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IMPORTED INPUT PRICE, THE CURRENT ACCOUNT AND MACROECONOMIC ADJUSTMENT*

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This paper provides an alternative possibility within an infinite-horizon optimizing framework of saving-investment and the current account by introducing a risk premium to the real cost of borrowing. The individual borrowing country faces an upward sloping supply schedule of foreign funds rather than a horizontal one in this framework. The introduction of a risk premium allows us to analyze the consumption/saving dynamics as well as production/investment dynamics simultaneously. The problem of flat consumption/saving dynamics found in the existing literature can also be avoided and more realistic adjustment pattern of saving/investment and the current account is obtained.

I. INTRODUCTION

Analyses of the current account based on intertemporal optimization model have been very popular in analyzing the disturbances in an open economy. This is because macroeconomic adjustments to changes in the economic environment are conditioned by the intertemporal choices of the economic agents, and moreover, the current account dynamics is one of the important aspects of the intertemporal choices.

This paper presents an optimizing model of saving-investment and the current account dynamics, and investigates the possible effects on the economy of external shocks such as increases in the price of an imported input.

Since the current account dynamics is the outcome of saving and investment decisions, we need at least a model which incorporates not only the consumption/saving decisions but also the production/investment decisions simultaneously. One part of the existing literature (e.g. Obstfeld, 1982a; Sachs, 1982; Pitchford, 1989)

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are based on the consumption/saving behavior only, and hence the analysis of the current account ends up with a partial analysis in the sense that investment behavior is not incorporated. The other part (Abel and Blanchard, 1983; Blanchard, 1983; Brock, 1988; Sen & Turnovsky, 1989) are the examples of infinite-horizon optimizing models which incorporate investment behavior.

However, many infinite-horizon models also suffer the disadvantage that consumption dynamics are flat in the sense that consumption is constant at the initial level throughout the planning horizon. That is because the time preference rate is assumed to be equal to the constant real interest rate, in order to guarantee the existence of a steady-state equilibrium. Matsuyama (1987) provides an example of a model which incorporates investment behavior into the finite-horizon Blanchard (1985) framework and deals with the effects of an imported input price shock which is the focus of this paper. Being a finite horizon model, Matsuyama (1987) guarantees the existence of a stable steady-state for a small open economy even with a constant rate of time preference which is different from the constant world real interest rate.

This paper provides an alternative possibility within the infinite-horizon optimizing framework of saving-investment by introducing a risk premium to the real cost of borrowing. The introduction of a risk premium is based on the fact that the world credit markets are not completely perfect for a small open developing economy. That is, the individual borrowing country faces an upward sloping supply schedule of foreign funds rather than a horizontal one (Snowden and Milner, 1992; p. 2). The rationale for this assumption is that increases in the level of debt increase the probability of default and hence drive the interest rate upward. The positive relationship between the level of indebtedness and the risk premium is confirmed in empirical analyses (e.g. Edwards, 1986). By introducing a risk premium to the real interest rate, the problem of flat consumption/saving dynamics can be avoided and more realistic results are obtained. The contributions and differences in the results from previous studies will be discussed throughout the analysis.

II. THE MODEL

The economy considered is a typical resource-poor small open developing economy, like Korea. Its main characteristics can be summarized as follows: the economy is highly open with large shares of tradable manufacturing sector and trade; most of the imports are producer goods which consist of intermediate inputs and capital goods; the liberalized world trade regime and access to the international credit markets are important to the economy's trade and growth.

The following simplifications are introduced in the model in order to capture the above characteristics.

(a) The economy produces only tradable goods which can be used for both consumption and investment.

(b) There is only one type of asset which can be traded internationally.

(c) The model is defined in real terms, i.e., there is no monetary sector in the economy.

(d) The economy consists of a fixed number of identical agents (consumer/producers). The government is assumed to keep budget balance, and hence the government expenditure is wasteful in the sense that it does not enter into the agent's utility.

(e) The agent has perfect foresight on the future exogenous variables.

The representative agent's decisions are determined by maximizing lifetime utility ($\widetilde{U}$) subject to the intertemporal budget constraint. Lifetime utility is assumed to be additively separable in time with a constant rate of time preference (ρ). The instantaneous utility function (U) is assumed to be non-negative, increasing and strictly concave in private consumption (C), and also to be twice continuously differentiable:

$$(1)\quad \widetilde{U} = \int_0^{\infty} U(C_t)e^{-\rho t}\, dt$$

$$\text{where } U \in C^{(2)},\ U'(C) > 0\ ,\ U''(C) < 0\ ,\ U'(0) = \infty.$$

The last condition, $U'(0) = \infty$, is postulated to avoid non-interior solutions to the agent's lifetime optimization problem. Then $\theta(C) = -U''(C)C/U'(C)$ denotes the elasticity of marginal utility or, in other words, the coefficient of relative risk aversion. The inverse of this coefficient, $1/\theta(C)$, is also called the agent's intertemporal elasticity of substitution in consumption.

Production (Q) of the tradable goods uses capital (K), labor (L), and imported intermediate input (N). $Q = Q(K, L, N)$ is a well behaved constant returns production function for gross output of the tradable goods. If we let the price of tradable output be unity as a numeraire, the relative price of imported input to the price of domestic output (π) is given. For simplicity, we assume that labor supply is inelastic and remains constant, which is normalized to be unity. The real wage is fully flexible to ensure the constant labor employment. Although the assumption of full employment with a flexible real wage is rather a restrictive assumption, we can focus on the adjustment mechanism which links the capital accumulation and the intermediate imported input use. The gross output function which is increasing and strictly concave in capital and imported intermediate input is expressed as equation (2):

$$(2)\quad Q_t = Q(K_t,\ N_t,\ \overline{L})$$

$$\text{where } Q_K > 0,\ Q_N > 0,\ Q_{KK} < 0,\ Q_{NN} < 0$$

$$Q_{KK}Q_{NN} - Q_{KN}^2 > 0.$$

The capital stock accumulates according to the following relationship (3), where I is gross investment and δ is the rate of depreciation of capital stock:

$$(3)\quad \dot{K}_t = I_t - \delta K_t, \qquad K_0 \text{ is given.}$$

It is well known that one of the main drawbacks of the standard model is the assumption of the instantaneous adjustment in capital stock to equate the marginal product of capital with the real interest rate. In the real world, however, it is reasonable to assume that capital formation is accompanied by adjustment (or installation) costs.

