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CONSUMER INVENTORY, PRICE BREAK, and PRICE DISPERSION

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This study offers a dynamic framework which formulates stockpiling behavior of consumers as a fundamental market state. Mixed pricing strategies are endogenously derived from the model as well as two extreme monopoly and marginal pricing as limiting cases. This model enables us to study both spatial and temporal price behavior using a simple dynamic programming method. The movement of price and sales can be examined at the level of the industry as well as the individual firm.

I. INTRODUCTION

The law of a single equilibrium price is empirically questionable even in a fairly competitive market. Quoted prices for standardized consumer electronics or brand name grocery items vary significantly within short periods of time, variation of which is highly unlikely to be attributed to a change in production costs. It appears impossible to attribute these differences to differences in service and ambience of the store as well since we can observe both a higher price and a lower price in one store over a short period of time on the identical items. Incumbent or new entrant cut the price drastically to attract more customers in the fast-moving product market, which gives rise to the price dispersion.

The goal of this research is to study the role of consumer stockpiling behavior in the market where consumers have imperfect information about prices. The existence of imperfect information provides consumers and firms with an incentive to improve on the market outcome by getting more information. The central implication of costly information is that the equilibrium price will not be the perfectly competitive price. Although the price distribution has been explored extensively on the various assumptions such as search mode, search cost and the nature of equilibrium, the empirical implications of price dispersion have not been studied as such.

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While most of the previous models are formulated as static models, it is clear that those models could be embedded in a dynamic framework, where periods are essentially independent, because a new cohort of consumers who are assumed to be identical to the previous generation enter the market each period. In such a model (Varian, 1983), it will remain an equilibrium for all the firms to use the same mixed strategy. Equilibrium price is not only random but also uncorrelated across time. For the product items that are purchased repeatedly and are storable, such a model fails to account for consumers' stockpiling behavior.

The present study offers a model in which, were consumers unable to carry inventories, there would not be a state transition, but the ability of consumers to hold inventories can create a state-contingent equilibrium price dispersion. This arises because a firm that charges its prices low enough to induce a consumer to purchase extra unit as inventory takes sales away from another firm that would have sold the consumer units in the future.

A large number of models have been offered to explain the existence of price dispersion. In a market for a homogeneous good, the identical constant return to scale firms, along with perfectly informed consumers, give rise to marginal pricing when the firms play a Bertrand price game. On the other hand, Diamond(1971) followed by Rothschild(1973) demonstrates that the unique Nash equilibrium price may end up with the pure monopoly pricing with each firm behaving as a monopolist to its customers due to the existence of search costs. These two outcomes depend on the structure of search costs among consumers and the information position firms and consumers are in. Dispersion by a mixed pricing strategy and nonexistence of Nash equilibrium, together with the single marginal pricing and the single monopoly pricing, cover all the possible pricing phenomena.[1]

To explain price dispersion, heterogeneity of consumers or producers needs to be generated either exogenously or endogenously. There are many models of price dispersion with different assumptions on buyer's behavior, seller's behavior and the market structure. The models generally can be divided into two classes depending on how heterogeneity is generated.[2] In the first class, such as Stigler, Diamond and Rothschild, differences in consumer search costs and firm production costs (Reinganum, 1979) account for the differences in offered prices. These models are, however, not adequate to explain variations in store prices across time, as any temporal variation in prices must be attributed fluctuations in production costs or consumer search costs, which seems highly unlikely.

The second class of models, beginning with Butters(1977), followed by Salop

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[1] For the classic paper on the nonexistence of Nash equilibrium, see J. Akerlof(1970). For a general survey of 'Imperfect information in the product market', see Ch. 13 in Handbook of Industrial Organization, Vol 1.

[2] Price dispersion can be explained mainly based on the assumptions on a search protocol, a shape of production cost, a concept of equilibrium, and a form of a demand function, etc. [D. Stahl(1992)]

and Stiglitz(1977, 1982), Burdett and Judd(1983) and Varian(1983), posits variation in the information held by consumers about offered prices. Robert-Stahl (1993) analyze the optimal sequential search model in which firms decide advertisements and price simultaneously. McAfee(1993) analyzes the generalized Butters advertising model in the context of cartel and merger games. The key feature or assumption of the second class of models is that some of the consumers are well informed of the prices and the others are not. In the generalized version, modeled mainly by advertising, some consumers become more informed than the others through a random process of a hazard rate function.

