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# POLLUTION ACCUMULATION AND ENDOGENOUS GROWTH IN A SUSTAINABLE EQUILIBRIUM

YONG JIN KIM *

*This paper presents an endogenous growth model, with a feature of pollution accumulated affecting production efficiency and preference altogether. This model presents the links between pollution level, economic growth, government policy and preference toward pollution.*

*With this model, the following three implications are derived: (1) As the government authority takes a tougher environmental measure, the pollution level, economic growth rate and R&D share in GNP for a less polluting energy source decrease. However, with this tougher policy, the optimal tax rate to finance environmental measures and pollution abatement spending share in GNP increase. (2) The countries with higher time preference rate show less pollution level, lower economic growth rate, higher abatement spending share and lower R&D spending share in income. (3) With a minimal use of calibration about the functions relating pollution and economic growth, I find that the relative shadow price of economic growth rate, with respect to one percent increase of pollution level accumulated, is very high over the broad range of parameter values.*

## I. INTRODUCTION

'What is the effect of an environmental policy on economic growth' is now emerging as an important question. It is not only because air, water and other environmental factors are no longer free goods, due to pollution, but also because it is now becoming a hot issue of Green Round in international trade.

This paper presents an endogenous growth model, with a feature of pollution accumulated affecting production efficiency and preference altogether. This model presents the links between pollution level, economic growth, government policy and preference toward pollution.

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* Department of Economics, Dongduck Women's University, Wolgok-dong, 23-1 Seoul, KOREA.
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As for the related literature, Nordhaus [1993, 94] focus on the green house effect on agriculture using DICE model without considering the effect on the economic growth. Additionally, Ligthart and van der Ploeg [1994] work on the endogenous growth model with pollution and policy variables. However, they do not consider the following facts: Pollution accumulates over time, and the accumulated pollution affects the production efficiency frontier. The model of this paper takes these two facts into consideration, and tries to capture the essence of the Nordhaus' DICE model. Finally, Bovenberg and Smulders [1995] is similar to our model in the setup of the model and its implications. However, they do not introduce the distortionary tax and the distinction of the flow and the accumulated stock of pollution, whereas our model does.

Because pollution accumulates, as a byproduct in the process of production, we need another concept, environment sustainability, in addition to the usual aspects of balanced growth path equilibrium. This sustainable balanced growth path equilibrium has an additional property that the accumulated pollution level does not change over time.

With this sustainable equilibrium concept, the following four implications are derived: (1) As the government authority takes a tougher environmental measure, the pollution level, economic growth rate and R&D share in GNP for a less polluting energy source decrease. However, with this tougher policy, the optimal tax rate to finance environmental measures and pollution abatement spending share in GNP increase. (2) The countries with higher time preference rate show less pollution level, lower economic growth rate, higher abatement spending share and lower R&D spending share in income. (3) With a higher initial capital stock, the direction of change of the endogenous variables is hard to determine. (4) With a minimal use of calibration about the functions relating pollution and economic growth, I find that the relative shadow price of economic growth rate, with respect to one percent increase of pollution level accumulated, is very high over the broad range of parameter values.

The rest of the paper is organized as follows: Section Ⅱ presents a general equilibrium model and the proof of it's existence and uniqueness. In Section Ⅲ, the implications of the model are pursued. Section Ⅳ tries to quantify the results of the model, using the calibration method. Section Ⅴ concludes.

## Ⅱ. THE MODEL

This section characterizes the solution of the model. Its main characterization is the proof of existence and uniqueness of the steady state solution. The model is summarized by the following relationships (1) to (6):

**Technology**

(1) $Y_t = f(B_t)AK_t$ : Production function

(2) $B_t = \int_0^t b_s e^{(s-t)\delta} ds$ : Pollution accumulation

(3) $b_t = g\left(\frac{Z_t}{Y_t}\right) e^{-\int_0^t d_\tau d\tau} Y_t$ : Pollution flow due to production

(4) $d_t = l\left(\frac{R_t}{Y_t}\right)$ : Reduction factor of pollution emission by developing energy substitute

**Preference**

(5) $\int_0^\infty e^{-\rho t}(\log C_t - \alpha B_t)dt$ : Intertemporal utility function.

