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FRAMING-BASED CHOICE: A MODEL OF DECISION-MAKING UNDER RISK

KOBI KRIESLER* · SHMUEL NITZAN**

In this study we propose an axiomatic theory of decision-making under risk that is based on a new approach to the modeling of framing that focuses on the subjective statistical dependence between prizes of compared lotteries. Unlike existing models that allow objective statistical dependence, as in Regret Theory, in our model the emphasis is on alternative subjective statistical dependence patterns that are induced by alternative descriptions of the lotteries, i.e., by alternative framing. A distinct advantage of the proposed general descriptive model of choice is its ability to adequately explain a wide variety of behaviors and, in particular, several well-known paradoxes of different types.

JEL Classification: D81
Keywords: Framing, Statistical Dependence, Non-expected Utility, Expected Value of Lottery Interchange

I. INTRODUCTION

The Expected Utility Model is widely accepted in the field of decision-making under risk. Its appeal is due to two advantages: it is based on logical and simple axioms and it yields powerful results. Nevertheless, descriptively this model is misleading and, at the least questionable, in light of the empirical data that reveal systematic violation of its

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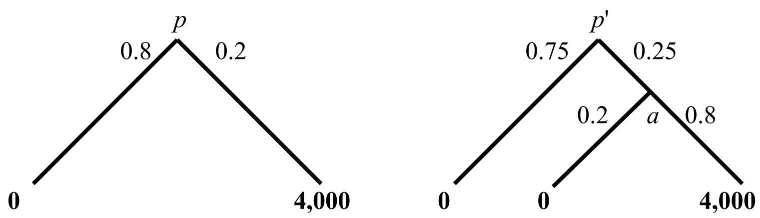
* Department of Economics, Bar-Ilan University, Ramat Gan, Israel 52900.
** Corresponding Author: Department of Economics, Bar-Ilan University, Ramat Gan, Israel 52900. Tel: 972-3-5318930, Fax: 972-3-5353180 E-mail: nitzans@ mail.biu.ac.il,.

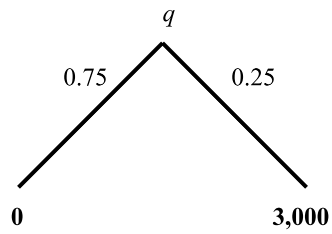
underlying assumptions (Starmer, 2000). The following are a number of examples:

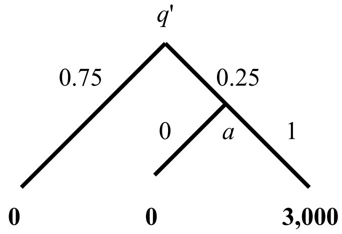
- "**Allais Paradox**", Allais (1953)
- "**The Two-Phase Lottery Paradox**", Kahneman & Tversky (1979)
- "**The Common Ratio Paradox**", Kahneman & Tversky (1979)
- "**The Duplex Gamble Paradox**", Slovic & Lichtenstein (1968), Payne & Braunstein (1971)
- "**Non-Transitive Preferences**", Tversky (1969), Fishburn (1970)
- "**Preference Reversal**", Grether & Plott (1979), Loomes et. al. (1991)
- "**Violations of Stochastic Dominance**", Tversky & Kahneman (1981, 1986)

Many models have been proposed to resolve the above paradoxes, usually by weakening, dispensing of, or replacing one or more of the original axioms. Most of these models are consistent with the Allais Paradox, some with Non-Transitive Preferences (for example, Fishburn's (1982) SSB model), and some with the Violations of Stochastic Dominance (the "Prospect Theory" of Kahneman & Tversky, 1979). The objective of this paper is to join the hunt for a descriptive theory of choice under risk by proposing a new axiomatic approach to the modeling of framing that resolves all the above paradoxes. A comprehensive discussion on the relationship between our approach and the existing models is left to the summary section.

The proposed model is based on several basic principles. First, the mode of presentation of a lottery is assumed to be an essential and inseparable part of it. For example, consider an individual who has to make two decisions between lotteries p and q and between lotteries p' and q'. The four lotteries are presented below:







Notice that while $p(q)$ is a simple lottery, $p'(q')$ is a complex two-stage lottery ('a' denoting the lottery's second stage). Although both lotteries share the same underlying distribution, that is, they give 4,000 with 20% probability (3,000 with 25% probability), they are considered in the proposed model to be two different lotteries.

Second, the means of modeling framing is the individual's perception of the dependence between the lotteries s/he confronts. Specifically, an individual facing a choice between two lotteries is assumed to form subjective statistical dependence between them according to their description and his or her subjective perception. Unlike subjective distortion of prizes or of probability of prizes, such formation of subjective statistical dependence between different lotteries' prizes has remained uncharted territory in the field of decision-making under risk. In the proposed model the subjective statistical dependence is represented by a matrix with rows and columns corresponding to prizes and entries that specify the joint probability of receiving the 'row' prize from the first lottery and the 'column' prize from the second lottery.