The coexistence of heterogeneous consumer groups rules out a pure strategy, as price must exceed marginal cost, and if prices are equal, it always pays to undercut slightly since the undercutting firm would get all the informed consumers. These models generate price dispersion because firms use a mixed strategy in pricing. Mixed strategy seems more consistent with casual empiricism about pricing strategies of grocery stores.

The model in this study provides a testable hypothesis on the implication of imperfect information on price and sales. This model also allows us to study various questions regarding the effect of stockpiling behavior on the equilibrium price distribution, and the effect of search cost on price dispersion. Once stockpiling behavior is formulated, both spatial and temporal price movement can be explained simultaneously.

II. THE MODEL

Any consumer knows that in the real world he/she is going to encounter a variety of different quoted prices for the same good. Buyers must search across different sellers in order to discover a low price and incur search cost. Costly information for the *ex ante* homogeneous consumers can sustain the price dispersion as an equilibrium phenomenon because the information cost can create heterogeneity among the consumers. A firm running a sale knows that consumers holding larger inventories than normal will not purchase this period, and thus has less incentive to run a sale now, that is, it could be more profitable to sell to the consumers who do not hold inventories or the spontaneous consumers who do not search. It is natural to suppose that a firm has a different policy depending on the inventory position of consumers if consumers can separate consumption and purchase.

As the theory of consumption examines the consumer's choice problem over time, so does the firm theory the dynamic pricing problem of maximizing its profits. Consumers can be better off by stockpiling depending on whether the current price is low enough to cover the discounted future prices. Firms can also take advantage of the inventory position of the consumers.

The confluence of consumers and firms generates the inventory position of the consumers, representing a market state in this study. It is commonly observed that retailers cut the price drastically (price break) and consumers stockpiling some items. In the model considered the pricing strategy of firms are, in part, dependent upon how many searchers are in the market and whether consumers have inventories storable for two periods. First, a critical cut-off price will be examined based on a consumer inventory position. Second, a pricing policy of a firm will be discussed. Finally, a pricing equilibrium will be derived from the interaction between consumer and firm behavior.

2.1 Consumer Behavior

Every period all the consumers are assumed to use one unit of good which is storable for two periods. Consumers can learn the price at any one store by visiting it. Nonsearchers buy one unit in their first random visit as long as price is lower than their reservation price. Searchers visit all the N stores and compare the prices over the stores. The proportion of the searchers $\alpha \in (0, 1)$ depends on the search costs of the consumer population. Proportion of searchers captures the structure of the distribution of search cost among the consumer population. Nonsearchers do not bother to stockpile possibly because the expenditure on the item takes up relatively a small portion of their income. Consumers know the range of prices that exists in the market, but they do not know which store charges what price. The consumers are assumed to have identical discount factor $\delta \in (0, 1)$.

With the price distribution in mind, consumers must visit a store each period to obtain information about the price in the individual store. Searchers make a decision on how many units to buy whereas nonsearchers buy one unit on their first visit. Stores must charge different prices to maximize their profits since there are both searchers and nonsearchers in the market. Equivalently, no pure equilibrium pricing strategy exists, for a firm can make more profits by undercutting a given non-competitive price charged by the other stores because only the lowest priced store can sell to the searchers.

Searchers can purchase zero, one or two units each period depending on the observed prices and the inventory they hold. The searchers, holding no inventory this period, who have arrived at a lowest priced store can buy two units for both immediate and future consumption, depending on the observed price and the discounted expected price of the next period. The assumption that consumers have unit consumption demand each period has the effect of introducing a limit price. There are N identical stores with marginal cost c, setting a price for the identical good each period before the consumers begin their shopping.