**Government Budget Constraint**

(6) $\tau Y_t = R_t + Z_t$,

where the variables are defined as

$Y_t$ = real GNP at $t$,
$K_t$ = capital stock at $t$,
$C_t = (1-\tau) Y_t - \dot{K}_t$; consumption at $t$,
$B_t$ = accumulated pollution level at $t$,
$b_t$ = pollution inflow at $t$,
$Z_t$ = government spending on abatement at $t$,
$R_t$ = government R&D spending to develop energy substitute at $t$,
$d_t$ = reduction factor of pollution emission due to R&D on new energy development to reduce pollution emission at $t$,
$\tau$ = the rate of income tax

and the parameters are defined as

$\delta$ = dissipation rate of pollution,
$\rho$ = time preference,
$\alpha$ = preference parameter against pollution.

Additionally, the above functions are characterized as[1]:

$f(0) = 1, f(B^*) = 0, f'(B) < 0, f''(B) < 0$ *and* $f'(B^*) = -\infty$ *for* $f(B)$, *where* $B^*$ *is the upper bound for* $B$;
$g(0) = 1, g(1) = 0, g'(z) < 0, g''(z) > 0$ *and* $g'(0) = -\infty$ *for* $g(z)$,
*and* $l(0) = -\rho, l(1) > 0, l'(r) > 0, l'(0) = \infty$ *and* $l''(r) < 0$ *for* $l(r)$.

[1] With the value imposed on $l(0)$, $\tau = 1$ is allowed, as we can see from (20).

Equation (1) represents a linear production function with the constant marginal productivity of physical capital, $f(B)\ A$, affected by the pollution level accumulated. Through this, the pollution accumulated affects not only the preference, but also the production efficiency. The term $g\left(\frac{Z_t}{Y_t}\right)\exp\left(-\int_0^t d_\tau d\tau\right)$ in (3) is the emission-output ratio. We can reduce this emission both by government spending on abatement and by R&D expenditure on new energy development.

Now to solve the above model at the steady state, we need to define an equilibrium concept, a sustainable balanced growth path equilibrium.

**Definition:** A sustainable balanced growth path equilibrium (SBGE) is defined to be a balanced growth path equilibrium with the constant level of accumulated pollution over time ($\dot{B}_t = 0$).

With this equilibrium concept, the model is solved in the rest of this section.

### 2.1 Consumer Utility Maximization

With the above equilibrium concept, the objective function (5) can easily be transformed into

$$(7)\quad \frac{1}{\rho}\left(\log\frac{C_0}{K_0} + \log K_0 - \alpha B_0\right) + \frac{1}{\rho^2}\ \frac{\dot{C}}{C}$$

To solve this consumer maximization problem, take derivatives on (7) with respect to $C_t$ and $K_t$ given $\tau$, $R_t$, $Z_t$ and $B_t$. Then we have the following Euler equations, dropping time subscripts.

$$(8)\quad \Pi \equiv \frac{\dot{C}}{C} = (1-\tau)\cdot A\cdot f(B) - \rho$$

$$(9)\quad \Pi \equiv \frac{\dot{K}}{K} = (1-\tau)\cdot A\cdot f(B) - \frac{C}{K}$$

The sustainable steady state equilibrium condition, $\dot{B}_t = 0$, gives us

$$(10)\quad \Delta B_{t+\Delta t} = g\left(\frac{Z_t}{Y_t}\right)\cdot A\cdot f(B_t)\cdot e^{-dt}\cdot K_0\cdot e^{\Pi t}\Delta t - B_t\cdot\delta\cdot\Delta t = 0$$

$$\Rightarrow g\left(\frac{Z_t}{Y_t}\right)\cdot A\cdot f(B_0)\cdot K_0 = B_0\cdot\delta$$

with

(11) $\Pi = d$ and constant $\frac{Z_t}{Y_t}$,

to have the sustainable steady state equilibrium. From (10), we know that the government abatement spending ratio in GNP increases with a decrease in an equilibrium pollution accumulation (B), in SBGE. However, the government R&D investment ratio in GNP moves with the equilibrium growth rate in the same direction as we can see from (11). In a word, abatement and R&D spending have different roles in SBGE.