The cost of installing physical capital is usually assumed to depend on the size of investment (I) relative to capital stock (K). Following the tradition[1] of introducing the adjustment costs associated with investment, we assume the adjustment costs per unit of investment will be an increasing and convex function of investment. Therefore, we introduce the adjustment costs per unit of net physical investment as $h\left(\frac{\dot{K}}{K}\right)K$, where $h\left(\frac{\dot{K}}{K}\right)$ is increasing and strictly convex in the relative size of net investment to capital stock, $\left(\frac{\dot{K}}{K}\right)$.[2] Then, the total adjustment costs $h\left(\frac{\dot{K}}{K}\right)K$ becomes increasing and strictly convex in net investment, $\dot{K}$. The total investment expenditure (TI) is, hence, the sum of gross investment and the adjustment costs:

$$(4)\quad TI_t = I_t + h\left(\frac{\dot{K}_t}{K_t}\right)K_t$$

$$\text{where } h\left(\frac{\dot{K}}{K}\right) > 0 \quad \text{for all } \frac{\dot{K}}{K} \neq 0, \quad h(0) = 0$$

$$h'\left(\frac{\dot{K}}{K}\right) \gtreqless 0 \quad \text{as } \frac{\dot{K}}{K} \gtreqless 0, \quad h''\left(\frac{\dot{K}}{K}\right) > 0 \quad \text{for all } \frac{\dot{K}}{K}.$$

It is noted from the properties of the $h(\cdot)$-function that the installation costs become the scrapping costs when the net investment is negative.

In addition to the capital formation constraint (3), the agent also faces a flow constraint which relates the difference between income and expenditure to his accumulation of net foreign assets, B(or net debt, $-B$). Another factor to note is

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[1] For example, see Lucas (1967). Alternative ways of introducing adjustment costs can be found in Uzawa (1969) and Hayashi (1982).

[2] If we assume the adjustment costs as a function of gross investment, $h\left(\frac{I}{K}\right)K$, rather than net investment, the qualitative result does not change. However we can gain analytical simplicity from the latter specification.

that even a small developing economy usually faces an upward sloping supply schedule of foreign funds rather than a horizontal one at the given world real interest rate. This is mainly because the interest rate charged includes a risk premium associated with potential default which is positively related to the level of debt outstanding. As discussed in the introduction, the positive effect of the level of indebtedness on the risk premium is confirmed in empirical cross-section analyses (e.g. Eaton and Gersovitz, 1981; Edwards, 1986). In order to reflect the above relationship, we assume an interest repayment (debt service) or receipt function $R(B)$ as in equation (5), where net interest $R(B)$ rises at a decreasing rate for lending and an increasing rate for borrowing:

$$\text{(5)}\quad R(B_t) = r(B_t)B_t \gtreqless 0 \quad \text{as } B_t \gtreqless 0$$
$$\text{where } r(B_t) > 0, \quad r'(B_t) < 0,$$
$$R'(B_t) > 0, \quad R''(B_t) < 0 \quad \text{for all } B_t.$$

This specification of interest repayments (or receipts) function (5) implies that each individual borrower imposes a negative externality on all other borrowers in the economy since additional borrowing drives up the interest cost for all.[3] Therefore, the marginal social cost (benefit) of borrowing (lending) becomes $R'(B) = r(B) + r'(B)B$, where $r(B)$ is the marginal private cost (benefit) of borrowing (lending) and $r'(B)B$ is the externality imposed by other borrower (lender). From this formulation of the interest rate function, we can also easily analyze the case of constant real interest rate by setting $r'(B) = 0$.

Then, the representative agent's flow budget constraint always coincides with the current account equation (6), where G is the balanced budget government expenditure:

$$\text{(6)}\quad \dot{B}_t = R(B_t) + Q(K_t, N_t, \bar{L}) - \pi N_t - C_t - I_t - h\left(\frac{\dot{K}_t}{K_t}\right)K_t - G_t$$
and B_0 is given.

The consolidated flow budget constraint (6) shows that the current account is determined by the relative size of income $[R(B) + Q(K, N, \bar{L}) - \pi N]$ and absorption $[C + I + h\left(\frac{\dot{K}}{K}\right)K + G]$, or saving $[R(B) + Q(K, N, \bar{L}) - \pi N$

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[3] On this point, see Bardhan (1967), Bruno (1976), Obstfeld (1982b), and Pitchford (1989). Among them, Obstfeld (1982b) and Pitchford (1989) are the examples of intertemporal optimizing models which analyze the effect of terms-of-trade deterioration on the current account with similar interest function as (5). However, they are partial analyses in the sense that they do not consider the production/investment decision, but focus only on consumption/saving behavior.

$- C - G$] and investment $[I + h\left(\frac{\dot{K}}{K}\right)K]$.

Finally, the following transversality conditions are imposed to ensure that both K and B remain bounded as time approaches infinity:

$$(7)\quad \lim_{t\to\infty} e^{-\rho t}\lambda_t \geq 0, \quad \lim_{t\to\infty} e^{-\rho t}\lambda_t K_t = 0, \quad \text{and}$$
$$\lim_{t\to\infty} e^{-\rho t}\psi_t \geq 0, \quad \lim_{t\to\infty} e^{-\rho t}\psi_t B_t = 0.$$

The optimizing problem of the agent is to choose $\{C_t, I_t, N_t\}$ which maximize the agent's utility (1) subject to the constraints (3), (6) and the transversality condition (7). The problem can be handled by the use of the current-value Hamiltonian, according to the Maximum Principle:

$$(8)\quad H = U(C_t) + \lambda_t[I_t - \delta K_t]$$
$$+ \psi_t[R(B_t) + Q(K_t, N_t, \overline{L}) - \pi N_t - C_t - I_t - h\left(\frac{\dot{K}_t}{K_t}\right)K_t - G_t].$$

The co-state variables λ and ψ can be interpreted as the shadow values on the respective constraints ; λ is the shadow price of newly installed capital, and ψ is the shadow price of the final output (capital goods or consumer goods). Therefore, we can define a new variable $q = \lambda/\psi$, which is the ratio of the market value of new additional capital to its replacement cost, i.e., Tobin's q.

The first-order optimality conditions with respect to the decision variables are found by letting $\partial H/\partial C = \partial H/\partial I = \partial H/\partial N = 0$, $d(\lambda_t e^{-\rho t})/dt = -e^{-\rho t}(\partial H/\partial K)$ and $d(\psi_t e^{-\rho t})/dt = -e^{-\rho t}(\partial H/\partial B)$, respectively. From (8) we have:

$$(9a)\quad \psi_t = U'(C_t)$$

$$(9b)\quad \lambda_t = \psi_t[1 + h'\left(\frac{\dot{K}_t}{K_t}\right)]$$

$$(9c)\quad \pi = Q_N(K_t, N_t, \overline{L}), \text{ or } N_t = N(K_t, \overline{L}; \pi)$$

$$(9d)\quad \dot{\lambda}_t = (\rho + \delta)\lambda_t - \psi_t[Q_K(K_t, N_t, \overline{L}) - h\left(\frac{\dot{K}_t}{K_t}\right) + \left(\frac{I_t}{K_t}\right)h'\left(\frac{\dot{K}_t}{K_t}\right)]$$

$$(9e)\quad \dot{\psi}_t = \psi_t[\rho - R'(B_t)].$$

Equations (9a) ~ (9c) indicate the usual necessary conditions for static optimization. That is, consumption continues until the marginal utility of consumption equals the shadow price of consumption. Investment continues until the marginal cost equals the marginal benefit of the investment. Also the demand for the imported intermediate input continues until the marginal physical product of the intermediate input equals the given real price of the imported intermediate input. Equation (9d) shows the dynamic optimal condition for investment that the capital gain ($\dot{\lambda}_t$) is the difference between the required return ($(\rho+\delta)\lambda_t$), and the marginal rentals from additional unit of capital [the second term on the right-hand side of (9d)]. In other words, the condition states that the actual return (capital gain plus the marginal rentals) should equal the required return. Finally, the equation (9e) shows the dynamic optimal condition for consumption that consumption changes until the time preference rate equals the real interest rate.