Searchers are assumed to have either zero or one unit of inventory at the beginning of the period. Assume that consumers know the distributions of the prices in each state, which rules out a learning process. There are two possibly different price distributions depending on the inventory position of the searchers. Symmetric Nash Equilibrium [SNE] will be analyzed where $F_i(p)$ denotes a SNE distribution of prices in a state with i unit of inventory. $F_0(p)$ and $F_1(p)$, assumed to be invariant to time, are the price distributions in each state where 0 and 1 represent the state without and with inventory. Each firm draws its price independently and randomly from the price distribution depending upon the consumer inventory position. The associated density functions $f_0(p)$, $f_1(p)$ indicate the probability with which each firm charges the price.

Consumers do not know *ex ante* the price each store is charging, although they know the pricing strategies or the state-contingent stationary distribution of price. Consumers cannot engage in arbitrage activity for some reason, so the best they can do is to buy an extra unit at a current price which is sufficiently low.

Thus far, major assumptions on consumer behavior are summarized as follows. Consumers have unit demand per period. Searchers buy 0, 1, and 2 units depending on the inventory position and price. Nonsearchers buy one unit on their first random visit. The portion of searchers, α, depends on the structure of the search cost among the consumer population. A per period discounter factor, δ, is between 0 and 1. No arbitrage is allowed.

To complete assumptions on consumer behavior, let $Supp(p)=[L_i,1]$ i.e., $F_i(L_i) = 0$ and $F_i(1)=1$. The last assumption means that no firm charges more than the good is worth to a consumer. Therefore, the upper limit can be interpreted as the normalized reservation price of the consumers.

The cut-off price p_i is defined as the one that induces searchers to buy extra unit for stockpiling when a price in state i is lower than p_i. In state 0, there will be a critical cut-off price p_0 inducing searchers to purchase two units at $p \leq p_0$. When inventory is held, the presence of nonsearchers implies that there is a mass point at $p=1$. Since nonsearchers should purchase one unit anyway even at a high price, a firm can increase its profits by increasing its price up to the highest possible price on the support. On the other hand, searchers buy one unit this period for inventory when the observed price is sufficiently low $p \leq p_1$.

Then the searchers who observe a price $p \leq p_0$ and $p \leq p_1$ they expect the price distribution in the next period to be $F_1(p)$, which leads to $p_0 = p_1$, as is shown in proposition 1. The current price in one state can be equivalently related to the price distribution next period in terms of the discounted expected price of the next state. The cut-off price in each state thus should satisfy the following Proposition 1. Note that rational expectation and the optimizing behavior of a consumer is embedded in a cut-off price for a stockpiling decision.

Proposition 1.

$$p_0 = \delta \int_{L_1}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp \tag{1.1}$$

and

$$p_1 = \delta \int_{L_1}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp \tag{1.2}$$

thus, $p_0 = p_1$.

Proof. Suppose $p \leq p_0$ in the state 0. Then a consumer faces the following two choices. The first choice is buying one unit now just for current consumption, and entering the subsequent market with no inventory. The second choice is buying two units now, one for current consumption and the other for inventory at a current price p, and facing the price distribution $f_1(p)$ in state 1 next period. The probability that next period price set by at least one firm is lower than a certain price is $1 - (1 - F_1(p))^N$.

Observing a price that is lower than the discounted expected price next period, i.e., $p \leq \delta \int_{L_1}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp$, a rational consumer should buy two units. Then the critical price must be equal to $\delta \int_{L_1}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp$. Therefore (1.1) holds.

To show that (1.2) holds, suppose a consumer sees a price $p \leq p_1$ in state 1. He must conjecture that the subsequent state will be state 1, i.e., every searcher comes to hold one unit of inventory. This means that he wants to buy an extra unit provided this is cheaper than a discounted expected price, which is $p \leq \delta \int_{L_1}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp$. Thus, $p_0 = p_1 = \delta \int_{L_1}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp$. Q.E.D.

A maximum value function is not needed to get p_0, p_1 because state transition does not need to be taken into consideration. In other words, individual consumer's decision does not cause any state transition. Note that the following issue arises. If the minimum of the two prices $p > p_0 = p_1$, then everyone expects 0 to be the subsequent state. Therefore it must be true that 0 is the subsequent state, so the current price should be higher than the discounted expected price of the subsequent period, i.e., $p > p_0 \rightarrow p > \delta \int_{L_0}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp$, otherwise a state 0 cannot occur in the subsequent period. This follows from the proposition 1 that $\delta \int_{L_0}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp < \delta \int_{L_1}^{1} Np(1 - F_1(p))^{N-1} f_1(p)dp$ is necessary. The last inequality comes from the inequality $L_1 < L_0$, implying that a higher price is charged to the customer holding no inventory.