Now, it is obvious, from (8) and (9), to have

(12) $\frac{C_0}{K_0} = \rho.$

Under SBGE, (11) gives us

(13) $\Pi = d = \frac{\dot{C}}{C} = \frac{\dot{K}}{K},$

and let us define $r \equiv \frac{R_t}{Y_t}$ and $z = \frac{R_t}{Y_t}$, then (6) yields

(14) $\tau = z + r.$

## 2.2 Government Welfare Maximization

Using (12) and (13), we can simplify the objective function as

(15) $\frac{1}{\rho}\log\rho + \frac{1}{\rho}\log K_0 - \frac{1}{\rho}\ \alpha B_0 + \frac{1}{\rho^2}\ \Pi.$

This relationship shows the tradeoff between the growth rate and the level of pollution accumulated. With an increase of time preference, agents want to substitute from economic growth to the lower level of pollution accumulation. Government maximizes (15) with respect to $\tau$, $z$, $r$, $\Pi$ and $B$, given $K_0$, $\rho$ and $\delta$, subject to (1), (2), (3), (4), (6), (8) and (12).

Now, we derive the first order condition with respect to $B$, considering that $\tau$ is a function of $B$, using relation (8).

(16) $-\frac{\alpha}{\rho} + \frac{1}{\rho^2}(1-\tau)\cdot A\cdot g'(B) - \frac{1}{\rho^2}A\cdot g(B)\cdot\frac{\partial\tau}{\partial B} = 0.$

At this point, we need more information about $\frac{\partial\tau}{\partial B}$. Using (4), (6) and (11), we get

(17) $\Pi = l\Big(\tau - g^{-1}\Big(\frac{B\delta}{A\cdot f(B)\cdot K}\Big)\Big) = (1-\tau)\cdot A\cdot f(B) - \rho$

Totally differentiating (17), we have

(18) $\frac{d\tau}{dB} = \Big[\frac{1}{g'(z)}\Big(\frac{\delta}{AfK} - \frac{B\delta f'}{Af^2K}\Big) + (1-\tau)Af'\Big]/(l'(r) + Af) < 0$

## 2.3 Existence

Under SBGE, we can summarize the model with the following five relationships.

(14) $\tau = z + r$

(8) $\Pi = (1-\tau)\cdot A\cdot f(B)\cdot K - \rho$

(10) $g(z)\cdot A\cdot f(B)\cdot K = B\cdot\delta$

(11) $l(r) = \Pi$

In addition to these, we need another relationship from the first order condition

(19) $\alpha\rho = (1-\tau)Af'(B) - Af(B)\frac{\partial\tau}{\partial B} \equiv \frac{\partial\Pi}{\partial B} > 0.$

By (6), (8) and (11), we have

(20) $(1-\tau) = \frac{l(r)+\rho}{Af(B)}.$

Now, we have a four equation system (6), (10), (19) and (20) with four variables, $B$, $z$, $r$ and $\tau$.

**Proposition 1**

If $l(0) = -\rho$ and $\lim_{z\to 1} g'(z)l'(r) = 0$, then there exists at least one solution to the above four equation system (6), (10), (19) and (20).