Then, the sufficiency theorem (Proposition 8 in Arrow and Kurz, 1970; p. 49) states that any policy solution which satisfies the conditions (7) and (9) is optimal if the maximized Hamiltonian is concave in its state variables (K and B in our case), for given t and costate variables (λ_t and ψ_t in our case) which are non-negative. Kamien and Schwartz (1971) also show that, if the current-value Hamiltonian is concave both in its state variables (K and B) and control variables (C, I, and N in our case), the maximized Hamiltonian is concave in its state variables. Therefore, the sufficiency condition can be applied by checking whether the current-value Hamiltonian is concave in the control and state variables. The concavity of the current-value Hamiltonian on the domain consisting of C, I, N, K, and B can easily be identified (see the mathematical appendix).

Since the sufficient condition is satisfied in our case, we can obtain the system of dynamic equations from the necessary conditions (7) and (9) together with equations (3) and (6). First, using the relationship $q = \lambda/\psi$, and equations (3) and (9b), we can obtain the motion of capital accumulation as a function of Tobin's q and capital stock:[4]

$$(10)\quad \dot{K}_t = I_t - \delta K_t = x(q_t - 1)K_t$$
$$\text{where } x(q_t - 1) \gtreqless 0 \ \text{ as } q_t - 1 \gtreqless 0,$$
$$x'(q_t - 1) > 0 \ \text{ for all } q_t - 1.$$

The function $x(\cdot)$ is the inverse function of the monotonic function $h'(\cdot)$ defined in (4). Equation (10) shows that net investment (capital accumulation) is encouraged when capital is valued more highly in the market than its replace-

[4] Since Tobin's q can be expressed as $q = \lambda/\psi = 1 + h'\left(\frac{\dot{K}}{K}\right)$ from (9b), $\frac{\dot{K}}{K}$ can also be expressed as the inverse function, x of the monotonic function, h′ defined in (4). See Hayashi (1982) for more details on the q-theory. The q-theory maintains that q contains important information about investment incentives that cannot be conveyed properly by traditional variables such as interest rates.

ment cost ($q > 1$), and discouraged when its valuation is less than its replacement cost ($q < 1$).

Secondly, the path of optimal consumption is obtained from equations (9a) and (9e):

$$\text{(11)}\quad \dot{C}_t = \frac{-C_t}{\theta(C_t)}\,[\,\rho - R'(B_t)]$$

$$\text{where}\quad \theta(C_t) = \frac{-U''(C_t)\,C_t}{U'(C_t)}.$$

The relationship (11) shows that consumption changes until the time preference rate is equated with the marginal cost of borrowing. If we rearrange equation (11), we obtain another expression (11a):

$$\text{(11a)}\quad R'(B_t) = \rho - \frac{U''(C_t)}{U'(C_t)}\,\dot{C}.$$

If we assume the economy is a borrower for convenience, the term $R'(B)$ is of course the marginal (social) cost of borrowing from abroad. The right hand side of equation (11a) is the marginal rate of premium (in utility) which the individual consumer places on present consumption over future consumption. If consumption is constant, this premium rate is simply the individual's time preference rate ρ. But if consumption is changing over time, an additional term arises from the assumed concavity of the function $U(C)$. In the case of constant real interest rate, however, the $R'(B)$ term in (11) will become r [$R'(B) = r(B) + r'(B)B = r$, since $r'(B) = 0$]. Hence, the consumption path should be flat from the beginning to guarantee the long run equilibrium where $\rho = r$.

Thirdly, equations (9d) and (9e) show the paths of shadow prices over time. Using these two equations together with (9b), (9c), (10), and the relationship $q = \lambda/\psi$, we can derive the behavior of Tobin's q over time:

$$\begin{aligned}\text{(12)}\quad \dot{q}_t &= [\,R'(B_t) + \delta]\,q_t - [\,Q_K(K_t,\ N(K_t,\ \overline{L};\ \pi),\ \overline{L}) \\ &\quad - h\left(\frac{\dot{K}_t}{K_t}\right) + \left(\frac{I_t}{K_t}\right)h'\left(\frac{\dot{K}_t}{K_t}\right)] \\ &= [\,R'(B_t) + \delta]\,q_t - [\,Q_K(K_t,\ N(K_t,\ \overline{L};\ \pi),\ \overline{L}) \\ &\quad - h(x(q_t-1)) + (q_t-1)\{x(q_t-1) + \delta\}].\end{aligned}$$

Equation (12) can also be rearranged as follows:

$$(12a)\quad R'(B_t) = [\dot{q}_t + Q_K(K_t,\ N(K_t,\ \bar{L};\ \pi),\ \bar{L}) - h(x(q_t-1)) + (q_t-1)\{x(q_t-1) + \delta\}]/q_t - \delta.$$

The right hand side of (12a) is the net marginal rentals (in utility) after allowing for both depreciation and adjustment cost. Again, if Tobin's q equals unity, this term is simply net marginal physical product of capital. If, again, we assume the economy is a borrower, the agent (producer) equates this adjusted net marginal rentals to the marginal social cost of borrowing, $R'(B)$, at the optimum.[5]

Finally, the path of assets/debt accumulation (6) can be rewritten using the relationships (9c), and (10) as follows:

$$\begin{aligned}(13)\quad \dot{B}_t &= R(B_t) + Q_K(K_t,\ N(K_t,\ \bar{L};\ \pi),\ \bar{L}) - \pi N(K_t,\ \bar{L};\ \pi) - C_t \\ &\quad - x(q_t-1)K_t - \delta K_t - h(x(q_t-1))K_t \\ &= R(B_t) + F(K_t,\ \bar{L};\ \pi) - C_t - x(q_t-1)K_t - \delta K_t - h(x(q_t-1))K_t.\end{aligned}$$

It is worthwhile to note that the second term (Q) and third term (πN) in the right-hand side of equation (13) constitute the real value-added(real GDP) function since the two terms altogether imply the gross output net of the value of optimal use of imported intermediate input, given the relative price of imported input (π).[6]

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[5] This is because we introduce a risk premium to the real rate of interest. If we did not introduce a risk premium, the representative atomistic producer (consumer) would equate the adjusted net marginal rentals (the marginal rate of premium) to r(B) which is the marginal private cost of borrowing. Therefore, by introducing a risk premium to the real rate of interest, we are implicitly assuming that an optimal tax on foreign borrowing or lending has been imposed. This optimal tax must equal the excess of the marginal social cost of borrowing over the marginal private cost.

