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OPTIMAL R&D COMPETITION AND ECONOMIC GROWTH*

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The optimality of R&D competition and its implications on growth and welfare are analyzed in a dynamic general equilibrium model. By explicitly incorporating pre-emptive R&D competition, the relationship between the intensity of competition and the growth rate can be investigated. This paper shows that in the competitive growth path, the number of research firms is larger and the rate of growth is lower than in the optimal growth path. By reducing entry into R&D competition by imposing an entry fee from which the winner of competition is subsidized, this sub-optimality arising from excessive competition could be overcome.

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I. INTRODUCTION

In most R&D based endogenous growth theories, only the aggregate level of R&D activities matters, due to their simplifying assumptions. In Romer (1987, 1990) an innovation occurs in a deterministic way to anyone who invests in R&D. In Segerstrom (1991) and Grossman and Helpman (1991a) a research firm is indifferent to entry by additional researchers or to changes in the effort levels of its competitors, because the probability per unit time of a successful innovation is proportional to the aggregate R&D investment. The same argument can be applied to

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Aghion and Howitt (1992), even though linearity is replaced by constant returns to scale in their model because two types of inputs are used in R&D.

Consequently, the roles played by market rivalry and competition among research firms in the determination of technological progress and economic growth have not been investigated thoroughly. This, however, can be misleading because market rivalry and competition determine the economic opportunities and constraints that research firms face when engaging in R&D activities. In fact, previous growth models cannot relate the intensity of R&D competition or the number of competitors in the R&D competition to the research firms' incentives to innovate, and thus to the rate of economic growth. In particular, they cannot provide insights concerning the social optimality of R&D competition and related R&D policies. On the other hand, a substantial number of papers analyzing R&D competition among firms and its implications for social welfare have been published in the field of industrial organization. The seminal work of D'aspremont and Jaquemin (1988), and many other papers built upon it, deal with such issues. Implications of their results, however, cannot be directly applied in the dynamic general equilibrium context. For example, we do not know from those studies how over-competition or under-competition in the R&D process is linked to the rate of economic growth.

In fact, the works by Peretto (1996, 1998, 1999) are the few exceptions that paid attention to the dependency of economic growth on the market rivalry and competition among individual firms. But his main concern is the interdependency of product market structure and economic growth when producers conduct in-house R&D. That is, the nature of the separate effect of R&D rivalry and competition on economic growth could not be investigated in his models. Moreover, the issue concerning the optimality of R&D competition was not dealt with in his studies.

This paper provides a dynamic general equilibrium model where endogenous technological progress is driven by pre-emptive R&D competition among research firms. The R&D competition takes the form of a simple patent race game[1], where a fundamental innovation is

[1] Loury (1979), Lee and Wilde (1980) and Reinganum (1982) pioneered the modeling of a

distinguished from the commercial application that is developed from the innovation. The patent protection over the innovation secures a monopoly position for the race winner in developing a new commercial application, which is sold to manufacturers in exchange for royalty fees. For example, DSL (Digital Subscriber Line) technology, which is an innovative technology for transmitting data over the phone line, is a fundamental innovation. Developing a blueprint for a new type of device in which this innovative method of transmission is realized is another stage of the R&D process. Successfully developed blueprints will be sold to device manufacturers in exchange for royalty fees.

In this R&D game, the losers' investments will be wasted, but not totally. That is, the investments will create an externality in that they will contribute to enhancing the productivity of the winner in developing the new commercial application. Therefore, the intensity of competition will affect the efficiency of the R&D sector both negatively and positively. By incorporating this preemptive R&D process into the model, the intensity of the competition or the number of research firms are related to the efficiency of R&D, and thus the optimality of economic growth. In case the decentralized equilibrium proves to be sub-optimal, designing a tax-subsidy scheme that induces optimal intensity of competition is also feasible using the model.

The R&D process of the model also effectively eliminates scale effects of economic growth. In R&D based endogenous growth models mentioned above, as the population rises, so does the rate of technological progress and the growth rate of per capita output. The size of an economy and its effect on growth is referred to as the "scale effect". Empirical studies by Jones (1995a), however, suggest that postwar U.S. data apparently lacks of such a scale effect. That is, economic growth has not accelerated in spite of substantial increase in R&D inputs over time. The findings of Kortum (1997) and Segerstrom (1998) also support that of Jones (1995a). These results motivate Jones (1995b), Young (1998), Segerstrom (1998), Howitt (1999) and so on to develop R&D based endogenous growth models without scale effects. Jones (1995b) eliminates scale effects by weakening the assumption of strong

patent race. See Reinganum (1989) for an extensive survey of this subject.

intertemporal spillovers in innovative activities in the previous models but can not allow for sustained growth. Young (1998) and Howitt (1999) solve the scale effect puzzle by assuming that the increase in size of an economy leads to product proliferation, which dissipates the increased reward to innovate. Segerstrom (1998) accounts for the same puzzle by assuming that R&D becomes progressively more difficult over time. In my paper, scale effects are eliminated by the underlying assumption that the larger the economy the more difficult it is, and the more resources it requires, to enhance the overall productivity.

The basic specifications of the model are presented in Section 2. A unique stable stationary equilibrium of the model is established in Section 3. In Section 4 the sub-optimality of the market equilibrium and a policy tool for correcting it will be discussed.

II. MODEL

II.1 Producers

The producers' side of the model consists of three sectors: a high-technology (hi-tech) goods sector, a traditional goods sector, and an R&D sector.

II.1.1 Final goods sectors

Firms in the hi-tech sector hire human capital factors (H) as well as intermediate inputs (M) to produce goods (Y), which are used both as intermediate inputs and as consumption goods. The model abstracts from ordinary capital accumulation because capital formation has a supportive role in the growth process in that it is not the source of self-sustained growth. The productivity of the hi-tech sector can be improved by adopting state-of-the-art technology for which a royalty fee (T) is paid to the winner of the patent race in the R&D sector. The aggregate production function of the hi-tech sector and the maximization problem of a typical firm are

$$\underset{H_t^Y, M_t^Y}{Max} \left(P_t^Y - T_t\right)Y_t - P_t^Y M_t^Y - W_t^H H_t^Y \quad \textit{subject to}$$
$$Y_t = A_t \left(M_t^Y\right)^{\alpha} \left(H_t^Y\right)^{1-\alpha}. \tag{1}$$

Let V denote the size of total population that is given exogenously. For simplicity assume that no population growth occurs. Equation (1) can be rewritten as follows:

$$\underset{h_t^Y, m_t^Y}{Max} \left[\left(P_t^Y - T_t\right)y_t - P_t^Y m_t^Y - W_t^H h_t^Y\right]V \quad \text{subject to}$$
$$y_t = A_t \left(m_t^Y\right)^{\alpha} \left(h_t^Y\right)^{1-\alpha}. \tag{2}$$

Small letter variables stand for the per capita amount of the corresponding capital letter variables. For example, we have $y_t \equiv \frac{Y_t}{V}$. The royalty fee T is charged per unit of output and is endogenously determined in the R&D sector.