2.2 Firm behavior

The model can be formulated by the simple argument of profit maximization over the period and the state. When adopting a firm's pricing strategy, each firm takes as given the pricing strategies chosen by the other firms and the purchasing behavior of the consumers. The presence of both searchers and nonsearchers implies that there should be a mixed pricing strategy. A necessary condition for price dispersion is that no firm can identify nonsearchers with searchers. The pattern of consumer demand should be a characteristic of interest to the sellers since it will affect the pricing policy. A high priced store can make a higher profit per unit sales, but makes lower sales. Lower sales can mean specifically loss of sales for consumer inventory in this model. Equilibrium should entail equal profits in each state, which is to be derived in this section. The presence of both searchers and nonsearchers implies that only a mixed pricing strategy can maximize the profits as long as the number of firms is finite. The pattern of an inventory policy of the consumers should be considered in formulating the pricing policy of the firms in this market.

There is always a gap of $(p_0, \overline{p}_0)$ in the support of $F_0(p)$, which will be shown later. The reason is that if all the stores charge p_0, any one store that raises its price a bit would lose all the sales for inventory. If one store raises its prices infinitesimally in $(p_0, 1)$, it could increase discretely its profit because there are nonsearchers in the market.

There is a mass point at $p = 1$ in the support of $F_1(p)$. Nonsearchers buy one unit for current consumption anyway whereas searchers opt to use their inventory and do not purchase at all, therefore firms can increase their profits by increasing their prices up to the highest possible 1 as long as $p > p_0$. Price is driven up to 1 since the firms know that they can sell only to nonsearchers in this price range when searchers hold inventory.

A simple description of the model can be done under the assumption that sales to nonsearchers are assumed to be split up evenly among all the firms. The maximum value function must satisfy the following functional depending on the state.

$$V_0(p) = (p-c)\left[\frac{(1-\alpha)}{N} + \alpha(1 - F_0(p))^{N-1}\right] + \delta[(1 - F_0(p_0))^{N-1} V_0 + (1-(1 - F_0(p_0))^{N-1}) V_1] \quad \text{if } \overline{p}_0 \le p \le 1. \tag{2.1}$$

$$V_0(p) = (p-c)\left[\frac{(1-\alpha)}{N} + 2\alpha(1 - F_0(p))^{N-1}\right] + \delta V_1 \quad \text{if } L_0 \le p \le p_0. \tag{2.2}$$

$$V_1(p) = (1-c)\left[\frac{(1-\alpha)}{N}\right] + \delta\left[(1-F_1(p_0))^{N-1}V_0 + (1-(1-F_1(p_0))^{N-1}V_1\right]$$

$$\text{if } p_0 \langle\, p \leq 1. \quad (2.3)$$

$$V_1(p) = (p-c)\left[\frac{(1-\alpha)}{N} + \alpha(1-F_1(p))^{N-1}\right] + \delta V_1 \quad \text{if } L_1 \leq p \leq p_0. \quad (2.4)$$

Take (2.1) for example to explain the meaning of the equations. When firm A randomizes a price in $\overline{p}_0 \leq p \leq 1$ in state 0, the value of the profits consists of the current and subsequent profits. Current profits are a profit margin $(p-c)$ per sales times sales volume. Sales can be made to $\frac{(1-\alpha)}{N}$ of nonsearchers since they are evenly split up among N firms. Sales can also be made to α of searchers with the probability of $(1-F_0(p))^{N-1}$ since searchers buy one unit at the lowest price available among the stores. Firm must charge the lowest price among N firms to sell to searchers, the probability of which is $(1-F_0(p))^{N-1}$.

If all the other stores charge a price in $\overline{p}_0 \leq p \leq 1$, state remains the same in the subsequent period as in 0, the probability of which is $(1-F_0(p))^{N-1}$. If at least one of the others charges a price lower than the cut-off price, state changes into 1, the probability of which is $[1-(1-F_0(p))^{N-1}]$. The total subsequent profits must be discounted as in (2.1).