[6] If we assume the optimal use of imported intermediate input in the production of gross output such that $\partial Q/\partial N = \pi$, the value-added output function (F) can be defined as following equation which is used in the latter part of expression in equation (13), that is,

$$F(K_t,\ \bar{L};\ \pi) = \max_{N_t} [Q(K_t,\ N_t,\ \bar{L}) - \pi N_t] = Q(K_t,\ N(K_t,\ \bar{L};\ \pi),\ \bar{L}) - \pi N(K_t,\ \bar{L};\ \pi)$$

where $\pi = Q_N(K,\ N,\ \bar{L})$,

$$F_\pi = -N = -N(K,\ \bar{L};\ \pi)$$

$$F_K = Q_K(K,\ N(K,\ \bar{L};\ \pi),\ \bar{L}) > 0$$

$$F_{KK} = Q_{KK} + Q_{KN}N_K = [Q_{KK}Q_{NN} - Q_{KN}^2]/Q_{NN} < 0.$$

Since the gross output function Q is assumed to be linearly homogeneous, the value-added function F is also linearly homogeneous in the primary factors L and K.

The dynamic system of the model economy comprises the four non-linear differential equations (10) ~ (13). The equilibrium path of the economy is obtained from solving this dynamic system simultaneously.

III. STEADY-STATE AND DYNAMICS

As shown in the previous section, this model is characterized by a system of four non-linear differential equations (10) ~ (13). Since the paths which converge to the steady-state satisfy the sufficient condition for optimality, we shall concentrate on the paths to long-run steady-state. It is more convenient to consider the steady-state behavior first since the paths to the steady-state require the existence of the steady-state. The steady-state defined by $\dot{K}_t = \dot{C}_t = \dot{q}_t = \dot{B}_t = 0$ is characterized as follows:

(14a) $q^* = 1$

(14b) $R'(B^*) = \rho$

(14c) $R'(B^*) = Q_K(K^*, N^*, \bar{L}) - \delta = F_K(K^*, \bar{L}; \pi) - \delta$

(14d) $C^* = R(B^*) + Q(K^*, N^*, \bar{L}) - \pi N^* - \delta K^* - G^0$
$= R(B^*) + F(K^*, \bar{L}; \pi) - \delta K^* - G^0$

(14e) $\pi = Q_N(K^*, N^*, \bar{L})$

(14f) $w^* \bar{L} = F(K^*, \bar{L}; \pi) - F_K(K^*, \bar{L}; \pi) K^*$.

Equations (14) determine the unique values of q^*, B^*, K^*, C^*, N^*, and w^* simultaneously, under the given level of the balanced budget government expenditure (G^0) financed by lumpsum tax(T^0). The steady-state Tobin's q is unity as in equation (14a). The steady-state level of net foreign assets is determined by the time preference rate as in equation (14b). Equations (14c) and (14e) determine the steady-state values of the capital stock and the use of the imported intermediate input. Since the net investment is zero in the steady-state, the total investment expenditure is equal to the amount of capital depreciation. The steady-state consumption, consequently, is equal to the after-tax net national product $[R(B^*) + F(K^*, \bar{L}; \pi) - \delta K^* - G^0]$ as in equation (14d). Another aspect worth noting is that equations (14c) and (14f) constitute a standard factor price frontier, from which the steady-state real wage (w^*) can be determined, given the relative price of the imported input (π).

Since we are interested in the path which converges to the long run steady-state, we linearize the system of equations (10) ~ (13) around the steady-state values of K^*, B^*, C^*, and $q^* = 1$, using the informations in (14):

$$(15)\quad \begin{bmatrix} \dot{K}_t \\ \dot{B}_t \\ \dot{C}_t \\ \dot{q}_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & a_1 K^* \\ \rho & \rho & -1 & -a_1 K^* \\ 0 & -a_2 C^*/\theta^* & 0 & 0 \\ a_3 & -a_2 & 0 & \rho \end{bmatrix} \begin{bmatrix} K_t - K^* \\ B_t - B^* \\ C_t - C^* \\ q_t - 1 \end{bmatrix}$$

$$\begin{aligned} \text{where } a_1 &= x'(q^* - 1) = x'(0) > 0 \\ a_2 &= -R''(B^*) > 0 \\ a_3 &= -F_{KK} = -[Q_{KK} + Q_{KN} N_K] = -[Q_{KK} Q_{NN} - Q_{KN}^2]/Q_{NN} > 0 \\ \rho &= R'(B^*) = F_K - \delta = Q_K - \delta > 0. \\ \theta^* &= -U''(C^*) C^*/U'(C^*) > 0. \end{aligned}$$

There are two stable (negative) eigenvalues corresponding to the two predetermined variables (K, B), and two unstable (positive) eigenvalues corresponding to the two non-predetermined jump variables (C, q). The eigenvalues of the transition (coefficient) matrix in (15) are:

$$(16)\quad \gamma_1 = \frac{\rho - \sqrt{\rho^2 + 2(\mathrm{P}-\mathrm{D})}}{2} < 0, \quad \gamma_2 = \frac{\rho - \sqrt{\rho^2 + 2(\mathrm{P}+\mathrm{D})}}{2} < 0,$$

$$\gamma_3 = \frac{\rho + \sqrt{\rho^2 + 2(\mathrm{P}-\mathrm{D})}}{2} > 0, \quad \gamma_4 = \frac{\rho + \sqrt{\rho^2 + 2(\mathrm{P}+\mathrm{D})}}{2} > 0,$$

$$\begin{aligned} \text{where } \mathrm{P} &= a_2 C^*/\theta^* + a_1 K^*(a_2 + a_3) > 0, \\ \mathrm{D} &= \sqrt{\mathrm{P}^2 - 4a_1 a_2 a_3 \mathrm{C}^* \mathrm{K}^*/\theta^*} \\ &= \sqrt{[a_2 \mathrm{C}^*/\theta^* + a_1 \mathrm{K}^*(a_2 - a_3)]^2 + 4a_1^2 a_2 a_3 \mathrm{K}^{*2}} > 0, \\ \mathrm{P} - D &> 0, \\ |\gamma_1| &< |\gamma_2|, \quad \gamma_3 < \gamma_4. \end{aligned}$$