For example, if individuals perceive the above p and q as statistically independent, the matrix representing these lotteries is $m(p,q)$. However, if they assume that they proceed to stage 'a' in p' iff they proceed to stage 'a' in q', then the matrix representing these lotteries is $m(p',q')$. These two matrices are:

$m(p,q)$

		q: 0	3000	4000
	0	60%	20%	
p	**3000**			
	4000	15%	5%	

$m(p',q')$

		q': 0	3000	4000
	0	75%	5%	
p'	**3000**			
	4000		20%	

Third, subjective statistical dependence is assumed to play an essential role in individual decision-making. The individual's preference relation over lotteries $\succsim$ is derived from basic preferences over matrices representing statistical dependence between the lotteries. Specifically, imposing several axioms on the basic preferences, it is shown that the individual's choice is based on the expected value corresponding to the interchange between the compared lotteries. That is, the following representation result is obtained:

$$p \succsim q \Leftrightarrow \sum_{X*X} m_{xy} \Phi(x, y) \geq 0 .$$

where m is the matrix describing the subjective statistical dependence between the lotteries; m_{xy} is the joint probability of receiving 'x' in the first lottery and 'y' in the second lottery, and Φ is a real-valued function representing the individual's preference for receiving 'x' instead of 'y'.

The model is presented in section 2. Section 3 contains the main representation result. We then illustrate in Section 4 how the model is consistent with the above-mentioned paradoxes. Section 5 concludes with a summary and discussion.

II. THE MODEL

Let X be a finite set of prizes $X = \{x_1, x_2, \ldots, x_n\}$, $n \geq 2$ and P the set of possible lotteries on X. An individual is defined by the pair < **M**, $\succ *$>, where **M** (**M** :PxP→M) is a function that assigns a single matrix from the matrix space M to every ordered pair of lotteries and $\succsim*$ is a binary relation on M.

A typical matrix m describes the statistical dependence between the two lotteries according to the individual's subjective perception. Given two lotteries $p, q \in P$, the matrix $m(p, q)$ assigned by the function **M** is referred to as the Subjective Statistical Dependence Matrix (SSDM) between p and q. This matrix has the following properties:

- it is of the order $n \times n$ (n being the number of prizes in X);

- $m_{xy}(p,q)$ is the probability of receiving prize x in p and prize y in q;
- $m_{xy}(p,q) \in [0,1], \forall\, x, y \in X$;
- $\sum_{XxX} m_{xy}(p,q) = 1$;
- $\sum_{y \in X} m_{xy}(p,q) = p(x)$; the sum of a row x is the probability of receiving prize x in p;
- $\sum_{x \in X} m_{xy}(p,q) = p(y)$; the sum of a column y is the probability of receiving prize y in q.

Note that in our setting one explicit characteristic of a lottery is the probabilities corresponding to the possible prizes. The remaining characteristics that are associated with framing are not explicitly defined. However, their impact is represented by the SSDM. In the special case of a certain lottery, these remaining framing characteristics do not affect the SSDM. In other words, when one of the lotteries is degenerate, the individual's decision-making is not affected by framing.[1]

$\succsim^*$ is a basic binary preference relation on *M*, which is assumed to be complete, transitive and reflexive, with the symmetric and asymmetric parts $\sim^*$ and $\succ^*$.

$\succsim^*$ has the following interpretation: for $p,q,r,w \in P$, $m(p,q) \succsim^*$ $m(r,w)$ means that receiving p instead of q is "preferred to" or "equivalent to" receiving r instead of w. For example, let p,q,r,w be the following certain lotteries (prizes):
$p = (50,100\%)$, $q = (10,100\%)$, $r = (1001,100\%)$ and $w = (1000,100\%)$.
That is, p is a lottery giving a prize 50 with probability 1, q gives 10 with probability 1, r gives 1001 with probability 1, and w gives the prize 1000 with probability 1. If $m(p,q) \succsim^* m(r,w)$, the individual values receiving 50 (instead of 10) more than receiving 1001 (instead of 1000).

Let $\succsim$ denote the individual's binary preference relation on *P*. $\succsim$ is induced from the basic binary relation $\succsim^*$ by assuming that $p \succsim q$ iff $m(p,q) \succsim^* m(q,p)$. This definition implies that the preference relation $\succsim$

[1] Our particular modeling of framing rules out this possibility, although, in general, even if lotteries are degenerate, the individual's decision-making may be affected by framing.

is affected by the statistical dependence between the two lotteries. Note that although $\succsim^*$ is assumed to be transitive, the binary relation $\succsim$ is not necessarily transitive.[2]

Given $m', m \in M$ and $\alpha \in [0,1]$, the matrix $[\alpha m' + (1-\alpha)m]$ denotes the $\alpha-$linear combination of m and m'. For example, let $p,q,r \in P$, and $p = (10,100\%)$, $q = (20,100\%)$ and $r = (30,100\%)$ be certain lotteries (prizes). In this case the matrices $m(p,q)$ and $m(r,r)$ are (if one or two of the lotteries are certain lotteries, the corresponding Subjective Statistical Dependence Matrix is unique):

m(p,q)

		q 10	20	30
	10		1	
p	**20**			
	30			

m(r,r)

		r 10	20	30
	10			
r	**20**			
	30			1

Now, let $p' = (10,50\%; 30,50\%)$, $q' = (20,50\%; 30,50\%)$. The subjective statistical dependence between p' and q' can be represented by various matrices and, in particular, by the matrices m^* and m^{**}:

*m**

	10	**20**	**30**
10		50%	
20			
30			50%

*m***

	10	**20**	**30**
10		25%	25%
20			
30		25%	25%

Note that the matrix m^* is a linear combination of $m(p,q)$ and $m(r,r)$. Thus, this matrix merges the lotteries while preserving the exact statistical dependence between p and q and between r and r, that is, the individual assumes s/he will get 10 in lottery p' iff s/he receives 20 in lottery q'. It is easy to verify that m^{**} is not a linear combination of $m(p,q)$ and $m(r,r)$.