$V_0(p)$ and $V_1(p)$ are the maximum value functions in state 0 and 1, defined by the following maximization problem of the firms.

$$V_0 = \max\left[\pi_0 + \delta\begin{cases} V_1, \text{ with probability of } (1-(1-F_0(p_0))^{N-1}) \\ V_0, \text{ with probability of } (1-F_0(p_0))^{N-1} \end{cases}\right], \quad (2.5)$$

$$V_1 = \max\left[\pi_1 + \delta\begin{cases} V_1, \text{ with probability of } (1-(1-F_1(p_0))^{N-1}) \\ V_0, \text{ with probability of } (1-F_1(p_0))^{N-1} \end{cases}\right]. \quad (2.6)$$

The value function in each state depends on the price distributions in both states. The price distribution as well as a critical price affects the value function in both states. Consumer behavior is affecting a pricing strategy of the firms through their stockpiling behavior. On the other hand, firms are adopting a pricing strategy, taking into account both a pricing policy of the other firms and the inventory position of the consumers. In this sense, one set of the mixed pricing strategies is not only a strategy of randomized prices to compete with the other firms as in Shilony [1977], but also one part of the pricing scheme designed to take advantage of the consumers' inventory position.

The problem facing a firm in this model is to solve the equation of the maximum value functions V_0, V_1 for the distributions $F_0(p)$, $F_1(p)$. That is, the firms have to maximize their expected profits by drawing a price from the price distribution that depends on the inventory position of the consumers. Only the case of a symmetric equilibrium will be examined, where each firm chooses the same mixed pricing strategy. The maximum value functions across the states turn out to be the same like the equal profit condition, which is usually necessary for the equilibrium price distribution. A set of possible prices chosen by the firms determines the price distribution. Note that the notion of a subgame perfection is subsumed in the value function.

The distribution of prices is an equilibrium distribution if no firm, taking the pricing behavior of all the other firms and the stockpiling behavior of the consumers as given, can increase its expected profits by choosing any other price deviant from the price distribution in question.

Behavior of consumers and sellers, i.e., both the demand and the supply sides of the market, are simultaneously affecting a market state, thereby determining a market equilibrium through state transition caused by stockpiling behavior of the consumers. Thus the market equilibrium is determined through the inventory position of the consumers and a pricing policy of the firms. An equilibrium pricing strategy in the model is a state-contingent price distribution that satisfies the dynamic programming functional. In order to solve this problem for $F_i(p)$, V_i and p_i, we need to invoke the properties of the probability distribution function.

$$V_0 - V_1 = \delta(V_0 - V_1)((1 - F_0(p))^{N-1} - (1 - F_1(p))^{N-1}) \quad \text{since } F_i(1) = 1. \quad (2.7)$$

$$V_0 - V_1 = (p_0 - c)\alpha[2(1 - F_0(p_0))^{N-1} - (1 - F_1(p_0))^{N-1}] \quad \text{for } p = p_0. \quad (2.8)$$

From (2.7) either $V_0 = V_1$, or

$$1 = \delta((1 - F_0(p))^{N-1} - (1 - F_1(p))^{N-1}), \quad (2.9)$$

but this is impossible since $\delta < 1$ and $((1 - F_0(p))^{N-1} - (1 - F_1(p))^{N-1}) \leq 1$. Therefore,

$$V_0 = V_1. \quad (2.10)$$

Equal profits for the firms are made at equilibrium in each state. Let V denote $V_0 = V_1$. From (2.8) using the fact of (2.7), $F_1(p_0) = 1 - 2^{\frac{1}{N-1}}(1 - F_0(p_0))$. Therefore,

$$V = \frac{(1-c)(1-\alpha)}{N(1-\delta)}. \tag{2.11}$$

Equilibrium profits are decreasing in the number of the firms in the market and approaching zero as the number of the firms goes to the infinite, reflecting zero profit property in the competitive market. Although a comparative static with respect to N cannot be performed due to the endogeneity of the equilibrium N when entry cost is fixed, it is quite instructive to see the implication of the parameters for the market structure. As the portion of searchers increases, the equilibrium number of firms decreases for a given number of the consumer population.