This proves that, in the neighborhood of the long run steady-state, there exists a unique convergent saddle path to the steady-state. Since the number of negative eigenvalues equals the number of predetermined variables, and the number of positive eigenvalues equals the number of jump variables, we can directly employ the solution method for the continuous perfect foresight saddle path problem described in Buiter (1984). Using the method described in the appendix, the saddle path to the long-run steady-state is given as (17):

$$\text{(17a)}\quad K_t - K^* = \frac{1}{D}\left[(Z_1 e^{\gamma_2 t} - Z_2 e^{\gamma_1 t})(K_0 - K^*) + a_1 a_2 K^*(e^{\gamma_1 t} - e^{\gamma_2 t})(B_0 - B^*)\right]$$

$$\text{(17b)}\quad B_t - B^* = \frac{1}{a_1 a_2 K^* D}\left[Z_1 Z_2(e^{\gamma_2 t} - e^{\gamma_1 t})(K_0 - K^*) + a_1 a_2 K^*(Z_1 e^{\gamma_1 t} - Z_2 e^{\gamma_2 t})(B_0 - B^*)\right]$$

$$\text{(17c)}\quad C_t - C^* = \frac{1}{\theta^* \gamma_1 \gamma_2 a_1 K^* D}\left[C^* Z_1 Z_2(\gamma_2 e^{\gamma_1 t} - \gamma_1 e^{\gamma_2 t})(K_0 - K^*) + a_1 a_2 C^* K^*(Z_2 \gamma_1 e^{\gamma_2 t} - Z_1 \gamma_2 e^{\gamma_1 t})(B_0 - B^*)\right]$$

$$\text{(17d)}\quad q_t - 1 = \frac{1}{a_1 K^* D}\left[(Z_1 \gamma_2 e^{\gamma_2 t} - Z_2 \gamma_1 e^{\gamma_1 t})(K_0 - K^*) + a_1 a_2 K^*(\gamma_1 e^{\gamma_1 t} - \gamma_2 e^{\gamma_2 t})(B_0 - B^*)\right]$$

$$\text{(17e)}\quad \dot{B}_t = \frac{1}{a_1 a_2 K^* D}\left[Z_1 Z_2(\gamma_2 e^{\gamma_2 t} - \gamma_1 e^{\gamma_1 t})(K_0 - K^*) + a_1 a_2 K^*(Z_1 \gamma_1 e^{\gamma_1 t} - Z_2 \gamma_2 e^{\gamma_2 t})(B_0 - B^*)\right],$$

where K_0 and B_0 are the capital stock and the net foreign assets at time zero, respectively. $Z_1 = a_1 a_3 K^* + \gamma_1 \gamma_3 > 0$, and $Z_2 = a_1 a_3 K^* + \gamma_2 \gamma_4 < 0$ are shown in the appendix. The current account path (17e) is derived by differentiating the foreign assets path (17b) with respect to time. It should be noted that the saddle path to the long run steady-state is governed by the state variables, K and B, as well as the stable eigenvalues, γ_1 and γ_2. In other words, the time path of the economy to the long run steady-state is affected by the difference between the steady-state level and initial level of state variables under the perfect foresight assumption. Additionally the speed of adjustment of the economy is affected by the absolute and relative size of the stable eigenvalues (γ_1 and γ_2).

IV. THE EFFECTS OF IMPORTED INPUT PRICE

1. Long Run Effects of Permanent Increase in π

Suppose that there is a permanent increase in the price of imported intermediate input at time $t = 0$. In order to examine the optimal responses to external disturbances, it is assumed that the economy is in the steady-state initially. The initial steady-state is denoted by superscript 0, and the new steady-state by superscript *. It is convenient to begin with a consideration of the long-run equilibrium effects of external disturbances, since the adjustment of the economy is determined in part by the expectations of long run steady-state because of the perfect foresight assumption.

From the long run steady-state relationship (14), it is clear that changes resulting from the increase in the imported input price (π) occur only in equations (14c), (14d), (14e), and (14f). The long run steady-state values of B^* and $q^* = 1$ are determined independently of the level of the imported input price, which is evident from equations (14a) and (14b).

The long run responses of the capital stock, consumption, imported input use, and real wage to a permanent increase in π can be obtained by differentiating equations (14) and solving for the endogenous variables:

$$\text{(18a)} \quad dq = q^* - q^0 = 0$$

$$\text{(18b)} \quad dB = B^* - B^0 = 0$$

$$\text{(18c)} \quad dK = K^* - K^0 = \frac{1}{a_3}\left(\frac{Q_{KN}}{Q_{NN}}\right) d\pi < 0$$

$$\text{(18d)} \quad dC = C^* - C^0 = \rho dK - N^* d\pi = [\frac{\rho}{a_3}\left(\frac{Q_{KN}}{Q_{NN}}\right) - N^*] d\pi < 0$$

$$\text{(18e)} \quad dN = N^* - N^0 = \left(-\frac{1}{a_3}\right)\left(\frac{Q_{KK}}{Q_{NN}}\right) d\pi < 0$$

$$\text{(18f)} \quad dw = w^* - w^0 = -\frac{1}{L} N^* d\pi < 0$$

Equation (18c) shows that, in the long run, the capital stock falls in response to the permanent increase in the price of imported intermediate input under the assumption that capital and imported intermediate input are cooperative factors ($Q_{KN} > 0$). Naturally the use of imported intermediate input falls in the long run in response to its own price increase as shown in equation (18e). Since the real wage is assumed to adjust flexibly to ensure the constant level of employment ($\overline{L}$) in our model economy, it also falls unambiguously in the long run as shown in equation (18f). Reflecting the long run fall in capital input and real wage income, the consumption falls as well. The magnitude of the long run fall in consumption coincides with the sum of the change in net national product caused by the decrease in capital stock (ρdK) and the change in the total real wage income ($dw\overline{L} = -N^{*}d\pi$). This is shown in equations (18d) and (18f).

Finally, equations (18a) and (18b) show that the Tobin's q and net foreign assets might be changed during the transitional period, but ultimately return to their initial steady-state values. Therefore, there are no long run effects of the increase in π on the Tobin's q and net foreign assets.