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[2] For example, let $X = \{a,b,c\}$. Since X is finite and $\succsim^*$ transitive, $\succsim^*$ can be represented by a real-value function denoted as Φ. Suppose now that $\Phi(m(a,b)) = \Phi(m(b,c)) = \Phi(m(c,a)) > 0$; that is, by definition, the individual prefers a over b, b over c and c over a, so $\succsim$ is not transitive.

III. THE REPRESENTATION RESULT

The individual preferences are assumed to satisfy the following axioms:

Axiom C (Continuity)

Let $m, m', m'' \in M$, such that $m \succsim^* m' \succsim^* m''$. Then there exists $\alpha \in [0,1]$ such that:

$$m' \sim^* \alpha m + (1-\alpha)m''.$$

That is, if *m* is preferred or equivalent to m' and m' is preferred or equivalent to m'', there is some linear combination of *m* and m'' that is equivalent to m'.

Axiom I (Independence)

Suppose that the matrix $\tilde{m}$ relates to two equivalent lotteries. Then for every $m \in M$ and $\alpha \in [0,1]$,

$$m \sim^* \alpha m + (1-\alpha)\tilde{m}.$$

That is, if the statistical dependence matrix between equivalent lotteries, say *r* and *w*, is merged to the matrix describing the dependence between any two lotteries, say *p* and *q*, while preserving complete statistical dependence (preserving the exact statistical dependence between *p* and *q* as well as between *r* and *w*), then the original preference relation between *p* and *q* is identical to the relation between the lotteries of the combined matrix $\alpha m + (1-\alpha)\tilde{m}$. Note that in the standard expected utility model, VNM (1944), it is assumed that, for every $p, q, r, w \in P$, such that $r \sim w$, $p \succsim q \Leftrightarrow [\alpha p + (1-\alpha)r] \succsim [\alpha q + (1-\alpha)w]$, where $\alpha \in [0,1]$ represents the probability assigned to the lottery *p*. Let $\mathbf{I}^s$ denote this standard independence axiom. In our model, the independence axiom **I** requires complete statistical dependence between the two relevant parts of the lotteries. This more demanding requirement means that one lottery materializes in probability α iff the other lottery materializes in the same probability. In other words, in the proposed model, independence requires that the matrix $m([\alpha p+(1+\alpha)r],[\alpha q+(1-\alpha)w])$ be a linear combination of $m(p,q)$ and $m(r,w)$.

Axiom S (Symmetry)

For every $p,q,r,w \in P$, $m(p,q) \succsim^* m(r,w) . \Leftrightarrow m(q,p) \precsim^* m(w,r)$.

That is, if getting *p* instead of *q* is preferred or equal to receiving *r* instead of *w*, then getting *w* instead of *r* is preferred or equal to receiving *q* instead of *p*.

In particular, since there are lotteries $\overline{p}, \underline{q} \in P$, such that $m(\overline{p},\underline{q}) \succsim^*$ $m(p,q)$ for every $p,q \in P$, axiom **S** implies that $m(q,p) \succsim^*$ $m(\underline{q},\overline{p})$ for every $p,q \in P$. That is, if getting $\overline{p}$ instead of $\underline{q}$ is perceived by the individual as the best result, then getting $\underline{q}$ instead of $\overline{p}$ is perceived by the individual as the worst result.

Let *x* denote any certain lottery that gives prize *x* with certainty.[3]

Axiom D (Dominance)

For every $p,q \in P$, if for every $x,y \in P$, $m_{xy}(p,q) \geq m_{yx}(p,q) \Leftrightarrow x$ $\succsim y$, then $p \succsim q$.

That is, if whenever *x* is preferred or equal to *y*, the probability of getting *x* in *p* and *y* in *q* is greater than the probability of getting *y* in *p* and *x* in *q*, then *p* is preferred or equal to *q*. For example, assume that for the two lotteries p,q, $m(p,q)$ is:

		q 10	20	30
	10		10%	
p	**20**	70%	20%	
	30			

In this case, by axiom **D**, if 20 $\succsim$ 10, then $p \succsim q$.

3 Since, as already noted, there is a single SSDM corresponding to any such *x* and *y*, the same relationship $\succsim^*$ holds between them.

Proposition 1: If $\succsim^*$ is a binary relation on M that satisfies axioms **I**, **C**, **S** and **D**, then there is a real valued function $\Phi: M \to \mathbb{R}$, such that for every $p, q \in P$,

$$p \succsim q \Leftrightarrow \sum_{X^*X} m_{xy}\Phi(x,y) \geq 0 .$$

Proof: Let p and q be any two lotteries in P. The matrix $m(p,q)$ has $k \in [0,\dots,n(n-1)/2]$ entries $m_{xy}(p,q)$, such that $x \succ y$.

If $k > 0$, let us take some entry $m_{xy}(p,q)$ for which $x \succ y$.