Firms in the traditional goods sector hire intermediate inputs and unskilled labor (L) to produce goods (X) that are used solely as consumption goods. The productivity of the traditional sector is influenced by old technology, which is available without the payment of a royalty fee. Note that this implies that the property right is guaranteed just for one period.

Let $P_t^X = 1$ for normalization. In terms of per capita variables, the maximization problem facing a typical firm is given as

$$\underset{l_t, m_t^X}{Max} \left[x_t - P_t^Y m_t^X - W_t^L l_t\right]V \quad \textit{Subject to} \quad x_t = A_{t-1} \left(m_t^X\right)^{\alpha} \left(l_t\right)^{1-\alpha} \tag{3}$$

where $\gamma_t = A_t / A_{t-1} > 1$ is the gross rate of productivity improvement from using the new technology existing at time t. This will be determined endogenously in the R&D sector.

Note that this specification of the production functions is special in that the two sectors are equally intensive in their use of the intermediate input. This not only makes the model tractable but also ensures the existence of a balanced growth path. Without this assumption, the importance of one sector would eventually disappear from the long-run equilibrium[2].

II.1.2 R&D sector and human capital allocation.

R&D activities are carried out in a separate sector and take the form of a simple patent race game that occurs in each period. In this race, a fundamental innovation is distinguished from the commercial application that is developed from the innovation. Corresponding to this distinction, the R&D process is conceptually decomposed into two stages: the patent race stage and the technology development stage. Firms that decide to enter the R&D sector should commit to three types of investment: two for the patent race stage, and one for the technology development stage conditional on winning the patent race. The two types of investment committed to the patent race stage are a fixed amount of entry investment $\overline{H}_F$, which is used to create an innovation, and variable amounts of competitive investment H_S^i, $i=1,\cdots,N$, which is required to beat competitors in the patent race. The size of the competitive investment of each firm, H_S^i, $i=1,\cdots,N$ determines the stochastic structure of the race. Each firm tries to maximize its expected profit given the number of firms in the race, and the zero expected profit condition determines the number of firms participating in the race. Given N, the expected profit maximization problem facing each firm can be expressed in terms of per capita variables as follows:

$$\begin{aligned}&\underset{h_S^i}{Max}\left\{\phi_i\left[\pi - W_H\left(h_S^i + \overline{h}_F\right)\right] - \left(1-\phi_i\right)\left[W_H\left(h_S^i + \overline{h}_F\right)\right]\right\}V \\ &= \left[\phi_i\pi - W_H\left(h_S^i + \overline{h}_F\right)\right]V\end{aligned} \qquad (4)^3$$

where ϕ_i is the probability of firm i winning the race, which is determined from the distribution of the H_S^i s. Specifically, we assume

[2] Similar arguments can also be found in Grossman and Helpman (1991b, Chapter 6).

[3] In what follows, for notational simplicity, time subscripts are omitted and superscripts are changed to subscripts (unless to do so would cause confusion).

$$\phi_i = \frac{H_S^i}{\sum_{j=1}^{N} H_S^j} = \frac{h_S^i}{\sum_{j=1}^{N} h_S^j}. \tag{5}$$

The term π is the monopoly profit of the race winner, say Π, divided by the population size, i.e. $\pi = \Pi / V$. The first-order condition for (4) becomes

$$\frac{\sum_{j=1}^{N} h_S^j - h_S^i}{\left(\sum_{j=1}^{N} h_S^j\right)^2} \pi - W_H = 0 \tag{6}$$

A monopolist winner develops a commercial application in the technology development stage with the following R&D function:

$$\gamma_t - 1 = \left[\sum_{j=1}^{N} \left(\bar{h}_F + h_S^j\right)\right]^{\delta} \left(h_R\right)^{1-\tau}, 0 < \tau < 1,\ 1 - \tau + \delta < 1. \tag{7}$$

Let $d \equiv \sum_{j=1}^{N} \left(\bar{h}_F + h_S^j\right)$ be the per capita amount of investments made in the patent race stage by all R&D firms[4]. The term h_R is the per capita amount of investment committed to developing the commercial application by the race-winning innovator. The term δ can be interpreted as the degree of the externality that is generated from the investments of research firms. It turns out that the constraint $1\text{-}\tau + \delta < 1$ or $\tau > \delta$ is required for the model to have an economically meaningful equilibrium[5]. It also turns out that this constraint implies that marginal productivity should be diminishing with respect to the increase of overall human capital inputs in the technology development stage. This

[4] The spirit of this specification is similar to that of rent seeking contest where the prize of contest increases with aggregate efforts of all participants. Refer to Chung (1996) for the rent seeking contest of this type.

[5] See the first paragraph of Appendix 1.

restriction is inherited from properties of the model but is a reasonable one in light of findings by Jones (1995a), Kortum(1993, 1997) and Segerstrom(1998) that show diminishing R&D performance per R&D inputs, measured by the number of patents per scientists and engineers employed in the R&D sector.

Note also that the magnitude of productivity improvement in the hi-tech sector that are achieved by adopting the new commercial application depends not on the total amount but on the per capita amount of investments undertaken by the firms in the R&D sector. This underlies the assumption that the larger the economy the more difficult it is, and the more resources it requires, to enhance the overall productivity. In fact, this assumption effectively removes scale effects. The reality of the assumption that might be in dispute depends on how closely the overall improvement of productivity in an economy is related to the per capita amount rather than the total amount of R&D investment.

We consider only a symmetric equilibrium in which every firm in the hi-tech sector adopts the new commercial application. Because the monopolist that wins the R&D race sets T at the highest level possible, provided firms in the hi-tech sector adopt the new technology, the reduction of unit costs achieved by using the new technology equals the royalty payment per unit, i.e., T, at the margin. Therefore, from the first-order conditions for (2), we have

$$T_t = Z_1\left(P_t^Y\right)^{\alpha}\left(W_t^H\right)^{1-\alpha}\left(A_{t-1}^{-1} - A_t^{-1}\right) = \left(P_t^Y - T_t\right)\left(\gamma_t - 1\right),$$
$$Z_1 = \alpha^{-\alpha}\left(1-\alpha\right)^{(1-\alpha)}. \tag{8}$$

From (1) and (8) we have

$$TY = \left(\gamma - 1\right)\left(P - T\right)Y = \frac{W_H H_Y}{1-\alpha}\left(\gamma - 1\right). \tag{9}$$

Therefore, the race winner that develops the new commercial application based on its innovation faces the following optimization problem:

$$\underset{h_R}{Max}\left[\frac{W_H h_Y}{1-\alpha}(\gamma-1)-W_H h_R\right]V \quad \textit{subject to} \quad \gamma-1=d^{\delta}\left(h_R\right)^{1-\tau} \tag{10}$$