2. Transitional Effects of Permanent Increase in π

The transitional time path of the system to a new long run steady-state following a permanent increase in the price of imported input can be obtained by using equations (17) and (18). The solution for the transitional path results from plugging (18b) and (18c) into equations (17), since the saddle path to the steady-state is expressed by only K, B, and other parameters including the stable eigenvalues. Hence, the transitional dynamics becomes (19):

$$\text{(19a)}\quad K_t - K^{*} = -\left(\frac{1}{\mathrm{D}}\right)(Z_1 e^{\gamma_2 t} - Z_2 e^{\gamma_1 t})\left(\frac{\mathrm{Q_{KN}}}{a_3\mathrm{Q_{NN}}}\right)d\pi$$

$$\text{(19b)}\quad B_t - B^{*} = -\left(\frac{\mathrm{Z_1 Z_2}}{a_1 a_2 \mathrm{K^{*}D}}\right)(e^{\gamma_2 t} - e^{\gamma_1 t})\left(\frac{\mathrm{Q_{KN}}}{a_3\mathrm{Q_{NN}}}\right)d\pi$$

$$\text{(19c)}\quad C_t - C^{*} = -\left(\frac{\mathrm{C^{*}Z_1 Z_2}}{\theta^{*}\gamma_1\gamma_2 a_1 \mathrm{K^{*}D}}\right)(\gamma_2 e^{\gamma_1 t} - \gamma_1 e^{\gamma_2 t})\left(\frac{\mathrm{Q_{KN}}}{a_3\mathrm{Q_{NN}}}\right)d\pi$$

$$\text{(19d)}\quad q_t - 1 = -\left(\frac{1}{a_1\mathrm{K^{*}D}}\right)(Z_1\gamma_2 e^{\gamma_2 t} - Z_2\gamma_1 e^{\gamma_1 t})\left(\frac{\mathrm{Q_{KN}}}{a_3\mathrm{Q_{NN}}}\right)d\pi$$

$$\text{(19e)}\quad \dot{B}_t = -\left(\frac{\mathrm{Z_1 Z_2}}{a_1 a_2 \mathrm{K^{*}D}}\right)(\gamma_2 e^{\gamma_2 t} - \gamma_1 e^{\gamma_1 t})\left(\frac{\mathrm{Q_{KN}}}{a_3\mathrm{Q_{NN}}}\right)d\pi.$$

where D, Z_1, a_1, a_2, a_3, θ^{*}, C^{*}, K^{*}, $Q_{KN} > 0$, Z_2, γ_1, γ_2, $Q_{NN} < 0$, and $|\gamma_1| < |\gamma_2|$. If appropriate numbers satisfying the conditions for sign and relative size are assigned to the above eigenvalues and parameters, the motion of the system (19) can be conveniently traced in Figure 1.

At time zero when the permanent increase in π occurs, the Tobin's q drops by the amount $[(Z_1\gamma_2 - Z_2\gamma_1)/a_1K^*D](Q_{KN}/a_3Q_{NN})d\pi$, giving the signal to reduce investment. As time evolves the Tobin's q starts to increase until it reaches the initial steady-state value, unity [see equation (19d)]. Until the economy arrives at the point $q = 1$, optimal capital stock decreases continuously from K^0 to K^*, since q is less than unity during the transitional adjustment periods [see equation (19a)]. The total amount of the decrease in the capital stock is given in (18c). Although the Tobin's q drops discontinuously at time zero, the capital stock does not drop discontinuously but starts to decrease continuously from the beginning.

Consumption also falls reflecting the fall in the net income component of wealth caused by both the capital loss and the increased imports bill of intermediate inputs. At time zero consumption drops by the amount $[(\gamma_1 - \gamma_2)\gamma_3\gamma_4/D - \rho]dK + Nd\pi$, which is smaller than the long run drop in consumption.[7] As time evolves, consumption declines further until it reaches the new steady-state level C^*.

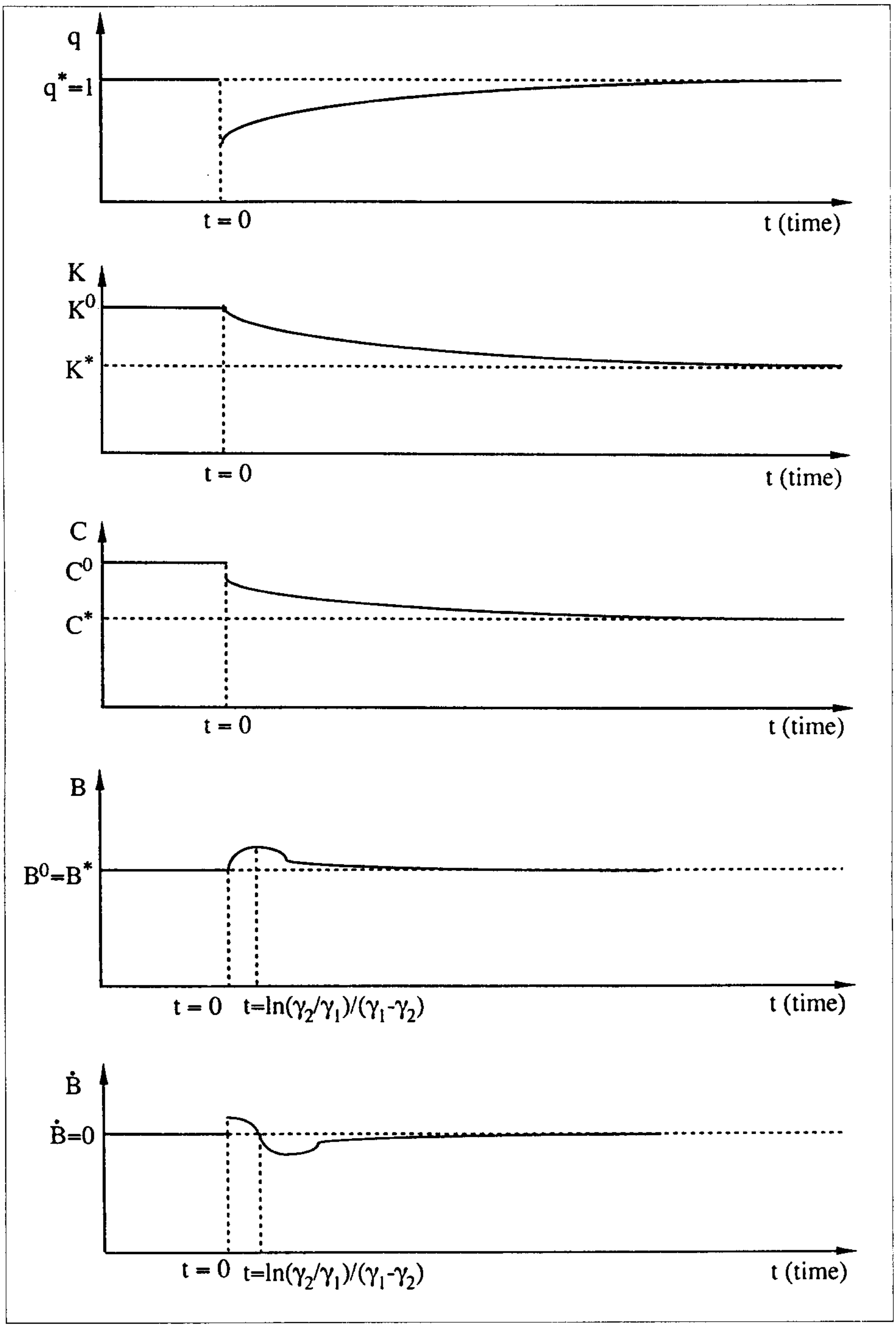
The impact effect on the current account is a surplus immediately after the permanent increase in π, reflecting the quick drop in consumption. As time evolves, however, income starts to decline with disinvestment. The current account turns into deficit when the decrease in saving caused by the continuous decline in income exceeds the decrease in investment. The time when the current account turns into a deficit depends on the relative size of the stable eigenvalues, γ_1 and γ_2 $[t = \frac{1}{\gamma_1 - \gamma_2} ln\left(\frac{\gamma_2}{\gamma_1}\right)]$.