Let $\overline{x}$ and $\underline{x}$ be certain lotteries (prizes) such that $m(\overline{x},\underline{x}) \succsim^*$ $m(x,y)$ for every certain lottery $x, y \in P$. The existence of such $\overline{x}$ and $\underline{x}$ is secured since X is finite and $\succsim^*$ is transitive. By axiom **C**, there exists a number $\Phi = \Phi(x,y)$, such that

$$m(x,y) \sim^* \Phi m(\overline{x},\underline{x}) + (1-\Phi)m(x,x)$$

(which implies that $\Phi(x,x) = 0$ and $\Phi(\overline{x},\underline{x}) = 1$).

Let m' be a matrix that differs from $m(p,q)$ only in the entry $m_{xy} = 0$.

By axiom **I**, the basic preference relation is unaffected if $m(x,y)$ is replaced with

$$\begin{aligned}
&\Phi m(\overline{x},\underline{x}) + (1-\Phi)m(x,x):\\
&m = m_{xy}[m(x,y)] + (1-m_{xy})[m'/(1-m_{xy})] \sim^*\\
&m_{xy}[\Phi(x,y)\, m(\overline{x},\underline{x}) + (1-\Phi(x,y))m(x,x)]\\
&+(1-m_{xy})[m'/(1-m_{xy})] = m_1 .
\end{aligned}$$

Now, a different entry in the matrix m_1, for which $x \succ y$, can be chosen and replaced in the same way, thereby creating the matrix m_2. After k such replacements, in the resulting matrix m_k, which is equivalent to m, *i.e.*, $m_k \sim^* m$, all the entries $m_{xy}(p,q)$ such that $x \succ y$ are equal to 0, except the entry $m_{\overline{x}\underline{x}}$, which is given by

$m_{\bar{x}\underline{x}} = \sum_{X*X} m_{xy}\Phi(x,y) \mid x \succ y$.

In a similar manner we can transform all the entries $m_{xy}(p,q)$ for which $x \prec y$. Specifically, let $\underline{x}', \bar{x}'$ be certain lotteries (prizes) such that $m(x,y) \succsim^* m(\underline{x}', \bar{x}')$ for all certain lotteries $x, y \in P$. The existence of $\bar{x}'$ and $\underline{x}'$ is secured since X is finite and $\succsim^*$ is transitive. By axiom **C**, there is a number $\Phi = \Phi(x,y)$, such that

$$m(x,y) \sim^* \Phi\, m(\underline{x}', \bar{x}') + (1-\Phi) m(x,x) .$$

There are $j \in [0,\ldots,n(n-1)/2]$ entries $m_{xy}(p,q)$ in the matrix m_k for which $x \prec y$.

If $j > 0$, let us take some entry $m_{xy}(p,q)$ for which $x \prec y$.

This entry may be replaced with 0 in the same manner as entries in the matrix for which $x \succ y$ were transformed to 0, until we obtain the matrix m_{k+j} in which all the entries such that $x \prec y$ or $x \succ y$ are equal to 0 except $m_{\bar{x}\underline{x}}$ and $m_{\underline{x}'\bar{x}'}$ where

$$m_{\bar{x}\underline{x}} = \sum_{X*X} m_{xy}\Phi(x,y) \mid x \succ y \quad \text{and} \quad m_{\underline{x}'\bar{x}'} = \sum_{X*X} m_{xy}\Phi(x,y) \mid x \prec y .$$

By axiom **S**, for every x, y, $m(\bar{x}, \underline{x}) \succsim^* m(x,y) \Leftrightarrow m(x,y) \succsim^* m(\underline{x}, \bar{x})$, thus

$(\underline{x}, \bar{x}) = (\underline{x}', \bar{x}')$. By axiom **D**, for every $p, q \in P$,

$$p \succsim q \Leftrightarrow \sum_{X*X} m_{xy}\Phi(x,y) \ge 0 . \qquad \blacksquare$$

Since the real-valued function $\Phi(x,y)$ can be interpreted as representing the individual's preference for receiving '*x*' instead of '*y*', we say that the choice between two lotteries *p* and *q* is based on the expected value corresponding to their interchange. The function $\Phi(x,y)$ is referred to as the value function of prize-differences or, in short, the value function.

IV. RESOLUTION OF THE PARADOXES

In this section the proposed model is shown to be consistent with the aforementioned paradoxes.

The Allais Paradox

Suppose that the individual chooses between lotteries p and q and between lotteries p' and q' given below:

Lottery	Probability		
	90%	9%	1%
p	1m	1m	1m
q	1m	5m	0
p'	0	1m	1m
q'	0	5m	0

Most individuals prefer p to q and p' to q', violating the standard independence axiom $\mathbf{I}^s$, Allais (1953). If one of the lotteries is a certain lottery (prize), then there is a unique Subjective Statistical Dependence Matrix, $m(p,q)$:

m(p,q)

p \ q	0	1m	5m
0			
1m	1%	90%	9%
5m			

However, there are various matrices that can represent the subjective statistical dependence between p' and q' and, in particular, the following matrices m^* and m^{**}:

*m**

p' \ q'	0	1m	5m
0	90%		
1m	1%		9%
5m			

*m***

p' \ q'	0	1m	5m
0	81.9%		8.1%
1m	9.1%		0.9%
5m			

If the individual's subjective perceptions regarding the interdependence between p' and q' are given by m*, then in our model, $p \succ q$ iff $p' \succ q'$.