Let us consider only the symmetric equilibrium, in which each firm undertakes the same level of competitive investment H_S. Then, from the first-order condition for (10), we have

$$h_R=\left(\frac{(1-\tau)}{(1-\alpha)}d^{\delta}h_Y\right)^{\frac{1}{\tau}} \qquad \text{and} \tag{11}$$

$$\pi=\frac{\tau}{1-\tau}W_H h_R\,. \tag{12}$$

From the zero expected profit condition for (4) we have

$$d=h_R\cdot\frac{\tau}{1-\tau}\,. \tag{13}$$

Plugging (13) into (7) shows that the constraint $1\text{-}\tau+\delta<1$ implies diminishing marginal productivity of human capital in technology development. With (11) and (13) we have the following equation that shows how human capital is allocated between the hi-tech sector and the R&D sector:

$$h_R+d=Zh_Y^{\frac{1}{\tau-\delta}}, \quad Z=\frac{1}{1-\tau}\left(\frac{1-\tau}{1-\alpha}\right)^{\frac{1}{\tau-\delta}}\left(\frac{\tau}{1-\tau}\right)^{\frac{\delta}{\tau-\delta}}. \tag{14}$$

II.2 Consumers

Unskilled labor and human capital are determined as a result of consumers' optimizing decisions over a two-period lifetime. To acquire human capital, each individual must take part in a costly schooling process when he or she is young, and must pay tuition fees and invest time during that process. Specifically, the model introduces a schooling sector that consists of teachers and students only, and ignores associated facilities. To obtain a meaningful equilibrium, the teacher–student ratio, denoted by φ, should be less than unity and greater than zero ($0<\varphi<1$).

We ignore management costs for this schooling system. Thus, the total tuition paid by students must equal the total wages paid to teachers. In a perfect consumption–credit market, each individual can make savings (S) or borrowings (B) at the market interest rate R. Thus, when young, those who want to participate in the labor force will save and those who want to obtain education will borrow. Each individual born at time t faces the following optimization problem:

$$\begin{aligned}
&Max\ {}_jU_t = \log\left\{\left({}_jC_{1,t}^X\right)^{\mu}\cdot {}_jC_{1,t}^Y\left[\left({}_jC_{2,t+1}^X\right)^{\mu}\cdot {}_jC_{2,t+1}^Y\right]^{\beta}\right\},\ j{=}H, L,\\
&0<\beta<1, \quad \textit{subject to}\ \ {}_HC_{1,t}^X + P_t\cdot {}_HC_{1,t}^Y = B_t - \varphi\cdot W_t^H,\\
&{}_HC_{2,t+1}^X + P_{t+1}\cdot {}_HC_{2,t+1}^Y = W_{t+1}^H - \left(1+R_t\right)\cdot B_t,\ if\quad j{=}H\\
&{}_LC_{1,t}^X + P_t\cdot {}_LC_{1,t}^Y = W_t^L - S_t,\\
&{}_LC_{2,t+1}^X + P_{t+1}\cdot {}_LC_{2,t+1}^Y = \left(1+R_t\right)\cdot S_t,\ if\quad j{=}L\,. \qquad (15)^{6}
\end{aligned}$$

Note that in (15), $\varphi\cdot W_t^H$ is the tuition cost per student. The term β represents a subjective time discount rate and μ turns out to be the ratio of expenditure on good X to expenditure on good Y. Subscripts 1 and 2 represent the young and old generations respectively. The index $j=H,L$ denotes the individual's career path as a unit of human capital or as an unskilled worker, between which each individual must be indifferent. This implies the following career arbitrage condition:

$$\begin{aligned}
&{}_LU_t = {}_HU_t \ \text{ or }\ {}_LC_{it}^G = {}_HC_{it}^G,\ {}_LC_{i,t+1}^G = {}_HC_{i,t+1}^G,\\
&i=1,2,\ \ G=X,Y\,. \qquad (16)
\end{aligned}$$

From the first-order conditions for (15) we have

$$\begin{aligned}
&{}_JC_{it}^X = \mu\cdot P_t\cdot {}_JC_{it}^Y \ \text{ or }\ C_{it}^X = \mu\cdot P_t\cdot C_{it}^Y,\\
&\text{where }\ C_{it}^G \equiv {}_HC_{it}^G + {}_LC_{it}^G,\ \ G=X,Y\,. \qquad (17)
\end{aligned}$$

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6 This specification of the consumer's problem is identical in nature to that in Eicher (1996).

Using (16) and (17) the optimal savings and borrowings can be derived from (15) as

$$S_t = \theta \cdot W_t^L , \quad B_t = \varphi \cdot W_t^H + \frac{1}{\beta} \cdot \theta \cdot W_t^L . \tag{18}$$

where $\theta = \dfrac{\beta}{1+\beta}$ can be interpreted as the marginal rate of savings. Note that $0 < \theta < \dfrac{1}{2}$.

II.3 Equilibrium

In equilibrium, the goods markets, the consumption credit market, and the labor markets are cleared in each time period. As for the credit market, in equilibrium, total savings equal total borrowings. This gives us

$$L_t \cdot S_t = H_{t+1} \cdot B_t \quad \text{or} \quad l_t \cdot S_t = h_{t+1} \cdot B_t . \tag{19}$$

The size of population born at each point of time is assumed to be constant and, thus, should be half the size of total population in our two period overlapping generations model. Therefore, we have the following

$$L_t + H_{t+1} = \tfrac{1}{2} \cdot V \quad \text{or} \quad l_t + h_{t+1} = \tfrac{1}{2} . \tag{20}$$

Then, with (18) – (20), the equilibrium paths for the relative wage and the intensity of human capital satisfy the following credit market equilibrium condition:

$$h_{t+1} = \frac{\tfrac{1}{2} \cdot \theta}{1 + \varphi \cdot \Delta_t} , \tag{21}$$

where $\dfrac{W_t^H}{W_t^L} \equiv \Delta_t$ is the relative wage of human capital.

For labor market equilibrium, human capital formed in each period should be employed in the hi-tech sector, the R&D sector or the schooling sector. Therefore, with (14), we have the following equation that shows the allocation of human capital among sectors:

$$\begin{aligned}&\varphi \cdot H_{t+1} + H_t^R + N \cdot \left(\overline{H}_t^F + H_t^S\right) + H_t^Y = H_t \Rightarrow \\ &Z\left(h_t^Y\right)^{\frac{1}{\tau-\delta}} + h_t^Y = h_t - \varphi h_{t+1}\end{aligned} \quad . \tag{22}$$

Let $C_{1t}^X + C_{2t}^X \equiv C_t^X$, $C_{1t}^Y + C_{2t}^Y \equiv C_t^Y$ and $M_t^X + M_t^Y \equiv M_t$. For the goods markets, we have

$$X_t = C_t^X , \;\; Y_t = C_t^Y + M_t . \tag{23}$$

The equilibrium path that satisfies the equilibrium conditions of the goods markets and the human capital market is

$$\Delta_t = \frac{\left(\frac{1}{\mu} + \alpha\right) \cdot \left(\frac{1}{2} - h_{t+1}\right)}{\left(1\text{-}\alpha\right) \cdot \left(h_t - \varphi h_{t+1}\right)} \tag{24}$$

The derivation of (24) is given in Appendix 1.