Shortly after the initial surplus, therefore, the current account turns into deficit during the remaining transitional adjustment period until the economy reaches the new steady-state.[8] This result reflects the larger decrease in saving relative to the decrease in investment during the second stage of adjustment. Reflecting the position of the current account, the net foreign assets (or debt) holding rises (falls) while the current account is in surplus, and then falls (rises) to return to its initial steady-state value while the current account is in deficit. This pattern of current account response shows a significant contrast to the existing optimization theory which yields an unambiguous surplus (Obstfeld, 1982a), surplus or balance

[7] The initial drop in consumption can be obtained by subtracting $C_0 - C^* > 0$ in (19c) from the long run drop in consumption $-dC = -\rho dK + Nd\pi > 0$ in (18d), since $C_t - C^*$ is positive for all t in (19c).

[8] If we assume that the capital and imported intermediate input are non-cooperative ($Q_{KN} < 0$), the result is reversed. For the Korean case, however, Ahn (1991) shows that the two factors are cooperative ($Q_{KN} > 0$).

[Figure 1] The Effects of a Permanent Increase in π.



(Pitchford, 1989), or ambiguous result (Svensson and Razin, 1983; Matsuyama, 1987). Because of the interest rate function associated with the risk premium, the economy behaves as if it had a certain level of net foreign assets or debt target to maintain.

Therefore, an exogenous economic shock exerts long-run effects on non-human wealth, but not on net foreign assets unless the agent's preference structure is changed. Given the preference structure with a constant time preference rate, the effects of an imported input price shock on the economy is dominated by its long-run effect on the capital stock. This wealth effect differs significantly from Matsuyama's (1987) where the wealth effect occurs through the changes in net foreign assets rather than the capital stock. The channel through which an external shock affects the current account is also different : being a finite-horizon model, saving decision depends on the agent's human wealth and investment decision depends on the agent's portfolio substitution effect in Matsuyama (1987) ; in the case of Sen and Turnovsky (1989), the current account is affected only by the investment decision regardless of whether it is driven by wealth effect or substitution effect. In our case, however, the saving, investment and current account decisions are dominated by the wealth effect. Due to the different speed of adjustments in consumption and investment, the current account shows hump-shape responses, and moreover has a turning point rather than a monotonic surplus or deficit.

V. CONCLUSIONS

This paper has analyzed the effects of an increase in the price of an imported input on a small economy using a simple intertemporal optimizing framework. In parallel with several models which analyze the current account adjustment based on intertemporal optimizing framework, this paper has attempted to provide an alternative infinite-horizon optimizing framework of saving, investment, and the current account. The introduction of a risk premium to the real interest rate allows us to analyze the consumption/saving dynamics as well as production/investment dynamics. A permanent increase in the price of imported intermediate input results in long-run decreases in the capital stock, consumption, the real wage, and intermediate input use. When the imported input price rises, consumption falls immediately reflecting the fall in real income (value-added output) and continues to fall until it reaches a lower level at the new equilibrium. The capital stock and the use of imported input in production also fall, and hence income falls. The current account responds with a surplus immediately after the shock, but turns into deficit during the remaining adjustment period. The speed of adjustment and the time when the current account turns into a deficit depends on the relative size of the stable eigenvalues of the system. Additionally, this framework can readily be extended to analyze the theoretical effects of various fiscal policies by introducing the government sector.

These results, however, should be interpreted with caution since some of the underlying assumptions are restrictive. One strong assumption is the absence of an adjustment mechanism of labor employment. Although the real sector of the economy is the main interest of this paper, the lack of price adjustment mechanism is another omission. The effects of an external shock will be observed not only in the real sector of the economy but also in the monetary sector. The world-wide stagflation after the second oil-shock is one example of this.

MATHEMATICAL APPENDIX

Sufficient Condition for Optimum

According to the sufficiency theorem in optimal control theory (Arrow and Kurz, 1970; Kamien and Schwartz, 1971), any policy solution which satisfies the necessary conditions (9) and transversality conditions (7) is optimal if the maximized Hamiltonian is concave in the state variables (K and B). The concavity of the maximized Hamiltonian in the state variables is satisfied if the current-value Hamiltonian is concave in both control variables (C, I, and N) and state variables.[9] Therefore it is sufficient to check the concavity of the current-value Hamiltonian on the domain which consists of the control and state variables of the problem. For given non-negative λ and ψ, the Hessian determinant ($|H|$) of the current-value Hamiltonian, H(C, I, N, K, B), and the principal minors, $|H_i|$ $(i = 1,\cdots,5)$ are given in (A1):

$$\text{(A1)}\quad |H| = \begin{vmatrix} U_{CC} & 0 & 0 & 0 & 0 \\ 0 & -\frac{\psi}{K} h''\left(\frac{\dot{K}}{K}\right) & 0 & \psi \frac{I}{K^2} h''\left(\frac{\dot{K}}{K}\right) & 0 \\ 0 & 0 & \psi Q_{NN} & \psi Q_{NK} & 0 \\ 0 & \psi \frac{I}{K^2} h''\left(\frac{\dot{K}}{K}\right) & \psi Q_{KN} & \psi\left[Q_{KK} - \frac{I^2}{K^3} h''\left(\frac{\dot{K}}{K}\right)\right] & 0 \\ 0 & 0 & 0 & 0 & \psi R''(B) \end{vmatrix}$$

[9] See the lemma in Kamien and Schwartz (1971), p. 211.

$$
\begin{aligned}
\text{where } |H_1| &= U_{CC} < 0 \\
|H_2| &= -U_{CC}\frac{\psi}{K}h''\left(\frac{\dot{K}}{K}\right) > 0 \\
|H_3| &= -U_{CC}Q_{NN}\frac{\psi^2}{K}h''\left(\frac{\dot{K}}{K}\right) < 0 \\
|H_4| &= -U_{CC}\frac{\psi^3}{K}h''\left(\frac{\dot{K}}{K}\right)(Q_{KK}Q_{NN}-Q_{KN}^2) > 0 \\
|H_5| &= -U_{CC}R''(B)\frac{\psi^4}{K}h''\left(\frac{\dot{K}}{K}\right)(Q_{KK}Q_{NN}-Q_{KN}^2) < 0.
\end{aligned}
$$

The alternating signs in the principal minors obtained from the Hessian determinant shows that the current-value Hamiltonian is concave on the domain. Therefore, the solution which satisfies the necessary and transversality conditions is optimal.

