 reports that the measure of skilled workers declines with the offer arrival rates.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_u, \lambda = (1.35, 0.3)$</th>
<th>$\lambda = (1.4175, 0.315)$</th>
<th>$\lambda = (1.485, 0.33)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u$</td>
<td>$e$</td>
<td>sum</td>
</tr>
<tr>
<td>unskilled</td>
<td>0.026</td>
<td>0.250</td>
<td>0.276</td>
</tr>
<tr>
<td>skilled</td>
<td>0.050</td>
<td>0.674</td>
<td>0.724</td>
</tr>
</tbody>
</table>

One important thing that I need to point out is that this simulation result is based on just local prediction. If we can eliminate the investment motive completely at a certain level of the offer arrival rates, we can see that the training intensity
Figure 8: Training Intensity under Different Levels of Search Friction

Figure 9: Training Intensity under Different Levels of Search Friction

may increase at higher offer arrival rates. Then, the economy may converge to the Beckerian economy in terms of training by focusing only on the compensation motive. However, as long as we are not able to completely eliminate the investment motive, we are not sure that we could encourage the training intensity by mitigating search friction.

Another interesting experiment is the case with training subsidy. Suppose that the government gives 10% of training cost to each training firm. Figure 9 shows that with the subsidy, firms invest more at each piece rate. In addition, Table 3 shows that the steady state with the subsidy has more skilled workers.

4 Conclusion

The paper demonstrates that firms provide general but informal training as a means of investment and compensation even when the training has not been specified in the labor contract. In particular, the investment motive encourages an intensive training by better paying firms, while the compensation motive discourages it. That is, a higher
Table 3: Subsidy and Steady State Measure of Skilled Workers

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0.05$</th>
<th></th>
<th>$\alpha = 0.045$</th>
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<tbody>
<tr>
<td></td>
<td>$u$</td>
<td>$e$</td>
<td>sum</td>
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</tr>
</tbody>
</table>

expected profit through a longer expected job duration encourages investment, while a higher wage payment which has been already specified on the labor contract protects the match from other poaching firms and reduces the training intensity. Consequently, the equilibrium wage-training pattern is hump-shaped. At low wages, the compensation motive dominantly generates a positive correlation between wage payment and training provision, but at high wages, the investment motive oppositely generates a negative correlation between them.

I have also conducted the numerical experiment to investigate the (long-run) relationship between search friction and training. Becker (1964) argue that perfect competition (hence frictionless labor market) could achieve the socially efficient level of training. This paper is not opposed to his insightful viewpoint based on perfect competition. However, it points out that given search friction, the economy may move in the opposite direction in terms of training and the Beckerian outcome might be the local phenomenon around the perfectly competitive limit. In particular, unless we are able to encourage the compensation motive without deteriorating the investment motive, we cannot encourage training by accelerating the offer arrival rates. The effort to mitigate search friction and the accelerate job turnover lowers training intensity before the economy converges to the limit point.

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11One can easily figure out that in Becker (1964), the investment motive is always zero so that it cannot be discouraged any more.
References


BECKER, G. (1964): *Human capital: A theoretical and empirical analysis, with special reference to education*, University of Chicago Press. 3, 18, 21