The perfect-foresight equilibrium of the model consists of the sequence $\left\{h_t, 0 \le h_t \le \frac{1}{2} \cdot \theta\right\}_0^\infty$ satisfying (21) and (24). Therefore, given the initial condition $0 < h_0 < \frac{1}{2} \cdot \theta$, the perfect-foresight equilibrium of the model is given by the following first-order quadratic difference equation:

$$h_{t+1} = \frac{\frac{1}{2} \cdot \theta}{1 + \varphi \cdot \dfrac{\left(\frac{1}{2} - h_{t+1}\right) \cdot \left(\frac{1}{\mu} + \alpha\right)}{\left(h_t - \varphi h_{t+1}\right) \cdot \left(1\text{-}\alpha\right)}} \tag{25}$$

A stationary equilibrium corresponds to a perfect-foresight equilibrium with constant h_t, defined as h^*. The existence of multiple stable stationary equilibria is established in the following two propositions whose proofs are provided in Appendix 2.

Proposition 1a. Given the necessary and sufficient condition $\frac{\varphi}{1-\varphi}\cdot\left[\frac{\frac{1}{\mu}+\alpha}{1-\alpha}\right]<\theta$, a unique positive stable stationary equilibrium intensity of human capital (h^*) exists such that $h_t = h_{t+1} = h^*$, $0 < h^* < \frac{1}{2}\cdot\theta$, where $\left\{h_t, 0 \le h_t \le \frac{1}{2}\cdot\theta\right\}_0^\infty$ is a perfect-foresight equilibrium for the model

Note that the condition $\frac{\varphi}{1-\varphi}\cdot\left[\frac{\frac{1}{\mu}+\alpha}{1-\alpha}\right]<\theta$ is satisfied when the schooling cost (φ) is not too high relative to the marginal savings rate (θ), given the parameters underlying the structures of production and preference (α,μ). As long as this condition is satisfied in an economy, it will have a positive level of human capital in the steady state and its hi-tech sector is viable in the long run.

The transitional dynamics of relative wage can be trivially figured out from (21). When the condition $\frac{\varphi}{1-\varphi}\cdot\left[\frac{\frac{1}{\mu}+\alpha}{1-\alpha}\right]<\theta$ can not be satisfied in an economy, the human capital can not be formulated in the steady state even if it is initially endowed with a positive level of human capital. The hi-tech sector of the economy will be degenerated as a result. This result is formally stated and proved in the following proposition.

Proposition 1b. $h_t = h_{t+1} = h^* = 0$ always satisfies the system but is a stable equilibrium if, and only if, $\frac{\varphi}{1-\varphi}\cdot\left[\frac{\frac{1}{\mu}+\alpha}{1-\alpha}\right]\ge\theta$.

Hereafter, we assume that $\frac{\varphi}{1-\varphi}\cdot\left[\frac{\frac{1}{\mu}+\alpha}{1-\alpha}\right]<\theta$, which guarantees a unique positive stable stationary equilibrium intensity of human capital formation. In this stationary equilibrium, we have the following from the consumption credit market equilibrium:

$$h = \frac{\frac{1}{2} \cdot \theta}{1 + \varphi \Delta} \quad \text{or} \quad \Delta = \frac{1}{\varphi} \cdot \left(\frac{\frac{1}{2} \cdot \theta}{h} - 1 \right). \tag{26}$$

From the goods markets and the human capital market, we have

$$\Delta = \frac{\frac{1}{\mu} + \alpha}{(1-\alpha)\cdot(1-\varphi)} \cdot \left(\frac{\frac{1}{2}}{h} - 1 \right). \tag{27}$$

Equations (26) and (27) together determine the stationary equilibrium intensity of human capital (h^*) and the corresponding relative wage (Δ^*) as follows:

$$h^* = \frac{1}{2} \cdot \left[\frac{(1-\alpha)\cdot(1-\varphi)\cdot\theta - \varphi\cdot\left(\frac{1}{\mu}+\alpha\right)}{(1-\alpha)\cdot(1-\varphi) - \varphi\cdot\left(\frac{1}{\mu}+\alpha\right)} \right],$$
$$\Delta^* = \frac{(1-\theta)\cdot\left(\frac{1}{\mu}+\alpha\right)}{(1-\alpha)\cdot(1-\varphi)\cdot\theta - \varphi\cdot\left(\frac{1}{\mu}+\alpha\right)} \tag{28}$$

This stationary equilibrium also implies a balanced growth path for intermediate inputs, GDP and consumption. That is, along the stationary equilibrium, the growth rates of intermediate inputs, GDP and consumption are all stationary. The following proposition formally characterizes the balanced growth rate along the steady state equilibrium path:

Proposition 2a. Along the stationary equilibrium given in Proposition 1a, the growth rates of intermediate inputs, GDP and consumption are all stationary. Furthermore, each grows at the same rate, say λ^*. That is, along the stationary equilibrium, we have

$$\frac{GDP_{t+1}}{GDP_t} - 1 = \frac{P_{t+1}^Y \cdot Y_{t+1} + X_{t+1}}{P_t^Y \cdot Y_t + X_t} - 1 = \frac{C_{t+1}}{C_t} - 1 = \frac{M_{t+1}}{M_t} - 1 = \lambda^*, \tag{29}$$

where $C_t \equiv C_t^X + P_t \cdot C_t^Y$. This stationary rate of growth, λ^*, can be derived as a function of steady state intensity of human capital and is given by the following proposition.

Proposition 2b. The stationary rate of growth λ^* is expressed as follows

$$\lambda^* = \gamma^{*\frac{1}{1-\alpha}} - 1 = \left(g\left[(1-\varphi)\cdot h^*\right]+1\right)^{\frac{1}{1-\alpha}} - 1 \quad \text{where} \quad g\left[0\right]=0$$
$$\text{and} \quad g' > 0. \tag{30}$$

Appendix 3 provides proofs for these propositions. Intuitively, Proposition 1a and Proposition 2b imply that, given the parameters underlying the structures of production and preference (α, μ), schooling costs should not be too high relative to the marginal savings rate, i.e., $\frac{\varphi}{1-\varphi}\cdot\left[\frac{\frac{1}{\mu}+\alpha}{1-\alpha}\right] < \theta$ is required for an economy to have a positive rate of long-run growth. Otherwise, as implied by Proposition 1b, a no-growth trap will emerge.

Note also that scale effects do not exist, in that population growth would not change the rate of economic growth because only the human capital intensity in the population, namely $h^* \equiv \frac{H^*}{V}$, matters in the determination of the rate. From (28) and (30), it is straightforward to show the following.