(pf) This maximization problem can easily be analyzed in the $\Pi - B$ plane. The indifference curve can be constructed from (5) and the opportunity set is also described by (8), (10), (11) and (14) in the $\Pi - B$ plane. The upper bound of $B$ ($B^r$) in the opportunity set is achieved with $z=0$ by (10). The remaining question is whether the other restrictions, (6), (8) and (11), can be satisfied. The relationship between $r$ and upper bound of $B$ from these restrictions can be described by $\frac{l(r)+\rho}{1-r} = A \cdot g(B^r)$. The LHS is an increasing continuous function with $r \in [0, 1)$ and covers $[0, \infty)$. Therefore, the other restrictions are satisfied with $B^r$ and $z=0$. When $z=0$, RHS of (19) is negative valued.

On the other hand, the lower bound of $B$ in the opportunity set is obtained through (10) when $z=1$. With $z=1$, we have $r=0$, $\tau=1$, $B=0$ and $\Pi = -\rho$ by the other relationships (6), (8) and (11). Additionally, if $\lim_{z \to 1} | g'(z) \cdot l'(r) | = 0$, then RHS of (19) goes to infinity. Moreover, we know that l, $f$ and $g$ are continuously differentiable. This is why there exists at least one solution of $B$ satisfying (19).///

To guarantee that the solution achieves a maximum, we need the second order condition. By (19), we can get SOC as

$$(21) \quad \frac{d^2\Pi}{dB^2} = -\frac{d^2\tau}{dB^2} Af(B) - 2\frac{d\tau}{dB} Af'(B) + (1-\tau) Af''(B) < 0.$$

From (6), (10) and (20), we derive

$$(22) \quad \frac{d^2\tau}{dB^2} = \left[ \frac{-g''(z)}{(g'(z))^3} \left( \frac{\delta}{AfK} - \frac{B\delta f'}{Af^2K} \right)^2 + \frac{1}{g'(z)} \left( \frac{-2\delta f'}{Af^2K} + \frac{2B\delta f'^2}{Af^3K} - \frac{B\delta f''}{Af^2K} \right) + (1-\tau)Af'' - Af'\frac{d\tau}{dB} \right] (l'(r) + Af)^{-1} - \left[ \frac{1}{g'} \left( \frac{\delta}{AfK} - \frac{B\delta f'}{Af^2K} \right) + (1-\tau)Af' \right] \left( l''(r)\frac{dr}{dB} + Af' \right) \cdot (l'(r) + Af)^{-2}.$$

If $f'' < 0$, $\frac{d^2\tau}{dB} > 0$ is a sufficient condition for (21). But as we can see from (22), it seems almost impossible to hypothesize a set of simple assumptions guaranteeing $\frac{d^2\tau}{dB^2} > 0$. Therefore, we will go around and present only the sufficient condition for a local maximum.

Using $l(r) = l(\tau - z) = \Pi$, FOC $\left( \frac{d\Pi}{dB} = \alpha\rho \right)$, and the implicit function theorem, we have

(23) $l'(r)(d\tau - dz) = d\Pi = \alpha\rho d\,B$

yielding

(24) $$\frac{d\tau}{dB} = \frac{dz}{dB} + \frac{\rho\beta}{l'(r)}$$

which also gives

(25) $$\frac{d^2\tau}{dB^2} = \frac{d^2 z}{dB^2} - \frac{\rho\beta}{(l'(r))^2}\,l''(r)\,\frac{dr}{dB}$$

$\frac{dr}{dB}$ is positive by (11) and FOC. We can also infer that if $\frac{d^2 z}{dB^2} > 0$, then $\frac{d^2\tau}{dB^2} > 0$ by (25).

Now, let us find the sufficient conditions to have $\frac{d^2 z}{dB^2} > 0$. Define and $\eta_f = \frac{df}{dB}\frac{B}{f}$ and $\eta_z = \frac{dz}{dB}\frac{B}{z}$. Then, we have

(26) $$\frac{d\eta_z}{dB} = \frac{d\left(\frac{z'(B)\cdot B}{z(B)}\right)}{dB} = \frac{z''\cdot B}{z} + \frac{z'}{z} - \left(\frac{z'}{z}\right)^2$$

$$= \frac{\eta_z}{B}\,(\eta_{z'} - \eta_z + 1),$$

where $z' = \frac{dz}{dB}$, $z'' = \frac{d^2 z}{dB^2}$ and $\eta_{z'} = \frac{dz'}{dB}\frac{B}{z'}$.