This is due to the fact that $m(p,q)$ and $m^*(p',q')$ have, respectively, the following 0.9-linear combination representations:

0.9

	0	**1m**	**5m**
0			
1m		100%	
5m			

+ 0.1

	0	**1m**	**5m**
0			
1m	10%		90%
5m			

and

0.9

	0	**1m**	**5m**
0	100%		
1m			
5m			

+ 0.1

	0	**1m**	**5m**
0			
1m	10%		90%
5m			

Therefore, by axiom **I**, $p \succ q$ iff $p' \succ q'$. However, if the individual's subjective perceptions regarding the interdependence between p' and q' are given by $m^{**}(p',q')$, $p \succ q$ does not imply $p' \succ q'$. In other words, the value function $\Phi(x,y)$ may be such that $\sum_{X*X} m_{xy}(p,q)\Phi(x,y) \geq 0$ and $\sum_{X*X} m_{xy}(p',q')\Phi(x,y) < 0$, that is, the individual prefers p to q and q' to p'. More specifically, given $m(p,q)$,

$$p \succ q \Leftrightarrow 0.01\Phi(1m,0) + 0.9\Phi(1m,1m) + 0.09\Phi(1m,5m) > 0 .$$

However, if the individual perceives the statistical dependence between p' and q' as in $m^{**}(p',q')$, then

$$p' \succ q' \Leftrightarrow 0.091\Phi(1m,0) + 0.819\Phi(0,0) + 0.009\Phi(1m,5m) \\ +0.081\Phi(0,5m) < 0 .$$

It is easy to verify that there are values of Φ such that these inequalities hold. For example, if Φ (1m, 0) = 100, Φ (1m, 1m) = Φ (0, 0)=0, Φ (1m, 5m) = -10, Φ (0, 5m) = -200, then $p \succ q$ (0.1 > 0) and $p' \succ q'$ (-7.19 < 0).

Moskowitz (1974) and Keller (1985) found that the way that problems like those resulting in the Allais paradox, are described; significantly affects the proportion of subjects making decisions in conformation with or in violation of the standard independence axiom. Different choices are possible if the problems are described in the standard matrix form (as above), in a decision-tree form, or in a form of minimally structured written statements. Such findings cannot be explained by most models of decision-making under risk. The proposed model assumes that different representations of lotteries differently affect the individual's perception of the statistical dependence between the lotteries he faces, and consequently may lead to different choices.

The Two-Phase Lottery Paradox

An individual may assign different statistical dependence matrices to two pairs of lotteries p, q and p' and q' that are characterized, respectively, by the same prize distributions, depending on whether the lotteries are presented as one-phase or two-phase lotteries (Kahneman & Tversky, 1979). Such an example of $m(p,q)$ and $m(p',q')$ is given in the introduction. By Proposition 1, in our setting the individual decision-making is based on expected changes between the lotteries, and the value function $\Phi(x,y)$ can be such that $\sum_{X*X} m_{xy}(p,q)\Phi(x,y) \geq 0$ and $\sum_{X*X} m_{xy}(p',q')\Phi(x,y) < 0$, which implies that $p \succ q$ and $q' \succ q'$. If the individual perceives $m(p,q)$ and $m(p',q')$, as stated in the introduction, then by Proposition 1,

$$p \succ q \Leftrightarrow 0.6\Phi(0,0) + 0.2\Phi(0,3000) + 0.15\Phi(4000,0)$$
$$+0.05\Phi(4000,3000) > 0,$$

and

$$p' \prec q' \Leftrightarrow 0.75\Phi(0,0) + 0.05\Phi(0,3000) + 0.2\Phi(4000,3000) < 0.$$

It is easy to verify that there are values of Φ such that these inequalities hold. For example, if $\Phi(0,0) = 0$, $\Phi(0,3000) = -100$, $\Phi(4000,0) = 200$, $\Phi(4000,3000) = 20$, then $p \succ q (11 > 0)$ and $p' \prec q'$ $(-1 < 0)$.

The Common Ratio Paradox

Consider the following four lotteries:

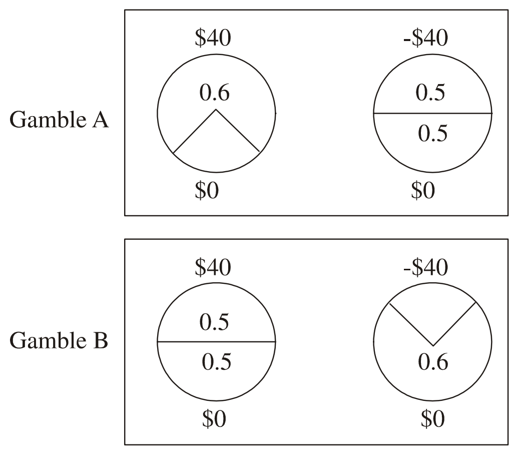
$$p = (\$4000, 80\% \ ; \ \$0, 20\%) \qquad q = (\$3000, 100\% \ )$$

$$p' = (\$4000, 20\% \ ; \ \$0, 80\%) \qquad q' = (\$3000, 25\% \ ; \ \$0, 75\% \ )$$

Many individuals making a choice between p and q and between p' and q' prefer q to p and p' to q', violating the standard independence axiom $\mathbf{I}^s$ (Kahneman & Tversky, 1979). However, in our model, such behavior is not paradoxical. The explanation of the observed behavior is essentially similar to that rationalizing the individual's behavior in the case of the Two-Phase Lotteries paradox.