Proposition 3. An increase of the marginal savings rate, a relative increase in expenditure on traditional goods, or a decrease in the schooling cost would raise the stationary rate of growth λ^*. That is, $\frac{\partial\lambda^*}{\partial\mu} > 0$, $\frac{\partial\lambda^*}{\partial\theta} > 0$, $\frac{\partial\lambda^*}{\partial\varphi} < 0$.

An increase in the marginal savings rate implies that future consumption becomes more valuable for consumers so that they are more willing to attend the schooling process, which raises human capital formation and improve the rate of economic growth. The intuition behind the effect of a lowering schooling cost on the economic growth is also similar. On the other hand, the increased relative expenditure on traditional goods raises the demand for low-skilled labor and hence lowers the relative wage for human capital. This makes schooling cheaper and stimulates human capital formation and economic growth.

III. EFFICIENCY OF R&D AND GOVERNMENT POLICY

Because technological progress is a fundamental ingredient of economic growth and welfare, governments in many countries tend to intervene in R&D activities. This intervention can be further justified if certain types of distortions exist in the R&D process that make R&D activities inefficient from the social standpoint. In this section we define a constrained optimal growth path and observe how the market results deviate from it.

III.1 Constrained optimal growth path.

In principle, we can obtain a benchmark social optimum by maximizing a well-defined social welfare function. However, in the presence of heterogeneous agents, any definition of a social welfare function is entirely arbitrary. Moreover, for an overlapping generations model, philosophical questions will always arise from the discounting of the intertemporal utility of generations[7]. Therefore, the goal of this section is not to investigate the overall optimality of the allocation of resources in the model constructed so far. Instead, the investigation is restricted only to the optimality of resource allocation in the R&D process along the balanced growth path. Due to difficulties associated with the choice of a social welfare function in models such as the one developed here, I turn to this weaker and narrower optimality criterion. This is relevant as long as the main concern is the efficiency of the R&D process in the steady state. Furthermore, this approach can at least provide qualitative policy advice.

Let us consider a problem facing the central planner who tries to

[7] See Blanchard and Fischer (1989, Chapter 3).

maximize the utility of a typical consumer[8] born at time *t* in the steady state, given a steady state level of human capital and an allocation of human capital between the hi-tech sector and the R&D sector. From (7), (15), (16) and proposition 2a, we can derive the utility of a typical consumer born at *t* in the steady state as follows:

$$\begin{aligned} U_t = {} & \frac{\beta(1+\mu)}{1-\alpha}\log\left(d^{\delta}h_R^{1-\tau}+1\right)+(1+\beta)\log\left(\left(C_{1t}^X\right)^{\mu}C_{1t}^Y\right) \\ & +(1+\beta)(1+\mu)\log\frac{1}{2} \end{aligned} . \tag{31}$$

Note that this is not stationary, because of the non-stationary consumption. Resource constraints facing the central planner in this specific problem are given as

$$d+h_R=(1-\varphi)h-h_Y\,, \tag{32}$$

$$C_t^X+PC_t^Y=V\left[W_t^L\left(\tfrac{1}{2}-h\right)+W_t^H(1-\varphi)h\right], \tag{33}$$

where h, h_Y, W_t^L, W_t^H and *P* are given. From the first order conditions, we have

$$\frac{d}{h_R}=\frac{\delta}{1-\tau} \tag{34}$$

$$\frac{C_{1t}^Y}{PC_{1t}^X}=\mu \tag{35}$$

Equation (34) gives the condition for the optimal allocation of human capital in the R&D process and (35) provides the condition for the optimal expenditure ratio. With (34) and (35) we define a constrained optimality in the sense that the utility of a typical consumer in the steady

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[8] Due to the career arbitrage condition, the steady state utility of a typical human capital factor should be equal to that of a typical unskilled labor factor.

state is maximized, given the constraints. From (13) and (17) we notice that the market results violate only (34).

Let us denote the rate of growth corresponding to this constrained optimal allocation by $\hat{\lambda}$. From (7) and Proposition 2b we have $\lambda^* < \hat{\lambda}$, where the term λ^* is the rate of growth corresponding to the market allocation given by (13). That is, the market equilibrium results in a lower than optimal rate of growth and a lower than optimal level of utility for a typical consumer. In this case, improving the rate of growth by optimizing the allocation of human capital in the R&D process will improve welfare.

Considering symmetry, we have the following from (6) and (12):

$$\left(1-\frac{1}{N}\right)\cdot\frac{h_R\cdot\frac{\tau}{1-\tau}}{N\cdot h_S}=1 \tag{36}$$

From (34) and (36), the number of firms corresponding to the optimal allocation of human capital in the R&D process is given as follows:

$$\hat{N}=\frac{\tau\left(h_S+\bar{h}_F\right)}{\tau\bar{h}_F+\left(\tau-\delta\right)h_S}\equiv N\left(\delta\right) \tag{37}$$

The number of firms corresponding to the market allocation given by (13) is $N(\tau)$. Note that $N(\delta)<N(\tau)$. Therefore, we have the following proposition.

Proposition 4. In the decentralized equilibrium**,** the number of research firms is strictly larger and the rate of growth is strictly lower than in the constrained optimal growth path.

Without the externality ($\delta=0$), the optimal number of R&D firms would be one as can be verified from (37). As the degree of externality becomes greater, it is socially desirable to have an appropriate level of R&D competition so that the optimal number of R&D firms becomes bigger than one. However, the social planner should take into account wasteful dissipation of rents as well as beneficial externalities from extra entrants into the R&D competition. In our dynamic general equilibrium model, the degree of externality from the R&D competition is limited by the restriction that the marginal productivity of the R&D sector should be diminishing with respect to overall R&D inputs in order for the model to have a meaningful equilibrium. Therefore, the optimal number of R&D firms is determined as the number for which the R&D sector can have positive profits, that is, rents associated with a new innovation are not fully dissipated. On the other hand, in the decentralized economy, entry continues to occur until the expected profits for entering the R&D sector become zero. As a result, the number of firms that enters the R&D sector becomes socially excessive in the decentralized economy.

The results do not critically depend on the specific form of R&D function (7) although it clearly helps to describe the equilibrium in the closed form. Instead, the main factor that leads to the above result is that the R&D sector in our model exhibits diminishing marginal productivity with respect to overall R&D inputs. As mentioned before, this restriction is required for the model to have a meaningful equilibrium but it is also a reasonable restriction judging from recent studies by Jones (1995a), Kortum (1993, 1997) and Segerstrom (1998). They all indicate that the productivity of R&D sector with respect to R&D inputs is apparently diminishing. Therefore, at least from a standpoint of a national economy, not of an individual industry, this restriction inherited from attributes of the model can be regarded as being realistic. Under such premise, the main result of the paper shows that socially excessive R&D competition is a natural outcome in a decentralized economy that has persuasive power.