Ⅲ. PRICING EQUILIBRIUM

Equilibria in the price dispersion usually embed profit maximization, utility maximization, optimum search, and long-run zero profit by entry. Proposition 1 embeds the maximizing behavior of the firms. To focus on the effect of stockpiling behavior, we assume that the relative portion of searchers is exogenously given in this model so as to preserve an N firm market structure, and hence rule out an endogenous process of optimal search.

Consumers and firms are interacting through their stockpiling behavior and pricing policies. Consumers have an incentive to buy extra unit as inventory when the current price they think is low compared to their expected price next period. Depending on the consumer stockpiling behavior, firms have an incentive to adopt different pricing strategies to take advantage of the inventory position of the consumers.

From the interaction between consumer behavior and firm behavior, an equilibrium price distribution is derived. A state-contingent pricing policy or a price distribution can be obtained using the information to date given in the previous sections. These distribution functions derived in the following are the unique SNE of the model.

$$F_0(p) = \begin{cases} 1 - \left(\frac{1-\alpha}{\alpha N}\right)^{\frac{1}{N-1}} \left[\frac{(1-p)}{(p-c)}\right]^{\frac{1}{N-1}} & \text{for } \overline{p}_0 \leq p \leq 1, \\ 1 - \left(\frac{1-\alpha}{2\alpha N}\right)^{\frac{1}{N-1}} \left[\frac{(1-p)}{(p-c)}\right]^{\frac{1}{N-1}} & \text{for } L_0 \leq p \leq p_0. \end{cases} \tag{3.1}$$

$$f_0(p) = \begin{cases} \frac{1}{N-1}\left(\frac{1-\alpha}{\alpha N}\right)^{\frac{1}{N-1}}\left[\frac{(1-p)}{(p-c)}\right]^{\frac{2-N}{N-1}}\frac{(1-c)}{(p-c)^2} & \text{for } \bar{p}_0 \le p \le 1, \\ \frac{1}{N-1}\left(\frac{1-\alpha}{2\alpha N}\right)^{\frac{1}{N-1}}\left[\frac{(1-p)}{(p-c)}\right]^{\frac{2-N}{N-1}}\frac{(1-c)}{(p-c)^2} & \text{for } L_0 \le p \le p_0. \end{cases} \tag{3.1$'$}$$

$$F_1(p) = \begin{cases} 1 \quad \text{at } p = 1, \\ 1 - \left(\frac{1-\alpha}{\alpha N}\right)^{\frac{1}{N-1}}\left[\frac{(1-p)}{(p-c)}\right]^{\frac{1}{N-1}} & \text{for } L_1 \le p \le p_0. \end{cases} \tag{3.2}$$

$$f_1(p) = \begin{cases} \text{None at } p = 1, \\ 0 \quad \text{for } p_0 < p < 1, \\ \frac{1}{N-1}\left(\frac{1-\alpha}{\alpha N}\right)^{\frac{1}{N-1}}\left[\frac{(1-p)}{(p-c)}\right]^{\frac{2-N}{N-1}}\frac{(1-c)}{(p-c)^2} & \text{for } L_1 \le p \le p_0. \end{cases} \tag{3.2$'$}$$

One of the key features of the model is that the equilibrium price distribution is derived from the model environment in which consumers and sellers interact in the market. A cut-off price can be obtained using those price distributions depending on the state and the price range. We can confirm the existence of the gap in the price distribution in state 0.

Proposition 2. There is *always* a gap in the price distribution in state 0 at the presence of enough searchers.

Proof. Note that from $F_0(p_0) = F_0(\bar{p}_0)$,

$$\left(\frac{1-\alpha}{\alpha N}\right)^{\frac{1}{N-1}}\left[\frac{(1-\bar{p}_0)}{(\bar{p}_0-c)}\right]^{\frac{1}{N-1}} = \left(\frac{1-\alpha}{2\alpha N}\right)^{\frac{1}{N-1}}\left[\frac{(1-p_0)}{(p_0-c)}\right]^{\frac{1}{N-1}}.$$

This reduces to $\frac{2(1-c)}{(\bar{p}_0-c)} - \frac{(1-c)}{(p_0-c)} = 1$, which implies $\bar{p}_0 = \frac{2p_0 - c - p_0 c}{1 + p_0 - 2c}$.