The Duplex Gamble Paradox

Due to alternative modes of lottery representation or "framing", individuals choosing between two probabilistically equivalent lotteries may strictly prefer one lottery to the other. For example, Payne & Braunstein (1971) used pairs of gambles that involve spinning the pointers on both "gain" and "loss wheels", with the gambler receiving the resulting sums. Two such gambles, A and B, are presented below.



Although A and B have an identical underlying distribution ($40,30% ; $0,50% ; $-40,20%), individuals tend to choose gamble A – the one with the greater probability of gain. If there is greater probability of losing than winning, individuals tend to choose the gamble with the lower probability of loss. Once again, one can easily find combinations of a subjective dependence matrix $m(A, B)$ and a value function $\Phi(x, y)$ that give rise to the observed behavior, which means that in our proposed setting these phenomena can be easily rationalized.

Non-Transitive Preferences

Several researchers, May (1954), Tversky (1969), and Fishburn (1970), among others, have collected a large body of evidence to indicate that individual behavior does not conform with the transitivity axiom. Due to the non-separable representation of preferences by the function $\Phi(x, y)$, our model is consistent with non-transitive preferences. An interesting example that is based on Fishburn (1987) is presented below. An individual is asked to make pair-wise choices between every two successive lotteries contained in the following table:

Lottery	Probability					
	1/6	1/6	1/6	1/6	1/6	1/6
L_1	100	200	300	400	500	600
L_2	200	300	400	500	600	100
L_3	300	400	500	600	100	200
L_4	400	500	600	100	200	300
L_5	500	600	100	200	300	400
L_6	600	100	200	300	400	500

Note that lottery L_2 is similar to L_1 except that the payoffs are all "rotated". Lotteries L_{j+1} *and* L_j, $j = 2, \ldots, 5$, are defined by a si milar successive payoff rotation. Now suppose that the individual pe rceives complete statistical dependence among the columns of the table. In particular this means that $m(L_2, L_1)$ is given by:

		L_1					
		100	200	300	400	500	600
	100						1/6
	200	1/6					
L_2	300		1/6				
	400			1/6			
	500				1/6		
	600					1/6	

And the matrices $m(L_{j+1}, L_j)$, $j = \{2,3\ldots5\}$, as well as $m(L_6, L_1)$ have a similar form.

Now let $\Phi(x, y) = c$, $c \neq 0$, when $x - y = 100$, $(\Phi(x, y) = -c$ when $x - y = -100)$, and $\Phi(x, y) = tc$ when $x - y = 500$ $(\Phi(x, y) = -tc$ when $x - y = -500)$. In such a case the expected value of the interchange between the lotteries L_{j+1} and L_j, $j = \{2,3,4,5\}$, as well as between the lotteries L_6 and L_1 is equal to $5/6c + 1/6(-tc)$. By Proposition 1, if $5/6c + 1/6(-tc) = 0$, that is, if $t = 5$, then the individual is indifferent about the lotteries. If $t < 5$, the individual would reveal the cyclical choice pattern: $L_2 \succ L_1, L_3 \succ L_2, \ldots, L_1 \succ L_6$, while if $t > 5$ the individual would reveal the reverse cyclical choice pattern: $L_1 \succ L_2, L_2 \succ L_3, \ldots, L_6 \succ L_1$. Hence, under the assumed statistical dependence among the lotteries and the assumed value function, two opposite cyclical choice patterns are possible in our setting, although the six simple lotteries share the same underlying probability distribution.

Preference Reversal

A special case of Non-Transitive Preference examined extensively in the literature is “Preference Reversal”, whereby lotteries assigned lower certainty equivalents are subsequently preferred over those associated with higher ones (Lichtenstein & Slovic, 1973; Grether & Plott, 1979; Tversky et al., 1990, and Loomes et al., 1989,1991). Preference reversals are most often observed in the “P”- “$” scenario. Individuals tend to choose option “P” (the high-probability low-payoff lottery) over “$” (the low- probability high-payoff lottery) in a pair-wise choice, however, they continue to assign a higher price to the “$” lottery.

For example, consider “P”= ($30,90%; $0,10%) and “$”=($100,30%; $0,70%). Individuals tend to prefer “P” over “$” although they report that “P” ~ $25 and “$” ~ $27. Another way of illustrating this phenomenon is by confronting individuals with a pair-wise choice between two of the following lotteries:

Lottery	40%	20%	40%
“C”	6	6	6
“P”	0	10	10
“$”	0	0	15

As in the former experiment, individuals tend to make cyclical choices that seem to prove that transitivity is violated. In fact, the behavior of some individuals seems to imply that $(C \succ P,\ P \succ \$$ and $\$ \succ C)$ and the behavior of other individuals seems to imply the reverse cyclical pattern, that is, $(C \prec P,\ P \prec \$$ and $\$ \prec C)$. Note that if convexity of the value function Φ is assumed, models that deal with non-transitive preferences like Regret Theory or SSA, can account only for the first cyclical pattern. In our proposed model, since the individual's behavior hinges also on his SSDM *m*, even if convexity of Φ is assumed, both cyclical patterns are possible. As mentioned above, Preference Reversal can be viewed as a special case of non-transitive preferences and can therefore be rationalized in our model.