Therefore, we have only to prove $\eta_z < 0$ to have $z'' > 0$.
By (10), we have

(27) $$\frac{dz}{dB}\frac{B}{z} = \eta_z = \left(\frac{g(z)}{g'(z)B} - \frac{f'g}{g'f}\right)\frac{B}{z} = \frac{1}{\eta_g}(1 - \eta_f),$$

where $\eta_g = \frac{dg}{dz}\frac{z}{g}$.

Also, (26) gives us

$$(28)\quad \eta_{z'} = \frac{B\dot{\eta}_z}{\eta_z} + \eta_z - 1,$$

where $\dot{\eta}_z = \dfrac{d\eta_z}{dB}$.

Using (26) and (27),

$$(29)\quad \frac{d\eta_z}{dB} = \dot{\eta}_z = \frac{-\dot{\eta}_f}{\eta_g} - \frac{(1-\eta_f)}{\eta_g^2}\,\dot{\eta}_g\,\frac{dz}{dB}$$

$$= \frac{-\eta_f}{\eta_g B}(\eta_{f'} - \eta_f + 1) - \frac{(1-\eta_f)^2}{\eta_g^2 B}(\eta_{g'} - \eta_g + 1)$$

where $\eta_{g'} = \dfrac{d^2 g}{dz^2}\dfrac{z}{g'}$

By (28) and (29), we derive

$$(30)\quad \eta_{z'} = -\frac{1}{\eta_z}\left(\frac{\eta_f}{\eta_g}(\eta_{f'} - \eta_f + 1) + \frac{(1-\eta_f)^2}{\eta_g^2}(\eta_{g'} - \eta_g + 1)\right) + \eta_z - 1$$

$$= -\frac{\eta_f \eta_{f'}}{1-\eta_f} - 2\eta_f - \frac{(1-\eta_f)\eta_{g'}}{\eta_g}.$$

Now, we get the sufficient condition to guarantee the existence of at least one local maximum solution.

**Proposition 2:** If $|\eta_f|$ and $|\eta_{f'}|$ are small enough compared to $\dfrac{\eta_{g'}}{\eta_g}$ at the point satisfying FOC, then this point achieves a local maximum.

Also, we can have a stronger result of uniqueness with a more strict assumption than in Proposition 2.

**Proposition 3:** If $|\eta_f|$ and $|\eta_{f'}|$ are small enough compared to $\dfrac{\eta_{g'}}{\eta_g}$ over the whole range of $B$ and $z$, then there exists only one unique solution.

(pf) The contour of the opportunity set is smooth and differentiable. Therefore, we know that, if there exist multiple equilibria, then there exists at least one local minimum or continuum of maxima. But the latter case is excluded by the SOC we assumed. Moreover, for the former case, we also know that we cannot have any local minimum, due to the fact that $\frac{d^2\Pi}{dB^2} < 0$ at any tangent point satisfying the FOC over the whole range. ///

## Ⅲ. IMPLICATIONS OF THE MODEL

This section explores several interesting implications of the model. Based on the comparative statics at the steady state, we try to answer the question: In which direction endogenous variables move in response to a change in exogenous parameter values, such as time preference, initial capital stock and the preference parameter of pollution.

### 3.1. Change of Preference Parameter $\alpha$

If the economic agents' disutility of pollution increases, or the government takes a tougher measure against pollution, such that $\alpha$ goes up, then the slope of indifference curve in $\Pi - B$ plane increases. But the opportunity set does not change at all. Therefore, with SOC, as $\alpha$ increases, both $\Pi$ and $B$ decrease. Accordingly, $r$ decreases and $z$ increases due to the relationships (10) and (20). Also, $\tau$ increases by (18).