But $\frac{2p_0 - c - p_0 c}{1 + p_0 - 2c} \ge p_0$, since $p_0 \le 1$. Therefore, $\bar{p}_0 \ge p_0$. Equality holds when $p_0 = 1$, i.e., when all the consumers are searchers. Q.E.D.

This result shows that the firms have an incentive to take advantage of the presence of nonsearchers in the state 0. In order for the stores to make an extra sales in state 0, not only must they charge a price lower than the critical price, also they must beat the prices charged by all the other stores. Once they choose to price higher than the critical price, they must compete for the consumers who must buy one unit for immediate need and don't have to consider the extra sales any more. It always pays a firm to increase its price by infinitesimal in this critical price.

In state 1, however, firms don't have to compete in the high price range because they can sell only to nonsearchers. Knowing that the customers who buy in their stores at a high price are nonsearchers without inventory, all the stores charge the highest possible price, that is, 1. The stores' pricing strategies reflect the market condition represented by the consumer inventory position and the proportion of searchers among the consumer population. In other words, the configurations of the pricing equilibrium is cleary dependent upon the state of the market. This state-contingent pricing policy would give rise to a certain pattern of sales not only at the industry level but also at the individual firm.

Since the state matters to both the consumers and the firms, the probabilities of state transition are needed to obtain the steady state variables of interest. Let p_{ij} denote the probability that a state i moves to a state j. The following theorem can be used to obtain the probabilities on each state in the steady state.

Theorem: For the ergodic distribution of Markov chain P, a limiting probability $\Pi_j = \lim_{n \to \infty} p_{ij}^{(n)}$ exists and P_{ij}, $0 \leq j \leq M$ are the unique nonnegative solutions of the following system of equation (i) and (ii).

$$\pi_j = \sum_{j=0}^{M} \sum_{k=0}^{M} \pi_j p_{kj} \tag{i}$$

$$\sum_{j=0}^{M} \pi_j = 1 \tag{ii}$$

with $M=2$ in this model. π_j is the limiting probability that, no matter what the initial state is, the state ends up with j. A transition matrix can be obtained using the distributions given in the previous section[3]. Specifically, each probability is characterized by c, α, δ and the critical price triggering state transition in the transition matrix (3.3).

[3] The sequence of random variables is said to form a Markov chain if each time the system is in state i there is some fixed probability p_{ij} that will next be in state j [W. Feller(1970)]. See also this reference for the ergodic distribution of Markov chain.

$$p = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = \begin{pmatrix} (1-F_0(p_0))^N & [1-(1-F_0(p_0))^N] \\ (1-F_1(p_0))^N & [1-(1-F_1(p_0))^N] \end{pmatrix} \tag{3.3}$$

It is possible to interpret p_{ij} as the conditional probability specifying the probability of the transition from one given state i to another state j. Following the conventional notation, let $p_{ij} = p(j \mid i)$. It can be easily shown that there exists α such that the probability of changing from one state to another state is greater than the probability of remaining at the same state, which is of the realistic features of the market.

Given the steady state distribution, the steady state expected price(EP) is defined as $EP = \Pi_0 \int_{L_0}^{1} PdF_0 + (1-\Pi_0)\int_{L_1}^{1} Pd F_1$. Noting that $\int_{L_0}^{1} Pd F_0$ and $\int_{L_1}^{1} Pd F_1$ are expected prices in each state, let's denote them μ_0 and μ_1, respectively. In passing, the limiting probability that a sale is run in the steady state is given by (3.4), denoted by $P(S)$.

$$P(S) = \{1-(1-F_0(P_0))^2\}\Pi_0 + \{1-(1-F_1(p_0))^2\}(1-\Pi_0) \tag{3.4}$$

Putting together all the information out of the model, we can examine, as an example, the price distribution in the setting of a duopoly market across time. The prices of the subsequent periods can fluctuate across time depending on the number of searchers.