Violations of Stochastic Dominance

Within our proposed setting, in general, the Stochastic Dominance Condition need not be satisfied. Consider, for example, the situation studied by Tversky & Kahneman (1986) where individuals had to make a choice between the following lotteries *p* and *q* and between p' and q':

p	90%	6%	1%	1%	2%
	\$0	**win \$45**	**win \$45**	**lose \$10**	**lose \$15**
q	90%	6%	1%	1%	2%
	\$0	**win \$45**	**win \$30**	**lose \$15**	**lose \$15**

p'	90%	7%	1%	2%
	\$0	**win \$45**	**lose \$10**	**lose \$15**
q'	90%	6%	1%	3%
	\$0	**win \$45**	**win \$30**	**lose \$15**

Lottery *p* stochastically dominates lottery *q*, and lottery $p'(q')$ has the same underlying distribution as lottery *p* (*q*). All the individuals preferred *p* to *q*, while 58% of the individuals preferred q' (the inferior lottery) to p'. If an individual perceives statistical dependence between *p* and *q* according to the columns of the table, that is, assume that s/he gets \$0 in lottery *p* iff s/he receives \$0 in lottery *q*, and so on with respect to the prizes corresponding to the other columns, then in our setting, by the dominance axiom, s/he must prefer p to q. If the individual does not perceive the same statistical dependence between p' and q' and between p and q, especially if p' is estimated to yield a \$10 loss iff q' yields a \$30 gain, then even if p' dominates q', according to the model, he or she may prefer q' to p'. That is, there exist combinations of an SSDM m and a value function Φ that give rise to the preference of q' over p'.

V. SUMMARY AND DISCUSSION

This paper proposes a descriptive theory of choice under risk based on a new axiomatic approach to the modeling of lottery framing. In the proposed model, decision-making is perceived as a two-phase process. Facing a choice between two lotteries, the individual first forms a Subjective Statistical Dependence Matrix (SSDM) between the two lotteries that depends on their description and on his or her subjective perceptions. Since framing is an integral part of lotteries that can affect the formation of the SSDM, in this model lotteries characterized by the same underlying probability distribution, yet different framing, are considered different. In a second phase the individual chooses one of the lotteries by comparing pairs of statistically dependent prizes that correspond to the lotteries. Under four axioms regarding the individual's basic preference relation over SSDMs, his or her preferences over lotteries are shown to be represented by the expected value of the interchange between the lotteries.

The proposed model bears some resemblance to two types of “Non-Expected Utility” models. One type includes models relaxing the transitivity assumption (see, for example, Bell, 1982; Loomes & Sugden, 1982,1987; Fishburn, 1982, 1984a, 1984b, 1987, 1990; Sugden, 1993, and Nakamura, 1998) and attributing relative and not absolute values to prizes. That is, individuals choosing between two lotteries do not value each prize independently but relative to what they would have received had they chosen the other lottery. The main difference between these models and ours is the source of the statistical dependence. In Fishburn’s (1982, 1984a) SSB (skew-symmetric bilinear) model, the lotteries are assumed to be statistically independent. In Loomes & Sugden's (1982) and Bell's (1982) Regret Theory, as well as in Fishburn's (1984b, 1987) SSA (skew-symmetric and additive) model, statistical dependence is assumed to be 'objective' and dependent on the states of the world. In our model, individuals may assume any subjective statistical dependence between the lotteries, which may depend on their description. In particular, statistical independence between the lotteries can be assumed. In such a case our representation result reduces to Fishburn's result in the SSB model. That is,

$$p \succsim q \Leftrightarrow \sum_{X*X} p(x)q(y)\Phi(x,y) \geq 0\,.$$

If the value function Φ is assumed to be separable, then our model reduces to the standard expected utility model. That is, the representation result takes the form:

$$p \succsim q \Leftrightarrow \sum_{x\in X} p(x)u(x) - \sum_{y\in X} q(y)u(y) \geq 0\,.$$

The other type of models (see, for example, Kahneman & Tversky, 1979; Tversky & Kahneman, 1992; Chateauneuf & Wakker, 1999, and Rubinstein, 1988) perceive decision-making as a two-phase process: an editing phase and a valuation phase. The difference between these models is embedded in the underlying principles of these phases. In Tversky (1972), Kahneman & Tversky (1979), and Tversky & Kahneman (1981,1986) the decision-making process hinges on the manner in which the choice problem is presented as well as on norms, habits and expectancies of the decision maker. In particular, the following principles or 'editing operations' were examined:

Coding: perceiving lottery prices in term of gains and losses from some reference point.

Cancellation: disregarding identical parts in the compared lotteries.

Isolation (pseudo certainty): this special case of *Cancellation* means that identical parts in compound lotteries are disregarded while tending to prefer the compound lottery with the pseudo certainty prize (as in the two stage lottery paradox).

Combination: combining the probabilities associated with identical outcomes. For example, the lottery (200,25%; 200,25%; 0,50%) is reduced to (200,50%; 0,50%).

Rounding probabilities or outcomes when convenient.

Weighting probabilities: probabilities are transformed; the individual over-weigh small probabilities and under-weigh large probabilities.

Rubinstein (1988) introduced a *similarity* principle claiming that when there is similarity between probabilities and/or prizes, the individual's choice is based on the probabilities/ prizes that are not similar.