With the increase of $\alpha$, lower pollution accumulation is valued more that economic growth. Therefore, government substitutes economic growth for lower pollution accumulation. Thus, agents will have lower growth and pollution accumulation. To have lower level of pollution, there will be higher government abatement spending. However, noting that government R&D spending is positively related to growth, we know that government R&D spending decreases.

This exercise implies: As one country becomes more conscious of environments with the increase of its income, its growth rate will decrease with the increase of environmental taxes and of government abatement spending.

Even in the case the government's parameter value is different from the economic agents', the above analysis still holds. It is because economic agents take the pollution level as an exogenously given constant. In other words, the term $\alpha B$ does not affect their marginal condition.

### 3.2. Change of Time Preference $\rho$

With an increase in $\rho$, the slope of the indifference curve increases, while the slope of the opportunity set does not change. However, the opportunity set itself shifts down as we can see from (8), (10), (11) and (14). So, $\Pi$ and $B$ decrease as $\rho$ increases.

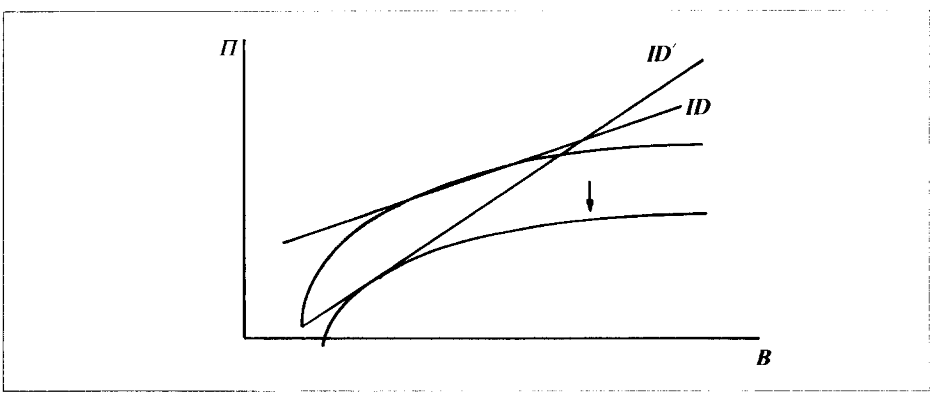
Exactly speaking, the opportunity set can be described by (8), (10), (11) and (14) as

$$(31) \quad \Pi = (1 - z(B) - l^{-1}(\Pi))Af(B) - \rho.$$

Therefore, as $\rho$ increases, the opportunity set shifts down along the $\Pi$ axis without changing its slope $\frac{d\Pi}{dB}$ at any value of B in plane $\Pi - B$ as in Figure 1. Moreover, as $\rho$ increases, the ratio of consumption over capital increases, because of (12).

Therefore, countries with a higher time preference show lower economic growth rate, less pollution, higher government abatement spending, less government spending on R&D on energy substitute, and higher consumption ratio to capital. The intuition behind this exercise is identical to the previous exercise of 3.1. The exercises with a change in initial capital stock or technology change do not provide us any easily discernible direction of movement of endogenous variables.

**[Figure 1]**




## Ⅳ. CALIBRATION

Using the estimated pollution model in W. D. Nordhaus [1993], a calibration work can be performed. We can obtain one number and one functional form, $\eta_f = -0.0133$ and $g(z) = a(1 - 2.53z^{0.3464})$. From these two pieces of information only, it can be figured out what is the trade-off between economic growth and environmental degradation in equilibrium.

From (18) and (19), we obtain

$$(32) \quad \frac{\frac{\partial \Pi}{\partial B}}{B} = \frac{1}{1 + \frac{l'(r)}{Af}} \left( - \frac{(1 - \eta_f)}{\eta_g} z \right) + \frac{l'(r)(1 - \tau)\eta_f}{1 + \frac{l'(r)}{Af}} = B\alpha\rho$$

$$\langle -\frac{(1-\eta_f)}{\eta_g} z,$$ [2]

where $\eta_g = \frac{dg}{dz}\frac{z}{g}$ and $\eta_f = \frac{df}{dB}\frac{B}{f}$.