Solution for the Unique Saddle Path

Using the solution method for the continuous time perfect foresight model described in Buiter (1984) and Murphy (1988), we can rewrite the linearized system (15) as follows:

$$
\text{(A2)} \quad \begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1(t) - X_1^* \\ X_2(t) - X_2^* \end{bmatrix},
$$

where $X_1 = [K\ B]'$ is the vector of predetermined variables, $X_2 = [C\ q]'$ is the vector of non-predetermined variables, and A is the coefficient (transition) matrix. The matrix A can be diagonalized as follows:

$$
\text{(A3)} \quad A = V\Gamma V^{-1}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{bmatrix}
$$

where Γ_1 and Γ_2 are diagonal matrices with the stable and unstable eigenvalues of matrix A, respectively, and V is a conformable matrix with the associated eigenvectors of matrix A as columns.

The eigenvalues of matrix A can be obtained by solving the characteristic equation of A, $A(\gamma) = |A - \gamma I| = \gamma^2(\gamma-\rho)^2 - [\frac{a_2 C^*}{\theta^*} + a_1 K^*(a_2+a_3)]\gamma(\gamma-\rho) + \frac{a_1 a_2 a_3 C^* K^*}{\theta^*} = 0.$

There are two negative eigenvalues (γ_1, γ_2) and two positive eigenvalues (γ_3, γ_4) as in equation (16) in the text. Hence, the matrix Γ is expressed as:

$$\text{(A4)} \quad \Gamma = \begin{bmatrix} \frac{\rho-\sqrt{\rho^2+2(P-D)}}{2} & 0 & 0 & 0 \\ 0 & \frac{\rho-\sqrt{\rho^2+2(P+D)}}{2} & 0 & 0 \\ 0 & 0 & \frac{\rho+\sqrt{\rho^2+2(P-D)}}{2} & 0 \\ 0 & 0 & 0 & \frac{\rho+\sqrt{\rho^2+2(P+D)}}{2} \end{bmatrix}$$

where $P = a_2 C^*/\theta^* + a_1 K^*(a_2+a_3) > 0$

$$D = \sqrt{P^2 - 4a_1 a_2 a_3 C^* K^*/\theta^*}$$

$$= \sqrt{[a_2 C^*/\theta^* + a_1 K^*(a_2 - a_3)]^2 + 4a_1^2 a_2 a_3 K^{*2}} > 0$$

$$P - D > 0, \quad |\gamma_1| < |\gamma_2|, \; \gamma_3 < \gamma_4.$$

The column eigenvectors V_i (i = 1, 2, 3, 4) associated with the eigenvalues γ_i (i = 1, 2, 3, 4) of matrix A can be obtained by solving the matrix equation $[A - \gamma_i I]\, V_i = 0$ for V_i (i = 1, 2, 3, 4) with the condition of normalization. The resulting matrix $V = [V_1\ V_2\ V_3\ V_4]$ is expressed as follows:

$$\text{(A5)} \quad V = \begin{bmatrix} \frac{-\theta^* a_1 a_2 K^* \gamma_1}{\sqrt{A_1}} & \frac{-\theta^* a_1 a_2 K^* \gamma_2}{\sqrt{A_2}} & \frac{\theta^* a_1 a_2 K^* \gamma_3}{\sqrt{A_3}} & \frac{\theta^* a_1 a_2 K^* \gamma_4}{\sqrt{A_4}} \\ \frac{-\theta^* Z_1 \gamma_1}{\sqrt{A_1}} & \frac{-\theta^* Z_2 \gamma_2}{\sqrt{A_2}} & \frac{\theta^* Z_1 \gamma_3}{\sqrt{A_3}} & \frac{\theta^* Z_2 \gamma_4}{\sqrt{A_4}} \\ \frac{a_2 C^* Z_1}{\sqrt{A_1}} & \frac{a_2 C^* Z_2}{\sqrt{A_2}} & \frac{-a_2 C^* Z_1}{\sqrt{A_3}} & \frac{-a_2 C^* Z_2}{\sqrt{A_4}} \\ \frac{-\theta^* a_2 \gamma_1^2}{\sqrt{A_1}} & \frac{-\theta^* a_2 \gamma_2^2}{\sqrt{A_2}} & \frac{\theta^* a_2 \gamma_3^2}{\sqrt{A_3}} & \frac{\theta^* a_2 \gamma_4^2}{\sqrt{A_4}} \end{bmatrix}$$

$$
\begin{aligned}
\text{where } A_1 &= Z_1^2(\theta^{*2}\gamma_1^2 + a_2^2 C^{*2}) + \gamma_1^2\theta^{*2}a_2^2(\gamma_1^2 + a_1^2 K^{*2}) > 0 \\
A_2 &= Z_2^2(\theta^{*2}\gamma_2^2 + a_2^2 C^{*2}) + \gamma_2^2\theta^{*2}a_2^2(\gamma_2^2 + a_1^2 K^{*2}) > 0 \\
A_3 &= Z_1^2(\theta^{*2}\gamma_3^2 + a_2^2 C^{*2}) + \gamma_3^2\theta^{*2}a_2^2(\gamma_3^2 + a_1^2 K^{*2}) > 0 \\
A_4 &= Z_2^2(\theta^{*2}\gamma_4^2 + a_2^2 C^{*2}) + \gamma_4^2\theta^{*2}a_2^2(\gamma_4^2 + a_1^2 K^{*2}) > 0 \\
Z_1 &= a_1 a_3 K^* + \gamma_1\gamma_3
\end{aligned}
$$

$$
= \frac{-[a_2 C^*/\theta^* + a_1 K^*(a_2 - a_3)] + \sqrt{[a_2 C^*/\theta^* + a_1 K^*(a_2 - a_3)]^2 + 4a_1^2 a_2 a_3 K^{*2}}}{2} > 0
$$

$$
Z_2 = a_1 a_3 K^* + \gamma_2\gamma_4
$$

$$
= \frac{-[a_2 C^*/\theta^* + a_1 K^*(a_2 - a_3)] - \sqrt{[a_2 C^*/\theta^* + a_1 K^*(a_2 - a_3)]^2 + 4a_1^2 a_2 a_3 K^{*2}}}{2} < 0
$$

Finally, given that Γ_1 is a diagonal matrix, the exponential matrix $e^{\Gamma_1 t}$ is defined as follows:

$$
\text{(A6)} \quad e^{\Gamma_1 t} = \begin{bmatrix} e^{\gamma_1 t} & 0 \\ 0 & e^{\gamma_2 t} \end{bmatrix} .
$$

Then, the solution for the unique saddle path is given by (A7), which yields the solution given as the system equations (17) in the text.

$$
\text{(A7a)} \quad [X_1(t) - X_1^*] = V_{11} e^{\Gamma_1 t} V_{11}^{-1} [X_1(0) - X_1^*]
$$

$$
\text{(A7b)} \quad [X_2(t) - X_2^*] = V_{21} e^{\Gamma_1 t} V_{11}^{-1} [X_1(0) - X_1^*]
$$

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