Let p_t, p_{t+1}, denote the state-contingent random prices in time t and $t+1$ and let μ_0, μ_1 be the expected price in state i. Note that $Ep_t = Ep_{t+1} = \Pi_0\mu_0 + (1-\Pi_0)\mu_1$ since the expected prices in any period in the steady state are the same. Considering the price distribution along with its subsequent state and $\mathrm{Cov}(p_t, p_{t+1}) = E(p_t\, p_{t+1}) - Ep_t Ep_{t+1}$, we can calculate the covariance of the prices in two steady states by the following procedure.

$$\mathrm{Cov}(P_t,\ P_{t+1}) \tag{3.5}$$

$$= \Pi_0\,[\int_{p_0}^{1} p \int_{L_0}^{1} pd\, F_0(p)d\, F_0(p) + \int_{L_0}^{p_0} p \int_{L_1}^{1} pd\, F_1(p)d\, F_0(p)]$$

$$+ (1-\Pi_0)\,[\int_{p_0}^{1} p \int_{L_0}^{1} pd\, F_0(p)\, d\, F_1(p) + \int_{L_1}^{p_0} p \int_{L_1}^{1} pd\, F_1(p)d\, F_1(p)]$$

$$- \{ \Pi_0 \mu_0 + (1 - \Pi_0) \mu_1 \}^2$$

$$= \Pi_0 \, [\mu_0 \int_{p_0}^{1} p d\, F_0(p) + \mu_1 \int_{L_0}^{p_0} p d\, F_0(p)]$$

$$+ (1 - \Pi_0) \, [\mu_0 \int_{p_0}^{1} p d\, F_1(p) + \mu_1 \int_{L_1}^{p_0} p d\, F_1(p)]$$

$$- \{ \Pi_0 \mu_0 + (1 - \Pi_0) \mu_1 \}^2$$

The only source of the correlation between the price distributions is the state in the model. A negative covariance in this model can be interpreted as a temporal price distribution, called sales in the same store, and as a spatial price distribution as well over the stores. As long as there is a certain portion of nonsearchers among the consumer population, the equilibrium doesn't occur at the perfectly competitive price. On the other hand, presence of searchers also prevents a monopoly pricing from persisting. Typically, searchers in the imperfect information models give an external economy to nonsearchers when stores cannot identify the type of consumers. In this sense, nonsearchers affect adversely the welfare of searchers, for searchers need to search because there are nonsearchers. An equilibrium could be the case where stores charge a very high price to nonsearchers and make a high profit per sale while charging a competitive price to searchers. Nonsearchers subsidize searchers in this case.

IV. CONCLUDING REMARKS

Although the model is very simple, it introduces some dynamics of price and quantity sold over time and space in the market. Price dispersion may be spatial, across brands of the same item, or in the industry in general, or at one firm over time which is called temporal dispersion. The ability to adjust a price distribution with a state transition provides the stores with a richer strategy set, being able to respond more flexibly to a changing demand. Unit demand doesn't preclude a welfare analysis, for the welfare comparison can be made using the average prices with and without inventory position.

The estimate of the demand function without consideration of consumer inventory can be biased because a demand function can be more or less sensitive to the price charged, depending upon whether consumers hold inventory or not. Since costly information prevents perfect substitutes between items in the basket and costly information has an income effect, price dispersion can also cause bias in estimating the price indices or the cost-of-living indices [Anglin and Baye (1987), Reinsdorf(1994)].

Some findings from the equilibrium price distribution are in order. The cutoff price and the value of a firm(V) are invariant to a state. There is a gap and a mass point in the price distribution. Intertemporal covariance of the price distribution is critically depending on the portion of searchers among the consumer population. One of the key feature of the model is that the model provides a testable hypothesis.

The welfare implication is that reduction of search cost can lead stores to charge a low price on the average, increasing the consumer surplus in general. Emergence of some dominant retailers or category killers can be attributed to reduction of search cost in addition to increasing return to scale in consumption. Empirical test can be done easily as the electronic transaction (e.g., Electronic Commerce, Point of Sale System) is getting rapidly prevalent. The direction of research can also take the form of a more general demand pattern or an endogenized ratio of informed consumers by the search or advertisement process. Ultimately, a learning process of firm and consumer should be incorporated to explain a more realistic market.

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