Additionally, from the indifference curve constructed from (15), we have

$$(33)\quad \frac{\frac{\partial K}{K}}{\partial \Pi} = -\frac{1}{\rho}.$$

Therefore (33) yields

$$(34)\quad \frac{\frac{\partial K}{K}}{\frac{\partial B}{B}} = -\frac{1}{\rho}\frac{\partial \Pi}{\frac{\partial B}{B}}.$$

From (32), (33) and (34) in addition to the above two pieces of information, the following numbers are generated.[3]

| $z$ | $\eta_f$ | $\eta_{f'}$ | $\eta_g$ | $\eta_{g'}$ | $\frac{-(1-\eta_f)}{\eta_g} z$ |
|---|---|---|---|---|---|
| 0.001 | −0.0133 | −1.0133 | −0.104 | −0.3464 | 0.0097 |
| 0.005 | | | −0.235 | | 0.0216 |
| 0.01 | | | −0.365 | | 0.0277 |
| 0.05 | | | −2.99 | | 0.0169 |
| 0.06 | | | −7.30 | | 0.0083 |

Because all the necessary numbers for calibration are not ready at hand, we cannot go further than this table. And the above table reads as: If the government is maximizing the welfare, and if $z = 0.01$, then the shadow relative price of one percent increase in pollution with respect to economic growth rate is at most 0.0216. When $z = 0.05$, its relative price is at most 0.0277. This relative price means $\frac{P_B}{P_\Pi}$, where $P_B$ is the shadow price of one percent increase of pollution,

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[2] If we assume $l'(r) \approx 0$, then $\frac{\partial \Pi}{\frac{\partial B}{B}} \approx -\frac{(1-\eta_f)}{\eta_g} z$

[3] Within this range of $z$, SOC is satisfied locally.

and $P_{\Pi}$ is that of one percentage increase in economic growth rate.

The fact, that these prices are low over the reasonable range of $z$, means that one percentage increase of economic growth rate is valued much higher than one percent increase of pollution in equilibrium. These relative prices can play the role of a bench mark, when we plan an environmental policy, vis-à-vis economic growth, if all the calibration numbers are measured correctly. Nordhaus [1993] estimated $g(z)$ and $f(B)$ considering only the effect of green house gases on agriculture. This is why the cost of pollution is measured to be very small such that excessive pursuit of economic growth at the cost of pollution will be considered optimizing.[4]

To have the exact estimates of the effect of pollution on preference and technology, extensive empirical work is necessary. However, the ecosystem is so much complicated that it is almost impossible to do this job. However, if we can have rough estimates for $\eta_g$ and $\eta_f$, we can calculate a benchmark number for an environmental policy as above.

## V. CONCLUSION

The model is set up to describe a mechanism of the interaction between pollution accumulation and economic growth. The preference of this model is shown to have a linear trade off between them, but its contour of the opportunity set is rather complicated, due to the complex process of pollution accumulation.

The model is shown to have the following implications: (1) Countries with tougher government measure against pollution or with higher time preference rate, have lower pollution level, lower growth rate, higher government spending on abatement and lower government spending of R&D for less polluting energy sources. (2) Calibration shows that the shadow relative price of one percent increase in pollution with respect to one percentage increase in economic growth rate, is very small. However, we should be more cautious to take (2) for granted, because more exact estimation of calibration numbers are needed to be done.

In the future, the current research can be extended to the model with: (1) additional variables, such as government consumption and investment, (2) simulation with more exact calibration, (3) comparison among different countries, based on the result of the model, using cross-country data, and, finally, (4) different equilibrium concepts, for example, such as $\dot{B} = \text{constant}$.

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[4] Nordhaus shows the simulation result that the welfare difference between optimal policy and no control policy is 0.039% of consumption. This difference is large in absolute size, but rather small relative to the economy size.

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