In the proposed model, in the editing phase the Subjective Statistical Dependence Matrix that reflects the effect of framing is formed. Not specifying how this matrix is related to framing implies that we can allow ample flexibility with respect to the relationship between the SSDM and the underlying principles or editing operations on which it is based. Some of the above editing operations can be conveniently presented in our framework. For example, *Cancellation* of identical parts of two lotteries can be captured by locating the corresponding joint probability on the diagonal of the SSDM. Similarly, *Isolation (pseudo certainty)* can be taken into account by locating the joint probability of the identical parts of two compound lotteries on the diagonal of the SSDM and by assuming complete statistical dependence between their non-identical parts. *Similarity* in our model can take the form of statistical dependence between similar probabilities/prizes. Other principles like *Coding* that stresses the significance of a single subjective reference point can also be captured by the proposed model, provided that the prizes in the lotteries are defined in terms of gains and losses relative to that reference point. However, principles like *Rounding* and *Weighting probabilities* are inconsistent with the proposed model; in the proposed model probabilities and prizes are undistorted.[4]

[4] To emphasize the difference between the basic features of our model (in particular, the explicit account of how the individual perceives statistical dependence between the compared lotteries) and those of related models, let us consider, for example, the issue of "probabilistic insurance" introduced by Kahneman & Tversky (1979). "Probabilistic insurance" differs from regular insurance by adding the risk that in the event the hazard occurs, the insurance company does not pay the due payment with some probability α. It has been shown (Wakker, Thaler & Tversky, 1997), that even for very small α, people are extremely risk averse and, contrary to the prediction

Although many models of decision-making under risk have been proposed, there is no widely accepted model resolving all the observed paradoxes (Machina, 1987; Loomes, 1999; Starmer, 2000). The proposed model is unique in solving such paradoxes in both scope and method: it is consistent with all seven paradoxes mentioned above. Most of the "Non-Expected Utility Models" are consistent with the paradoxes violating the standard independence axiom, such as the Allais Paradox. Some models are consistent with non-transitive preferences, while others, like "Prospect Theory", rationalize behavior that takes into account various types of framing effects. None of these models can account for all the paradoxes on which we have focused.

We have proposed a new approach to taking framing into account. If choice is affected not only by the underlying probability distributions of the possible prizes, but also by the description of the compared lotteries, it is very difficult and it may even be impossible to find a formal general definition of a lottery (simple lottery, compound lottery, n-tuples, etc.) that relates to all of its characteristics. In the proposed model, this problem is overcome by setting the Subjective Statistical Dependence Matrix as the primitive of the model and by defining the individual's basic preference relation on such SSDMs. By adopting this approach we avoid the introduction of a formal general definition for "lotteries" and, in turn, the need to impose any restrictions on their nature. This allows the proposed model to be consistent with phenomena not explained by other

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based on the standard expected utility theory, their willingness to pay for the probabilistic insurance is considerably reduced relative to their willingness to pay for the standard insurance. Segal (1988) showed that this phenomenon is consistent with rank-dependent utility models; that is, models (including prospect theory) that allow a weighting function of probabilities and, in particular, over-weighting of small probabilities. An alternative approach could be based on subjective statistical dependence. Suppose that the individual believes that "when it rains it pours" and, similarly, that "troubles come in bunches", that is, if the house burns down, then the unfortunate event of not receiving the due payment from the insurance company is realized (the probably of this event is α). If complete statistical dependence between those two events is perceived, the value of the probabilistic insurance is equal to \$0. In the less extreme case, where some statistical dependence between those two events is perceived, the probabilistic insurance is worth to the individual only a fraction of the original price he is ready to pay for the standard insurance policy. Although the "probabilistic insurance" phenomenon is not modeled in our current study (for simplicity, we assumed subjective statistical dependence between two lotteries, but not within the components of the lottery), we suggest that the possibility of subjective statistical dependence rather than distorted probabilities should be studied as the possible cause of the "probabilistic insurance" phenomenon.

models such as changes in preferences when decision problems are presented in different forms, for example, using tables, graphs, or just verbally (Moskowitz, 1974; Keller, 1985; Harless, 1992 and Humphrey, 2000,2001).

One may argue that the proposed model is too flexible, allowing for “irrational” or “implausible” preferences, such as non-transitive preferences or allowing choice of stochastically-dominated lotteries. Non-transitive preferences are a well-documented phenomenon and various researchers claim that its existence can be rationalized within the scope of rational behavior (Loomes & Sugden, 1982; Fishburn, 1984, and Starmer & Sugden, 1998). As for stochastically-dominated choices, on one hand, if one alternative is perceived as stochastically dominating the other, then according to the proposed model, the decision-maker has to choose the dominant one. On the other hand, if he or she fails to recognize that an alternative is stochastically dominant, he or she may choose the inferior alternative. Such behavior can be described, at worst, as boundedly rational but not irrational. In any event, the proposed theory constitutes another link in the long chain of attempts to develop a reasonable unconventional general descriptive theory that offers a consistent account of observed behavior such as cyclical choice or choice of stochastically-dominated lotteries.

Finally, one may argue that the proposed model is too flexible and therefore its explanatory and predictive strength is limited. Our response is that indeed our proposed basic framework requires further study, firstly, of particular statistical dependence patterns and their implications in terms of the individual’s predicted behavior and, secondly, of experimental work that examines the relationship between particular forms of framing and such particular statistical dependence patterns (subsets of dependency matrices).

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