III.2 Efficiency of the R&D process and government policy

An appropriate tax-subsidy tool can be designed to improve efficiency in the R&D process. Consider a tax-subsidy scheme that imposes an entry fee from which the winner of competition is subsidized. Let σ denote the subsidy rate. Then the problem facing the race winner becomes

$$\underset{h_R}{Max}\left[\frac{W_H h_Y}{1-\alpha} d^{\delta} h_R^{1-\tau} - (1-\sigma) W_H h_R\right] V\,, \quad \textit{subject to}$$

$$\gamma - 1 = d^{\delta}\left(h_R\right)^{1-\tau} \tag{38}$$

In per capita terms, the monopoly R&D firm's profit gross of setup cost will be given as

$$\pi = \frac{\tau}{1-\tau}(1-\sigma)W_H h_R . \tag{39}$$

The zero expected profit condition facing an entry firm becomes

$$\phi_i \cdot \left[\frac{\tau}{1-\tau}(1-\sigma)W_H h_R\right] - W_H \cdot \left[\left(\overline{h}_F + h_S\right) + \sigma\frac{h_R}{N}\right] = 0 \tag{40}$$

Note that a lump sum entry fee $\sigma\frac{W_H h_R}{N}$ is imposed on each firm entering the R&D sector. With (40) we have the following instead of (13):

$$\frac{\tau-\sigma}{1-\tau} = \frac{d}{h_R} \tag{41}$$

With (41) we have

$$h_Y \equiv \overline{f}\left[(1-\varphi)h,\sigma\right],\ \overline{f}_{\sigma} < 0 \tag{42}$$

$$\gamma - 1 = \overline{g}\left[(1-\varphi)h,\sigma\right],\ \overline{g}_{\sigma} > 0\,, \tag{43}$$

We can show that (27) still remains unchanged. That is, we have

$$\Delta = \frac{\frac{1}{\mu}+\alpha}{\left(1-\alpha\right)\cdot\left(1-\varphi\right)}\cdot\left(\frac{1/2}{h}-1\right),\ \frac{\partial\Delta}{\partial\sigma} = 0 \tag{44}$$

This is because the decreased amount of human capital employed in the hi-tech sector is exactly offset by the increased amount of human capital employed in the R&D sector. Note that (26) also remains unchanged.

Therefore, the tax-subsidy scheme considered here will not change the stationary equilibrium intensity of human capital and the relative wage but will alter the allocation of human capital in the R&D sector. From (34) and (41), it is straightforward to derive the optimal tax-subsidy scheme that leads to the optimal allocation of human capital in the R&D sector. This optimal tax-subsidy scheme is given by the following proposition.

Proposition 5. The subsidy rate ($\hat{\sigma}$) and the corresponding lump sum tax ($\hat{\Omega}$) that lead to the optimal allocation of human capital in the R&D process is given as $\hat{\sigma} = \tau - \delta = 1 - (1 - \tau + \delta)$ and $\hat{\Omega} = \hat{\sigma} \frac{W_H h_R}{N}$, where N is the number of entrants in the patent race who pay the tax.

With this tax subsidy scheme, the rate of growth is increased to the optimal rate and the intensity of R&D competition is diminished to the optimal intensity. Note that the term $(1 - \tau + \delta)$ can be interpreted as indicating the efficiency of the R&D sector. In fact, we have the following from (7) and (13).

$$\frac{\partial(\gamma - 1)}{\partial h_R} \cdot \frac{h_R}{(\gamma - 1)} = (1 - \tau + \delta) \tag{45}$$

That is, the term $(1 - \tau + \delta)$ is the elasticity of R&D performance with respect to R&D inputs. The term $(1 - \tau + \delta)$ becomes closer to one with more efficient R&D sector and the optimal subsidy rate becomes closer to zero.

IV. CONCLUSION

Most R&D-based growth theories are abstracted from market rivalry and competition among research firms by relating only the aggregate level of R&D investment to economic growth. Due to this, previous growth models cannot relate the intensity of R&D competition or the number of competitors to the rate of economic growth. In particular, they cannot provide any insights concerning the socially optimal intensity of R&D competition or related R&D policies.

This paper has provided a dynamic general equilibrium model that explicitly incorporates pre-emptive R&D competition and deals with the relationship between the intensity of competition and economic growth. To evaluate the optimality of competition in a decentralized economy, a concept of constrained optimality was adopted. It was found that in the decentralized equilibrium, the number of research firms is larger and the rate of growth is lower than in the constrained optimal growth path. That is, excessive R&D competition contributes to sub-optimality in the decentralized equilibrium. A constrained optimal tax and subsidy scheme that corrects this sub-optimality was also identified. The sub-optimality resulting from excessive competition could be overcome by reducing entries into the R&D sector by imposing an entry fee with which the winner of the competition is subsidized.

In fact, these results resemble those in the patent race literature including Loury (1979), Lee and Wilde (1980) and Reinganum (1982) where excessive number of firms compete in the competitive patent race in comparison with in the cooperative patent race. In these papers, cooperation among firms, like central planning in this paper, enables them to coordinate to reduce duplications and to internalize externalities resulted from R&D competition. However, these standard industrial organization studies cannot provide evaluation of such excessive competition from the perspectives of social welfare and economic growth. One of the main contributions of this paper is to illuminate how over-competition in the R&D process is linked to the rate of economic growth and social welfare.

In this paper, I did not fully exploit the implications of the dynamic general equilibrium model in general but I focused on the implications of the optimality in the resource allocation of the R&D process. I would like to conclude the paper by pointing out other interesting issues that could be explored in quite straightforward ways within the model developed so far.

Instead of describing the equilibrium mainly in terms of the level of human capital, I could have focused on the relative wage. In such case, the model could be used to examine determinants, dynamics and comparative static analyses of wage premium. In relation to this, the model could be used to exploit the implications of educational policies. For example, the effects of subsidizing students and schooling institutes on economic growth and wage premium could be analyzed. The model could also be easily extended to analyze open economy issues such as technology trade, global R&D competition, and growth convergence.

Appendix 1. Derivation of Eq. (24)

Let $k\left(h_t^Y\right) \equiv Z\left(h_t^Y\right)^{\frac{1}{\tau-\delta}} + h_t^Y - \left(h_t - \varphi h_{t+1}\right)$. Under the constraint $\tau > \delta$, we have $k(0) = -\left(h_t - \varphi h_{t+1}\right)$, $k\left(h_t - \varphi h_{t+1}\right) = Z \cdot \left(h_t - \varphi h_{t+1}\right)^{\frac{1}{\tau-\delta}}$ and $k'(\cdot) > 0$ within the interval $\left[0, h_t - \varphi h_{t+1}\right]$ so that equation (22) has a unique solution h_Y^* within the interval $\left(0, h_t - \varphi h_{t+1}\right)$ when $h_t - \varphi h_{t+1} > 0$, and $h_Y^* = 0$ when $h_t - \varphi h_{t+1} = 0$. (Note also that, in case of $\tau < \delta$, we have $k(0) = \infty$, $k\left(h_t - \varphi h_{t+1}\right) = Z \cdot \left(h_t - \varphi h_{t+1}\right)^{\frac{1}{\tau-\delta}}$, $k'(\cdot) < 0$ within the interval $\left[0, h_t - \varphi h_{t+1}\right]$ so that no solution exists within the interval $\left[0, h_t - \varphi h_{t+1}\right]$ when $h_t - \varphi h_{t+1} > 0$. And with $\tau = \delta$, from (11) and (13) we have $h_Y^* = (1-\alpha) \cdot \tau^{-\tau} \cdot (1-\tau)^{-(1-\tau)}$ that is a positive constant but this is an economically meaningless solution for any h_0 subject to $h_0 < h_Y^*$. Therefore, the constraint $\tau > \delta$ is required for the model to have a meaningful equilibrium.) We have the following.

$$h_Y^* \equiv f\left[h_t - \varphi h_{t+1}\right],\ f' > 0,\ f[0] = 0. \tag{A1}$$

With (13) and (14), equation (7) can be written as

$$\begin{aligned} &\gamma_t - 1 = k(1-\alpha) f\left[h_t - \varphi h_{t+1}\right]^{\frac{1}{\tau-\delta}-1} \equiv g\left[h_t - \varphi h_{t+1}\right], \\ &g' > 0,\ g[0] = 0 \end{aligned} \tag{A2}$$

From the first order conditions for (2) and (3), we have

$$P_t \equiv \frac{P_t^Y}{P_t^X} = \Delta_t^{1-\alpha} \tag{A3}$$

Let $\omega_t \equiv \dfrac{h_t^Y}{l_t} = \dfrac{f\left[h_t - \varphi h_{t+1}\right]}{\frac{1}{2} - h_{t+1}}$. (A3), together with (17) and (23), gives us

$$\Delta_t^{1-\alpha} = \frac{x_t}{\mu\left[y_t - (m_t^X + m_t^Y)\right]} = \frac{1}{\mu}\left[\left(\frac{y_t}{x_t}\right) - \left(\frac{m_t^X + m_t^Y}{x_t}\right)\right]^{-1}$$

$$= \frac{1}{\mu}\left[\left(\frac{W_t^H h_t^Y}{(P_t - T_t)W_t^L l_t}\right) - \alpha\left(\frac{W_t^L l_t / P_t + W_t^H h_t^Y / P_t}{W_t^L l_t}\right)\right]^{-1} \qquad \text{(A4)}$$

From (8), we have $P_t - T_t = \frac{P_t}{\gamma_t}$. Using this, we obtain the following from (A4):

$$\Delta_t^{1-\alpha} = \frac{1}{\mu}\left[\Delta_t^{1-\alpha}\gamma_t\Delta_t\omega_t - \alpha\Delta_t^{1-\alpha}\left(1 + \Delta_t\omega_t\right)\right]^{-1}$$

$$\Rightarrow \frac{1}{\mu} = \gamma_t\Delta_t\omega_t - \alpha - \alpha\Delta_t\omega_t \Rightarrow \Delta_t = \frac{\frac{1}{\mu} + \alpha}{\omega_t\left(\gamma_t - \alpha\right)} \qquad \text{(A5)}$$

From (22) and (A2), we can show that

$$f\left[h_t - \varphi h_{t+1}\right]\cdot\left(g\left[h_t - \varphi h_{t+1}\right] + 1\text{-}\alpha\right) = (1\text{-}\alpha)\cdot\left[h_t^Y + Z_2\left(h_t^Y\right)^{\frac{1}{\tau-\delta}}\right]$$

$$= (1-\alpha)\left(h_t - \varphi h_{t+1}\right) \qquad \text{(A6)}$$

With this, (A5) can be rewritten as

$$\Delta_t = \frac{\frac{1}{\mu} + \alpha}{\omega_t\left(\gamma_t - \alpha\right)} = \frac{\frac{1}{\mu} + \alpha}{\dfrac{f\left[h_t - \varphi h_{t+1}\right]\left(g\left[h_t - \varphi h_{t+1}\right] + 1 - \alpha\right)}{\left(1/2 - h_{t+1}\right)}}$$

$$= \frac{\left(\frac{1}{\mu} + \alpha\right)}{\dfrac{\left(h_t - \varphi h_{t+1}\right)\cdot\left(1\text{-}\alpha\right)}{1/2 - h_{t+1}}} = \frac{\left(\frac{1}{\mu} + \alpha\right)\cdot\left(1/2 - h_{t+1}\right)}{\left(h_t - \varphi h_{t+1}\right)\cdot\left(1\text{-}\alpha\right)} \qquad \text{(A7)}$$

Appendix 2. Proof of Proposition 1a and 1b

Lemma 1. $\dfrac{dh_{t+1}}{dh_t} > 0$.

Rewriting (25) gives us

$$f(h_{t+1}) \equiv \left(\varphi \cdot \frac{\frac{1}{\mu} + \alpha}{1-\alpha} + \varphi \right) \cdot (h_{t+1})^2$$
$$- \left(\tfrac{1}{2} \cdot \varphi \cdot \frac{\frac{1}{\mu} + \alpha}{1-\alpha} + \tfrac{1}{2} \cdot \varphi \cdot \theta + h_t \right) h_{t+1} + \tfrac{1}{2} \cdot \theta \cdot h_t = 0 \tag{A8}$$

From (20), we have

$$h_{t+1} < \tfrac{1}{2} \cdot \theta \tag{A9}$$

From the total differentiation, we have

$$\frac{dh_{t+1}}{dh_t} = \frac{\tfrac{1}{2} \cdot \theta - h_{t+1}}{\left(\tfrac{1}{2} \cdot \varphi \cdot \frac{\frac{1}{\mu} + \alpha}{1-\alpha} + \tfrac{1}{2} \cdot \varphi \cdot \theta + h_t \right) - \left(\varphi \cdot \frac{\frac{1}{\mu} + \alpha}{1-\alpha} + \varphi \right) \cdot 2h_{t+1}} \tag{A10}$$

From (A10) and $h_t - \varphi h_{t+1} \geq 0$, it is straightforward to show the Lemma 2 from (A9).

Lemma 2. There exists a unique continuous mapping ζ such that $h_{t+1} = \zeta(h_t)$ where (h_t, h_{t+1}) satisfies (25) and $0 \leq \zeta(h_t) < \tfrac{1}{2} \cdot \theta$ for $0 \leq h_t \leq \tfrac{1}{2} \cdot \theta$.

Given $0 \leq h_t \leq \tfrac{1}{2} \cdot \theta$, (A8) gives two roots $h_{t+1}^1 = \zeta_1(h_t)$, $h_{t+1}^2 = \zeta_2(h_t)$. From $\theta \cdot h_t \geq 0$ and (A-9), the smaller root, say $h_{t+1}^1 = \zeta_1(h_t)$, lies in $[0, \tfrac{1}{2} \cdot \theta)$ and the larger root, $h_{t+1}^2 = \zeta_2(h_t)$, lies in $(\tfrac{1}{2} \cdot \theta, \infty)$. Both mappings are continuous mappings from lemma 1 and the implicit function theorem, which proves the lemma.

We must find whether there exists a fixed point for the mapping ζ such that

$$h = \zeta_1(h), \quad 0 \le h < \tfrac{1}{2} \cdot \theta \tag{A11}$$

Let $\varphi \cdot \dfrac{\frac{1}{\mu} + \alpha}{1-\alpha} \equiv k$. Equivalently, we must find a root that satisfies the following equation:

$$(k+\varphi)\cdot(h)^2 - \left(\tfrac{1}{2}\cdot k + \tfrac{1}{2}\cdot\varphi\cdot\theta + h\right)h + \tfrac{1}{2}\cdot\theta\cdot h = 0, \quad 0 \le h < \tfrac{1}{2}\cdot\theta \tag{A12}$$

From (A12), $h = 0$ is one solution to the equation. The other solution can exist if and only if $k < (1-\varphi)\cdot\theta$ and it is given by

$$h = \frac{1}{2}\cdot\left[\frac{(1-\varphi)\cdot\theta - k}{(1-\varphi) - k}\right], \tag{A13}$$

To determine whether these solutions are stable, we must look at $\dfrac{dh_{t+1}}{dh_t}$ around the solution. Rewrite (A10) as

$$\frac{dh_{t+1}}{dh_t} = \frac{\tfrac{1}{2}\cdot\theta - h_{t+1}}{\left(\tfrac{1}{2}\cdot k + \tfrac{1}{2}\cdot\varphi\cdot\theta + h_t\right) - 2\cdot h_{t+1}\cdot(k+\varphi)} \tag{A14}$$

Case 1. $h = 0$

$$\left.\frac{dh_{t+1}}{dh_t}\right|_{h_t = h_{t+1} = h = 0} = \frac{\theta}{(k+\varphi\cdot\theta)} < 1 \ \text{ if } \ (1-\varphi)\cdot\theta < k \tag{A15}$$

$\left.\dfrac{dh_{t+1}}{dh_t}\right|_{h_t = h_{t+1} = h = 0} = \dfrac{\theta}{(k+\varphi\cdot\theta)} = 1$ if $(1-\varphi)\cdot\theta = k$ but in this case we have

$$\left.\frac{d^2 h_{t+1}}{d(h_t)^2}\right|_{h_t = h_{t+1} = h = 0} = \frac{\theta(k+\varphi-1)}{(k+\varphi\cdot\theta)^2} < 0, \tag{A16}$$

Note that $(1-\varphi)\cdot\theta = k$ implies $k+\varphi=\theta+\varphi(1-\theta)<1$. (A15) and (A16) imply that if $h_0>0$, h_t will settle at zero as time passes.

Case 2. $h=\dfrac{(1-\varphi)\cdot\theta-k}{(1-\varphi)-k}$ and $0\le k<(1-\varphi)\cdot\theta$

$$\left.\frac{dh_{t+1}}{dh_t}\right|_{h_t=h_{t+1}=h}=\frac{\frac{1}{2}\cdot\theta-h}{\left(\frac{1}{2}\cdot k+\frac{1}{2}\cdot\varphi\cdot\theta+h\right)-2h\cdot(k+\varphi)}=\frac{\frac{1}{2}\cdot\theta-h}{\frac{1}{2}\cdot\theta-h\cdot(k+\varphi)}<1 \quad \text{(A17)}$$

This is because $0\le k<(1-\varphi)\cdot\theta$ also implies that $k+\varphi<1$. Therefore, from lemma 1, (A15), (A16) and (A17), we can conclude that, given $0<h_0<\frac{1}{2}\cdot\theta$, $h=\dfrac{(1-\varphi)\cdot\theta-k}{(1-\varphi)-k}$ is the unique stable stationary solution to (A1) if and only if $0\le k<(1-\varphi)\cdot\theta$, and $h=0$ is the unique stable stationary solution to (A1) if and only if $k\ge(1-\varphi)\cdot\theta$.

Appendix 3. Proof of Proposition 2a and 2b

Given the stationary equilibrium level of h , γ is also stationary from (A2). From the first order conditions for (2) and (3), we have

$$\left[\frac{W_{t+1}^H}{W_t^H}\right]^{1-\alpha}=\gamma\cdot\left[\frac{P_{t+1}}{P_t}\right]^{1-\alpha},\quad \left[\frac{W_{t+1}^L}{W_t^L}\right]^{1-\alpha}=\gamma\cdot\left[\frac{P_{t+1}}{P_t}\right]^{1-\alpha} \quad \text{(A18)}$$

$$P_t=\left(\frac{W_t^H}{W_t^L}\right)^{1-\alpha},\quad P_{t+1}=\left(\frac{W_{t+1}^H}{W_{t+1}^L}\right)^{1-\alpha} \quad \text{(A19)}$$

$$\frac{P_{t+1}}{P_t}=\gamma\cdot\left[\frac{M_{t+1}^X}{M_t^X}\right]^{\alpha-1}=\gamma\cdot\left[\frac{M_{t+1}^Y}{M_t^Y}\right]^{\alpha-1} \quad \text{(A20)}$$

From (A18) and (A19), we have

$$\frac{P_{t+1}}{P_t} = 1, \quad \frac{W_{t+1}^H}{W_t^H} = \frac{W_{t+1}^L}{W_t^L} = \gamma^{\frac{1}{1-\alpha}} \equiv \lambda^* + 1 \tag{A21}$$

With (A20) and (A21), we have

$$\frac{M_{t+1}^X}{M_t^X} = \frac{M_{t+1}^Y}{M_t^Y} = \frac{M_{t+1}}{M_t} = \lambda^* + 1 \tag{A22}$$

Using the consumers' budget constraints and (17), we can show that

$$C_t = W_t^L \cdot L_t + W_t^H \cdot \left[H_t - \varphi H_{t+1} \right] \tag{A23}$$

From (A23), we have

$$\frac{C_{t+1}}{C_t} = \frac{W_{t+1}^L}{W_t^L} \cdot \frac{L_{t+1} + \Delta_{t+1} \cdot \left(H_{t+1} - \varphi H_{t+2} \right)}{L_t + \Delta_t \cdot \left(H_t - \varphi H_{t+1} \right)} \tag{A24}$$

Because L_t, H_t, Δ_t are all stationary in the stationary equilibrium, we have

$$\frac{C_{t+1}}{C_t} = \frac{W_{t+1}^L}{W_t^L} = \lambda^* + 1 \tag{A25}$$

From (23), we have

$$P_t \cdot M_t = P_t \cdot Y_t + X_t - C_t \tag{A26}$$

From (A22), (A25) and (A26), we have

$$\frac{P_{t+1} \cdot Y_{t+1} + X_{t+1}}{P_t \cdot Y_t + X_t} = \frac{P_{t+1} \cdot M_{t+1} + C_{t+1}}{P_t \cdot M_t + C_t} = \lambda^* + 1 \tag{A27}$$

(A22), (A25) and (A27) prove the proposition 2a. (A-2) and (A-24) prove the proposition 2b.

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